

EFT and the Geometry of EWSB

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Based in part on

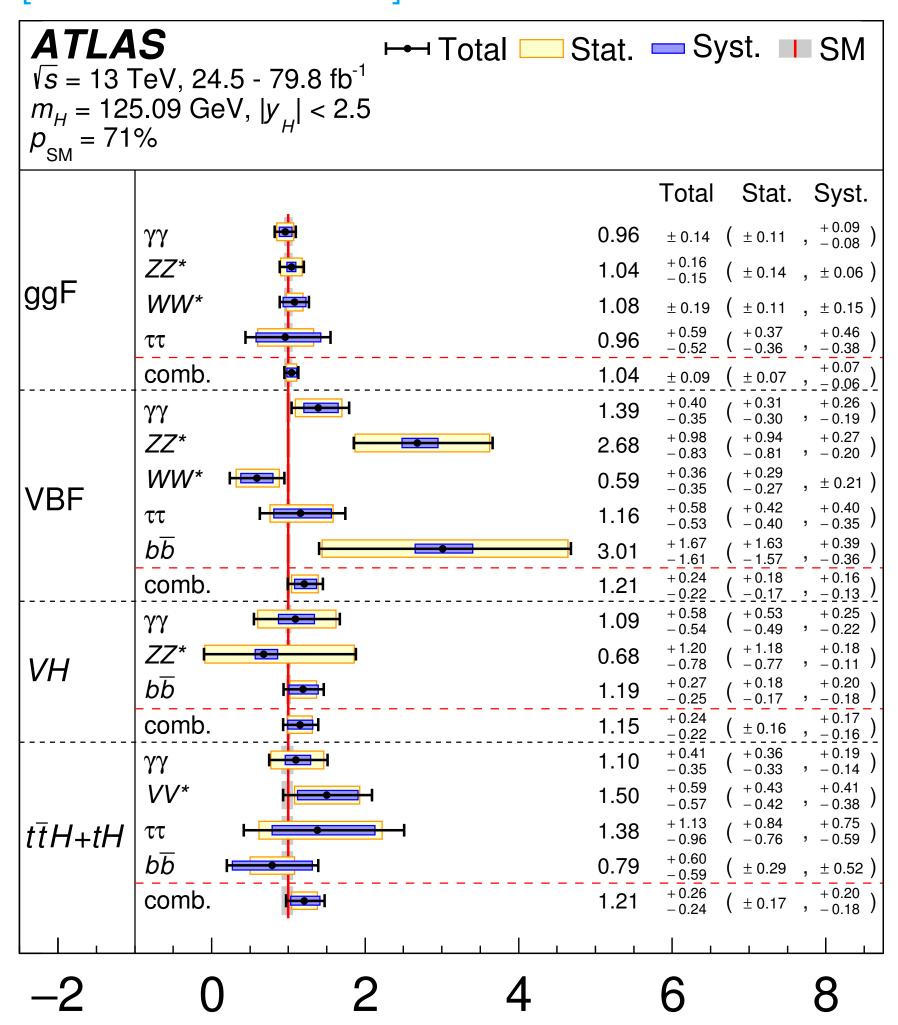
"Is SMEFT enough?" [2008.08597] w/ Tim Cohen, Xiaochuan Lu, and Dave Sutherland + work appearing soon w/ same + lan Banta

Inspired by

R. Alonso, E. Jenkins, A. Manohar [1511.00724, 1605.03602]

Measurements → Meaning

[ATLAS 1909.02845]



Precision Higgs measurements a key program of LHC3/HL-LHC.

Anticipated 5-10% precision provides unprecedented tests. Future colliders to ~0.5%

Interpreting either agreement or disagreement with SM invites an EFT framework.*

Strong motivation to develop and understand Higgs EFTs!

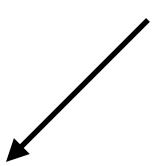
This talk: which EFT?

Higgs EFTs

SM

$SU(2)_L \times U(1)_Y$

$$(D_{\mu}H)^{\dagger}(D^{\mu}H) - m^{2}|H|^{2} - \lambda|H|^{4}$$



HEFT*

*U(1)*_{em}

[Feruglio '93, Bagger et al. '93, ...]

$$\frac{1}{2}(\partial_{\mu}h)^{2} + \frac{1}{2} \left[vF(h/v) \right]^{2} (\partial \vec{n})^{2}$$
$$-V(h) + \dots$$

