

Matter-Antimatter Asymmetry in Neutral Kaons

Fermilab Theory Seminar

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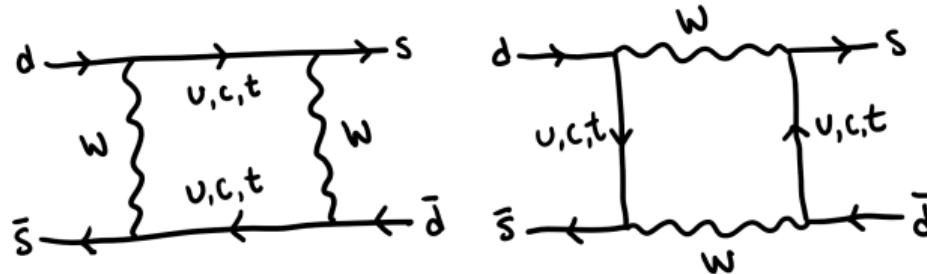
Based on 2108.00017 with J. Brod and S. Kvedaraitė

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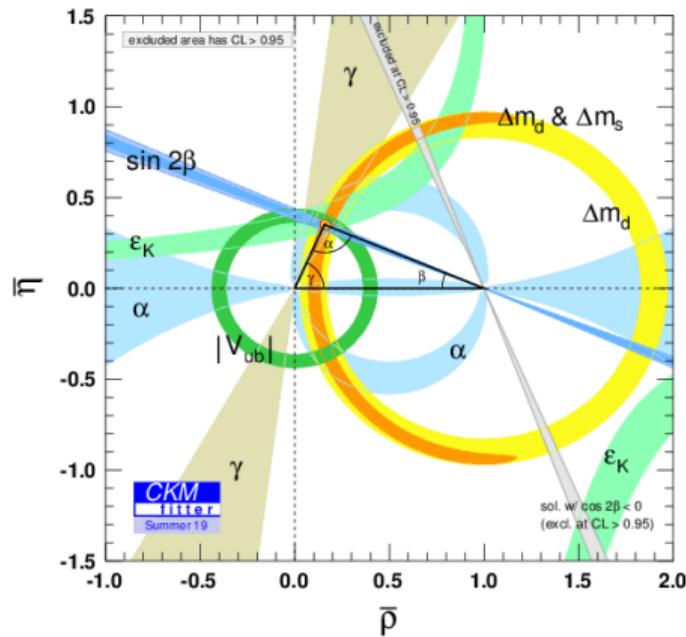
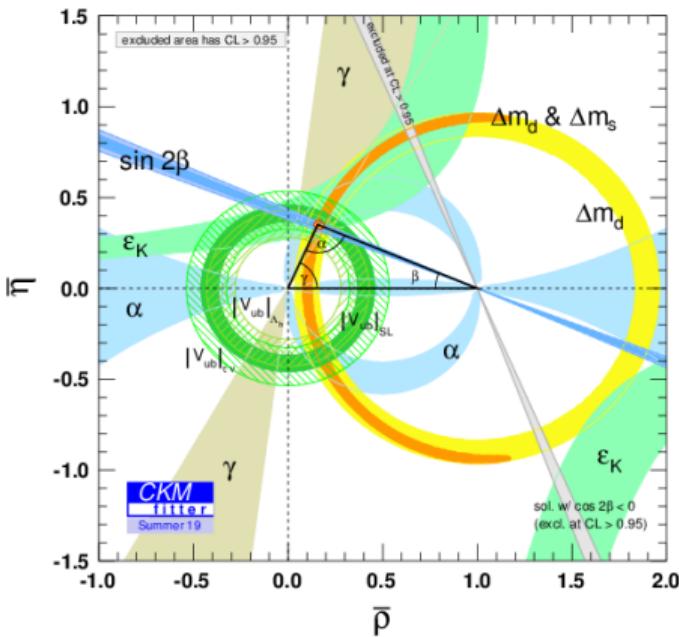
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CP Violation in Neutral Kaons

- *CP* violation first discovered in decays of kaons (1964)
- $K^0 - \bar{K}^0$ mixing can lead to (indirect) *CP*-violation → parameterized by ϵ_K
- Sensitive probe of new physics, input for global CKM fit
- Recently, perturbative theory error greatly reduced → electroweak effects now important!



Reduction of Perturbative Error



Overview

- CP violation in Neutral Kaon System
- Effective Field Theories and Matching
- Results and Discussion

CP Violation in the Neutral Kaon System

Diagonalizing the Hamiltonian

- Time evolution of $K^0 - \bar{K}^0$ system described by

$$i\frac{d}{dt} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right) \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

- Diagonalized by linear combinations

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \quad |K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

- If $|p/q| = 1$, K_L and K_S CP eigenstates $\Rightarrow K_L$ (CP odd) $\rightarrow \pi\pi$ (CP even) forbidden

CP Violation in $K^0 - \bar{K}^0$ System

- Several ways to violate CP :

$|\langle f | K^0 \rangle| \neq |\langle \bar{f} | \bar{K}^0 \rangle|$ (Different decay amplitudes) \rightarrow *Direct / Decay*

$|p| \neq |q|$ (Mass eigenstates not CP eigenstates) \rightarrow *Indirect / Mixing*

$\frac{q \langle f | \bar{K}^0 \rangle}{p \langle f | K^0 \rangle} \neq \pm 1$ (Relative phase between mixing/decays) \rightarrow *Interference*

Definition of ϵ_K

- Indirect CP violation suppressed by GIM → sensitive probe of high energies

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle}$$

- Experimentally¹:

$$|\epsilon_K|_{\text{ex}} = 2.228 \pm 0.011 \times 10^{-3}$$

¹PDG 2020.

Definition of ϵ_K

- After some work...

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left(\frac{\text{Im } M_{12}}{2\Delta M_K} + \xi \right)$$

$$\phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta \Gamma / 2}, \quad \xi \equiv -\frac{\text{Im } \Gamma_{12}}{2 \text{Re } \Gamma_{12}}, \quad \Delta M_K = M_L - M_S, \quad \Delta \Gamma = \Gamma_S - \Gamma_L$$

Definition of ϵ_K

- Combine ξ and “long-distance” part of M_{12} (along with prefactor) into κ_ϵ
- “Short-distance” effects calculated from 3-quark $|\Delta S| = 2$ Hamiltonian

$$\epsilon_K = \frac{\kappa_\epsilon e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\text{Im } M_{12}^{\text{short-dist}}}{\Delta M_K}, \quad M_{12}^{\text{short-dist}} = \frac{1}{2m_K} \langle K^0 | \mathcal{H}_{n_f=3}^{|\Delta S|=2} | \bar{K}^0 \rangle$$

