Appendix B Neutrino-Nucleon Scattering Kinematics

The following explanation of neutrino-nucleon scattering kinematics is adapted from [1]:

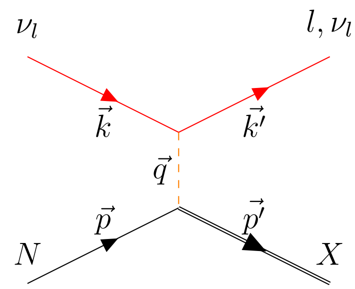

Figure B.1: A schematic diagram of a neutrino-nucleon scattering process

The expression $\nu_l + N \longrightarrow l, \nu_l + X$ describes the scattering of a neutrino, ν_l off a nucleon, N as shown in Figure B.1. This interaction proceeds through the exchange of a $W^{\pm}$ or Z^0 boson, depending on whether it is a CC or NC interaction, respectively. For the case of neutrino scattering, the incoming lepton is a neutrino and the outgoing lepton is either a neutrino (NC) or a charged lepton, l (CC). X denotes the resultant hadronic system.

The nucleon mass, M, is neglected where appropriate; the lepton mass is neglected throughout. The following kinematic variables describe the momenta and energies involved in the scattering process:

- $\vec{k}, \vec{k}'$ are the four-momenta of the incoming and outgoing lepton.
- $\vec{p}$ is the initial four-momentum of the nucleon.
- E_ν is the energy of the incoming neutrino.
- E_N is the energy of the nucleon.

The Lorentz invariants are the following:

- The squared ν+N collision energy is $s = (|\vec{p} + \vec{k}|)^2 = 4E_N E_\nu$.
- The squared momentum transfer to the lepton $Q^2 = -q^2 = -(|\vec{k} - \vec{k}'|)^2$ is equal to the virtuality of the exchanged boson. Large values of Q^2 provide a hard scale to the process, which allows resolution of quarks and gluons in the nucleon.

- The Bjorken variable $x_{Bj} = Q^2/(2\vec{p} \cdot \vec{q})$ is often simply denoted by x. It determines the momentum fraction of the parton (quark or gluon) on which the boson scatters. Note that $0 < x < 1$ for ν+N collisions.

- The inelasticity $y = (\vec{q} \cdot \vec{p})/(\vec{k} \cdot \vec{p})$ is limited to values $0 < y < 1$ and determines in particular the polarization of the virtual boson. In the lab frame, the energy of the scattered lepton is $E_l = E_\nu(1 - y) + Q^2/(4E_\nu)$; detection of the scattered lepton thus typically requires a cut on $y < y_{max}$.

These invariants are related by $Q^2 = xys$. The available phase space is often represented in the plane of x and Q^2. For a given ν+N collision energy, lines of constant y are then lines with a slope of 45 degrees in a double logarithmic $x - Q^2$ plot.

Two additional important variables are:

- The squared invariant mass of the produced hadronic system (X) is denoted by $W^2 = (|\vec{p} + \vec{q}|)^2 = Q^2(1 - 1/x)$. Deep-inelastic scattering (DIS) is characterized by the Bjorken limit, where Q^2 and W^2 become large at a fixed value of x. Note: for a given Q^2, small x corresponds to a high W, Z - N collision energy.

- The energy lost by the lepton (i.e., the energy carried away by the virtual boson) in the nucleon rest frame, is denoted $\nu = \vec{q} \cdot \vec{p}/M = ys/(2M)$.

For scattering on a nucleus of atomic number A, the nucleon momentum $\vec{p}$ would be replaced by $\vec{P}/A$ in the definitions, where $\vec{P}$ is the momentum of the nucleus. Note that the Bjorken variable is then in the range $0 < x < A$.

References

1. A. Accardi, J. Albacete, M. Anselmino, N. Armesto, E. Aschenauer, *et al.*, "Electron Ion Collider: The Next QCD Frontier - Understanding the glue that binds us all," BNL-98815-2012-JA, JLAB-PHY-12-1652, `arXiv:1212.1701 [nucl-ex]`, 2012. Cited in Section B (pg.233).