Block Instrumentation for the Far Detector at NOνA

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Block Instrumentation for the *NOvA* Far Detector

I. Abstract

The NO*v*A experiment is a long-baseline neutrino experiment currently under construction. It consists of two detectors: the Near Detector (ND) at Fermilab in Illinois, and the Far Detector at Ash River, Minnesota. At the completion, the Far Detector is designed to weigh 14 kilo tons, and the neutrino beam will travel a distance of 810 km from the ND to the FD. The Far detector is composed by 28 blocks made out of PVC modules that consist of tubular cells filled with liquid scintillator. Since this detector is made out of plastic modules, they undergo some deformation in form of strain as blocks gets hot, and the distance between blocks changes as the PVC modules contract or expand. Physicists record the reading measurement for this data (distance and strain) at any time in order to follow and understand exactly the physical deformation of blocks. Having this data available in the NO*v*A control room helps scientists to control and monitor these parameters any time. This research summarizes how the distance and the strain are measured at the Far Detector and most importantly, it gives details about the work done to build displays in forms of GUIs using a graphical program known as Synoptic to provide a quick and convenient way of reading out this data (strain and distance between blocks at the Far Detector) in the control room.

II. Introduction and background information

II.1. Neutrinos and Nova

At the intensity frontier of Fermilab, many experiments have been designed to study neutrinos and explore more about their properties. Among many similar experiments such as MINERVA, MICROBOONE, MINOS, is the NOVA. The acronym, "Nova" stands for Numi [neutrinos at main injector] off-axis [2 Gev mono-energetic beam], and *V*e appearance. The nova experiment consists of two detectors: the near detector located at Fermilab, and the far detector located in Ash River, MN. At the time of completion in January of 2014, the near detector will weigh 222 metric ton while the far detector will be 14,000-ton detector. The near detector will be installed in a cavern 350 feet underground, connected to an existing underground hall of the Fermilab site. The far detector is currently under construction and half-complete, scheduled to be completed by next year.

Neutrinos are among the most abundant particles in the universe, a billion times more abundant than the particles that make up stars, planets and people. Unimaginably large numbers of neutrinos from the first moments of the universe are still present today. Though a trillion naturally occurring neutrinos from the sun and other bodies in the galaxy pass through us each second, t*hey interact so rarely with other particles that they are very difficult to detect*. That is why researchers strive to create intense beams packed with as many neutrinos as they can produce and to build large, precise detectors that can spot them when they interact. Neutrinos have no electric charge and come in three kinds, or "flavors," *muon, electron,* and *tau* neutrinos. They have a mass, but the heaviest neutrino is nearly a million times lighter than the lightest charged particle.

The NOvA experiment will use the existing "Neutrinos at the Main Injector" (NuMI) beam at Fermilab that is currently producing neutrinos for the MINOS experiment. Unlike MINOS, which is located on the centerline of the neutrino beam, NOvA will locate its detector slightly off the centerline. This off-axis location produces a large neutrino flux that peaks at 2 GeV, the energy where oscillations to electron neutrinos is expected to be a maximum. Nova is designed to answer three fundamental questions in neutrino physics: observing the oscillation of *muon* neutrinos to *electron* neutrinos, the ordering of the neutrino masses, and the symmetry between matter and antimatter. Scientists know that neutrinos oscillate or change from one type to another, and have seen the oscillation from *muon* neutrinos to *tau* neutrinos, but they have not seen *muon* neutrinos oscillating into *electron* neutrinos. Indeed, scientists know that the masses of neutrinos are about a million times lighter than the masses of other particles in the standard model of physics. However, they do not know yet the masses of the different neutrino types, nor they do they know the neutrino mass hierarchy. That is, which kind of neutrino is the lightest and which is the heaviest. Scientists think that neutrinos get their masses through a different process than other particles. Therefore, they cannot be sure that neutrino masses follow the same pattern as other particle.   Lastly, as the theory suggests, the big bang created equal amounts of matter and antimatter. When corresponding particles of matter and antimatter meet, they annihilate one another. But somehow we're still here, and antimatter, for the most part, has vanished. Indeed, physicists once theorized that nothing would change about the laws of physics if every particle were replaced with its antiparticle. This is known as “the charge-parity symmetry”. However, it turns out that matter and antimatter are not exactly mirror images, and this could explain why they exist in unbalanced quantities. Breaking charge-parity symmetry is called “CP violation”. Therefore, if the NOvA collaboration discovers that muon antineutrinos oscillate at a different rate than muon neutrinos, they will know the symmetry between the neutrinos and antineutrinos is broken. This could be a clue to why the universe has more matter than antimatter – the reason we exist.

II.2. Block Instrumentation for the Far Detector

As already described, the Far Detector, one of the two detectors of Nova, is located at Ash River, Minnesota. As of now, the far detector is half way through the completion, and is scheduled to be completed by May of 2014. It is composed of 385, 000 cells of extruded PVC plastic. Each cell is 3.9 cm wide by 6.0 cm deep and is 15.5 meters long. The cells are filled with 3.3 million gallons of liquid scintillator and scintillation light will be guided to APD photo-detectors using wavelength shifting fiber. At the far end of a module, NOvA will collect an average of 28 photo-electrons per muon crossing about the APD threshold of 15 photoelectrons. Upon completion, the far detector will be 15.6 m wide 15.6 m tall and 78 m long, weighing in at approximately 14 kilotons whereas the near detector will be 2.9 m x 4.2 m x 14.3 m, weighing

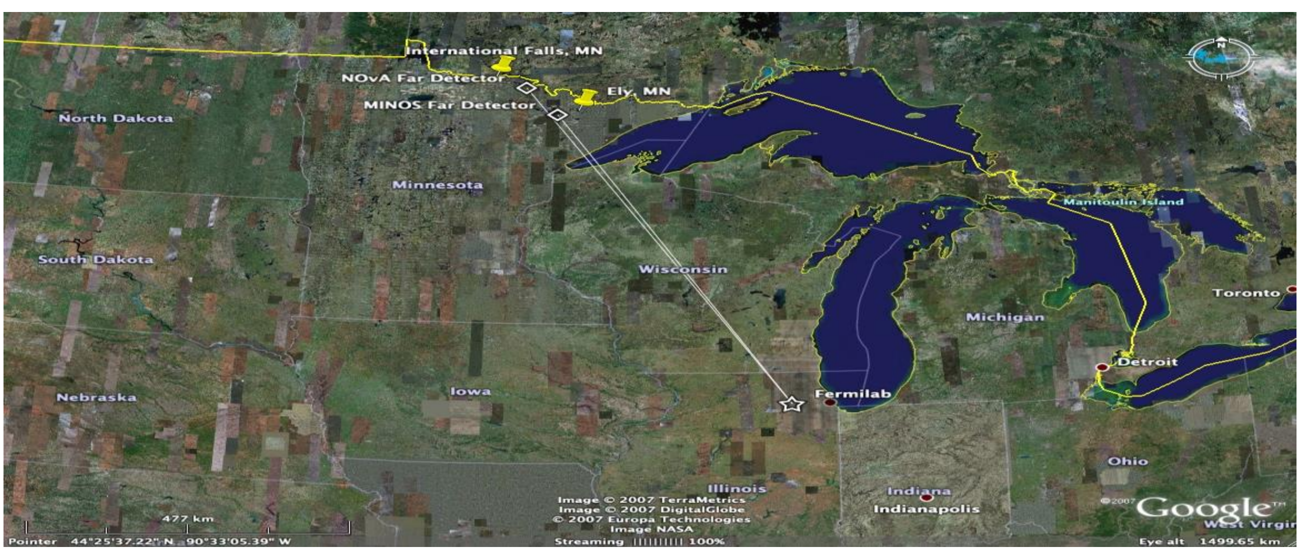


Figure 1 shows the neutrino baseline from Fermilab, IL to Ash River, MN.

in at a total of 222 tons.