Effective Field Theories and Matching

EFT Basics

- General EFT written as

$$\mathcal{L} = \sum_i \mathcal{C}_i^{(0)} \mathcal{O}_i^{(0)}$$

- $\mathcal{C}_i^{(0)}$ → Wilson coefficients (WCs) (“coupling constants”)
- $\mathcal{O}_i^{(0)}$ → local operators (typically products of fields)
- Renormalize in same way as dimension-4 theory: $\mathcal{C}_i^{(0)} = Z_{ij} \mathcal{C}_j$
- Renormalization Group Equations (RGE) determine scale-dependence of renormalized WCs in terms of anomalous dimension matrix (ADM) and evolution matrix, U

$$\frac{d\mathcal{C}_i}{d \log \mu} = \mathcal{C}_j(\mu) \gamma_{ji} \quad \rightarrow \quad \mathcal{C}_i(\mu) = \mathcal{C}_j(\mu_0) U_{ji}(\mu_0, \mu)$$

EFT Matching Calculations

- Need initial conditions for $\mathcal{C}_i(\mu)$

- Found by requiring

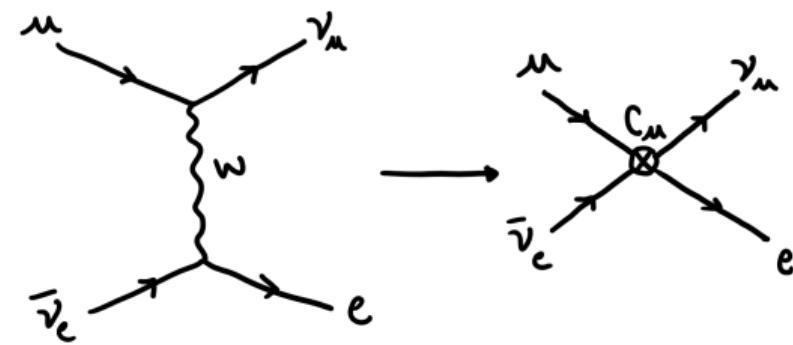
$$\mathcal{A}_{\text{full}}(\mu_{\text{match}}) = \mathcal{A}_{\text{EFT}}(\mu_{\text{match}})$$

- E.g. LO μ -decay:

$$\mathcal{L}_\mu = -\frac{4}{\sqrt{2}} G_F (\bar{\nu}_\mu \mu)_{V-A} (\bar{e} \nu_e)_{V-A}$$

- Calculate SM $\mu \rightarrow \nu_\mu e \bar{\nu}_e$ amplitude ($p_{\text{ext}}^\mu = 0$) and match to EFT amplitude:

$$G_F^{(0)} = \frac{\pi \alpha}{\sqrt{2} M_W^2 s_w^2}$$



Evanescence Operators

- Higher orders \rightarrow Dirac structures like $\gamma^\mu \gamma^\nu \gamma^\rho \otimes \gamma_\mu \gamma_\nu \gamma_\rho$, etc.
- Use dimensional regularization ($d = 4 - 2\epsilon$) \Rightarrow can't use γ_5 relations!
- Introduce *evanescent operators* (unphysical) which vanish when $d \rightarrow 4$, e.g.

$$E = \gamma^\mu \gamma^\nu \gamma^\rho \otimes \gamma_\mu \gamma_\nu \gamma_\rho - (16 - a_{11}\epsilon - a_{12}\epsilon^2 - \dots) \gamma^\mu \otimes \gamma_\mu$$

- Subtlety: After renormalization, $Z_{iE} E$, $1/\epsilon^n$ terms in Z_{iE} cancel ϵ^n term in $E \rightarrow$ values of a_{ij} define renormalization scheme (arbitrary)

Weak Hamiltonian

- Recall...

$$M_{12}^{\text{short-dist}} = \frac{1}{2m_K} \langle K^0 | \mathcal{H}_{n_f=3}^{|\Delta s|=2} | \bar{K}^0 \rangle$$

- Effective Hamiltonian given by

$$\mathcal{H}_{n_f=3}^{|\Delta s|=2} = \frac{G_F^2 M_W^2}{4\pi^2} [\lambda_u^2 C_{S2}''^{uu}(\mu) + \lambda_t^2 C_{S2}''^{tt}(\mu) + \lambda_u \lambda_t C_{S2}''^{ut}(\mu)] Q_{S2}'' + h.c. + \dots$$

$$\lambda_i = V_{is}^* V_{id}, \quad Q_{S2}'' = (\bar{s}_L^\alpha \gamma^\mu d_L^\alpha)(\bar{s}_L^\beta \gamma_\mu d_L^\beta)$$

Why Electroweak?

- Conventional parameterization:

$$\mathcal{H}_{n_r=3}^{|\Delta s|=2} = \frac{G_F^2 M_W^2}{4\pi^2} [\lambda_c^2 C''_{S2}^{cc}(\mu) + \lambda_t^2 C''_{S2}^{tt}(\mu) + \lambda_c \lambda_t C''_{S2}^{ct}(\mu)] Q''_{S2} + h.c. + \dots$$

- Related via $\lambda_c = -\lambda_u - \lambda_t$ (CKM unitarity)
- Drastic reduction of perturbative theory errors²³

$$|\epsilon_K|_{\text{th}} = 1.81(16)_{\text{pert}}(5)_{\text{non-pert}}(23)_{\text{param}} \times 10^{-3} \rightarrow |\epsilon_K|_{\text{th}} = 2.16(6)_{\text{pert}}(7)_{\text{non-pert}}(15)_{\text{param}} \times 10^{-3}$$

- Perturbative errors \lesssim expected e/w effects \rightarrow relevant!

²Brod, Gorbahn, Stamou, Phys.Rev.Lett. 125 (2020) 17, 171803, 1911.06822 [hep-ph].

³Brod, Gorbahn, Phys.Rev.Lett. 108 (2012) 121801, 1108.2036 [hep-th].

EFT Normalization

- Including e/w effects \Rightarrow EFT normalizations no longer equivalent
 - Different normalizations have different dependence on matching scale

$$\frac{\alpha(\mu)^2}{8M_W(\mu)^2 s_w(\mu)^4} \neq \frac{\alpha(\mu)G_F}{4\sqrt{2}\pi s_w(\mu)^2} \neq \frac{G_F^2 M_W(\mu)}{4\pi^2}$$

