VUV and X-ray Free-Electron Lasers

FEL_4
Harmonic Generation, HGHG & EEHG

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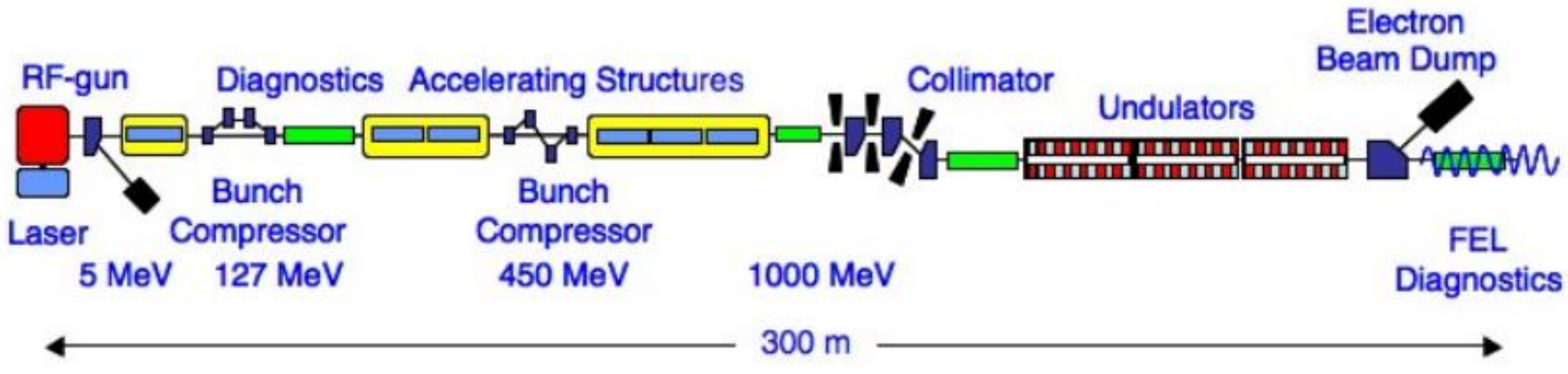
Tuesday (Jan 26) Lecture Outline

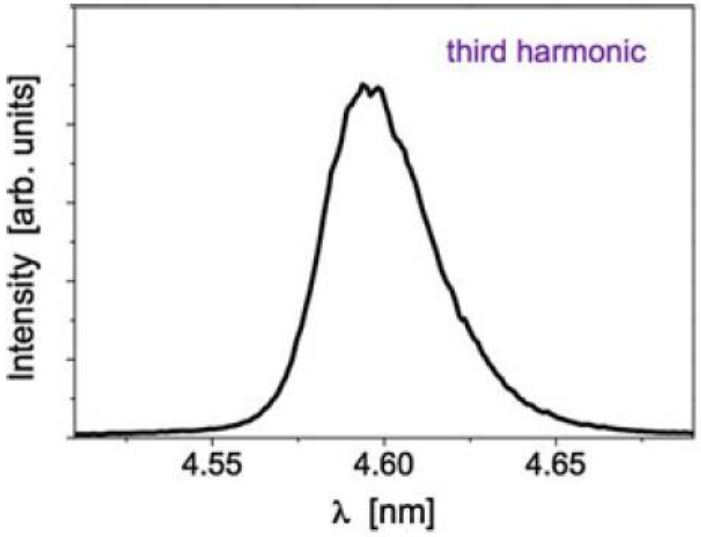
	Time
• Nonlinear Harmonic Generation	10:00 – 10:30
• High-Gain Harmonic Generation (HGHG)	10:30 – 11:00
• Break	11:00 – 11:10
• Echo-Enabled Harmonic Generation (EEHG)	11:10 – 11:40

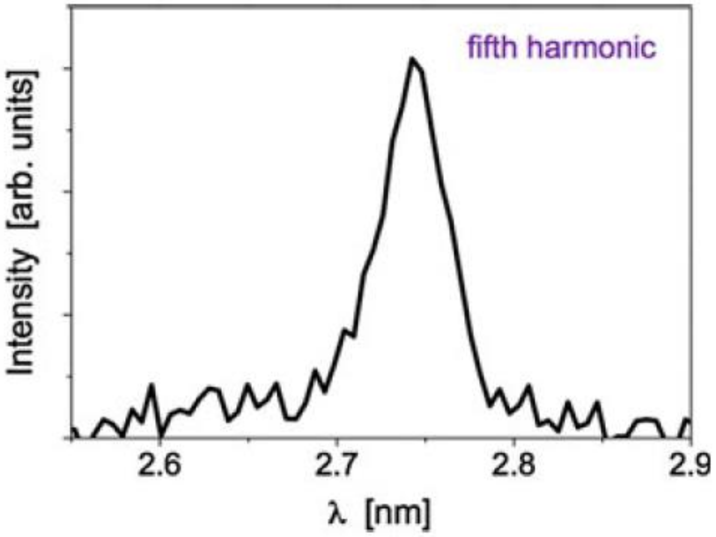
Nonlinear Harmonic Generation

Harmonic Generation at FLASH EUV FEL



Fundamental wavelength = 13.7 nm





h	λ (nm)	Pulse energy (μJ)
1	13.7	40
3	4.6	0.25
5	2.74	0.01

Harmonic Generation in an FEL

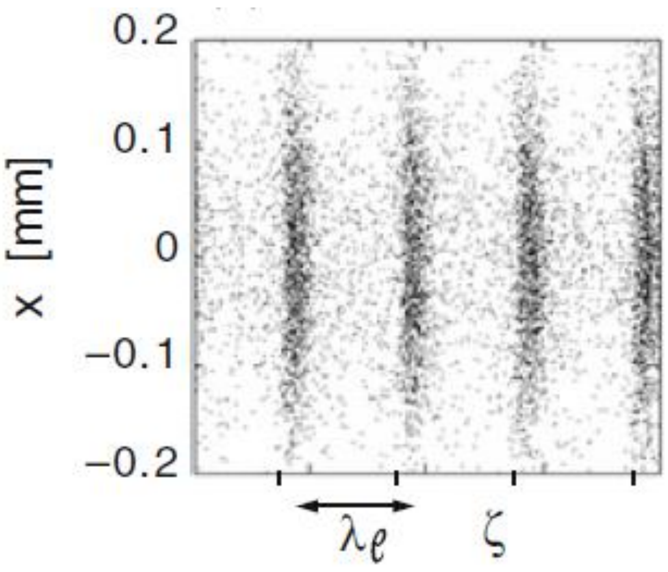
In the special case where the longitudinal distribution is periodic in phase

$$S(\psi) = \frac{a_0}{2} + Re\left\{\sum_{k=1}^{\infty} c_k \exp(ik\psi)\right\}$$

Complex Fourier coefficients

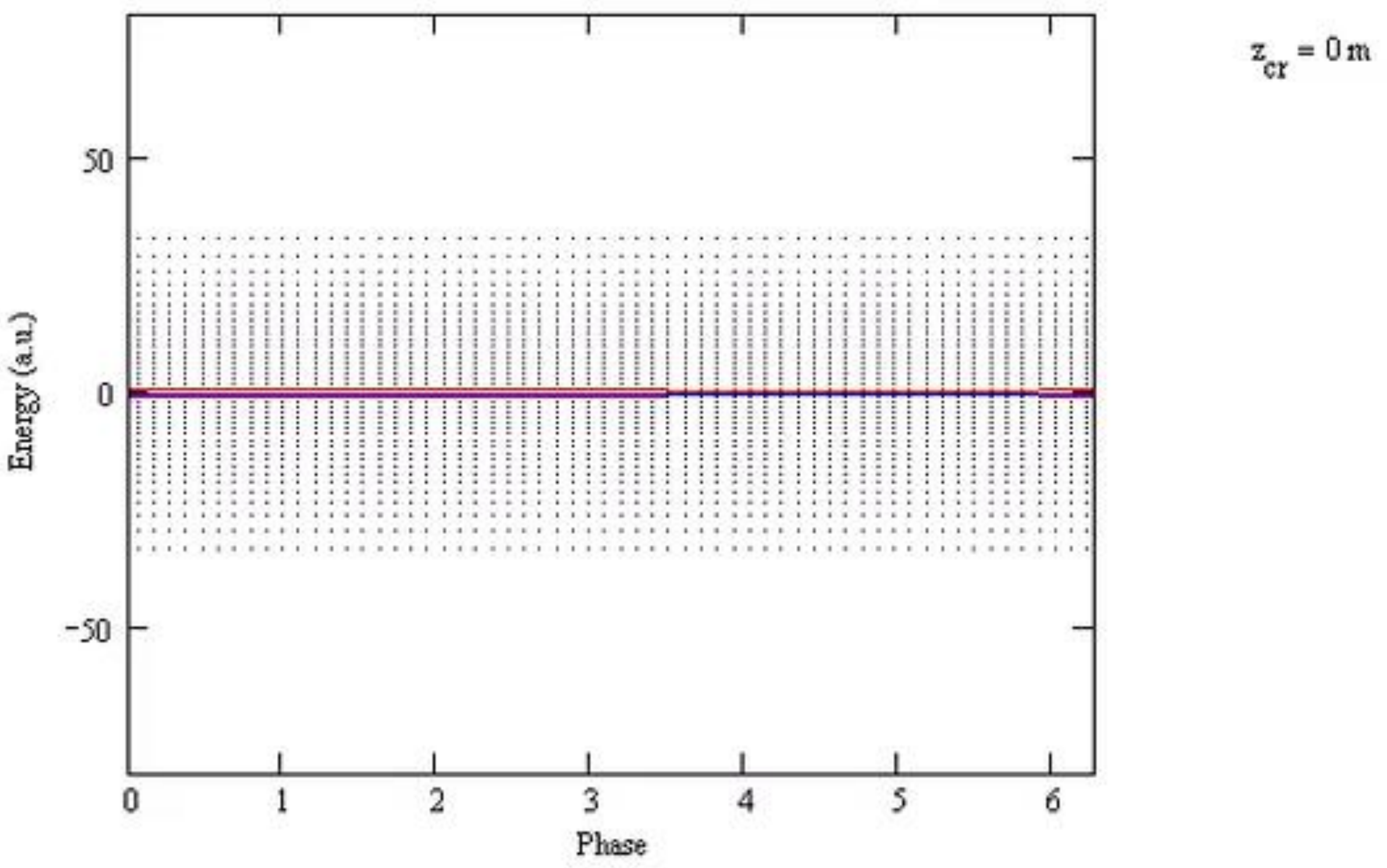
$$c_k = \frac{1}{\pi}\int_0^{2\pi} S(\psi)\exp(-ik\psi_n)d\psi$$