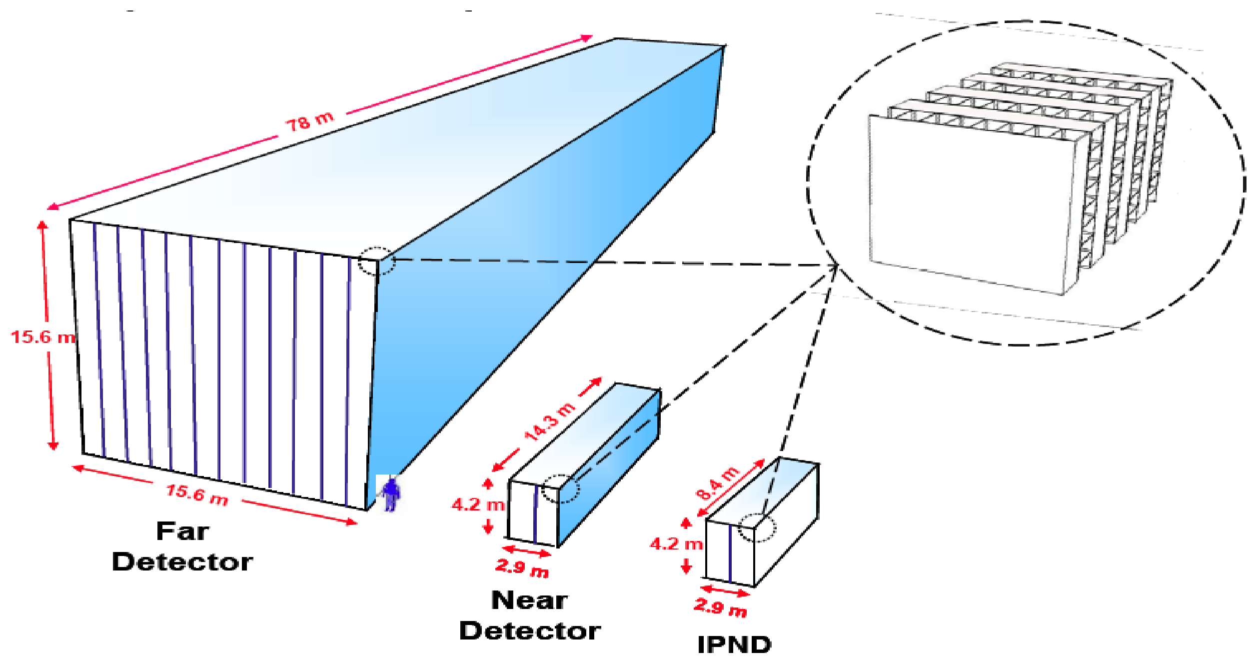


Figure 2 shows the sizes of the Near Detector (ND) and the Far Detector (FD)

The far detector will consist of 28 blocks, where each block is composed by 32 planes, and each plane has 12 modules. Thus, one block is composed by 384 PVC modules.

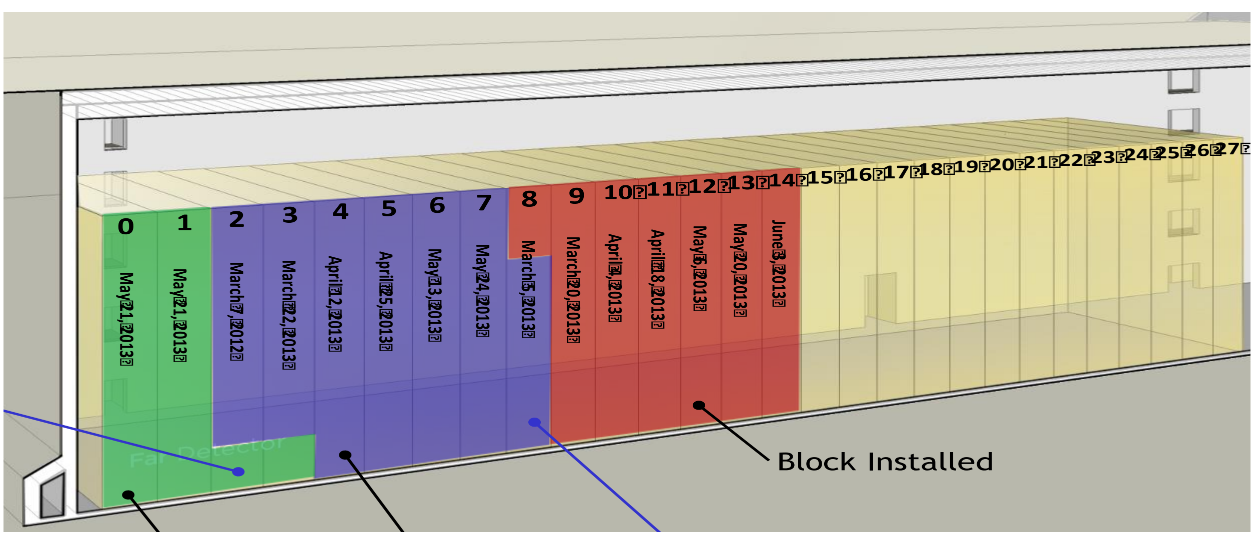


Figure 3 shows the Far Detector Overview and the progress made by far toward the installation of all blocks

My whole project was related to measuring the distance sensors and strain sensors that are found between blocks at the Far Detector and use a program called Synoptic to have this data available in forms of graphical user interfaces in the control room. The distance sensors measure the distance between blocks while the strain sensors measure the strain due to the change in elongation of the modules. As already mentioned, there are 28 blocks noted from B00 to B027. Between every two consecutive blocks, there are 24 distance sensors: 18 sensors are called north side sensors and 6 are called south side sensors. The distance between two consecutive blocks is called "gap".

	B01-S61	B01-S51	B01-S41	B01-S31	B01-S21	B01-S11	
B00-N19							B00-N29
B00-N18							B00-N28
B00-N17							B00-N27
B00-N16							B00-N26
B00-N15							B00-N25
B00-N14							B00-N24
B00-N13			Block 00/01 Gap				B00-N23
B00-N12							B00-N22
B00-N11							B00-N21

Figure 4: This figure is the example layout for all the distance sensors of the block gap 00/01 or more explicitly the distance sensors between the first and second block.

Each distance sensor is given a special notation to differentiate it from others. The north side sensors are vertically aligned while the south side sensors are horizontally aligned. Thus, the 18-north side sensors are denoted from N11 to N19 on one side and N21 to N29 on the other. The 6 south side sensors are denoted from S11, S21, S31, S41, S51, and S61. The general notation is as follows *DIS- B# # - N/S XY* with the following explanation B: [Block], ##: [block number], N or S indicates north or south side sensor respectively, and X, Y indicates the position of each sensor. It is important to mention that north side sensors on each side have the same *X* since they are vertically aligned together. For similar reasons, the south side sensors have the same *Y* since they are horizontally aligned together.

