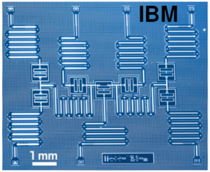
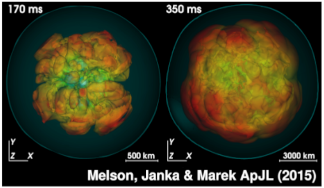
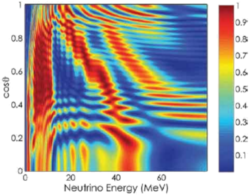
# Entanglement in collective neutrino oscillations: quantum simulations and tensor networks


Alessandro Roggero
(UNITN)

Duan et al. (2006)

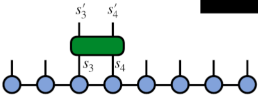
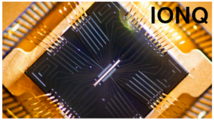
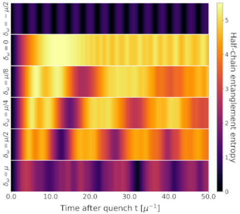


image from itensor.org



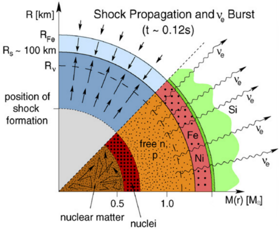




FNAL – 02 Sep, 2021

# Neutrino's roles in supernovae

figures from Janka et al. (2007)

- efficient energy transport away from the shock region (burst)

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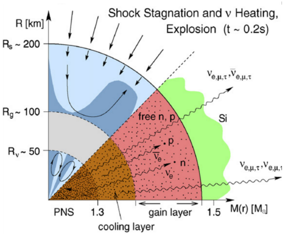
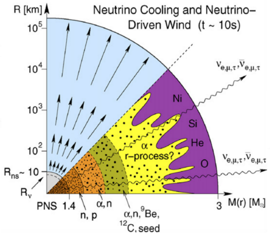



- regulation of electron fraction in $\nu$-driven wind (nucleosynthesis)



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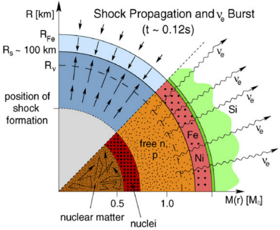


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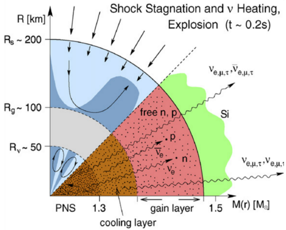


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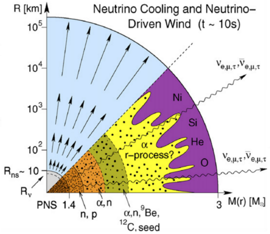
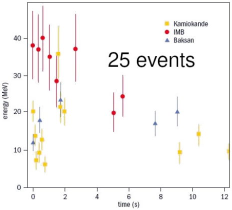
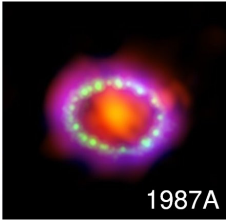



$\approx 10^{58}$ neutrinos emitted in few sec.



# Neutrino oscillations in astrophysical environments

We know that neutrinos can display flavor oscillations in vacuum (and FNAL/DUNE will tell us more about it), does it matter in an SN?

- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)

# Neutrino oscillations in astrophysical environments

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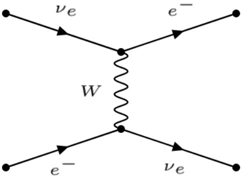
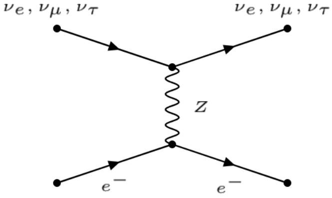
- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)
- neutrino oscillation rates can get enhanced through elastic forward scattering with high density external matter (MSW effect)

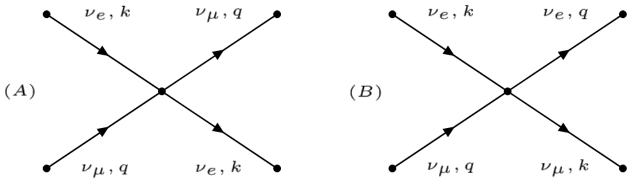



Can a similar effect happen also when neutrinos scatter on neutrinos?

# Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, . . .




- diagonal contribution (A) does not impact flavor mixing
- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
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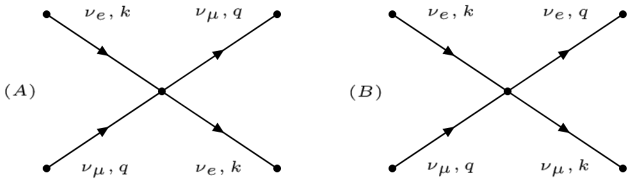


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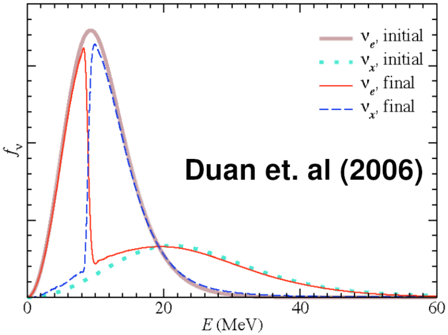
Important effect if initial distributions are strongly flavor dependent



# Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors ($\nu_e$,$\nu_x$) and encode flavor amplitudes for a neutrino with momentum $p_i$ into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

A system of $N$ interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}\left(1-\cos(\phi_{ij})\right)\vec{\sigma}_i\cdot\vec{\sigma}_j$$

- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

for a full derivation, see e.g. Pehlivan et al. PRD(2011)

# The mean-field approximation

The equations of motion for the polarization vector $\vec{P}_i = \langle \vec{\sigma}_i \rangle$ are

$$\frac{d}{dt}\vec{P}_i = \left( \frac{\Delta m^2}{4E_i}\vec{B} + \lambda(0,0,1) \right) \times \vec{P}_i + \frac{\mu}{2N}\sum_{j\neq i}\left(1 - cos(\phi_{ij})\right) \langle \vec{\sigma}_j \times \vec{\sigma}_i \rangle$$

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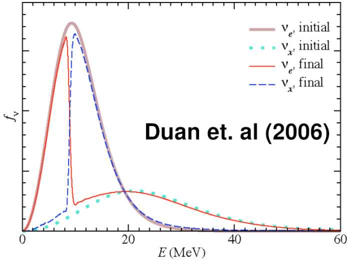
In this way we obtain a closed system of $3N$ coupled differential equations

- efficient solutions for systems containing $N \approx \mathcal{O}(10^{4-5})$ neutrino amplitudes [$\approx \mathcal{O}(100)$ energies and $\approx \mathcal{O}(100)$ angles ]



# The MF approx. neglects **entanglement**, does it matter?

| | | |
|---|---|---|
| April '03 | speedup through entanglement $\tau \sim \mu^{-1}$ | Bell et al. PLB (2003) |
| July '03 | in a highly symmetric limit the MF prediction is qualitatively correct $\tau \propto \mu^{-1}\sqrt{N} \to \infty$ | Friedland&Lunardini JHEP (2003) |
| August '04 | some models seem to produce $\tau \propto \mu^{-1}\log(N)$ | Sawyer (2004) |
| Summer '19 | exact simulations for systems with small $N$ show substantial entanglement buildup | Cervia et al. PRD(2019), Rrapaj PRC(2020) |

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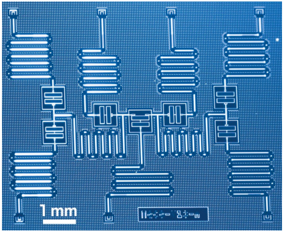
Two approaches for today: quantum simulations and tensor networks



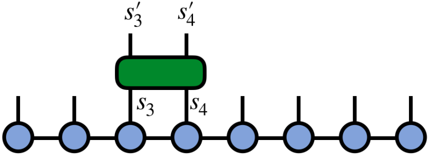




image from itensor.org

# Quantum Computing and Quantum Simulations

| | |
|---|---|
| P.Benioff (1980) | quantum mechanical Hamiltonians can be used to represent universal digital computational devices |
| R.Feynman(1982) | we can use a controllable quantum system to simulate the behaviour of another quantum system |