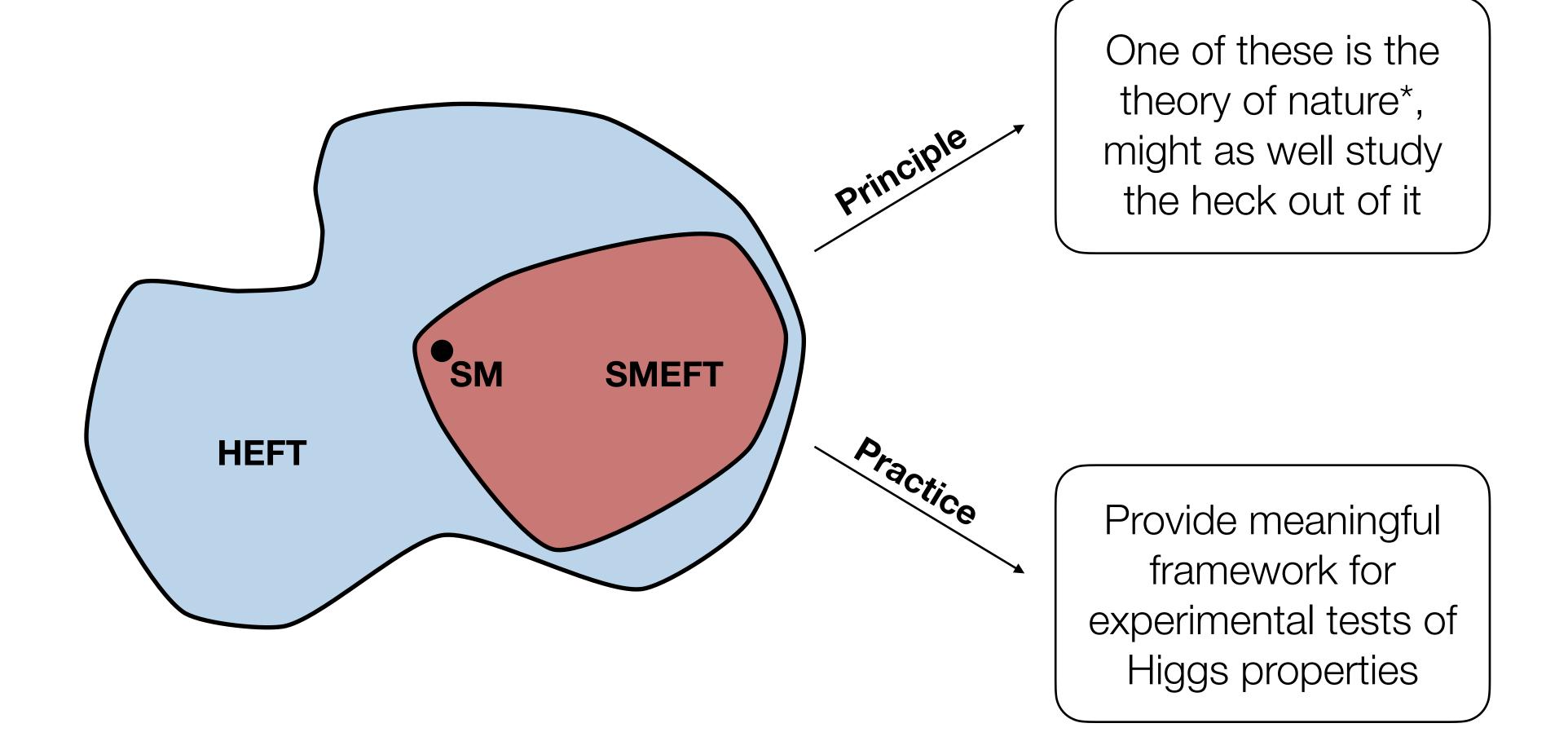
SMEFT

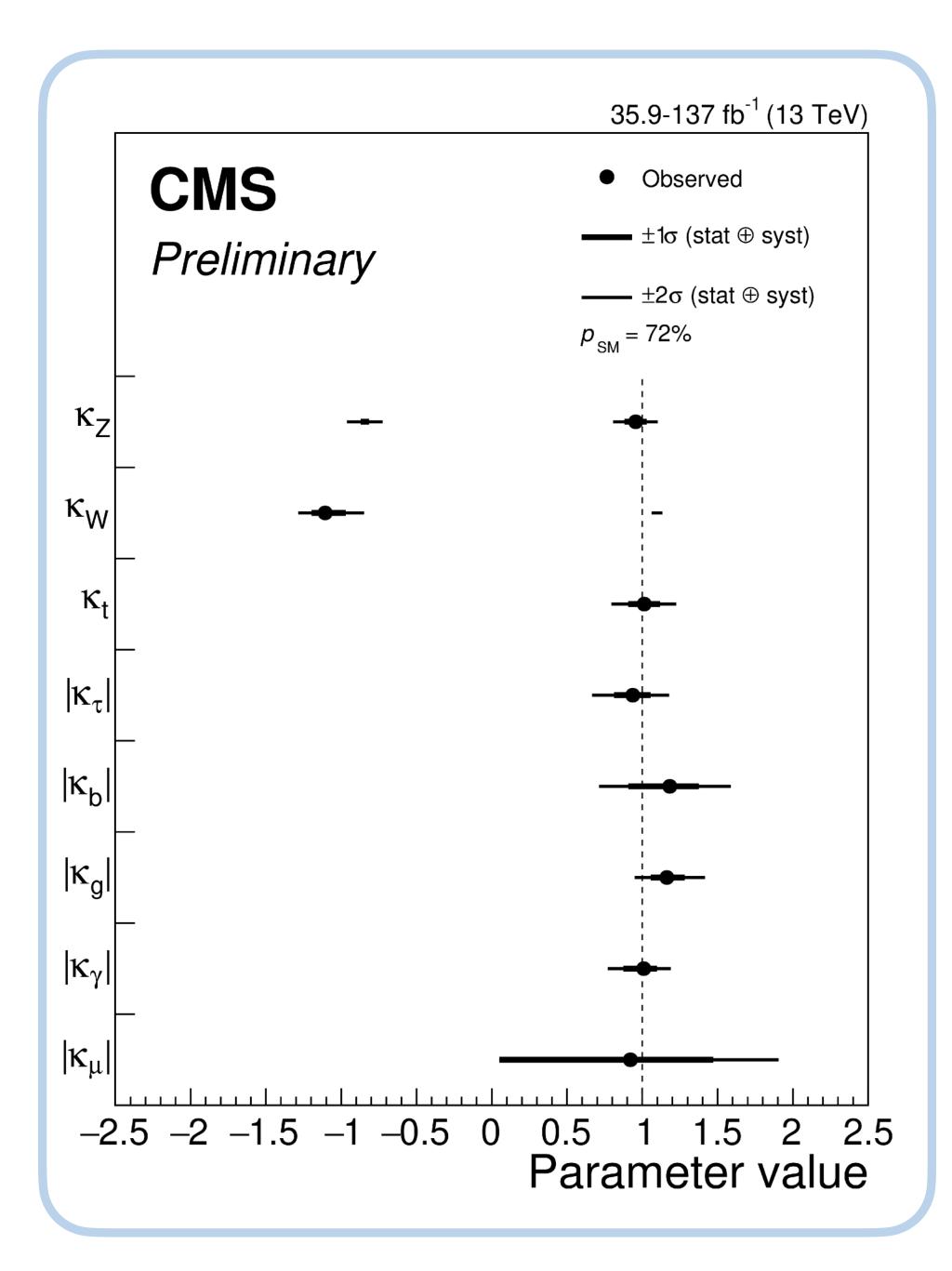
$SU(2)_L \times U(1)_Y$

[Weinberg '79, Buchmuller, Wyler '86, ...]

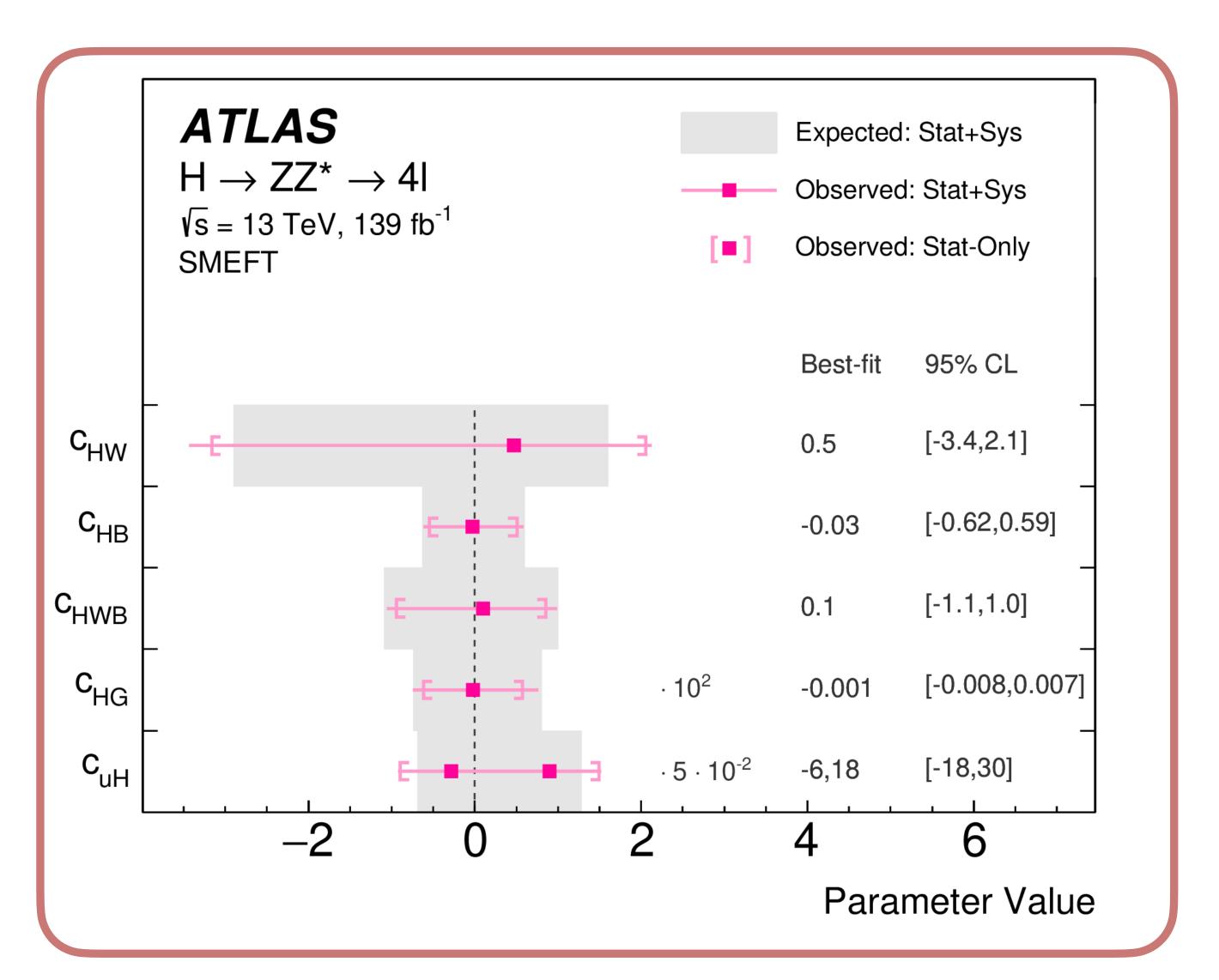
$$(D_{\mu}H)^{\dagger}(D^{\mu}H) - m^{2}|H|^{2} - \lambda|H|^{4}$$
$$+ \frac{c_{H}}{2\Lambda^{2}}(\partial_{\mu}|H|^{2})^{2} + \frac{c_{6}}{\Lambda^{2}}|H|^{6} + \dots$$

Why bother



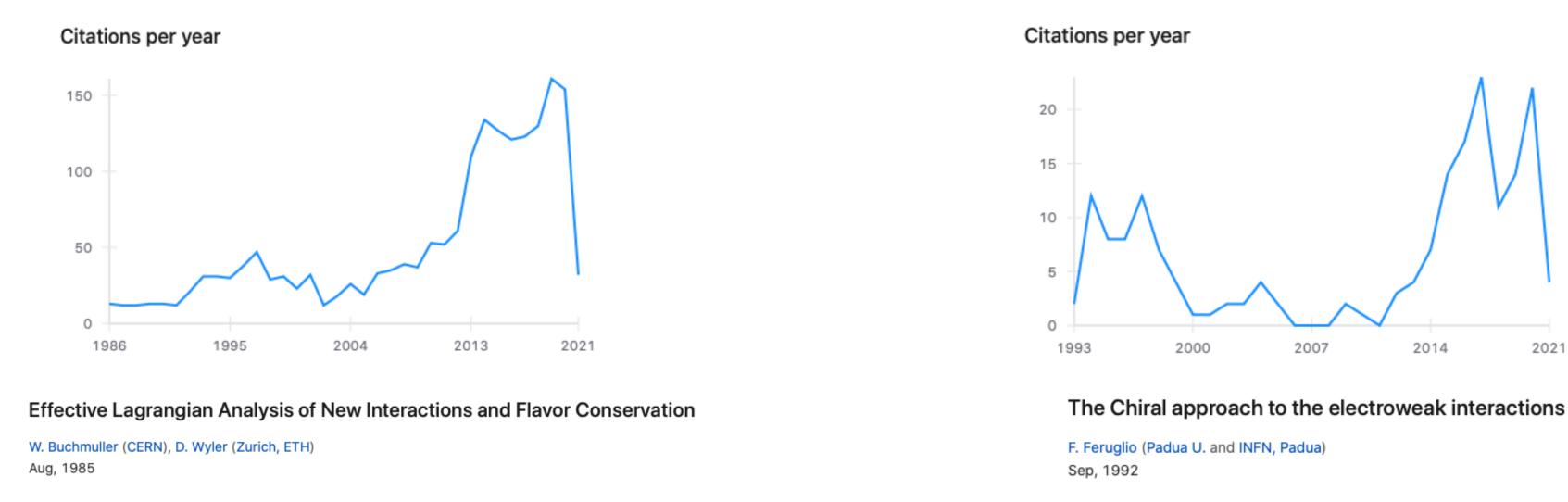


Data!



