Finetuning, naturalness, and the next good theory

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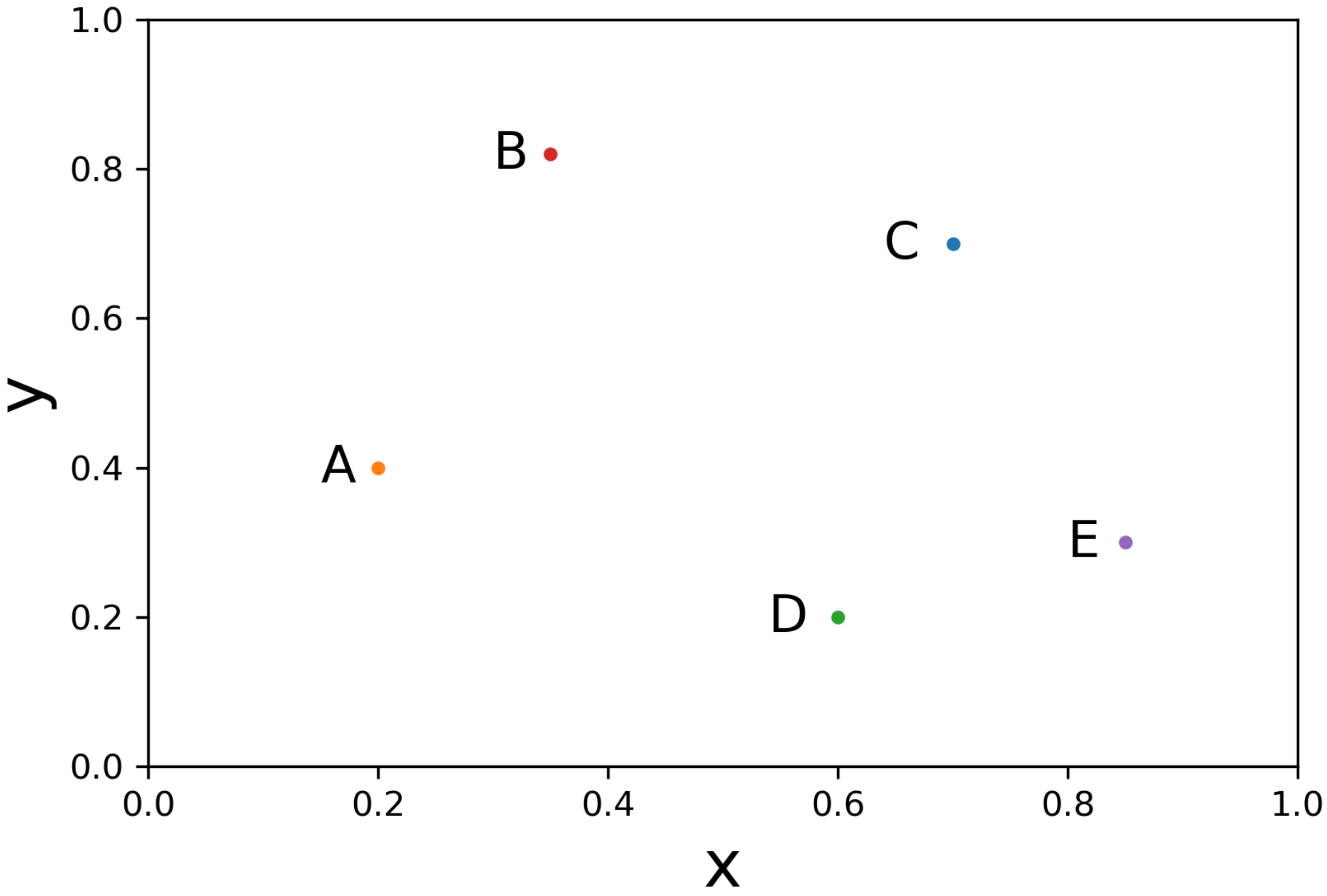
Imagine two random variables x and y, flatly distributed [0,1]

