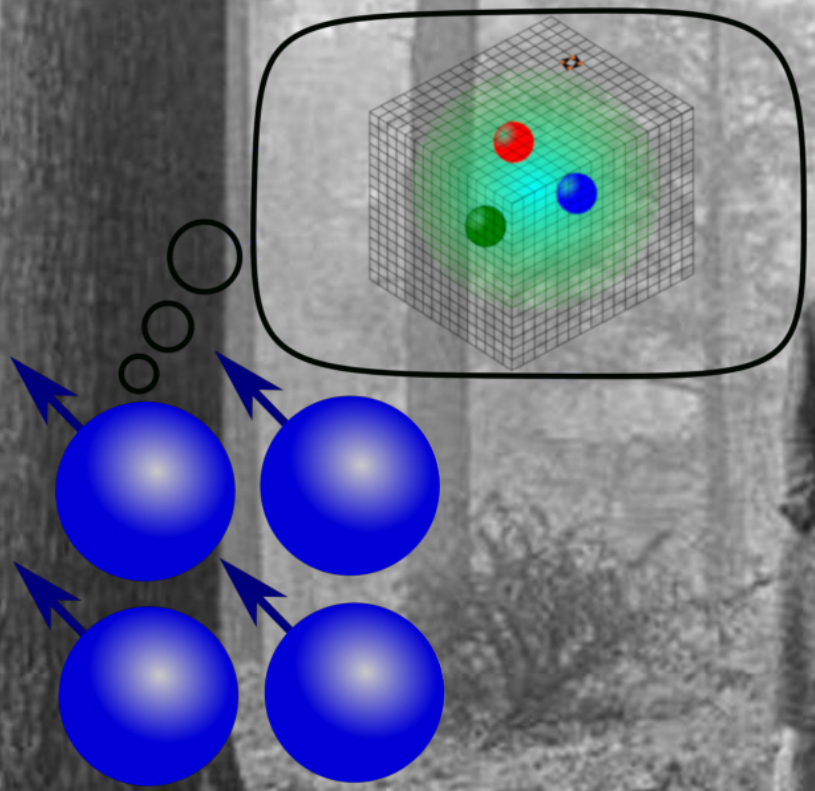
Pushed from the Precipice: Lattice Field Theory for Quantum Computers

Hank Lamm



I just wanna say thank you to my colleagues

General Methods for Digital Quantum Simulation of Gauge Theories

Henry Lamm,[1,*] Scott Lawrence,[1,†] and Yukari Yamauchi[1,‡]
(NuQS Collaboration)
[1] *Department of Physics, University of Maryland, College Park, MD 20742, USA*
(Dated: August 13, 2019)

A general scheme is presented for simulating gauge theories, with matter fields, on a digital quantum computer. A Trotterized time-evolution operator that respects gauge symmetry is constructed, and a procedure for obtaining time-separated, gauge-invariant correlators is detailed. We demonstrate the procedure on small lattices, including the simulation of a 2+1D non-Abelian gauge theory.

hep-lat/1903.08807

Gluon Field Digitization for Quantum Computers

Andrei Alexandru,[1,2,*] Paulo F. Bedaque,[2,†] Siddhartha Harmalkar,[2,‡] Henry Lamm,[2,§] Scott Lawrence,[2,¶] and Neill C. Warrington[2,**]
(NuQS Collaboration)
[1] *Department of Physics, The George Washington University, Washington, D.C. 20052, USA*
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*
(Dated: November 18, 2019)

Simulations of gauge theories on quantum computers require the digitization of continuous field variables. Digitization schemes that uses the minimum amount of qubits are desirable. We present a practical scheme for digitizing $SU(3)$ gauge theories via its discrete subgroup $S(1080)$. The $S(1080)$ standard Wilson action cannot be used since a phase transition occurs as the coupling is decreased, well before the scaling regime. We proposed a modified action that allows simulations in the scaling window and carry out classical Monte Carlo calculations down to lattice spacings of order $a \approx 0.08$ fm. We compute a set of observables with sub-percent precision at multiple lattice spacings and show that the continuum extrapolated value agrees with the full $SU(3)$ results. This suggests that this digitization scheme provides sufficient precision for NISQ-era QCD simulations.

