Higgs boson pair production at NNLO in QCD with finite top quark mass effects

Florian Herren

March 25, 2021

Fermilab

Why are we interested in Higgs boson pair production?

Why Higgs boson pair production?

- Scalar potential of the SM:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

$$\xrightarrow{EWSB} V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4} \lambda H^4$$

- Where the self-coupling λ can be expressed through known quantities:

$$\lambda = \frac{\mu^2}{v^2} = \frac{m_H^2}{2v^2} \approx 0.13$$

Why Higgs boson pair production?

- Scalar potential of the SM:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2 - \frac{C_H}{\Lambda^2} \left(\Phi^\dagger \Phi\right)^3$$

$$\xrightarrow{EWSB} V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4} \lambda_4 H^4 + \ldots$$

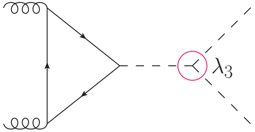
- Where the self-coupling λ_3 gets modified:

$$\lambda_3 = \lambda \left(m_H^2 - \frac{C_H v^2}{\lambda} \right)$$

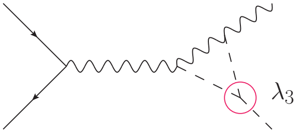
- Virtually any new physics which couples to the Higgs will induce such an operator

Why Higgs boson pair production?

- In which processes do we have access to λ_3?
- The direct way, Higgs boson pair production:

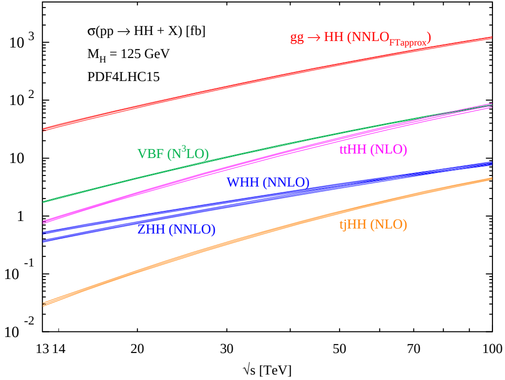


- The indirect way, corrections to single Higgs boson production:



Experimental challenges

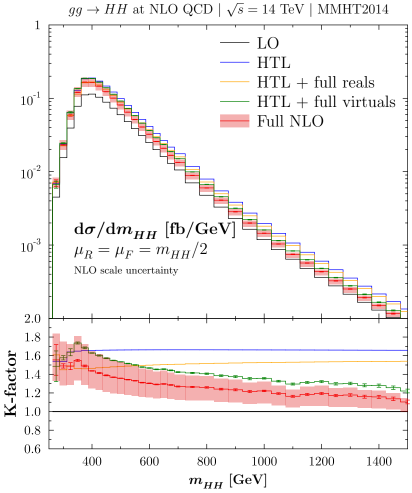
Production cross sections



taken from arXiv:1910.00012 [hep-ph]

- Cross sections three orders of magnitude smaller than for single Higgs boson
- Only $gg \to HH$ accessible

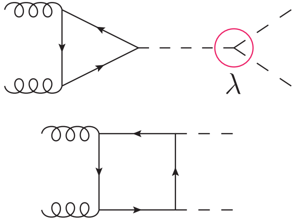
$gg \to HH$ cross section

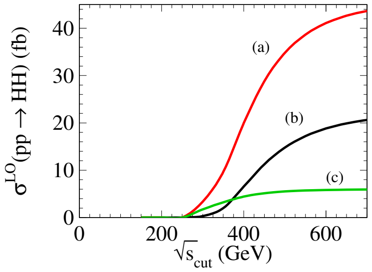


taken from arXiv:1910.00012 [hep-ph]