**Quantum System**
**we have control over**

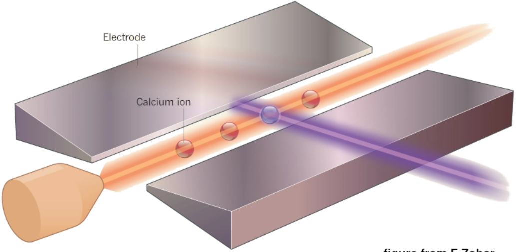
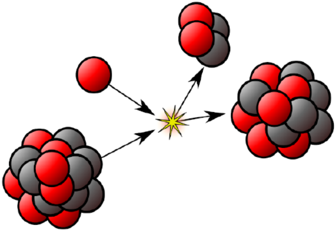



figure from E.Zohar

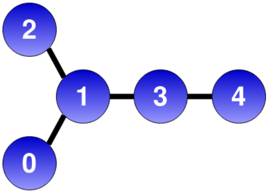
**Quantum System**
**we want to simulate**



Both **digital** and **analog** simulations are possible in principle

# Quantum simulation of collective neutrino oscillations

$$H_\nu = \sum_i \omega_i \vec{B}\cdot\vec{\sigma}_i + \frac{\mu}{2N}\sum_{i<j} J_{ij}\vec{\sigma}_i\cdot\vec{\sigma}_j$$




- with only 2 flavors direct map to spin 1/2 degrees of freedom (qubits)
- only one- and two-body interactions $\Rightarrow$ only $\mathcal{O}(N^2)$ terms
- all-to-all interactions are difficult with reduced connectivity

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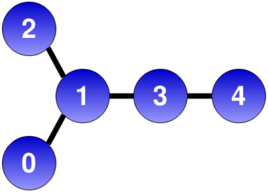


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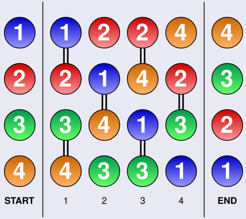
## SWAP network



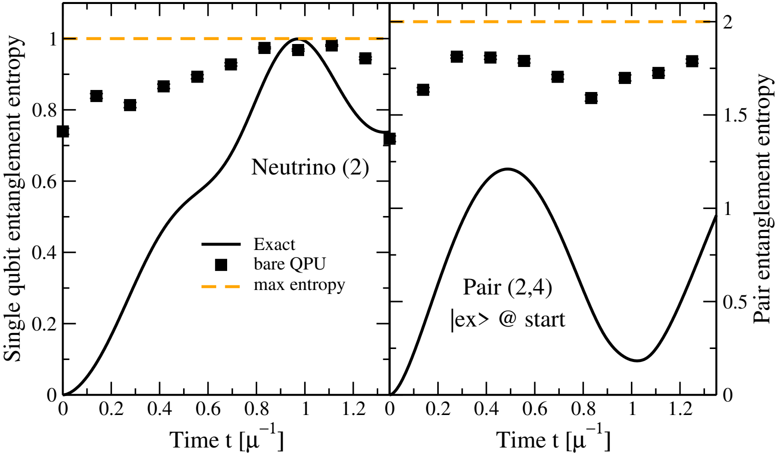

- SWAP qubits every time we apply time-evolution to neighboring terms
- in $N$ steps we perform full evolution using only $\binom{N}{2}$ two qubit gates
  - NOTE: final order will be reversed

Kivlichan et al. PRL (2018)

B.Hall, AR, A.Baroni, J.Carlson (2021), AR (2021)

# Entanglement evolution and error mitigation with $N = 4$

B.Hall, AR, A.Baroni, J.Carlson (2021)

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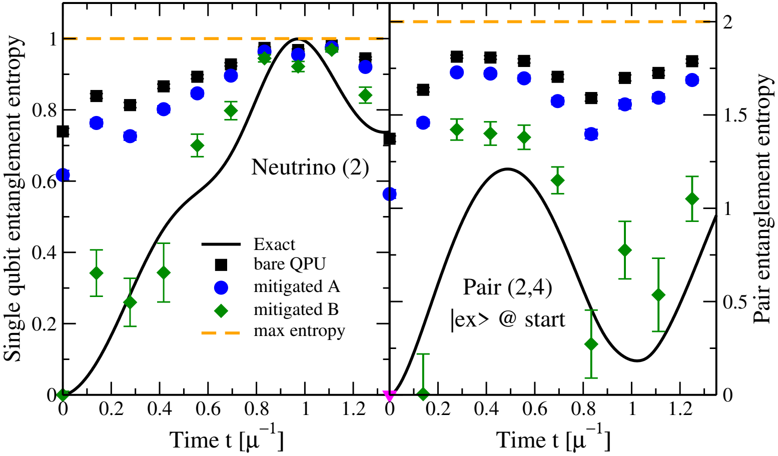


