

Flavor-specific Neutrino Self-interaction in Cosmology

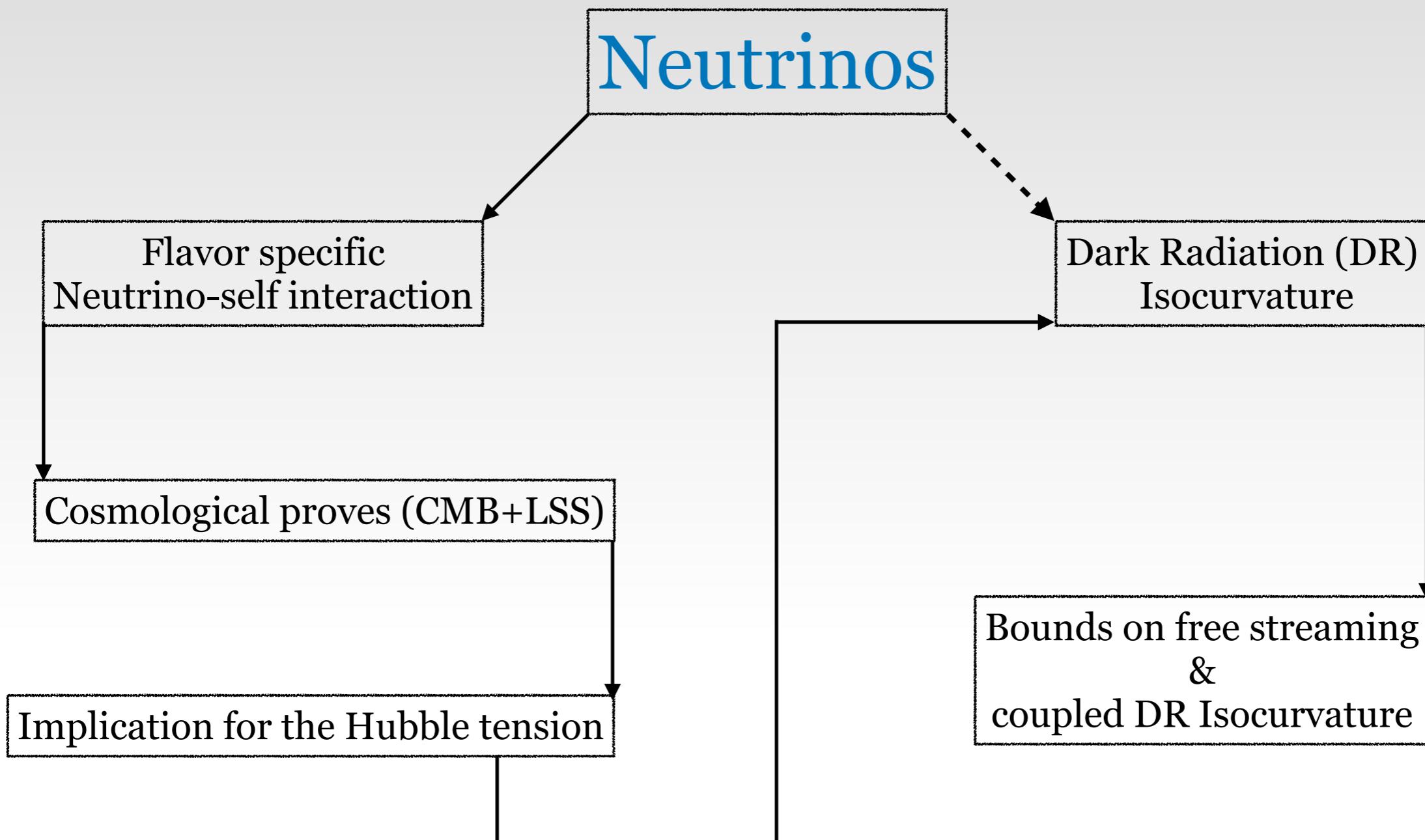
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Theoretical Physics Seminar | Fermilab | 4 Nov, 2021

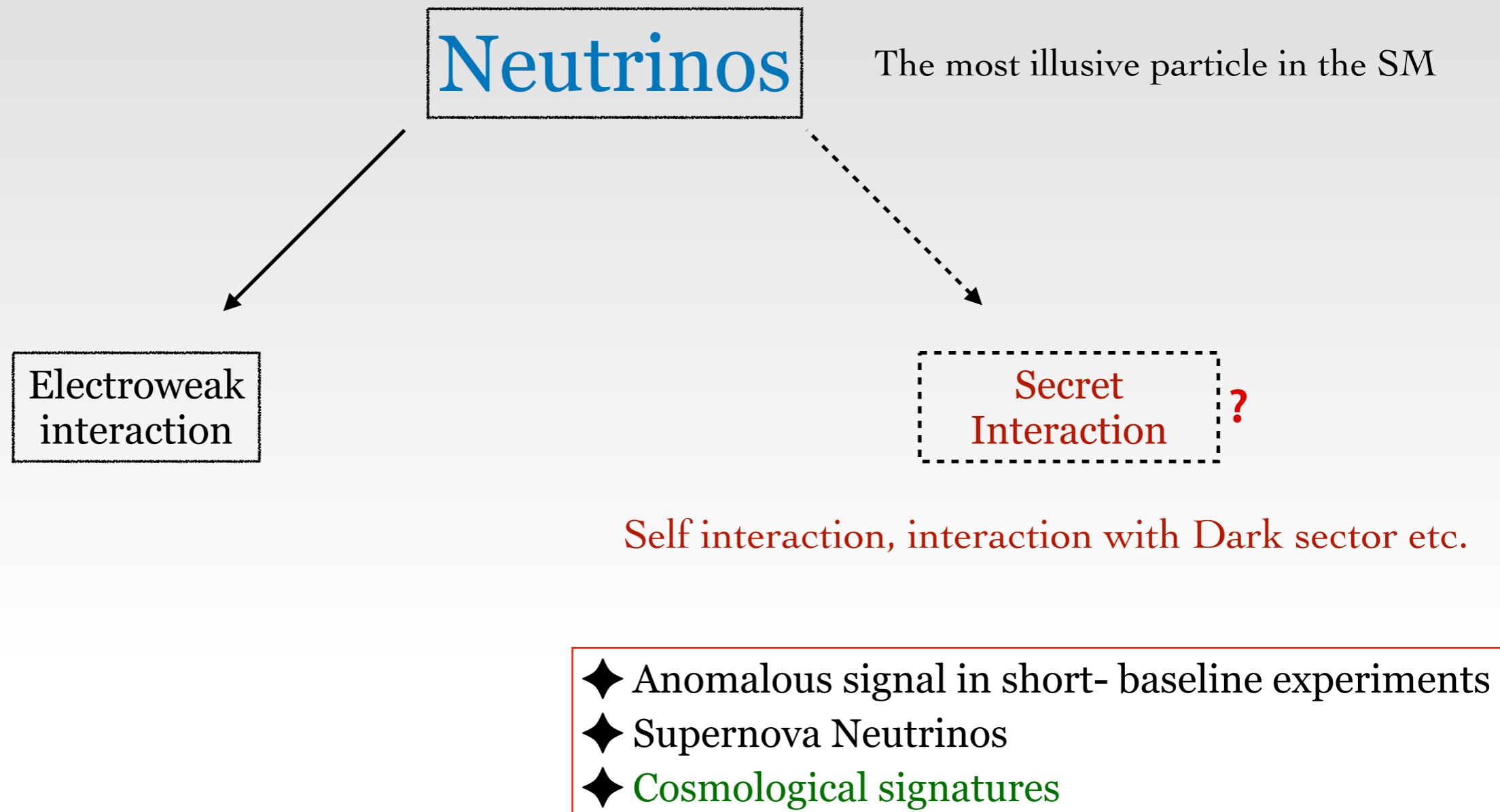
Overview



Part I

Part II

Introduction



Cosmological signatures of Neutrino self interaction

$$\mathcal{L}_{\text{int}} \supset \frac{1}{2} g_{ij} \bar{\nu}_i \nu_j \phi, \quad g_{ij} = g \delta_{ij}$$

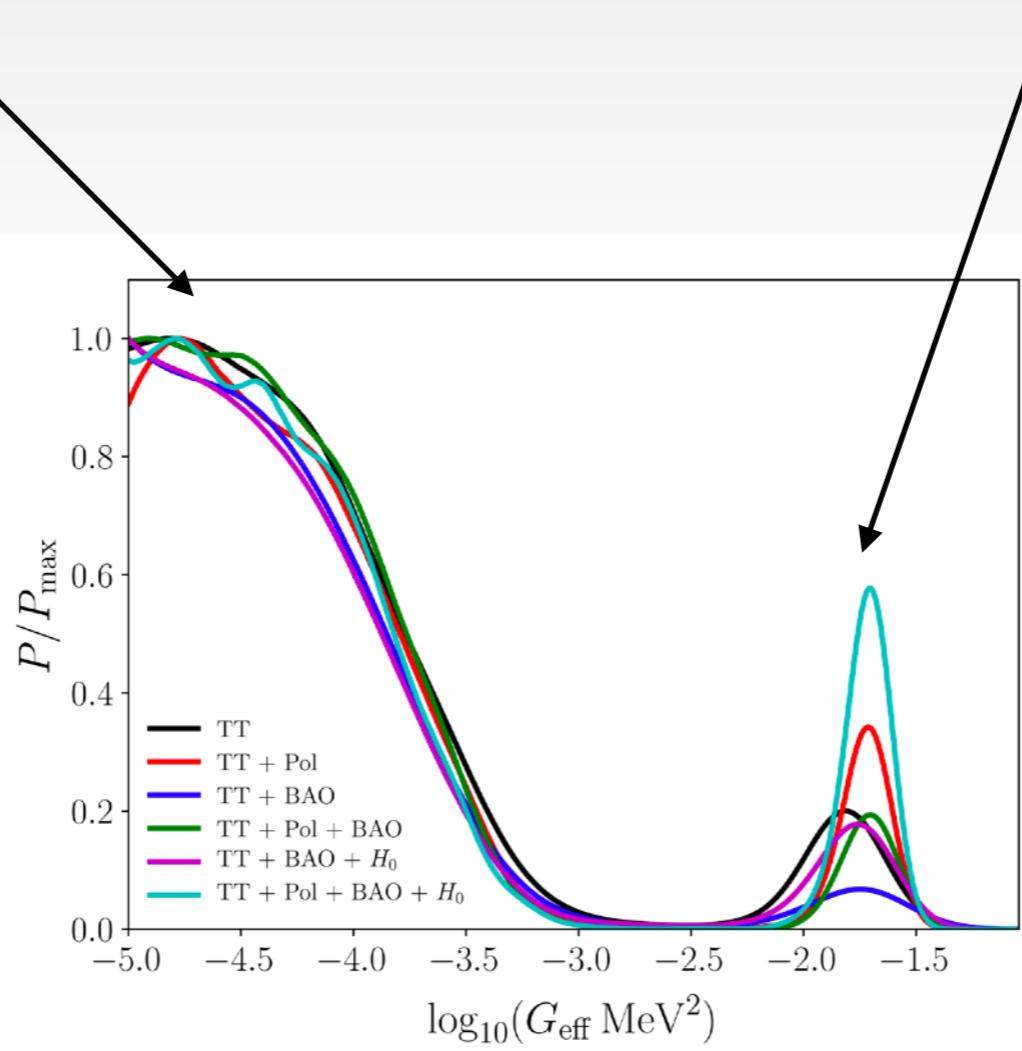
Flavor universal
Self-interaction

(Mediator can be vector)

$$\mathcal{L}_{\text{eff}} = G_{\text{eff}} (\bar{\nu} \nu) (\bar{\nu} \nu), \quad G_{\text{eff}} = \frac{g^2}{m_\phi^2}$$

Moderately interacting
(MI)

Strongly interacting
(SI)

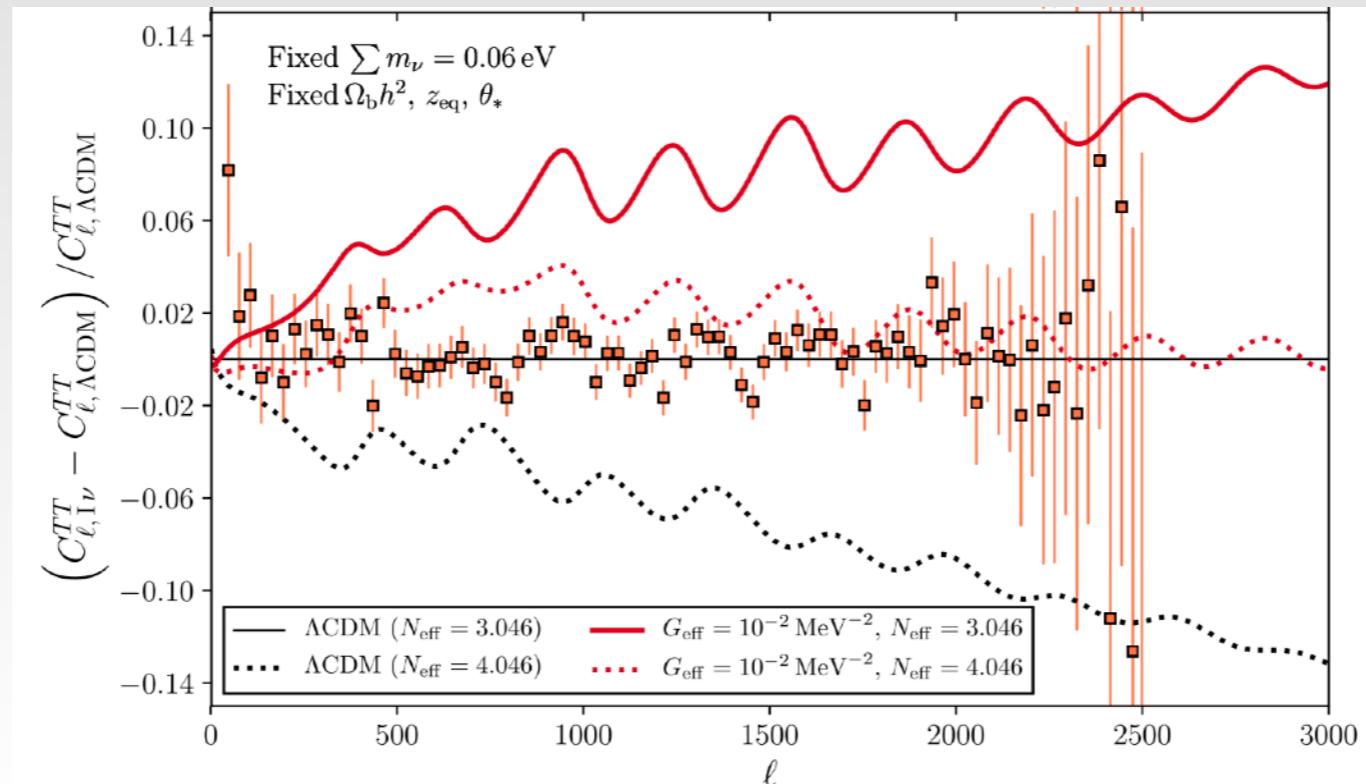


Lancaster et. al. (1704.06657)

(Using Planck 2015 data)

Cosmological signatures of Neutrino self interaction

$$\mathcal{L}_{\text{eff}} = G_{\text{eff}}(\bar{\nu}\nu)(\bar{\nu}\nu), \quad G_{\text{eff}} = \frac{g^2}{m_\phi^2}$$



Kreisch et al (1902.00534)

Degeneracy of G_{eff} with N_{eff} (H_0)

CMB (Planck)
 $H_0 = 67.36 \pm 0.54$

$\sim 4.4\sigma$

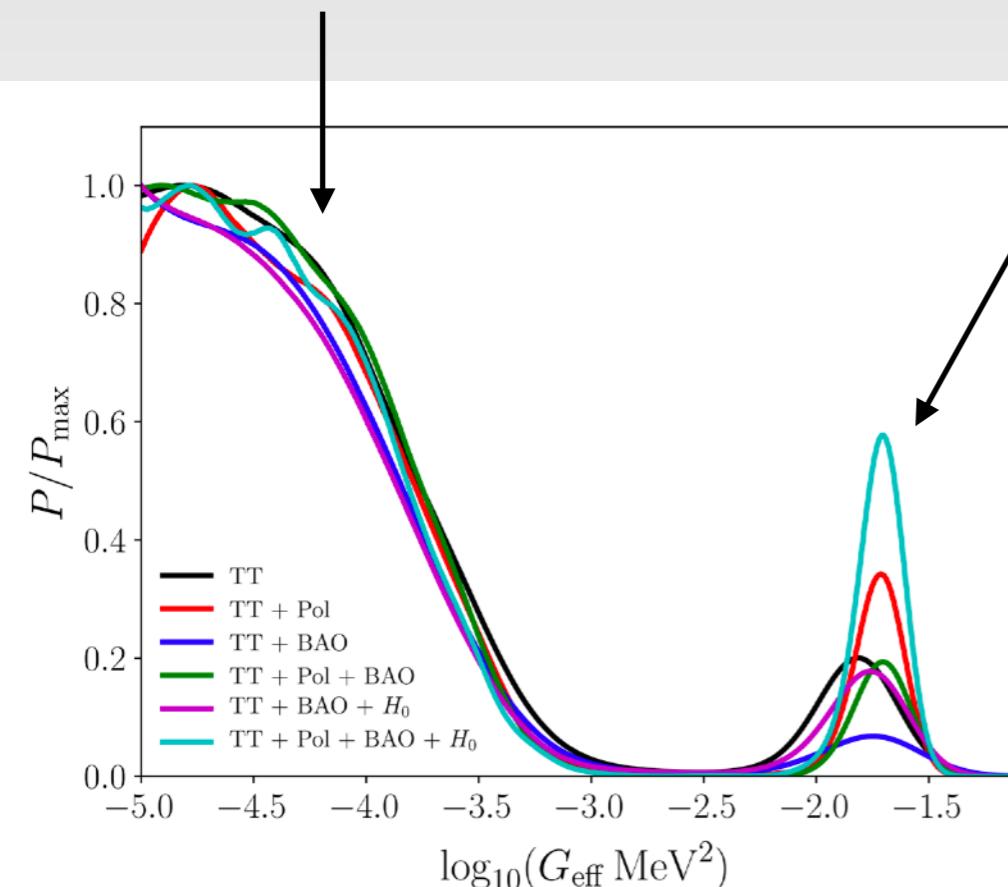
Local measurement (Reiss et. al.)
 $H_0 = 74.03 \pm 1.42$