hep-lat/1906.11213

Parton Physics on a Quantum Computer

Henry Lamm,[1,*] Scott Lawrence,[1,†] and Yukari Yamauchi[1,‡]
(NuQS Collaboration)
[1] *Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
(Dated: August 29, 2019)

Parton distribution functions and hadronic tensors may be computed on a universal quantum computer without many of the complexities that apply to Euclidean lattice calculations. We detail algorithms for computing parton distribution functions and the hadronic tensor in the Thirring model. Their generalization to QCD is discussed, with the conclusion that the parton distribution function is best obtained by fitting the hadronic tensor, rather than direct calculation. As a side effect of this method, we find that lepton-hadron cross sections may be computed relatively cheaply. Finally, we estimate the computational cost of performing such a calculation on a digital quantum computer, including the cost of state preparation, for physically relevant parameters.

hep-lat/1908.10439

Quantum Simulation of Field Theories Without State Preparation

Siddhartha Harmalkar,[1,*] Henry Lamm,[2,†] and Scott Lawrence[1,‡]
(NuQS Collaboration)
[1] *Department of Physics, University of Maryland, College Park, MD 20742, USA*
[2] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
(Dated: January 31, 2020)

We propose an algorithm for computing real-time observables using a quantum processor while avoiding the need to prepare the full quantum state. This reduction in quantum resources is achieved by classically sampling configurations in imaginary-time using standard lattice field theory. These configurations can then be passed to quantum processor for time-evolution. This method encounters a signal-to-noise problem which we characterize, and we demonstrate the application of standard lattice QCD methods to mitigate it.

hep-lat/2001.11490

Suppressing Coherent Gauge Drift in Quantum Simulations

Henry Lamm,[1,*] Scott Lawrence,[2,†] and Yukari Yamauchi[2,‡]
(NuQS Collaboration)
[1] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[2] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

Simulations of field theories on noisy quantum computers must contend with errors introduced by that noise. For gauge theories, a large class of errors violate gauge symmetry, and thus may result in unphysical processes occurring in the simulation. We present a method, applicable to non-Abelian gauge theories, for suppressing coherent gauge drift errors through the repeated application of pseudorandom gauge transformation. In cases where the dominant errors are gauge-violating, we expect this method to be a practical way to improve the accuracy of NISQ-era simulations.

hep-lat/2005.12688

Gluon Field Digitization via Group Space Decimation for Quantum Computers

Yao Ji,[1,2,*] Henry Lamm,[3,†] and Shuchen Zhu[4,‡]
(NuQS Collaboration)
[1] *Theoretische Physik I, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, D-57068 Siegen, Germany*
[2] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[3] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[4] *Department of Computer Science, Georgetown University, Washington, DC 20057, USA*
(Dated: June 2, 2020)

Efficient digitization is required for quantum simulations of gauge theories. Schemes based on discrete subgroups use fewer qubits at the cost of systematic errors. We systematize this approach by deriving a single plaquette action for approximating general continuous gauge groups through integrating out field fluctuations. This provides insight into the effectiveness of these approximations, and how they could be improved.

hep-lat/2005.14221

Toward Quantum Simulations of $\mathbb{Z}_2$ Gauge Theory Without State Preparation

Erik J. Gustafson[1,*] and Henry Lamm[2,†]
[1] *Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242, USA*
[2] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*

Preparing strongly-coupled particle states on quantum computers requires large resources. In this work, we show how classical sampling coupled with projection operators can be used to compute Minkowski matrix elements without explicitly preparing these states on the quantum computer. We demonstrate this for the 2+1d $\mathbb{Z}_2$ lattice gauge theory on small lattices with a quantum simulator.

hep-lat/2011.11677

THE UNIVERSITY OF CHICAGO — Wanqiang Liu

University of Colorado Boulder — **Scott Lawrence**

UNIVERSITÄT SIEGEN — **Yao Ji**

THE GEORGE WASHINGTON UNIVERSITY, WASHINGTON, DC — INSTITUTE for NUCLEAR THEORY — **Andrei Alexandru**, **Neill Warrington**

GEORGETOWN UNIVERSITY — **Shuchen Zhu**

IOWA — Erik Gustafson→ (Fermilab)