Which EFT?

Vastly more progress in SMEFT since c. 2012 (precision, fits, projections, theorems,...)



Seems justified: SU(2)xU(1) an apparently good symmetry, no O(1) deviations or custodial symmetry violation

2021

(When) Is HEFT necessary?

See also: [Burgess, Matias, Pospelov '99; Grinstein & Trott '07; Alonso, Gavela, Merlo, Rigolin, Yepes '12; Espriu, Mescia, Yencho '13; Buchalla, Cata, Krause '13; Brivio et al. '13; Chang & Luty '19; Falkowski & Rattazzi ' 19; Abu-Ajamieh, Chang, Chen, Luty '20] On-shell perspective: [Durieux, Kitahara, Shadmi, Weiss '19]

For this talk: focus exclusively on scalar sector in the global limit, assume custodial symmetry, restrict to 2-derivative order.

The Standard Model EFT

SMEFT: EFT where 4 scalar d.o.f. are arranged into an SU(2) doublet (equivalently, O(4) fundamental; assuming custodial symmetry):

"Electroweak symmetry is linearly realized."

$$\mathcal{L}_{SM} = \frac{1}{2} (\partial \vec{\phi} \cdot \partial \vec{\phi}) - \frac{1}{4} \lambda (\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\mathcal{L}_{\text{SMEFT}} = \frac{1}{2} A (\vec{\phi} \cdot \vec{\phi}) (\partial \vec{\phi} \cdot \partial \vec{\phi}) + \frac{1}{2} B \left(\vec{\phi} \cdot \vec{\phi} \right) (\vec{\phi} \cdot \partial \vec{\phi})^2 - V \left(\vec{\phi} \cdot \vec{\phi} \right) + \mathcal{O} \left(\partial^4 \right)$$

Reminder: only worrying about scalars up to 2 derivatives...

The Higgs EFT

Alternately, HEFT:

construct EFT out of singlet h and Goldstones π_i

No presumed relation between h, π

$$h \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}$$

$$h \to h , \vec{n} \to O\vec{n} , O \in O(4)$$

"Electroweak symmetry is nonlinearly realized."

$$\mathcal{L}_{SM} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} (v + h)^2 (\partial \vec{n})^2 - \frac{1}{4} \lambda (h^2 + 2vh)^2$$

$$\mathcal{L}_{HEFT} = \frac{1}{2} [K(h)]^2 (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

(K(h)) redundant, conventional to redefine h to set K(h) = 1; retaining K(h) clearer for matching)

SM c SMEFT c HEFT

SM SMEFT
HEFT

[R. Alonso, E. Jenkins, A. Manohar 1511.00724 & 1605.03602]

Relate the two by field redefinition:

$$\vec{\phi} = (v+h)\vec{n}(\pi); \quad \vec{\phi} \cdot \vec{\phi} = (v+h)^2$$

SMEFT can always be written as HEFT:

HEFT cannot always be written as SMEFT:

$$\mathcal{L} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial \vec{n})^2 - V(h)$$

$$= \frac{1}{2}\frac{v^2F}{\vec{\phi}\cdot\vec{\phi}}(\partial \vec{\phi})^2 + \frac{1}{2}(\vec{\phi}\cdot\partial \vec{\phi})^2\frac{1}{\vec{\phi}\cdot\vec{\phi}}\left(K^2 - \frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}\right) - \tilde{V}(\vec{\phi}\cdot\vec{\phi})$$

$$\uparrow \quad \text{Generically non-analytic} \quad \uparrow \quad \text{at the origin}$$

What defines the HEFTs that cannot be written as SMEFTs? What is the UV physics that produces them?

HEFT or SMEFT?

When can a theory be written as HEFT but not SMEFT?

Maybe you can always just tell by eye...

$$\mathcal{L} = \frac{1}{2} \left(1 + \frac{h}{2v} \right)^2 (\partial h)^2 + \frac{1}{2} (v + h)^2 \left(\frac{3}{4} + \frac{h}{4v} \right)^2 (\partial \vec{n})^2 - V$$

Definitely HEFT, right?

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But now let's perform the field redefinition

$$h \to \tilde{h} \equiv h + \frac{1}{4v}h^2$$

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$$\mathcal{L} = \frac{1}{2} (\partial \tilde{h})^2 + \frac{1}{2} (\tilde{v} + \tilde{h})^2 (\partial \vec{n})^2 + \dots = |\partial \tilde{H}|^2 + \dots$$

Actually the SM

Field redefinitions readily obscure the distinction at the level of the Lagrangian.

A Geometric Perspective

Instead: classify EFTs based on geometry.

Two-derivative terms define a metric on the scalar field manifold

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial \phi^i \partial \phi^j - V(\phi)$$

Field space corresponds to a (possibly curved) manifold with functions (e.g. V) defined on it; the field parameterization corresponds to charts on the manifold. Use geometric invariants* to classify EFTs.

Long history (primarily) applied to nonlinear sigma models, e.g. [Honerkamp '72; Tataru '75; Alvarez-Gaume, Freedman, Mukhi '81, ...]

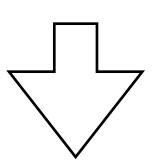
Application to HEFT: [Alonso, Jenkins, Manohar 1511.00724 & 1605.03602] (Applied within SMEFT: [Helset, Martin, Trott 2001.01453])

SM: flat manifold **HEFT:** curved manifold **SMEFT:** curved manifold w/ O(4) invariant point

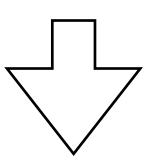
*2-derivative theory sufficient unless one considers derivative field redefinitions; then Finsler geometry required

Geometry & EFT

For example,



gives metric



& Ricci scalar

$$\mathcal{L}_{\text{HEFT}}^{(2)} = \frac{1}{2} [K(h)]^2 (\partial h)^2 + \frac{1}{2} [v F(h)]^2 (\partial \vec{n})^2$$

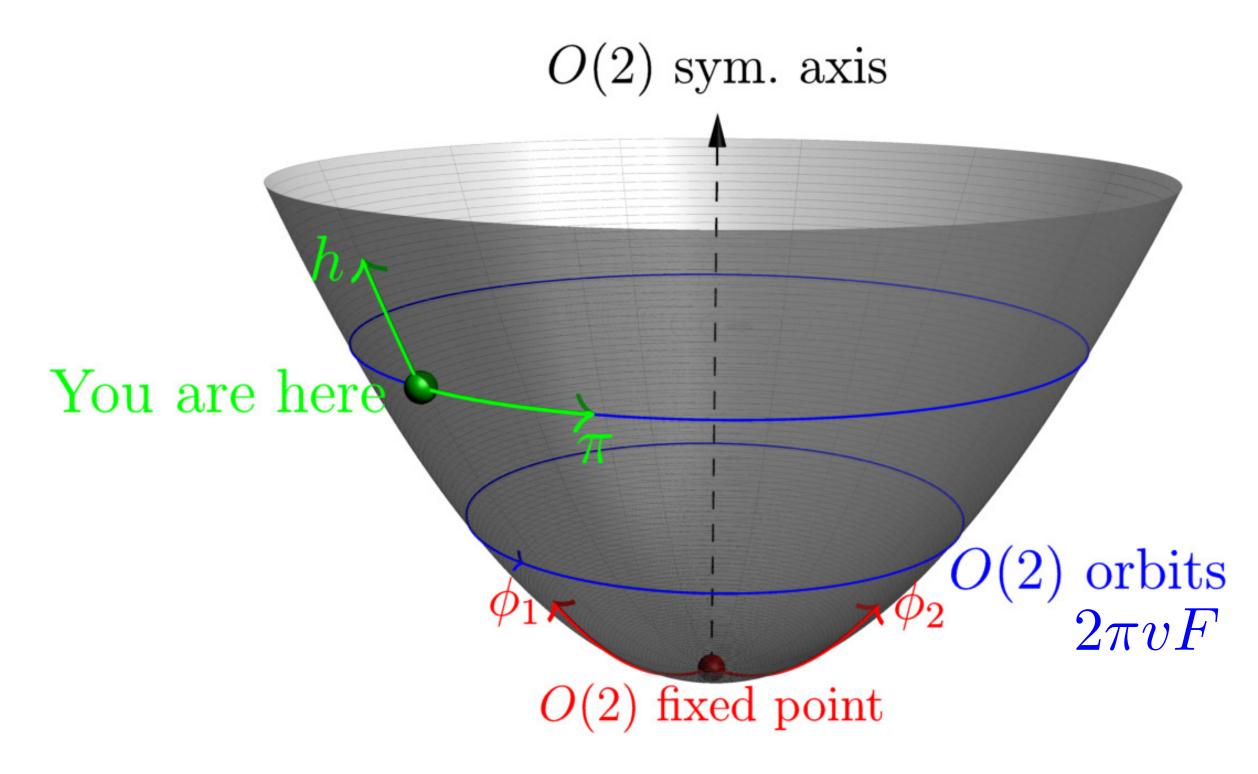
$$g_{hh} = K^2$$
 $g_{ij} = v^2 F^2 \left(\delta_{ij} + \frac{n_i n_j}{1 - n^2} \right)$

$$R = -\frac{2N_{\varphi}}{K^{2}F} \left[\left(\partial_{h}^{2}F \right) - \left(\partial_{h}K \right) \left(\frac{1}{K} \partial_{h}F \right) \right] + \frac{N_{\varphi} \left(N_{\varphi} - 1 \right)}{v^{2}F^{2}} \left[1 - \left(\frac{v}{K} \partial_{h}F \right)^{2} \right]$$