- Relate e/w parameters and G_F through NLO muon decay

Factorization of Evolution Matrix

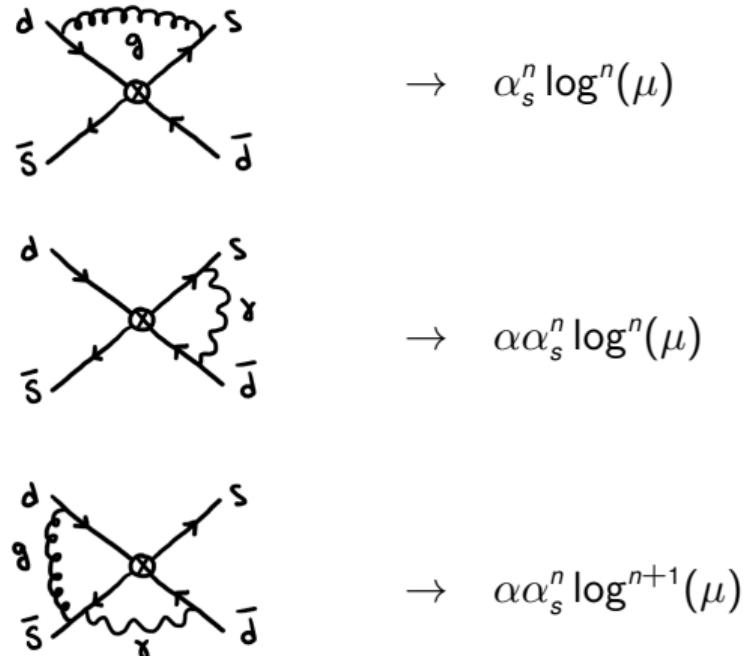
- With QCD, *amplitude factorizes into two separately scheme- and scale-independent pieces*

$$\lambda_i \lambda_j C^{ij}(\mu) U(\mu, \mu_0) \langle Q_{S2} \rangle (\mu_0) = \lambda_i \lambda_j \left[C^{ij}(\mu) J(\mu) U^{(0)}(\mu) \right] \left[\left(U^{(0)}(\mu_0) \right)^{-1} J^{-1}(\mu_0) \langle Q_{S2} \rangle (\mu_0) \right]$$

- Split up as:
 - Perturbative piece** with $C^{ij}(\mu) \rightarrow \eta_{ij} S(x_i, x_j)$
 - Non-perturbative piece** with $\langle Q_{S2} \rangle (\mu_0) \rightarrow \hat{B}_K$
- When including QED, equations for $J(\mu)$ singular \rightarrow can't write analogous expression!

EFT at NLO

- QCD contribution to initial conditions from Buras, Jamin, Weisz⁴
- Two-loop QCD + QED ADM needed to resum all logs
- Downside: can no longer formally define η_{tt} to include QED
- Upside: EFT scheme-dependence $\sim \mathcal{O}(\alpha)$ and can be neglected

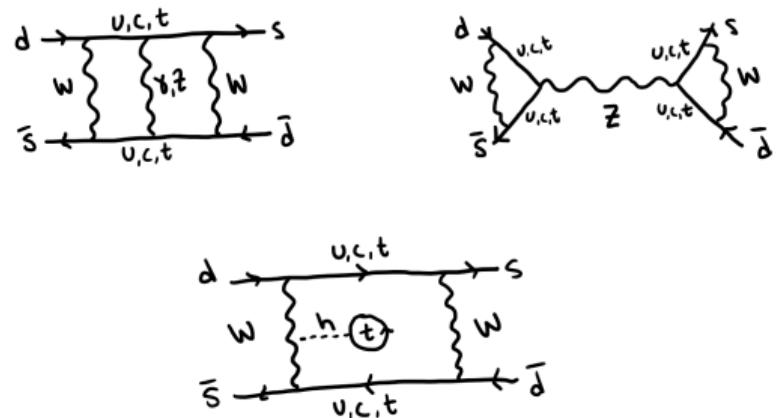


⁴Nucl. Phys. B 347 (1990) 491-536.

Calculation of C_{S2}^{tt} : Results and Discussion

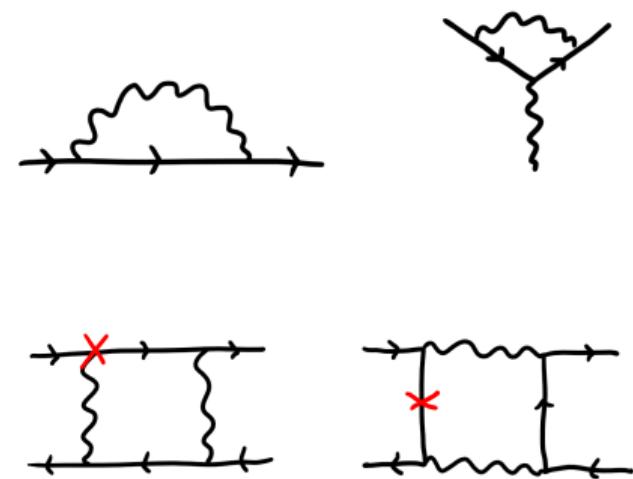
Calculation Above E/W Scale

- Calculate all $O(30,000)$ two-loop Feynman diagrams including Goldstones, ghosts, tadpoles and “Z-bridges”
- Diagrams generated by qgraf and calculated using self-written FORM routines
- Separately calculated and verified agreement



Renormalization Above E/W Scale

- Full one-loop renormalization of all *independent* e/w parameters
 - E.g. $\{M_W, m_t, M_Z, M_h, \alpha\}$ or $\{M_W, m_t, \sin^2 \theta_W, M_h, \alpha\}$
 - Different sets of input parameters better suited for different e/w renorm. schemes
- Two options for NLO:
 1. Replace bare parameters \rightarrow renormalized in LO result, expand in α
 2. Calculate all diagrams with counterterm insertions up to desired order in α
- Explicitly showed these are identical
- All UV divergences cancel, leftover IR divergences match those in EFT



A “Small” Correction

$$C_{S2}^{ll,(sc)} = [128y - 896yz - 384xyz + 288xz^2 + 2752y^2 + 1504xyz^2 + 448x^2yz^2 - 1080x^2z^3 - 7536xyz^3 + 872xy^2z^3 - 256x^2yz^3 - 576y^2z^3 + 144x^4z^4 - 864x^3z^4 + 608yz^4 - 1240xyz^4 + 1564x^2yz^4 - 1664x^3yz^4 + 64x^4yz^4 + 2016x^2y^2z^4 + 288y^2z^4 - 540x^2z^4 + 6408xyz^3 - 1632xyz^5 + 4740x^2yz^5 - 10694x^3yz^5 + 184x^4yz^5 - 3438x^2y^2z^5 - 720xy^3z^5 + 432x^3z^6 - 5184x^2y^3z^6 + 2117x^2yz^6 - 1668x^3yz^6 - 4056x^2y^2z^6 + 1872x^3y^2z^6 + 7652x^2y^3z^6 - 36x^4z^7 + 432x^6z^7 - 373x^8yz^7 + 364x^4yz^7 + 1080x^2yz^7 - 198x^3yz^7 - 225x^2y^2z^7] / [288x(z-1)(xz-1)^3]$$

$$+ \pi^2 [-16x + 128xz + 16x^2z - 404xz^2 - 24x^2z^2 + 8x^3z^2 + 472xz^3 - 156x^2z^3 - 8x^3z^3 + 32x^4z^3 - 32x^5z^3 + 148x^2yz^3 - 134x^3yz^3 + 48x^4yz^3 + 8x^5yz^3 - 16x^5 - 64x^2z^5 - 24x^3z^5 + 44x^2yz^5 - 24x^4z^5 + 8x^5z^5 + 2x^6z^5 - 96x^2y^6z^6 + 2x^3y^6z^6 - 18x^2yz^6z^6 - 6x^2z^7 - 108x^2yz^7 - x^2y^2z^7] / [216x^2(z-1)^3]$$