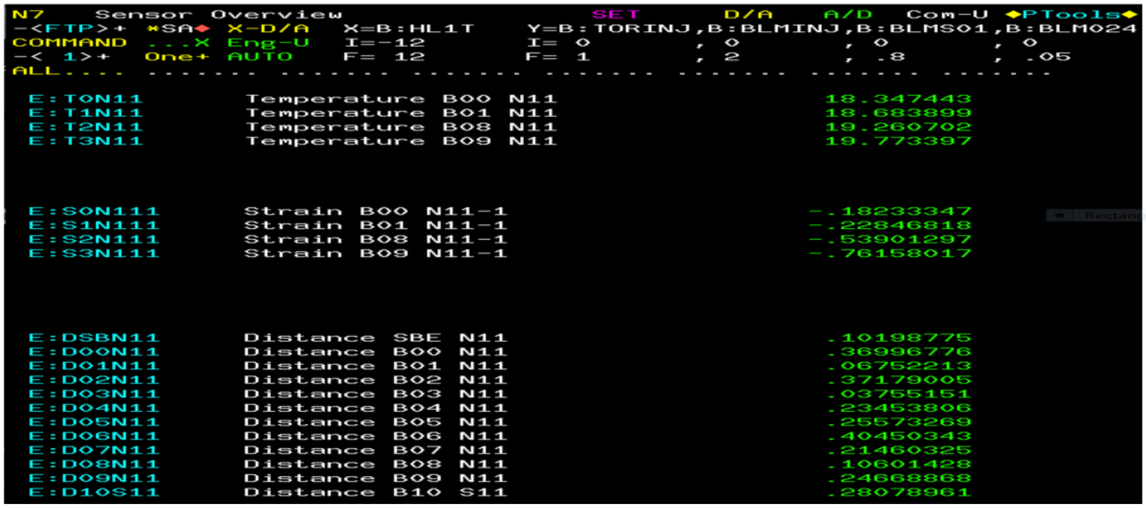


Figure 5 shows ACNET and various device names for strain sensor and distance sensors and their read out data

Like distance sensors, strain sensors also get their special notation. Unlike the distance sensors, they are not placed on every block gap. Instead, they are placed on only 8 few selected blocks: Block 00, Block 00, Block 01, Block 08, Block 09, Block 18, Block 19, Block 28, and Block 29. The general notation for the strain sensor variable is as follows: *S-#-N-XY-1/2/3*. *S* stands for strain, # corresponds to one digit for the block, *N* stands for north side, *X* and *Y* for the *x*-position, and *y-position* respectively and digits 1, 2, and 3 represents the gauge number respectively. Below is the table summarizing the block number mapping to their corresponding digits.

S1N11	S1N21	S1N31	S1N41	S1N51	S1N61
S1N12	S1N22	S1N32	S1N42	S1N52	S1N62
S1N13	S1N23	S1N33	S1N43	S1N53	S1N63
S1N14	S1N24	S1N34	S1N44	S1N54	S1N64

Table 1 shows all 24 rosettes on the north side of each strain block

Digit	Block Number
0	B00
1	B01
2	B08
3	B09
4	B18
5	B19
6	B26
7	B27

Table 2 show all blocks that have strain sensors on them and their respective digit number represented in ACNET.

For example, the strain sensor variables *S2N11 (1)* represents a strain sensor attached to *Block 08* at the position *-11*. Each one of the above block has 24 strain sensors called "rosettes". Each rosette has 3 gauges (which are measured to provide 3 strain measurements that get transformed into two principal strains.

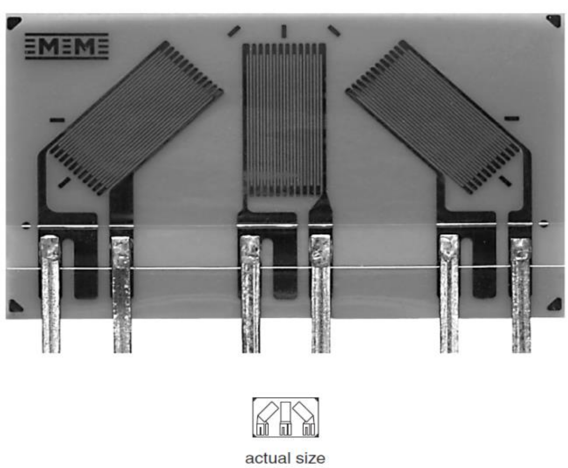


Figure 6 shows a rosette (strain sensor) with 3 gauges

III. Materials and Methods

III.1. Measuring the distance between blocks

To measure the distance between blocks at the far detector, we use a distance sensor. This sensor is designed such that it has a changing resistor (R_{DIS}) inside which gets connected to the voltage divider with a known resistance of about $20K\Omega$. Thus, the distance gets measured just by measuring the output voltage of the circuit.

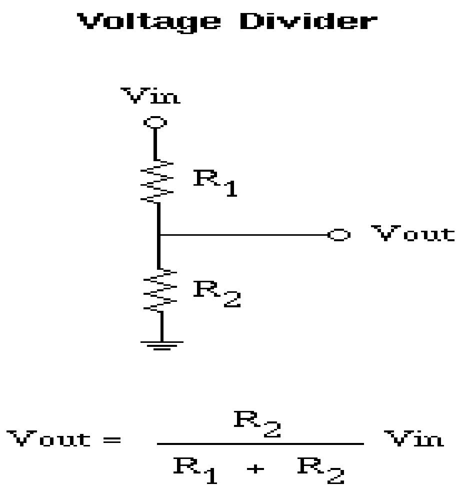


Figure 7 shows a voltage divider circuit

For our experiment, $\mathbf{R_1}$ is denoted by $\mathbf{R_{DIS}}$, *the resistance across the sensor,* and $\mathbf{R_2}$ is denoted by $\mathbf{R_{VD}}$**,** *the resistance across the voltage divider***.** Thus, we have

$V_{out} = V_{in} \cdot \frac{R_{VD}}{R_{DIS}+R_{VD}}$ (1), and

$The\ resistance\ across\ the\ sensor, R_{DIS}\ is\ linearly\ proportional\ to\ the\ distance\ between\ blocks$

$$(R_{DIS} = mx + b).$$

Thus,

$X\ = \frac{1}{V_{out}}\left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right] - \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$(5)

And finally,

$X = \frac{1}{V_{out}}[C] + [D]$ (6),

with $C = \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$ $and\ D = \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$ being constants.

[See appendix [1] for complete derivation of equation (6)]

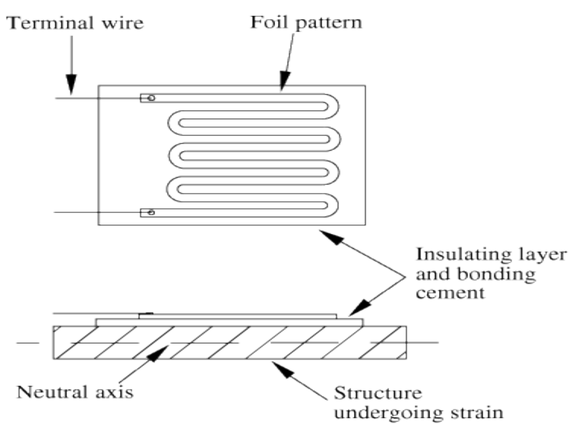
To get the real distance L, we take into account of the offset_sensor and offset_detector.