SM: flat manifold
$$K(h) = 1$$
 $F(h) = 1 + \frac{h}{n} \implies R = 0$

Geometry & SMEFT

(Think O(4), but O(2) is easier to illustrate)



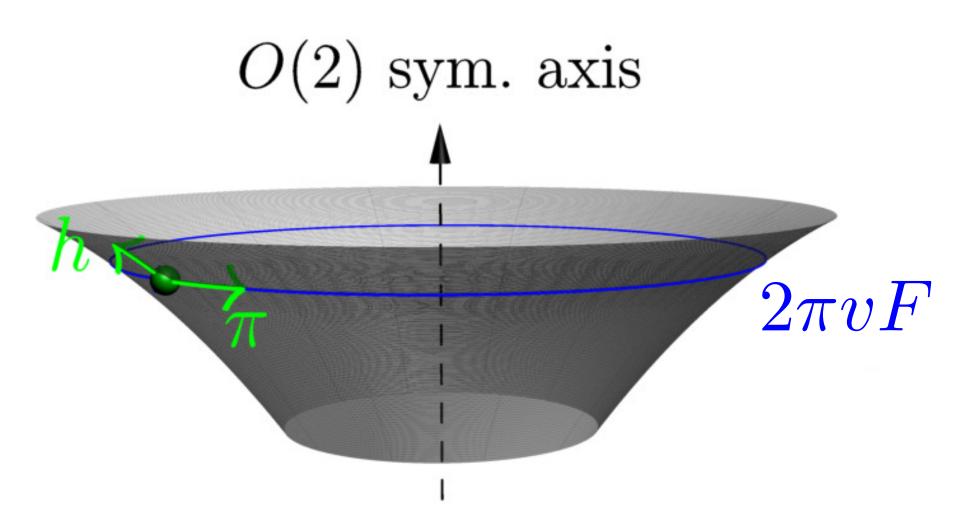
$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

SMEFT if O(4) fixed point on manifold \rightarrow F(h) = 0 somewhere (say, h = -v)

HEFT not SMEFT: Case I

[Alonso, Jenkins, Manohar 1605.03602]

When there's a hole s.t. h = -v is not on the manifold (no O(4) fixed point about which to expand in SMEFT coordinates)



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

Corresponds to $F(h) \neq 0$ everywhere

HEFT not SMEFT: Case I

How does this arise? When UV physics also breaks the symmetry.

A toy example: 2AHM, i.e. two Higgses charged under a U(1) gauge symmetry

Acquire vevs s.t.
$$v^2 \equiv 4v_1^2 + v_2^2$$

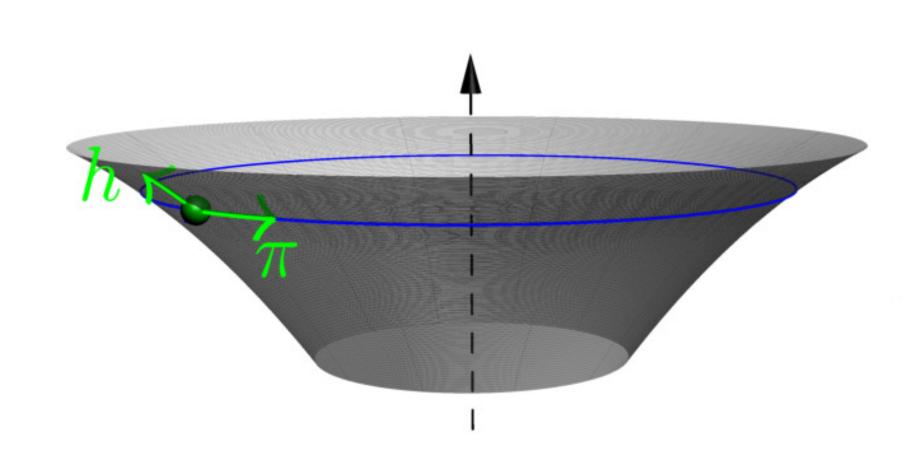
$$\begin{array}{c|c} \text{Field} & Q \\ \hline H_1 & +2 \\ H_2 & +1 \\ \end{array}$$

Spectrum: light Higgs h, goldstone π , heavy fields H, Π

Integrate out H, Π to obtain EFT of h, π

$$K(h) = 1,$$
 $F(h) = \frac{1}{v}\sqrt{4(v_1 + c_{\alpha}h)^2 + (v_2 + s_{\alpha}h)^2}$

Generically F(h) \neq 0 everywhere for nonzero v_1, v_2



HEFT not SMEFT: Case II

When there's a cone or cusp at h=-v

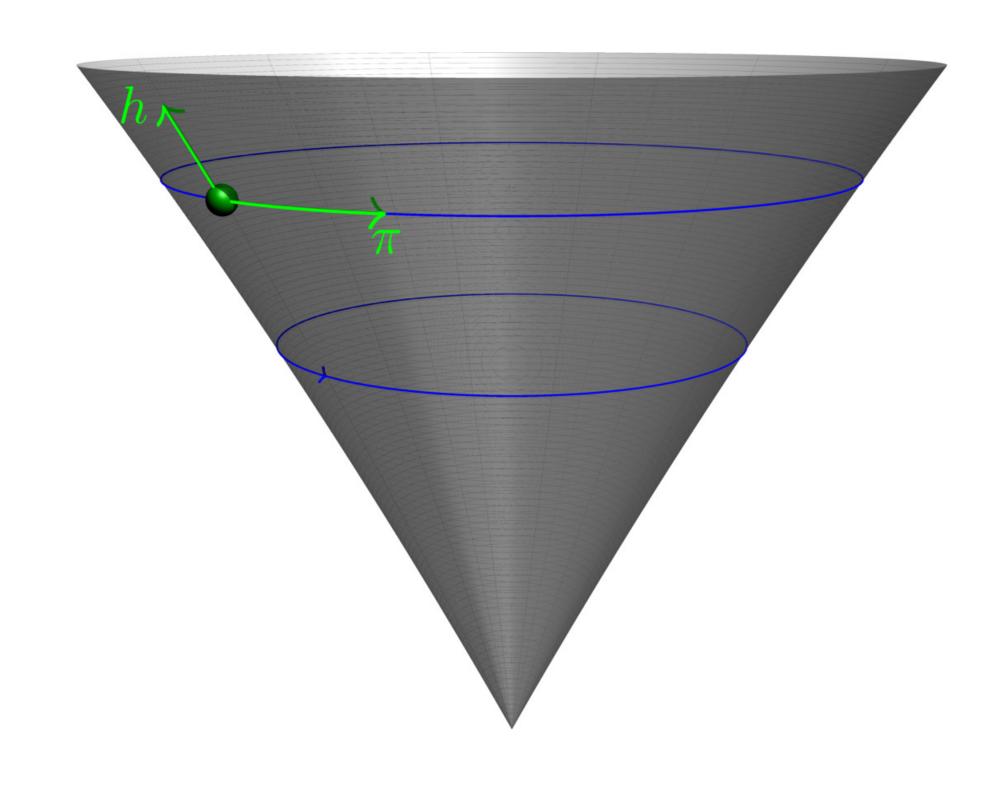
Can often tell by inspecting F(h), V(h) for non-analyticities, but this does not always work.

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h)$$

But can diagnose singularities as in GR:

If
$$(\nabla^2)^n R$$
 and $(\nabla^2)^{n+1} V$

are finite at h=-v, then can write HEFT as SMEFT (gives the requisite infinite set of conditions!)



Otherwise, there is a cone/cusp and HEFT is required.

HEFT not SMEFT: Case II

How does this arise? When a field becomes massless.