$$c_0 = a_0 = \frac{N}{\pi}$$



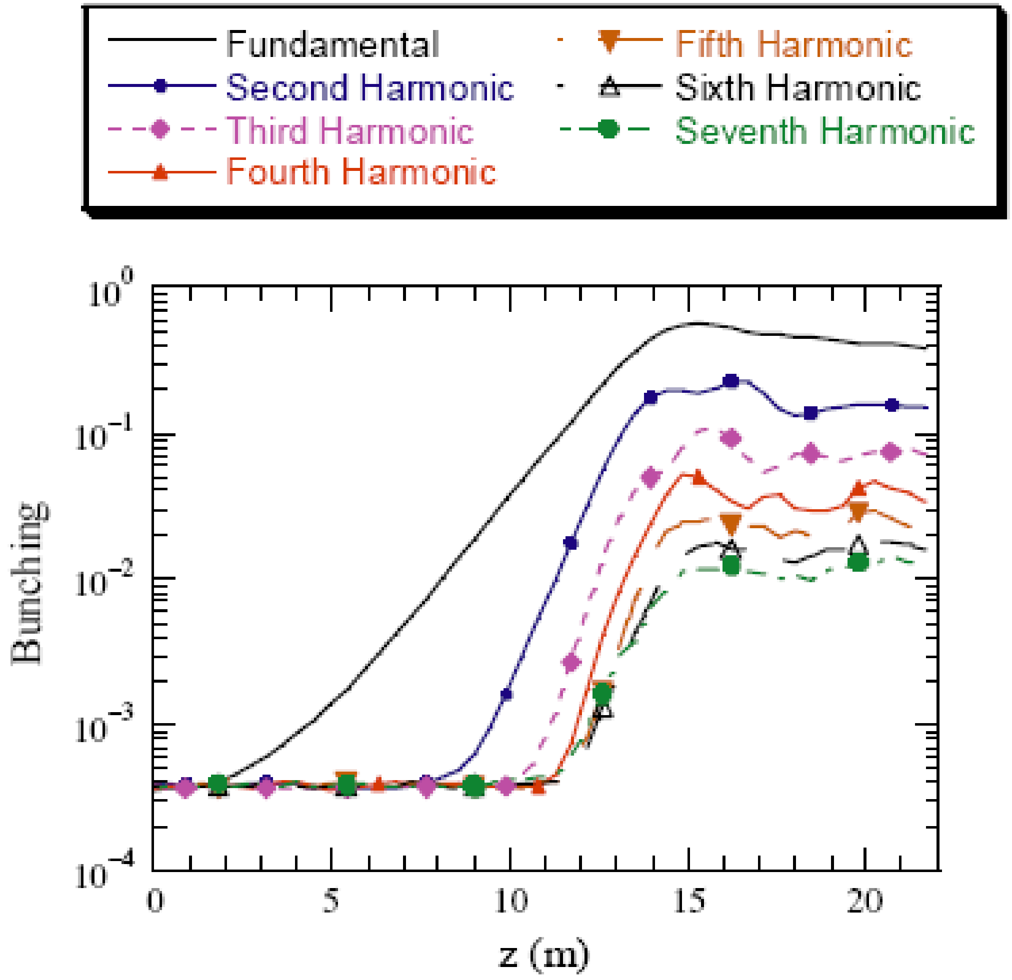
Strong bunching along the longitudinal coordinate ζ of the electron bunch translates into non-zero Fourier coefficients at multiple harmonic frequencies

1D FEL Simulation with Third Harmonic



Harmonic Generation in a Single-pass FEL

Harmonic bunching occurs at larger z but the harmonic bunching exponential growth rates are greater than the fundamental bunching growth rate. However, the bunching factor decreases with harmonic number. As a result, the FEL harmonic power also decreases with harmonic number.



H. Freund et al., IEEE J. Quantum Elec. 36(3), 275 (2000).

High-Gain Harmonic Generation (HGHG)

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Longitudinal Motion Transfer Matrix

The 6D phase-space transfer matrices

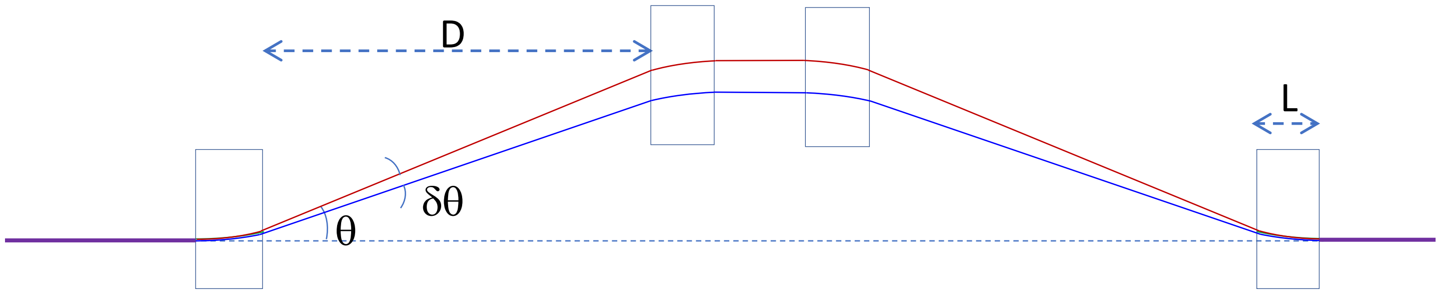
$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix}_{n+1} = \begin{pmatrix} \boxed{\mathbf{M}_{xx'}} & 0 \quad 0 & 0 \quad 0 \\ 0 \quad 0 & \boxed{\mathbf{M}_{yy'}} & 0 \quad 0 \\ 0 \quad 0 & 0 \quad 0 & \boxed{\mathbf{M}_{z\delta}} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix}_{n}$$

It is customary to denote the transfer matrix for the longitudinal motions (z and relative energy deviation δ) as $\mathbf{R}_{z\delta}$

$$\boldsymbol{R}_{z\delta} = \begin{bmatrix} R_{55} & R_{56} \\ R_{65} & R_{66} \end{bmatrix}$$

R_{56} matrix element transforms the relative difference in energy δ into position z along the bunch

Chicane R_{56}



At E_0, the beam follows the red path with an extra pathlength relative to the straight path (dashed) as given by

$$l = \left(\frac{2}{3}L + D\right)\theta^2$$

If we raise the beam energy to $E_0 + \delta E$, the beam follows a slightly shorter path (blue) with an extra pathlength

$$l + \delta l = \left(\frac{2}{3}L + D\right)(\theta - \delta\theta)^2 \approx \left(\frac{2}{3}L + D\right)(\theta^2 - 2\theta\delta\theta)$$