Sample these two variables five times and plot in 2D plane

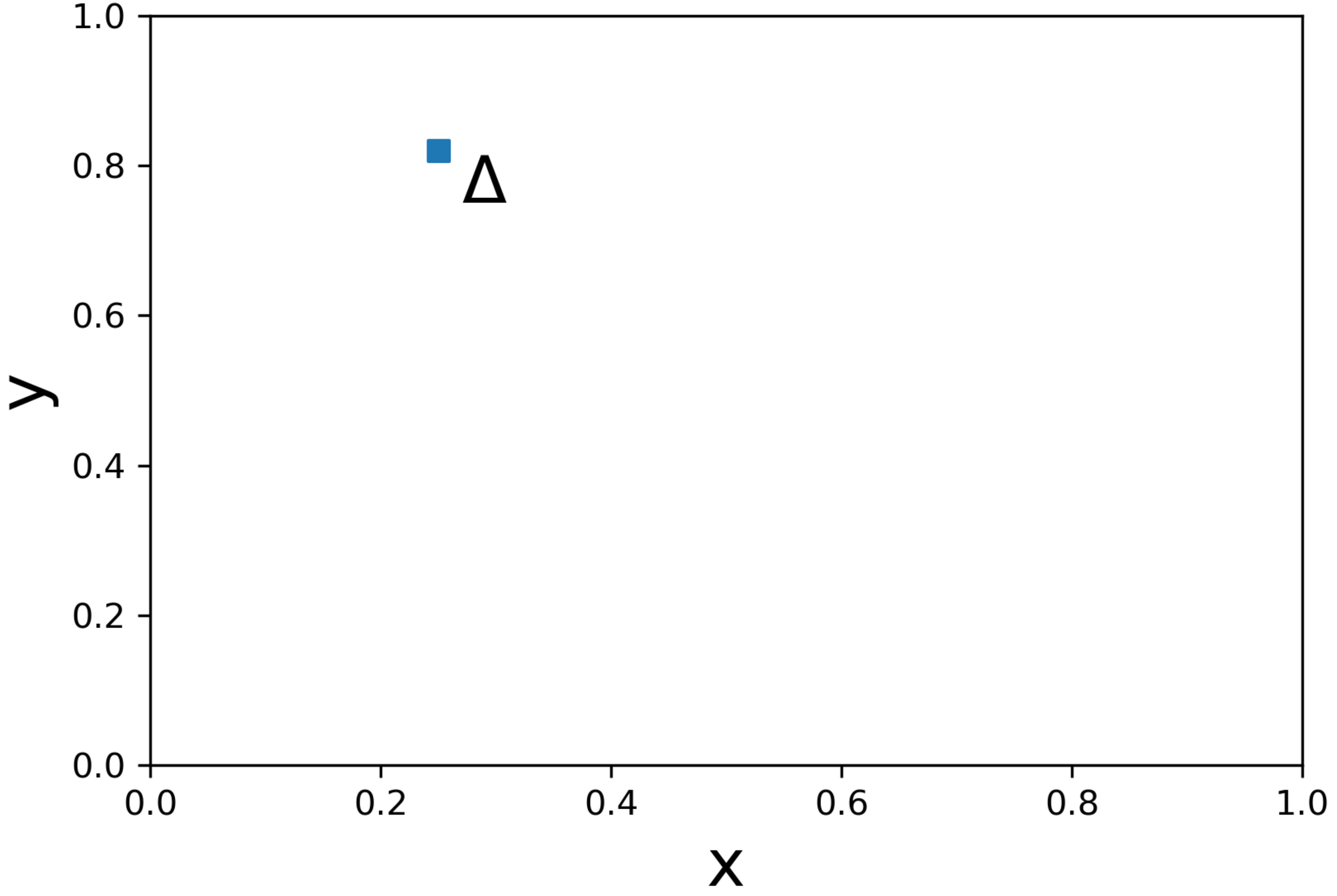
(x1,y1)
(x2,y2)
(x3,y3)
(x4,y4)
(x5,y5)



Nothing looks weird about these points, eh? There is nothing alarming here.

Now, start over and draw a tiny box of volume Δ in the x-y plane but do not show anyone.

Then, ask somebody to pick two numbers between 0 and 1: (x,y).



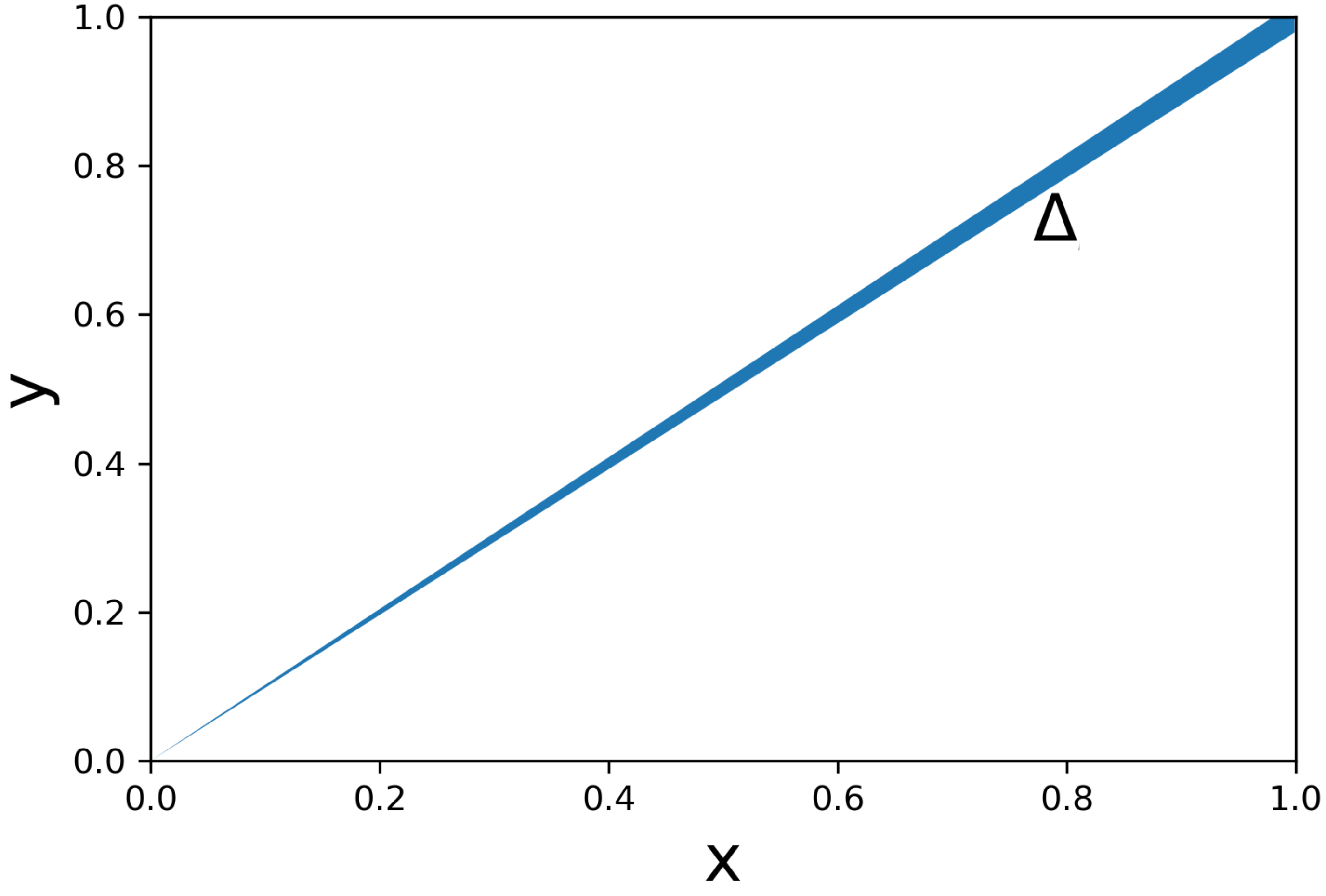
What is the probability that the randomly chosen (x,y) will fall into Δ box?