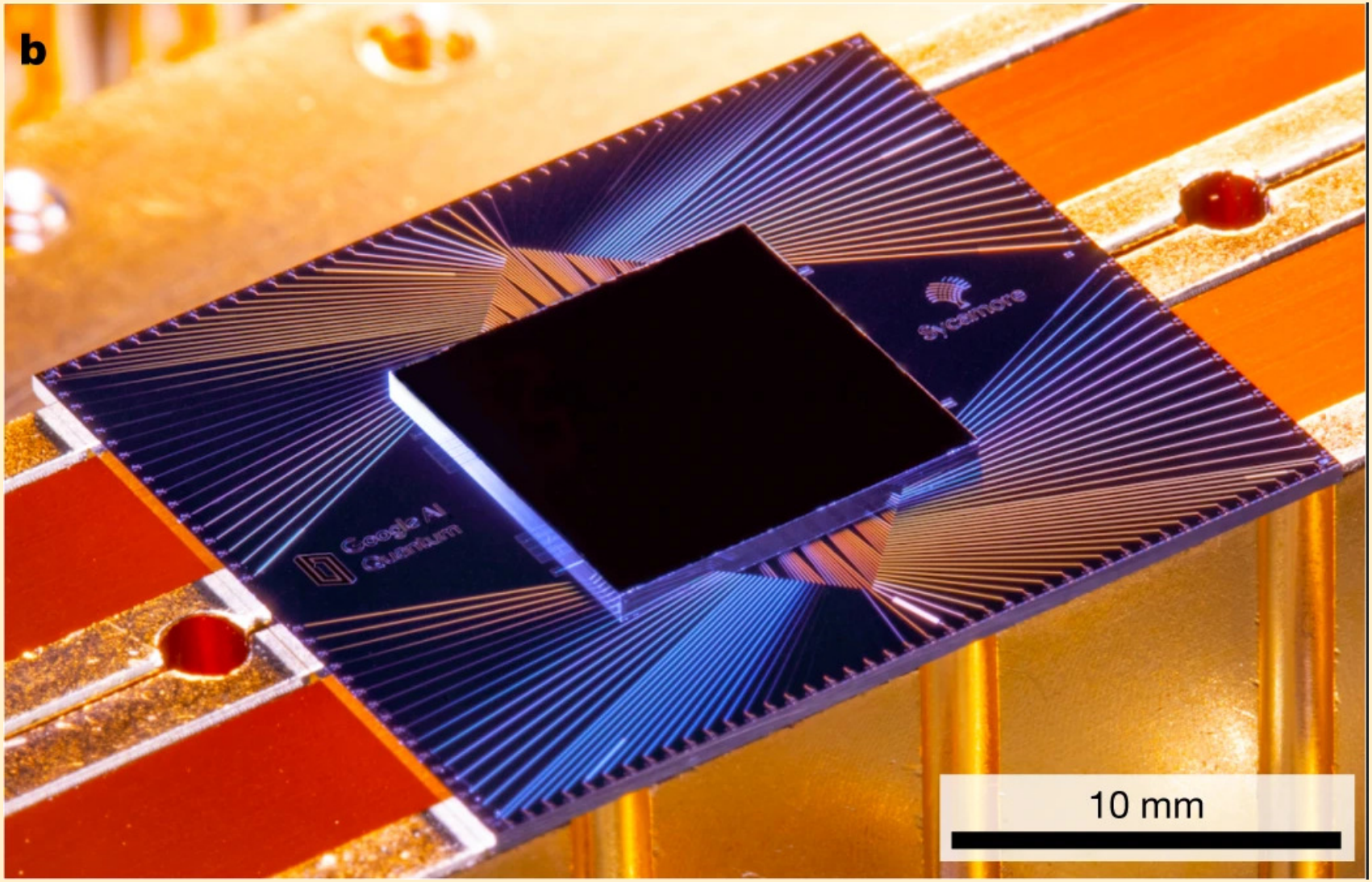
Fermilab — Marcela Carena, **Hank Lamm**, Yingying Li, Joe Lykken