Relative energy change

$$\delta = \frac{\delta E}{E_0}$$

Relative angle change

$$\frac{\delta\theta}{\theta} = -\delta$$

Pathlength change relative to δ

$$R_{56} = -2\left(\frac{2}{3}L + D\right)\theta^2$$

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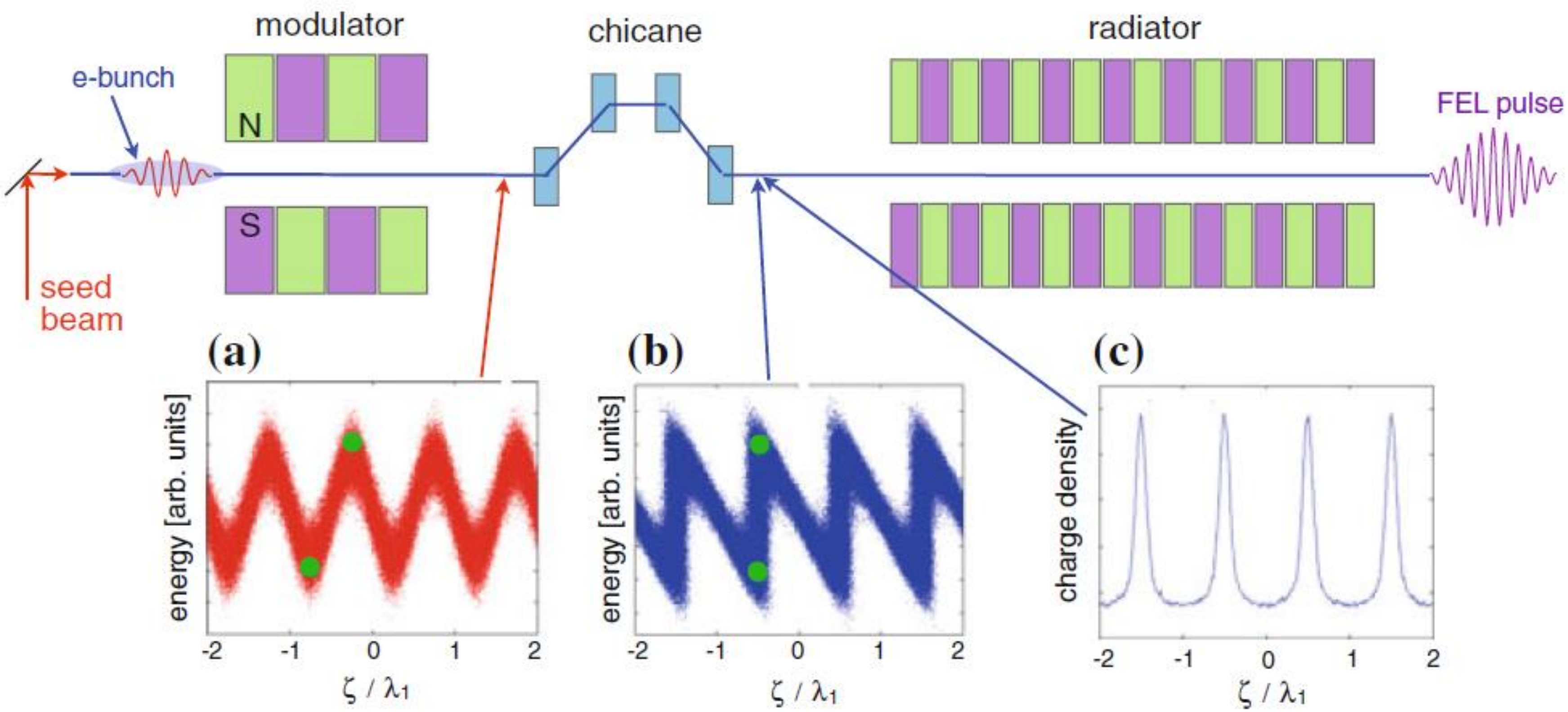
Principle of HGHG FEL



HGHG uses an external laser to modulate the electron energy in the **modulator**, followed by a **chicane** to converts the energy modulations into density modulations, and finally the bunched beam with high Fourier coefficients radiates coherently at a harmonic frequency in the **radiator**.

Dimensionless Variables

Dimensionless longitudinal position, ξ, which is defined as where s is the longitudinal position in meters, and λ is the wavelength of the laser used to modulate the beam.

$$\xi = \frac{2\pi s}{\lambda}$$

The energy of the electrons is described by the dimensionless energy deviation p which is given by:

$$p = \frac{\gamma - \gamma_0}{\sigma_\gamma}$$

Here σ_γ is the rms energy spread in the electron beam before the beam is modulated.

We will assume that the initial electron distribution is Gaussian in energy and is independent of the longitudinal coordinate. The initial dimensionless distribution is:

$$f_0(p) = \frac{N_0}{\sqrt{2\pi}} e^{-\frac{p^2}{2}}$$

Dimensionless Modulation Strength *A*

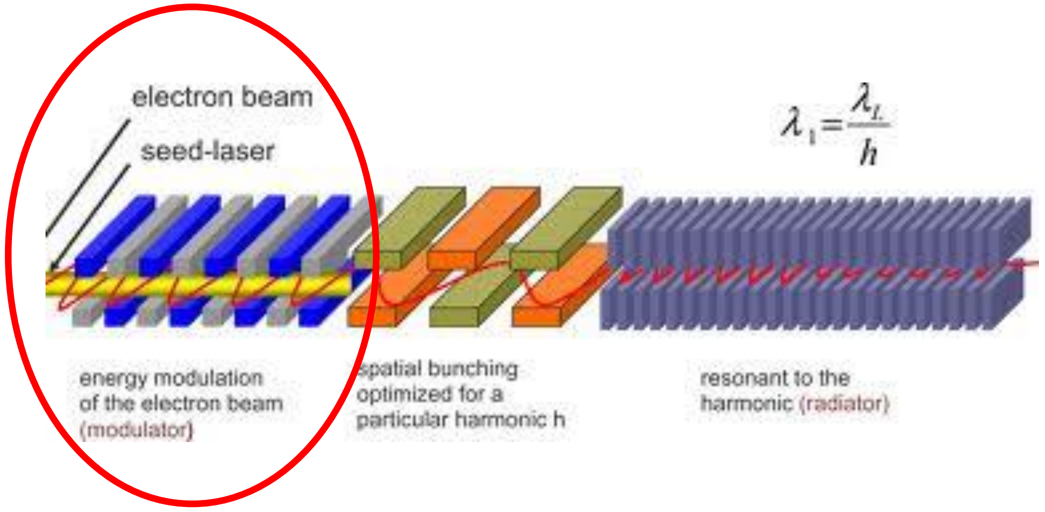
The prime denotes the phase space after modulation, while the unprimed coordinates are the phase space before modulation. The dimensionless parameter A represents the energy modulation strength as given by:

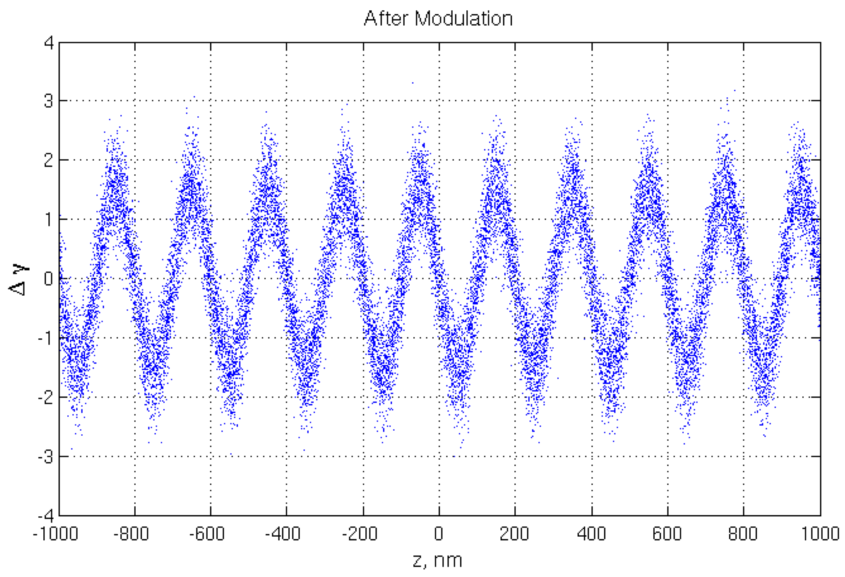
Electron phase space after the modulator

$$p' = p + A\sin(\xi)$$

$$A = \frac{\Delta\gamma}{\sigma_\gamma}$$

The energy modulation $\Delta\gamma$ depends on the laser power, laser transverse size, undulator length and undulator K





Dimensionless Buncher Strength *B*

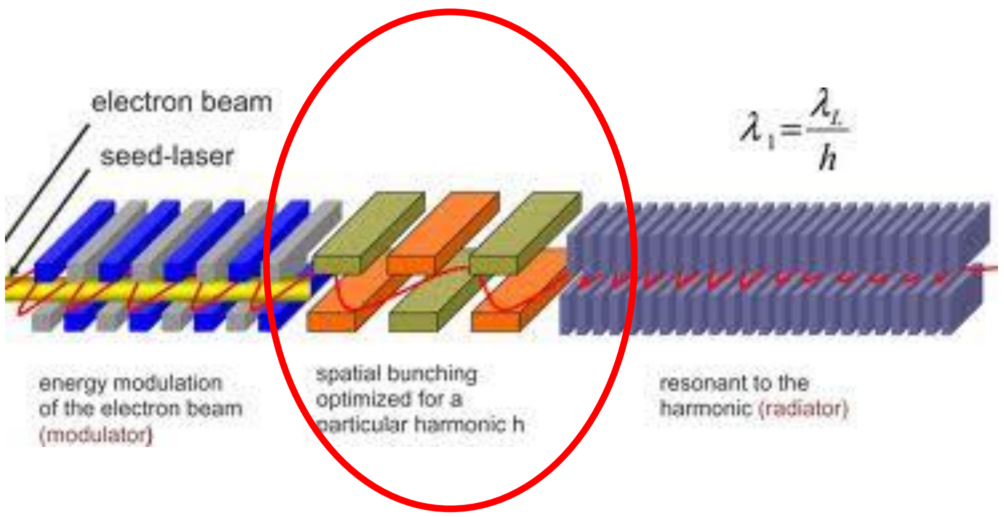
Next the electron beam passes through a chicane where it undergoes bunching. The new phase space is:

