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VUV and X-ray Free-Electron Lasers

Time-independent 3D FEL Simulations

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Thursday (Jan 28) Lecture Outline

- Transverse dynamics and natural focusing
- Q&A
- Strong focusing and a FODO lattice
- Q&A
- Ming Xie analysis
- Q&A
- Numerical simulator LUME-Genesis
- Q&A





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Transverse dynamics and natural focusing





Planar Undulator





 $\boldsymbol{B} = B_0 \hat{\boldsymbol{y}} \sin k_u z \cosh k_u y$ $+ B_0 \hat{\boldsymbol{z}} \cos k_u z \sinh k_u y$

0

- A previously assumed magnetic field does not satisfy $abla \cdot {m B} = 0!$
- The vertical sinusoidal field that satisfies Maxwell's equation has the vector potential $A_x = -\frac{B_0}{k_u} \cos k_u z \cosh k_u y$ that results in $B_z \neq 0$!
- Lorentz force (written as the second derivative with respect to z) results in:

•
$$x'' = k_u \frac{K}{\gamma} \cosh k_u y \sin k_u z$$
, which is a standard FEL equation (2.7) for $y \approx$
• $y'' + k_u \frac{K^2}{\gamma^2} \cos^2 k_u z \frac{\sinh 2k_u y}{2} = -p_x k_u \frac{K}{\gamma} \sinh k_u y \cos k_u z \approx 0$

According to S.Y. Lee *Accelerator Physics*, 4th ed, chapter 4.III.2





Planar Undulator





 $\boldsymbol{B} = B_0 \hat{\boldsymbol{y}} \sin k_u z \cosh k_u y$ $+ B_0 \hat{\boldsymbol{z}} \cos k_u z \sinh k_u y$

• The nonlinear magnetic field can be neglected since $k_u y \ll 1$:

•
$$y'' + \left(k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z\right) y = 0$$

• Many FEL codes assume that dynamics is *averaged over the undulator period*:

•
$$y'' = -\left\langle k_u^2 \frac{K^2}{\gamma^2} \cos^2 k_u z \right\rangle_u y = -k_u^2 \frac{K^2}{2\gamma^2} y$$
, which correspond to "natural focusing" of the undulator and the Hill's equation $y'' + K_p y = 0$, with the focusing function $K_p = k_u^2 \frac{K^2}{2\gamma^2}$

• The helical undulator has $K_{\chi} = K_{\chi} = \frac{1}{\sqrt{2}}K_p$





Matrix transformation

• Solution of the Hill's equation for the planar undulator is







Matrix transformation

• Solution of the Hill's equation for the planar undulator is

$$\begin{bmatrix} y \\ y' \end{bmatrix}_{z \to z+L} \approx \begin{bmatrix} 1 - \frac{K^2}{4\gamma^2} k_u^2 L^2 & L \\ -\frac{K^2}{2\gamma^2} k_u^2 L & 1 - \frac{K^2}{4\gamma^2} k_u^2 L^2 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}_z$$

- This matrix transformation corresponds to a thick lens of the thickness L and the focal length $f = \frac{2\gamma^2}{K^2 k_u^2 L} = \frac{6825[m^2]}{L[m]}$ for MaRIE-like x-ray FEL;
- The natural focusing is not sufficient for controlling transverse beam dynamics in FELs.





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Strong focusing and FODO lattice



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 \widehat{x}



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Focusing in Quadrupole

$$B = B_1(y \,\hat{x} + x \,\hat{y}), \text{ where } B_1 = \frac{\partial B_y}{\partial x} \left[\frac{T}{m}\right] \text{ used by Genesis}$$

in order to calculate the focusing strength: $k[m^{-2}] = 0.299 \frac{B_1\left[\frac{T}{m}\right]}{E[GeV]}$
with a sign-convention $k > 0$ for horizontal focusing ('QUADF');
$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+l} = \begin{bmatrix} \cos \sqrt{k}l & \frac{\sin \sqrt{k}l}{\sqrt{k}} \\ -\sqrt{k} \sin \sqrt{k}l & \cos \sqrt{k}l \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z, \text{ focusing plane}$$
$$\begin{bmatrix} y \\ y' \end{bmatrix}_{z \to z+l} = \begin{bmatrix} \cosh \sqrt{k}l & \frac{\sinh \sqrt{k}l}{\sqrt{k}} \\ \sqrt{k} \sinh \sqrt{k}l & \cosh \sqrt{k}l \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix}_z, \text{ defocusing plane}$$
In the limit of $\sqrt{k}l \ll 1$, a thin quadrupole represents a thick lens of the length l and the focal length $f = (kl)^{-1}$.