III.2. Measuring strain on blocks

Strain is the amount of deformation of a body due to an applied force. It can be either positive (tension) or negative (compressive). It is expressed by

- $\varepsilon = \frac{\Delta L}{L}$ (1) and it is measured in micro-strain ($\mu\varepsilon$).

Strain is a very small quantity, so it requires very precise measurement of very small changes in electrical resistance in order to measure it. The nova experiment uses one of the most popular strain gauges known as the bond metallic strain gauge. This device is able to change a mechanical motion to an electrical signal.



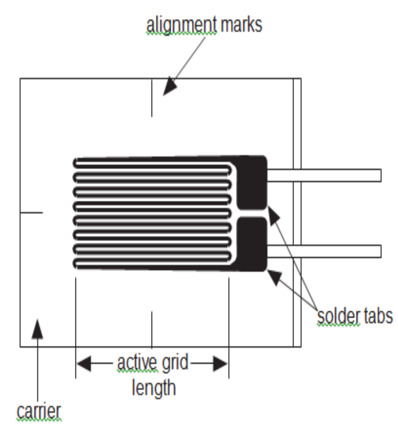


Figure 8 shows the internal part of a strain gauge

Figure 9 shows the inner part of a strain gauge

If a wire is held under tension, it gets slightly longer and its cross-sectional area is reduced.

$R = \rho \frac{L}{A}$, with

$\rho: resistivity, L: length\ of\ the\ wire, and\ A: the\ cross - sectional\ area.$

This changes its resistance (R) in proportion to the strain sensitivity (S) of the wire's resistance. When a strain is introduced, the strain sensitivity (GF) is given by

- GF= $\frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon}$, thus, $\Delta R = \varepsilon \cdot R \cdot GF$

To measure strain with a bonded resistance strain gage, it must be connected to an electric circuit that is capable of measuring the minute changes in resistance corresponding to strain. That circuit is the Wheatstone bridge, and consists of 4 resistive arms with *an excitation voltage*, V_{EX}, applied across the bridge.

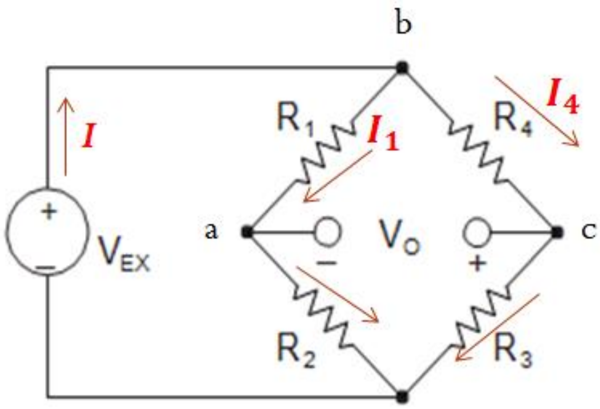


Figure 10 shows a Wheatstone bridge circuit

- The output voltage of the Wheatstone bridge is expressed by the following equation,

- $V_0 = \left[\frac{R_3}{(R_3+R_4)} - \frac{R_2}{(R_1+R_2)}\right] . V_{EX}$ (7)

Equation (7) is the expression represented by the output voltage as function the excitation voltage. *[See appendix [2] for derivation and further explanation]*

The Wheatstone bridge gets connected to the strain gauge with a slight modification below. This circuit is called a quarter-bridge circuit.

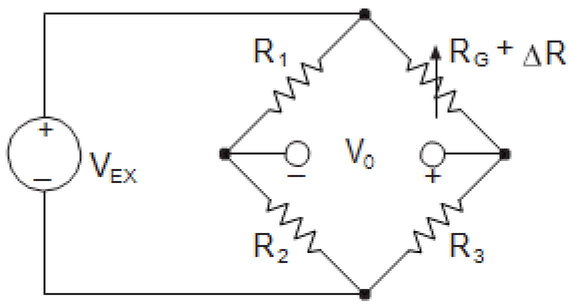


Figure 11 shows a Quarter Bridge Circuit

When the bridge is balanced or ($V_0 = 0$), any change in resistance in any arm of the bridge will result in a nonzero output voltage. Therefore, by replacing R_4 with *a strain gauge*, *any changes in the strain gauge resistance* will unbalance the bridge and *produce a nonzero output voltage*. If R_G is the nominal resistance of the strain gauge, then *the strain-induced change in resistance*, $\Delta R = R_G . GF . \varepsilon$. Then, by assuming that $R_1 = R_2$ and setting, $R_3 = R_G$, then the bridge equation can be written to express $\frac{V_0}{V_{EX}}$ as a function of strain.

$$\frac{V_0}{V_{EX}} = -\frac{GF . \varepsilon}{4}\left(\frac{1}{1 + GF . \frac{\varepsilon}{2}}\right) (8)$$

[See appendix [3] for derivation]

We want to assume that, $V_r = \frac{V_0}{V_{EX}}$, and find the equation expressing the strain (ε) in terms of the V_r.

We find that $\varepsilon = \frac{-4 \cdot V_r}{GF(1+2V_r)}$ (9)

[See appendix [4] for derivation]

Equation (8) shows how strain is measured just by measuring both the excitation voltage and the output voltage.

Each rosette has 3 strain readings $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ from which we find two principal strains $(\varepsilon_p, \varepsilon_q)$ expressed by the following equations:

$$\varepsilon_p = \frac{\varepsilon_1+\varepsilon_2}{2} + \frac{1}{\sqrt{2}}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2} \text{ (9),}$$

$\varepsilon_q = \frac{\varepsilon_1+\varepsilon_2}{2} - \frac{1}{\sqrt{2}}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2}$ (10), as well as, $\emptyset = \frac{1}{2}tan^{-1}(\frac{\varepsilon_1-2\varepsilon_2+\varepsilon_3}{\varepsilon_1-\varepsilon_3})$, which is the angle between the two principal strains.

