USPAS FEL 2021 Homework Set 2

2.1 The complete undulator magnetic fields for a planar undulator are shown below:

$$B_x = 0$$

$$B_y = -B_0 \cosh(k_u y) \sin(k_u z)$$

$$B_z = -B_0 \sinh(k_u y) \cos(k_u z)$$

Using Lorentz force law for relativistic acceleration in the y direction, show that the planar undulator provides, on average, a transverse focusing on the electron beam in the y dimension along the undulator, i.e., the electron motion in the y direction has following 2nd order derivative with respect to z

$$\langle y'' \rangle = -k_\beta^2 y$$

where $y'' = \frac{d^2 y}{dz^2}$

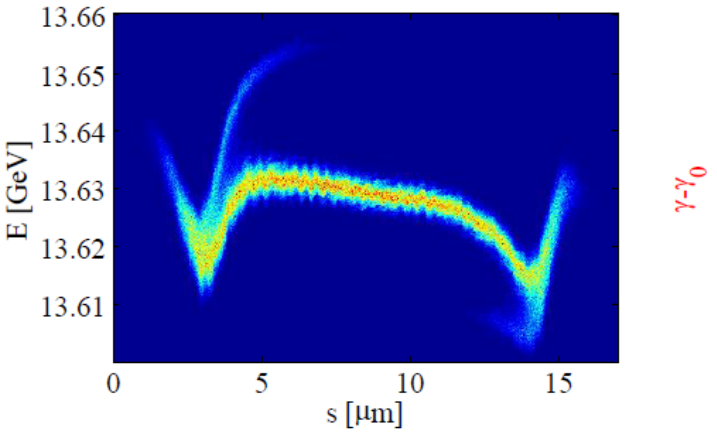
Also show that the force constant is given by $k_\beta = \frac{k_u K}{\gamma\sqrt{2}}$

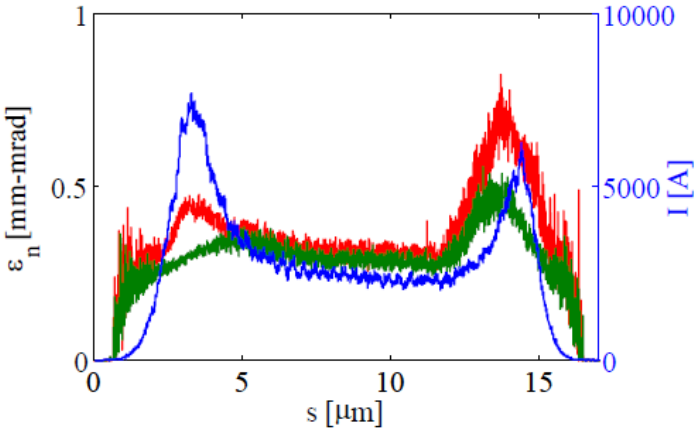
2.2 In the exponential regime, we only consider the root of the cubic equation corresponding to the exponentially growing mode. In the lethargy regime, all three roots of the cubic equation are comparable in amplitude. Derive the expression for the FEL power as a function of z taking into account all three roots corresponding to the growing, decaying and oscillating modes.

2.3 a. Express the FEL rho in terms of beta function and normalized rms emittance, instead of rms beam radius.

b. For a fixed wavelength, express rho in terms wavelength instead of undulator period.

2.4 The following simulations are for a hard x-ray FEL lasing at 20 keV (0.62 Angstrom). The undulator period is 2.6 cm, K = 2.18 and the average beta function is 24 m.





a. Calculate the 1D rho, using the slice energy (top trace), emittance (bottom red trace) and peak current (bottom blue trace) of the two figures above
b. Calculate the SASE wavelengths at the head (s=3 um), core (s=10 um) and tail (s=14 um) of the electron bunch.
c. Calculate the required normalized emittance to satisfy the condition $\frac{4\pi\varepsilon_u}{\lambda_r} \leq 1$

d. Using what you learned in class about the 3D effects of slice energy spread and emittance (ignoring diffraction), calculate the 3D gain lengths of the horns (s=3 um and 14 um) and the core (s=10 um).
e. Calculate the 3D peak power for the horns and the core.