FODO lattice – focusing in both planes



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FODO lattice in thin lens approximation

• A single FODO cell in matrix notations for x coordinate is $\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z+L_c} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L_c}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_z$

that simplifies to

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{z \to z + L_{cell}} = \begin{bmatrix} 1 - \frac{L_c^2}{8f^2} & L_c \left(1 + \frac{L_c}{4f} \right) \\ -\frac{L_c}{4f^2} \left(1 - \frac{L_c}{4f} \right) & 1 - \frac{L_c^2}{8f^2} \end{bmatrix} = U^T \begin{bmatrix} e^{i\phi_c} & 0 \\ 0 & e^{-i\phi_c} \end{bmatrix} U \begin{bmatrix} x \\ x' \end{bmatrix}_z$$

- The matrix transformation for y coordinate requires $f \rightarrow -f$ in the Eq. above;
- Phase advance is related to the transfer matrix by $\sin \frac{\phi_c}{2} = \frac{L_c}{4|f|}$, which results in a real phase and a stable solution if $|f| > L_c/4$.





FODO lattice in thin lens approximation



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Beta function study for MaRIE-like x-ray FEL







Courant-Snyder Parametrization

• The most general form for matrix M_{FODO} with *unit modulus* can be parametrized as

$$M_{FODO} = \begin{bmatrix} \cos \phi_c + \alpha \sin \phi_c & \beta \sin \phi_c \\ -\gamma \sin \phi_c & \cos \phi_c - \alpha \sin \phi_c \end{bmatrix} = \mathbf{I} \cos \phi_c + \mathbf{J} \sin \phi_c ,$$

where α , β and γ are Courant-Snyder parameters of the periodic solution, ϕ_c is the phase advance, **I** is the unit matrix, and

$$\mathbf{J} = \begin{bmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{bmatrix}, \text{ with } \mathrm{Tr}(\mathbf{J}) = 0, \, \mathbf{J}^2 = -\mathbf{I} \text{ or } \beta\gamma = 1 + \alpha^2$$

- One can thus solve for FODO matched beam parameters;
- Using the property of matrix , we obtain the De Moivere's theorem:

$$M^n = \mathbf{I} \cos n\phi_c + \mathbf{J} \sin n\phi_c$$
 and $M^{-1} = \mathbf{I} \cos \phi_c - \mathbf{J} \sin \phi_c$

that describe an oscillation if the beam parameters are not FODO matched.





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Genesis Description

FODOlattice.ipynb Input parameters

UNDULATOR parameters XLAMD = 0.0186 # undulator wavelength ku = 2*np.pi/XLAMD AW0 = 0.86 # rms Undulator parameter K = np.sqrt(2)*AW0 # FODO parameters F1ST = 5 QUADF = 30 FL = 10 QUADD = 30 FD = 10 DRL = 100 # Beam parameters

GAMMA0 = 12e9/0.511e6 # beam energy in mc2

http://genesis.web.psi.ch/Manual/parameter_focusing.html http://genesis.web.psi.ch/Manual/parameter_undulator.html http://genesis.web.psi.ch/Manual/parameter_beam.html

Derived parameters

Focusing quads
lf1st = F1ST*XLAMD
lf = FL*XLAMD
kf = 585*QUADF/GAMMA0
Defocusing quads
ld = FD*XLAMD
kd = -585*QUADD/GAMMA0
Undulator focusing
L = DRL*XLAMD
Ku = K**2*ku**2/(2*GAMMA0**2)

Thin lens approximation
f = (lf*kf)**-1
phi_c = 2*np.arcsin((lf+ld+2*L)/(4*f))
print(f"Simple phi_c={phi_c}")
print(f"Simple beta_max={(lf+ld+2*L)*(1+np.sin(phi_c/2))/np.sin(phi_c)}")
print(f"Simple beta_min={(lf+ld+2*L)*(1-np.sin(phi_c/2))/np.sin(phi_c)}")

Simple phi_c=0.2859559977103042
Simple beta_max=16.57388512332508
Simple beta_min=12.43970009250435





Horizontal FODO in a thick lens approximation PR-21-20607

```
[[ 0.96200859 4.64624157]
[-0.01604296 0.96200859]]
```

```
phi_c = (np.arccos(np.trace(FODOx)/2))
print(phi_c)
```

```
0.27653006629258703
```

```
beta_x = F0D0x[0,1]/np.sin(phi_c)
print(beta_x)
```

```
17.018003459945035
```

```
np.sqrt(beta_x*0.2e-6/GAMMA0)
```

1.2038964357474105e-05

- The phase advance is now smaller;
- The maximum of the beta function is now greater.