- Bulk of cross-section near $\sqrt{s} = 2m_t$

$gg \to HH$ cross section

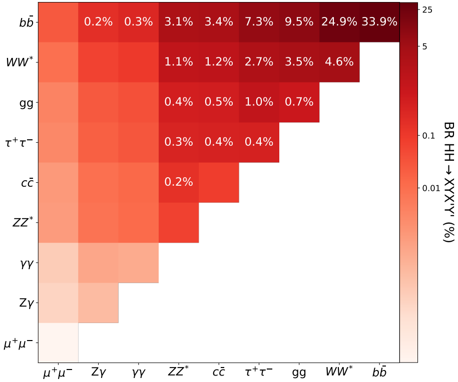




taken from arXiv:1305.7340 [hep-ph]

- Destructive interference
  $\Rightarrow$ sensitivity to λ near threshold

Branching ratios



taken from arXiv:1910.00012 [hep-ph]

- Clean channels have even smaller branching ratios than in the SM
  $\Rightarrow$ always one $H \to b\bar{b}$ decay for most searches

Branching ratios

$HH \to b\bar{b}b\bar{b}$

- 33% of all events
- Large QCD multi-jet background, especially at low m_{hh}

$HH \to b\bar{b}\gamma\gamma$

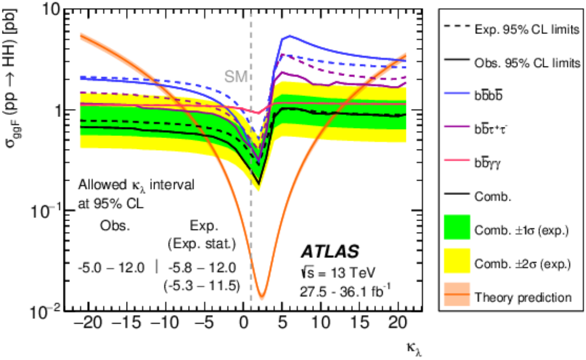
- Only 0.3% of the events
- High signal-to-background ratio

$HH \to b\bar{b}\tau^+\tau^-$

- Lower backgrounds than 4-b final state
- Use of semi-hadronic and fully hadronic tau decays
- τ_h identification, $Zb\bar{b}$, $t\bar{t}$ as backgrounds

Other final states searched for:
$b\bar{b}VV^{(*)}$, WW^*WW^*, $\gamma\gamma WW^*$, $\tau^+\tau^-\tau^+\tau^-$

Status and outlook

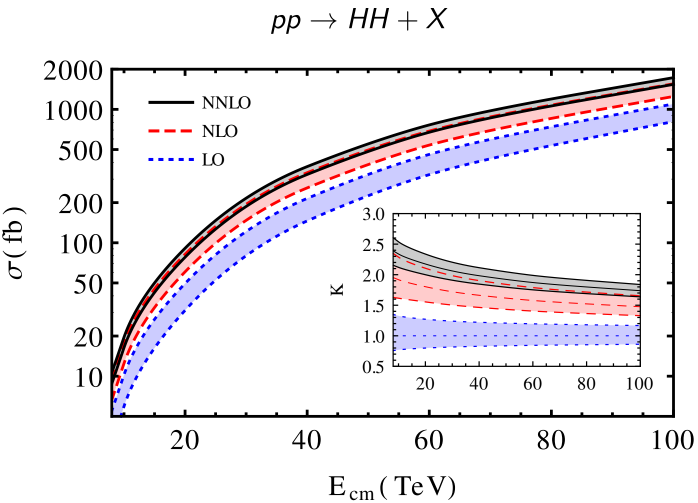


taken from arXiv:1906.02025 [hep-ex]

- current limit: $-5.0 < \kappa_\lambda < 12.0$
- HL-LHC: $0.5 < \kappa_\lambda < 2.4$ [WG2 Report '19]
- FCC-hh: 5% accuracy [FCC CDR '19]

Theoretical Status

Why higher orders?