III.3. Synoptic and other related programs

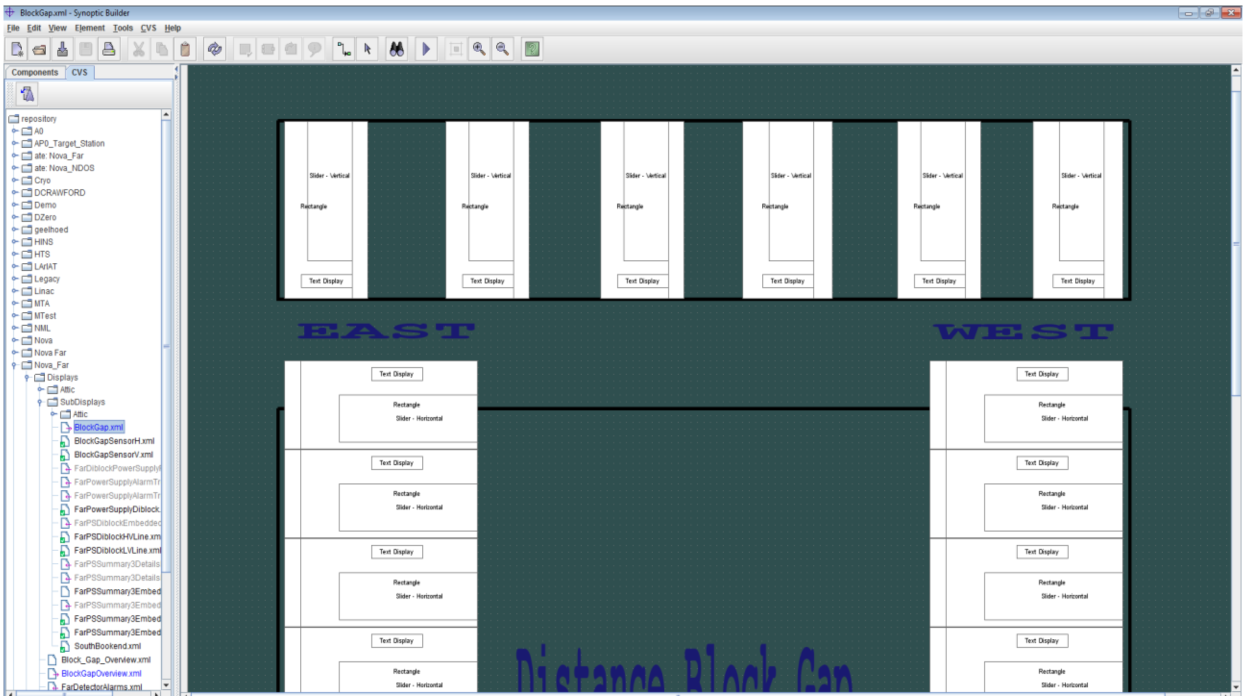


Figure 12 shows Synoptic Interface

In order to show the measurement of the distance between blocks and the strain on specific blocks, we used a program called SYNOPTIC to display these values in a very convenient way by displaying real time data anywhere at any time. To read off data (distances and strains) in Synoptic, sensor variables are imported in ACNET, the accelerator network control at Fermilab.

Synoptic is a graphical display tool used in the accelerator control system of Fermilab. It is basically a system for graphical representation of real-time data in the control system. It creates diagrams representing a certain machine or process along with actual reading from the control system indicating its current state. Indeed, it also supports setting data back from the control system. By using Synoptic, we are not only getting real time data from ACNET, but also we are able to get nice (GUI) graphical user interface for our displays, which allow users to monitor changes in distances and strain readings between blocks easily and efficiently.

IV. Results and Conclusion

IV.1. Displaying the Distances between blocks

The displays measuring the distances between every two consecutive blocks was successfully built and was added on the “Nova synoptic viewer” section under “Nova_Far” in the central repository. Basically, we have one display named “BlockGapOverview” that has all 28 blocks of the far. Then, by clicking the area between two blocks from which you want to view the readings, you access the display named, “BlockGap”, which show all 24 readings of the distance sensors: 18 north side sensors and 6 south side sensors. At this point, we are able to broadcast sensors up to B10.

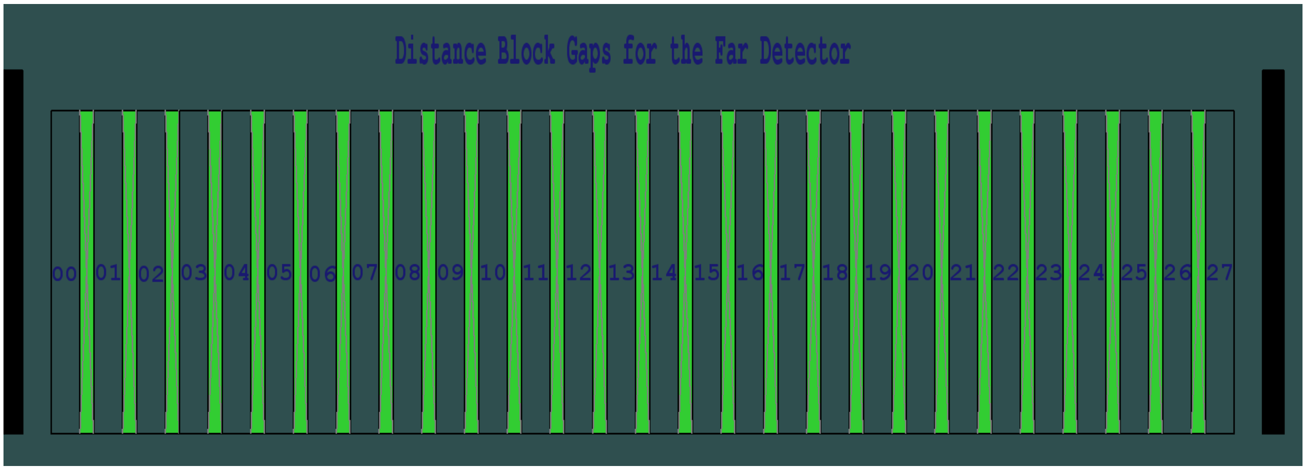


Figure 13 shows the synoptic display for the whole far detector overview showing all 28 blocks and the block gaps are embedded inside.

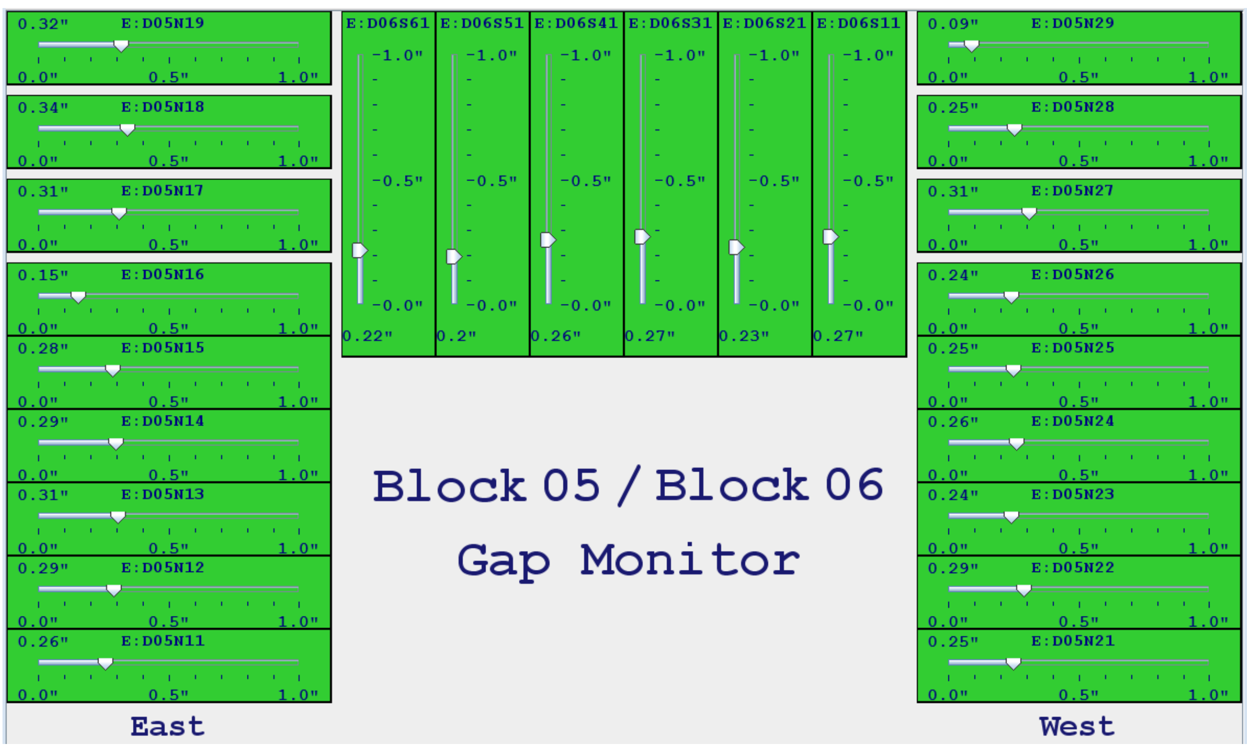


Figure 14 shows the distance sensor readings between BLOCK 05 and BLOCK 06. This display is accessed by clicking between the two blocks on the overview display.