Vertical FODO



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Without "natural focusing"

```
[[ 0.96200859 3.53536912]
[-0.02108393 0.96200859]]
```

```
phi_c = (np.arccos(np.trace(FODOy)/2))
print(phi_c)
```

0.2765300662925874

```
beta_y = F0D0y[0,1]/np.sin(phi_c)
print(beta_y)
```

12.949159647934803

np.sqrt(beta_y*0.2e-6/GAMMA0)

1.0501603512555186e-05

With "natural focusing"

```
FODOy = focus(-kf, lf1st) @ \
        focus(Ku, L) @ focus(-kd, ld) @ focus(Ku, L) @ \
        focus(-kf, lf1st)
print(F0D0y)
[[ 0.96085158 3.53385877]
 [-0.0217225
               0.96085158]]
phi_c = (np.arccos(np.trace(F0D0y)/2))
print(phi_c)
0.2807367077713572
beta_y = FODOy[0,1]/np.sin(phi_c)
print(beta_y)
12.754684818958047
np.sqrt(beta_y*0.2e-6/GAMMA0)
1.042244688359981e-05
```





Betatron Envelope Equation

- The focusing function K(s) is real and hence the amplitude and phase functions satisfy $w'' + K w - \frac{1}{w^3} = 0$, and $\psi' = \frac{1}{w^2}$, where the normalization is chosen such that w^2 is exactly the Courant-Snyder β -function and the Courant-Snyder function $\alpha = -\frac{\beta'}{2} = -w w'$.
- We want to use the numerical formalism in order to calculate the β -function in the FODO cell instead of a single point value obtained by the matrix formalism;
- We then use the numerical formalism in order to evaluate the average β -function in the FODO cell and compare this value to arithmetic average between max and min values of the β -function:

$$\bar{\beta} = \frac{\beta_{max} + \beta_{min}}{2}$$
 vs $\bar{\beta} = \frac{1}{L_c} \int_0^{L_c} \beta(s) ds$





Betatron envelope equation results

BetatronEnvelopeEquation.ipynb



 $\bar{\beta}_x = 14.898 \text{ m and } \bar{\beta}_y = 14.675 \text{ m are both less than}$ the arithmetically average $\bar{\beta}$ -function.

- The thick lens solution has smooth turns inside the quadrupoles instead of sharp ones in the thin lens approximation;
- The vertical solution is focused more due to the natural focusing in the undulator;
 - Horizontal solution:
 - The maximum x beta function 17.018 m.
 - Phase advance is 0.27653 rad.
 - The minimum x beta function 12.949 m.
 - Vertical solution:
 - The maximum y beta function 12.755 m.
 - Phase advance is 0. 28074 rad.
 - The minimum y beta function 16.763 m.





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$\bar{\beta}$ -function matching

$$\boldsymbol{\beta}$$
-function mismatch $\left(\bar{\beta}_{x}-\bar{\beta}_{y}
ight)^{2}$



- The matched solution requires a focusing quad, which defocuses in the vertical plane, to be stronger than a defocusing quad in order to compensate for the natural focusing of the undulator:
 - QUADF = 30.061 T/m;
 - QUADD = -30 T/m;





$\bar{\beta}$ -function matching

$$\boldsymbol{\beta}$$
-function mismatch $\left(\bar{\beta}_x - \bar{\beta}_y\right)^2$



- Horizontal solution:
 - The phase advance is 0.27892 rad.
 - Average x beta function is 14.771 m.
 - The maximum x beta function is 16.876 m.
 - The minimum x beta function is 12.837 m.
- Vertical solution:
 - The phase advance is 0.27892 rad.
 - Average y beta function is 14.771 m.
 - The maximum y beta function is 16.874 m.
 - The minimum y beta function is 12.836 m.
- The $\overline{\beta}$ -functions are matched but the maximum and minimum are not the same due to the natural focusing in one plane modifying the beta function evolution.





