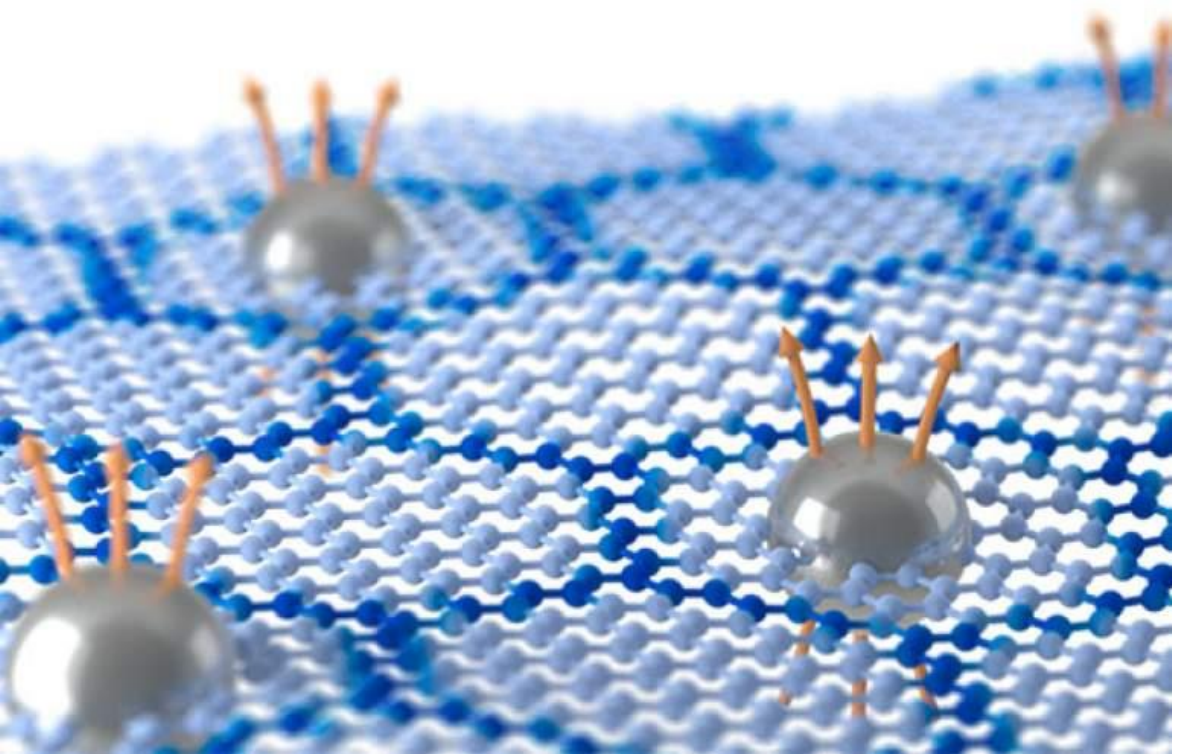
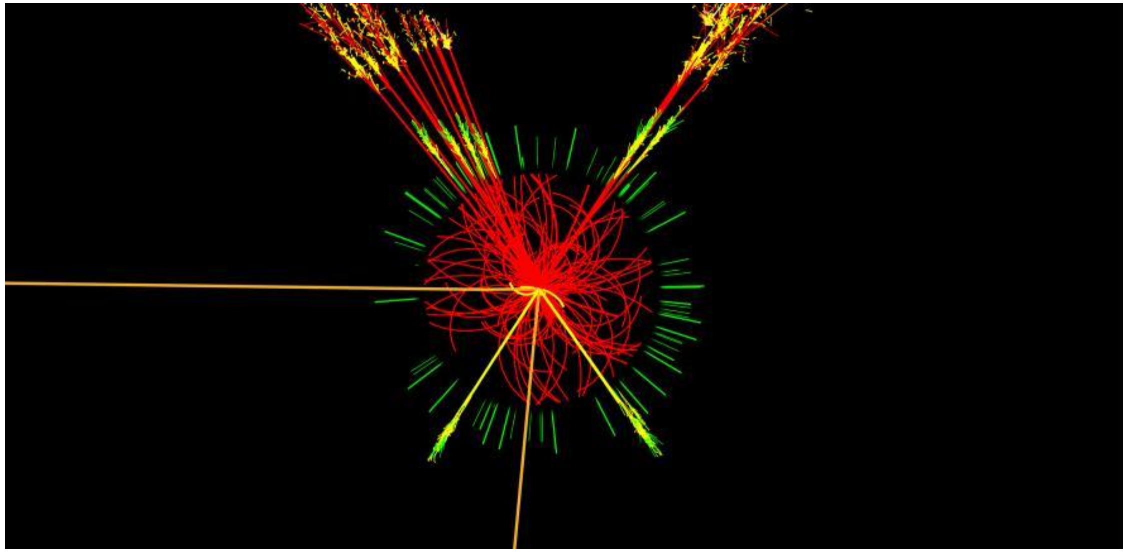
Fractional quantum Hall effect in a relativistic QFT



Srimoyee Sen,
Iowa State University
In collaboration with David B. Kaplan
PRL 124 (2020) 13, 131601

Figure courtesy: Nationalmaglab

Questions of principle that span different areas of physics



Courtesy: U of Cambridge

Courtesy: sciencealert.com

Anderson-Higgs mechanism

Questions of principle that span different areas of physics

BF theory and metallic superconductors:

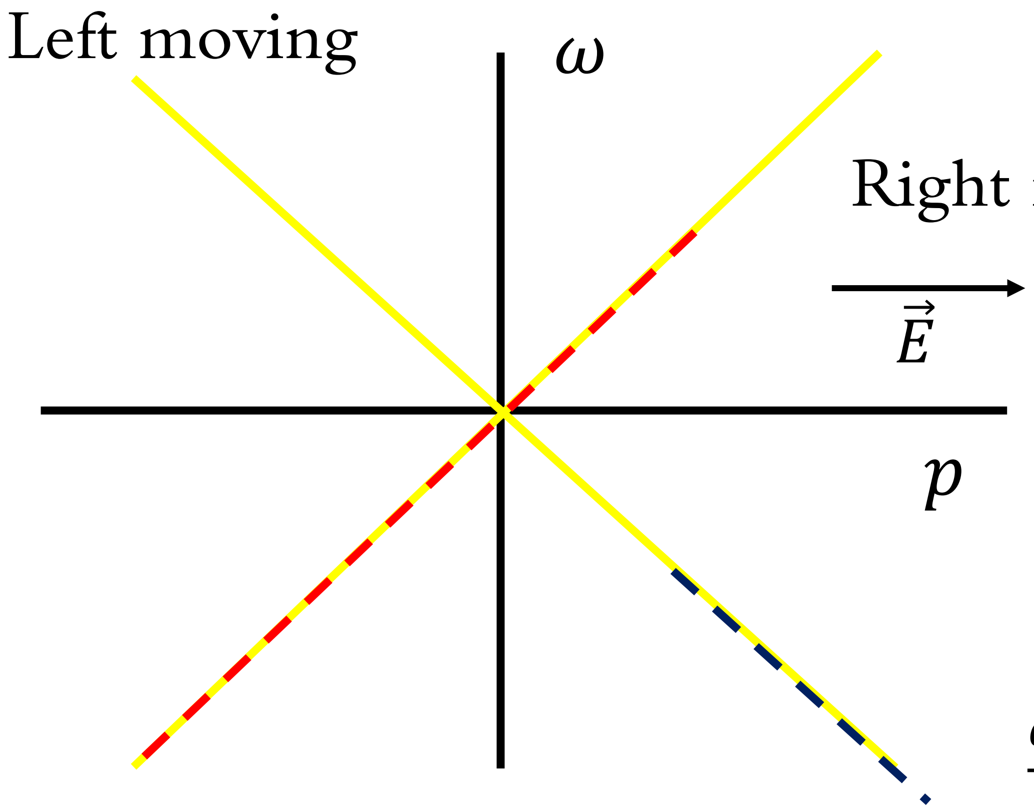
T. H. Hansson, Vadim Oganesyan, S. L. Sondhi

$$\mathcal{L}_{BF} = \frac{i}{2\pi}\epsilon^{\mu\nu\rho}B_\mu\partial_\nu A_\rho$$

Courtesy: sciencealert.com

Intrinsic topological order and topological QFTs

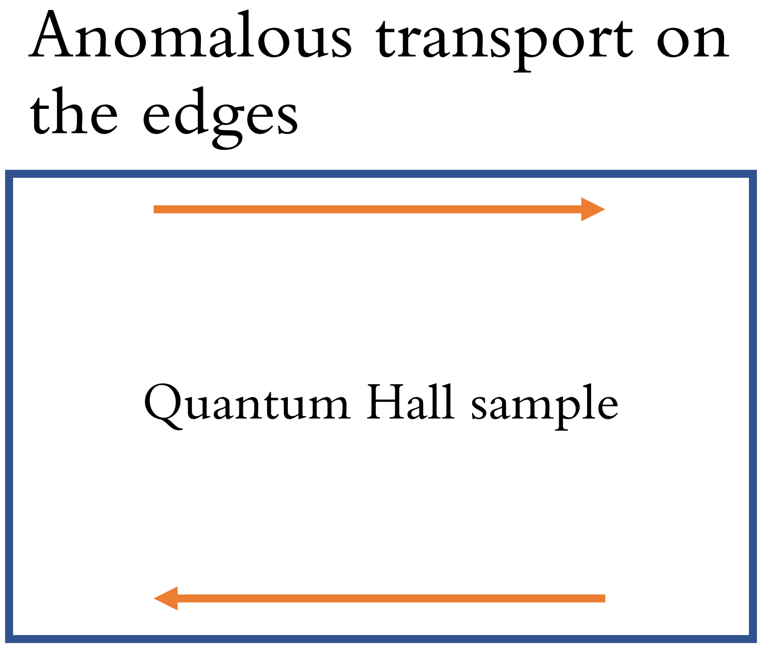
Ideas that span different areas of physics



$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

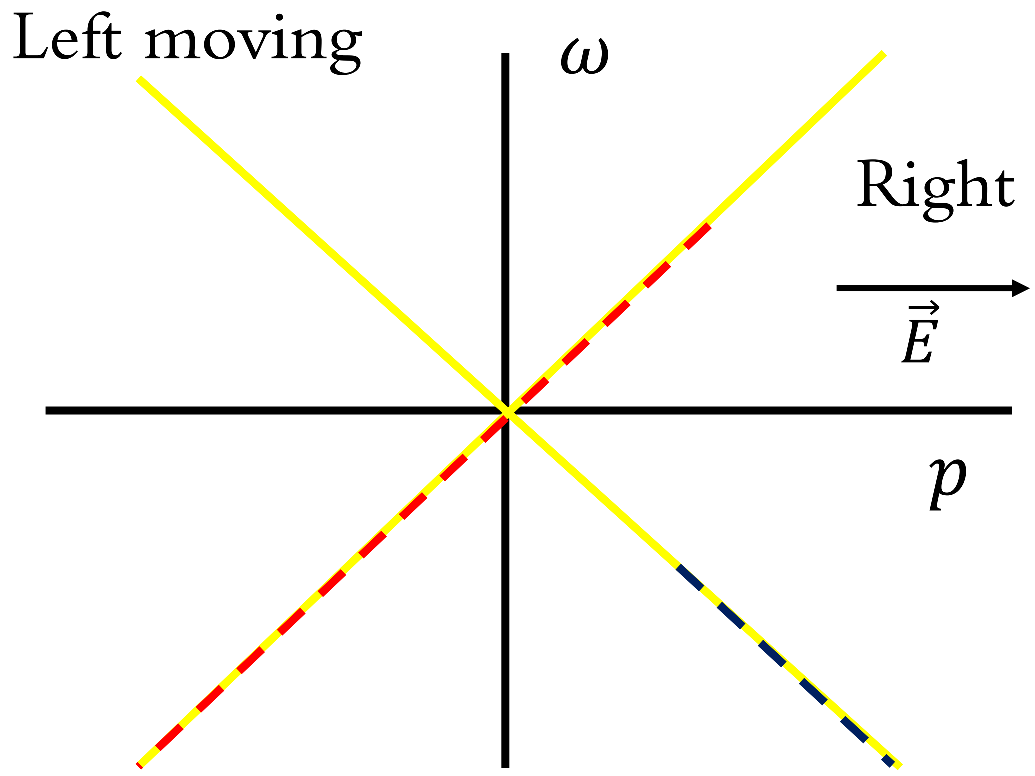
$$\frac{dn_L}{dt} = -\frac{eE}{2\pi}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{eE}{\pi}$$