UNIVERSITY OF MARYLAND — **Paulo Bedaque**, **Yukari Yamauchi**, **Sid Harmalkar**, **Hersh Kumar**

rigetti — Sohaib Alam

NuQS member
Student

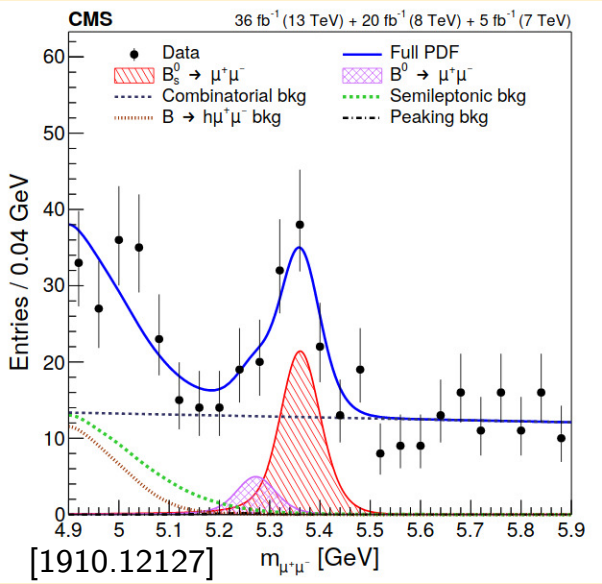
What is the state of QC? Nasty, brutish and short



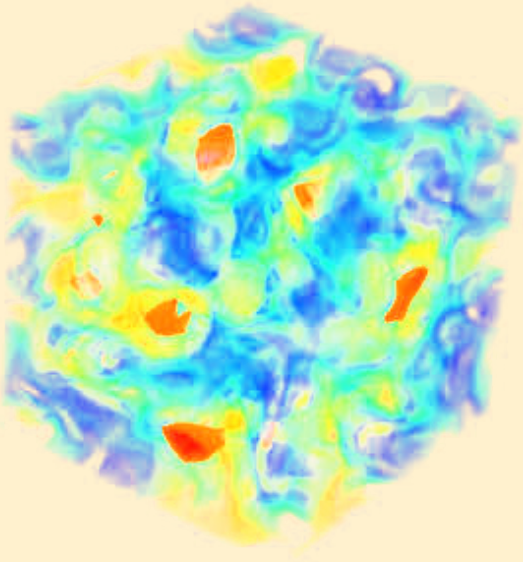
20 cycles of gates with 53 qubits

Third floor on the west side, me and you

$\mathcal{B}(B_s^0 \to \mu\mu) = [2.9 \pm 0.7(\text{exp}) \pm \mathbf{0.2(frag)}] \times 10^{-9}$

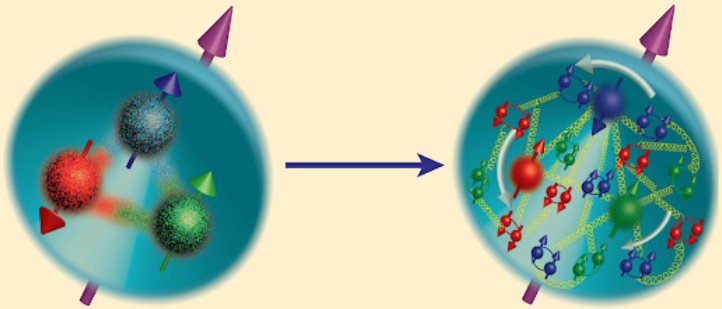


[1910.12127]



$\frac{\eta}{s}(T) = ???$

$H^q(x, \xi, t) = ???$ $E^q(x, \xi, t) = ???$



Examples: Hadronization, QGP Dynamics, Hadron Tomography[1]

Practical Quantum Advantages in High Energy Physics

Marcela Carena,[1,2,3,*] Henry Lamm,[1,†] Scott Lawrence,[4,‡] Ying-Ying Li,[1,§] Joseph D. Lykken,[1,¶] Lian-Tao Wang,[2,**] and Yukari Yamauchi[5,††]