Tiny! Probability = Δ

You would win bets against people who thought the probability is large.

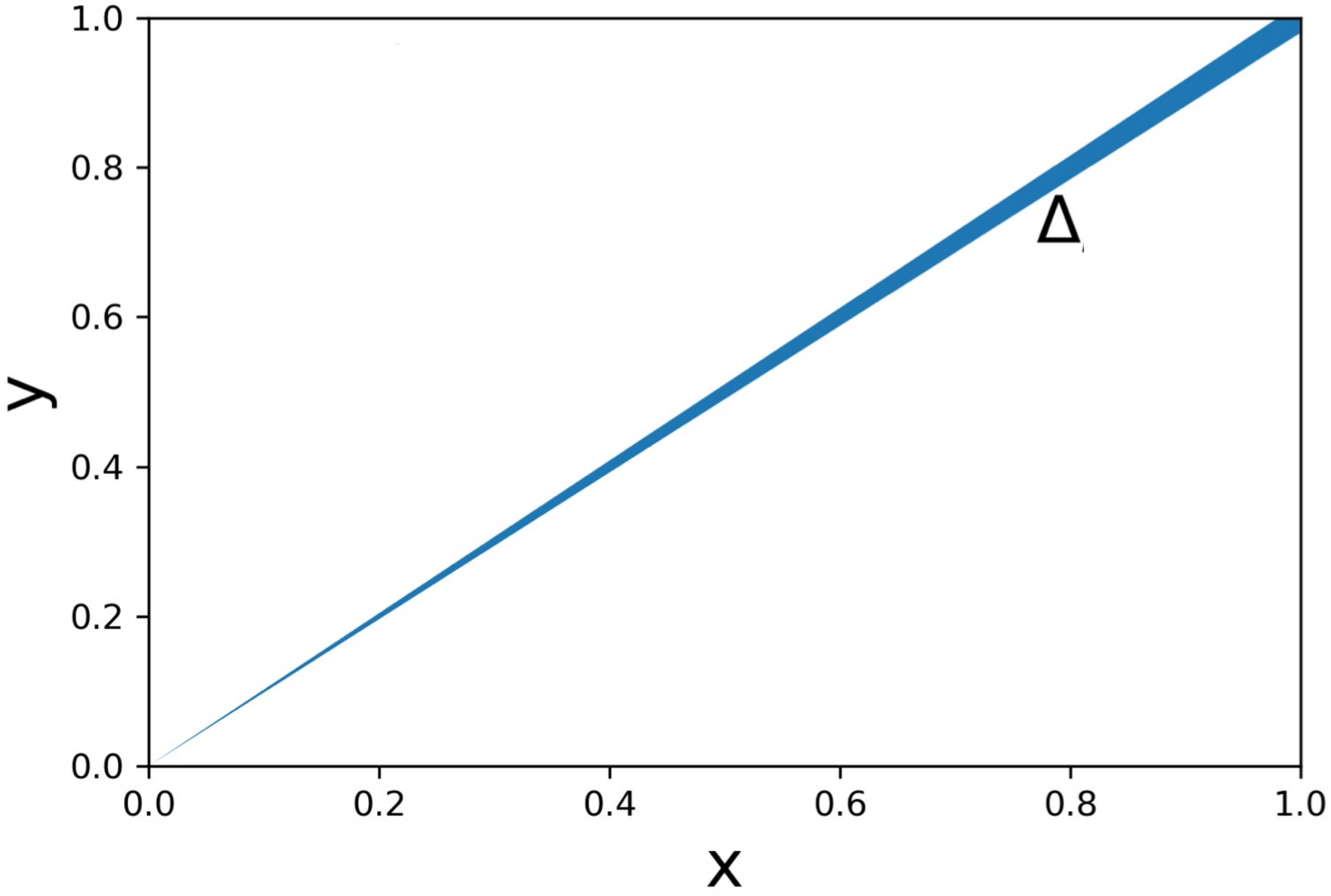
But the Δ box is not the only tiny volume in the (x,y) plane that I can draw.