Chiral anomaly and the quantum Hall effect

Ideas that span different areas of physics



$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

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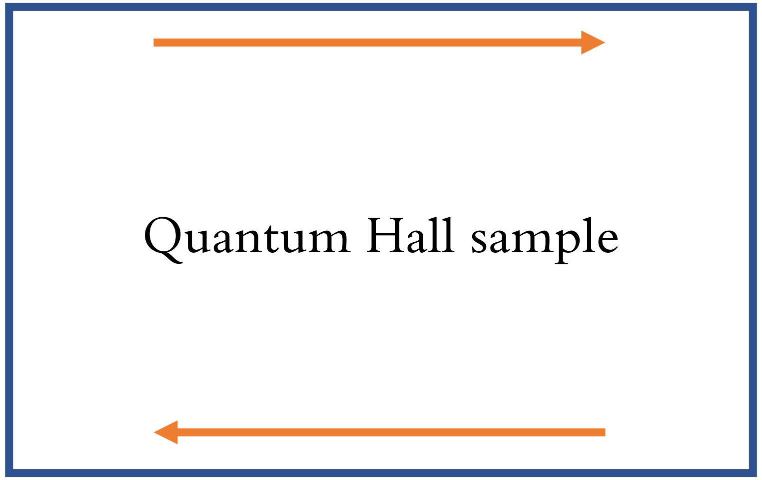
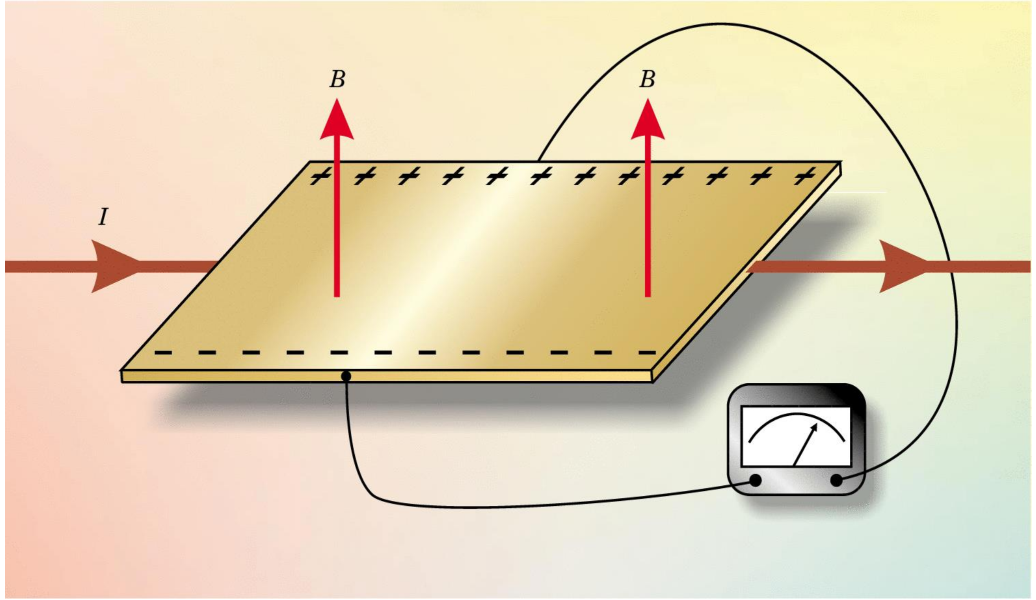
Anomalous transport on the edges



Chiral anomaly and the quantum Hall effect

Let's focus on this.

Anomaly and Quantum Hall Effect(QHE)



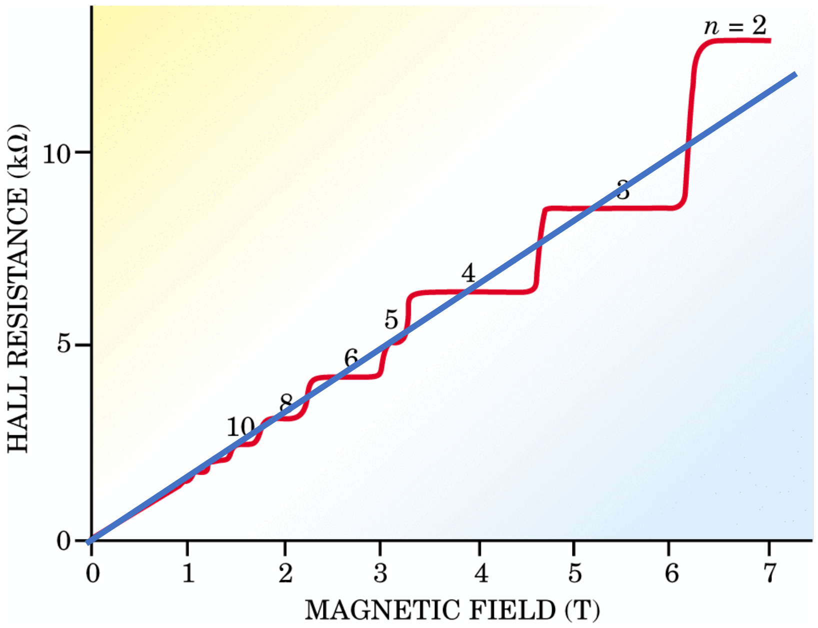
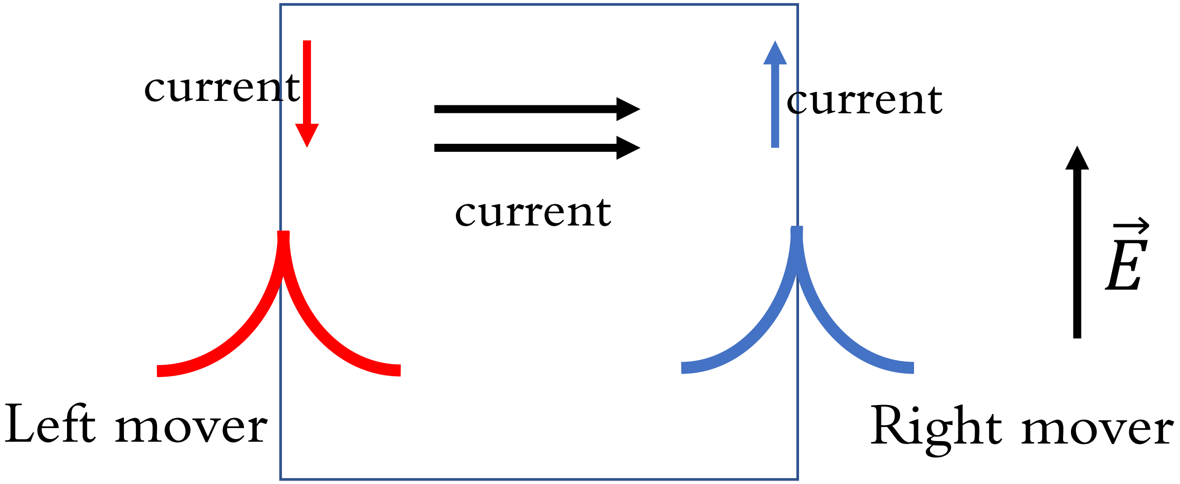