Proposed as a solution (?) of Hubble tension

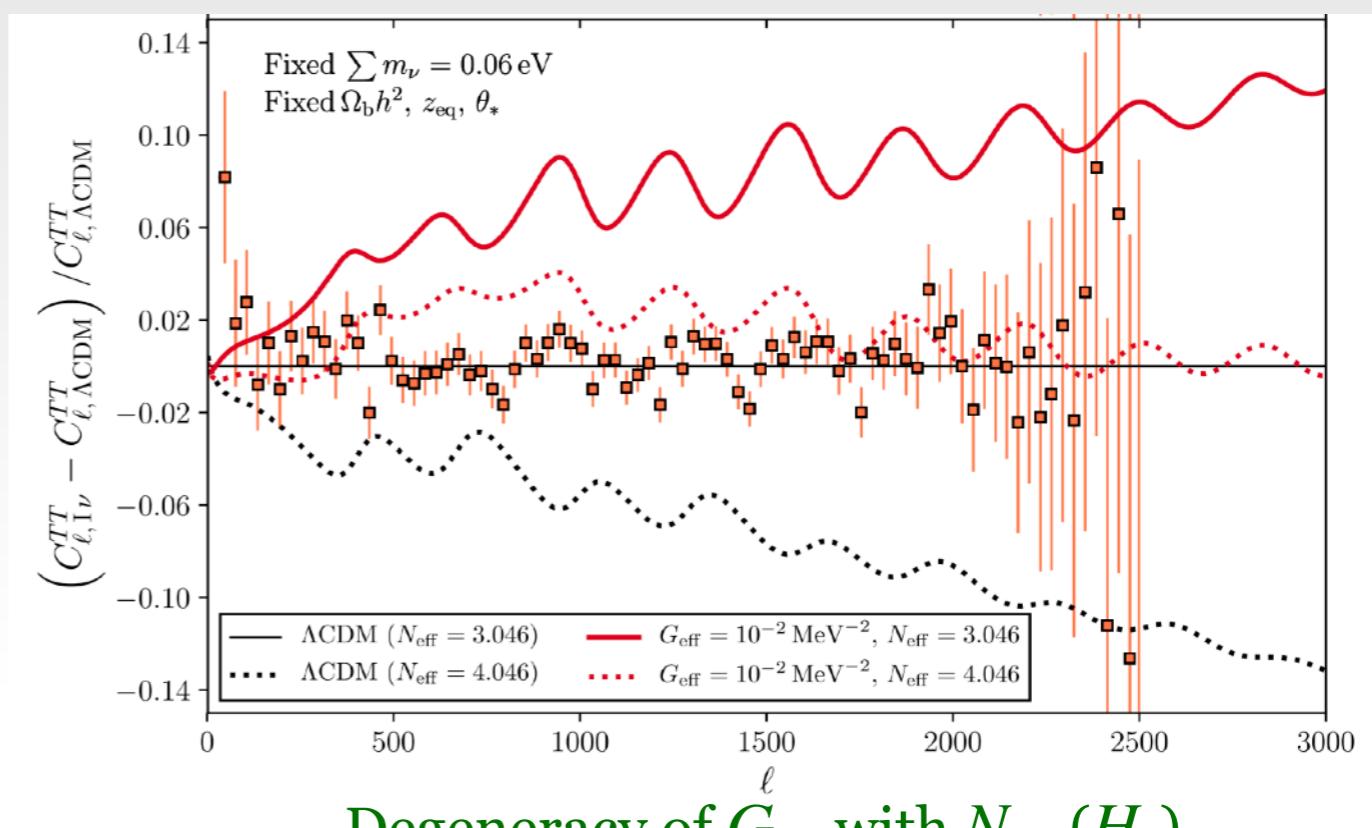
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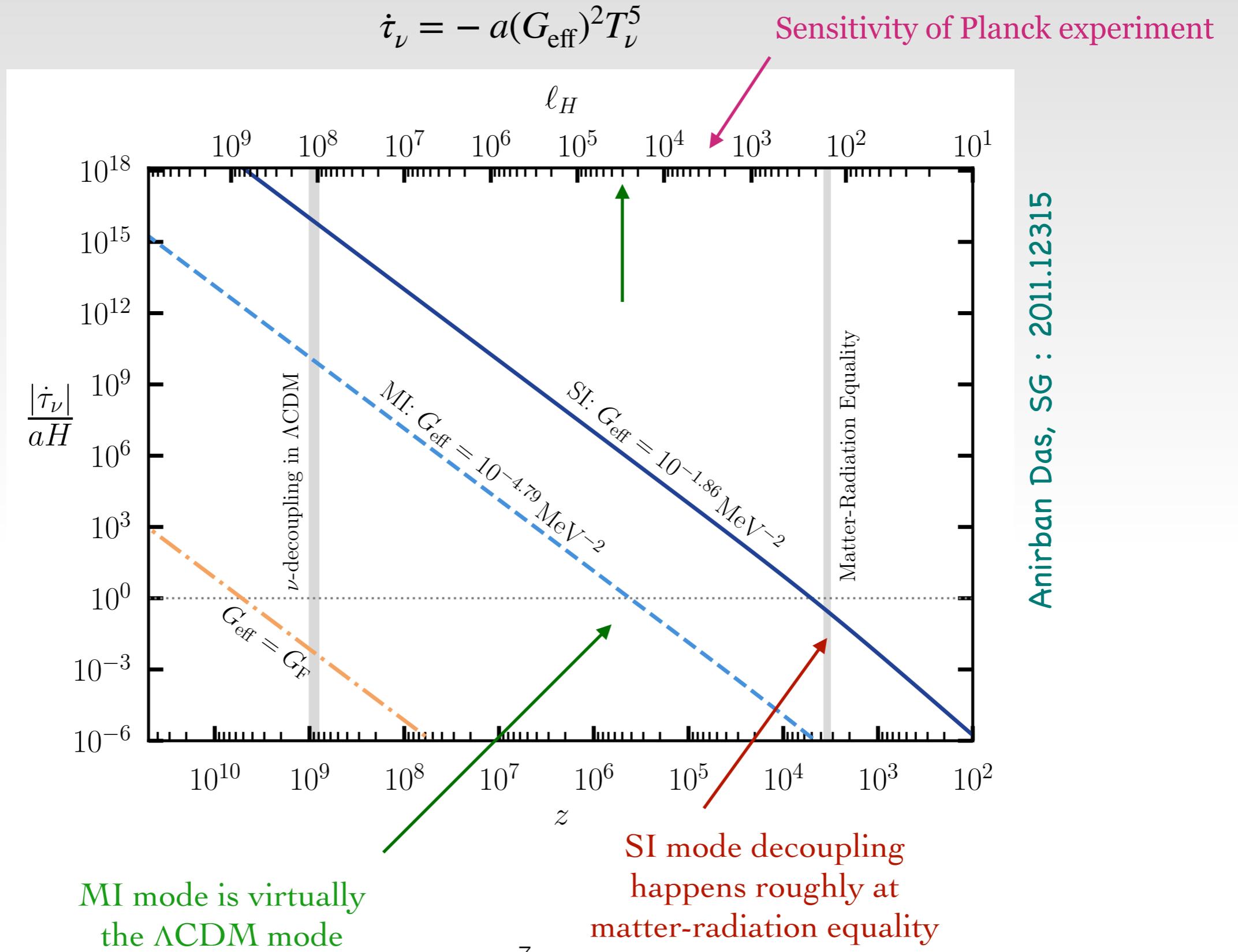
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Proposed as a solution (?) of Hubble tension

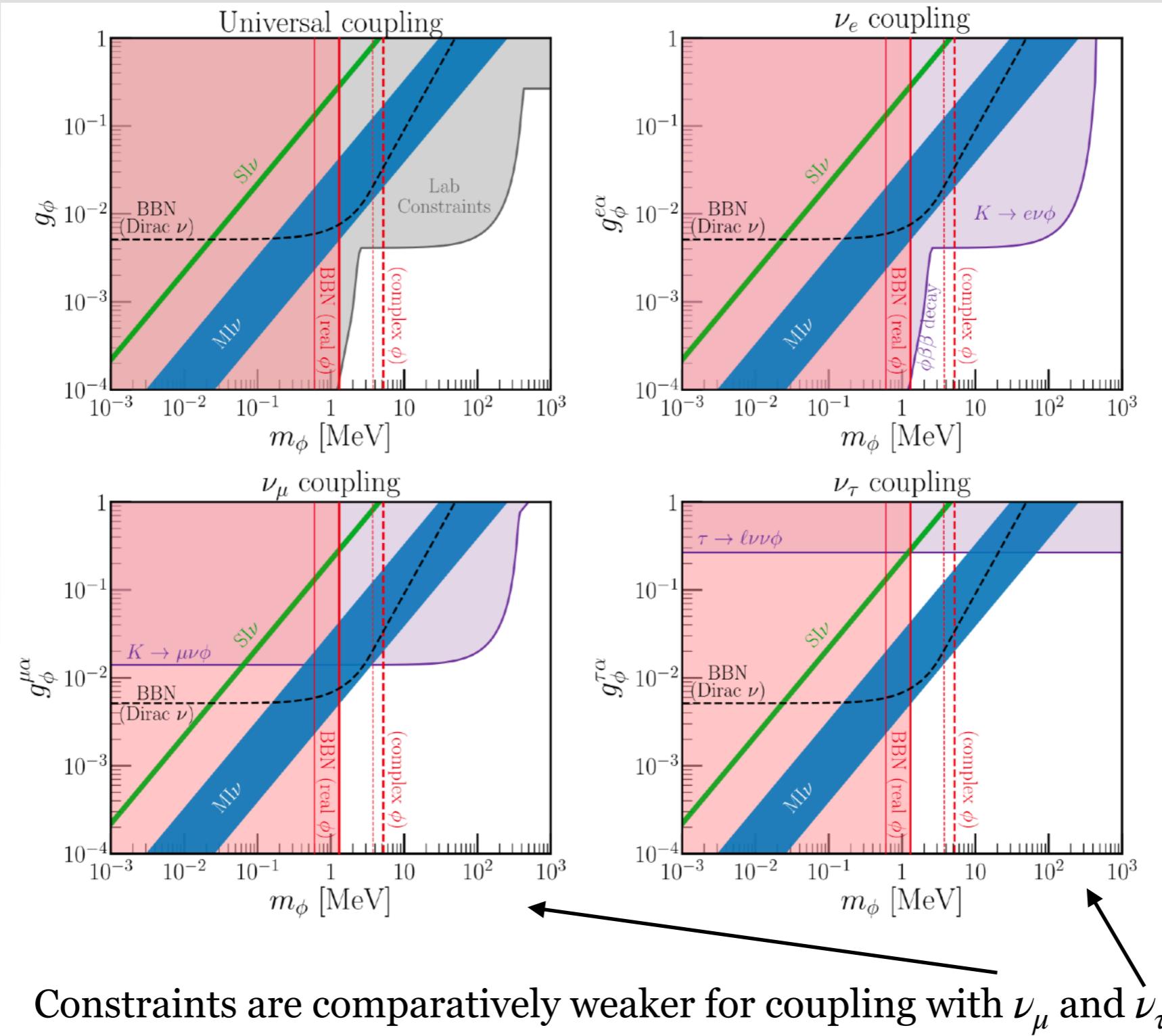
(Doesn't work when CMB polarisation data is included)

Laboratory constraint



Laboratory constraint

Universal coupling is strongly ruled out by laboratory constraints



Blinov et. al., 1905.02727

(Also see,
Lyu et. al., 2004.10868)

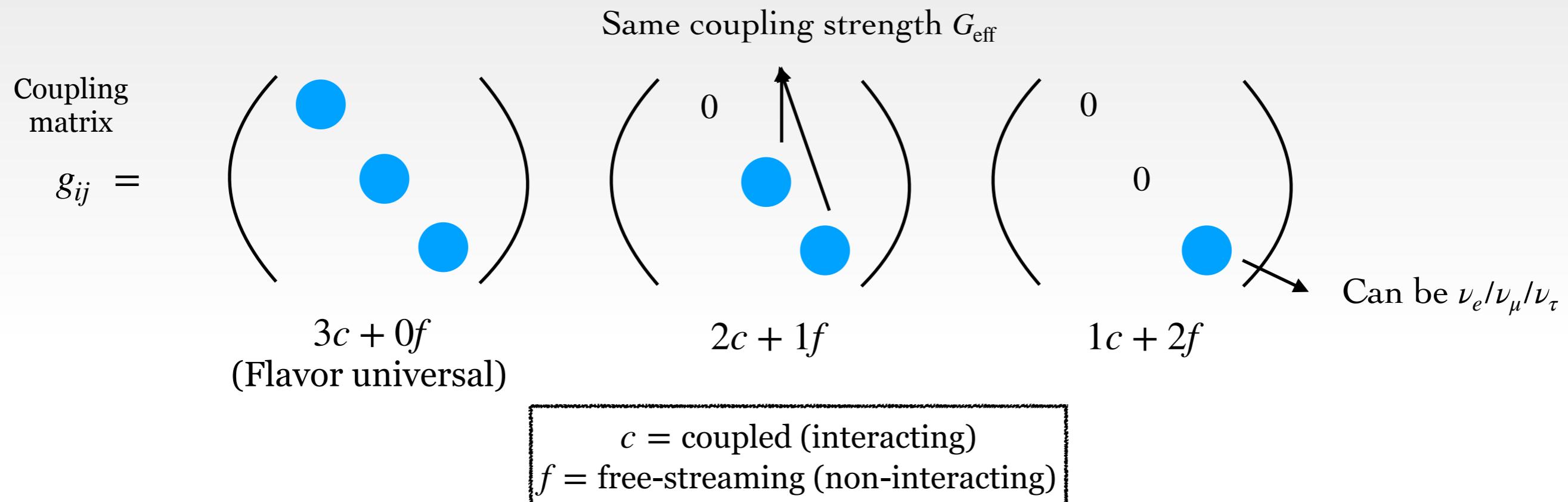
Need for cosmological analysis of Flavor specific neutrino self interaction

Flavor Specific Self Interaction

Flavor specific neutrino self interaction in cosmology

CMB is insensitive to specific flavor (ν_e, ν_μ, ν_τ) of Neutrino
(- Not sensitive to weak interaction)

CMB is sensitive to flavor specific interaction ‘collectively’ though free-streaming properties



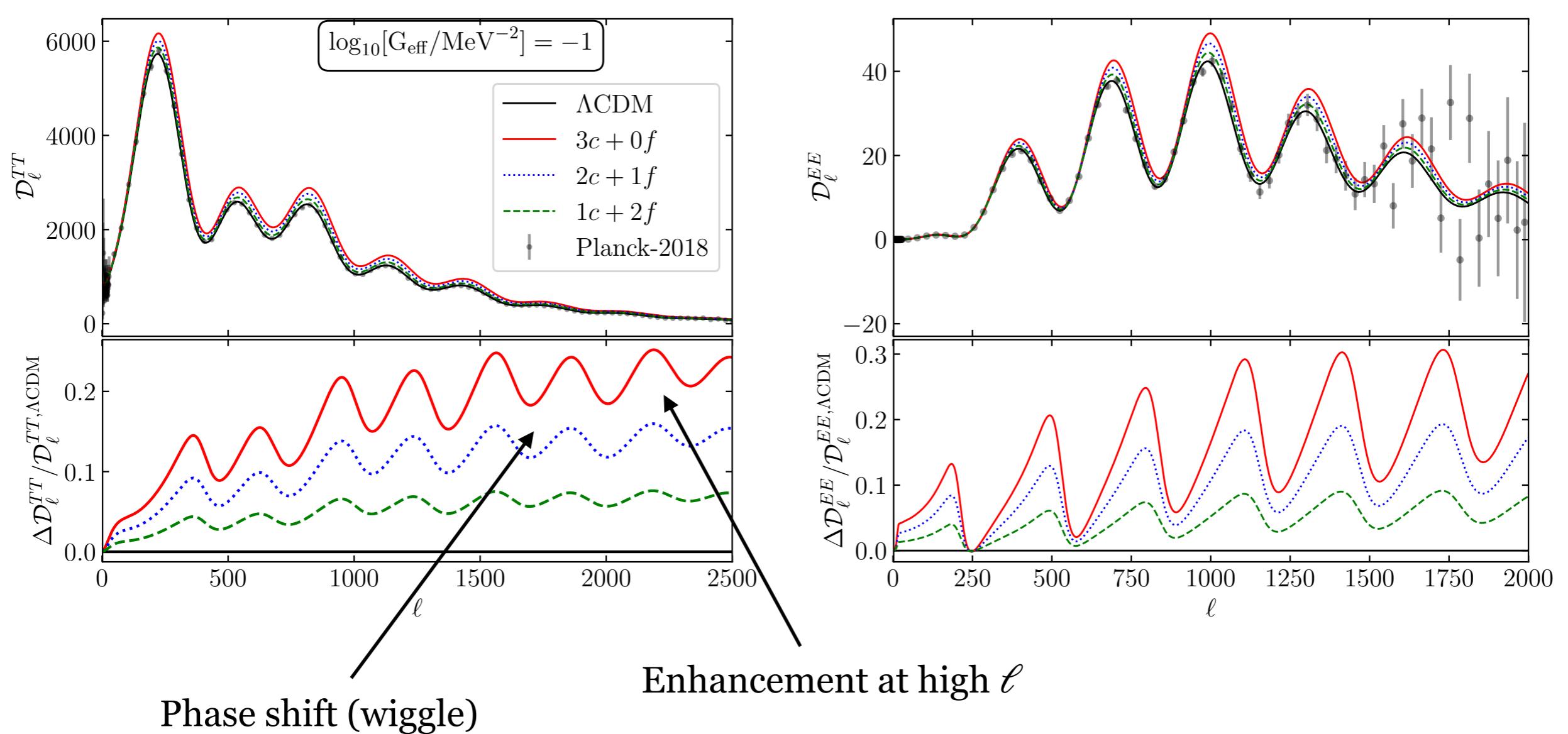
Common coupling strength G_{eff} for coupled flavors (CMB insensitive to specific flavor)