An example: integrating out anything that acquires all of its mass from EWSB, e.g. M=0 limit of

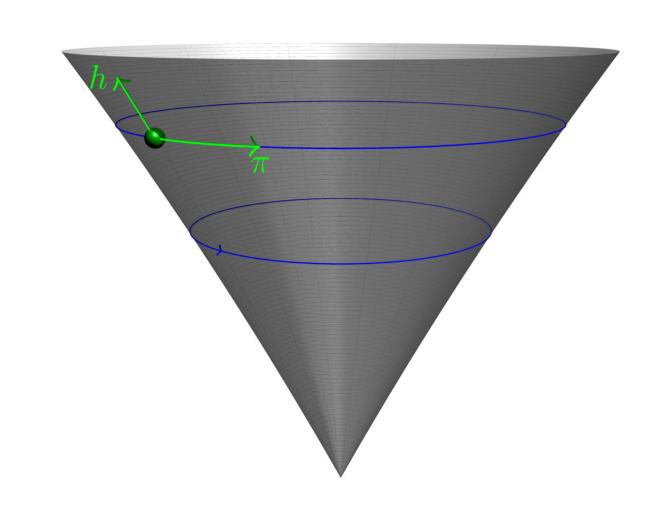
$$\mathcal{L} \supset \bar{\psi}_1(i \not \partial - M)\psi_1 + \bar{\psi}_2(i \not \partial - M)\psi_2 - y\bar{\psi}_1H\psi_2 + \text{h.c.}$$

F(h=-v)=0, so okay according to Case I

Compute Ricci scalar:
$$R(h=-v) \propto \frac{|y|^4}{16\pi^2} \frac{1}{M^2}$$

When $M \neq 0$, curvature finite and SMEFT is consistent

For M=0, curvature blows up. K, F, and V all non-analytic at h=-v due to log(v+h)



HEFT as SMEFT IFF

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

- 1. $F(h^*) = 0$ at some $h=h^*$ (candidate O(4) f.p.)
- 2. *Metric is analytic* at h=h*: F(h), K(h) admit convergent Taylor expansions here, and curvature invariants $\sim \nabla^n R$ are finite for n ≥ 0 .
- 3. **Potential is analytic** at $h=h^*$: V(h) admits convergent Taylor expansion here, and invariants $\sim \nabla^n V$ are finite for $n \ge 0$.

Satisfying these conditions ensures the theory admits a SMEFT expansion around the O(4) fixed point. However, a further consideration: that expansion should converge at our vacuum (h=0).

SMEFT Convergence

Even when SMEFT exists, the SMEFT expansion may not converge at our vacuum.

Clear example: for SMEFT with
$$\Lambda < v$$
, $\mathcal{L} \supset \sum_{n=1}^{\infty} c_n \frac{|H|^{4+2n}}{\Lambda^{2n}}$ diverges, w/out optimal truncation

To make this more concrete...

Consider a singlet scalar with nonzero bare mass,

$$\mathcal{L}_{UV} = |\partial H|^2 + \mu_h^2 |H|^2 - \frac{1}{2} \lambda_h |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2 \right) S$$

Integrating out the scalar gives 0- & 2-derivative effective lagrangian for H:

$$\delta \mathcal{L}_{\text{Eff}}^{(0)} = \frac{1}{(4\pi)^2} \frac{1}{4} \left(m^2 + \kappa |H|^2 \right)^2 \left(\ln \frac{\mu^2}{m^2 + \kappa |H|^2} + \frac{3}{2} \right) \qquad \delta \mathcal{L}_{\text{Eff}}^{(2)} = \frac{1}{(4\pi)^2} \frac{1}{4} \frac{1}{6} \frac{\kappa^2}{m^2 + \kappa |H|^2} \left(\partial |H|^2 \right)^2$$

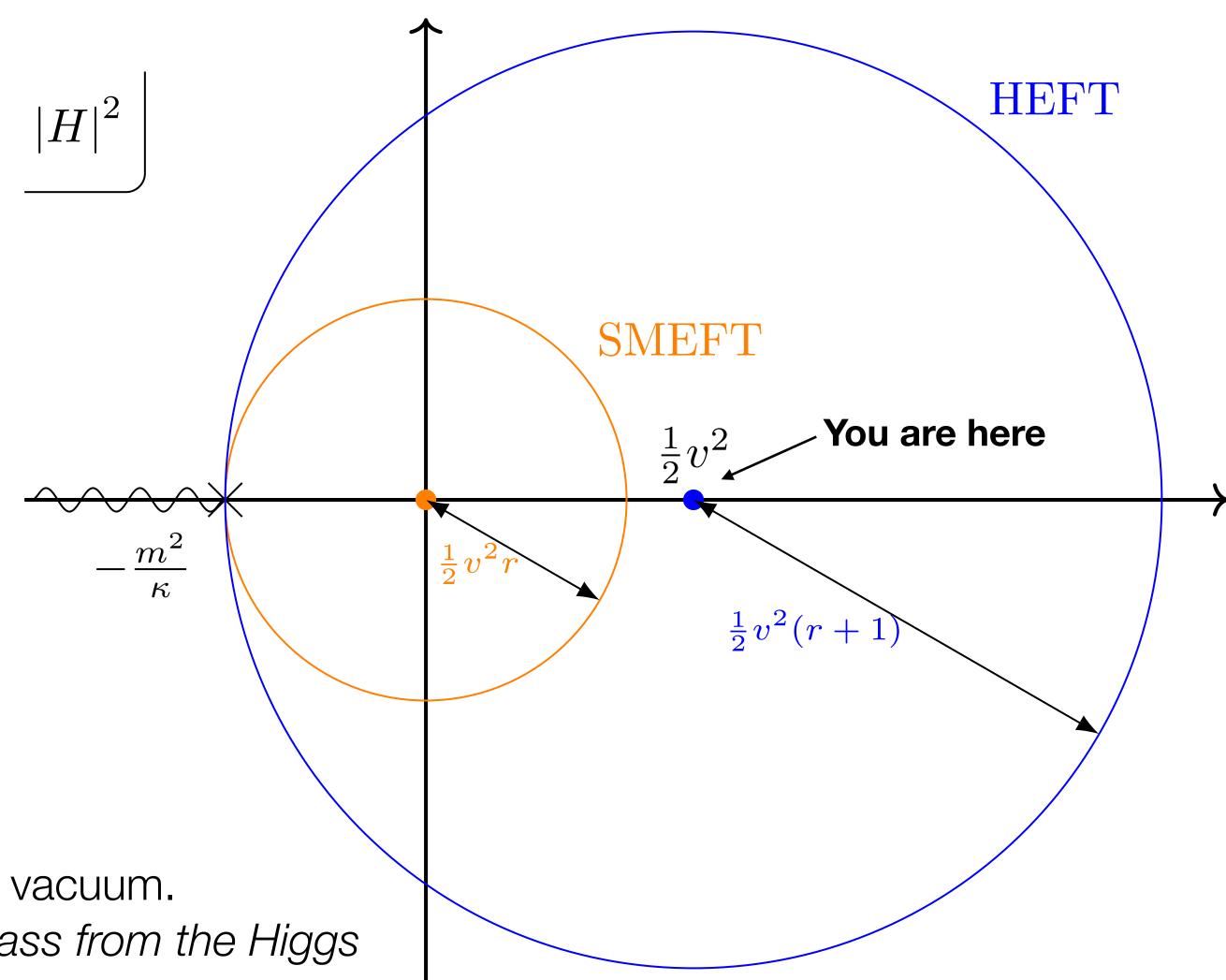
SMEFT Convergence

Consider analytic structure of the effective Lagrangian in the complex |H|² plane

$$r \equiv \frac{\text{bare mass}^2}{\text{mass}^2 \text{ from Higgs}} = \frac{m^2}{\frac{1}{2}\kappa v^2}$$

Branch cut at $|H|^2 = -\frac{m^2}{\kappa} \Rightarrow$

SMEFT radius of convergence is $v^2r/2$ HEFT radius of convergence is $v^2(r+1)/2$



r < 1: SMEFT expansion does not converge at our vacuum. HEFT required by states w/ more than half of their mass from the Higgs

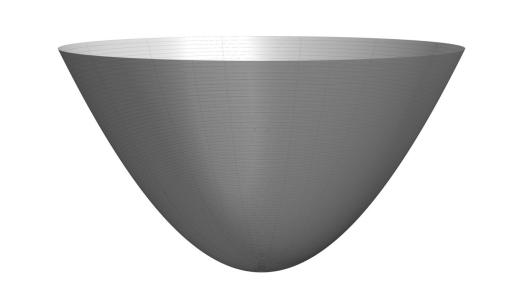
SMEFT Convergence

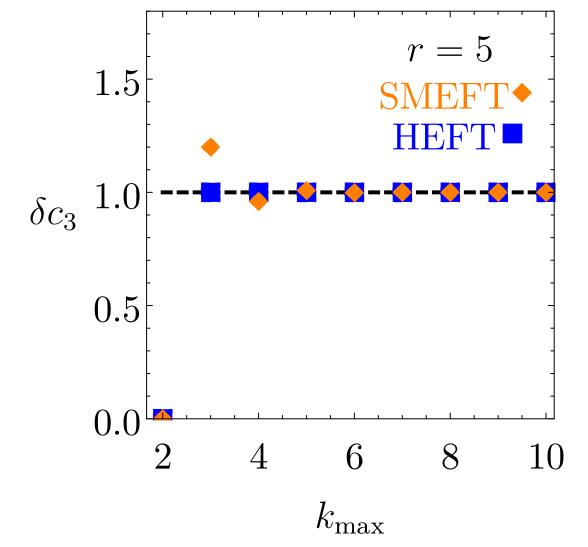
Even for $r \gtrsim 1$, HEFT can capture true corrections to SM using fewer terms than SMEFT.