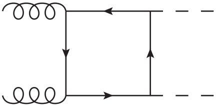


Hadronic c.o.m energy, taken from [de Florian, Mazzitelli '13]

Status at NLO

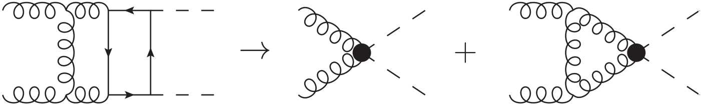
Exact LO results known for 3 decades:

[[Glover, van der Bij '88],[Plehn, Spira, Zerwas '96]



Various approximations used at NLO, mostly in the limit of (infinitely) heavy top quark:

- Born-improved HEFT [Dawson, Dittmaier, Spira '98]
- LME [Grigo, Hoff, Melnikov, Steinhauser '13], [Degrassi, Giardino, Gröber '16]
- $\mathrm{FT}_{\mathrm{approx}}$, $\mathrm{FT}'_{\mathrm{approx}}$ [Maltoni, Vryonidou, Zaro '14]
- LME + Threshold expansion [Gröber, Maier, Rauh '17]



Status at NLO

Furthermore there exist approximations addressing other parts of the phase-space:

- High-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18 + '19],[Mishima '18]: valid for $s, t \gg m_t^2 \gg m_h^2$
- Small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]: valid for $p_T^2 < 4m_t^2$ (this is always true for $s <$ 750 GeV)

$\rightarrow$ Combining LME and these two expansions should cover the full phase-space

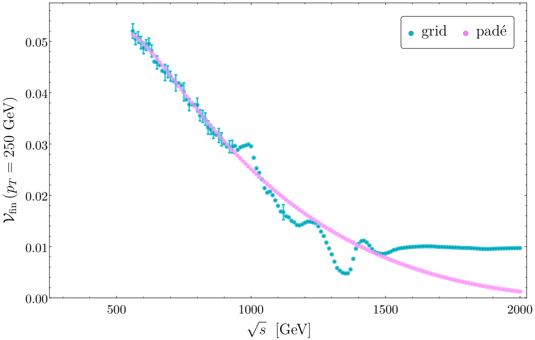
Status at NLO

Exact numerical calculations available, covering the whole phase-space

[Borowka, Greiner, Herinrich, Jones, Kerner, Schlenk, Zirke '16],
[Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '18]

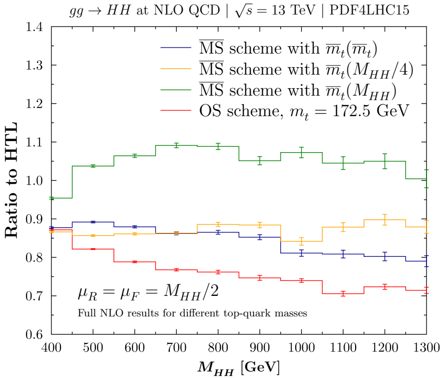
Recently have been combined with high-energy expansion

[Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann '19]



Status at NLO

There is one issue....

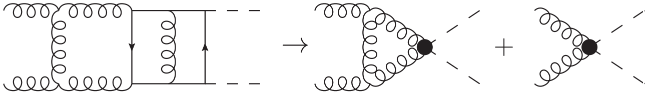


taken from arXiv:2008.11626 [hep-ph]

Status beyond NLO

The next step is to improve the situation at NNLO, here only few results exist:

- HEFT [de Florian, Mazzitelli '13], [Grigo, Melnikov, Steinhauser '14]
- $1/m_t^2$ corrections for virtual parts [Grigo, Hoff, Steinhauser '14]
- $1/m_t^2$ corrections for virtual parts combined with exact real radiation where available [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]

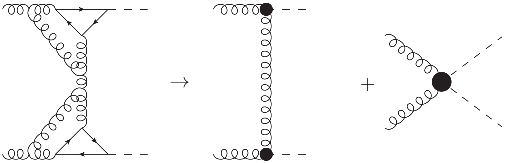


$\rightarrow$ our goal is to obtain $1/m_t^2$ corrections for real radiation

Status beyond NLO

At N^{3}LO the basic building differing from single Higgs production recently have been computed for HEFT:

- Wilson coefficient C_{HH} [Spira '16],[Gerlach, FH, Steinhauser '18]
- 2 loop box-type diagrams [Banerjee, Borowka, Dhani, Gehrmann, Ravindran '18]



$\rightarrow$ HEFT result recently obtained [Chen, Li, Shao, Wang '19] $\rightarrow 2-3\%$ increase of total cross-section

Towards a better NNLO description

What are we computing?