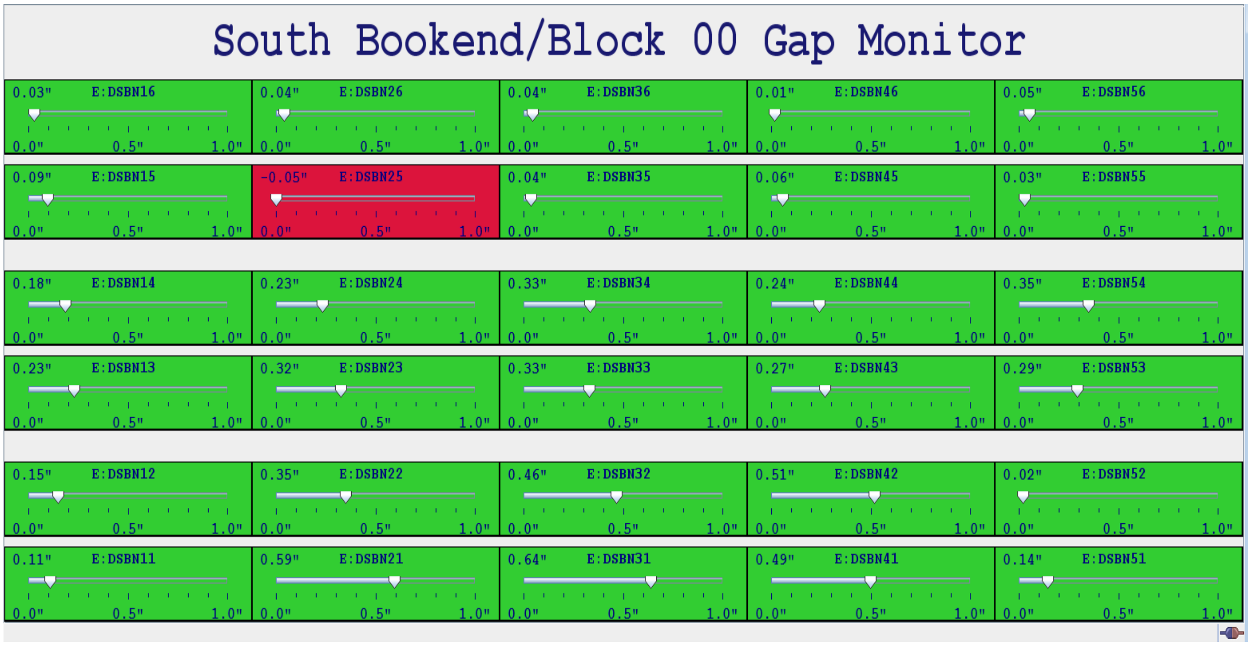


Figure 15 shows the gap between the sound bookend and the first block

The green colors indicate that the sensors were displaying normal or expected values and the red colors meant an alarm condition where the values are out of range or a broken sensor. For example, the sensor DSBN25 marks “red” and a negative value, which means that there was a calibration issue with this specific sensor. Other sensors such those of block gap 05/06 are displaying normal values and are marked in green. This helps users to have a quick look at what is going on with the sensors.

IV.2. Displaying strain readings on blocks

For the strain display, we have created 3 main files: “BlockStrainOverview”, “BlockStrain”, and “StrainRosette”. The “BlockStrainOverview” is basically the overview of the whole Far Detector with strain blocks. Once one block is clicked upon in the overview, the ‘BlockStrain’ file is accessed. This display shows the readings for the 24 rosettes found on the north side of specific blocks and 6 on the west side. Then, the ‘StrainRosette’ file takes 3 strain readings of each rosette and changes it into one horizontal principal strains. It is important to remember that the strain is measured in micro-strain ($\varepsilon \times 10^{-6}$). For each rosette, we are displaying one principal strain in the horizontal motion, so we have a total of 30 readings: 24 north side readings and 6 west side readings for each block with strain sensors.

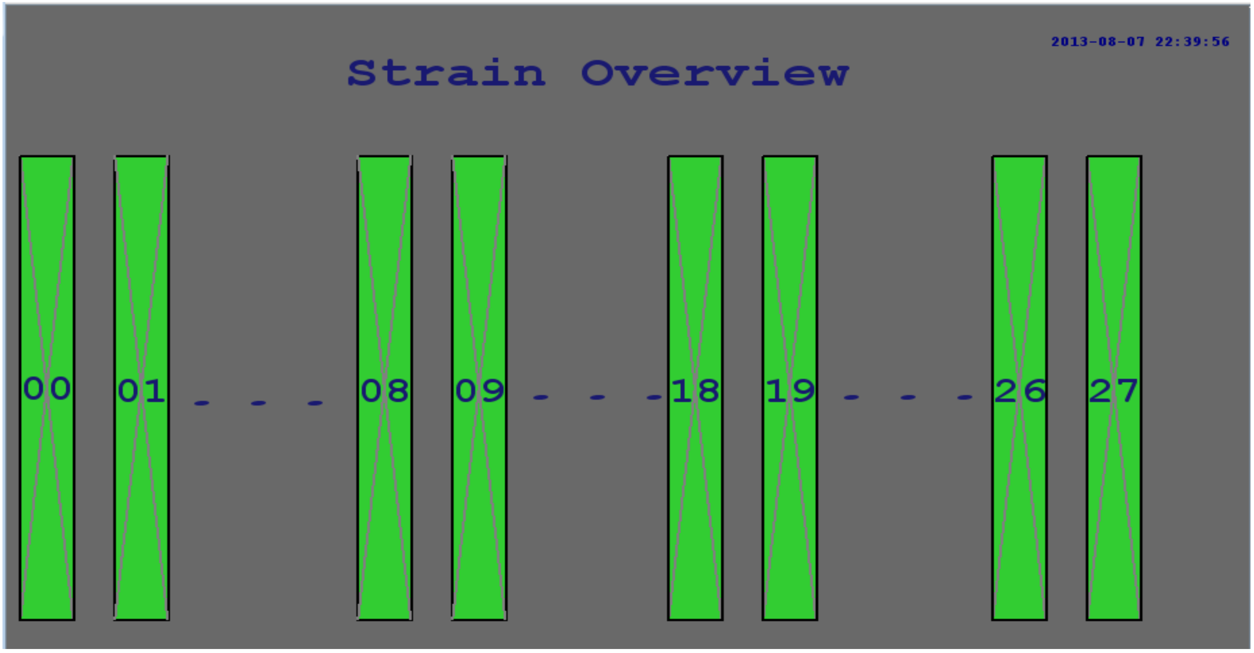


Figure 16 shows the overview of all strain blocks

The above display is the display that shows all 8 blocks that have strain sensors on them. By clicking on a specific block, the strain readings are displayed. Below is the strain display for just BLOCK=08 showing all 30 readings: 24 north side and 6 west side as measured in micro-strain.