[1] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
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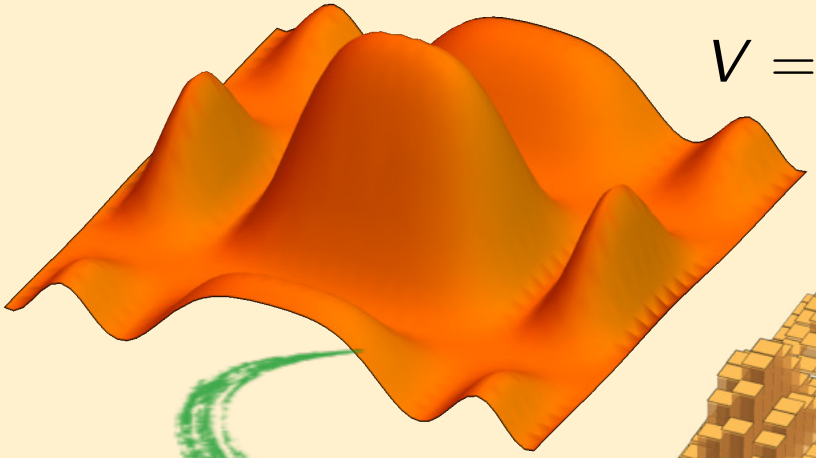
[3] *Kavli Institute for Cosmological Physics, University of Chicago, Chicago, Illinois, 60637, USA*
[4] *1Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
[5] *Department of Physics, University of Maryland, College Park, MD 20742, USA*

In this LOI, we discuss open questions in cosmology and particle physics whose solutions would demonstrate practical quantum advantage – solving a problem of *interest* using quantum hardware that is impractical for classical resources. Arriving at these calculation will require theoretical developments in nonperturbative and nonequilibrium physics along side improved quantum algorithms.

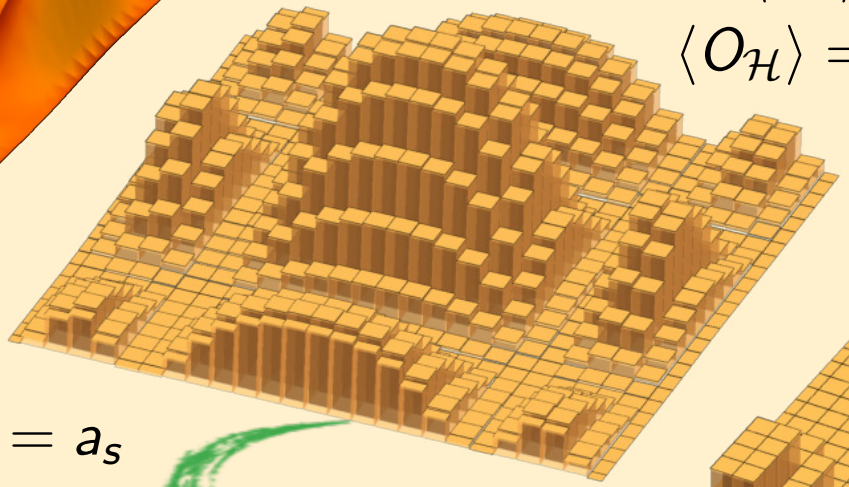
[1] Carena, M. et al. In: *Snowmass 2021 LOI* TF10-077 (2020).

QFT is about infinities and how to regulate them

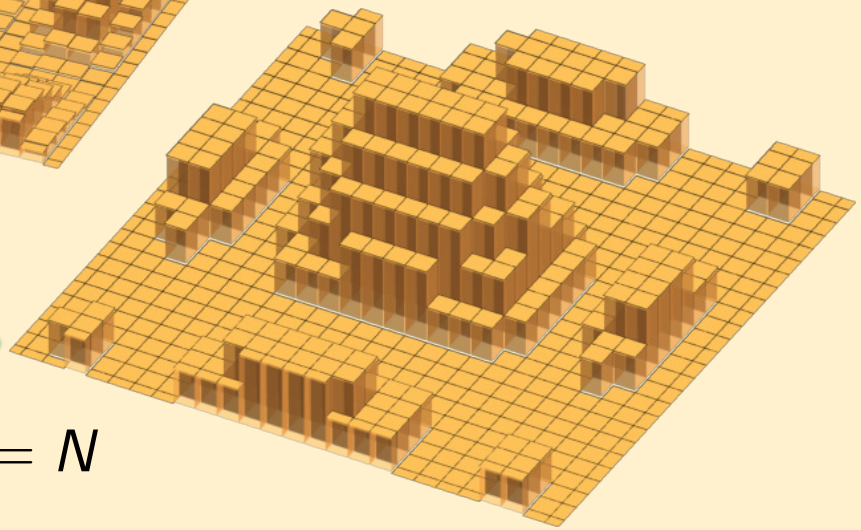


$V = \infty \to V = L^d$

$$\langle O_\infty \rangle$$
$$\langle O_L \rangle = \langle O_\infty \rangle + f(L)$$
$$\langle O_a \rangle = \langle O_\infty \rangle + f(L, a)$$
$$\langle O_{\mathcal{H}} \rangle = \langle O_\infty \rangle + f(L, a, \{\mathcal{H}\})$$