- Inclusive, partonic cross-section at NNLO

$$\sigma_{ij}(s, m_H, m_t) = \delta_{ig}\delta_{jg}\sigma^{(0)}_{gg} + \frac{\alpha_s}{\pi}\sigma^{(1)}_{ij} + \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(2)}_{ij} + \ldots$$

$$\sigma^{(2)}_{ij} = \underbrace{\delta_{ig}\delta_{jg}\sigma^{(2)}_{gg,\mathrm{virt}}}_{\text{[Grigo,Hoff,Steinhauser '14]}} + \underbrace{\sigma^{(k)}_{ij,\mathrm{real}}}_{\text{this work}}$$

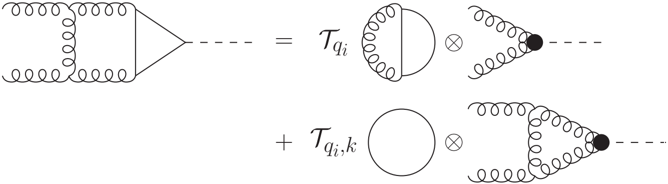
- So far only known in $m_t \to \infty$ limit, full computation not feasible
  $\to$ asymptotic expansion in large top quark mass

$$\sigma^{(l)}_{ij}(s, m_H, m_t) \approx \sum_k \rho^k \sigma^{(l),k}_{ij}(x)$$

$$\rho = m_H^2/m_t^2, \quad x = m_H^2/s$$

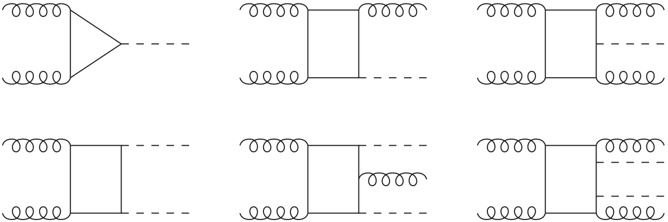
How are we computing this?

We need to systematically compute higher-order terms in the large mass expansion.
The method of asymptotic expansion lets us separate hard scales (m_t) and soft scales (q_i, m_h) by taking all relevant regions of the loop integration into account:



How are we computing this?

We need to systematically compute higher-order terms in the large mass expansion.
Perform a Taylor expansion of all possible one loop building blocks:

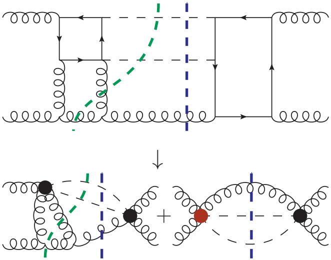


$$F^{\mu\nu}\left(q_1, q_2, m_t\right) \rightarrow f_{00}\left(\frac{q_i \cdot q_j}{m_t^2}\right) q_1 \cdot q_2 g^{\mu\nu} + f_{kl}\left(\frac{q_i \cdot q_j}{m_t^2}\right) q_k^\mu q_l^\nu$$

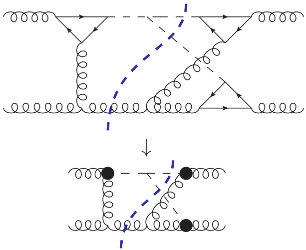
$\rightarrow$ Pre-compute all building blocks in LME, insert them in amplitudes

How are we computing this?

We use reverse unitarity to express the real corrections through cuts of 5-loop diagrams with 2 masses in forward scattering kinematics.