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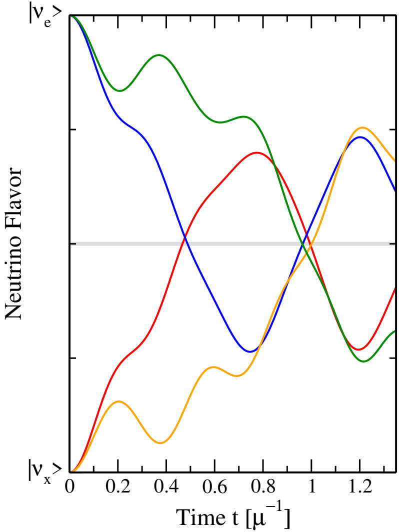
Entanglement is useful to understand collective oscillation mechanism but priority is to correctly predict flavor evolution. How's the current accuracy?



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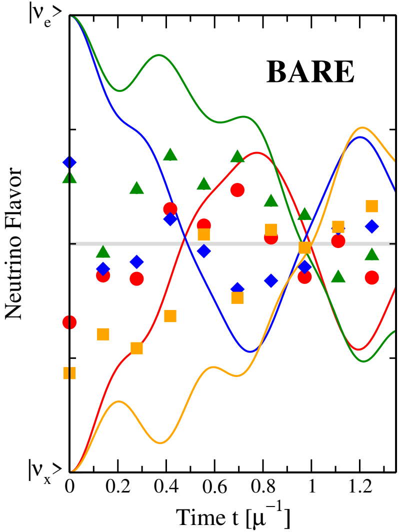

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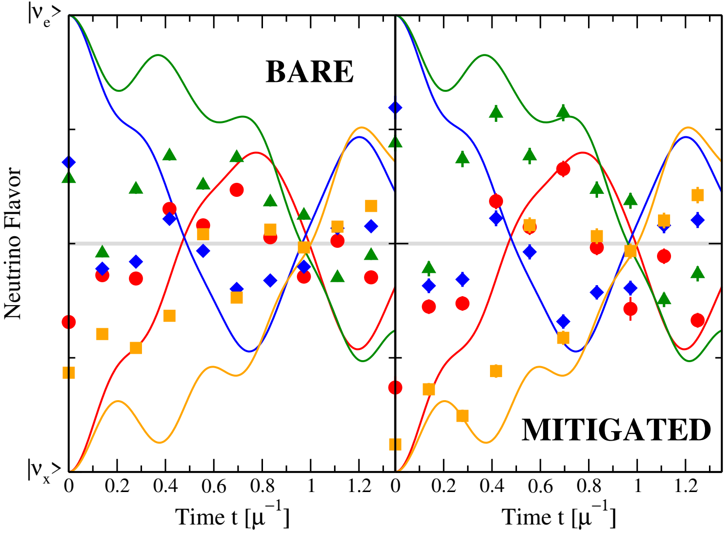