$a = 0 \to a = a_s$

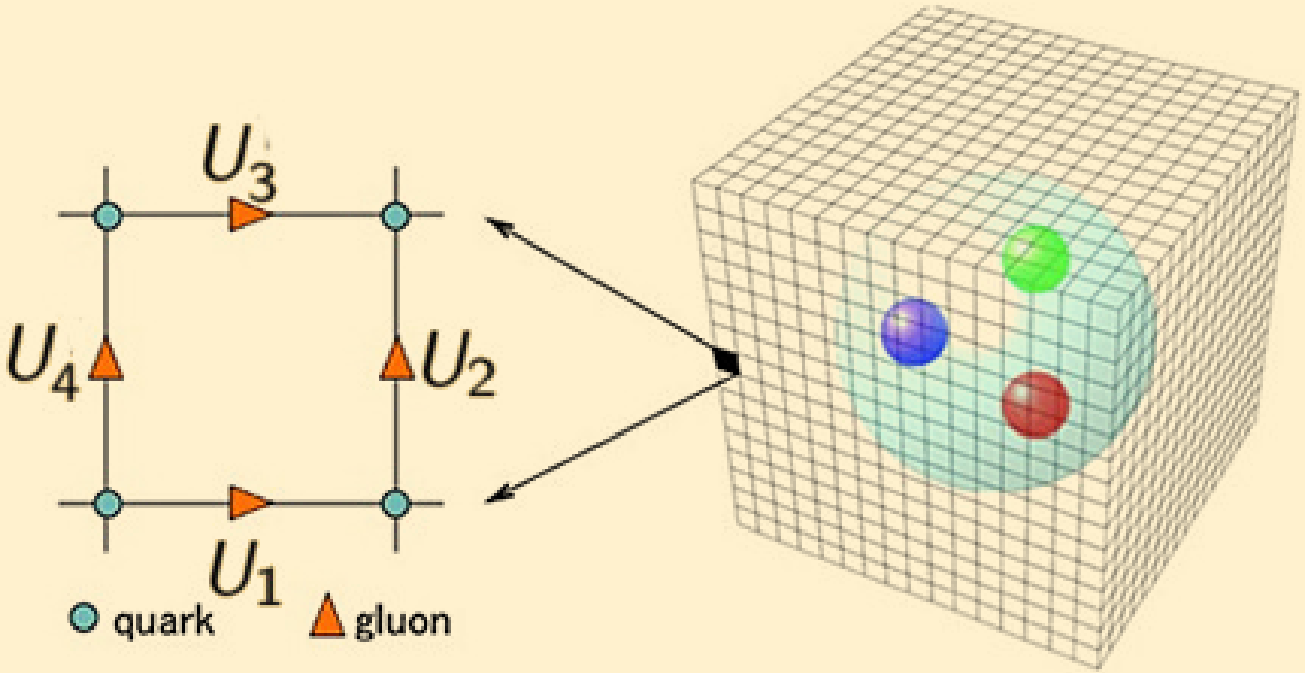


$\{\mathcal{H}\} = \infty \to \{\mathcal{H}\} = N$

LFT has been successful beyond our wildest dreams

$$S_\infty = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{q}(i\not{D} - m)q \right]$$

$$S_W = \sum_x \left[\beta \,\mathrm{Re}\,\mathrm{Tr}(1 - U_p) + S_f \right] \text{ with } U_p = U_1 U_2 U_3^\dagger U_4^\dagger \text{ and } U_i = e^{ia_\mu A^\mu}$$



Wick rotate $t \to i\tau$ then **sample** from e^{-S_R}

LFT can compute **most** $\langle \psi_i | \prod_n \mathcal{O}_n(\tau_n) | \psi_i \rangle$

Sign(al-to-noise) problems stymie Monte Carlo

$S_I = 0$, $\langle \sigma \rangle_{S_R} = 1$

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \; e^{-iS_I} \mathcal{O} e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R}} \frac{\int \mathcal{D}\phi \; e^{-S_R}}{\int \mathcal{D}\phi \; e^{-S_R} e^{-iS_I}}$$