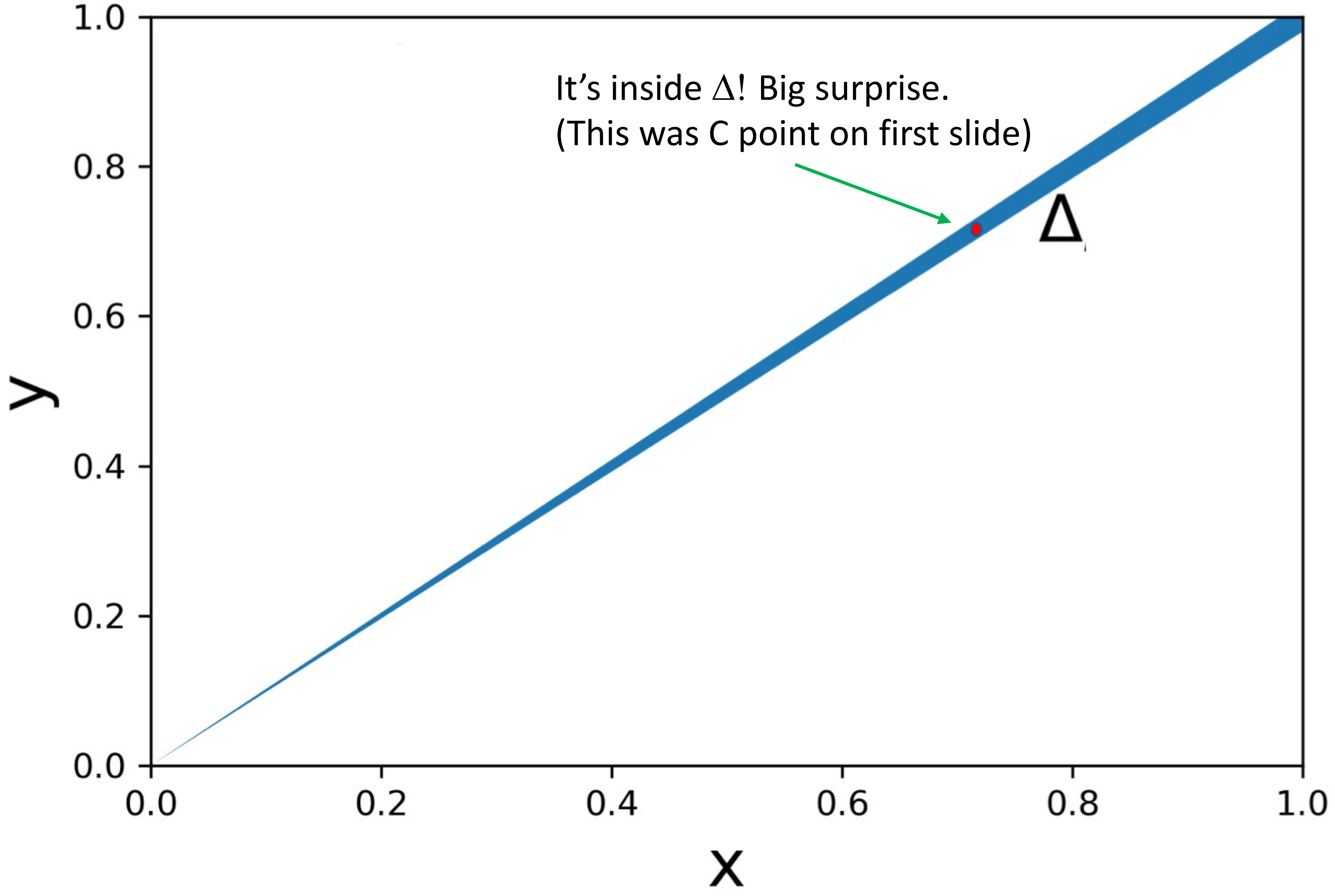
Consider the "flaring thin diagonal", which is also small volume.



I would not bet much that a random choice of (x,y) would fall into the blue Δ volume.

Now imagine making a bet, and then asking someone to randomly choose (x,y)