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Ming Xie analysis of 3D FELs





Ming Xie solution

- Ming Xie solution has been implemented in zfel.old_scripts.mingxie()
- Inputs keyword arguments:

 sigma_x # RMS beam size
 und_lambda # Undulator period (m)
 und_k # Undulator K
 current # Beam current (A)
 gamma # Relativistic gamma
 norm_emit # Normalized emittance (m-rad)
 sigma_E # RMS energy spread (eV)
- Output as dictionary:

gain_length # Gain length (m
saturation_length # Saturation length (m)
saturation_power # Saturation power (W)
fel_wavelength # FEL wavelength (m)
pierce_parameter # Pierce parameter (rho)

• My Mathematica implementation is presented here.

LA-UR-21-20607 SaturationL[$\sigma\eta_{-}, \varepsilon_{-}, \beta_{-}, \lambda u_{-}: 1.86 \times 10^{-2}, \lambda 1_{-}: 0.3 \times 10^{-10}, bEnergy_{-}: 12 \times 10^{9}, Ib_{-}: 3000$] := $Block [\{ku, k1, \gamma b, K, \xi, JJ, \sigma r, IA = 17050, \rho, LG0, \eta \gamma, \eta \varepsilon, \eta d, \Lambda, Pb, Psat, \alpha, Pn \},$ (*Design Optimization for an X-ray Free Elecgron Laser Driven by SLAC Linac Ming Xie, Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA http://accelconf.web.cern.ch/AccelConf/p95/ARTICLES/TPG/TPG10.PDF*) ku = $2\pi/\lambda u$; k1 = $2\pi/\lambda 1$; yb = 1 + bEnergy / 510 999.; $K = \sqrt{2} \sqrt{2 \gamma b^2} \lambda 1 / \lambda u - 1;$ $\boldsymbol{\xi} = \frac{\mathbf{K}^2}{\mathbf{A} + 2\mathbf{K}^2}; \quad JJ = BesselJ[0, \boldsymbol{\xi}] - BesselJ[1, \boldsymbol{\xi}];$ $\sigma r = \sqrt{\varepsilon \beta / \gamma b}$; $\rho = \left(\frac{1}{16} \frac{Ib}{IA} \frac{K^2 J J^2}{\sqrt{h^3} \sigma r^2 k u^2}\right)^{1/3}; \text{ (*FEL parameter*)}$ $LG0 = \frac{\lambda u}{4\pi\sqrt{3}\rho}; (*1D \text{ Gain Length}*)$ (*Ming Xie Parametrizaton*) $\eta d = \frac{LG0}{2 k_1 - r^2}; \quad \eta \varepsilon = LG0 \frac{(4 \pi) \varepsilon / \gamma b}{\beta \lambda_1}; \quad \eta \gamma = 4 \pi \frac{LG0}{\lambda_1} \sigma \eta;$ $\Lambda = 0.45 \, nd^{0.57} + 0.55 \, n\epsilon^{1.6} + 3 \, n\chi^2 + 0.35 \, n\epsilon^{2.9} \, n\chi^{2.4} + 51 \, nd^{0.95} \, n\chi^3 + 5.4 \, nd^{0.7} \, n\epsilon^{1.9} + 1140 \, nd^{2.2} \, n\epsilon^{2.9} \, n\chi^{3.2};$ (*Conclusions*) Pb = Ib bEnergy; Psat = $1.6 \rho Pb / (1 + \Lambda)^2$; $\alpha = 1/9;$ Pn = $\rho^2 bEnergy 1.60217662 \times 10^{-19} \times 299792458 / \lambda 1;$ Return [LG0 (1 + Λ) Log [Psat / Pn / α]]; 23 |;





Ming Xie evaluation – single value

XLAMD = 1.86e-2 # undulator period XLAMDS = 0.3e-10 # desired x-ray wavelength CURPEAK = 3e3 # peak current GAMMA0 = 12e9/0.511e6 # energy in mc2 energy = 0.511e6*GAMMA0 # beam energy in eV DELGAM = 1.5e-4 sigma_e = DELGAM*energy # energy spread in eV AWO = np.sqrt(2*GAMMA0**2*XLAMDS/XLAMD-1) # we use resonant condition formula here K = np.sqrt(2)*AWO EMITX = 0.2e-6 # normalized emittance in x beta = 15 # beta function in meters RXBEAM = np.sqrt(beta*EMITX/GAMMA0) # the corresponding beam size

```
# Some parameters
params = {
    'sigma_x':RXBEAM,
    'und_lambda':XLAMD,
    'und_k':K,
    'current':CURPEAK,
    'gamma':GAMMA0,
    'norm_emit':EMITX,
    'sigma_E':sigma_e}
```

mingxie(**params)