$$+ \Phi\left(\frac{1}{4x}\right) [-256z + 2816xz - 10624x^2z + 15232x^3z - 8192x^4z - 2048x^5z + 128x^6z - 512x^7z - 3584x^8z^2 + 21888x^9z^2 - 27520x^10z^2 + 11264x^11z^2 + 10240x^12z^2 - 16z^3 - 32x^3z + 1304x^2z^3 - 4752x^3z^3 + 2928x^4z^3 - 12704x^5z^3 + 1024x^2yz^3 - 14848x^2z^4 - 32x^4z^4 - 10562x^2z^4 + 5936x^3z^4 - 4640x^4z^4 - 27952x^2z^5 + 72256x^3z^5 - 3840x^4z^5 + 7168x^5z^5 - 8x^2z^5 + 2304x^3z^5 - 19252x^4z^5 + 50320x^5z^5 - 35164x^6z^5 - 40352x^7z^5 + 19456x^8z^5 - 512x^9z^5 - 1532x^10z^5 + 11756x^11z^5 - 23860x^12z^6 + 4184x^13z^6 + 13696x^14z^6 - 2816x^15z^6 + 41x^16z^7 - 198x^17z^7 - 250x^18z^7 + 1612x^19z^7 - 252x^20z^7 - 1280x^21z^7] / [576x^2(4x-1)(z-1)(xz-1)^4]$$

$$+ \Phi\left(\frac{y}{4x}\right) [-176x^2z^3 + 256xyz^3 - 56y^2z^3 + 656x^3z^4 - 1088x^2yz^4 + 304xy^2z^4 - 16y^3z^4 - 1164x^4z^5 + 1740x^3yz^5 - 564x^2y^2z^5 + 48xy^3z^5 + 728x^5z^6 - 948x^2y^3z^6 + 32x^4y^2z^6 - 32x^5y^2z^6 - 44x^6y^2z^7 + 112x^7yz^7 - 66x^4y^2z^7 + 14x^3y^3z^7 - x^2y^4z^7] / [64(z-1)(xz-1)^4]$$

$$+ \Phi\left(\frac{z}{4}\right) [256x^2z + 7872x^2z^2 - 512x^3z^2 - 23072x^2z^3 - 17216x^3z^3 + 256x^4z^3 + 5664x^2z^4 - 1680x^3z^4 + 2048x^4z^4 + 5688x^2z^5 - 6240x^3z^5 - 2768x^4z^5 - 1222x^2z^6 - 1636x^3z^6 + 2032x^4z^6 - 73x^5z^7 + 900x^2z^7 + 322x^3z^7 + 32x^5z^8 - 208x^4z^8 - 8x^5z^9] / [576(z-1)(xz-1)^4]$$

$$+ \Phi\left(\frac{yz}{4}\right) [144x^2z^3 + 48x^3z^4 - 224x^2yz^4 - 192x^4z^5 + 336x^3yz^5 + 132x^2y^2z^5 - 88x^4yz^6 - 228x^3y^2z^6 - 18x^2y^3z^6 + 98x^4y^2z^7 + 36x^3y^3z^7 - x^2y^4z^7 - 16x^4yz^8] / [64(z-1)(xz-1)^4]$$

$$+ \Phi\left(\frac{1}{xz}, \frac{1}{x}\right) [64 + 576z + 384xz - 2128z^2 - 2848xz^2 - 928x^2z^2 + 3824z^3 + 8040xz^3 + 5664x^2z^3 + 1120x^3z^3 - 3312z^4 - 12752xz^4 - 14056x^2z^4 - 5816x^3z^4 - 672x^4z^4 + 992z^5 + 9472xz^5 + 14904x^2z^5 + 11672x^3z^5 + 3544x^4z^5 + 224x^5z^5 + 272z^6 - 3032xz^6 - 6904x^2z^6 - 98666x^3z^6 - 6800x^4z^6 - 1840x^5z^6 - 224x^6z^6 - 144z^7 - 424xz^7 + 2232x^2z^7 + 4678x^3z^7 + 6914x^4z^7 + 4536x^5z^7 + 1104x^6z^7 + 288x^7z^7 - 16z^8 + 456x^8z^8 - 592x^2z^8 - 607x^3z^8 + 644x^4z^8 + 368x^5z^8 - 2664x^6z^8 - 408x^7z^8 - 160x^8z^8 + 40x^9z^9 - 304x^2z^9 - 1787x^3z^9 + 2891x^4z^9 + 2268x^5z^9 - 2720x^6z^9 + 1000x^7z^9 - 8x^8z^9 + 32x^9z^9 - 16x^2z^10 + 565x^3z^10 - 1766x^4z^10 + 2963x^5z^10 - 2524x^6z^10 + 1018x^7z^10 - 272x^8z^10 + 32x^9z^10 + 41x^3z^11 - 403x^4z^11 + 1093x^5z^11 - 1279x^6z^11 + 694x^7z^11 - 154x^8z^11 + 8x^9z^11] / [288x^2(z-1)z(xz-1)^4]$$

$$+ \Phi\left(\frac{1}{xz}, \frac{y}{x}\right) [-16 + 104xz + 64yz - 296x^2z^2 - 296xyz^2 - 96y^2z^2 + 476x^3z^3 + 544x^2yz^3 + 344xy^2z^3 + 64y^3z^3 - 436x^4z^4 - 568x^3yz^4 - 416x^2y^2z^4 - 216xy^3z^4 - 16y^4z^4 + 160x^5z^5 + 326x^2yz^5 + 271x^3y^2z^5 + 192x^2y^3z^5 + 64xy^4z^5 + 80x^6z^6 - 106x^2y^6z^6 - 185x^3y^2z^6 + 19x^3y^3z^6 - 88x^4y^2z^6 - 100x^7z^7 + 114x^8yz^7 - 31x^5y^2z^7 - 24x^4y^3z^7 + 41x^4yz^7 + 28x^8z^8 - 78x^7yz^8 + 65x^6y^2z^8 - 7x^5y^3z^8 - 9x^4y^4z^8 + x^3y^5z^8] / [32x^2(z-1)z(xz-1)^4]$$

$$+ \text{Li}_2\left(1 - \frac{1}{z}\right) [16 - 112z - 48xz + 292z^2 + 232xz^2 + 40x^2z^2 - 260z^3 - 420xz^3 - 128x^2z^3 + 16z^4 + 236xz^4 + 114x^2z^4 - 40x^3z^4 + 44x^5 - 64x^2z^5 + 186x^2z^5 + 72x^3z^5 + 4x^6 - 42xz^6 + 8x^2z^6 - 155x^2z^6 - 2xz^7 - 54x^2z^7 + 31x^3z^7 - 4x^2y^2z^8 + 36x^3z^8 + 2x^3z^9] / [36x^2(z-1)^2]$$