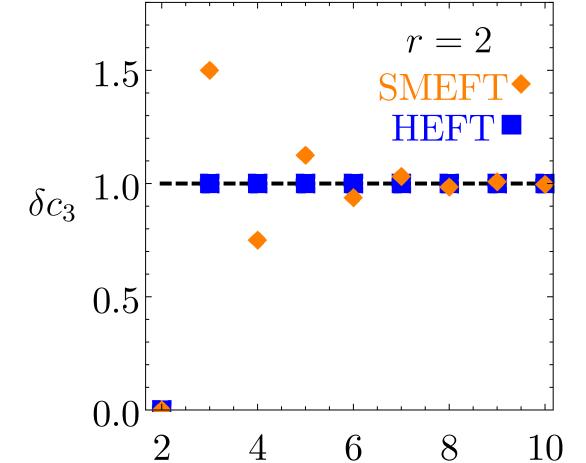
[Englert et al. 1403.7191; Brehmer et al. 1510.03443]

Improve agreement in SMEFT by defining matching scale as physical mass of new particles ("v-improved matching").

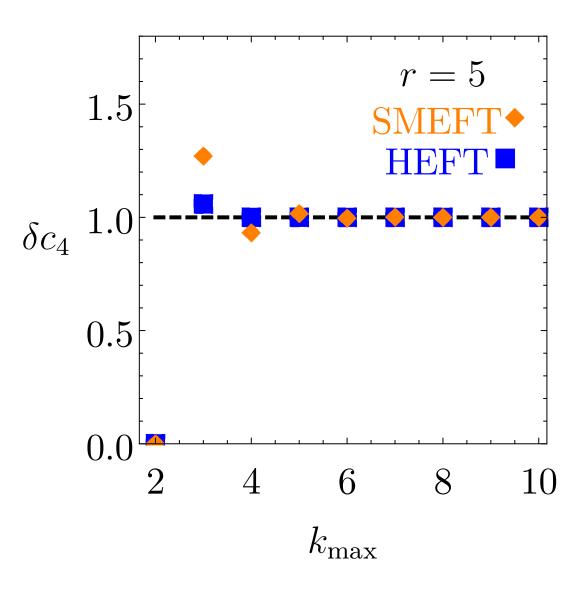
Practically amounts to matching in HEFT, converting to SMEFT coordinates.

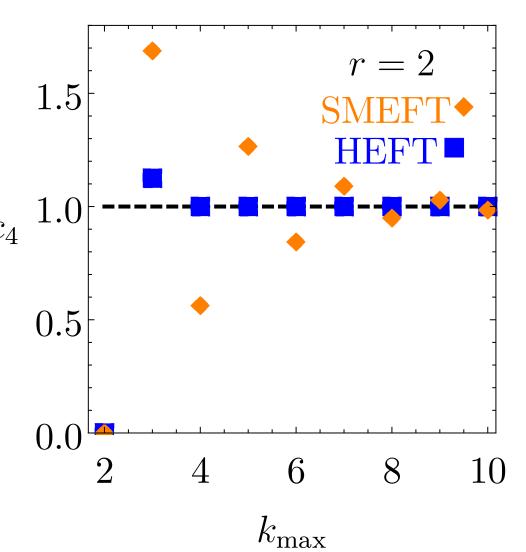






 k_{\max}



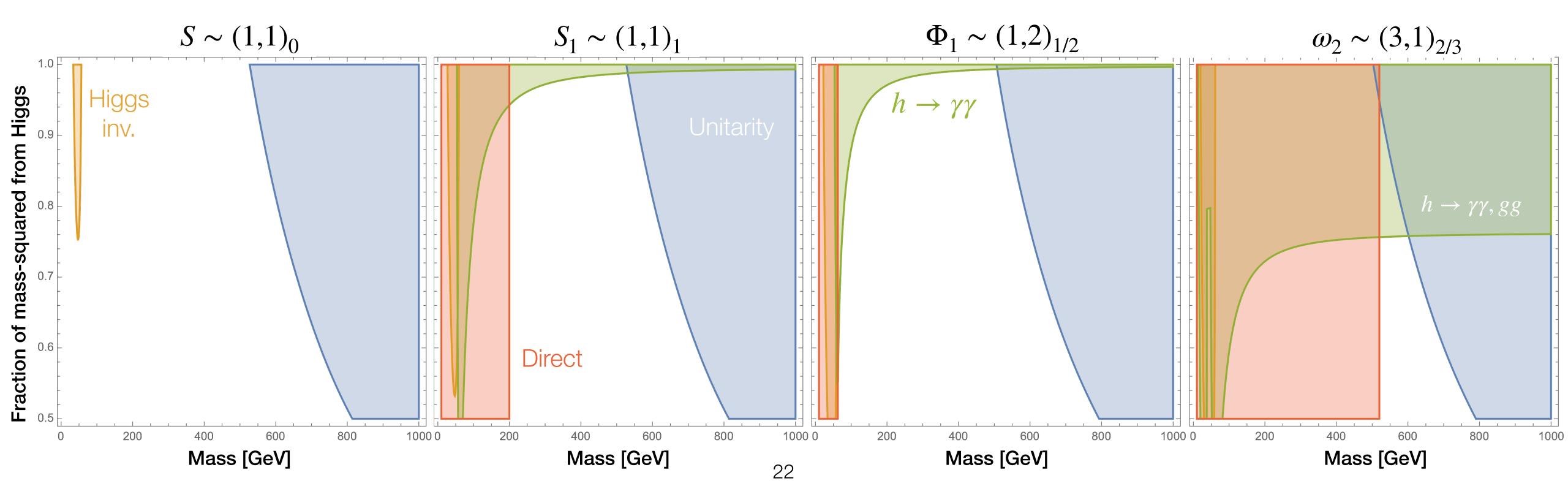


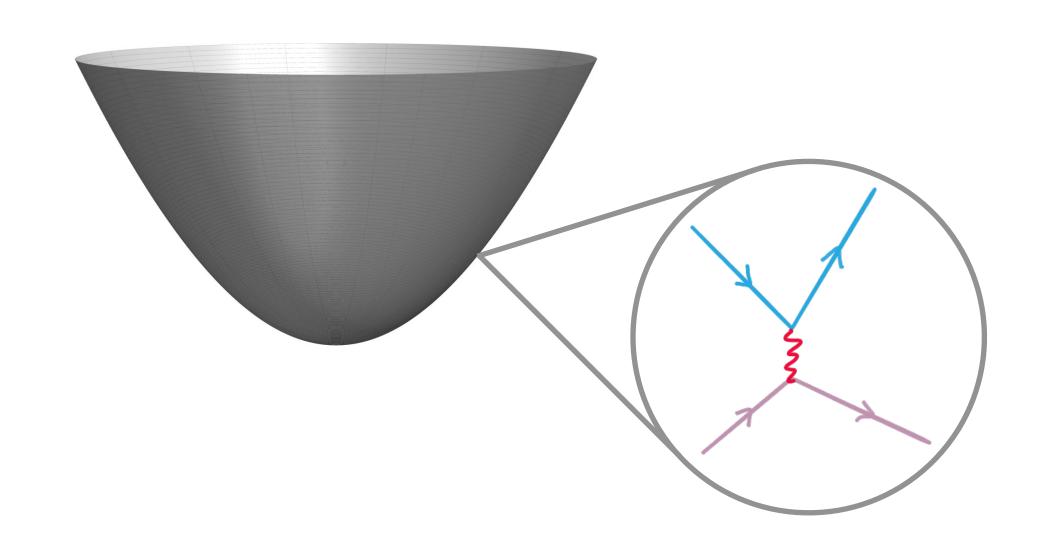
Enter the Loryon

(Following Gell-Mann, from Finnegan's Wake: "with Pa's new heft...see Loryon the comaleon.")

HEFT required whenever a new particle ("Loryon") acquires more than half of its mass from the Higgs.

Many such Loryons viable, consistent with all existing data (see also [Bonnefoy et al. 2011.10025]). A few examples here, but there are many more [Banta, Cohen, NC, Lu, Sutherland, to appear]...





How do we measure the EFT geometry?