{'gain_length': 3.21,
'saturation_length': 54.91,
'saturation_power': 9.916e9,
'fel_wavelength': 3e-11,
'pierce_parameter': 4.125e-4}





Ming Xie evaluation – scan



- We can use Ming Xie predictions in order to find the optimum beam size for MaRIE-like x-ray FEL;
- We can see that, although $\bar{\beta} = 10$ m results in a shorter saturation length (higher gain), the highest saturated power is reached at $\bar{\beta} = 15$ m;
- Picking up $\overline{\beta}$ provides guidance on the FODO lattice design, which we discussed in the previous section;
- Please recall the value of the previously discussed $\overline{\beta}$ values; they were closer to the power maximum than to the gain maximum, which is inversely related to the saturation length.





Ming Xie evaluation – scan



- The saturation length measured in the number of gain length is less than predicted by 1D theory;
- The saturated power is less than 50% of that predicted by 1D theory;
- These degradations are due to the 3D effects.





ZFEL execution – 2 variable scan



- $\bar{\beta} = 15 m$ case is studied here;
- This analysis shows that XFEL performance is highly sensitive to emittance and energy spread;
- The studied design here is emittance dominated and one would require a low emittance beam!





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3D Numerical simulator – LUME-Genesis





Genesis v2

- We will focus on Genesis v2, which is a stable version of a very popular FEL modeling code (<u>http://genesis.web.psi.ch/</u>) written in Fortran;
- It uses undulator averaged approximation and thus expresses all the distances in the units of undulator period XLAMD;
- Electromagnetic fields are expressed on the Cartesian grid;
- Electrons are represented by an equal number of macroparticles arranged in slices, one resonant wavelength XLAMDS long;
- Slices are ZSEP wavelength apart;
- LUME-genesis is python interface to setup, run and analyze Genesis v2 simulations;
- Genesis v4, rewritten in C, is under active development now (<u>https://github.com/svenreiche/Genesis-1.3-Version4</u>);





Configuring Genesis executable

- Genesis v2 is written in Fortran and does not support dynamic array allocations;
- Maximum array sizes have to be provided prior to compilation of an executable;
- 'genesis.def' file contains these variables https://github.com/slaclab/Genesis-1.3-Version2/blob/master/genesis.def, which is a modification of the original Genesis source code.

| parameter(| | | #endif | #endif | | |
|--------------|------------------|---|--------------|------------------------------|---|--|
| + | genver = 2.3, | !genesis version | + | ndmax = 1250000, | !maximum of particle in imported distributio | |
| #if WIN == 1 | | | + | keepdist = 1, | <pre>!<> 0 keeps distribution in memory</pre> | |
| + | platf ='Win ', | !platform | + | eev = 510999.06d0, | !energy units (mc^2) in ev | |
| #else | | | + | vacimp = 376.73d0, | <pre>!vacuum impedence in ohms</pre> | |
| + | platf ='Unix', | !platform | + | ce = 4.803302d-11, | <pre>!speed of light * electron charge</pre> | |
| #endif | | | + | pi = 3.14159265358979d0, | , !pi | |
| + | original = 1 , | <pre>!indicator for original filetype</pre> | + | <pre>pihalf = pi/2.d0,</pre> | !pi/2 | |
| + | f_sdds = 2 , | <pre>!indicator for sdds filetype</pre> | + | twopi = 2.d0*pi, | !2*pi | |
| + | npmax = 1000001, | !# of particles | + | tiny = 1.d-25, | !check i for precission | |
| + | nzmax = 10000, | <pre>!# of integration steps</pre> | + | small = $1.d-7$, | check ii for precission | |
| #if WIN == 1 | | | + | nrgrid = 1000, | number of radial points for s. c. | |
| + | nsmax = 50000, | !# of slices | #if WIN == 1 | 5 | · | |
| + | nhmax = 7, | !maximum of harmonics | + | ncmax = 261) | !# of gridpoints of cartesian mesh | |
| #else | | | #else | | 5 . | |
| + | nsmax = 150000, | !# of slices | + | ncmax = 503) | !# of gridpoints of cartesian mesh | |
| + | nhmax = 5, | !maximum of harmonics | #endif | | | |
| | | | | | VC | |



Genesis v2: Input file http://genesis.web.psi.ch/Manual/files1.html

- Concise description of the input parameters could be found in <u>https://github.com/slaclab/Genesis-1.3-Version2/blob/master/input.f#L1272</u> as well as <u>https://github.com/ocelot-collab/ocelot/blob/master/ocelot/adaptors/genesis.py#L202</u>
- Genesis generates 'template.in' if ran without an input file provided!