$$+ \text{Li}_2(1 - yz) [4 - 14xz - 12yz + 20x^2z^2 + 24xyz^2 + 12y^2z^2 - 13x^3z^3 - 12x^2yz^3 - 18xy^2z^3 - 4y^3z^3 + 6x^3yz^4 - 6x^2y^2z^4 + 8xy^3z^4 + 9x^3y^2z^5 - 2x^2y^3z^5 - 2x^3y^3z^6] / [4x(z-1)(xz-1)^2]$$

$$+ \text{Li}_2(1 - xz) [- 32 + 256z + 128xz - 808z^2 - 816xz^2 - 176x^2z^2 + 944z^3 + 2112xz^3 + 896x^2z^3 + 80x^3z^3 - 2504xz^4 - 1788x^2z^4 - 396x^3z^4 + 1968x^2z^5 + 616x^3z^5 + 164x^4z^5 + 32x^5z^5 - 3x^3z^6 - 644x^4z^6 - 164x^5z^6 - 48x^6z^6 - 375x^{4,7} + 332x^5z^7 + 44x^6z^7 + 16x^7z^7 + 25x^8z^8 - 112x^6z^8 + 16x^7z^8 + 449x^6z^9 + 4x^7z^9 - 216x^7z^{10}] / [72x(z-1)^3(zx-1)^3]$$

$$+ \text{Li}_2\left(1 - \frac{1}{y}\right) [9x^2z^3 - 18xyz^3 + 9y^2z^3 - 20x^3z^4 + 42x^2yz^4 - 24xy^2z^4 + 2y^3z^4 + 8x^4z^5 - 18x^3yz^5 + 12x^2y^2z^5 - 2xy^3z^5] / [4(z-1)(xz-1)^2]$$

$$+ \text{Li}_2(1 - x) [- 64z + 192xz - 160z^2 + 16x^3z + 16x^5z^3 + 32z^2 - 32xz^2 - 72x^2z^2 - 16x^3z^2 + 200x^4z^2 - 96x^5z^2 - 16x^6z^2 - 4z^3 - 2xz^3 + 141x^2z^3 - 256x^3z^3 + 165x^4z^3 - 84x^5z^3 + 32x^6z^3 + 8x^7z^3 + 4xz^4 - 98x^2z^4 + 136x^3z^4 - 64x^4z^4 - 16x^5z^4 - 28x^6z^4 + 8x^7z^4 + 84x^3z^5 - 188x^4z^5 + 126x^5z^5 - 24x^6z^5 + 2x^7z^5] / [36x^2(z-1)(xz-1)^2]$$

$$+ \log^2(x) [256yz - 1280xyz + 1408x^2yz - 128x^3yz - 128y^2z^2 - 512xy^2yz - 4096x^2yz^2 - 4352x^3yz^2 - 1216x^4yz^2 + 768x^5yz^2 + 512xy^2z^2 + 4096x^2y^2z^2 - 4352x^3y^2z^2 - 1216x^4y^2z^2 + 768x^5y^2z^2 + 512x^6yz^2 + 16y^3z^2 + 256xy^2z^3 - 376x^2yz^3 - 3767x^2yz^3 + 8160x^3yz^3 - 2560x^4yz^3 - 832x^5yz^3 + 1296x^3y^2z^3 - 504x^2y^2z^3 - 48xy^3z^4 + 736x^2yz^4 + 72x^3yz^4 - 3904x^4yz^4 - 7296x^5yz^4 - 12608x^6yz^4 + 3008x^7yz^4 + 704x^8yz^4 - 6480x^4y^2z^4 + 2952x^3y^2z^4 - 144x^2y^2z^4 - 5184x^7z^5 + 40x^2yz^5 - 3120x^3yz^5 + 6228x^4yz^5 + 968x^5yz^5 - 13272x^6yz^5 + 6000x^7yz^5 - 1344x^8yz^5 - 320x^9yz^5 + 13284x^2yz^5 - 6660x^4yz^5 + 576x^3yz^5 + 10368x^2y^2z^6 - 8x^3yz^6 + 3788x^4yz^6 - 8232x^5yz^6 + 2092x^6yz^6 - 2760x^7yz^6 + 48x^2y^2z^7 + 64x^2yz^6 + 64x^10yz^6 - 12528x^6yz^6 + 6552x^5yz^6 - 720x^4yz^6 - 5184x^9yz^7 - 41x^4yz^7 - 1580x^5yz^7 + 2772x^6yz^7 + 804x^7yz^7 - 5584x^8yz^7 - 272x^9yz^7 + 64x^10yz^7 + 4824x^7yz^7 - 2736x^6y^2z^7 + 396x^5y^4z^7 - 9x^4y^5z^7 + 41x^5yz^8]$$