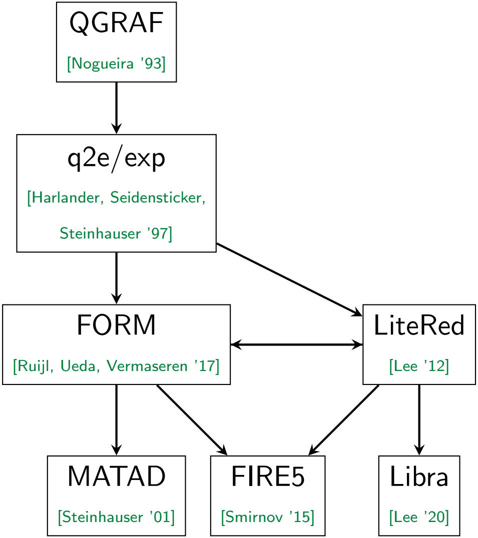


- 2 1-loop tadpoles + 3-loop PS integral
- 1-loop + 2-loop tadpoles + 2-loop PS integral



- 3 1-loop tadpoles + 2-loop PS integral

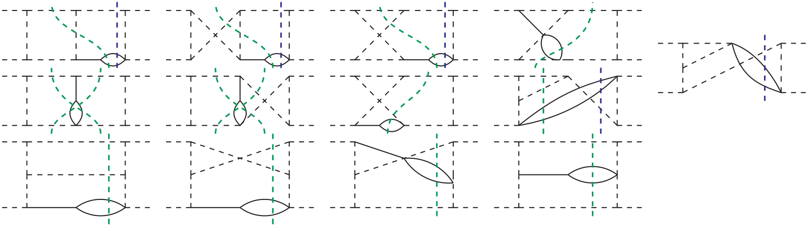
Setup


QGRAF
[Nogueira '93]
q2e/exp
[Harlander, Seidensticker, Steinhauser '97]
FORM
[Ruijl, Ueda, Vermaseren '17]
LiteRed
[Lee '12]
MATAD
[Steinhauser '01]
FIRE5
[Smirnov '15]
Libra
[Lee '20]

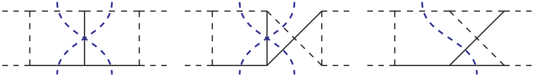
Families

We use LiteRed [Lee '12] for minimizing families, identifying common sectors and performing the reduction

We end up with 13 three-loop,



and 3 two-loop phase space integral families:



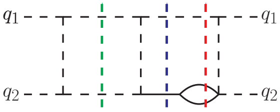
Differential equations

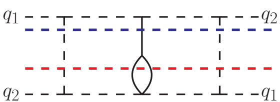
- 3 systems of differential equations
- We bring them to Fuchsian form using Epsilon [Prausa '17]:

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I} \qquad x = m_h^2/s$$

- They contain the letters:

$x_i =$	0	1	$\frac{1}{4}$	-1	$-\frac{1}{4}$	$e^{\pm i\frac{\pi}{3}}$	$-\frac{1}{3}$
2 loop, 3 particle	✓	✓	✓			✓	✓
3 loop, 3 particle	✓	✓	✓				
3 loop, 4 particle	✓	✓	✓	✓	✓		





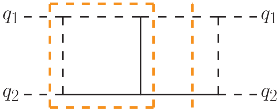
Differential equations

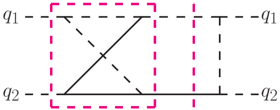
- 3 systems of differential equations
- We bring them to Fuchsian form using Epsilon [Prausa '17]:

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I} \qquad x = m_h^2/s$$

- They contain the letters:

$x_i =$	0	1	$\frac{1}{4}$	-1	$-\frac{1}{4}$	$e^{\pm i \frac{\pi}{3}}$	$-\frac{1}{3}$
2 loop, 3 particle	✓	✓	✓			✓	✓
3 loop, 3 particle	✓	✓	✓				
3 loop, 4 particle	✓	✓	✓	✓	✓		