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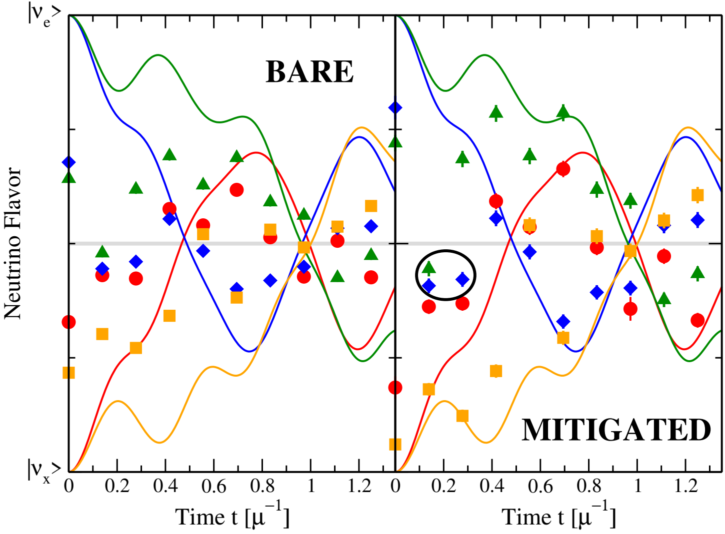
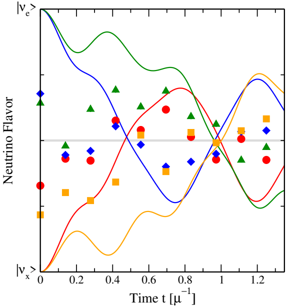
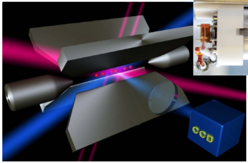
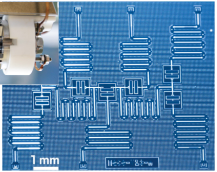


# Current limitations of digital quantum simulations




current and near term digital quantum devices have limited fidelity and might not scale much beyond $N = \mathcal{O}(10)$ neutrinos in next years



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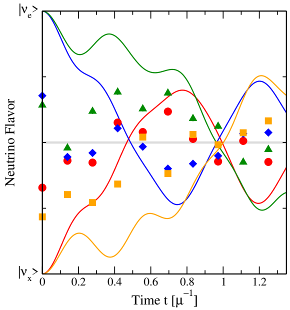


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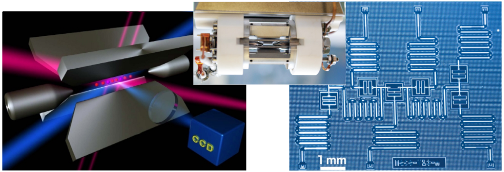


## Possible paths to scalability in the meantime

- Analog Quantum Simulators

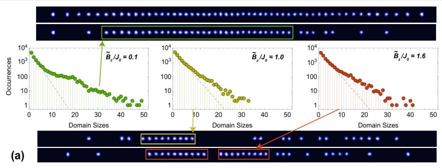


figure from Zhang et al Nature(2017)

- Describe low entanglement states with Tensor Networks

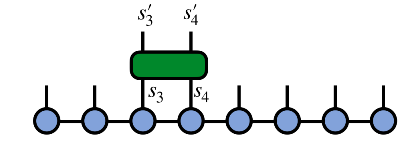


image from itensor.org

# One slide intro to Matrix Product States

Vidal PRL(2003)

**General state of $N$ spins $1/2$**

$$|\Psi\rangle = \sum_{\sigma_1,\dots,\sigma_N} \Psi^{\sigma_1\cdots\sigma_N} |\sigma_1 \cdots \sigma_N\rangle$$

$\Psi^{\sigma_1\cdots\sigma_N}$ is an $N$-component tensor

- using SVD we can write it as a product of $(2)N$ matrices

$$\Psi^{\sigma_1\cdots\sigma_N} = \mathbf{A}^{[1]\sigma_1}\mathbf{A}^{[2]\sigma_2}\dots\mathbf{A}^{[N]\sigma_N}$$

exact representation if $dim(\mathbf{A}) = 2^{\frac{N}{2}}$

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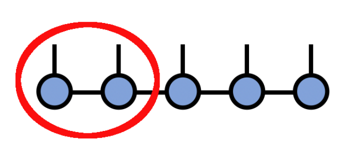

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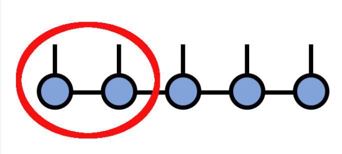

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- $dim(\mathbf{A}) = 1 \Rightarrow$ mean-field state

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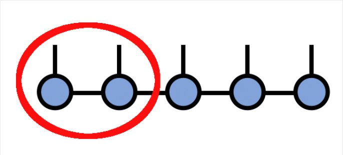
- using SVD we can write it as a product of $(2)N$ matrices

$$\Psi^{\sigma_1\cdots\sigma_N} = \mathbf{A}^{[1]\sigma_1}\mathbf{A}^{[2]\sigma_2}\dots\mathbf{A}^{[N]\sigma_N}$$

exact representation if $dim(\mathbf{A}) = 2^{\frac{N}{2}}$

image from itensor.org

Entanglement entropy of a subsystem of size $m < N$ will be bounded by



$$\mathcal{S}(m) \leq \log_2(dim(\mathbf{A})) \leq m$$

- $dim(\mathbf{A}) = 1 \Rightarrow$ mean-field state
- $dim(\mathbf{A}) = 2^m \Rightarrow$ generic state