A “Small” Correction

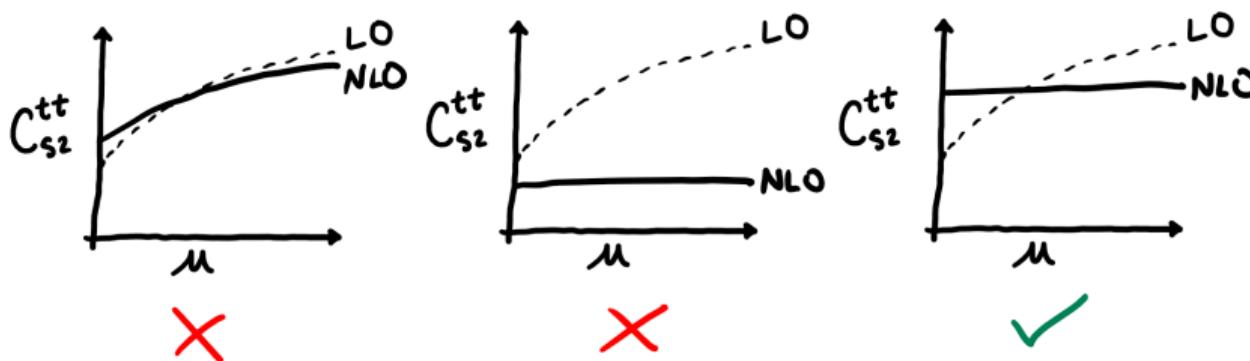
$$\begin{aligned}
 & + 48x^6yz^6 - 208x^7yz^5 - 88x^8yz^4 + 3696x^9yz^3 + 16x^{10}yz^2 \\
 & - 396x^9y^2z^3 + 396x^8y^3z^2 - 108x^8y^4z^3 + 9x^8y^5z^3 - 864x^{10}y^2z^3 \\
 & / [576x^6y(z-1)(xz-1)^3] \\
 & + \log^2(y)[9x^3z^2 - 18xyz^3 + 7y^2z^3 - 20x^4z^4 + 42x^2yz^4 - 20xy^2z^4 + 2y^3z^4 \\
 & + 8x^2z^5 - 18x^2yz^5 + 10x^2y^2z^5 - 2xy^3z^5] / [(z-1)(xz-1)^2] \\
 & + \log(z)[128y - 832yz - 448yz^2 + 1776yz^3 + 297xyz^2 + 512x^2yz^3 - 192yz^3 \\
 & - 6256xyz^3 - 4784x^2yz^3 - 160x^3yz^3 - 144y^2z^3 + 216xz^4 \\
 & - 648x^2z^4 - 16yz^4 + 232xy^2z^3 + 10336x^2y^2z^3 + 4032x^4y^2z^3 \\
 & - 64x^4yz^3 - 684x^2yz^3 - 1026x^2yz^4 + 3588x^3yz^4 + 40xy^5z^4 \\
 & + 268x^2yz^5 - 5374x^3yz^5 - 1576x^4yz^5 + 32x^5yz^5 - 1134x^3y^2z^5 \\
 & + 1566x^4y^2z^5 - 7128x^5y^2z^5 - 491x^3yz^6 + 9007x^4yz^6 + 1168x^5yz^6 \\
 & + 184x^3y^3z^6 + 1377x^4y^3z^6 - 171x^5y^3z^6 - 810x^3yz^7 + 4104x^4yz^7 \\
 & + 2031x^3yz^8 + 340x^4yz^8 - 1938x^5yz^8 - 648x^3y^2z^7 + 189x^3y^3z^7 \\
 & + 54x^3y^4z^7 - 216x^3y^5z^7 - 96x^3y^6z^7 - 78x^3y^7z^7 + 567x^3y^8z^7 \\
 & + 27x^3y^9z^7 - 72x^3y^10z^7] / [(144y(z-1)(xz-1)^4] \\
 & + \log^2(z) - 288y + 1872yz - 560x^2yz + 864x^2z - 5328x^3yz^2 + 1424x^2yz^2 \\
 & + 2096x^3yz^3 + 128x^3yz^4 - 4320x^2y^2z^2 - 864x^2y^3z^2 + 2348x^3y^2z^2 \\
 & - 2672x^4yz^2 - 3168x^3yz^5 - 448x^3yz^6 + 864x^2y^2z^3 + 3888x^3yz^3 \\
 & + 288y^2z^3 - 240x^4yz^3 - 5916x^2y^2z^4 + 1440x^3y^2z^4 + 2404x^4y^2z^4 \\
 & + 576x^2yz^4 - 986x^2y^2z^4 - 6048x^2y^3z^4 + 1040x^3y^3z^4 - 2928x^4y^3z^4 \\
 & - 5184x^3y^4z^4 + 2806x^2y^4z^4 - 16040x^3y^2z^5 + 20288x^4y^2z^5 - 2928x^5y^2z^5 \\
 & - 944x^3y^5z^5 - 320x^3y^6z^5 - 7920x^3y^7z^5 + 3618x^3y^8z^5 + 2736x^3y^9z^5 \\
 & - 2502x^3y^10z^5 + 10368x^3y^11z^5 - 1368x^3y^12z^5 + 2450x^3y^13z^5 + 9708x^3y^14z^5 \\
 & - 11280x^3y^15z^5 + 2240x^3y^16z^5 + 48x^3y^17z^5 + 64x^3y^18z^5 - 4212x^3y^19z^5 \\
 & - 270x^3y^20z^5 - 2484x^3y^21z^5 + 1296x^3y^22z^5 - 5184x^3y^23z^5 - 731x^3y^24z^5 \\
 & + 2332x^3y^25z^5 - 498x^3y^26z^5 - 1916x^3y^27z^5 - 3060x^3y^28z^5 - 480x^3y^29z^5 \\
 & + 64x^3y^30z^5 + 972x^3y^31z^5 - 378x^3y^32z^5 + 1080x^3y^33z^5 - 9x^3y^34z^5 \\
 & + 105x^3y^35z^5 - 1188x^3y^36z^5 + 142x^3y^37z^5 + 3524x^3y^38z^5 + 167x^3y^39z^5 \\
 & + 54x^3y^40z^5 - 180x^3y^41z^5 + 9x^3y^42z^5 - 40x^3y^43z^5 + 224x^3y^44z^5 \\
 & - 864x^3y^45z^5 - 82x^3y^46z^5] / [576x^6y(z-1)(xz-1)^5] \\
 & + \log\left(\frac{\mu}{M_W}\right)\left\{ \left[(432xz - 1408xyz - 1620x^2z^2 + 1072yz^2 + 5280x^3yz^2 \right. \right. \\
 & + 216xz^3 + 432x^2z^3 - 312xyz^3 - 3732x^2yz^3 - 4224x^3yz^3 \\
 & + 108xyz^4 - 810x^2yz^4 + 3132x^2yz^5 + 1323x^3yz^5 - 312x^3yz^6 \\
 & + 352x^2yz^7 - 405x^3yz^7 + 648x^3yz^8 - 2592x^4yz^8 - 171x^5yz^8 \\
 & \left. \left. + 137x^3yz^9 - 324x^4yz^9 - 54x^5yz^9 + 216x^6yz^9 + 78x^7yz^9 - 135x^8yz^9 \right] \right. \\
 & \left. - 27x^9yz^9 / [72y(z-1)(xz-1)^6] \right\} \\
 & + [108x^2z^3 - 352x^3yz^3 - 108x^4z^4 + 436x^5yz^4 + 352x^4yz^4 \\
 & + 54x^4z^5 - 216x^5z^5 - 180x^3yz^5 + 113x^4yz^5 + 27x^5yz^5 \\
 & - 54x^4z^6 + 216x^5z^6 + 27x^4yz^6 - 54x^5yz^6 - 27x^4y^2z^6] \\
 & \times \log(xz) / [12y(z-1)(xz-1)^6] \Big\} \\
 & + \log(y)\left[- 16y^2z^3 + 40xy^3z^3 - 16y^3z^3 - 30x^2yz^4 - 10xy^2z^4 + 7x^3yz^3 \right. \\
 & + 43x^2y^2z^3 - 10xy^3yz^3 - 13x^3y^2z^3] / [16(z-1)(xz-1)^3] \\
 & + [16 - 88xz - 48xy^2 + 208x^2z^2 + 192xyz^2 + 48y^2z^2 - 268x^2z^3 \\
 & - 288x^2yz^3 - 168xy^2z^3 - 16y^2z^3 + 168x^3z^4 + 284x^2yz^4 \\
 & + 168x^2y^2z^4 - 61xy^3z^4 + 8x^3yz^4 - 210x^2yz^5 - 39x^3y^2z^5 \\
 & - 88x^2y^3z^5 - 72x^3yz^5 - 120x^2yz^6 - 43xy^3yz^6 - 58x^3y^2z^6 \\
 & + 28x^2y^4z^6 - 50x^3yz^6 + 21x^3y^2z^7 - 12x^3y^3z^7] \\
 & \times \log(z) / [32z(z-1)(xz-1)^4] \\
 & + [- 64y + 512yz + 384xyz - 1616yz^2 - 2656xy^2z^2 - 928x^2yz^2 \\
 & + 2064yz^3 + 7456xyz^3 + 5568x^2yz^3 + 1120x^3yz^2 - 1104yz^4 \\
 & - 8312xyz^4 - 13640x^2yz^4 - 6008x^3yz^4 - 672x^4yz^4 + 432y^2z^4 \\
 & - 112y^3z^5 + 4512xyz^5 + 11544x^2yz^5 + 12608x^3yz^5 + 3704x^4yz^5 \\
 & + 224x^3yz^6 - 2160xy^2z^5 - 432y^3z^5 + 160y^4z^6 + 248xyz^6 \\
 & - 6200x^2yz^6 - 6630x^3yz^6 - 3968x^4yz^6 - 1776x^5yz^6 - 224x^6yz^6 \\
 & + 4320x^7yz^6 + 1944x^8yz^6 + 144y^4z^6 + 16y^7z - 640xyz^7 \\
 & + 176x^2yz^7 + 5732x^3yz^7 - 10218x^4yz^7 - 2248x^5yz^7 + 1072x^6yz^7 \\
 & + 288x^7yz^7 - 4716x^8yz^7 - 3024x^2y^2z^7 - 720xyz^4z^7 + 648x^4z^8 \\
 & - 5184x^6z^8 - 56xyz^8 + 760x^2yz^8 + 1197x^3yz^8 - 6256x^4yz^8 \\
 & + 25032x^5yz^8 + 520x^6yz^8 - 472x^7yz^8 - 160x^8yz^8 + 3474x^4y^2z^8 \\
 & + 1755x^2y^3z^8 + 1368x^3y^2z^8 - 1296x^5z^9 + 10368x^7z^9 + 56x^8yz^9 \\
 & - 870x^3yz^9 - 53x^4yz^9 + 2044x^5yz^9 - 11776x^6yz^9 + 1120x^7yz^9 \\
 & + 24x^8yz^9 + 32x^9yz^9 - 1242x^5y^2z^9 - 351x^4y^3z^9 - 1170x^3y^4z^9 \\
 & + 648x^5z^10 - 5184x^8z^10 - 57x^3yz^10 + 952x^4yz^10 - 2187x^5yz^10 \\
 & + 1820x^6yz^10 - 2962x^7yz^10 - 240x^8yz^10 + 32x^9yz^10 \\
 & - 558x^6yz^10 + 243x^5y^2z^10 + 450x^4y^4z^10 - 9x^2yz^10 \\
 & + 41x^4yz^{11} - 362x^5yz^{11} + 731x^6yz^{11} - 548x^7yz^{11} \\
 & + 3418x^8yz^{11} + 8x^9yz^{11} + 450x^7y^2z^{11} - 135x^6y^3z^{11} \\
 & - 72x^8y^2z^{11} + 9x^2yz^{11} - 864x^9yz^{12}] \\
 & \times \log(z) / [288xy(z-1)^2(zx-1)^2] \Big\} \\
 & \times \log(y) / [32z(z-1)(xz-1)^4]
 \end{aligned}$$