Assumptions

Massless neutrinos
3 flavor ($N_{\text{eff}} = 3.046$)
Flavor diagonal interaction

$\Lambda\text{CDM} \equiv 0c + 3f$

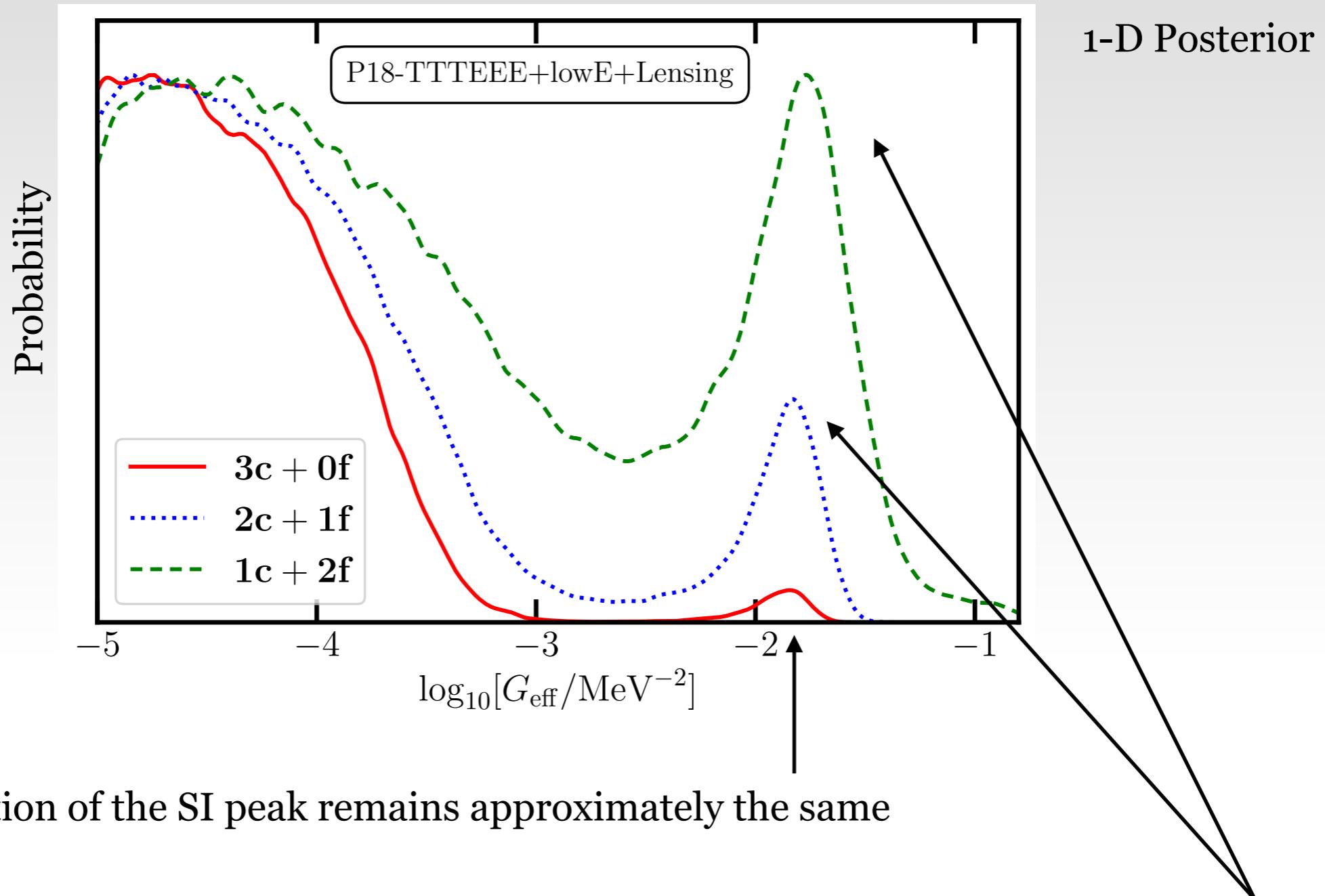
Effect on CMB spectrum



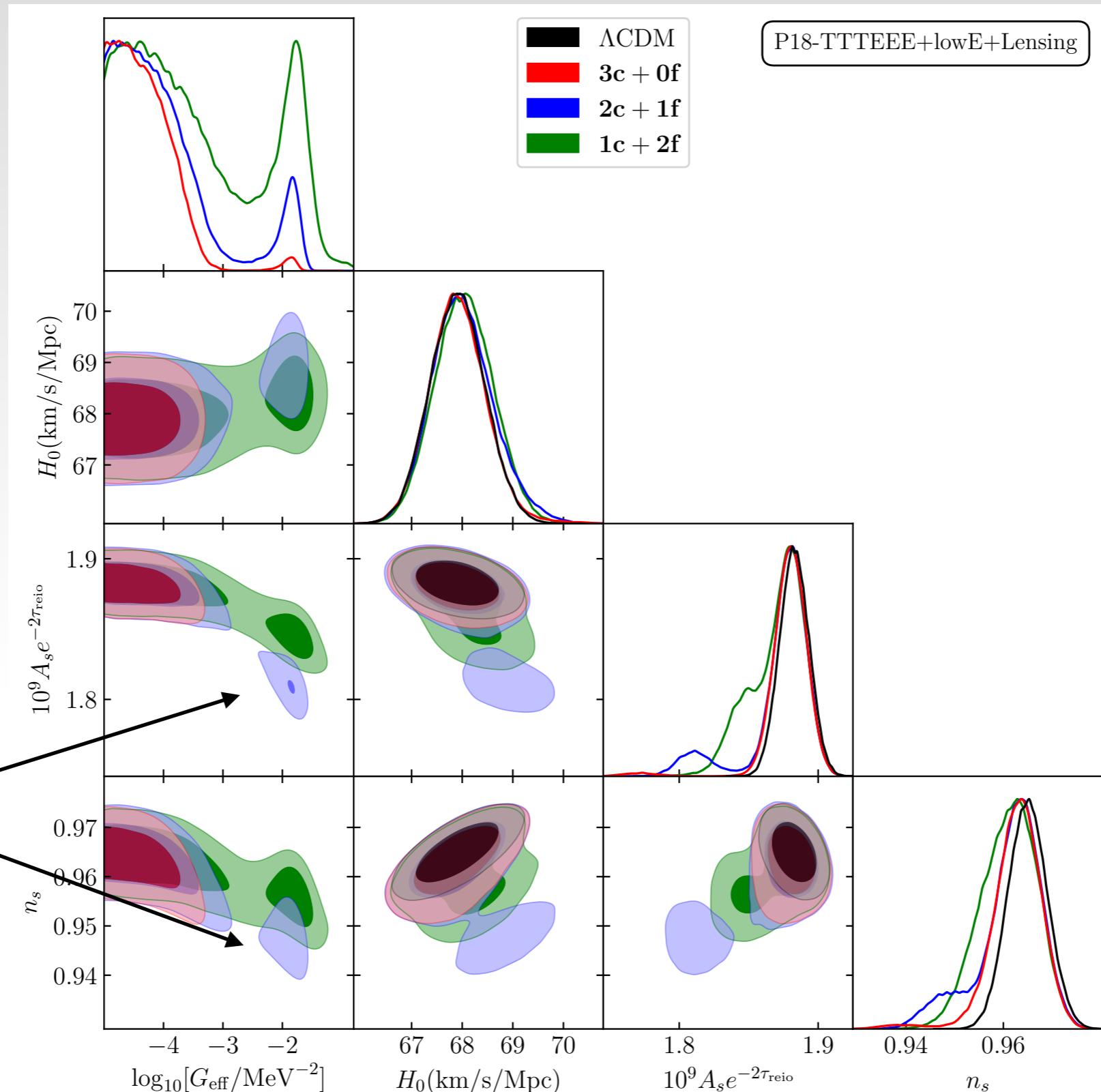
Changes are milder with less number of coupled neutrinos

$\Lambda\text{CDM} \equiv 0c + 3f$

Strong flavor specific interaction preferred by CMB



Strong flavor specific interaction preferred by CMB



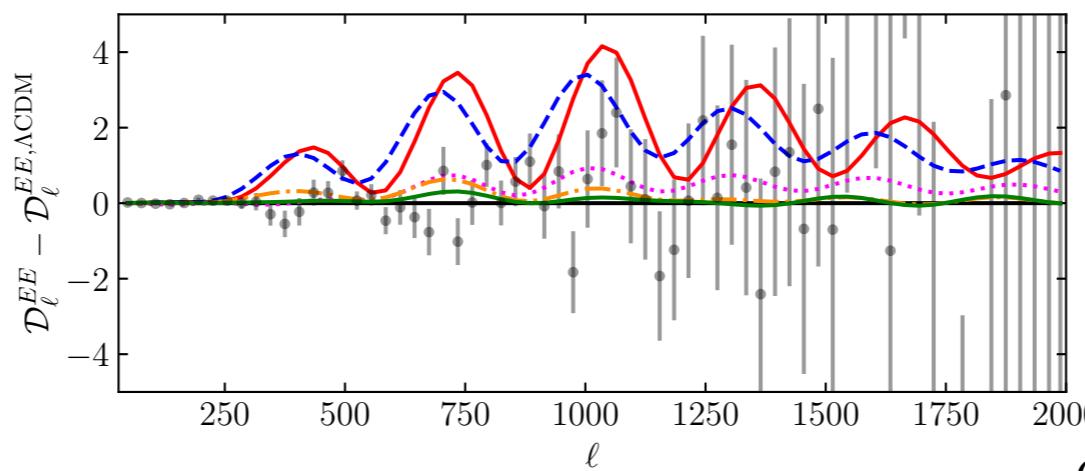
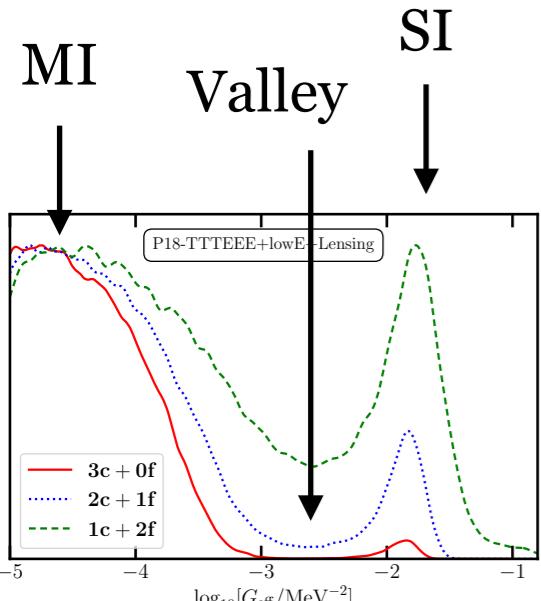
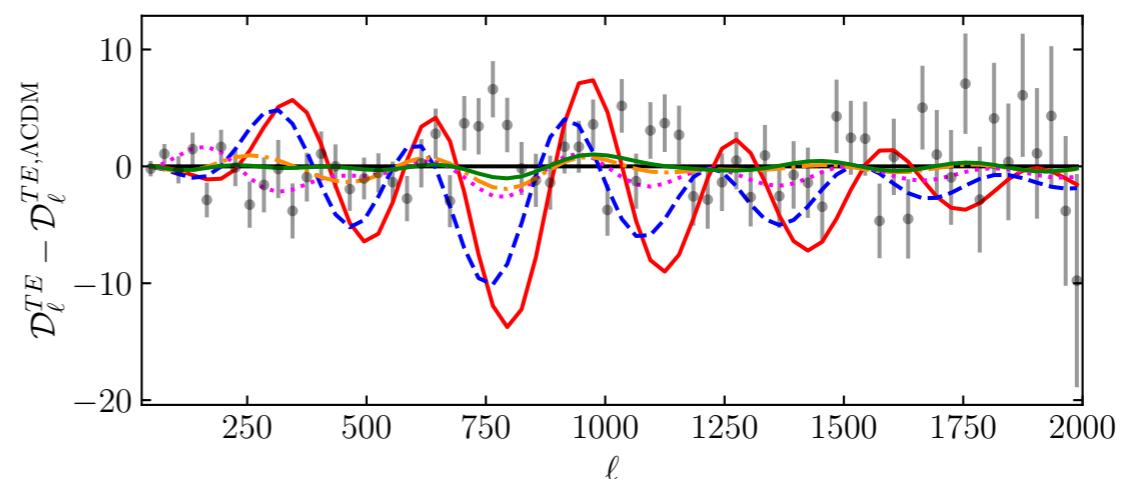
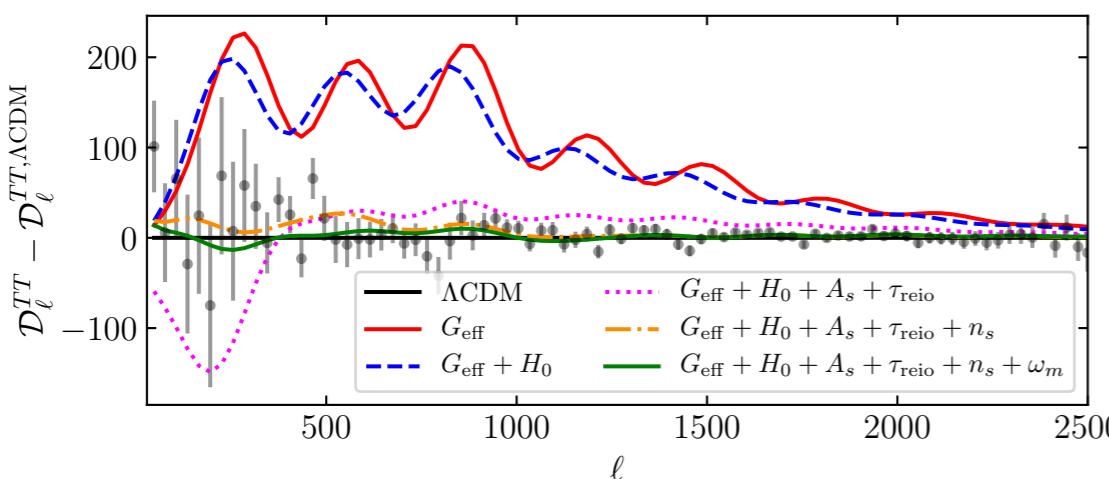
Why SI mode is a good fit to CMB data?