Differential equations

- We proceed to transform the differential equations to ϵ-form using Libra [Lee, unpublished]:

$$\partial_x \vec{I} = \sum_i \frac{M^{(i)}(\epsilon)}{x - x_i} \vec{I} \quad \rightarrow \quad \partial_x \vec{J} = \epsilon \sum_i \frac{\tilde{M}^{(i)}}{x - x_i} \vec{J}$$

- The solution to DEs in this form can be written as Goncharov Polylogarithms
- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

Differential equations

- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

$\rightarrow$ Need to find change of variable to transform square roots into rational functions

Differential equations

- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

- In the DE for the three-loop, three-particle cuts only $\sqrt{1-4x}$ appears

$$x = \frac{y}{(1+y)^2} \quad x \in (0, 1/4] \to y \in (0, 1]$$

Differential equations

- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

- In the DE for the three-loop, four-particle cuts both $\sqrt{1-4x}$ and $\sqrt{1+4x}$ appear

$$x = \frac{t^4+1}{8t^2} \quad x \in (0, 1/4] \rightarrow t = e^{i\phi} : \phi \in (1/4, 0]$$

Differential equations

- Some of the $M^{(i)}(\epsilon)$ contain Eigenvalues of the form $\epsilon \pm 1/2$:

	$\sqrt{1-4x}$	$\sqrt{1+4x}$	$\sqrt{(1+3x)(1-x)}$
2 loop, 3 particle	✓		✓
3 loop, 3 particle	✓		
3 loop, 4 particle	✓	✓	

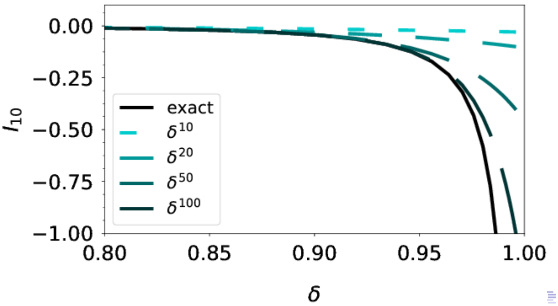
- In the DE for the two-loop, three-particle cuts $\sqrt{1-4x}$ and $\sqrt{(1+3x)(1-x)}$ appear
- Can not be rationalized together, however integrations over $\sqrt{(1+3x)(1-x)}$ do not contribute to cross-section

$$x = \frac{y}{(1+y)^2} \quad x \in (0, 1/4] \rightarrow y \in (0, 1]$$

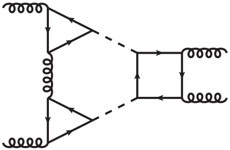
Boundary conditions and δ-expansion

- Parametrize necessary phase-space integrals through $\delta = 1 - 4x$
- Expand around threshold $\delta \approx 0$
- Minimization and other manipulation of Goncharov Polylogarithms with PolyLogTools [Duhr, Dulat '19]
- We also obtained solutions to the DE with the ansatz

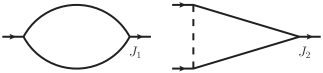
$$\vec{I} = \sum_{i,j,k} \vec{c}_{i,j,k} \epsilon^i \delta^{(2j+1)/2} \ln^k \delta$$



n_h^3: NLO



- Only gg channel contributes
- Already known in expansion around threshold [Grigo, Hoff, Melnikov, Steinhauser '13]
- Recomputed exactly in $m_h^2/s \to 2$ MIs

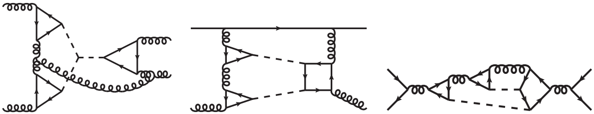


- collinear counterterm

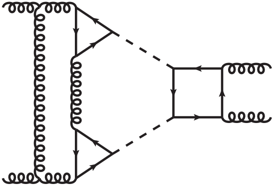
$$\sigma_{gg,\mathrm{coll}}^{(2),n_h^3} = 2\int_{1-\delta}^{1} \mathrm{d}z P_{gg}(z)\sigma_{gg}^{(1),n_h^3}(x/z)$$

n_h^3: NNLO

- Three contributing channels gg, gq and $q\overline{q}$:

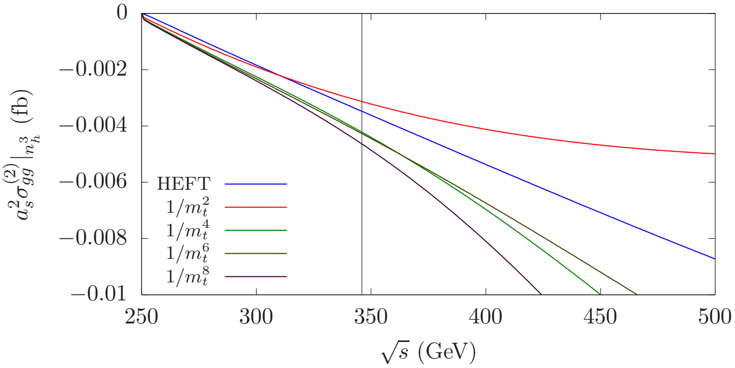


- Expanded to $1/m_t^8$
- Combine them with virtual corrections



Results

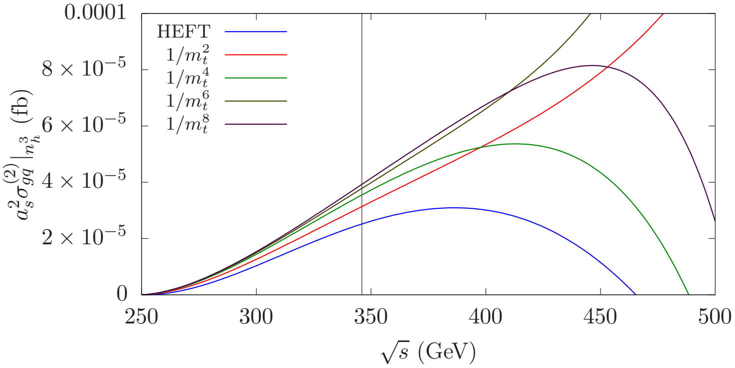
[Davies, FH, Mishima, Steinhauser JHEP 1905 (2019) 157]



- Convergence only below $2m_t$

Results

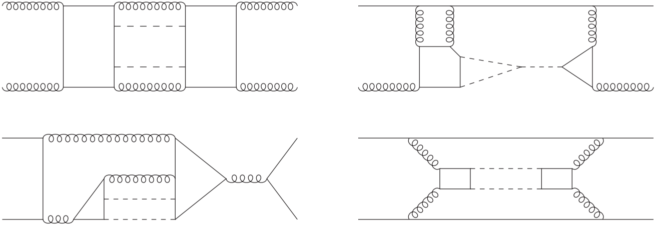
[Davies, FH, Mishima, Steinhauser JHEP 1905 (2019) 157]



- Convergence only below $2m_t$

Full NNLO

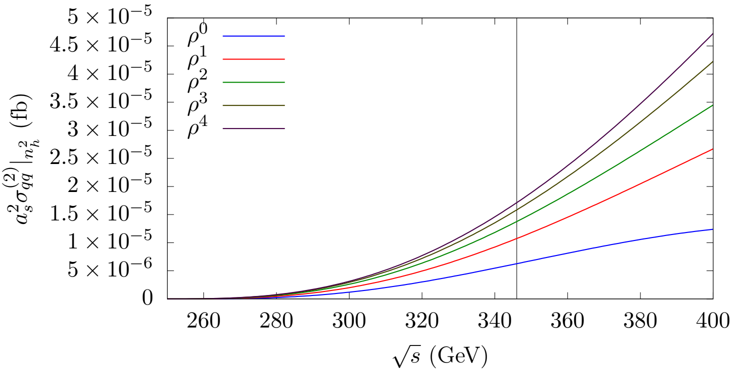
- Five contributing channels gg, gq, $q\bar{q}$, qq and qq':