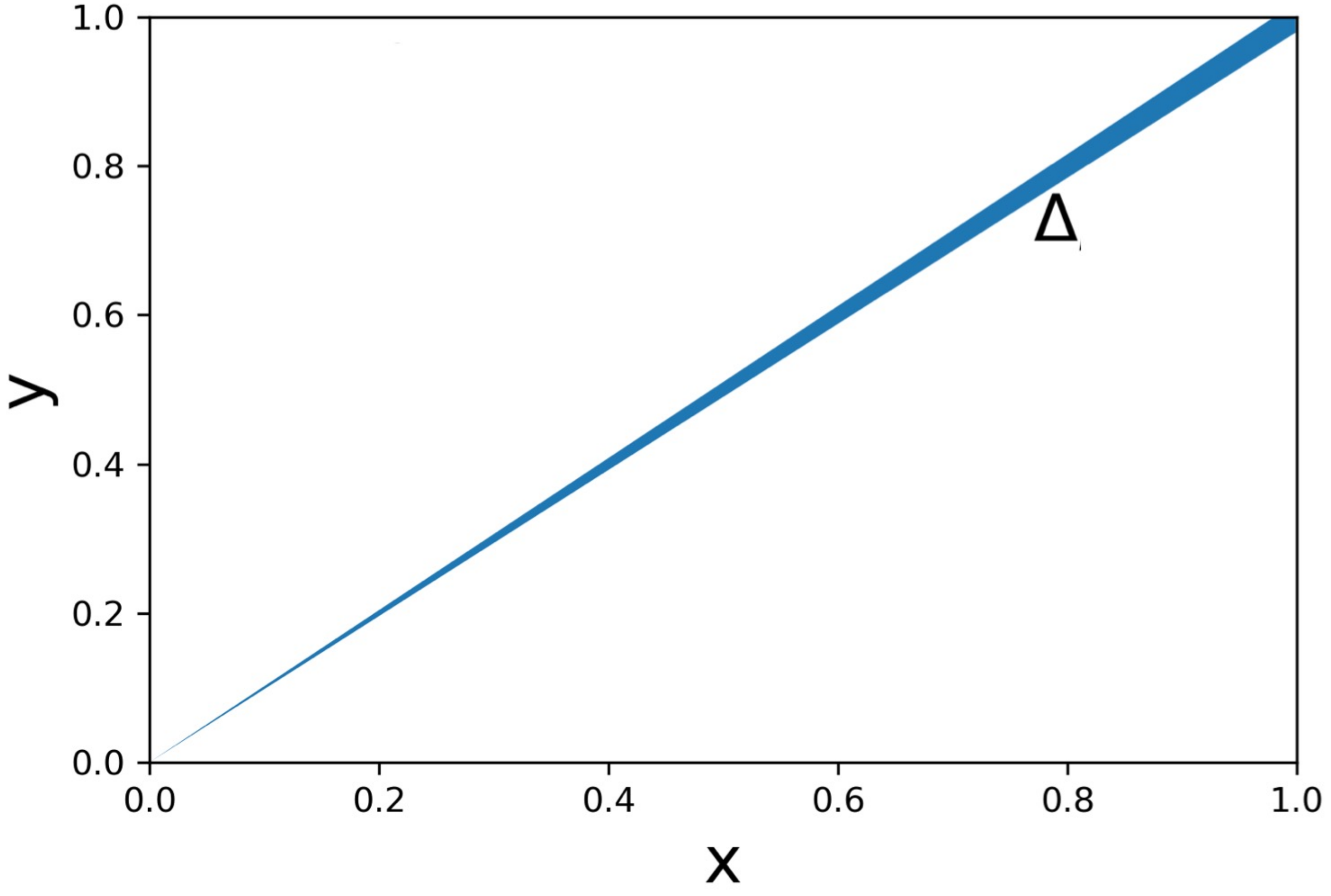




It’s inside Δ! Big surprise.
(This was C point on first slide)
Δ
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x

What does this have to do with physics? Finetuning considerations.

The blue flare region is the region of “high finetuning of Z” in Z = X – Y.



In general, consider

$$\xi = \xi(x_1, x_2, \ldots, x_n)$$

Finetuning is defined as

$$FT = \sum_{i=1}^{n} \left| \frac{x_i}{\xi} \frac{\partial \xi}{\partial x_i} \right| .$$

FT is evaluated for given values of x_1, x_2, …, x_n.

For Z = X – Y ($z=\xi$, $x=x_1$, $y=x_2$)

$$FT = \left| \frac{x}{z} \frac{\partial z}{\partial x} \right| + \left| \frac{y}{z} \frac{\partial z}{\partial y} \right| = \frac{x+y}{|x-y|}$$

The Δ region of FT > 10^2 is in blue.

$$V_{\Delta_{\mathrm{FT}}} = \frac{2}{\mathrm{FT}+1}$$

Many physics eqs. Map to Z = X – Y.

If I a priori define a large FT region in the (X,Y) space, I can entertain bets on whether nature’s choice lands in the tiny blue region (i.e., high FT region).

We will do examples.

But keep in mind: NOTHING IS WRONG WITH A HIGH FINETUNED THEORY.
I just wouldn’t bet on it.

Let me do one example first: the singlet Higgs added to the SM

Then I want to make grander, general statements about Naturalness and the Hierarchy problem, etc.

And then talk about more theories, including the SM itself.

One of the simplest ways to extend the SM is to add a real singlet scalar σ to the spectrum. One can call this theory SM+σ for short. The lagrangian is

$$\mathcal{L}_{SM+\sigma} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2 \sigma^2 - \frac{\eta_\sigma}{2} H^\dagger H \sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 \tag{12}$$

Let us suppose that the mass of the σ-particle is higher than the masses of the other particles in the spectrum, and let's also call the effective theory that includes the σ particle $\mathcal{L}_{\sigma+} = \mathcal{L}_{SM+\sigma}$.

Given the high mass of the σ particle we can integrate it out and are left with a low energy lagrangian $\mathcal{L}_{\sigma-}$ below the σ-mass threshold which is the SM lagrangian plus many higher dimensional operators, such as $\mathcal{O}_6 = |H|^6$. After some analysis we can see that no operator in $\mathcal{L}_{\sigma-}$ suffers from a finetuning of matching across the m_σ threshold except possibly the coefficient m^2 of the operator $|H|^2$. In that case the matching is

$$m^2_{(-)} = m^2_{(+)} - \frac{\eta_\sigma m_\sigma^2}{16\pi^2}\left[1 - \ln\left(\frac{m_\sigma^2}{\mu^2}\right)\right] \tag{13}$$

where for clarity we have defined

$$m^2_{(\pm)} = m^2 \text{ evaluated at } q^2 = m_\sigma^2(1 \pm \epsilon),\ \text{where } \epsilon \ll 1. \tag{14}$$

In other words $m^2_{(-)}$ is the coefficient of $|H|^2$ in the low-energy effective theory just below the m_σ threshold after the σ-particle has been integrated out, and $m^2_{(+)}$ is the coefficient of $|H|^2$ in the high-energy theory above the m_σ threshold that includes the σ particle.

$$\begin{aligned}
\mathrm{FT}[m^2] &= \mathrm{FT}[m^2_{(-)}\,|\,m^2_{(+)}] = \left|\frac{m^2_{(+)}}{m^2_{(-)}}\frac{\partial m^2_{(-)}}{\partial m^2_{(+)}}\right| = \left|\frac{m^2_{(+)}}{m^2_{(-)}}\right|_{\mu^2=m^2_\sigma} = \left|1-\frac{\eta_\sigma m^2_\sigma}{8\pi^2 m^2_h}\right| \\
&\simeq \frac{\eta_\sigma m^2_\sigma}{8\pi^2 m^2_h} \quad (\text{for large } m^2_\sigma)
\end{aligned}$$

Thus, there is a large finetuning of EFT matching at this threshold if m_σ is large.