SI mode interaction strength keep neutrino coupled till matter-radiation equality

Affects all the CMB peaks ($\ell \gtrsim 100$)

Λ CDM parameter ($A_s, n_s \dots$) compensate for the changes

P18 : TTTEEE + lowE | 3c + 0f



Spectral changes due to modes between MI & SI cannot be compensated by changes in Λ CDM parameters

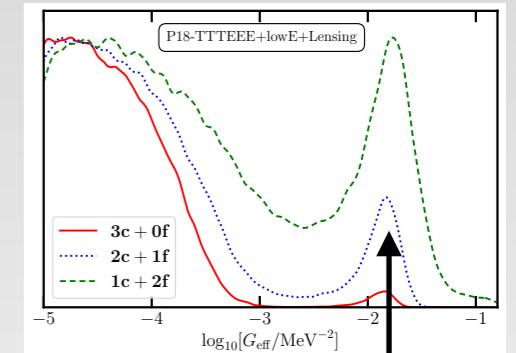
Existence of the 'Valley'
(The dip in between MI and SI mode)

SI mode enhancement in flavor specific scenario

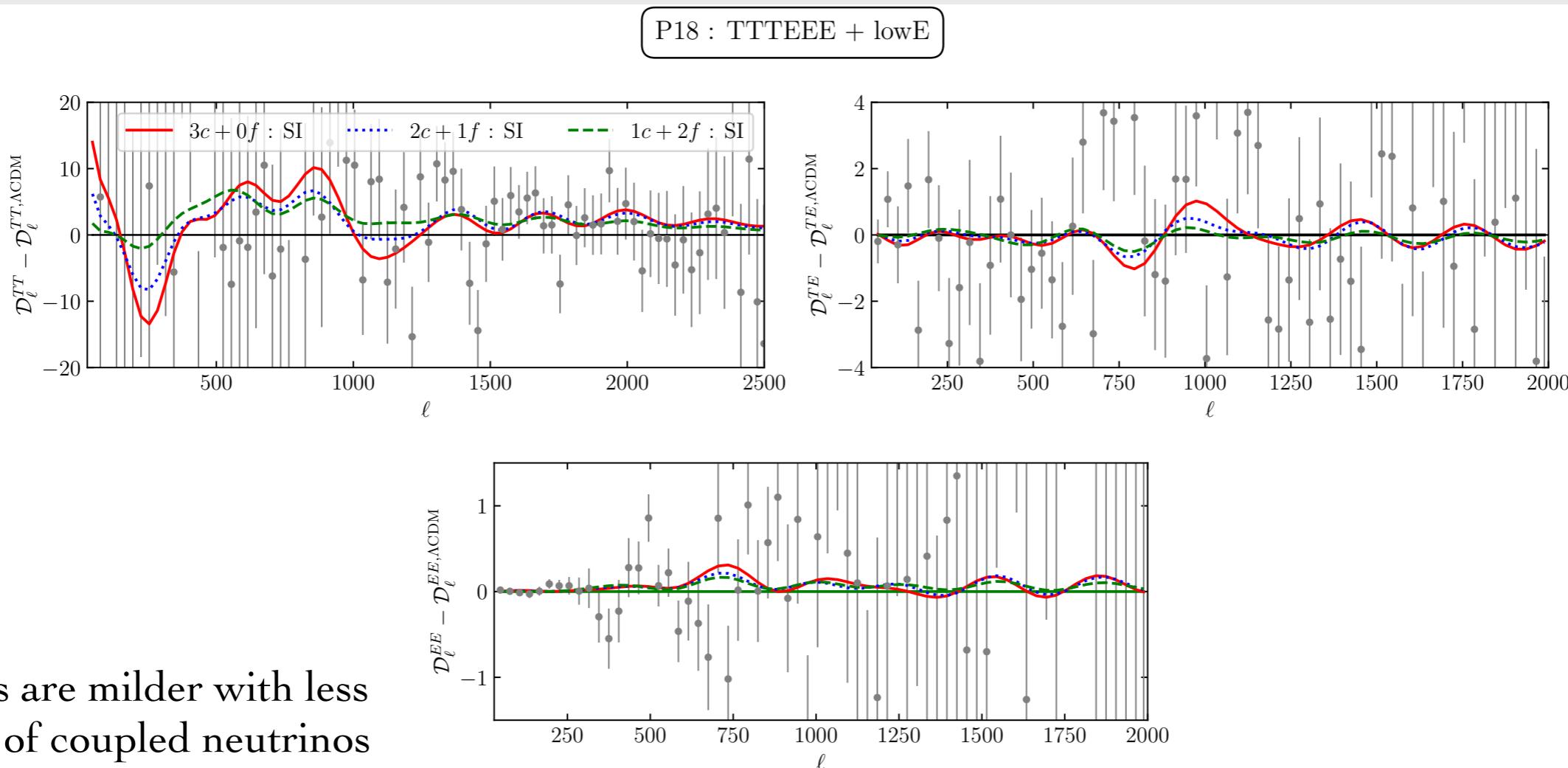
The existence of SI mode is connect with the value of G_{eff}



In flavor specific cases SI mode value of G_{eff} does not change



SI



Changes are milder with less number of coupled neutrinos

More freedom to fit
The residual
(smaller χ^2)

SI mode fits some features of the residual

$\Lambda\text{CDM} \equiv 0c + 3f$

*MI mode residual is virtually equivalent to ΛCDM

*Planck 2018 data with error bar are shown

Parameter Values

Planck 2018: TTTTEEE+lowE

Parameters	3c + 0f		2c + 1f		1c + 2f	
	SI	MI	SI	MI	SI	MI
$\Omega_b h^2$	0.022 ± 0.00016	0.022 ± 0.00015	0.022 ± 0.00016	0.022 ± 0.00015	0.022 ± 0.00015	0.022 ± 0.00015
$\Omega_c h^2$	0.1205 ± 0.0015	0.1201 ± 0.0014	0.1205 ± 0.0014	0.1201 ± 0.0013	0.1203 ± 0.0014	0.1201 ± 0.0013
$100\theta_s$	1.0464 ± 0.00087	1.0419 ± 0.0003	1.045 ± 0.00076	1.0419 ± 0.00031	1.043 ± 0.00058	1.0419 ± 0.0003
$\ln(10^{10} A_s)$	2.984 ± 0.017	3.042 ± 0.0161	3 ± 0.0167	3.042 ± 0.0161	3.024 ± 0.0166	3.042 ± 0.016
n_s	0.9386 ± 0.004	0.9626 ± 0.005	0.9473 ± 0.0046	0.9628 ± 0.005	0.9553 ± 0.0049	0.963 ± 0.005
τ_{reio}	0.0543 ± 0.0077	0.0537 ± 0.0077	0.0538 ± 0.0077	0.0538 ± 0.0077	0.0539 ± 0.0076	0.0539 ± 0.0077
$\log_{10}(G_{\text{eff}}/\text{MeV}^{-2})$	-1.92 ± 0.18	-4.35 ± 0.42	-1.93 ± 0.24	-4.24 ± 0.5	-1.9 ± 0.37	-4.06 ± 0.6
$H_0(\text{ km s}^{-1}\text{Mpc}^{-1})$	69.44 ± 0.64	67.82 ± 0.61	68.81 ± 0.63	67.83 ± 0.6	68.3 ± 0.62	67.83 ± 0.61
$r_s^*(\text{Mpc})$	144.54 ± 0.35	144.84 ± 0.32	144.64 ± 0.34	144.85 ± 0.32	144.76 ± 0.32	144.84 ± 0.31
σ_8	0.834 ± 0.008	0.824 ± 0.0075	0.829 ± 0.0079	0.824 ± 0.0075	0.825 ± 0.0083	0.824 ± 0.0075
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	5.14	0.18	1.8	0.28	0	0.1

Significance of the SI mode is increasing

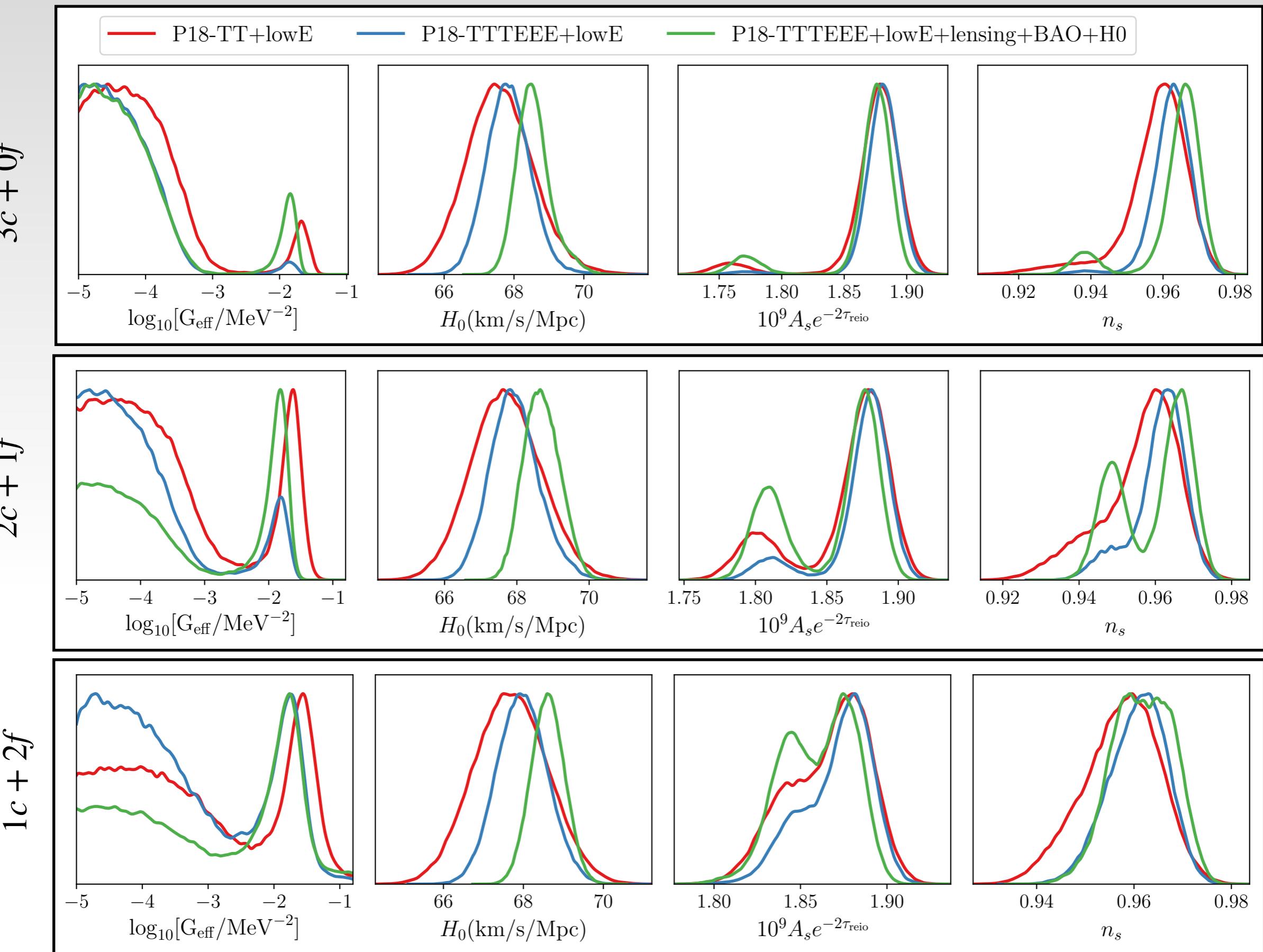
Parameter Values

Planck 2018: TTTTEEE+lowE+lensing+BAO+ShoES

Parameters	$3c + 0f$		$2c + 1f$		$1c + 2f$	
	SI	MI	SI	MI	SI	MI
$\Omega_b h^2$	0.023 ± 0.00014	0.022 ± 0.00013	0.022 ± 0.0001	0.022 ± 0.00013	0.022 ± 0.0001	0.022 ± 0.00013
$\Omega_c h^2$	0.1206 ± 0.001	0.1188 ± 0.0009	0.12 ± 0.001	0.1188 ± 0.0009	0.12 ± 0.0009	0.1188 ± 0.0009
$100\theta_s$	1.0465 ± 0.00079	1.042 ± 0.00029	1.045 ± 0.00068	1.042 ± 0.00029	1.043 ± 0.00056	1.042 ± 0.00029
$\ln(10^{10} A_s)$	2.98 ± 0.0153	3.044 ± 0.0144	3.0 ± 0.0151	3.044 ± 0.0145	3.0 ± 0.0151	3.045 ± 0.0142
n_s	0.9383 ± 0.004	0.966 ± 0.0045	0.9483 ± 0.004	0.966 ± 0.0046	0.9572 ± 0.004	0.966 ± 0.0042
τ_{reio}	0.0532 ± 0.007	0.0563 ± 0.0071	0.0544 ± 0.007	0.0565 ± 0.0071	0.0554 ± 0.007	0.0566 ± 0.0071
$\log_{10}(G_{\text{eff}}/\text{MeV}^{-2})$	-1.91 ± 0.16	-4.34 ± 0.43	-1.91 ± 0.22	-4.22 ± 0.52	-1.86 ± 0.36	-4.03 ± 0.61
$H_0(\text{ km s}^{-1}\text{Mpc}^{-1})$	69.45 ± 0.42	68.46 ± 0.41	69.08 ± 0.42	68.47 ± 0.4	68.75 ± 0.41	68.48 ± 0.41
$r_s^*(\text{Mpc})$	144.5 ± 0.26	145.12 ± 0.24	144.73 ± 0.26	145.12 ± 0.23	144.93 ± 0.24	145.12 ± 0.24
σ_8	0.833 ± 0.0065	0.821 ± 0.006	0.827 ± 0.0065	0.821 ± 0.0059	0.822 ± 0.0071	0.821 ± 0.006
$\chi^2 - \chi^2_{\Lambda\text{CDM}}$	1.99	0.17	-1.35	0.25	-1.67	0.33

Better fit than ΛCDM

Constraints with other dataset



Effect on H_0 : Phase shift

Neutrino self interaction can enhance H_0 even when N_{eff} is kept fixed

Photon transfer function — $\cos(kr_s^* + \phi_\nu)$

Bashinsky et. al. , astro-ph/0310198

Baumann et. al. , 1508.06342

Ghosh et. al. , 1908.09843



Phase shift due to
free-streaming neutrinos

$$\ell \approx kD_A^* = (m\pi - \phi_\nu) \frac{D_A^*}{r_s^*}$$

$$\phi_\nu \simeq 0.19\pi R_\nu$$

$$D_A^* = \int_0^{z^*} \frac{1}{H(z)} dz$$

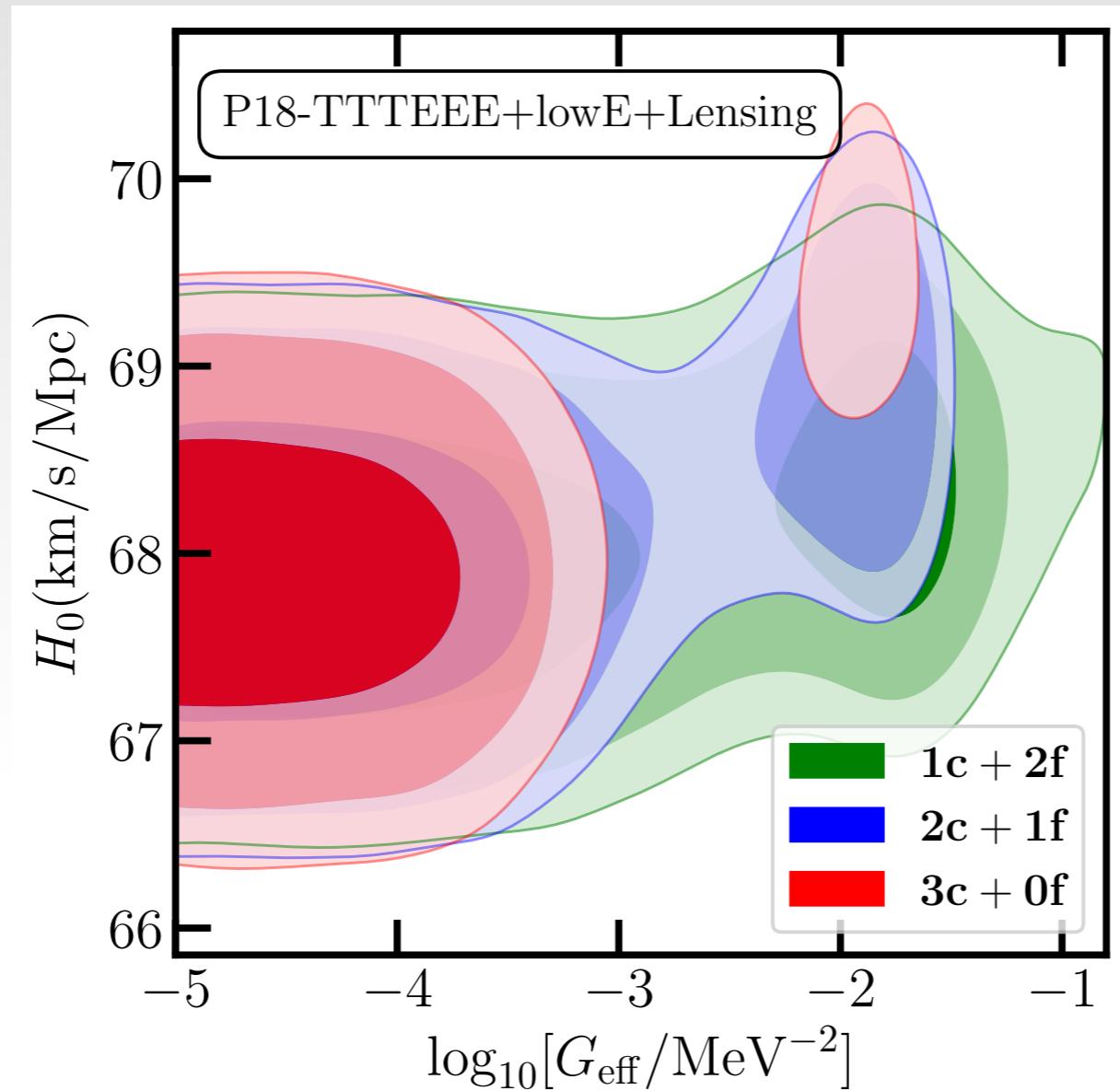
$$R_\nu = \frac{\rho_\nu}{\rho_\gamma + \rho_\nu}$$

$$r_s^* = \int_{z^*}^{\infty} \frac{c_s(z)}{H(z)} dz$$

$$R_\nu = R_\nu^{\Lambda\text{CDM}} \times \begin{cases} 0, & \text{for } 3c + 0f \\ \frac{1}{3}, & \text{for } 2c + 1f \\ \frac{2}{3}, & \text{for } 1c + 2f \end{cases}$$

Change in ϕ_ν is compensated (mostly) by change
 D_A^* — through change in Ω_Λ and H_0