Genesis v2: MaRIE input file

genesis_bin='/home/vagrant/.local/bin/genesis2-mpi'
gen = Genesis('template.in', genesis_bin=genesis_bin)
gen.binary_prefixes = ['mpirun', '-n', '4']

undulator

gen['xlamd'] = 0.0186 # undulator wavelength, m
gen['aw0'] = gen['awd'] = 0.86 # rms undulator parameter
gen['nwig'] = int(80/gen['xlamd']) # undulator length in xlamd

focusing

gen['f1st'] = 5 # half F length in FODO measured in xlamd
gen['fl'] = 10 # full F length in FODO measured in xlamd
gen['quadf'] = 30 # focusing in x quadrupole gradient, T/m
gen['dl'] = 10 # full D length in FODO measured in xlamd
gen['quadd'] = 30 # defocusing in x quadrupole gradient, T/m
gen['drl'] = 100 # full 0 length in FODO measured in xlamd

electron beam

gen['curpeak'] = 3000 # current, A
gen['curlen'] = 0 # negative for flattop; positive for Gaussian
gen['gamma0'] = 12e9/0.511e6 # beam energy, mc^2
gen['delgam'] = 1.5e-4*gen['gamma0'] # relative energy spread
gen['rxbeam'] = 1.2038964357474105e-05 # rms size, m
gen['rybeam'] = 1.042244688359981e-05 # rms size, m
gen['emitx'] = gen['emity'] = 0.2e-6 # normalized emittance, m rad
gen['npart'] = 2**10 # number of macroparticles in a bucket

```
# radiation at resonant condition
gen['xlamds'] = gen['xlamd']*(1+gen['aw0']**2)/(2*gen['gamma0']**2)
gen['prad0'] = 1e4 # shot noise power, W
gen['zrayl'] = 24 # Rayleigh length, m
gen['zwaist'] = 0 # focul point location, m
```

mesh

```
gen['ncar'] = 151 # number of mesh points, ODD is advised
gen['dgrid'] = 100e-6 # [-dgrid, dgrid], m
```

```
# simulation
gen['delz'] = 1 # integration step measured in xlamd
gen.run()
gen.output['run_info']
```

```
{'start_time': 1611354311.8145232,
    'run_script': 'mpirun -n 4 /home/vagrant/.local/bin/genesis2-mpi ge
    nesis.in',
    'run_time': 6.585921049118042,
```

```
'run_error': False}
```





Particle loading – 'quiet start'

The first 10000 points in the same sequence. These 10000 comprise the first 1000, with 9000 more points.



The first 10000 points in a sequence of uniformly distributed pseudorandom numbers. Regions of higher and lower density are evident.



Genesis uses Hammersley sequence for quiet start instead of pseudorandom numbers that introduces bunching! <u>https://en.wikipedia.org/wiki/Low-discrepancy_sequence#Hammersley_set</u>





Results of the no SASE simulation



Electron beam size is FODO matched but not $\overline{\beta}$ matched. Generated power is less than expected from Ming Xie. Why?





Power optimization



gen['iscan'] = 4
gen['nscan'] = 52
gen['svar'] = 0.0025
gen.run()
gen.output['run_info']

- The steady state simulation keeps the radiation wavelength fixed in order to determine the size of the slice;
- This type of simulation assumes no slippage of the radiation with respect to the electrons;
- Any electrons that fall behind get reintroduced into the slice assuming that an identical slice exists ahead of the simulated slice;
- The expected wavelength was 2.9337e-11 m but we have found 2.9384e-11 m, which is 0.0016 relative shift.





Gain optimization



We look at the power growth along the undulator and see oscillations at the start; We then estimate the Gain coefficient at the several points along the undulator; The gain peaks at the resonant wavelength and not where the power picked!





$\bar{\beta}$ -function matching



 $\bar{\beta}$ -function matched solution produces a round beam in the undulator but not necessary the highest power! Keeping a round beam may allow a smaller undulator gap and a stronger undulator parameter; Our solution saturates at a longer undulator distance than Ming Xie predicted.





Gain guiding of the optical mode