- $q\bar{q}$, qq and qq' done
- gg and gq significantly more complicated and WIP

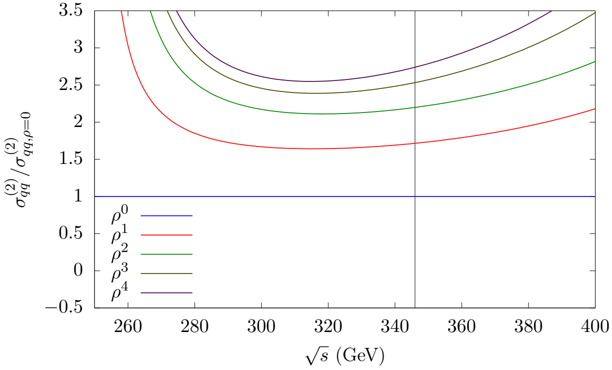
Full NNLO

- qq results:



Full NNLO

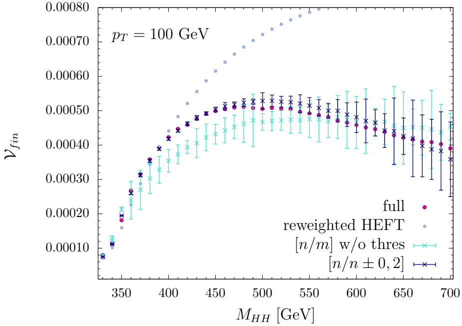
- qq results:



Conclusions and Outlook

Outlook

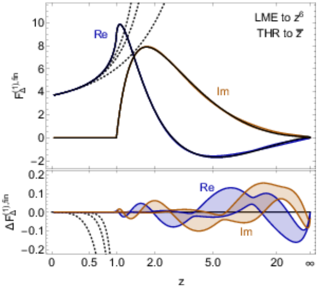
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part:



taken from [Gröber, Maier, Rauh '17]

Outlook

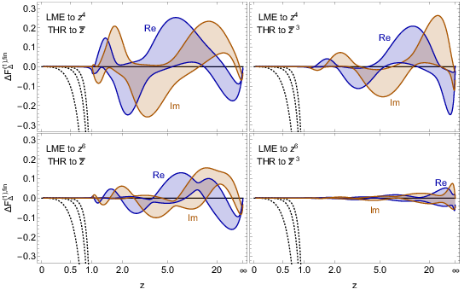
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part
- Higher orders in LME crucial, e.g. for NLO ggH formfactor:



taken from [Davies, Gröber, Maier, Rauh, Steinhauser '19]

Outlook

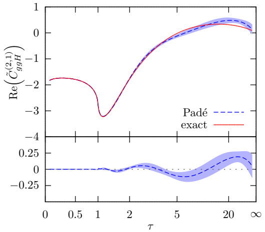
- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part
- Higher orders in LME crucial, e.g. for NLO *ggH* formfactor:



taken from [Davies, Gröber, Maier, Rauh, Steinhauser '19]

Outlook

- LME is only useful in a limited part of phase space
- Can be combined with threshold expansion
- Works well for virtual part
- Higher orders in LME crucial
- Recently contributions with light fermion loops at NNLO confirmed by analytic computation:



taken from [Harlander, Prausa, Usovitch '19]

- All other bits confirmed by numerical methods
- Expansions needed, in particular for $gg \to HH$

Summary

- Higgs Boson pair production is a very interesting process
- Higher order QCD corrections necessary
- We obtained top-mass suppressed terms at NNLO
- Computed phase-space MIs, both, in an expansion around threshold and exactly
  $\rightarrow$ rather generic to any pair-production process of massive particles
- Work in progress: cross-sections for double real and remaining real-virtual contributions
- More work needs to be done: minimize scheme dependence, look at electroweak contributions