EFT Scheme Dependence

- Any physical observable must be independent of renormalization scheme!!!
- EFT scheme dependence in C_{S2}^{tt} must cancel with similar dependence in either
 1. Evolution matrix, $U(\mu, \mu_0)$ (QCD case)
 2. Hadronic matrix element, $\langle Q_{S2} \rangle$
- QED ADM scheme-independent \Rightarrow must be case (2)
- Neglect scheme-dependence ($\mathcal{O}(\alpha)$)

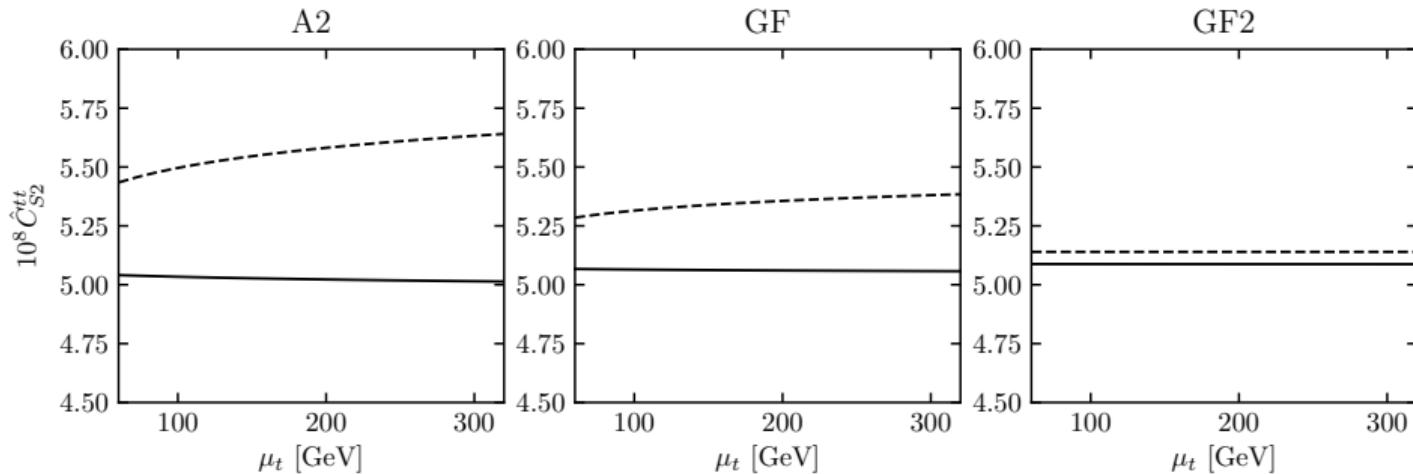
What We Want to See

- Matching scale dependence drops out (up to higher-order effects) \rightarrow NLO line should be flat
- Good perturbative convergence \rightarrow NLO corrections not large compared to LO