Figure credit: physicstoday



Integer (k) quantum Hall effect

Low energy EFT in the bulk: Chern-Simons theory of level k in $2+1$ dimensions.

$$S_{CS} = \frac{k}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

Magnetic field breaks parity

Integer quantized

Current transverse to the electric field (and magnetic field)

$$\text{Hall conductivity} = \frac{k\, e^2}{2\pi} \longleftarrow J_i = -\frac{k}{2\pi}\epsilon_{ij}E_j$$

Current conservation works due to k edge modes on the boundary ⟵ Integer quantized

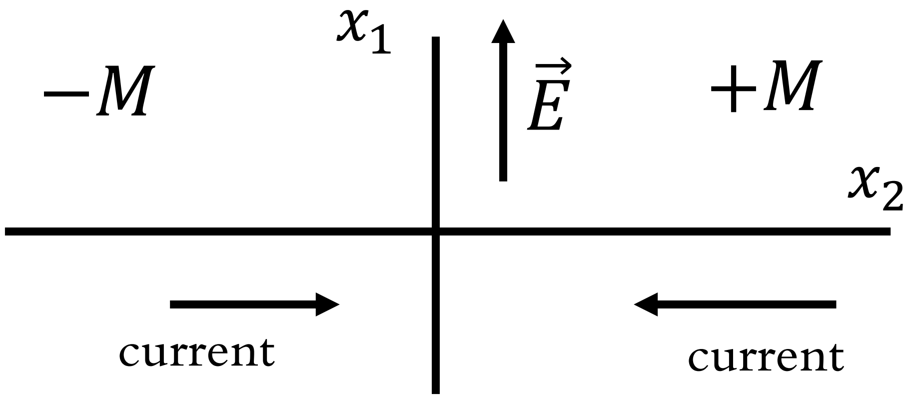
Relativistic QFT parallel for QHE (Callan-Harvey 1985)

Don't need magnetic fields.

Take abelian gauge theory coupled to k Heavy Dirac fermion in $2+1$ D.

Consider a domain wall in the fermion mass.

CS theory of '–' ve level



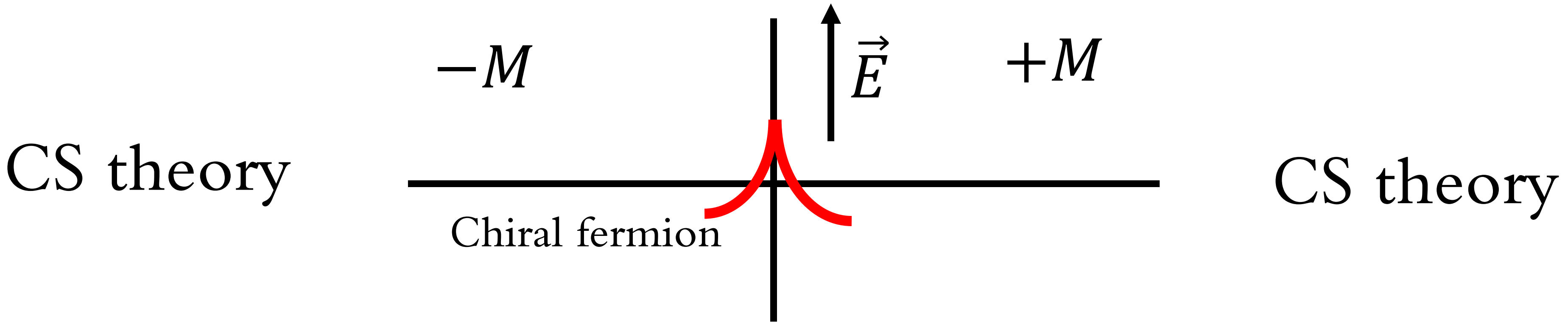
CS theory '+' ve level

Relativistic QFT parallel for QHE (Callan-Harvey 1985)

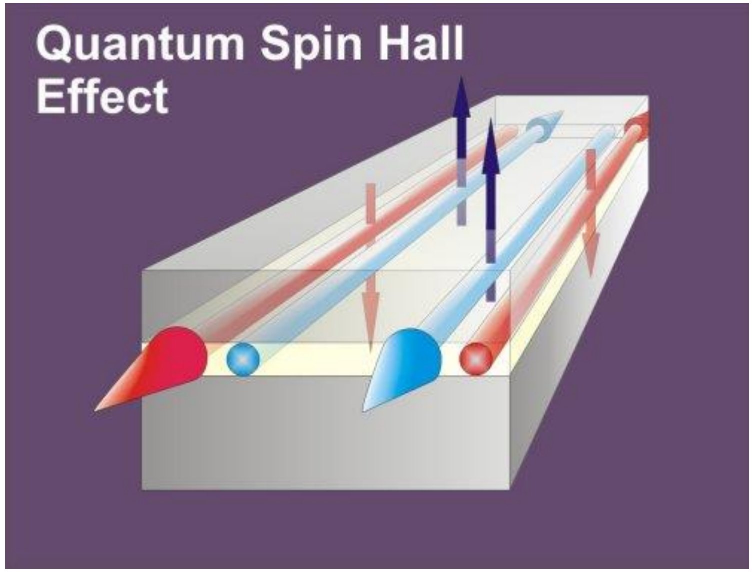
Solve the Dirac equation in the domain wall background.