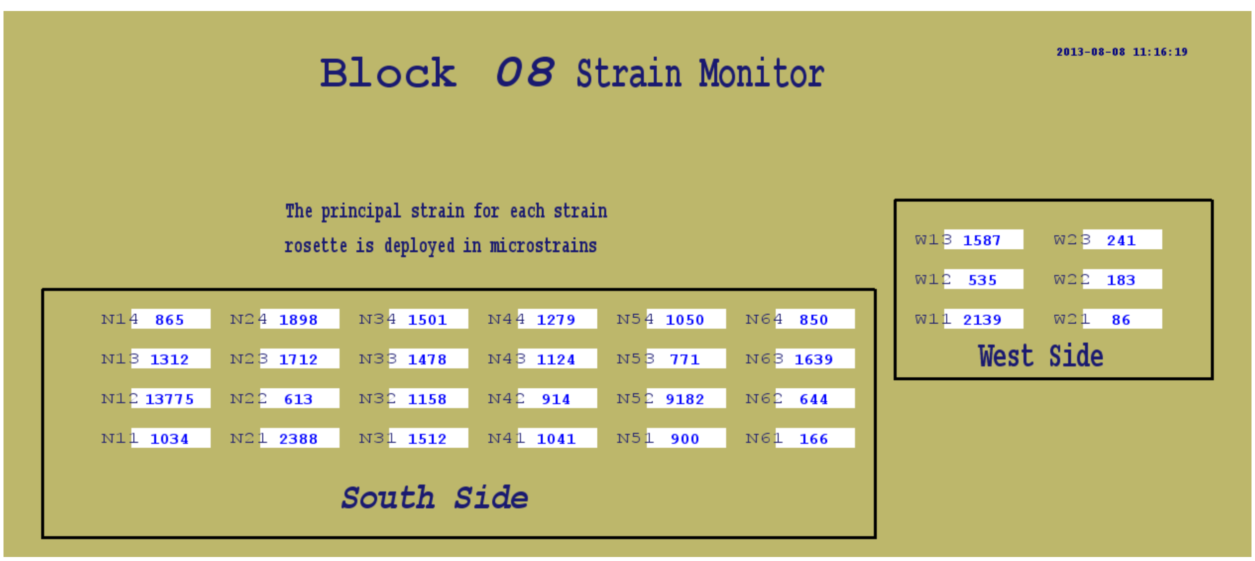


Figure 17 shows the principal strain for the BLOCK=8

By displaying the reading measurements, these displays are also not only supposed to show normal readings, but also they are supposed to show some inconsistencies in the strain such as when sensors are not working properly or dead. For example, when the sensor indicates, ‘NaN’, it means, ‘not an end’, which indicates that this rosette has some sort of issues, which alerts users in the control room and ensure a quick monitoring of the sensors as well the blocks.

V. Conclusion

Briefly, both displays showing the distance and strain measurements have been successfully made and they are up and can be accessed through the control room of NOvA. The display, “BlockGapOverview” showing the distance readings between blocks is almost finished and it is up and running in the *Nova* control room. Since the project is under development, only block up to B10 are displaying and other measurements will be displayed later as more blocks get deployed to ACNET.

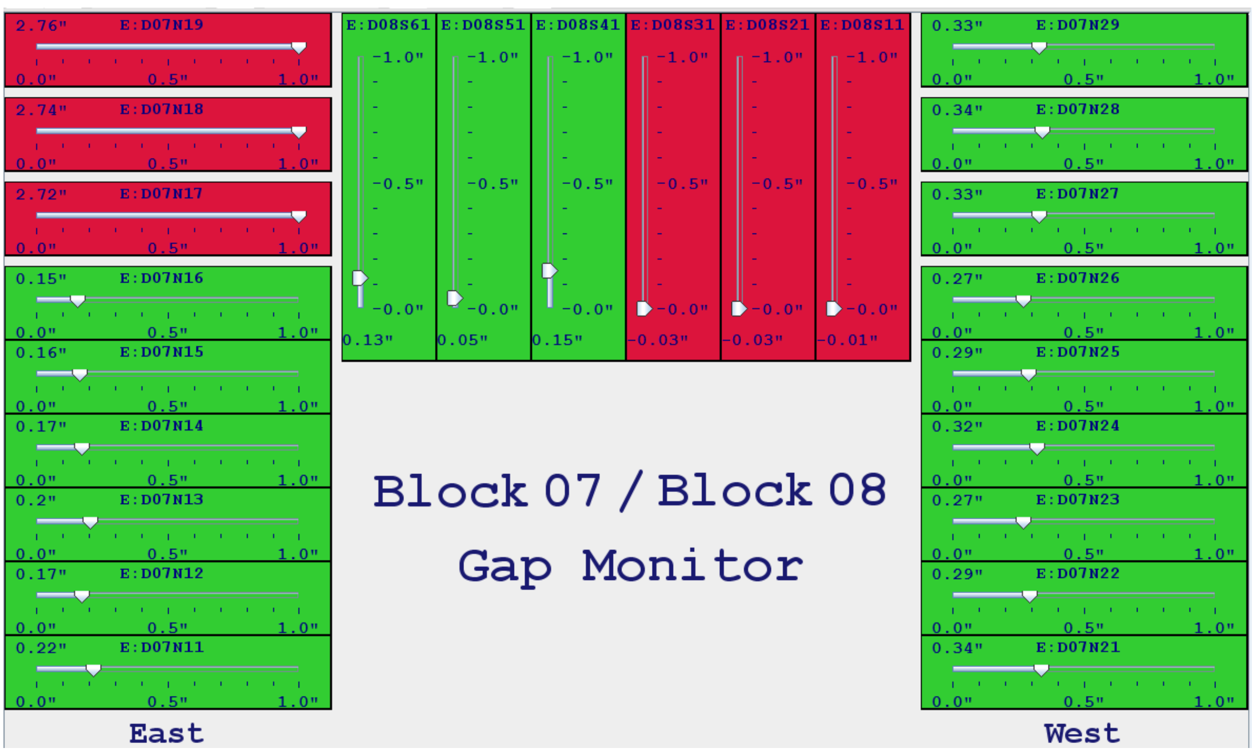


Figure 18 shows the gap sensor readings between BLOCK 07/08. The reds states an alarming condition while the greens state a normal condition.

Using synoptic displays to show the distance and strain readings between blocks in form of graphical user interfaces allow users to monitor and to control these facilities. These GUIs provide easy and convenient ways to read out the real time data any time. These displays help users to know the exact state of the sensor: whether good or bad. They also show when measurement readings are normal and identify issues that might be going on with the sensor readings such as identifying broken sensors, showing and revealing errors that might have happened during the calibration, or other related issues. This helps users to facilitate 24/7 monitoring system for the facility in the control room of NO*ν*A.