Amplitudes can be written in terms of geometric quantities on scalar manifold, e.g. for HEFT [Alonso, Jenkins, Manohar 1511.00724, Nagai, Tanabashi, Tsumura, Uchida 1904.07618]

$$\mathcal{A}(\pi_i \pi_j \to hh) = -\delta_{ij} \mathcal{K}_h (h = \pi_k = 0) E^2 + \dots$$

Sectional curvatures:

$$\mathcal{K}_h \equiv \frac{R_{\pi_i h h \pi_j}}{-g_{hh} g_{\pi_i \pi_j}}$$

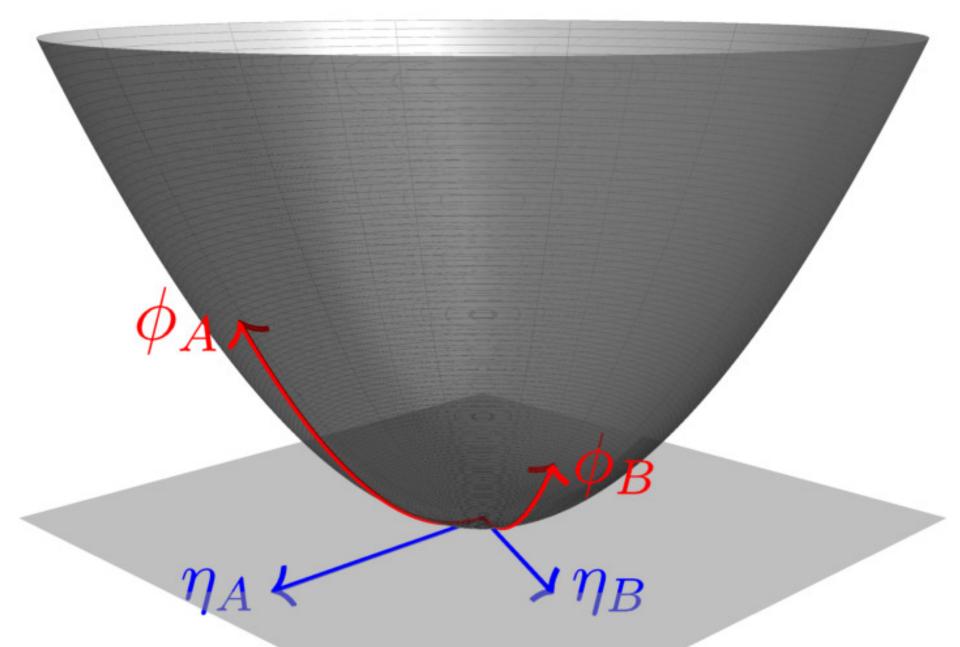
$$\mathcal{K}_{\pi} \equiv \frac{R_{\pi_i \pi_k \pi_l \pi_j}}{g_{\pi_i \pi_l} g_{\pi_k \pi_j} - g_{\pi_i \pi_j} g_{\pi_k \pi_l}}$$

$$R = 6(\mathcal{K}_h + \mathcal{K}_\pi)$$

Scattering amplitudes measure (local) curvature and its derivatives.

Connection is transparent in **normal coordinates**

$$\phi^{i} = \eta^{i} - \frac{1}{2} \Gamma^{i}_{jk} \eta^{j} \eta^{k} + (\frac{1}{3} \Gamma^{i}_{jk} \Gamma^{j}_{lm} - \frac{1}{6} \Gamma^{i}_{kl,m}) \eta^{k} \eta^{l} \eta^{m} + O(\eta^{4})$$



$$\mathcal{L}_{\phi} = \frac{1}{2} \left(\sum_{n \neq i} \frac{1}{n!} g_{ij,k_1...k_n}^{(\phi)} \phi^{k_1} \dots \phi^{k_n} \right) \partial \phi^i \partial \phi^j$$

$$\mathcal{L}_{\eta} = \frac{1}{2} \partial \eta^{i} \partial \eta^{j} \left(g_{ij} - \frac{1}{3} (R_{ikjl} + R_{jkil}) \eta^{k} \eta^{l} + O(\eta^{3}) \right)$$

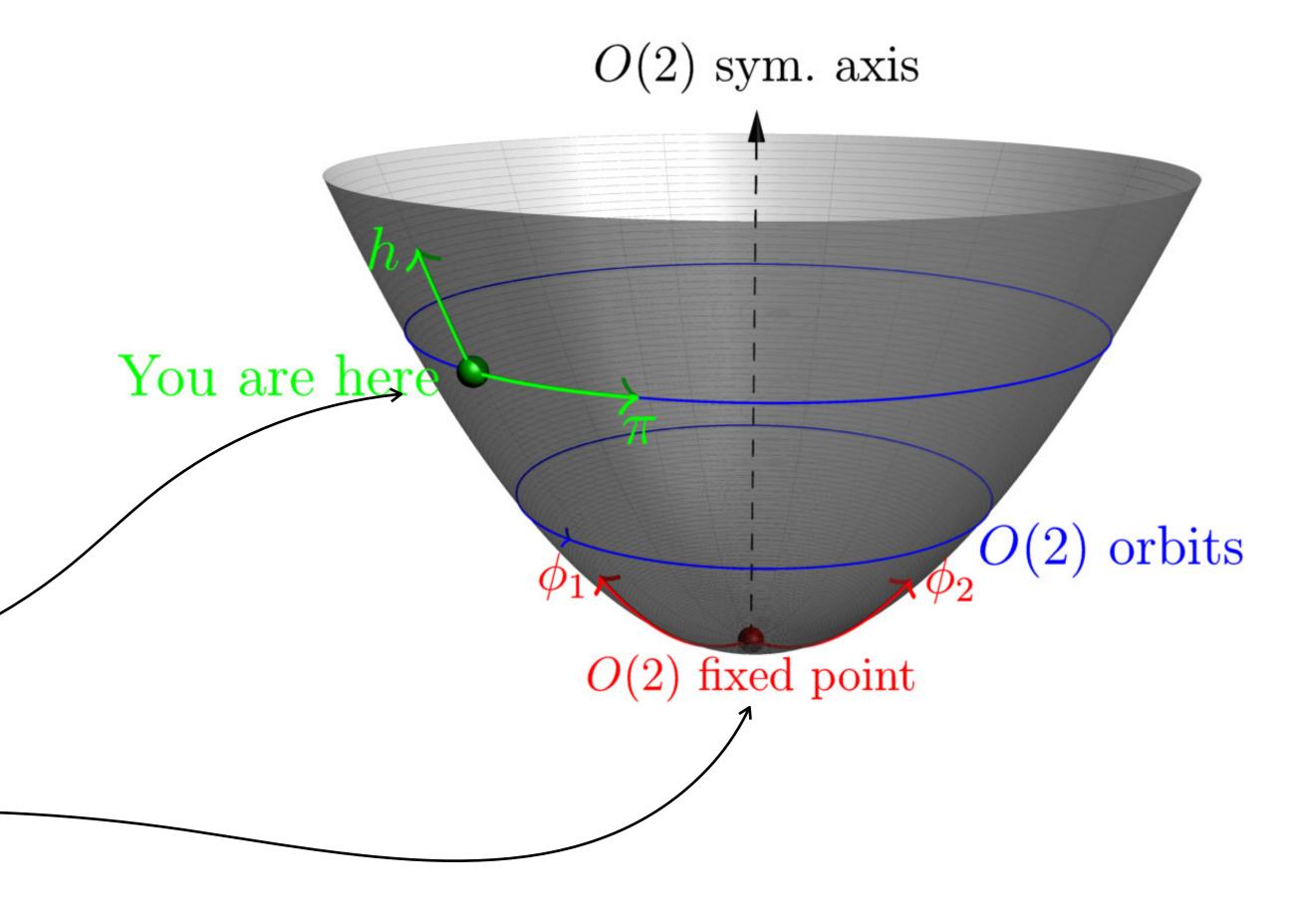
Lagrangian on tangent plane manifests geometric quantities

Long history of unitarity bounds in electroweak sector, *a la* [Lee, Quigg, Thacker '77].

Expect true HEFTs to violate unitarity at $\sim 4\pi v$, but 2-to-2 amplitudes only violate unitarity at $\sim 4\pi/\sqrt{\mathcal{K}_h(h=0)}$;

local sectional curvature needn't be $\mathcal{O}(1)/v^2$

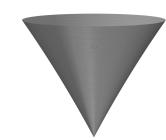
If amplitudes measure **local geometry**, how can we probe **geometry far from our vacuum**?



I.e., how do amplitudes test whether our EFT looks like





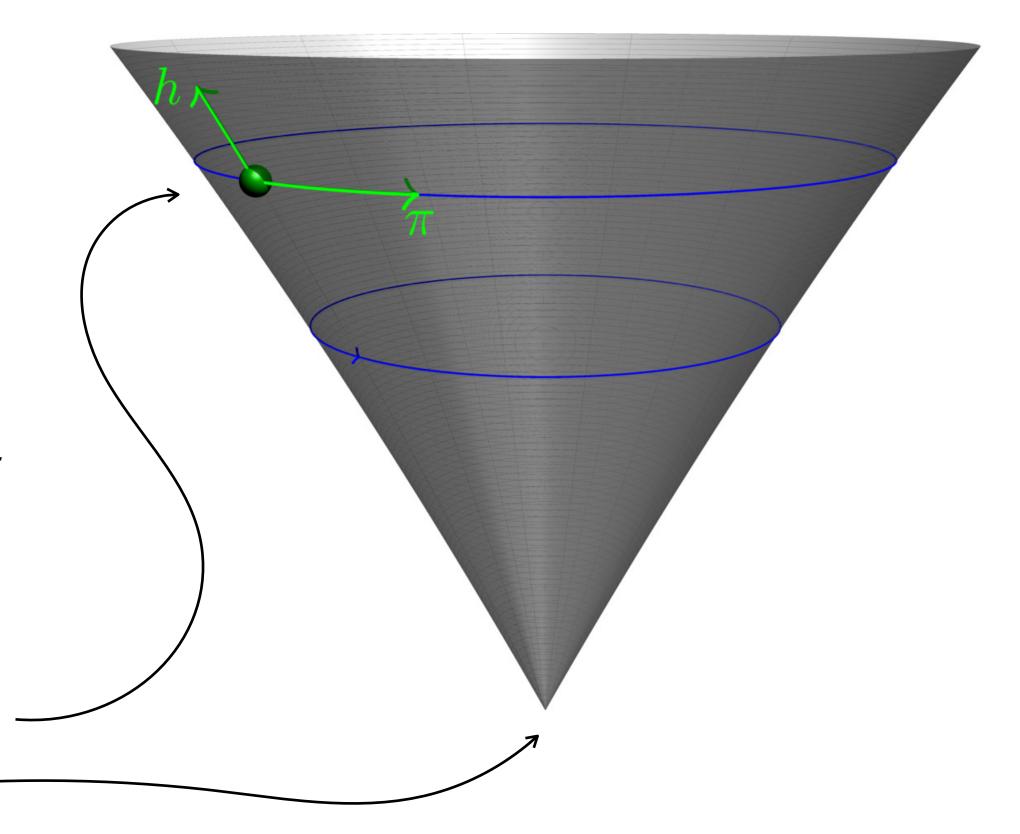


Parts of n>2 amplitudes that grow with energy are *derivatives* of sectional curvatures:

$$\mathcal{A}\left(\pi_i \pi_j \to h^n\right) = -E^2 \,\delta_{ij} \,\partial_h^{n-2} \mathcal{K}_h|_{h=0} + \mathcal{O}(E^0)$$

Higher-point amplitudes reconstruct coefficients in the Taylor expansion of geometric invariants on the EFT manifold.