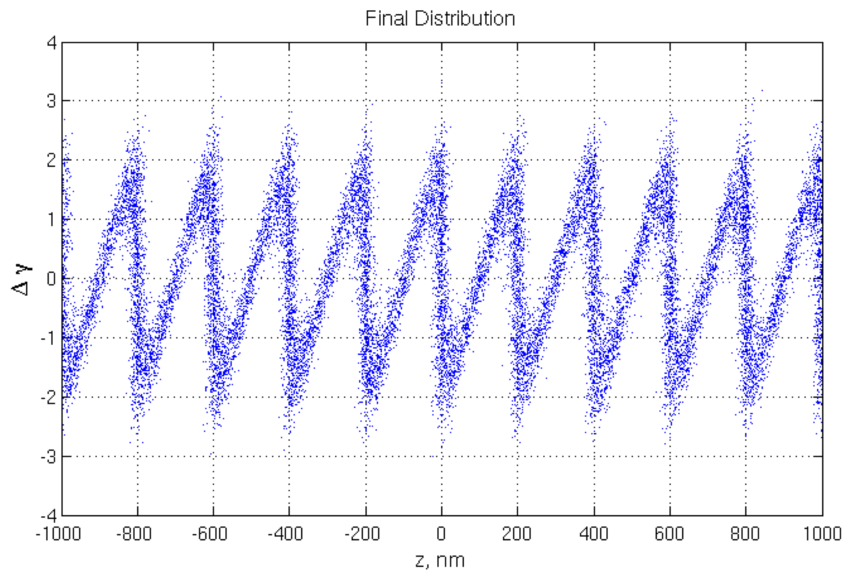
$$\xi' = \xi + Bp'$$

$$\xi' = \xi + B(p + A\sin\xi)$$

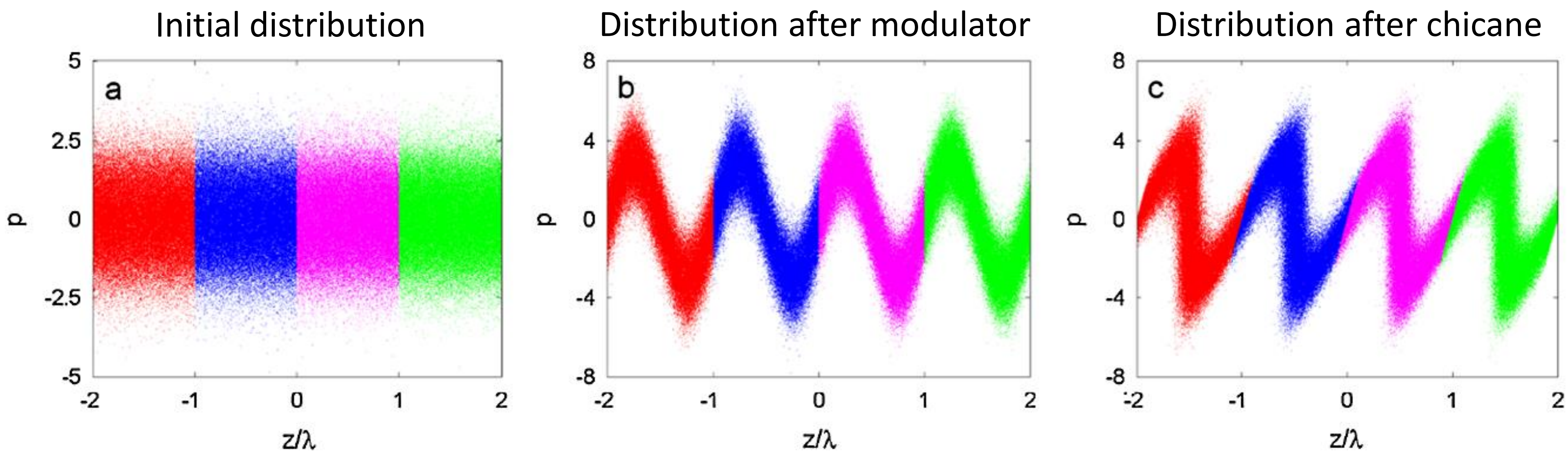
The dimensionless parameter *B* describes the strength of the chicane, and is given by:

$$B = \frac{2\pi R_{56}\sigma_\gamma}{\lambda_1 \gamma_0}$$





Phase-space Distribution

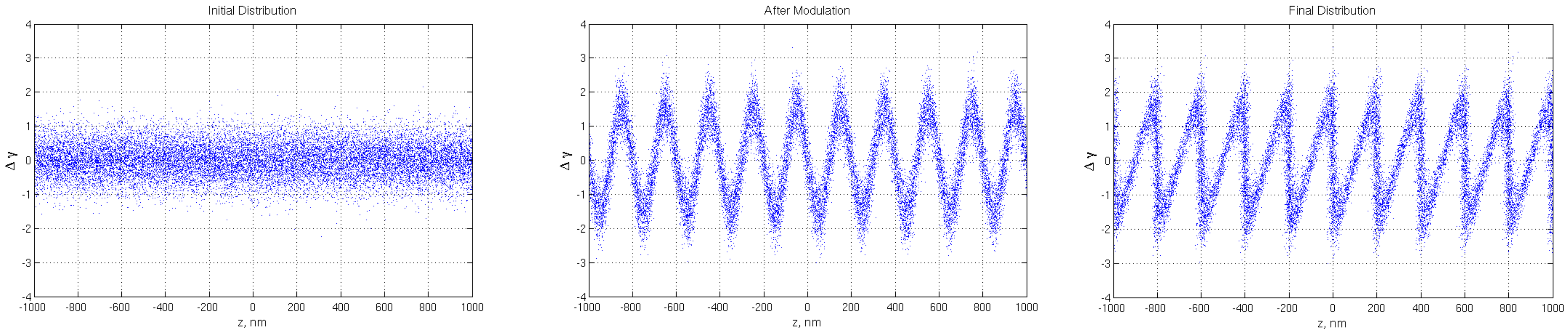


$$f_0(p) = \frac{N_0}{\sqrt{2\pi}} e^{-\frac{p^2}{2}}$$

$$f_1(\xi, p) = \frac{N_0}{\sqrt{2\pi}} e^{-\frac{(p - Asin\xi)^2}{2}}$$

$$f_2(\xi, p) = \frac{N_0}{\sqrt{2\pi}} e^{-\frac{\left(p - Asin(\xi - B_1 p)\right)^2}{2}}$$

Harmonic Bunching Factor



Initial electron line density

$$N_0 = \frac{I_p}{ec}$$

Bunching factor at the n^{th} harmonic

$$b(n) = \frac{1}{N_0}\left|\left\langle e^{-in\xi'} N(\xi')\right\rangle\right|$$

$$A = \frac{\Delta\gamma}{\sigma_\gamma}$$

Yu's formula for HGHG bunching

$$b(n) = J_n(nAB)\exp\left[-\frac{1}{2}n^2B^2\right]$$

$$B = \frac{2\pi R_{56}\sigma_\gamma}{\lambda_1\gamma_0}$$

Courtesy of Quinn Marksteiner (LANL)

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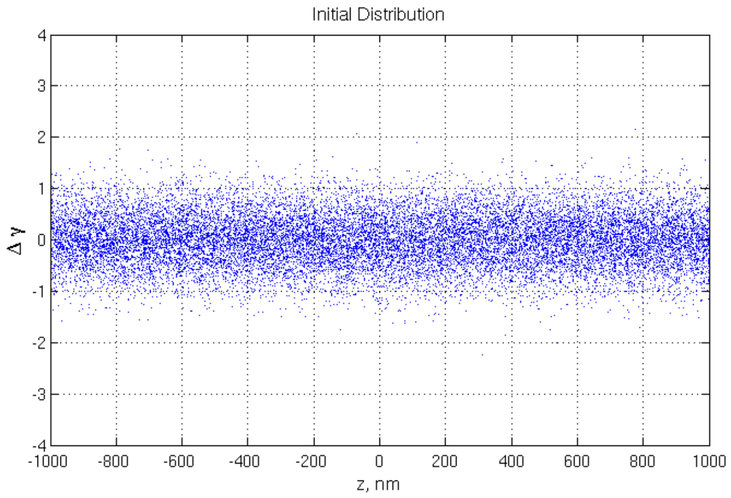
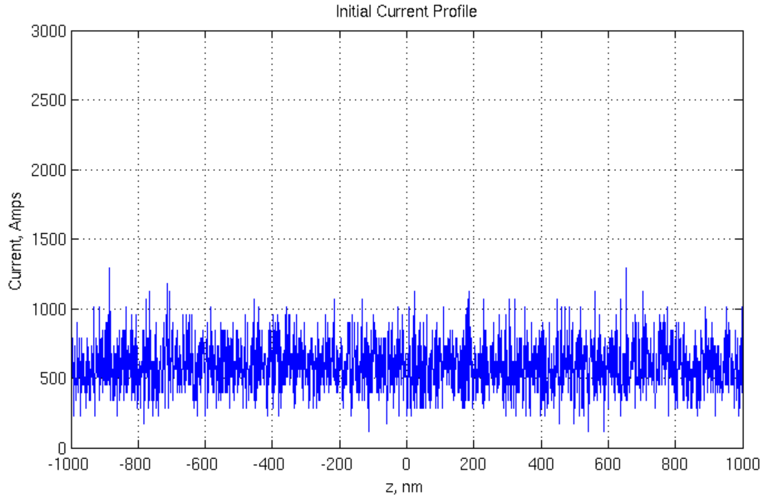
Current Enhancement