Chiral fermions stuck on the $1 + 1$ D wall. Appears anomalous.

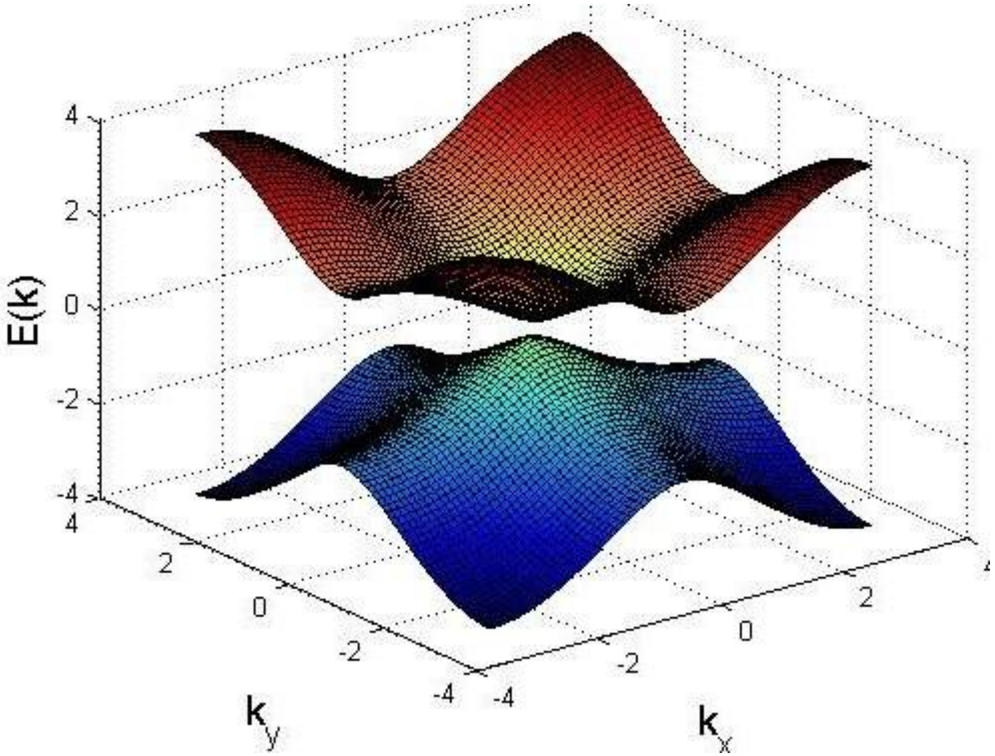
Not so when embedded in higher dimension like here.



Many other examples :



Spin Hall effect and DWF in anomaly free rep.



Chern insulators and lattice DWF.

DBK, PLB 288(1992) 342. K Jansen, PLB 288 (1992) 348. CL Kane, EJ. Mele PRL 95(2005) 226801. Golterman, Jansen, Kaplan, PLB 301 (1993) 219.

Figure courtesy: physicsworld.com, Rafeeq Syed

UV complete IR topological phenomena

- The hope is that we will discover new IR phenomena for topological materials in the process.

- In fact Chern insulators first appeared in lattice gauge theory.. but went unnoticed.

- New topological phenomena in HEP ?

Hope of cross pollination.

How about a UV completion of fractional quantum Hall effect (FQHE) ?

Let's first list the features of FQHE.

The setup is similar to IQHE.

fraction

But you have fractional currents. $$J_i = -\frac{\nu}{2\pi}\epsilon_{ij}E_j$$

As if you have fractional charges on the edges.

What kind of UV completion do we want ?

- We are in 2+1 D.

- Want Lorentz invariance.

- We don't want to use magnetic field to break parity: so we will stick with heavy fermions.

- Want the fermions to be integer charged.. i.e. fractional charge should emerge in the IR.

- A perturbative UV completion.

Take inspiration from FQHE EFT.

The EFT looks like

Integer

$$L = \int d^3x \left(\frac{1}{2\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho - \frac{m}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

Use the equations of motion for a, (same as integrating out a)

$$a^\mu = \frac{A^\mu}{m}$$

David Tong, arXiv:1606.06687

Take inspiration from FQHE EFT.

Substitute back in the action such that after integrating out a, we are left with

$$L = \int d^3x \left(\frac{1}{4\pi m} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \right)$$

Fraction

The current is :

$$J_i = -\frac{1}{2\pi m} \epsilon_{ij} E_j$$

David Tong, arXiv:1606.06687

Take inspiration from FQHE EFT.

We will discuss more phenomenology later.

But for now we want to design a UV completion for the two blocks:

$$L = \int d^3x \left(\frac{1}{2\pi} \boxed{\epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho} - \frac{m}{4\pi} \boxed{\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho} \right)$$

Mixing term

Quadratic term: level m CS term

Recap UV completion of level m CS theory in 2+1 D

$$\mathcal{L} = \int d^3x \left(\frac{m}{2\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho\right)$$

How do you arrive at this ?

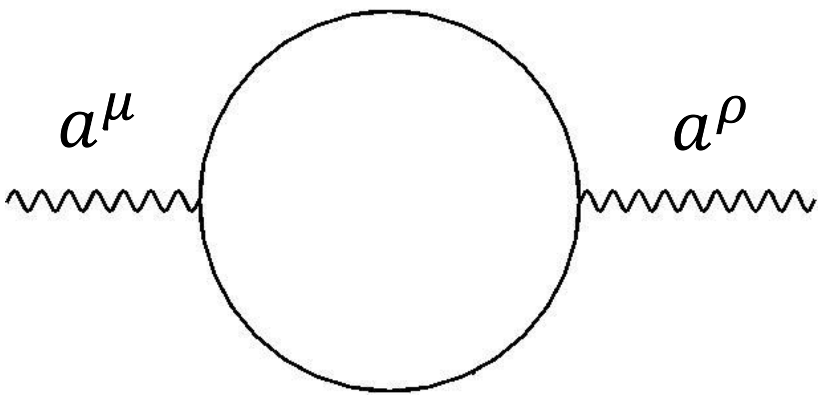
Take an abelian gauge field a^μ coupled to m flavors of massive fermions of charge 1 and mass $M > 0$. (2+1 D)

$$\mathcal{L} = \int d^3x \left(-\frac{1}{4g^2}f^2 + \sum_{i=1,..m} \overline{\psi_i}\{i\gamma^\mu D_\mu - M\}\psi_i\right)$$

Recap UV completion of level m CS theory in 2+1 D

Coupling constant g^2 is dimensionful.