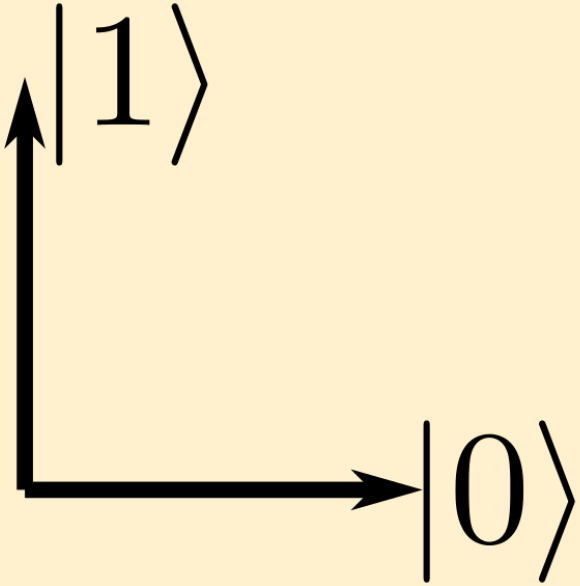
$$= \frac{\left\langle \mathcal{O} e^{-iS_I} \right\rangle_{S_R}}{\langle \sigma \rangle_{S_R}}$$

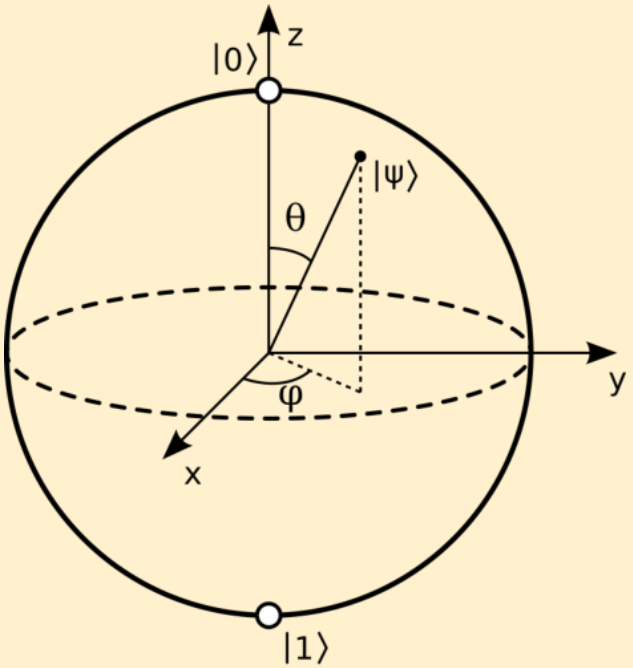
$S_I \neq 0$, $\langle \sigma \rangle_{S_R} \leq 1$

For **real-time** dynamics, $S_R = 0$!

What do I gain with a quantum computer?[2]

$$\langle\psi_i|\prod_n \mathcal{O}_n(t_n)|\psi_i\rangle = \langle\psi_i|e^{iHt_0}\mathcal{O}_0 e^{iH\delta t}\mathcal{O}_1 \ldots e^{-iHT}|\psi_i\rangle$$





QC can **efficently represent** superpositions and entanglement

Digital QC provide entangled qubits and gates, **not** field theories.

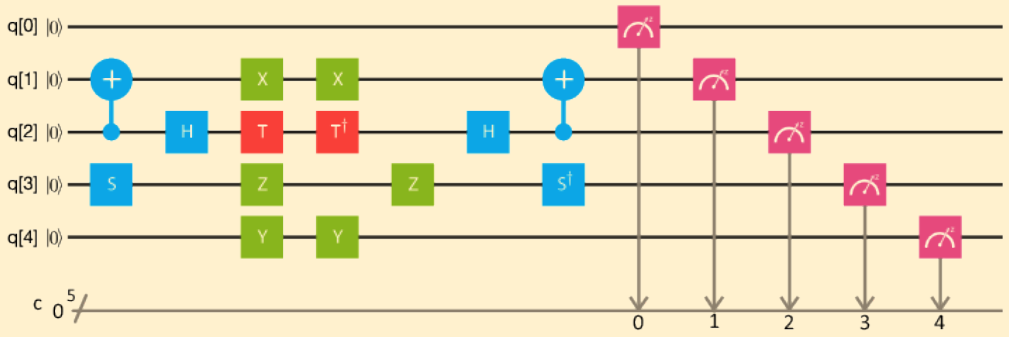
[2] Feynman, R. P. In: *Int.J.Theor.Phys.* 21 (1982).