For $\eta = 1$ the finetuning is FT > 10^3 (level-3 finetuning) if m_σ > 36 TeV.

I would bet against arbitrarily massive singlets that couple to the SM Higgs, unless there is a new principle at play!

- Supersymmetry
- ”separation mechanism” that enforces $\eta \ll 1$
- …

General principles I wish to advocate:

- FT computations across EFT thresholds is an *a priori* well-defined algorithm for determining tiny volumes Δ (high FT) that I would generally bet against.

- If from our perspective a speculative theory is viable only if it has very high finetuning (level 4 or higher, say) then the theory is likely to be wrong or there is a deeper idea that is yet to be invoked or discovered that explains the high FT.

- Finetuning is uniquely interesting method to determine a priori tiny volumes since its probability interpretation is largely independent of the range of values the underlying parameters can gave. This is certainly true for Z = X – Y, flat prior model.

$$V_{\Delta_{\rm FT}} = \frac{2}{{\rm FT}+1} = P$$

To be a little more precise

Endo-Natural theory: An endo-Natural theory is one where the finetunings are not high (all are, say, level-4 or lower[4]) across all its particle thresholds when matching EFTs above and below the thresholds, according to the *a priori* defined algorithms of assessment discussed above.

Exo-Natural theory: A theory may have large finetuning across threshold(s), but those finetunings are explained in principle and are not accidental. This case has no implication of low probability despite its large finetuning, and we call the corresponding theory exo-Natural.

Wilsonian Natural theory: A Wilsonian Natural theory is one that is either endo-Natural or exo-Natural.

Exo-Natural theory possibilities: Landscape + anthropics kind of approaches, or theories with large UV/IR correlations. (Note both of these categories evoke new principles)

Wilsonian Naturalness conjecture: Large accidental finetunings in EFT matching across a particle threshold is highly improbable. Any such finetuning that may occur should be pursued as a sign for the existence of new particle(s) or principle(s) that render the large finetuning as a non-accidental result (i.e., it is secretly an exo-Natural theory). Furthermore, any conjectured theory that relies on large, unexplained accidental finetuning in EFT matching across particle threshold(s) is unlikely to be a good description of nature. Summary: Wilsonian Naturalness is expected to be satisfied by the next useful theory of nature beyond the Standard Model[5].

<u>Very important</u>: There may be other definitions of naturalness that are useful to demote theories from having otherwise high status. Agnostic to that.
But ideas must give argument for WHY such and such property is bad. Only way to do that is connect to probability/likelihood. Without that, I'm uninterested.

The Standard Model is a Wilsonian Natural Theory

Statements abound in the literature that "the SM suffers from the Naturalness problem", or more or less equivalently, the hierarchy problem and the finetuning problem [11]. We need to find a theory that "cures the SM's naturalness problem" is another common refrain. However, there is no place for such talk when it comes to a highly successful theory like the SM. If some dreamt-up criteria ends up labeling the SM with a Naturalness problem, then I want every other theory I come up with to also have a Naturalness problem just like it.

$$m_H^2 = m_{\rm bare}^2 + \frac{y_t^2}{16\pi^2}\Lambda^2 + \delta\mathcal{O}(m_{\rm weak}^2)$$

Equations like the above have no obvious connection to probability.

Perhaps the best opportunity for the finetuning to have manifest itself is across the top quark threshold. The matching of the m^2 above and below the top mass after electroweak symmetry breaking requires us to inspect the m_h^2 coefficient of $\frac{m_h^2}{2}h^2$ operator above and below the top mass. One finds the leading term to be