Integrate out the fermions in the bulk when $g^2 < M$.



$$S_{CS} = \frac{l}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

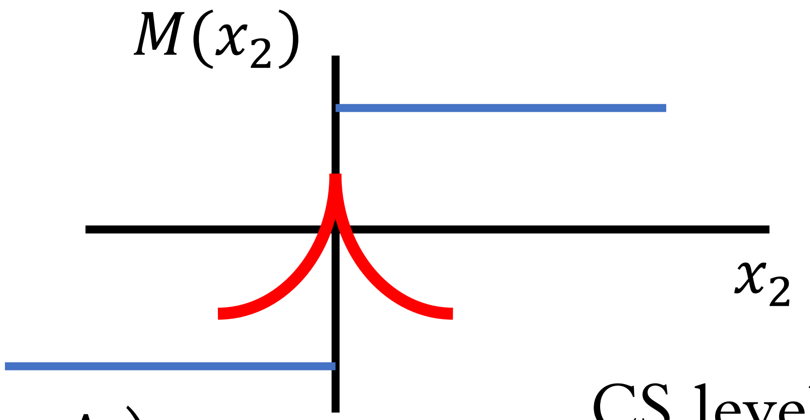
Pauli Villars mass

$$l = \frac{m}{2}\left(\frac{M}{|M|} - \frac{\Lambda}{|\Lambda|}\right) = m,$$ for $M > 0, \Lambda < 0$

$$S = \frac{m}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho - \int d^3x \frac{f^{\mu\nu} f_{\mu\nu}}{4g^2}$$

Forget in the long wavelength low energy limit

Domain wall in fermion mass

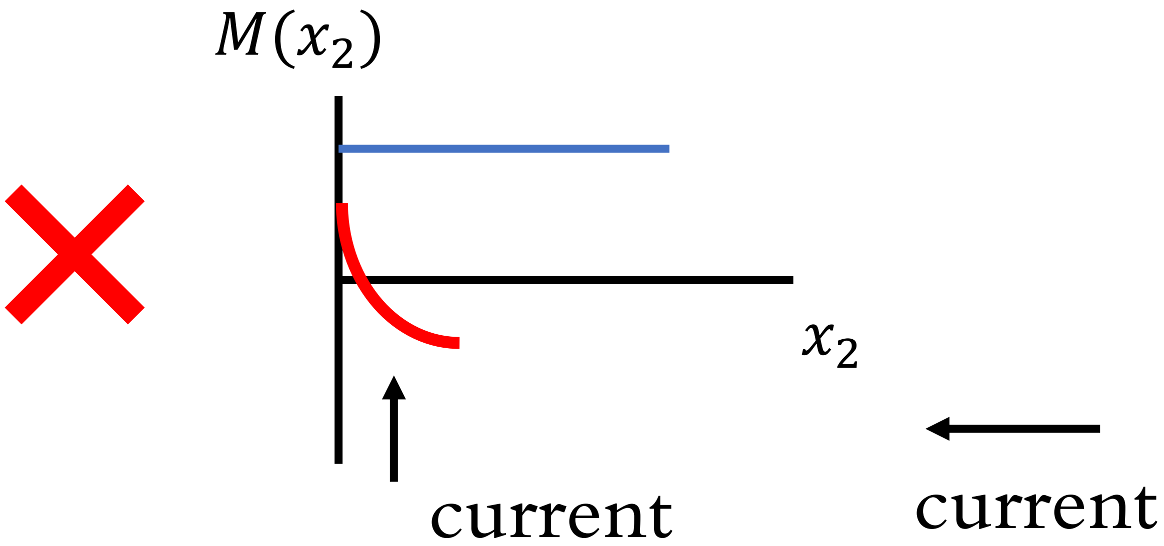


$M > 0, \qquad \Lambda < 0$

CS level of

$$\frac{m}{2}\left(-\frac{M}{|M|}-\frac{\Lambda}{|\Lambda|}\right)=0$$

CS level of $\frac{m}{2}\left(\frac{M}{|M|}-\frac{\Lambda}{|\Lambda|}\right)=m$



Back to FQHE: UV completion

We will need two $U(1)$ gauge fields. Call these A and Z.

The anticipated mapping: $A \to A$ and $Z \to a$ in

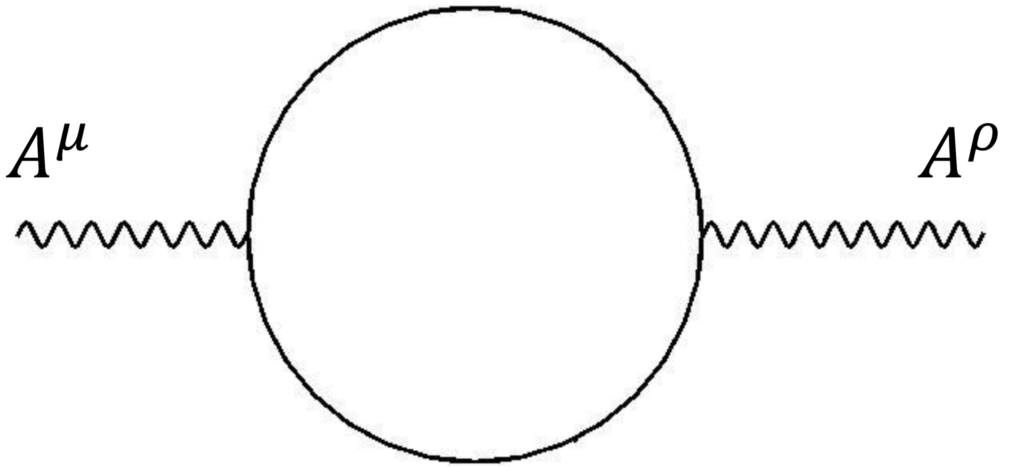
$$L = \int d^3x \left(\frac{1}{2\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a_\rho - \frac{m}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right)$$

There are two coupling constants e and g corresponding to the two gauge fields.