VI. References

[1] http://www-nova.fnal.gov/

[2] http://soliton.ae.gatech.edu/people/jcraig/classes/ae3145/Lab2/bridge-measure.pdf

[3] http://www.omega.com/literature/transactions/volume3/strain.html

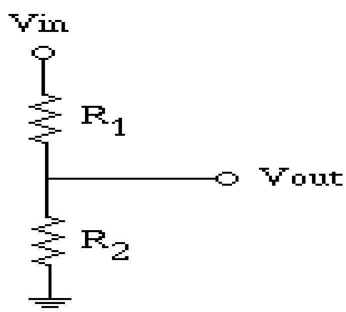
VII. Acknowledgements

This research was conducted at Fermi National Accelerator Laboratory during the summer of 2013. I would like to express my gratitude to all people who made this internship a very intellectual and educational experience for me. My appreciation goes to Dianne Engram, Linda Diepholz as well as the entire SIST committee for funding this project. I would also like thank my supervisor for this project, Dr. Martin Frank, and his assistant, Dr. Gavin Davies for teaching all the knowledge this summer. My thanks also go to Dr. James Davenport, and my mentors: Mccrory Elliott, and Bradly Verdant for their support throughout this project.

VIII. Appendix

[1] Explanation and derivation of the distance formula

Voltage Divider



$$Vout = \frac{R_2}{R_1 + R_2} Vin$$

For our experiment, $\mathbf{R_1}$ is denoted by $\mathbf{R_{DIS}}$, *the resistance across the sensor*, and $\mathbf{R_2}$ is denoted by $\mathbf{R_{VD}}$**,** *the resistance across the voltage divider***.** Thus, we have

$V_{out} = V_{in} \cdot \frac{R_{VD}}{R_{DIS}+R_{VD}}$ (1)

$\frac{V_{out}}{V_{in}} = \frac{R_{VD}}{R_{DIS} + R_{VD}}$ (2)

The resistance across the sensor, R_{DIS} *is linearly proportional to the distance between blocks*

$$(R_{DIS} = mx + b).$$

Thus, $eq.$ (2) becomes

$\frac{V_{out}}{V_{in}} = \frac{R_{VD}}{(mx+b)+R_{VD}}$ (3)

Then, $V_{out} \cdot (mx + b)$+ $R_{VD} \cdot V_{out}$=$R_{VD} \cdot V_{in}$ (4) and

$V_{out} \cdot (mx) + V_{out} \cdot b + R_{VD} \cdot V_{out}$=$R_{VD} \cdot V_{in}$

Therefore, $X = \frac{1}{m}\left[\frac{V_{in}}{V_{out}} \cdot R_{VD}\right] - \frac{1}{m}[b + R_{VD}]$, and finally

$X = \frac{1}{V_{out}}\left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right] - \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$(5)

The terms $\left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$ and $\left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$ are constants, so

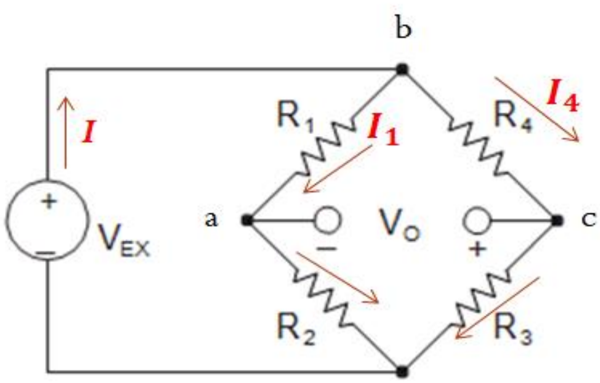
$X = \frac{1}{V_{out}}[C] + [D]$ (6), with $C = \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$ expressed in $\mathrm{Volt} - \mathrm{meter}$ $[\mathrm{V.m}]$

$and\ D = \left[\frac{1}{m} \cdot V_{in} \cdot R_{VD}\right]$, expressed in meter [m]

Therefore, in order to measure the distance between two consecutive blocks (X), all we need is to measure the output voltage of the circuit.

$$X = \frac{1}{V_{out}}[C] + [D] \ (2)$$

[2] Explanation and derivation of the Wheatstone bridge



<u>Explanation</u>

- At b, $I = I_1 + I_4$ (1)
- Thus, $I_1 = \frac{V_{EX}}{R_1+R_2}$ (2) and $I_4 = \frac{V_{EX}}{R_3+R_4}$ (3)
- By Ohm's Law: $V_a - V_b = I_1 \cdot R_1 = \frac{V_{EX}}{R_1+R_2} \cdot R_1$ (4) and

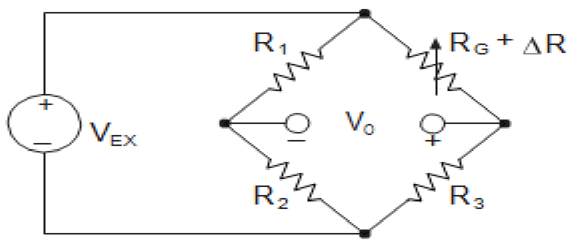
$$V_c - V_b = I_4 \cdot R_4 = \frac{V_{EX}}{R_3+R_4} \cdot \ R_4 \ (5)$$

- By definition, $V_0 = V_c - V_a = \frac{V_{EX}}{R_3+R_4} \cdot \ R_4 - \frac{V_{EX}}{R_1+R_2} \cdot \ R_1$ (6)

- Therefore,

- $V_0 = \ \left[\frac{R_3}{(R_3+R_4)} - \frac{R_2}{(R_1+R_2)}\right]. \ V_{EX} \ \ (7)$

[3] Derivation of the strain equation as a function of voltage in a quarter bridge circuit



Following equation (7) and knowing that $\Delta R = R_G.GF.\varepsilon$ in this circuit, while assuming that

$R_1 = R_2$, and setting, $R_3 = R_G$,

Then we get,

$$\text{Thus, } \frac{V_0}{V_{EX}} = \left(\frac{R_G}{R_G+R_G+\Delta R} - \frac{1}{2}\right) = \left(\frac{R_G}{2R_G+\Delta R} - \frac{1}{2}\right)$$

$$\frac{V_0}{V_{EX}} = \left(\frac{R_G}{2R_G + \varepsilon.R_G.GF} - \frac{1}{2}\right)$$

$$\text{Then, } \frac{V_0}{V_{EX}} = \left(\frac{1}{2+\varepsilon.\ GF} - \frac{1}{2}\right) = -\frac{GF.\varepsilon}{4+2GF.\varepsilon} = -\frac{GF.\varepsilon}{4}\left(\frac{1}{1+GF.\frac{\varepsilon}{2}}\right)$$

Therefore,

$$\frac{V_0}{V_{EX}} = -\frac{GF.\varepsilon}{4}\left(\frac{1}{1+GF.\frac{\varepsilon}{2}}\right) (8)$$

[4] Derivation of the strain equation as a function of $V_r = \frac{V_0}{V_{EX}}$

From equation (8), we know that $V_r = -\frac{GF.\varepsilon}{4+2GF.\varepsilon}$

$$\text{Then, } -GF \cdot \varepsilon = V_r \cdot (4 + 2 \cdot GF.\varepsilon)$$

$$-GF \cdot \varepsilon - 2 \cdot GF.\varepsilon = 4 \cdot V_r$$

$$\varepsilon(GF + 2 \cdot GF \cdot V_r) = -4 \cdot V_r$$

$$\varepsilon = \frac{-4 \cdot V_r}{GF(1+2V_r)} \text{ (9)}$$