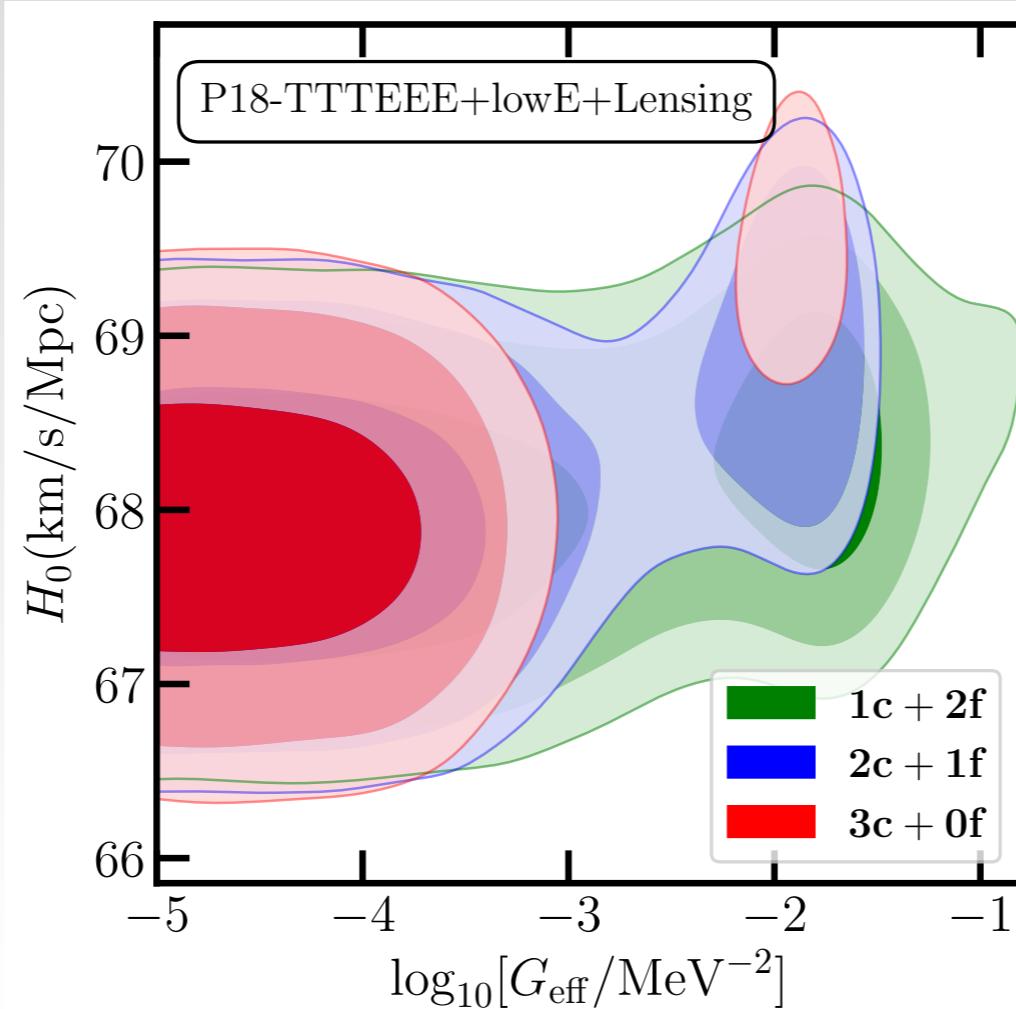
Effect on H_0 : Phase shift



Effect on H_0 : Phase shift

$$\ell \approx kD_A^* = (m\pi - \phi_\nu) \frac{D_A^*}{r_s^*}$$

$$D_A^* = \int_0^{z^*} \frac{1}{H(z)} dz$$



	SI: 3c + 0f	SI: 2c + 1f	Λ CDM
H_0 (km s ⁻¹ Mpc ⁻¹)	69.47 ± 0.59	68.87 ± 0.58	67.90 ± 0.54
Ω_Λ	0.7035 ± 0.0071	0.6989 ± 0.0072	0.6912 ± 0.0073
$100\theta_s$	1.0463 ± 0.00094	1.0447 ± 0.00079	1.04186 ± 0.00029
r_s^* (Mpc)	144.58 ± 0.32	144.69 ± 0.31	144.87 ± 0.29
D_A^* (Mpc)	12.69 ± 0.036	12.72 ± 0.034	12.773 ± 0.028

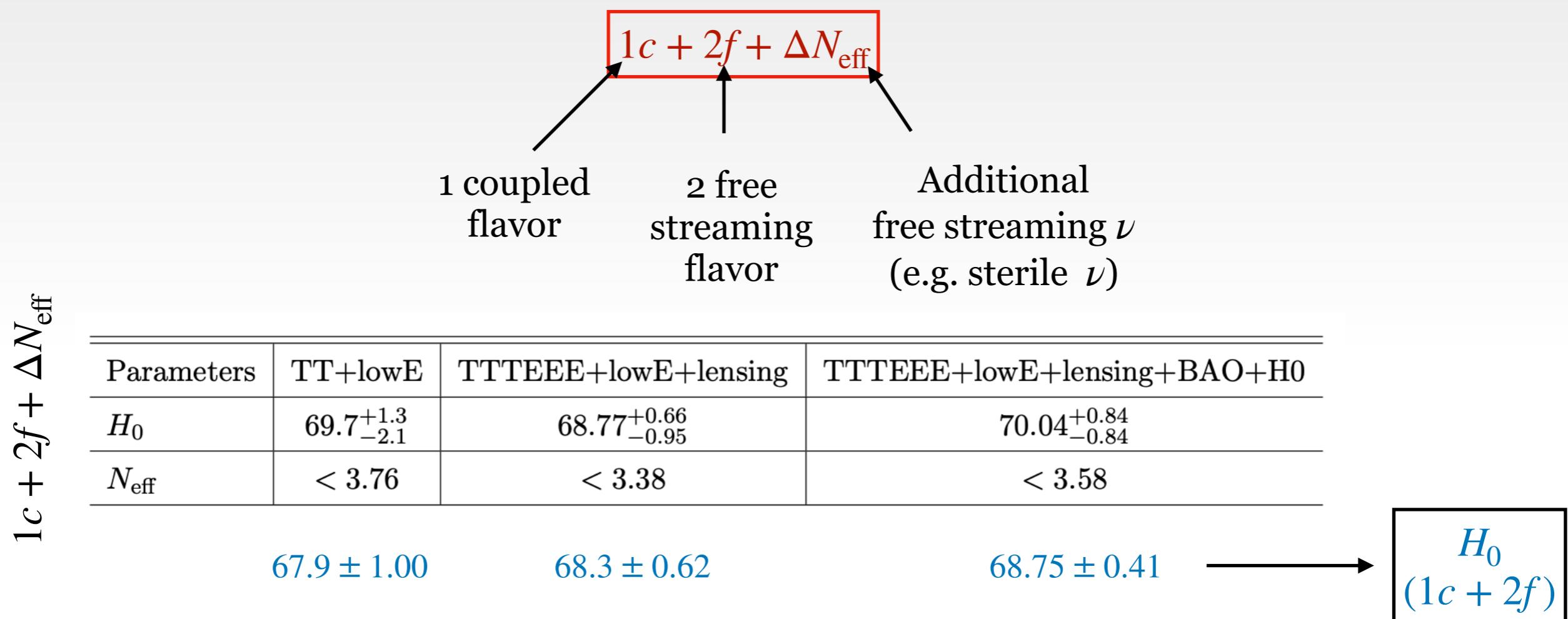
$$\theta_s \approx \theta_* \equiv \frac{r_s^*}{D_A^*}$$

Flavor specific SINU - with varying N_{eff} : $1c + 2f + \Delta N_{\text{eff}}$

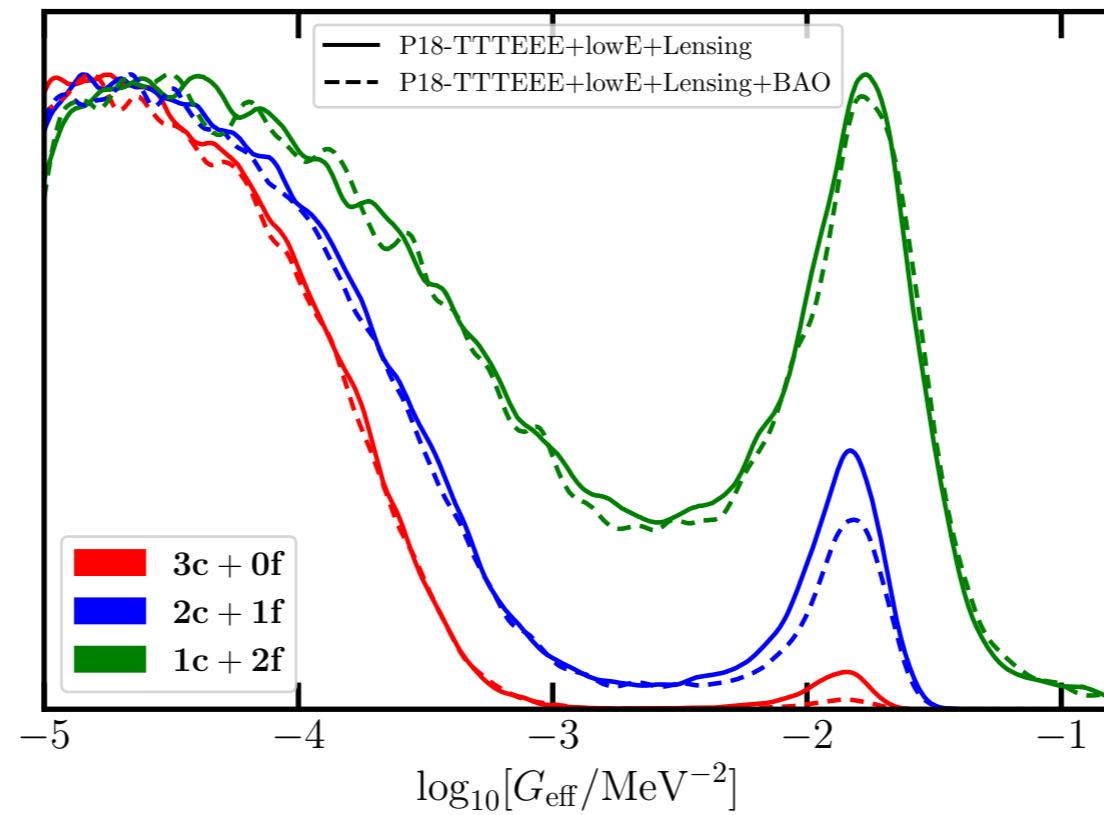
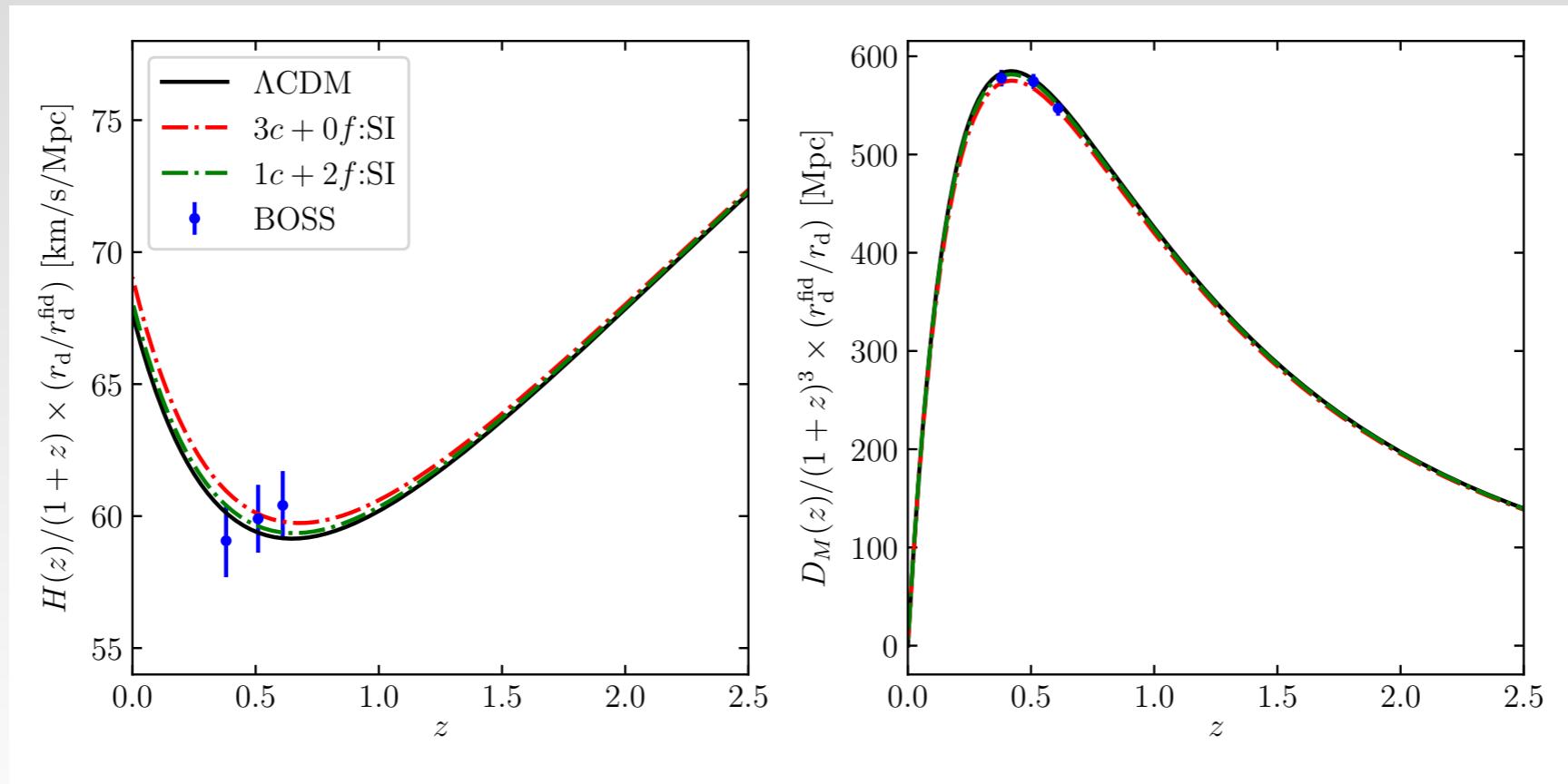
SINU can accommodates larger N_{eff}

← Enhancement of CMB spectra due to self-interaction compensates additional silk damping

In addition to enhancement of H_0 due to phase shift , larger N_{eff} can also boost the H_0 value



Effect of BAO data



Strong self interaction is
disfavored by
BAO data

Part I : Summary : CMB favors Flavor specific self-interaction

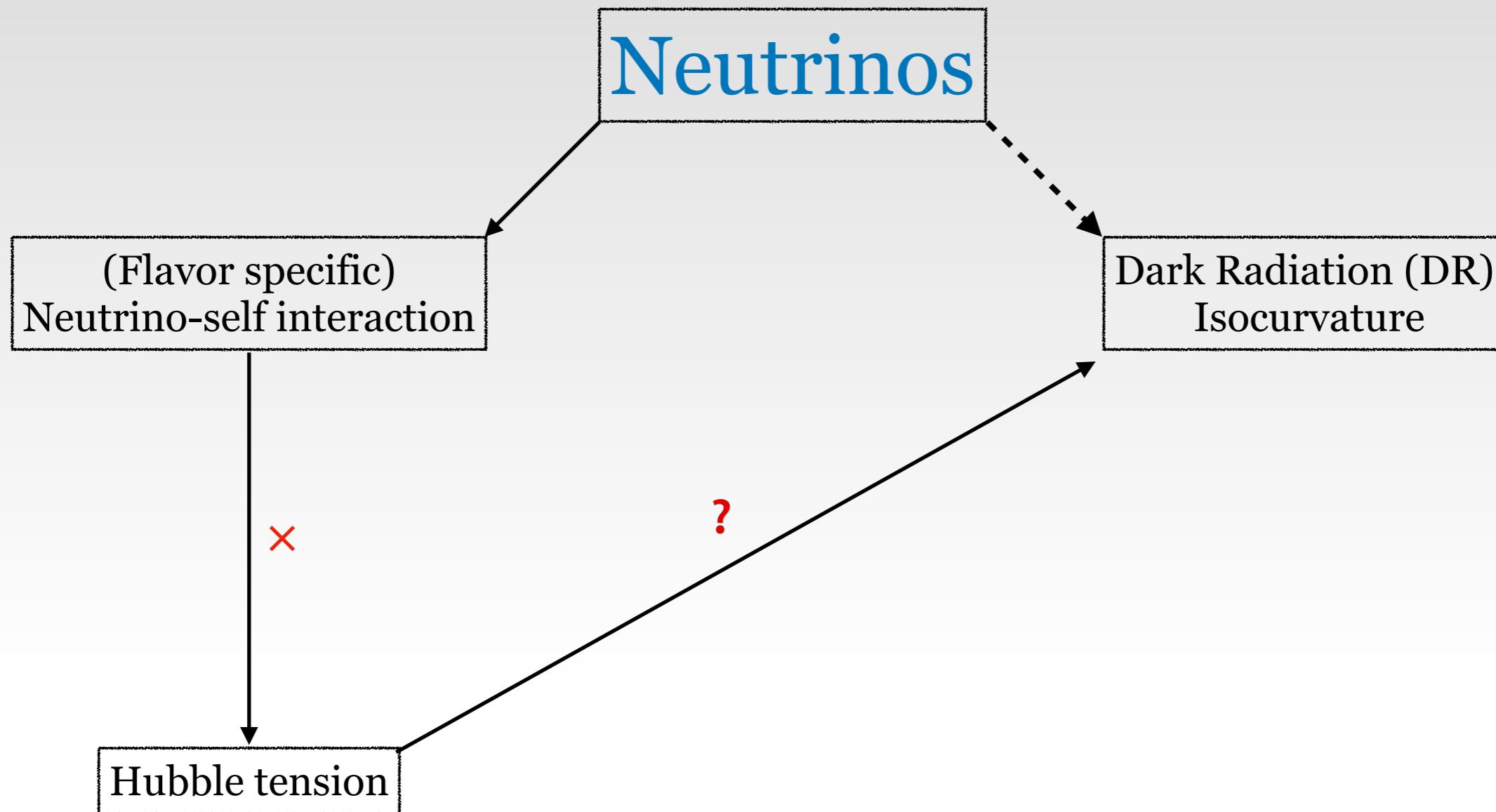
- Flavor specific neutrino self interaction is phenomenologically motivated
 - takes into account laboratory constraints
- The significance of the SI mode is increased dramatically
 - similar in χ^2 to Λ CDM fit
- The position of the SI mode peak in **Flavor specific interaction** remains almost the same in Flavor universal case
- However, does not predict a larger H_0 than flavor universal case