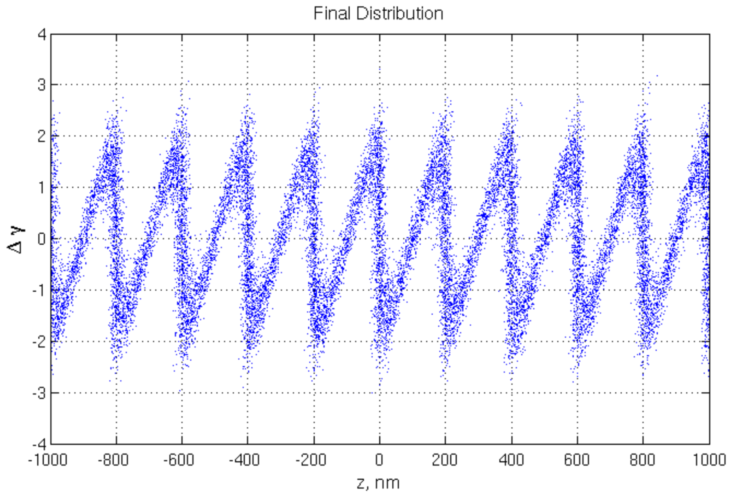
Initial phase space

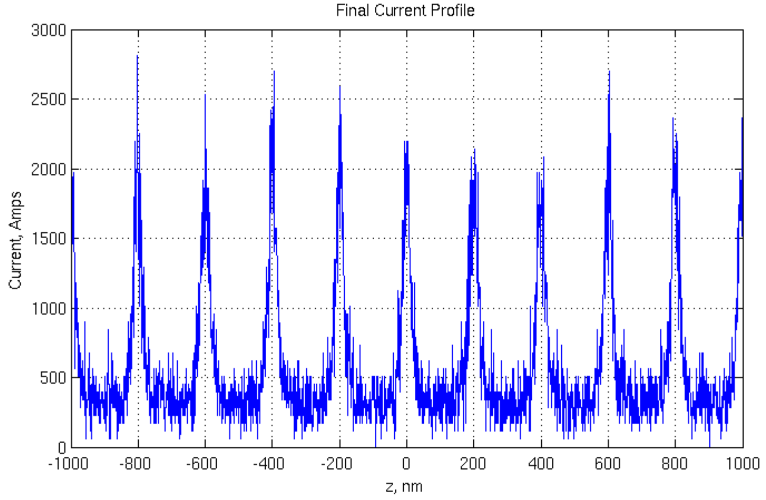




Initial current

Final phase space

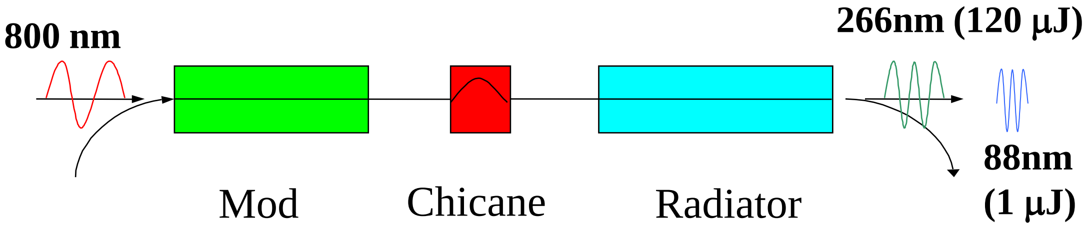




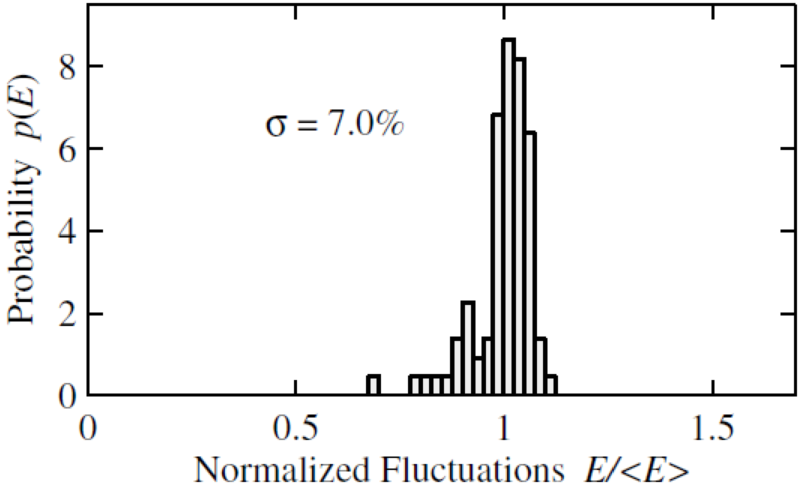
Final current

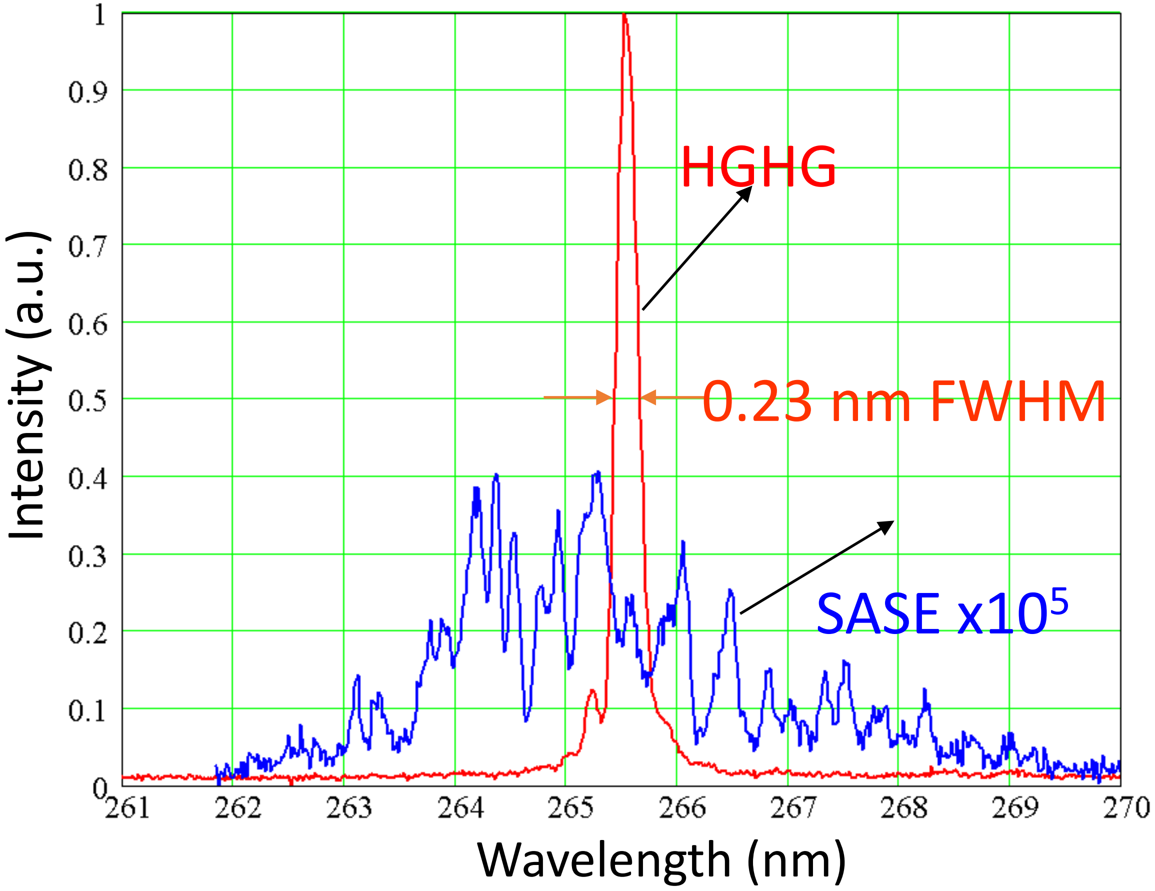
Courtesy of Quinn Marksteiner (LANL)

HGHG Demonstration in the UV and VUV



Spectra of HGHG (red) and unsaturated SASE (blue) under the same conditions. HGHG exhibits small shot-to-shot pulse energy fluctuations.



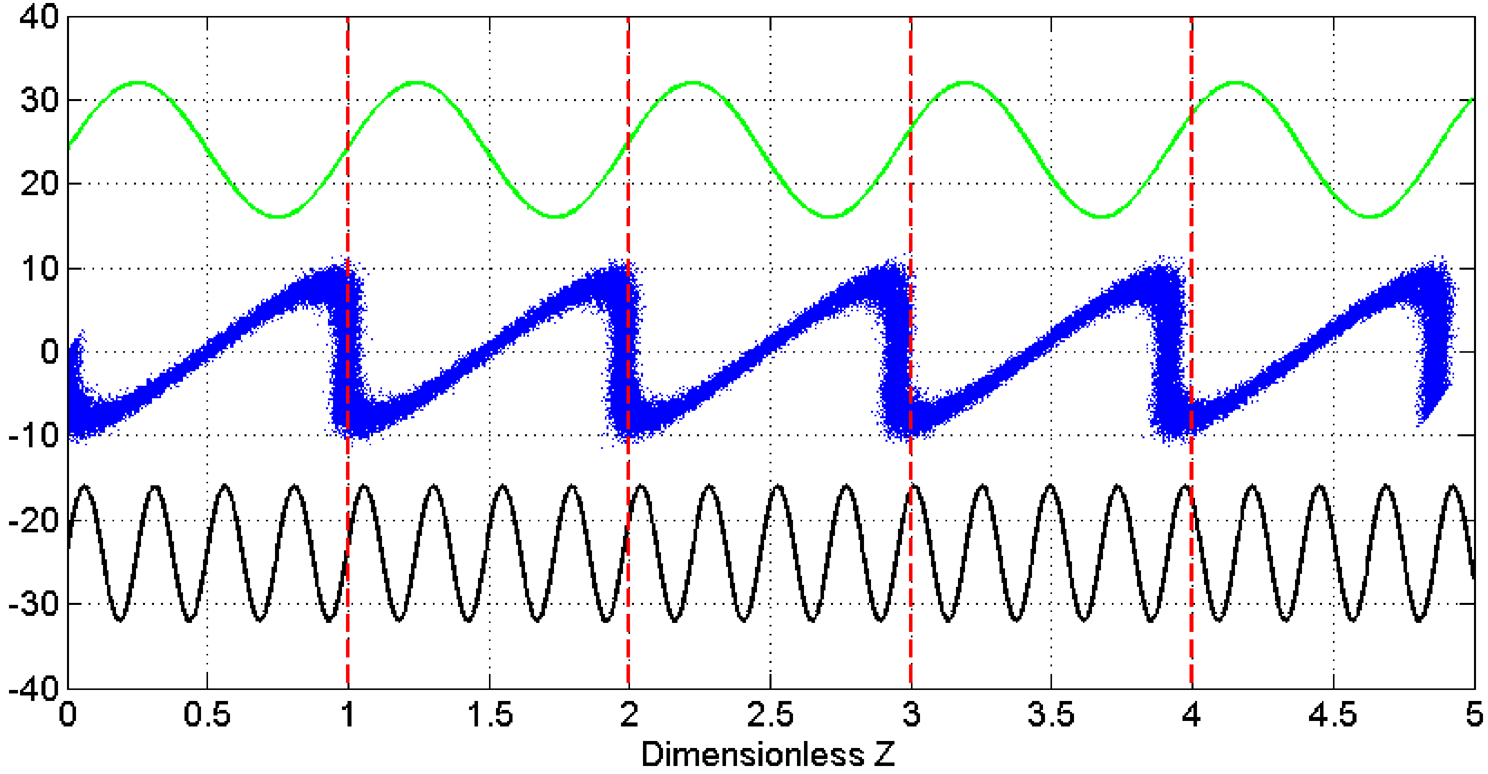


L. Yu et al., Phys. Rev. Lett. **91**, 074801-1 (2003).

Limitations of HGHG

Maximum harmonic bunching factor $$b(n) < \exp\left[-\frac{1}{2}n^2\left(\frac{\sigma_\gamma}{\Delta\gamma}\right)^2\right]$$