Mixing term

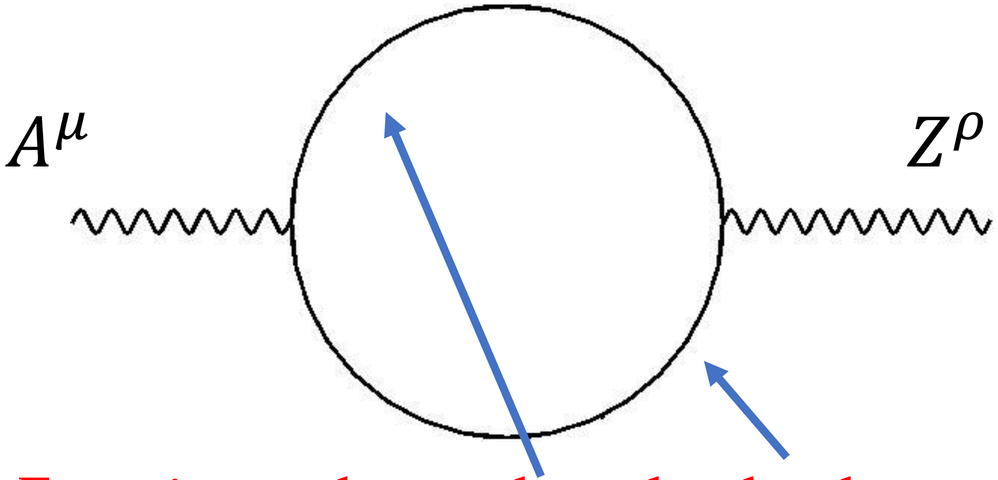
We know how to get
a quadratic CS term.

$$S_{CS} = \frac{l}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$



To get the mixed CS term
we need a diagram:

$$S_{CS} \sim \frac{\#}{4\pi}\int d^3x\, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu Z_\rho$$



Fermions charged under both A and Z

Fermions (remember we want integer charges)

We will need a few different species of fermions of multiple flavors charged under the two gauge fields :
Call them ψ, χ, ω.

q_A= charge under the gauge field A

q_Z= charge under the gauge field Z

	n_{flavor}	q_A	q_Z
ψ	n_ψ	1	0
χ	n_χ	0	1
ω	n_ω	1	1

Masses of the fermions

ψ, χ, ω have different masses. (choose the PV regulator to have the opposite sign for mass)

More specifically the three species of fermions can have different signs for their masses.

$|n_i|$ are the number of fermions of species i.

$\frac{n_i}{|n_i|}$ is the sign of the mass of species i

The hierarchy of scales

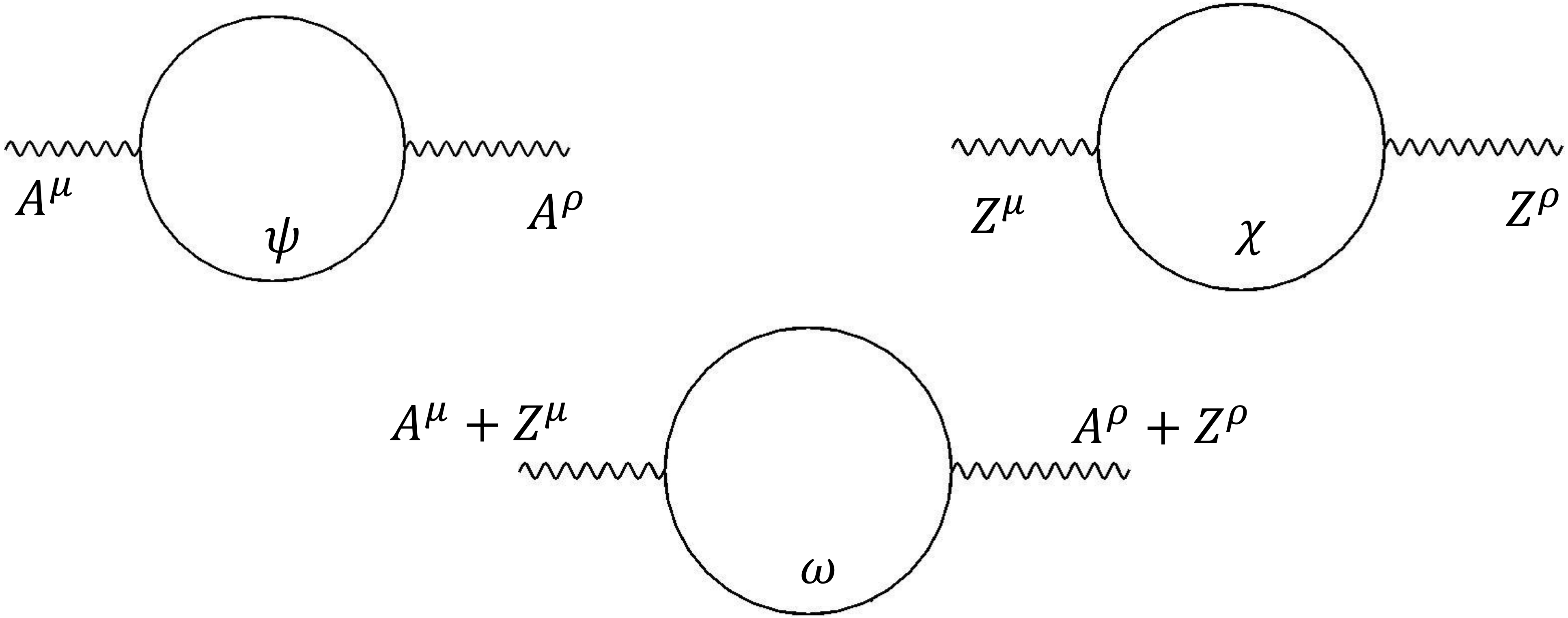
We will want the following hierarchy of scales

$$|\Lambda| > |M| > g^2 > e^2$$

I want to integrate out the fermions first in a weakly coupled regime to end up with CS theory.