Flavor specific neutrino self interaction can provide similar (in some case better) fit to the CMB (& LSS) data

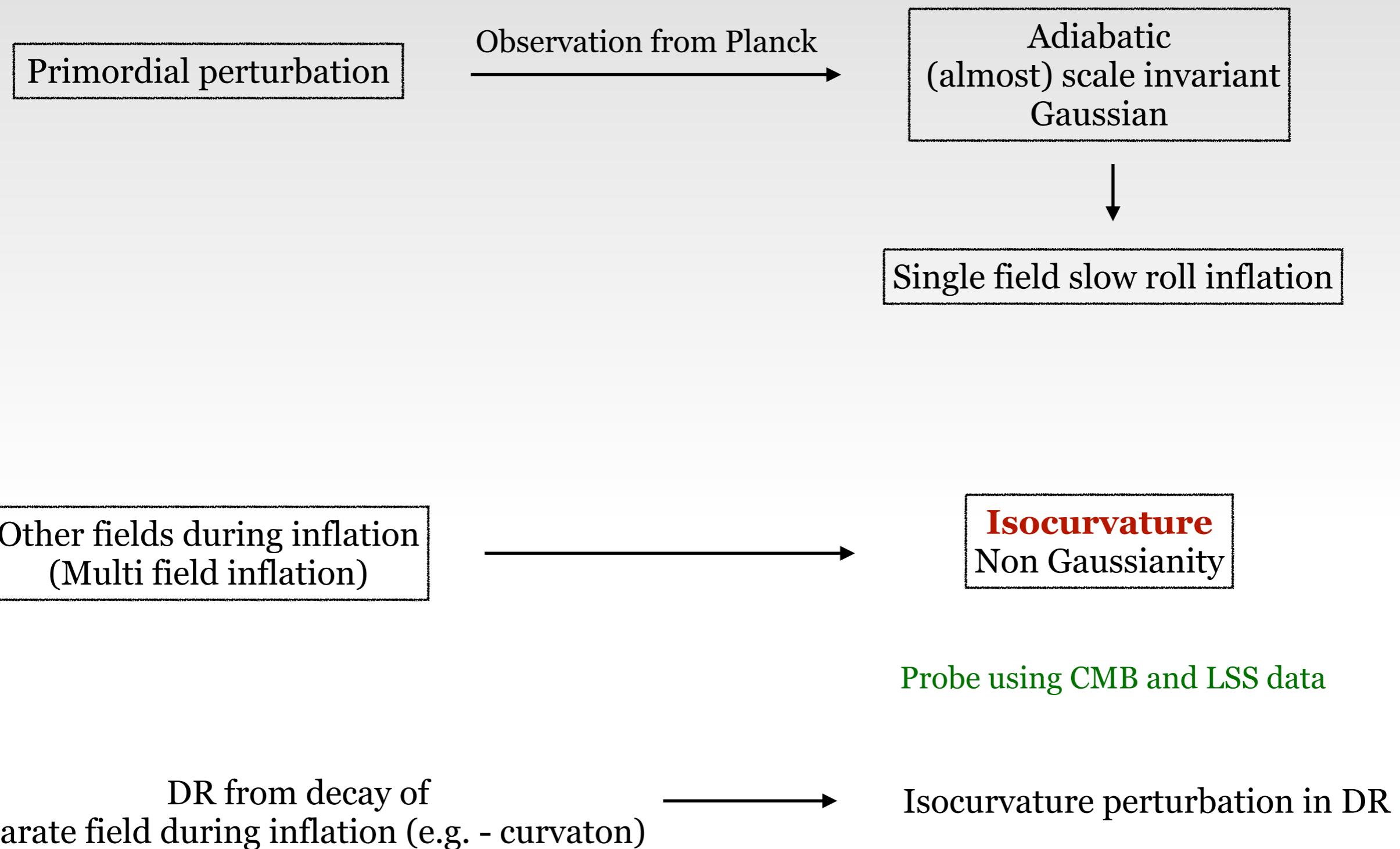
- Effect of non-diagonal interaction
 - arises naturally when considering massive Neutrinos
- Phenomenological model for flavor specific interaction

Cosmology favors Flavor specific neutrino self interaction

Part II : Dark Radiation Isocurvature



Isocurvature Perturbation in CMB



Dark Radiation (DR)

Parametrized by ΔN_{eff}

Free-streaming DR (FDR)

Similar to (SM/free-streaming) neutrinos

Non zero anisotropic stress

Coupled/fluid DR (CDR)

Similar to (strongly) self-interacting neutrinos

Zero anisotropic stress

Isocurvature
parameters

$A_{\text{iso}}(k_*)$ [or $f_{\text{iso}} \equiv A_{\text{iso}}/A_{\text{adia}}$]

n_{iso}



Or

$P_{II}^{(1)} (\equiv A_{\text{iso}}(k_1))$

$P_{II}^{(2)} (\equiv A_{\text{iso}}(k_2))$

$$k_1 = 0.002 \text{ Mpc}^{-1}$$
$$k_2 = 0.1 \text{ Mpc}^{-1}$$

Isocurvature Initial conditions

(In synchronous gauge)

variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(\omega k^2 \tau^3)$
δ_γ	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
θ_γ/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
δ_ν	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
θ_ν/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
σ_ν	0	0	$-\frac{19R_{\text{DR}}}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
δ_{DR}	1	0	$-\frac{1}{6}$	
θ_{DR}/k	0	$\frac{1}{4}$	0	
σ_{DR}	0	0	$\frac{15-15R_{\text{DR}}+4R_\nu}{30(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
η	0	0	$\frac{-R_{\text{DR}}+R_{\text{DR}}^2+R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_{\text{DR}}+4R_\nu)}$	
h	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$
δ_b	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	
δ_c	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$

Adiabatic initial condition : $\delta_\gamma = \delta_\nu = \delta_{\text{DR}}$

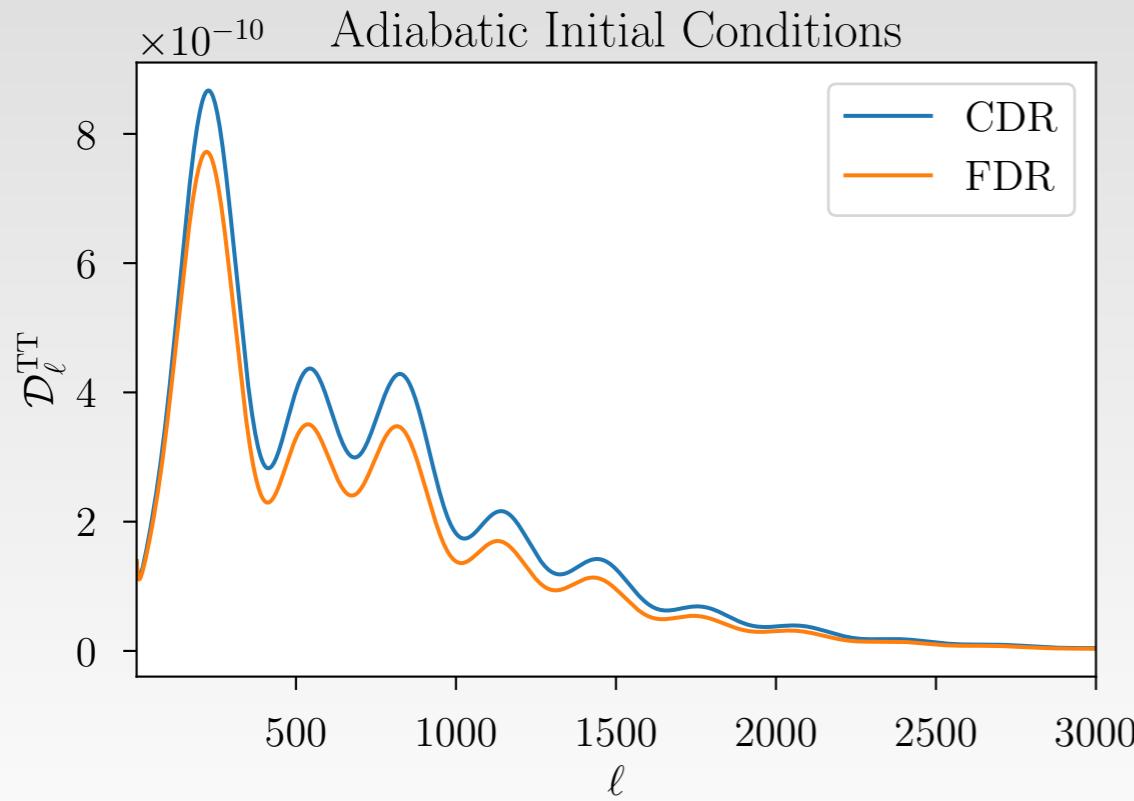
FDR - Isocurvature

For both CDR & FDR isocurvature: $\sum_i R_i \delta_i = 0$

$$R_i = \bar{\rho}_i / (\bar{\rho}_\gamma + \bar{\rho}_\nu + \bar{\rho}_{\text{DR}})$$

variable	$\mathcal{O}(0)$	$\mathcal{O}(k\tau)$	$\mathcal{O}((k\tau)^2)$	$\mathcal{O}(\omega k^2 \tau^3)$
δ_γ	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
θ_γ/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
δ_ν	$-\frac{R_{\text{DR}}}{1-R_{\text{DR}}}$	0	$\frac{R_{\text{DR}}}{6(1-R_{\text{DR}})}$	
θ_ν/k	0	$-\frac{R_{\text{DR}}}{4(1-R_{\text{DR}})}$	0	
σ_ν	0	0	$-\frac{R_{\text{DR}}}{2(1-R_{\text{DR}})(15+4R_\nu)}$	
δ_{DR}	1	0	$-\frac{1}{6}$	
θ_{DR}/k	0	$\frac{1}{4}$	0	
η	0	0	$\frac{R_{\text{DR}}R_\nu}{6(1-R_{\text{DR}})(15+4R_\nu)}$	
h	0	0	0	$\frac{R_{\text{DR}}R_b}{40(1-R_{\text{DR}})}$
δ_b	0	0	$\frac{R_{\text{DR}}}{8(1-R_{\text{DR}})}$	
δ_c	0	0	0	$-\frac{R_{\text{DR}}R_b}{80(1-R_{\text{DR}})}$

FDR vs CDR Isocurvature spectrum



$$\text{Adiabatic} : \delta_\gamma = \delta_\nu = \delta_{\text{DR}}$$

FDR free-streams out of potential well
 \rightarrow Smaller potential \rightarrow Smaller CMB anisotropy

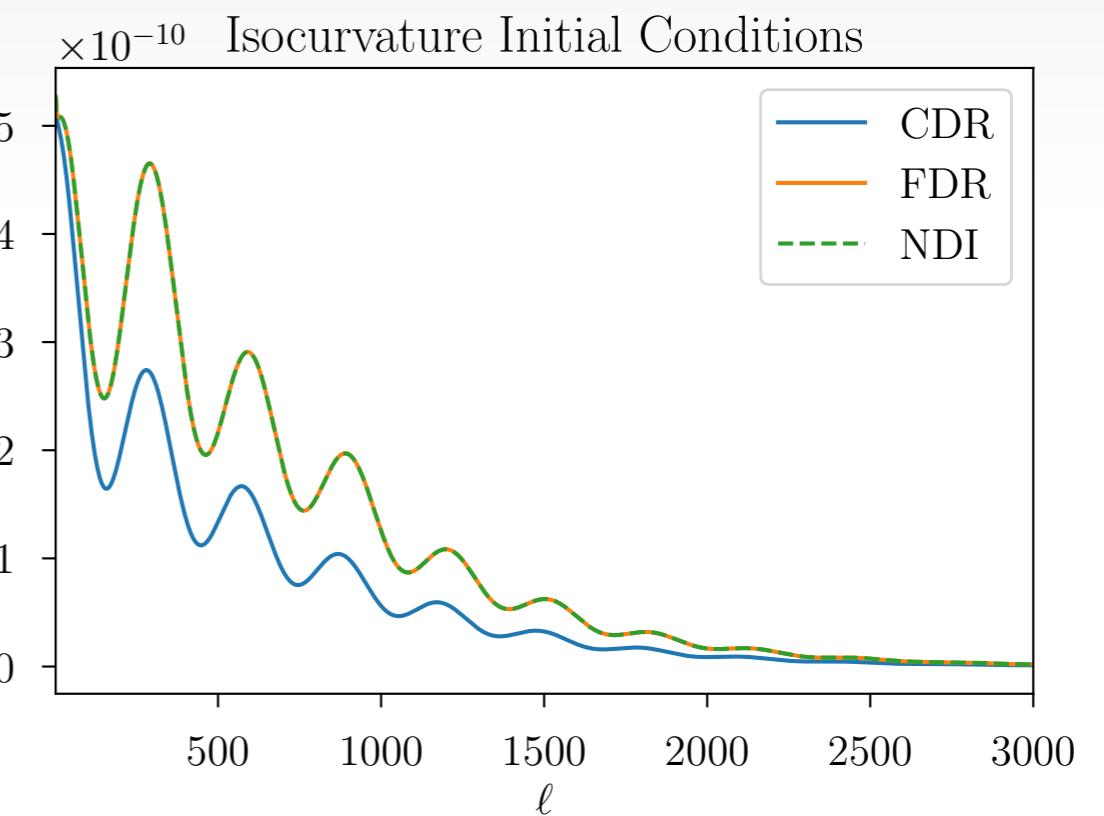
CDR does not free-stream
 \rightarrow larger CMB anisotropy

$$\sum_i R_i \delta_i = 0$$

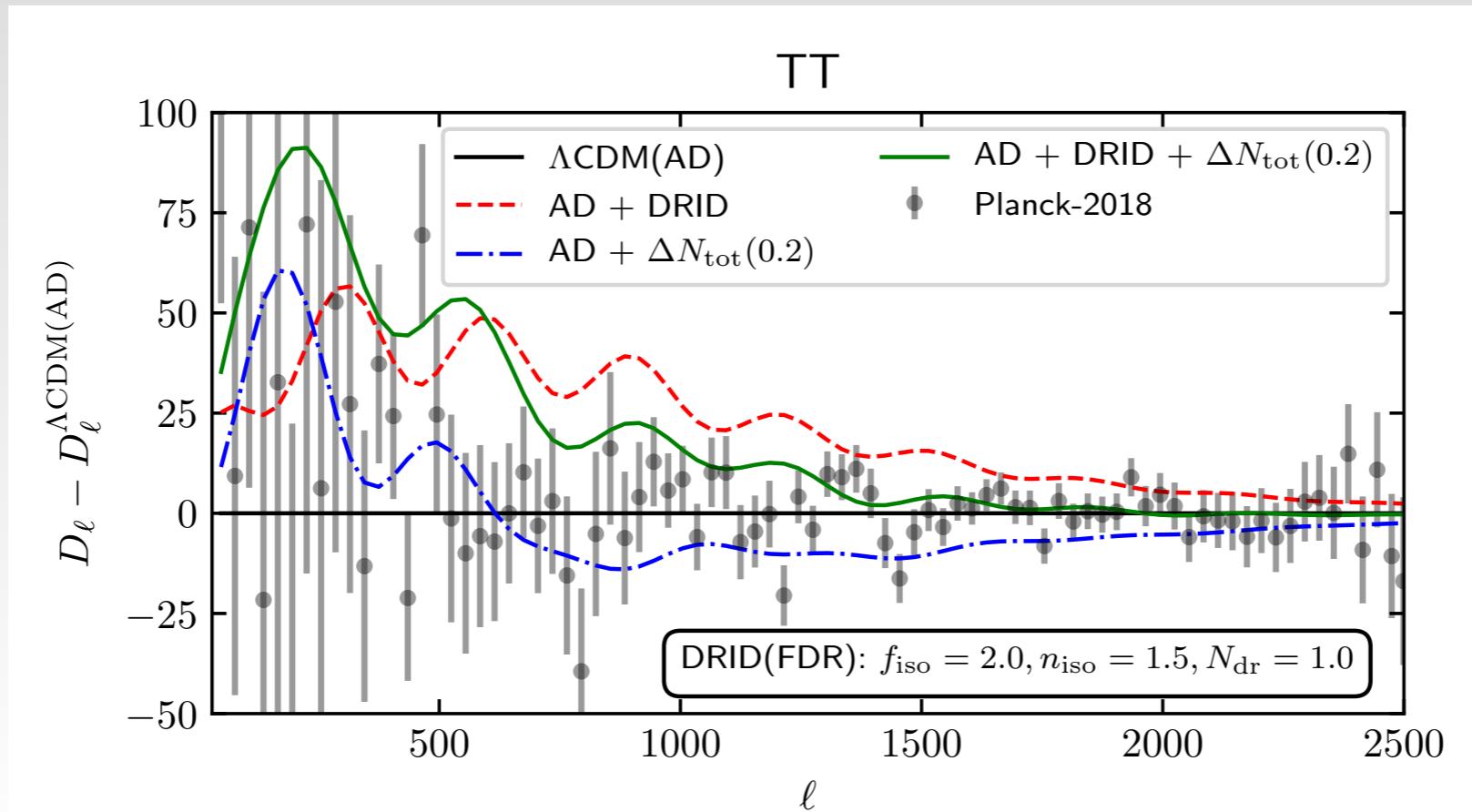
For isocurvature metric fluctuation is sourced
by anisotropic stress (σ) at leading order