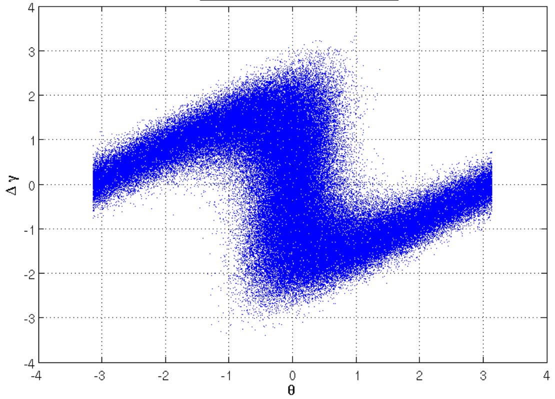
Small changes in phase of the laser translates to large phase changes in the harmonic



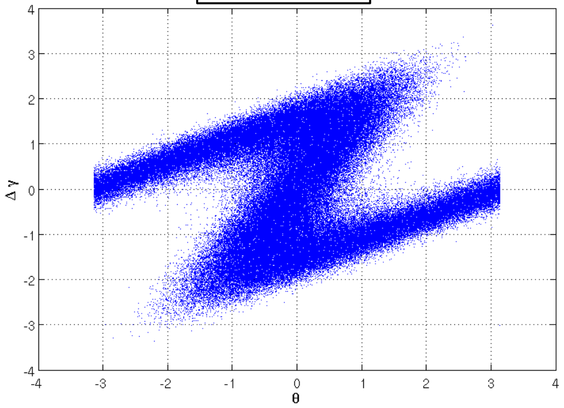
Echo Enabled Harmonic Generation

Shearing the Distribution with Large R_{56} (B)