It will be apparent in high-point amplitudes measured here if something pathological is happening over there



Ideally leads to a connection between geometry and $\sim 4\pi v$ scale of unitarity violation...

Geometry & Unitarity

Applying unitarity bound to suitably normalized s-wave state (e.g. [Abu-Ajamieh, Chang, Chen, Luty, 2009.11293])

$$E < 4\pi \times \left| \frac{\partial_h^{n-2} \mathcal{K}_h}{n!} \right|_{h=0}^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}} = \begin{cases} 8^{\frac{1}{4}} \sqrt{16\pi} \times |\mathcal{K}_h|_{h=0}^{-\frac{1}{2}} & n = 2\\ 4\pi v_* \times (n!)^{\frac{1}{n}} & n = \text{`a few'} \end{cases}$$

 v_* is the **radius of convergence** of \mathcal{K}_h , linked to $\partial_h^{n-2}\mathcal{K}_h$ by Cauchy-Hadamard theorem

Lesson: radius of convergence of sectional curvatures sets unitarity bound! As we have seen, HEFT $v_* \sim v$, SMEFT $v_* \equiv \Lambda \gg v$

Comments:

- 1. $2 \rightarrow 2$ scattering does not always access optimal unitarity bound [Falkowski & Rattazzi 1902.05936; Chang & Luty 1902.05556; Abu-Ajamieh, Chang, Chen, Luty 2009.11293].
- 2. Can sum over different final states to obtain only logarithmic dependence on prefactors, reproducing [Falkowski & Rattazzi 1902.05936] in geometric terms.

$$b_n \sim 1$$
 is a fudge factor of the form $\left(\frac{1}{b_n}\right)^{2n} = \frac{(4\pi)^2}{8(n-1)} \left(1 - \frac{2m_h^2}{(n+1)E^2}\right)^2 \times \frac{\text{Vol. } n \text{ body Higgs PS}}{\text{Vol. } n \text{ body massless PS}}$

Our Mission

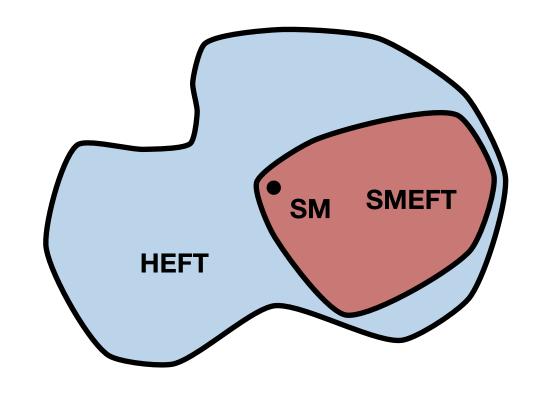
Should we choose to accept it...

To answer this question:

"Is electroweak symmetry linearly realized by the known fundamental particles?"

Equivalently: can we rule out pure HEFTs?

- It is a sharply defined, bounded question.
- We don't currently know the answer.
- We might be able to find out @ the LHC.
- Null results (agreement w/SM) only help.



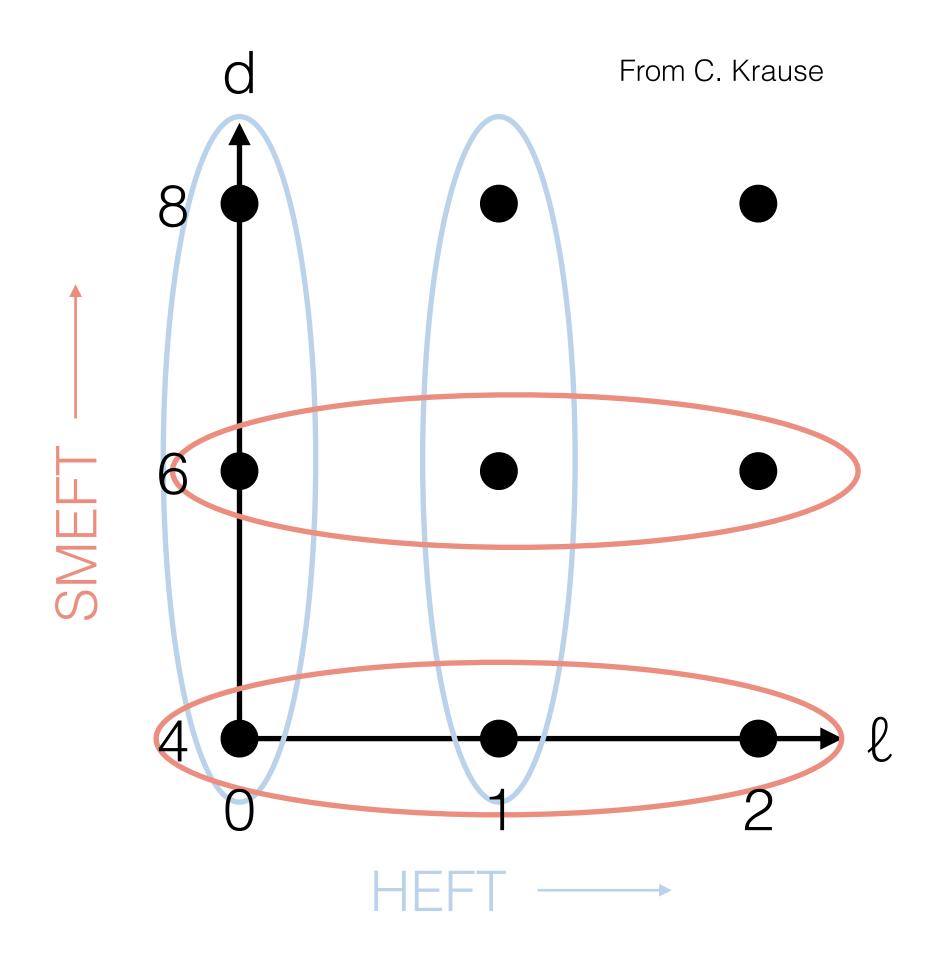
Top-down: rule out the perturbative scenarios forcing HEFT (less satisfying)

Bottom up: "check unitarity in a complete set of channels up to $4\pi v$ " (specifics TBD)

This is a "big" question that we can potentially answer even if the LHC sees no departures from SM.

HEFT Surprises?

Another reason to study HEFT: expect surprises analogous to those observed in SMEFT.



For example, HEFT 1-loop anomalous dimensions [Buchalla et al. 2004.11348]

One loop divergences due to some SMEFT operators reproduced in HEFT, e.g.

$$\mathcal{O}_{\phi\square} = \phi^{\dagger}\phi\square\phi^{\dagger}\phi$$

Such operators give "surprising zeroes" in SMEFT matrix of anomalous dimensions; should manifest in HEFT.

More generally, expect to discover rich structure of 1and 2-loop surprises mirroring that of SMEFT (e.g. [Bern, Parra-Martinez, Sawyer 2005.12917])...

Conclusions

Universal geometric criteria for HEFT vs. SMEFT:



- Many ways to get U(1)_{em} Higgs EFT starting from SU(2)xU(1) UV symmetry, consistent w/ data. HEFT can be
 the preferred EFT for data even when both HEFT & SMEFT expansions valid. Perhaps premature to focus
 heavily on SMEFT interpretations.
- Interesting connections between geometric picture & scattering amplitudes, e.g. [Alonso, Jenkins, Manohar '16; Nagai, Tanabashi, Tsumura, Uchida '19; Cohen, NC, Lu, Sutherland to appear]. Links approaches to HEFT/SMEFT based on unitarity, analyticity, and geometry.
- There is a "big" question we should ask, which to my knowledge is not being systematically explored: "Is electroweak symmetry linearly realized by the known fundamental particles?"
- Motivates giving HEFT more thorough attention, both "in principle" and "in practice." Plethora of structural
 questions currently being explored in SMEFT can also be addressed in HEFT...