So, clearly, I need fermion masses to be large compared to other physical scales $|\Lambda| > |M| > g^2,\ e^2$

Integrating out the fermions: $(|M| \gg e^2, g^2)$



$$\longrightarrow \frac{\epsilon^{\alpha\beta\gamma}}{4\pi}\left(n_\psi A_\alpha \partial_\beta A_\gamma + n_\chi Z_\alpha \partial_\beta Z_\gamma + n_\omega (A_\alpha + Z_\alpha)\partial_\beta (A_\gamma + Z_\gamma)\right)$$

Next, we anticipate integrating out the *Z* gauge field

Hierarchy of scales

This is the same as using the EOM for the a gauge field earlier.

What enables the integrating out of the gauge field ?

Note the gauge field kinetic terms in a CS + Maxwell theory

$$L_Z = \frac{Z_{\mu\nu} Z^{\mu\nu}}{4g^2} + k\, \epsilon^{\alpha\beta\gamma} Z_\alpha \partial_\beta Z_\gamma$$

There is a pole at $\sim k\, g^2$. So, the gauge theory is gapped.

Hierarchy of scales

The Z gauge field mass $\sim g^2$.

The A gauge field mass $\sim e^2$.

We want to integrate out the Z gauge field.

So, $e^2 < g^2$.

Integrating the Z gauge field out

The Z dependent part of the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left[\left(Z - \frac{n_\omega}{2\pi} A \Delta Q^{-1}\right) Q \left(Z - \frac{n_\omega}{2\pi} Q^{-1} \Delta A\right) - \left(\frac{n_\omega}{2\pi}\right)^2 A \Delta Q^{-1} \Delta A\right]$$

$$Q = -\frac{n_\chi + n_\omega}{2\pi}\Delta + \frac{\Xi}{\xi^2}$$ ← Gauge fixing

$$\Delta^{\alpha\beta} = \epsilon^{\alpha\beta\gamma}\partial_\gamma \,, \Xi^{\alpha\beta} = \partial_\alpha \partial_\beta, \quad Q^{-1} = \frac{2\pi}{n_\chi + n_\omega}\frac{\Delta}{\partial^2} + \frac{\xi \Xi}{(\partial^2)^2}$$

Integrating the Z gauge field out: the EFT

$$\Delta Q^{-1}\Delta = -\frac{2\pi\Delta}{n_\chi + n_\omega}$$

$$\mathcal{L} = \frac{1}{2}\left[\left(Z - \frac{n_\omega}{2\pi}A\Delta Q^{-1}\right)Q\left(Z - \frac{n_\omega}{2\pi}Q^{-1}\Delta A\right) - \left(\frac{n_\omega}{2\pi}\right)^2 A\Delta Q^{-1}\Delta A\right]$$

$$\mathcal{L}_{EFT} = -\frac{1}{4e^2}F^2 + \nu\frac{\epsilon^{\alpha\beta\gamma}}{4\pi}A_\alpha\partial_\beta A_\gamma + \cdots$$

$$\nu = \left(n_\psi + \frac{n_\chi n_\omega}{n_\chi + n_\omega}\right)$$

Edge modes

Consider a half plane

Choose the PV regulator to have the opposite sign of mass as that of the fermion which it is regulating.

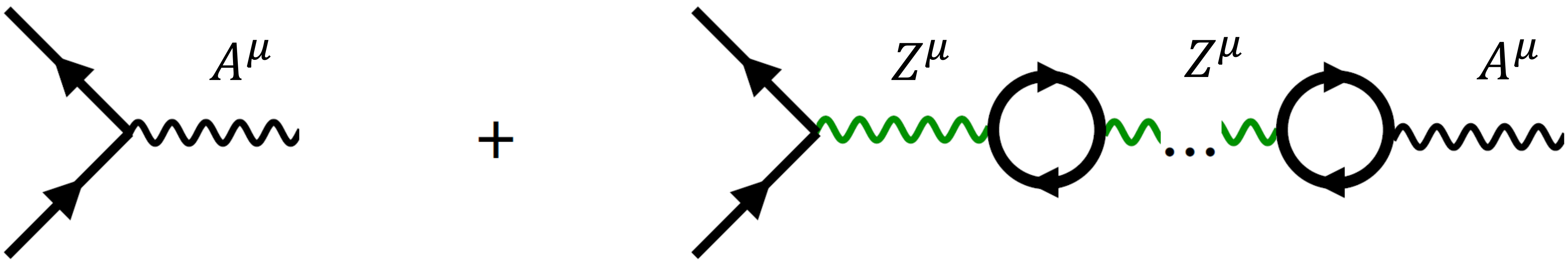
Solve Dirac equation to get edge modes on the half plane boundary.

These zero modes have integer charges under the gauge fields.

How do you conserve the fractional current then ?

Edge modes

After integrating out the Z gauge field, the edge modes get fractional charge



The fractional charges

$$q'_\psi = 1, \quad q'_\chi = -\frac{n_\omega}{n_\chi + n_\omega}, \qquad q'_\omega = 1 - \frac{n_\omega}{n_\chi + n_\omega}$$

Current conservation works out with these fractional charges.

Few more consequences (in the bulk)

Add a bosonic particle ϕ of mass smaller than the fermions and with charge q_ϕ under Z gauge transformation.

The current term in the action:

$$\int d^3x\, Z^\mu j_\mu$$

After integrating out the Z gauge field, we are left with

$$\mathcal{L}_{EFT} = \nu \frac{\epsilon^{\alpha\beta\gamma}}{4\pi} A_\alpha \partial_\beta A_\gamma + A_\mu j^{\mu\prime}$$

Charge q'_ϕ

Few more consequences (in the bulk)

Place a ϕ charge density at $x_2 = 0$. Take $j^0 \sim \delta^2(x)$

EOM: $\frac{1}{2\pi} F^{\mu\nu} = \frac{1}{\nu} \epsilon^{\mu\nu\rho} j_\rho, \rightarrow \frac{1}{2\pi} F^{12} = \frac{q'_\phi}{\nu} \delta^2(x).$

Leads to Aharonov-Bohm phase of $\frac{{q'_\phi}^2}{\nu}(2\pi)$ when you take one ϕ particle around another.

Results in anyons with fractional statistics

$$|\phi_1 \phi_2 >= e^{i \frac{{q'_\phi}^2}{\nu} \pi} |\phi_1 \phi_2>$$

Summary and open questions

We have a relativistic UV completion for the FQHE.

What about nonabelian quantum Hall states ?

Topological insulators in $3+1$ dimensions ?

Gauge theories in broken phases ?