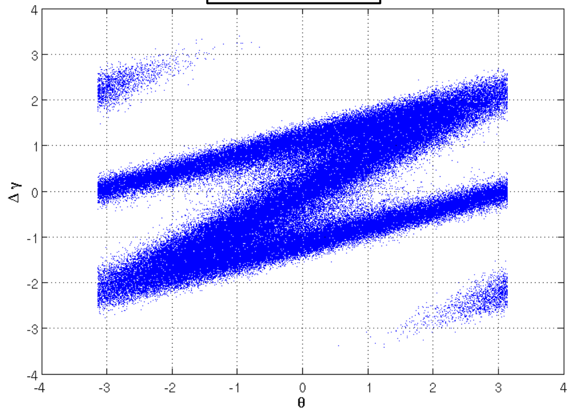
B = 0.34



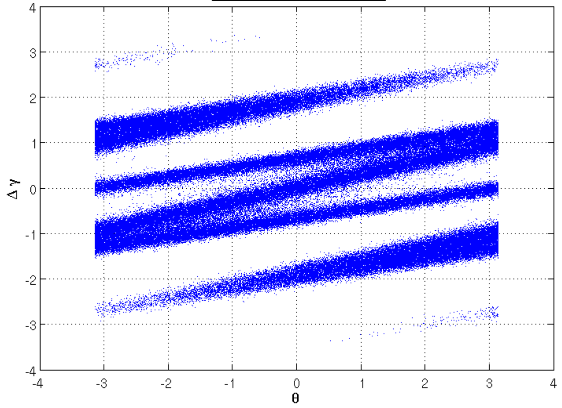
B = 0.6



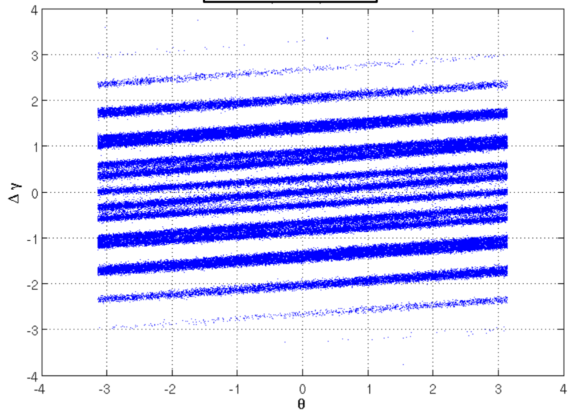
B = 1.0



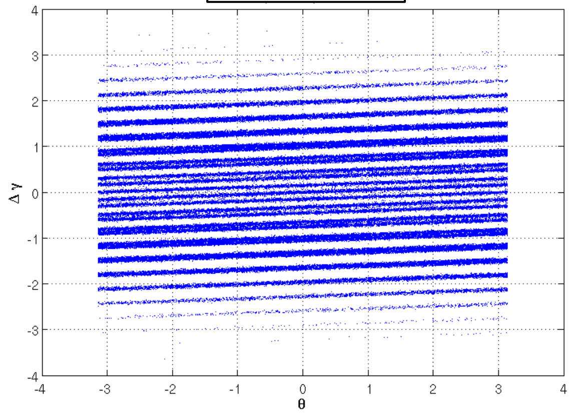
B = 2.0



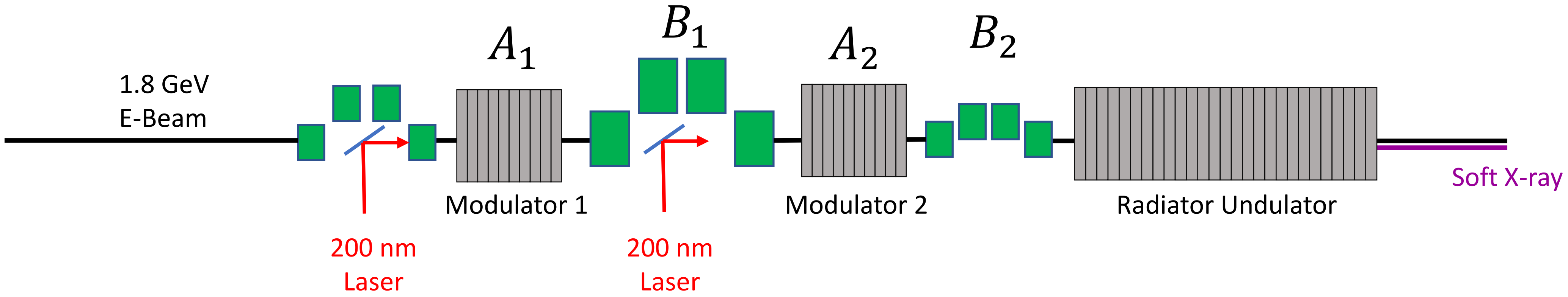
B = 5.0

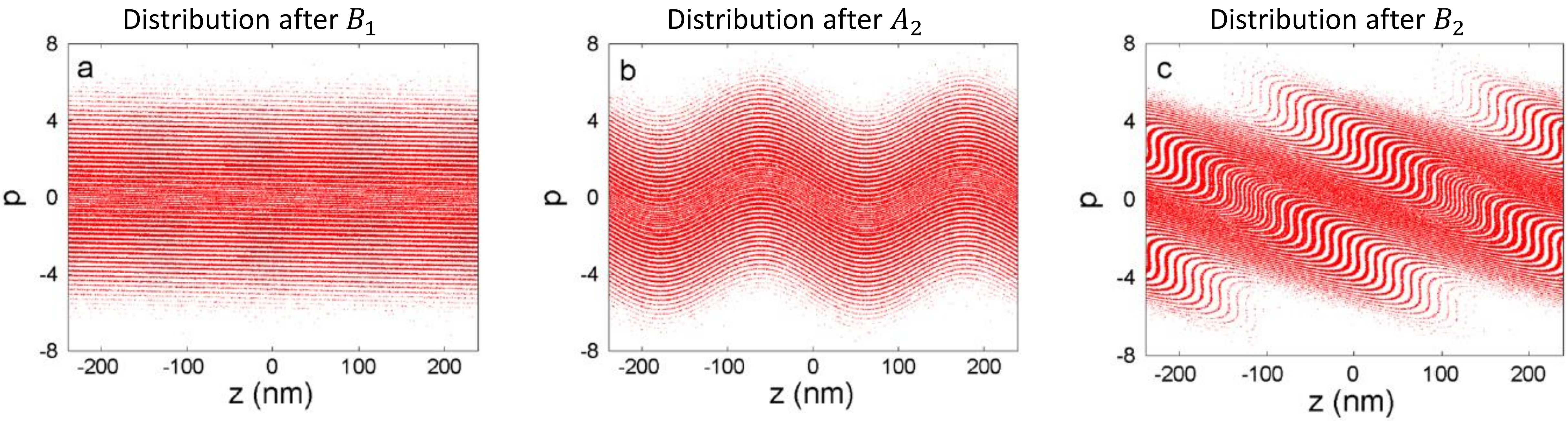


B = 10.0



Principle of EEHG - 1

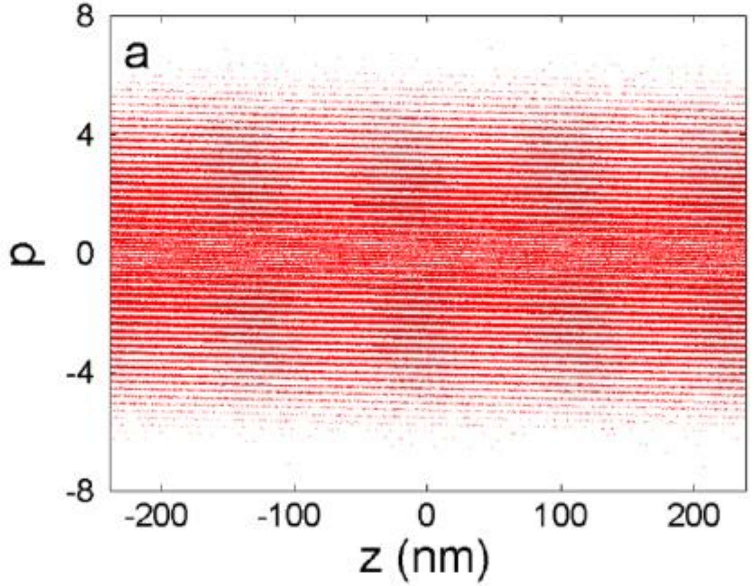




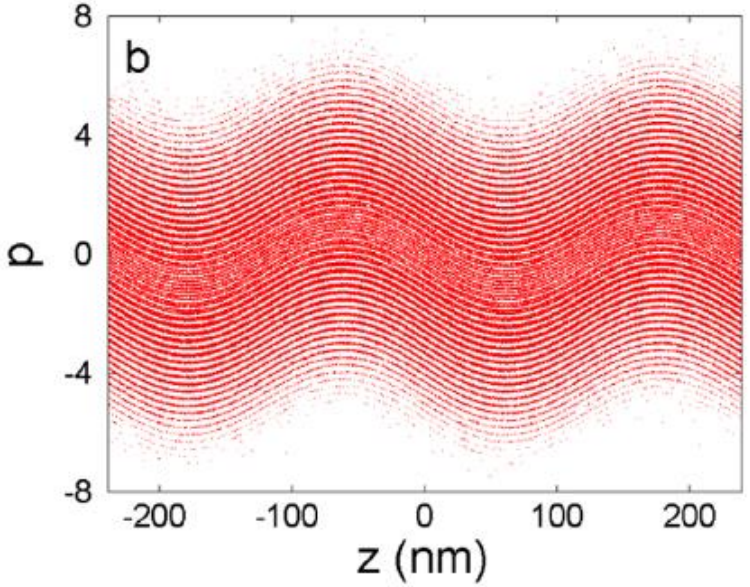
D. Xiang and G. Stupakov, Phys. Rev. ST Accel. Beams **12**, 030702 (2009).

Principle of EEHG - 2

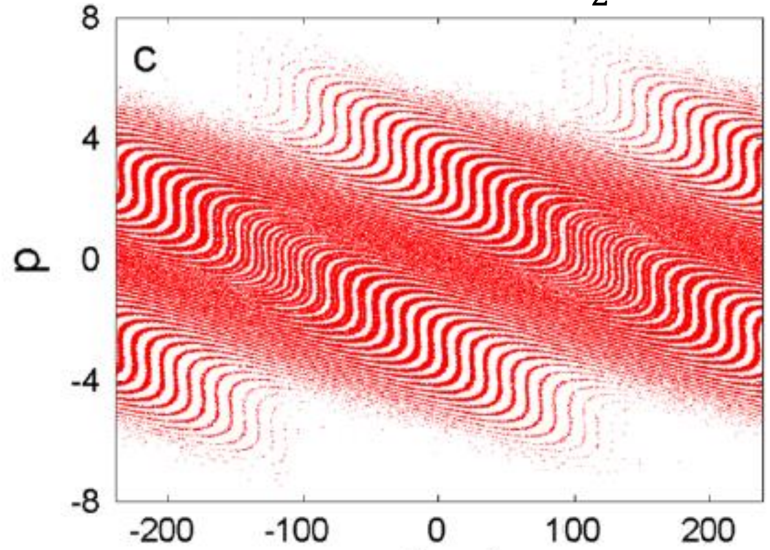
Distribution after B_2

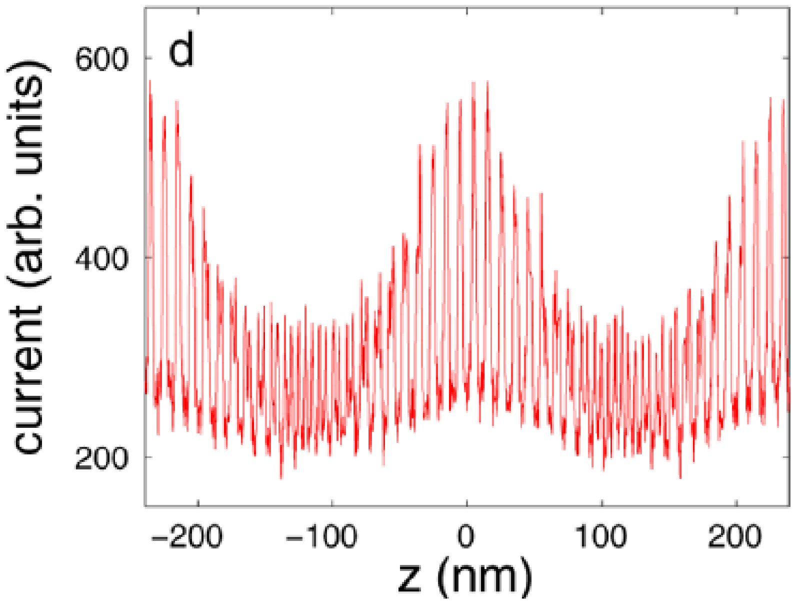


Distribution after B_1



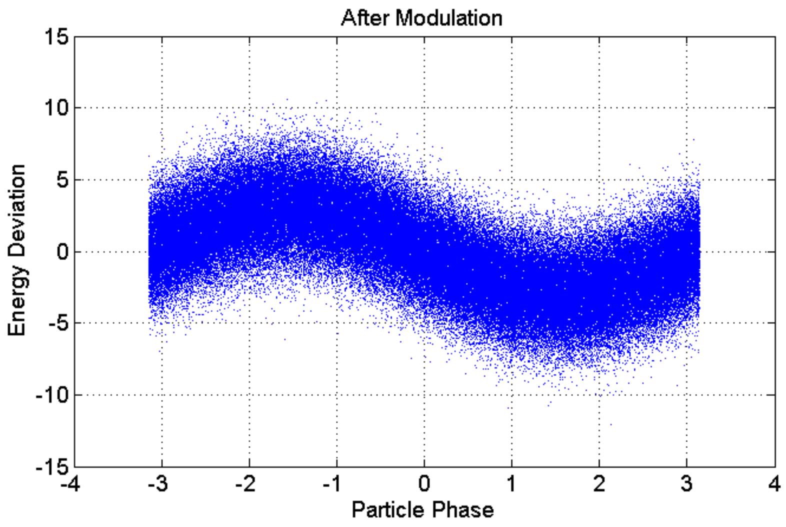
Distribution after A_2





D. Xiang and G. Stupakov, Phys. Rev. ST Accel. Beams **12**, 030702 (2009).

Phase Space after the First Modulator A_1

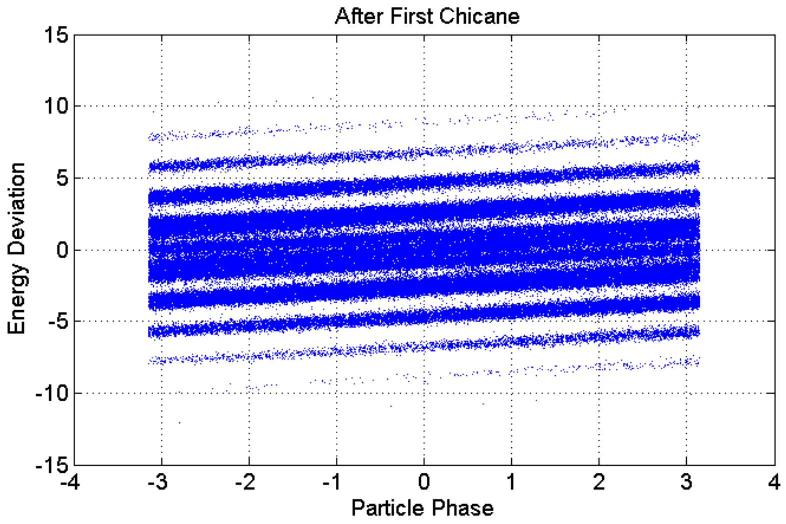


The first modulator imparts a sinusoidal modulation in energy, identical to the modulator in HGHG. Typical value of modulation is $A_1 \approx 3$.