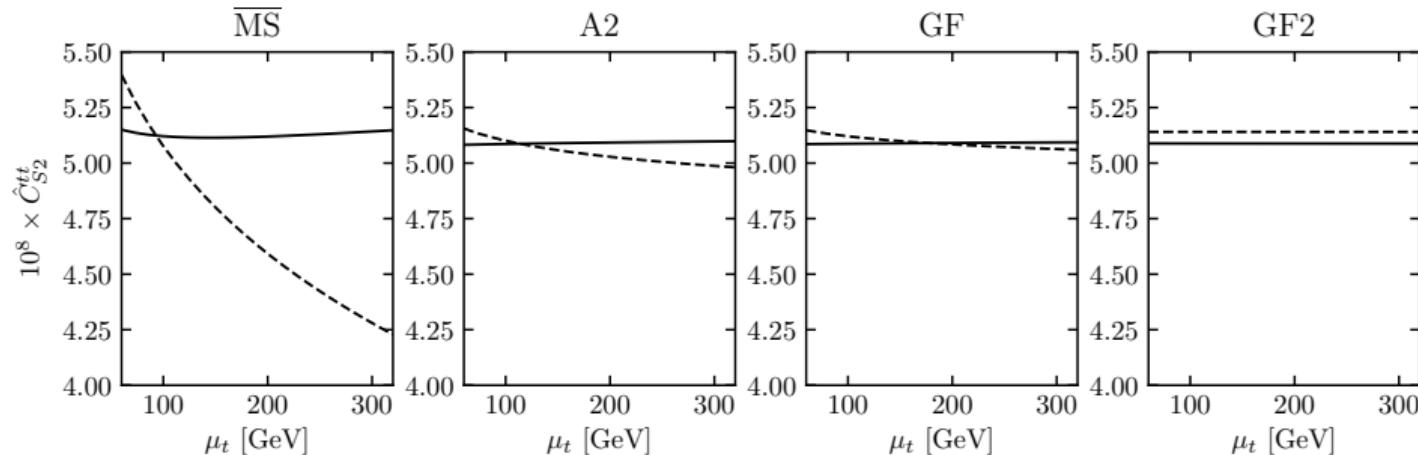


- Test several e/w renormalization schemes to ensure smallest matching scale dependence and perturbative convergence:
 1. Masses and s_w on-shell, couplings \overline{MS} \equiv On-shell
 2. All e/w parameters \overline{MS} \equiv \overline{MS}
 3. Masses on-shell, couplings and s_w \overline{MS} \equiv Hybrid

E/W Contribution: On-Shell Scheme

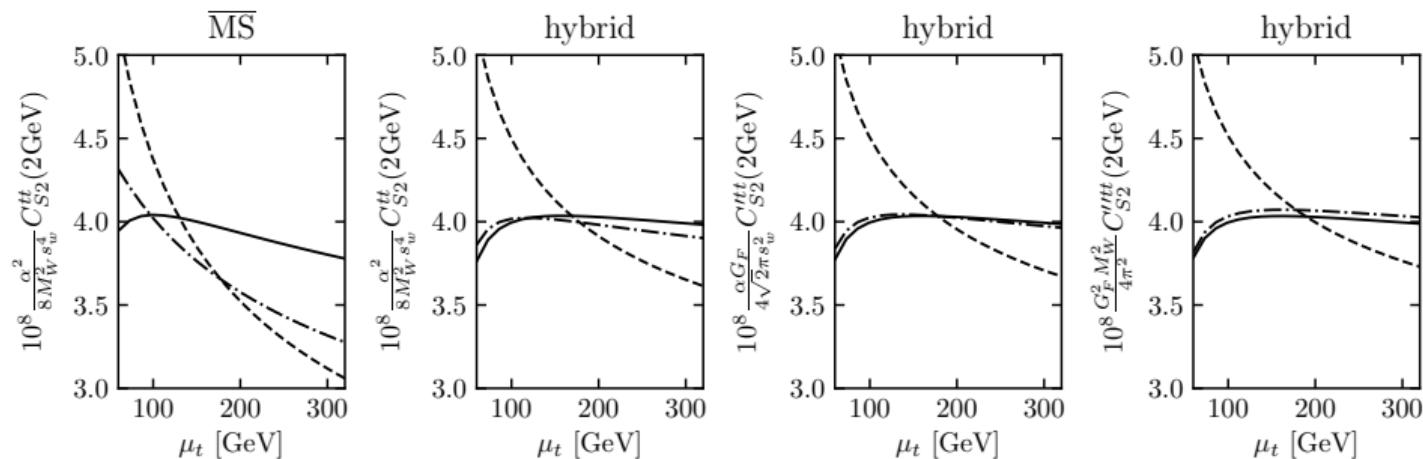


- Small matching-scale dependence
- Poor perturbative convergence → don't use this scheme



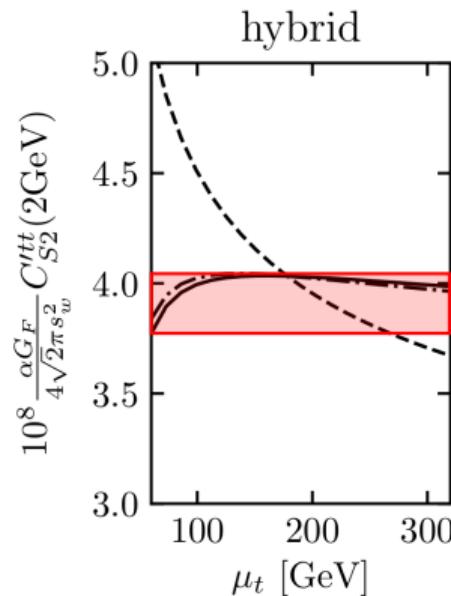
- Matching-scale dependence $\sim \pm 0.4\%$
- GF normalization shows better perturbative convergence than conventionally used GF2
- $\sim 1\%$ shift in GF2 normalization

NLO QCD + E/W Contribution



- MS scheme gets large corrections: poor perturbative convergence
- Most attractive: single G_F normalization

Perturbative Theory Error



- Higher order effects “leak in” (e.g. solving RGEs)
- Residual scale variation give estimation of size of these effects → Perturbative error

NLO QCD + E/W Contributions (Hybrid)

	NLL QCD	NLL QCD & NLL QED
A2 ($\times 10^8$)	3.96(6)	3.98(6)
GF ($\times 10^8$)	4.00(4)	3.98(5)
GF2 ($\times 10^8$)	4.02(5)	3.98(5)

- A2 and GF normalizations get $\sim \pm 0.5\%$ corrections, while GF2 gets $\sim -1.0\%$
- Central values of all three normalizations perfectly coincide

Contribution to ϵ_K

- Excellent approximation: include shift of $-1.0\% \pm 0.04\%$ to η_{tt} (exact result must include $\mathcal{O}(\alpha)$ corrections from hadronic ME)

$$|\epsilon_K|_{\text{th}} = 2.15(6)_{\text{pert}}(7)_{\text{non-pert}}(15)_{\text{param}} \times 10^{-3}, \quad |\epsilon_K|_{\text{ex}} = 2.228 \pm 0.011 \times 10^{-3}$$

- Previous theory value⁵

$$|\epsilon_K| = 2.16(6)_{\text{pert}}(7)_{\text{non-pert}}(15)_{\text{param}} \times 10^{-3}$$

⁵ Brod, Gorbahn, Stamou, Phys.Rev.Lett. 125 (2020) 17, 171803, 1911.06822 [hep-ph].

Conclusions

- Presented NLO e/w corrections to top-quark contributions to ϵ_K
- Analogous to calculation for $B^0 - \bar{B}^0$ system⁶ (First independent re-calculation, reproduced numerics)
- Discussed scheme-dependence → need non-perturbative ME including QED
- See $\sim -1.0\%$ shift in central value of Wilson coefficient (interesting direction)
- Upcoming three-loop QCD top contributions⁷, two-loop e/w charm contributions⁸, and possible updated lattice calculations will give more accurate prediction of ϵ_K

⁶Gambino, Kwiatkowski, Pott, Nucl.Phys. B544 (1999) 532-556, 9810400 [hep-th].

⁷Brod, Gorbahn, Stamou, Yu, WIP.

⁸Brod, Kvedaraite, Polonsky, Youssef, WIP.