# One slide intro to Matrix Product States

Vidal PRL(2003)

**General state of $N$ spins $1/2$**

$$|\Psi\rangle = \sum_{\sigma_1,\ldots,\sigma_N} \Psi^{\sigma_1\cdots\sigma_N} |\sigma_1 \cdots \sigma_N\rangle$$

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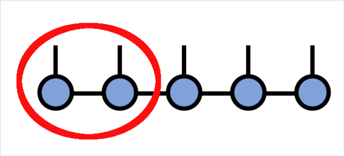

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- $dim(\mathbf{A}) = 1 \Rightarrow$ mean-field state
- $dim(\mathbf{A}) = 2^m \Rightarrow$ generic state
- $dim(\mathbf{A}) = m^\gamma \Rightarrow$ low-entanglement state

# Single angle approximation and no vacuum term

$$H = \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j = \frac{\mu}{N}S^2 + const.$$

Initialize system in $|\Psi(0)\rangle = |\downarrow\rangle^{\otimes N/2}\otimes|\uparrow\rangle^{\otimes N/2}$ and compute the flavor persistence $p(t) = (1-\langle\Psi(t)|\sigma^z_1|\Psi(t)\rangle)/2$ for increasing system size

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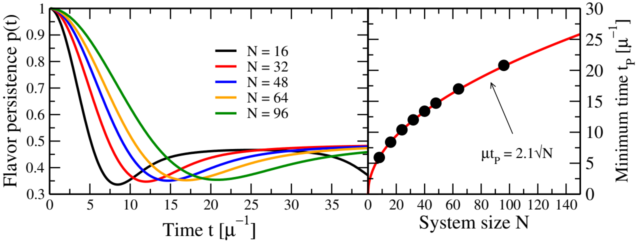
MPS simulations converge quickly as $\mathcal{S}(m) \approx \log_2(m) \Rightarrow dim(\mathbf{A}) \approx N/2$




AR arXiv:2103.11497(2021)

# Collective oscillations with many body speedup

$$H = -\frac{\omega_A}{2} \sum_{i \in \{1,\dots,N/2\}} \sigma_i^z - \frac{\omega_B}{2} \sum_{i \in \{N/2+1,\dots,N\}} \sigma_i^z + \frac{\mu}{2N} \sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j \ ,$$

Conservation of total $S^z$ means only $\delta_\omega = (\omega_A - \omega_B)/2$ matters

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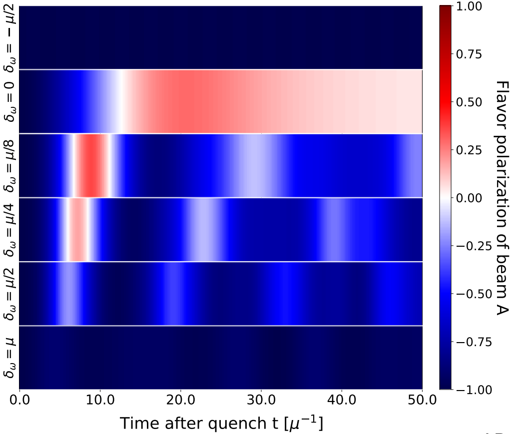


AR arXiv:2102.10188(2021)

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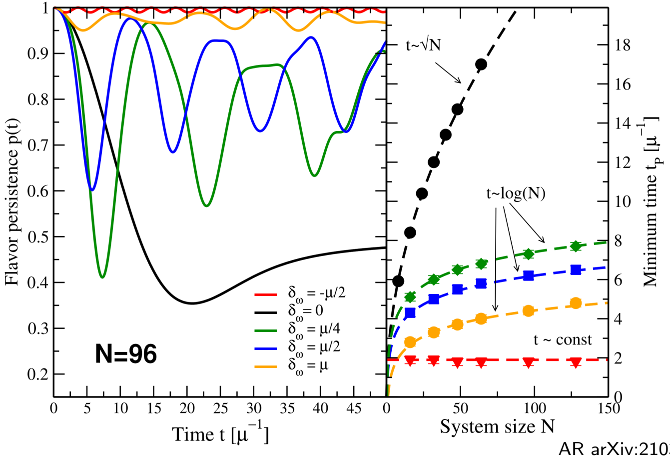