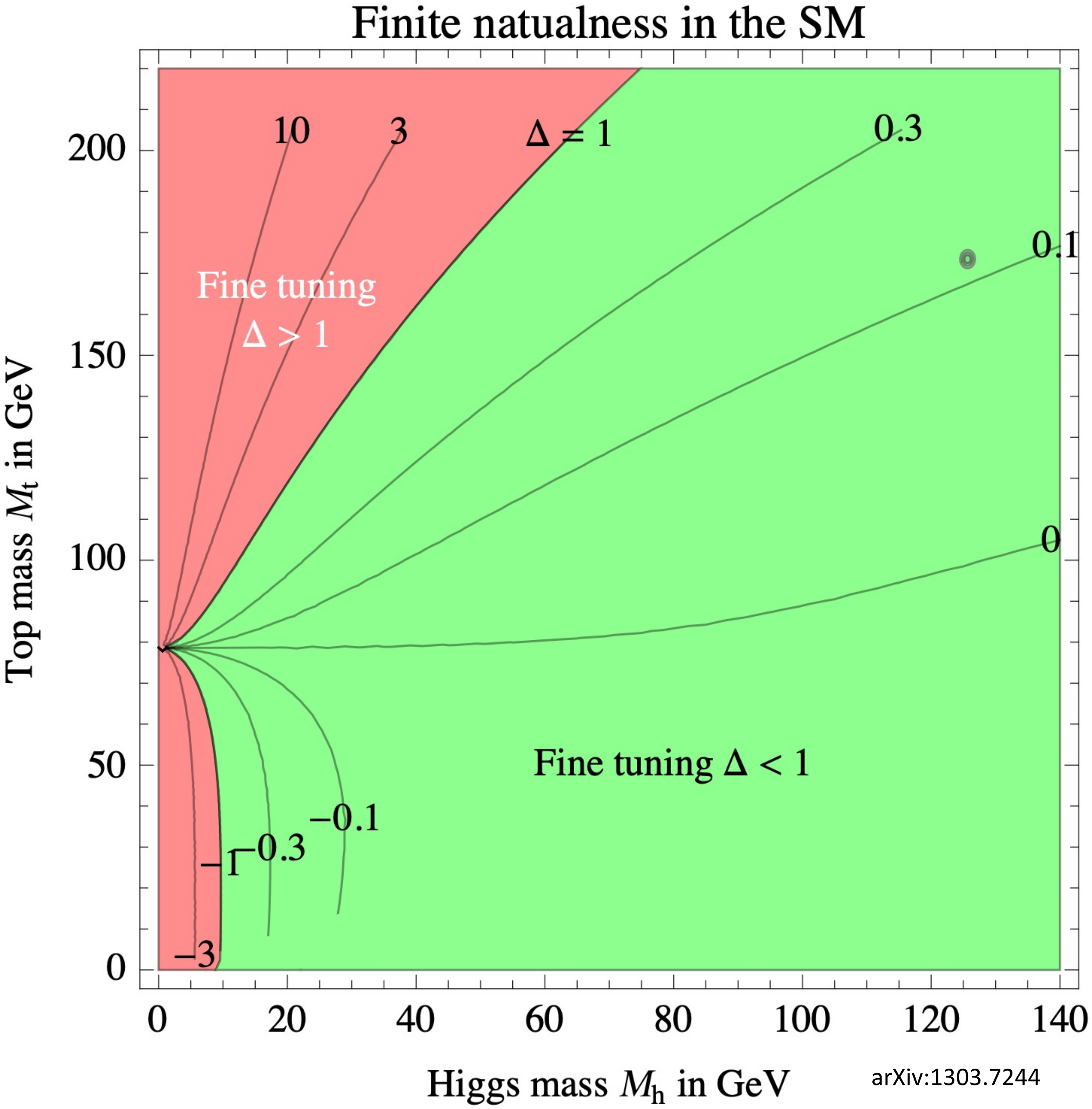
$$m_h^2(m_t)_L \;=\; m_h^2(m_t)_H + \frac{3m_t^4}{4\pi^2 v^2} + \mathcal{O}(y_t^2 m_h^2) \tag{10}$$

where $v \simeq 246\,\mathrm{GeV}$ and y_t is the top-quark Yukawa coupling.

We can compute the finetuning across the m_t threshold and we find

$$\mathrm{FT}[m_h^2|m_t] \simeq \frac{3m_t^4}{\pi^2 v^2 m_h^2} \tag{11}$$

Inserting $m_t = 173\,\mathrm{GeV}$ and $m_h = 125\,\mathrm{GeV}$ into this equation one finds $\mathrm{FT} = 0.3$ which is $\mathcal{O}(1)$ as we expect most finetunings to be across thresholds. This is a low finetuning that is consistent with a Natural theory.



arXiv:1303.7244

Algorithms and definitions are different but result matches intuitions of Farina et al.'s finite naturalness discussion.

Although the SM has no Naturalness problem or finetuning problem, it does have a **hierarchy problem**.

Hierarchy problem: a theory has a hierarchy problem if it immediately develops a Naturalness problem if generically expected new states (e.g., heavy singlet scalars) are added to it in standard ways that the theory would otherwise see no problem in doing.

As we saw before, adding a few massive singlets to the SM immediately creates a theory that has a Naturalness problem. No reason not to expect them.

→ SM has a hierarchy problem

Example: Supersymmetry

With these considerations we first write the theory above the heavy Higgs doublet threshold [14]:

$$
\begin{aligned}
V(H_u, H_d) &= (|\mu|^2 + m_{H_u}^2)|H_u|^2 + (|\mu|^2 + m_{H_d}^2)|H_d|^2 - bH_u \cdot H_d + \text{c.c.} \\
&\quad + \frac{1}{8} g'^2 \left(|H_u|^2 - |H_d|^2\right)^2 + \frac{1}{8} g^2 \left(H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d\right)^2
\end{aligned}
\tag{18}
$$

This then needs to be matched to the theory below the heavy Higgs doublet threshold

$$
V(H) = m^2|H|^2 + \lambda|H|^4 \tag{19}
$$

After some manipulations one finds

$$
\begin{aligned}
m^2 &= -\left(\frac{1-\sin^2 2\beta}{2}\right)\left[\frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1-\sin^2 2\beta}} - m_{H_d}^2 - m_{H_u}^2 - 2|\mu|^2\right] \qquad (20)\\
\lambda &= \frac{1}{8}\left(g'^2 + g^2\right)\left(1-\sin^2 2\beta\right) \qquad (21)
\end{aligned}
$$

where

$$
\sin 2\beta = \frac{2b}{m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2}. \qquad (22)
$$

Given this definition of the angle β, in the limit of large superpartner mass scale we can identify the light SM Higgs boson H and the heavy decoupled doublet state Φ with

$$
\begin{aligned}
H &= \cos\beta H_d + \sin\beta \overline{H}_u \qquad (23)\\
\Phi &= -\sin\beta H_d + \cos\beta \overline{H}_u. \qquad (24)
\end{aligned}
$$

where $\overline{H}_u = i\sigma^2 H_u^*$.