The first modulator modifies phase space distribution according to:

$$p' = p + A_1 \sin(\xi)$$

Phase Space after the First Chicane B_1



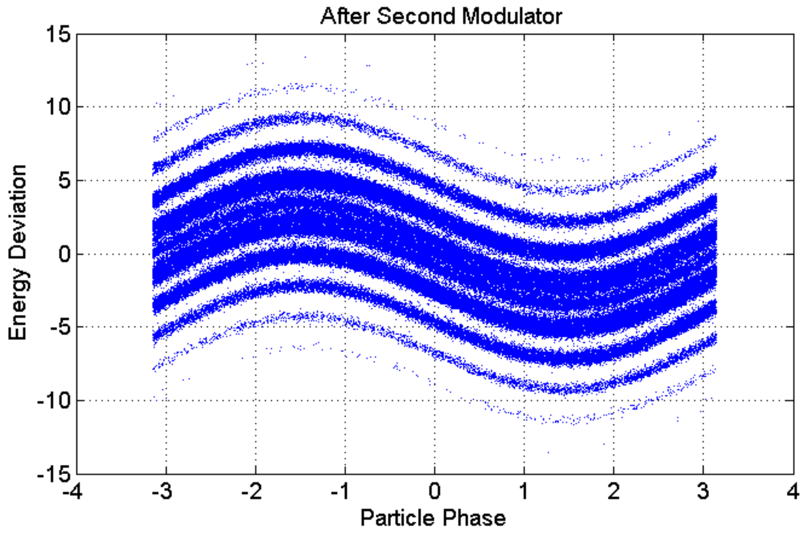
The first chicane following the first modulator produces energy stripes. The larger the chicane, the thinner each energy stripe will be. For the 24th harmonic, a value of bunching is $B_1 \approx 26.8$.

The first chicane modifies phase space distribution according to:

$$\xi' = \xi + B_1 p'$$

$$\xi' = \xi + B_1(p + A_1 \sin \xi)$$

Phase Space after the Second Modulator A_2



The second modulator gives each stripe a sinusoidal modulation.

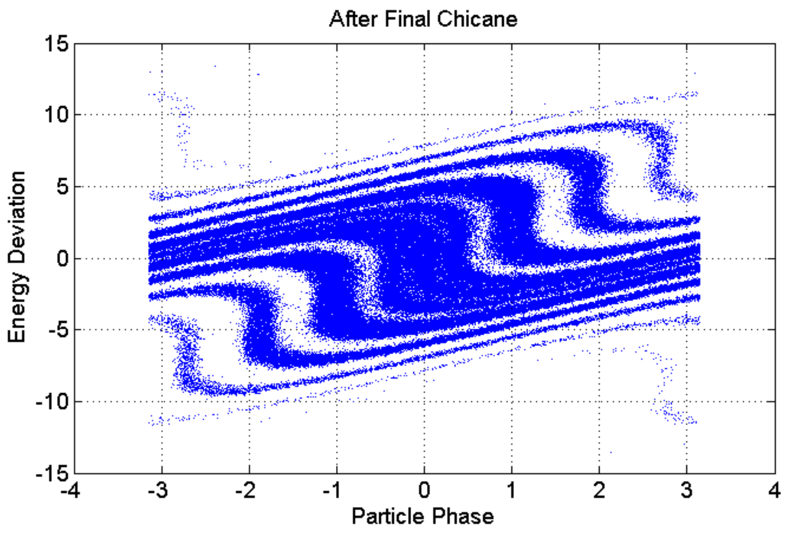
$$\kappa = \frac{k_n}{k_1}$$

The second modulator modifies phase space distribution according to:

$$p'' = p' + A_2 \sin(\kappa \xi')$$

$$p'' = p + A_1 \sin \xi + A_2 \sin\{\kappa[\xi + B_1(p + A_1 \sin \xi)]\}$$

Phase Space after the Final Chicane B_2



The final chicane rotates each of the stretched out energy bands in phase space, and produces bunching at very high harmonic.

The final chicane modifies phase space distribution according to:

$$\xi'' = \xi + B_1(p + A_1 \sin \xi)$$

$$\xi'' = \xi + (B_1 + B_2)p + A_1(B_1 + B_2)\sin \xi + A_2 B_2 \sin(\kappa\xi + \kappa B_1 p + \kappa A_1 B_1 \sin \xi + \phi)$$

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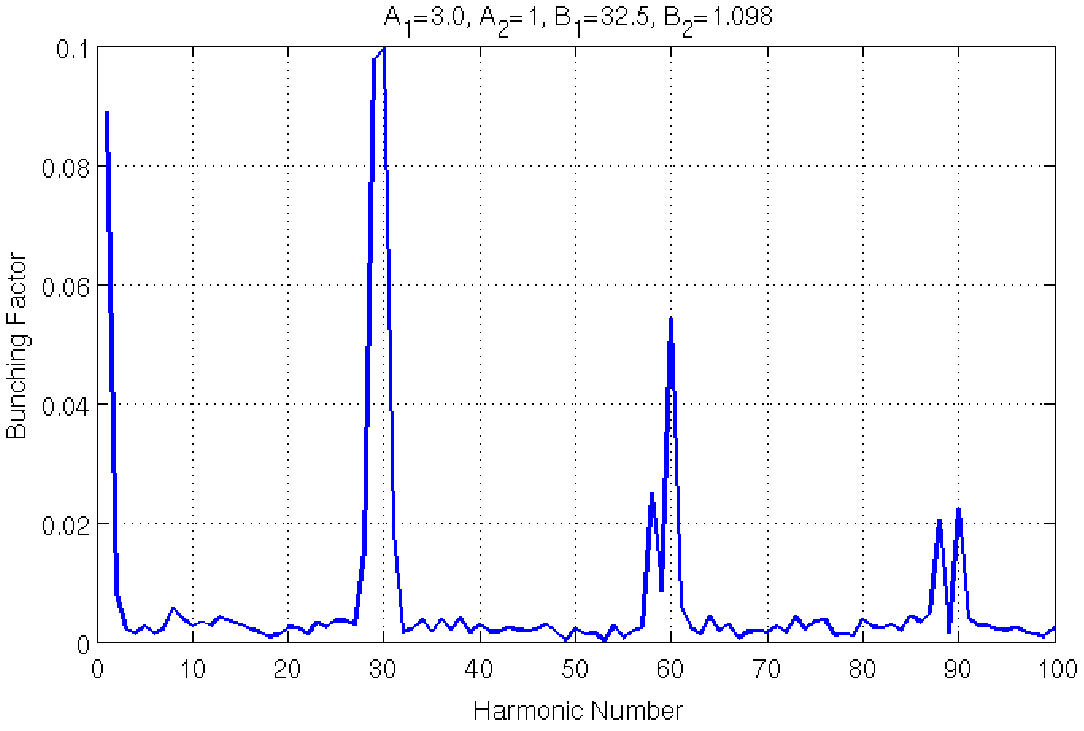
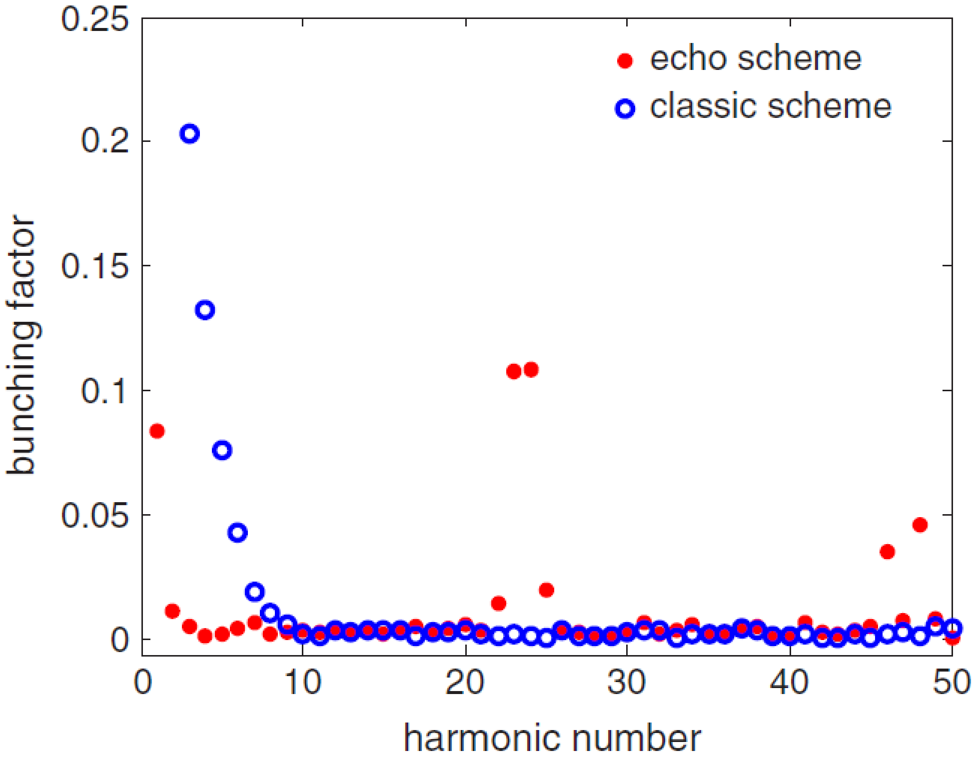
EEHG is better than HGHG at high harmonics





Scaling of EEHG bunching factor is favorable toward high harmonic generation.

$$b_n \approx \frac{0.39}{n^{1/3}}$$

Summary of Harmonic Generation

- FELs produce radiation at the fundamental and odd harmonics along the electron beam axis. The harmonic power decreases rapidly with harmonic number.

- High-Gain Harmonic Generation (HGHG) produces narrow-linewidth radiation at the harmonic of the injected laser. HGHG is sensitive to energy spread as well as phase and frequency changes in the laser.

- Echo-Enabled Harmonic Generation (EEHG) is an attractive technique to the generation of very high harmonic from the laser-driven harmonic FEL.