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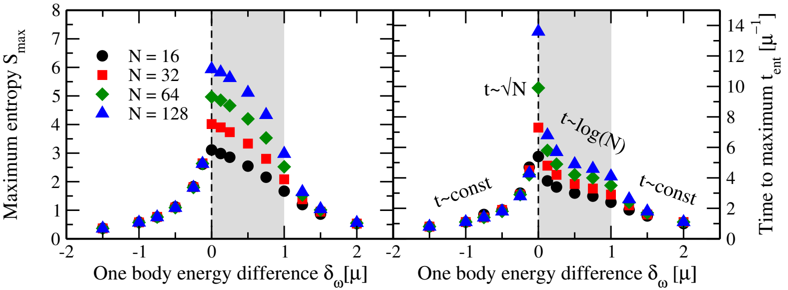
# Collective oscillations with many body speedup II

AR arXiv:2103.11497(2021)




## Why is this interesting?

- entanglement scaling provides general criterion for appearance of collective modes in full many-body treatment
- entropy scaling as $\log(N) \Rightarrow$ large ab-initio simulations possible
- MPS method fails when entanglement too large $\Rightarrow$ we can use this to detect interesting regime to study on quantum simulators!

# Summary

- collective neutrino oscillations are phenomena with possible important impact in flavor dynamics in extreme environments like supernovae
- even the basic 2-flavor model for collective oscillations poses a challenging many-body problem well suited to quantum technologies
- first calculations on small scale digital devices show promise in studying entanglement evolution but fidelity is not at the desired level
- tensor network methods like Matrix Product States can help push the boundary of what can be computed in a controllable way
  - correlation between entanglement and presence of collective modes
  - can we use MPS to find interesting regimes for quantum simulations?

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**Thanks to my collaborators on digital quantum simulations**

- Benjamin Hall (MSU)
- Alessandro Baroni (LANL)
- Joseph Carlson (LANL)

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# Error mitigation with zero-noise extrapolation

Li & Benjamin PRX(2017), Temme, Bravy, Gambetta PRL(2017), Endo,Benjamin,Li PRX(2018)

**Zero noise extrapolation**

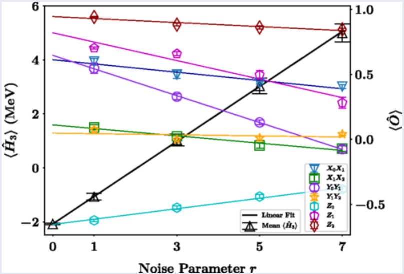
For small enough noise we can write

$$M(\epsilon) = M_0 + \epsilon M_1 + \frac{\epsilon^2}{2} M_2 + \dots$$

Using two points $\epsilon_2 = \eta \epsilon_1$ we have

$$M_0 \approx M(\epsilon_1) - \frac{M(\epsilon_1) - M(\epsilon_2)}{\eta - 1}$$

picture from Dumitrescu et al. PRL(2018)




- for moderate $\epsilon$ other parametrizations (like exp) might be more useful

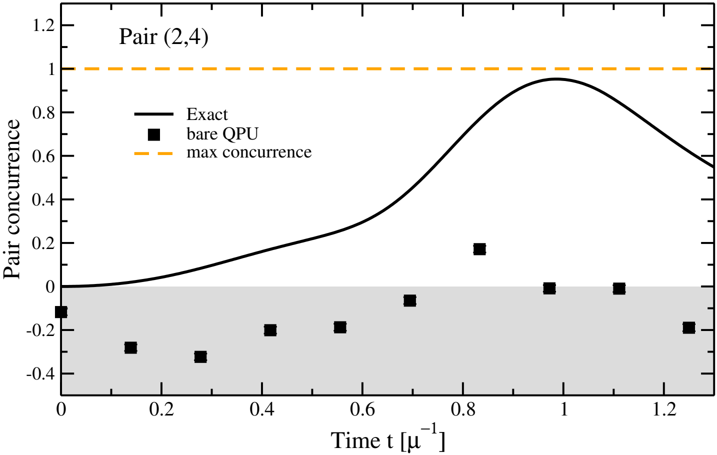
$$M(\epsilon) = M_0 e^{-\alpha\epsilon} \Rightarrow M_0 \approx M(\epsilon_1) \left( \frac{M(\epsilon_2)}{M(\epsilon_1)} \right)^{\frac{\epsilon_1}{\epsilon_1 - \epsilon_2}}$$