We are left with the below-threshold lagrangian of eq. 19 and its matching equations (eqs. 20 and 21) entirely in terms of the parameters of the above-threshold theory once the dependence of that angle β on the supersymmetry parameters are substituted (eq. 22). If we identify the heavy supersymmetry masses as the collection of $\tilde{m}_k^2 = \{m_{H_u}^2, m_{H_d}^2, |\mu|^2, b\}$ that have typical scale values of $\Lambda_{\rm susy}$, one can see readily from eq. 20 that

$$\mathrm{FT}[m^2] = \max_k \left| \frac{\tilde{m}_k^2}{m^2} \frac{\partial m^2}{\partial \tilde{m}_k^2} \right| \sim \frac{\Lambda_{\rm susy}^2}{m_Z^2}. \tag{25}$$

The precise values of the FT depends on the exact choices of parameters but it is generic that the result is as shown, $\mathrm{FT} \sim \Lambda_{\rm susy}^2/m_Z^2$.

Level-3 (level-4) finetuning suggests that superpartners should be below ~ few TeV (few tens of TeV) if standard susy is correct approach.

This scaling of finetuning matches the intuitions that have been present in the supersymmetry community for quite some time now. Traditionally the calculation was to check on the finetuning of the small value of m_Z^2 given all the heavy superpartner masses in the scalar potential. The equation for m_Z^2 for electroweak symmetry breaking at leading order is

$$m_Z^2 = -2|\mu|^2 + \frac{2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)}{\tan^2\beta - 1} \tag{26}$$

Finetunings are then computed and the result is generically $\mathrm{FT} \sim \Lambda_{\mathrm{susy}}^2/m_Z^2$. So, although the $\mathcal{O}(1)$ factors will be different between our algorithm for computing the threshold finetuning and the finetuning computed from considering superpartner mass dependences on m_Z, the results are the same within $\mathcal{O}(1)$ factors. The main reason for this is that at tree-level $m^2 = -\frac{1}{2}m_Z^2\cos^2 2\beta$, and so computations of finetuning on $\tilde{m}^2$ should be very similar to that of m_Z^2.

Example: Doublet-triplet splitting in GUTs (this approach reproduces intuitions)

Another example of an improbability of parameter cancellations that has been discussed in the literature for years is the so-called doublet-triplet splitting problem in grand unified theories [16, 17]. For example, minimal $SU(5)$ theory breaks down to the SM gauge groups via the condensation of the 24 dimensional representation Σ. The vacuum expectation value of this field is

$$\langle \Sigma \rangle = v_\Sigma \cdot \mathrm{diag}(2, 2, 2, -3, 3) \tag{28}$$

where the value of the vev v_Σ is determined by parameters $\vec{w}$ in GUT-scale Higgs potential: $v_\Sigma = v_\Sigma(\vec{w})$.

In the supersymmetric case the Σ also couples to the 5- and $\bar{5}$-dimensional Higgs representation H_5 and $H_{\bar{5}}$ respectively. Within the $H_{5,\bar{5}}$ are the Higgs doublets $H_{u,d}$ and the Higgs triplet $H_{3,\bar{3}}$ representations. The relevant GUT-scale superpotential for H_5 is

$$W_{(+)} = \mu_5 H_{\bar{5}} H_5 + \lambda H_{\bar{5}} \Sigma H_5 \tag{29}$$

After symmetry breaking the superpotential splits the $H_{5,\bar{5}}$ into $H + u, d, 3, \bar{3}$ terms:

$$W = \mu_3 H_{\bar{3}} H_3 + \mu H_u H_d \Longrightarrow W_{(-)} = \mu H_u H_d + \cdots \tag{30}$$

where

$$\mu_3 \quad = \quad \mu_5 + 2\lambda v_\Sigma, \quad \text{and} \tag{31}$$

$$\mu \quad = \quad \mu_5 - 3\lambda v_\Sigma. \tag{32}$$

We know that $v_\Sigma \simeq 10^{16}\,\mathrm{GeV}$ for the unification of couples, and we also know that μ needs to be $10^{2-3}\,\mathrm{GeV}$ for weak scale supersymmetry. Thus, there is an extraordinary finetuning in the cancellation that must occur in eq. 32 to realize these constraints. Upon symmetry breaking and assessing the finetuning of μ with respect to the high-scale theory parameter μ_5 one finds

$$\mathrm{FT}[\mu] = \left|\frac{\mu_5}{\mu}\frac{\partial\mu}{\partial\mu_5}\right| = \left|\frac{\mu_5}{\mu}\right| \simeq \left|\frac{2\lambda v_\Sigma}{\mu}\right| \sim 10^{13} \tag{33}$$

Thus, minimal supersymmetric $SU(5)$ GUTs have level-13 finetuning and do not pass their Wilsonian Naturalness test. This is why it is often referred to in the literature as the doublet-triplet splitting problem. It really is simply a Wilsonian Naturalness problem of the theory across matching EFT thresholds.

Conclusions

Naturalness is an extra-empirical assessment on a theory: are there reasons why a theory may be unlikely even though at present it is empirically fine. (E.g. susy is empirically perfectly fine today – LHC has not changed that)

Wilsonian Naturalness is one of (possibly) many ways to assess a theory's naturalness status. It relies on computing finetuning of EFT parameters across particle mass thresholds.

The Standard Model is Wilsonian Natural.

Other theories (supersymmetry, extra singlet theory, etc.) are Wilsonian Natural only in a limited region of parameter space.

I'm willing to bet that the next good theory is Wilsonian Natural.