FDR: $\sigma_{\text{tot}} > 0 \rightarrow$ More anisotropy

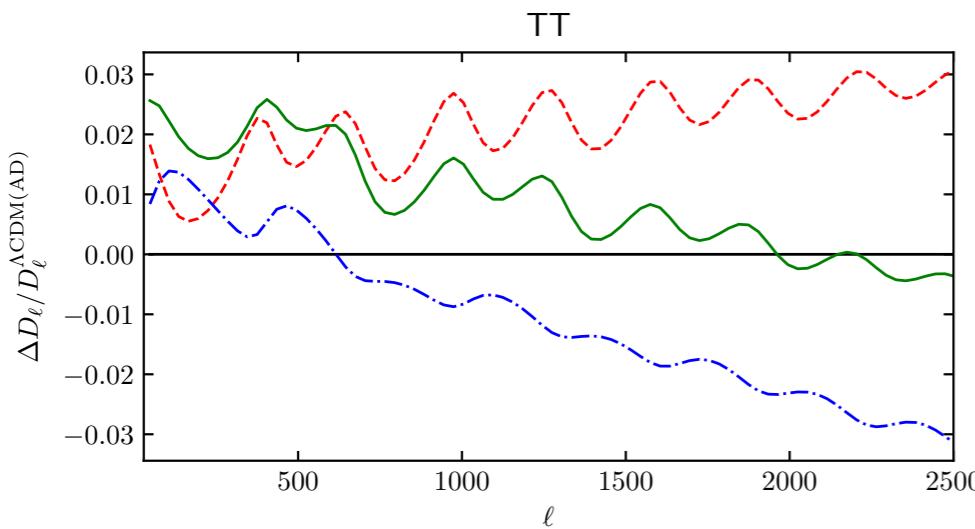
CDR: $\sigma_{\text{tot}} < 0 \rightarrow$ Less anisotropy



Isocurvature accommodate larger N_{eff} ($\equiv N_{\text{tot}}$)

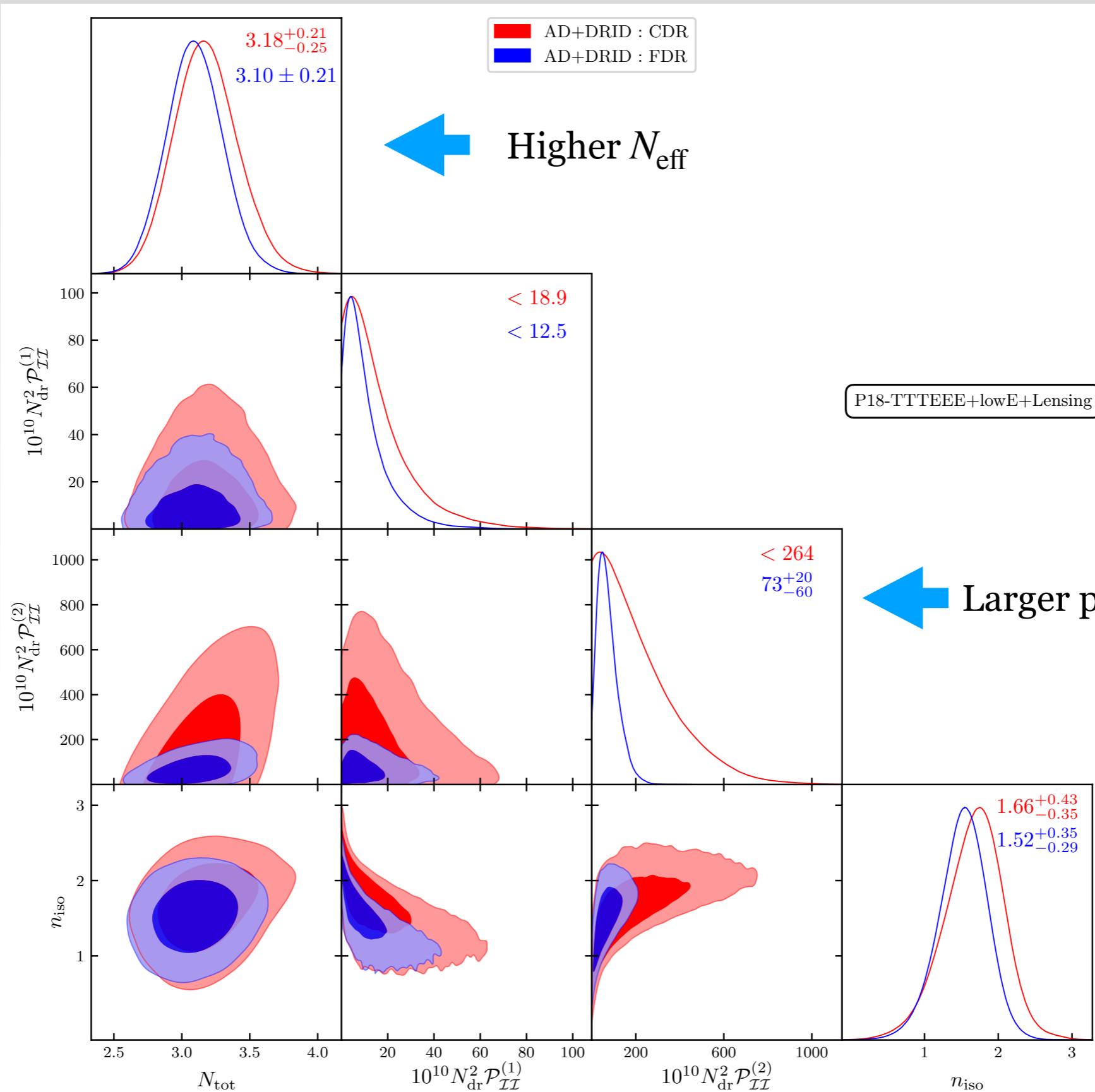


Blue tilted ($n_{\text{iso}} > 1$) isocurvature compensates for the larger silk damping due to higher N_{eff}

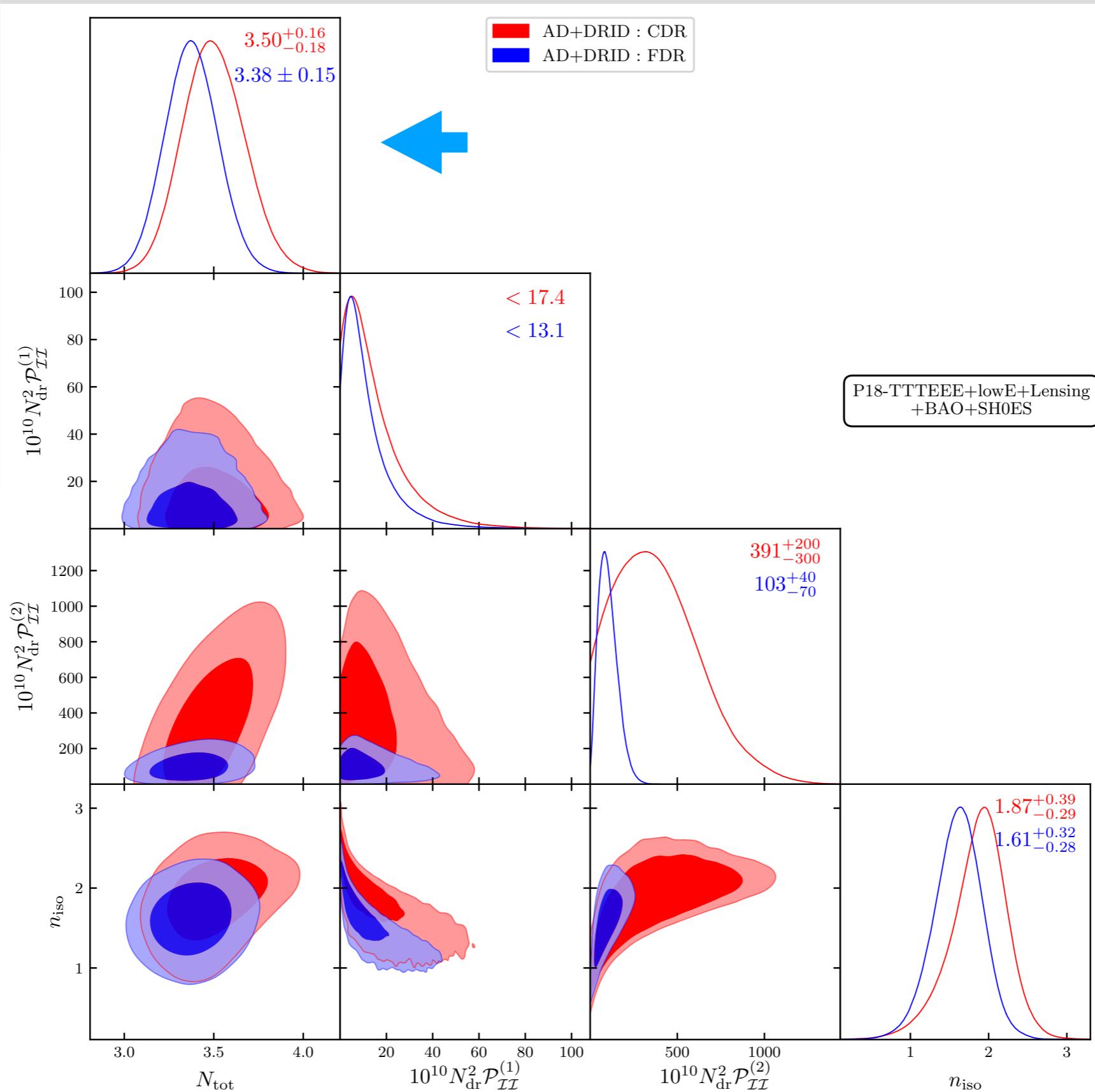


DRID \equiv Dark Radiation density Isocurvature

MCMC results

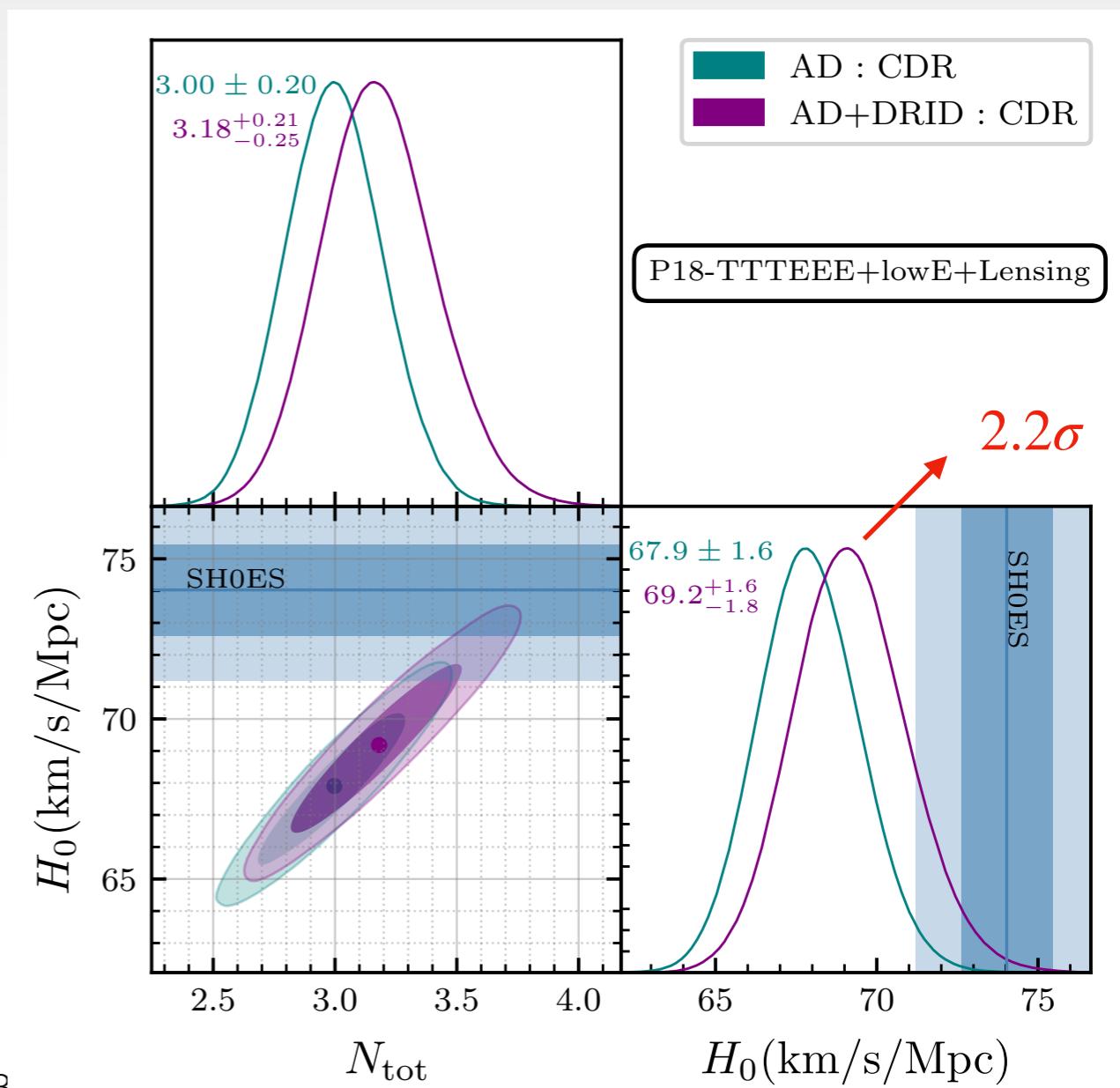
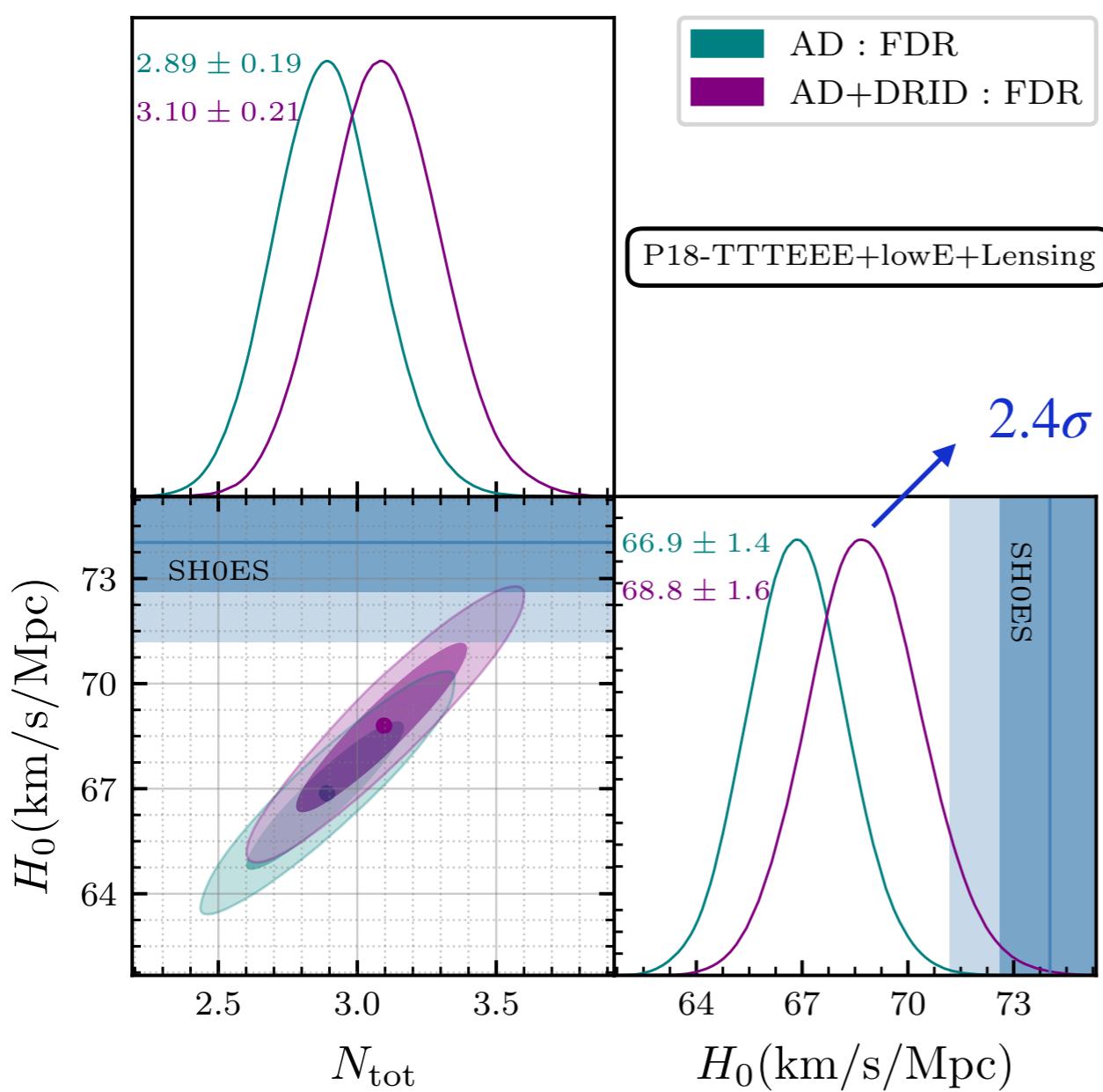