In that case it is very beneficial to ensure $M(\epsilon \to \infty) \to 0$ (mitigated B)

# Alternative entanglement measure: two qubit concurrence

W.K. Wootters (2001)

$c(\rho) = \max[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4]$ $\vec{\lambda}$ eigvals. of $M = \sqrt{\rho Y \otimes Y \rho^* Y \otimes Y}$




- decoherence has opposite effect ⇒ cross-validation with entropy

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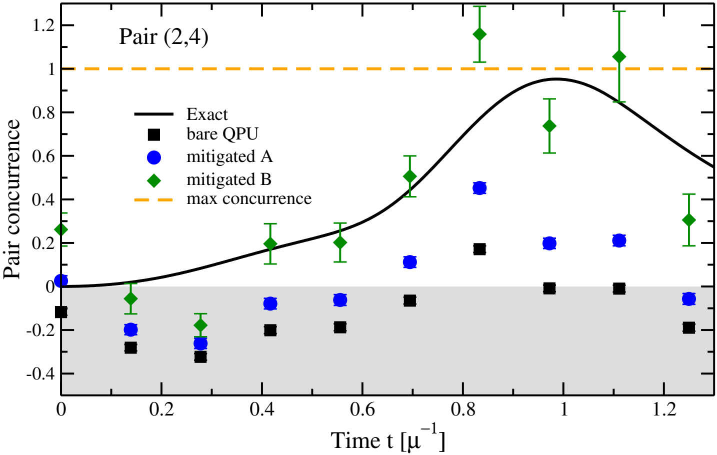


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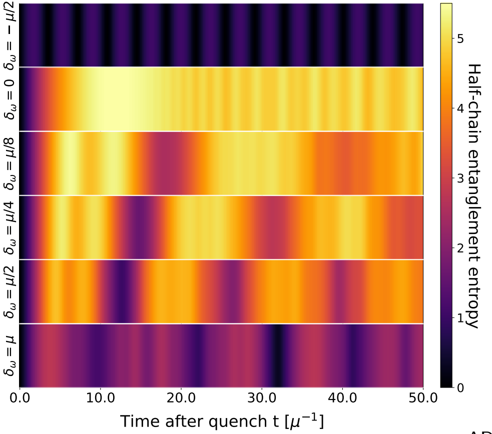



AR arXiv:2102.10188(2021)

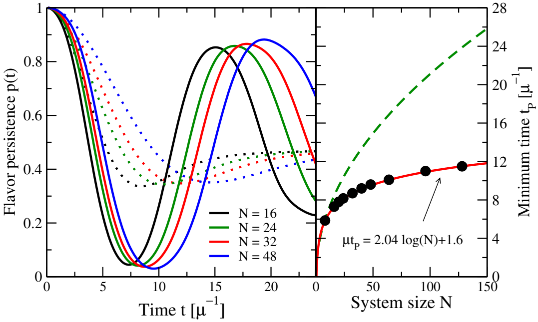
# Collective oscillations with many body speedup II

Bell, Rawlinson, Sawyer (2003), Sawyer (2004)

$$H_{BRS} = \frac{\mu}{2N} \sum_{i<j} \mathcal{J}_{ij} \left( X_i X_j + Y_i Y_j + \Delta Z_i Z_j \right)$$

with $\mathcal{J}_{ij} = J_{\parallel}$ for $(i,j) \in \mathcal{A}$ or $(i,j) \in \mathcal{B}$ and $\mathcal{J}_{ij} = J_{\perp}$ otherwise.

For $\Delta < 1$ observation of speedup: $t \approx \mu^{-1} \log(N) \ll \mu^{-1}\sqrt{N}$




- entropy scaling is similar to before $\mathcal{S} \approx \log_2(N)$
- effect requires explicit $SU(2)$ breaking ($\Delta < 1$)

Origin of effect related to a Dynamical Phase Transition

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