MCMC results

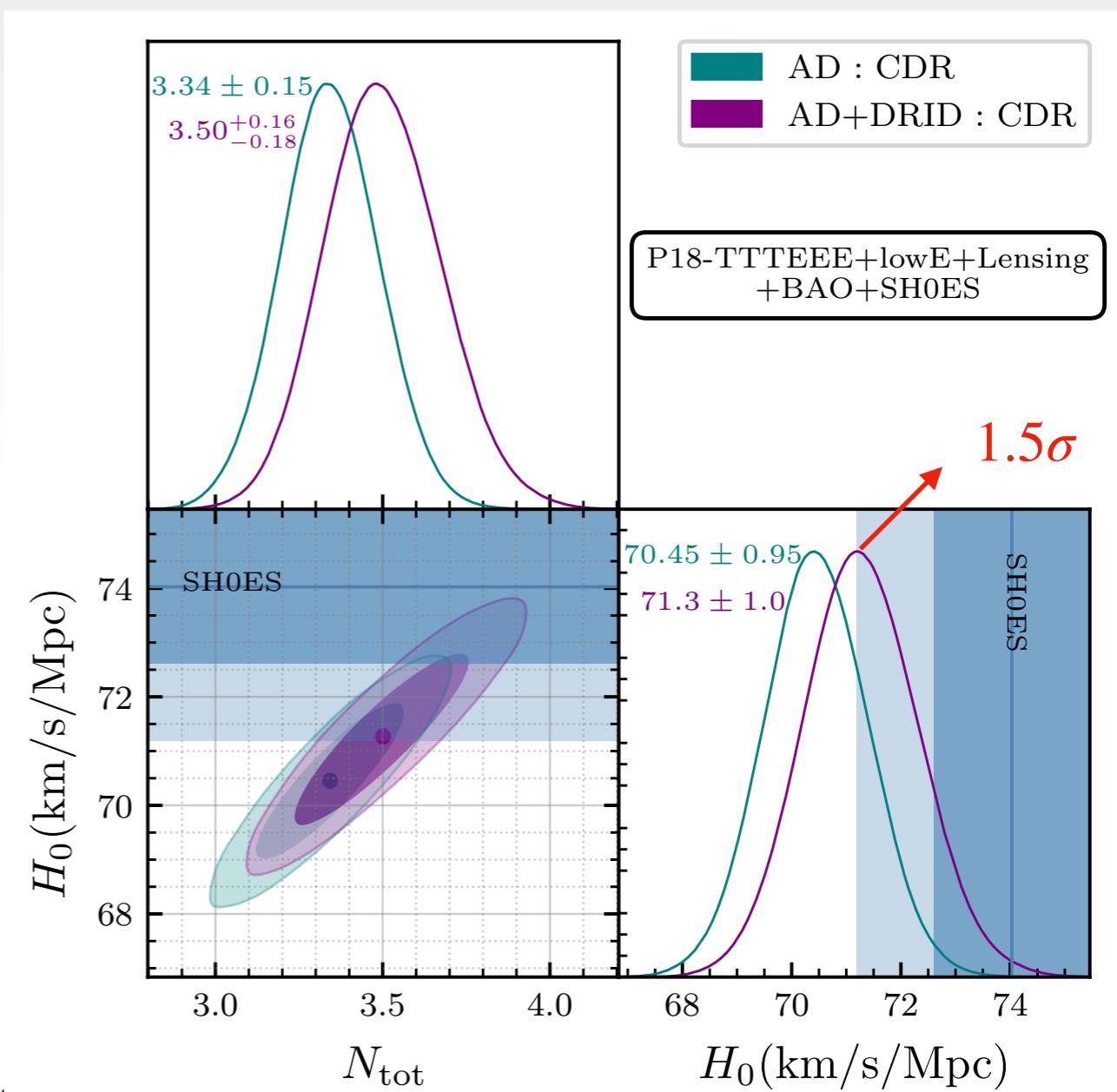
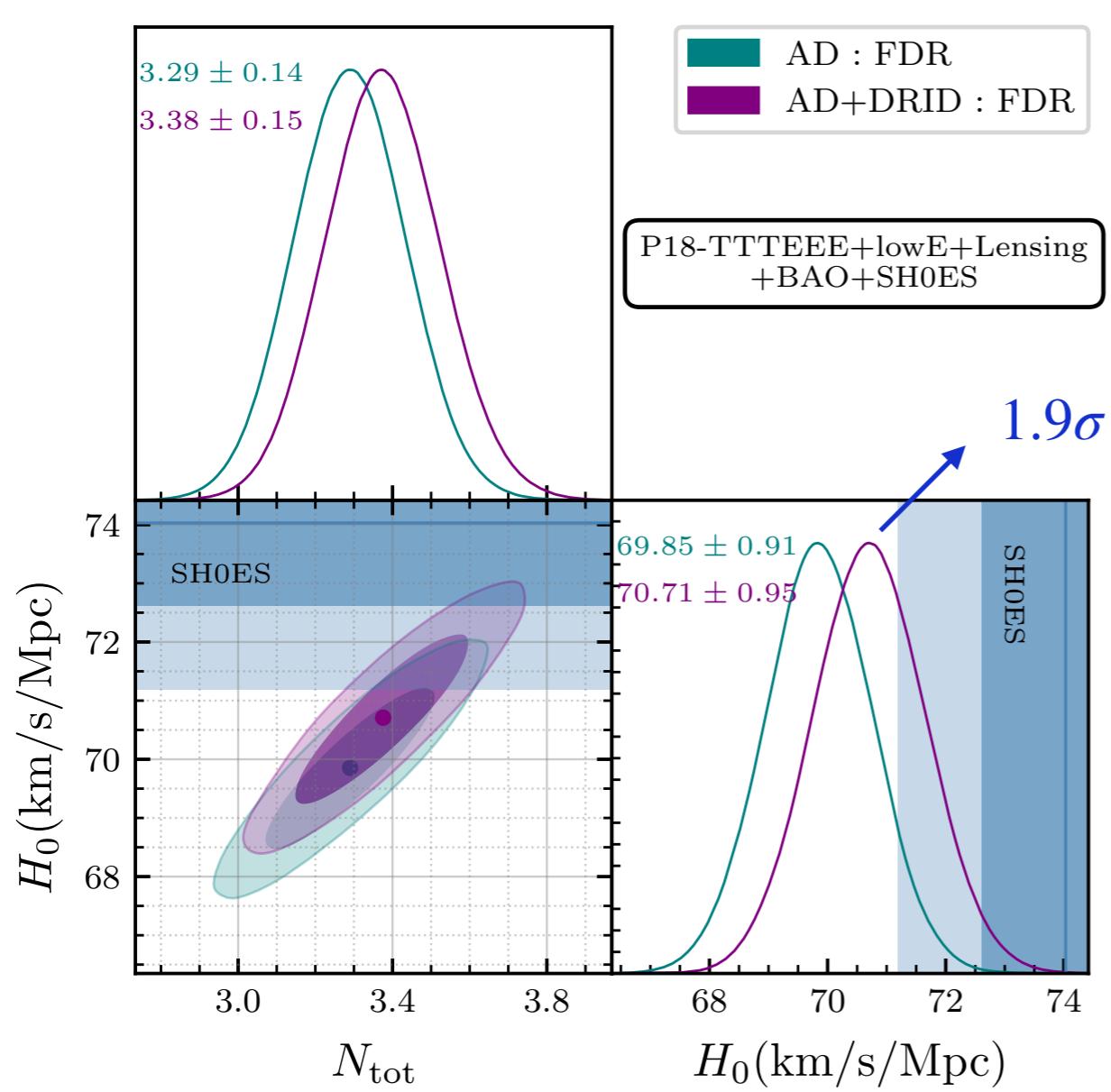


Isocurvature accommodates larger N_{eff} \rightarrow larger H_0

$$H_0 (\text{SH0ES}) = 74.03 \pm 1.42 \text{ km/s/Mpc}$$



Isocurvature accommodates larger N_{eff} \rightarrow larger H_0



Parameter values: FDR DRID

FDR	P18-TT+lowE	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES
$100 \omega_b$	$2.3^{+0.052}_{-0.064}$	$2.253^{+0.025}_{-0.026}$	$2.278^{+0.017}_{-0.017}$
ω_{cdm}	$0.1252^{+0.0046}_{-0.0058}$	$0.12^{+0.0031}_{-0.0031}$	$0.1241^{+0.0027}_{-0.0028}$
$100 * \theta_s$	$1.042^{+0.00073}_{-0.00077}$	$1.042^{+0.00052}_{-0.00053}$	$1.042^{+0.0005}_{-0.0005}$
τ_{reio}	$0.05416^{+0.0085}_{-0.0091}$	$0.05534^{+0.0075}_{-0.008}$	$0.05594^{+0.007}_{-0.0075}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(1)}$	$22^{+1.1}_{-1.1}$	$23.32^{+0.57}_{-0.57}$	$22.88^{+0.49}_{-0.49}$
$10^{10} P_{\mathcal{R}\mathcal{R}}^{(2)}$	$20.55^{+0.57}_{-0.63}$	$20.37^{+0.43}_{-0.46}$	$20.68^{+0.37}_{-0.39}$
$10^{10} N_{dr}^2 P_{\mathcal{I}\mathcal{I}}^{(1)}$	$17.48^{+2.1}_{-17}$	$11.22^{+1.7}_{-11}$	$11.84^{+1.7}_{-12}$
$10^{10} N_{dr}^2 P_{\mathcal{I}\mathcal{I}}^{(2)}$	$228.9^{+61}_{-2.2e+02}$	73.91^{+26}_{-60}	102.1^{+37}_{-67}
N_{ur}	$2.469^{+1.3}_{-0.79}$	$2.031^{+1.1}_{-0.49}$	$2.265^{+1.1}_{-0.47}$
N_{dr}	$1.19^{+0.34}_{-1.2}$	$1.066^{+0.32}_{-1.1}$	$1.111^{+0.33}_{-1.1}$
H_0	$74.03^{+3.9}_{-5.1}$	$68.8^{+1.6}_{-1.7}$	$70.71^{+0.97}_{-0.98}$
σ_8	$0.8231^{+0.015}_{-0.015}$	$0.82^{+0.01}_{-0.01}$	$0.8302^{+0.009}_{-0.0092}$
$10^{+9} A_s$	$2.079^{+0.045}_{-0.049}$	$2.087^{+0.036}_{-0.038}$	$2.105^{+0.033}_{-0.034}$
n_s	$0.9828^{+0.017}_{-0.017}$	$0.9654^{+0.0091}_{-0.0092}$	$0.9741^{+0.0068}_{-0.0068}$
n_{iso}	$1.72^{+0.36}_{-0.32}$	$1.52^{+0.35}_{-0.29}$	$1.61^{+0.32}_{-0.28}$
f_{iso}	$17.4^{+7.0}_{-17}$	$11.9^{+5.2}_{-11}$	$13.0^{+5.3}_{-12}$
N_{tot}	$3.66^{+0.4}_{-0.54}$	$3.097^{+0.21}_{-0.21}$	$3.376^{+0.15}_{-0.16}$
f_{dr}	$0.3285^{+0.097}_{-0.33}$	$0.3444^{+0.1}_{-0.34}$	$0.3293^{+0.095}_{-0.33}$
$\chi^2 - \chi^2_{\Lambda CDM}$	-0.36	-3.54	-9.24

AIC

+7.64

+4.46

-1.24

4 extra parameter
compared to Λ CDM:
 $N_{dr}^2 P_{II}^{(1)}, N_{dr}^2 P_{II}^{(2)}, N_{dr}, N_{ur}$

$$AIC = \Delta\chi^2 + 2n$$

Parameter values: CDR DRID

CDR	P18-TT+lowE	P18-TTTEEE +lowE+lensing	P18-TTTEEE+lowE+ lensing+BAO+SH0ES
$100 \omega_b$	$2.267^{+0.039}_{-0.05}$	$2.257^{+0.026}_{-0.028}$	$2.285^{+0.018}_{-0.018}$
ω_{cdm}	$0.1301^{+0.0053}_{-0.011}$	$0.122^{+0.0034}_{-0.004}$	$0.1272^{+0.0031}_{-0.0035}$
$100 * \theta_s$	$1.042^{+0.0011}_{-0.0012}$	$1.043^{+0.00064}_{-0.00075}$	$1.042^{+0.00065}_{-0.00081}$
τ_{reio}	$0.05327^{+0.0079}_{-0.0087}$	$0.0561^{+0.0075}_{-0.0084}$	$0.05643^{+0.007}_{-0.0077}$
$10^{10} P_{\mathcal{RR}}^{(1)}$	$23.06^{+0.93}_{-0.95}$	$23.46^{+0.55}_{-0.57}$	$23.14^{+0.52}_{-0.54}$
$10^{10} P_{\mathcal{RR}}^{(2)}$	$20.32^{+0.69}_{-0.67}$	$20.19^{+0.45}_{-0.48}$	$20.34^{+0.45}_{-0.44}$
$10^{10} N_{dr}^2 P_{\mathcal{II}}^{(1)}$	25.54^{+4}_{-26}	$16.43^{+2.9}_{-16}$	$15.39^{+2.6}_{-15}$
$10^{10} N_{dr}^2 P_{\mathcal{II}}^{(2)}$	$662.7^{+91}_{-6.6e+02}$	$218.7^{+50}_{-2.2e+02}$	$390.6^{+1.6e+02}_{-3.2e+02}$
N_{ur}	$3.408^{+0.46}_{-0.72}$	$2.938^{+0.24}_{-0.26}$	$3.164^{+0.27}_{-0.24}$
N_{dr}	$0.2589^{+0.051}_{-0.26}$	$0.2444^{+0.064}_{-0.24}$	$0.3372^{+0.14}_{-0.27}$
H_0	$71.79^{+3}_{-4.7}$	$69.19^{+1.7}_{-1.9}$	$71.27^{+1}_{-1.1}$
σ_8	$0.8341^{+0.017}_{-0.023}$	$0.8205^{+0.01}_{-0.011}$	$0.8298^{+0.0096}_{-0.0096}$
$10^{+9} A_s$	$2.077^{+0.054}_{-0.051}$	$2.073^{+0.038}_{-0.04}$	$2.081^{+0.038}_{-0.038}$
n_s	$0.9677^{+0.016}_{-0.016}$	$0.9617^{+0.0092}_{-0.0094}$	$0.9671^{+0.0086}_{-0.0079}$
n_{iso}	$1.83^{+0.45}_{-0.41}$	$1.66^{+0.43}_{-0.35}$	$1.87^{+0.39}_{-0.29}$
f_{iso}	< 31.7	58^{+22}_{-53}	49^{+23}_{-44}
N_{tot}	$3.666^{+0.37}_{-0.71}$	$3.182^{+0.22}_{-0.26}$	$3.501^{+0.17}_{-0.19}$
f_{dr}	$0.07186^{+0.014}_{-0.072}$	$0.07591^{+0.021}_{-0.076}$	$0.09615^{+0.04}_{-0.077}$
$\chi^2 - \chi^2_{\Lambda CDM}$	2.72	0.46	-5.8

AIC

+10.72

+8.46

+2.2

4 extra parameter
compared to Λ CDM:
 $N_{dr}^2 P_{\mathcal{II}}^{(1)}, N_{dr}^2 P_{\mathcal{II}}^{(2)}, N_{dr}, N_{ur}$

$$AIC = \Delta\chi^2 + 2n$$

Constraint on isocurvature parameters

$$\frac{\delta\sigma}{\sigma} \lesssim 2 \times 10^{-4}$$

Isocurvature constraints at 95 % C.L.

	Planck	Planck +BAO+ SHoES
FDR	$\leq 2 \times 10^{-8}$	$\leq 2.2 \times 10^{-8}$
CDR	$\leq 6 \times 10^{-8}$	$\leq 10 \times 10^{-8}$

$$\frac{\delta\sigma}{\sigma} \lesssim 5 \times 10^{-4}$$

95 % C.L. limits of $N_{\text{dr}}^2 P_{II}^{(2)}$ for $N_{\text{dr}} = 0.4$

Planck \equiv TTTEEE+lowE+lensing

$$P_{II}^{(2)} = A_{\text{iso}} \quad (k = 0.1 \text{Mpc}^{-1})$$

Part II : Conclusion: DR isocurvature alleviates Hubble tension

- DR isocurvature is a very generic in multi field inflation models
- In presence of isocurvature perturbation :
FDR gives more anisotropy than CDR
- Blue tilted isocurvature accommodates a larger Hubble constant
- For CDR isocurvature - the Hubble tension is reduced to 1.5σ

THANK YOU