What should a theorist be aware of?

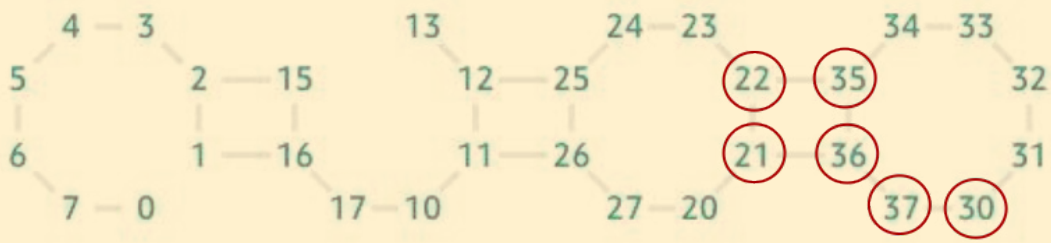
Think assembly language and bits...

+**Noise** limits fideltiy of gates to $95 - 99\%$ today

Connectivity matters!



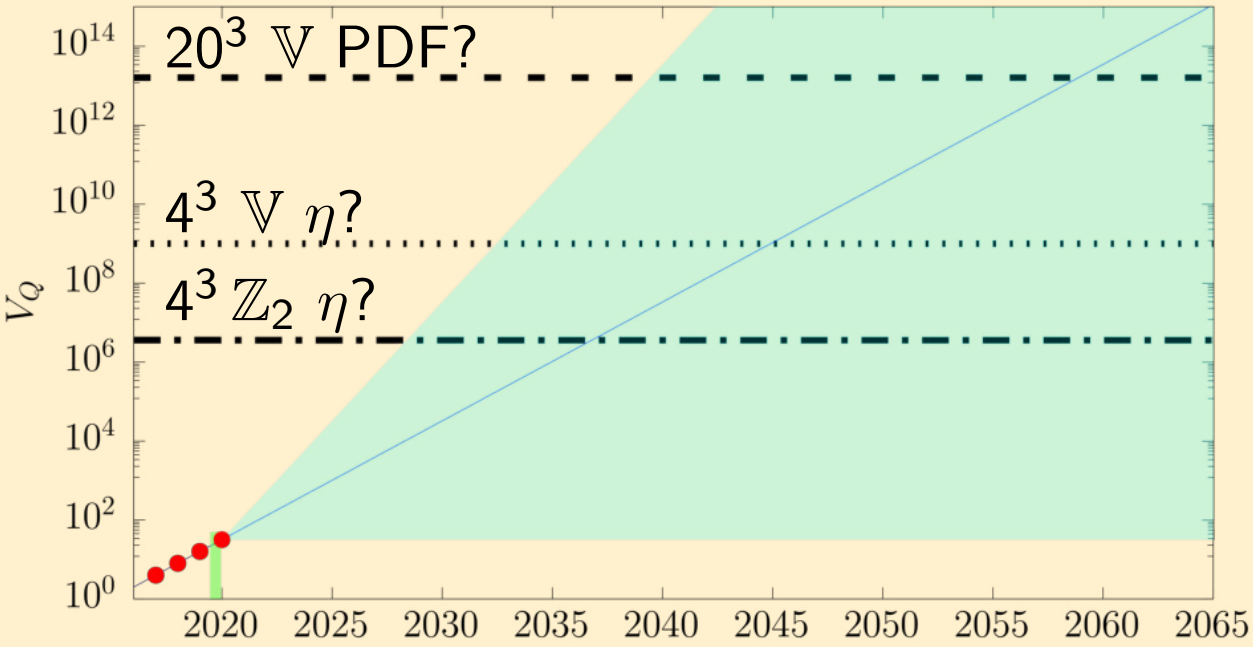
Aspen-8 lattice



$\mathcal{O}(10)$ **physical** qubits with $\mathcal{O}(10)$ gates

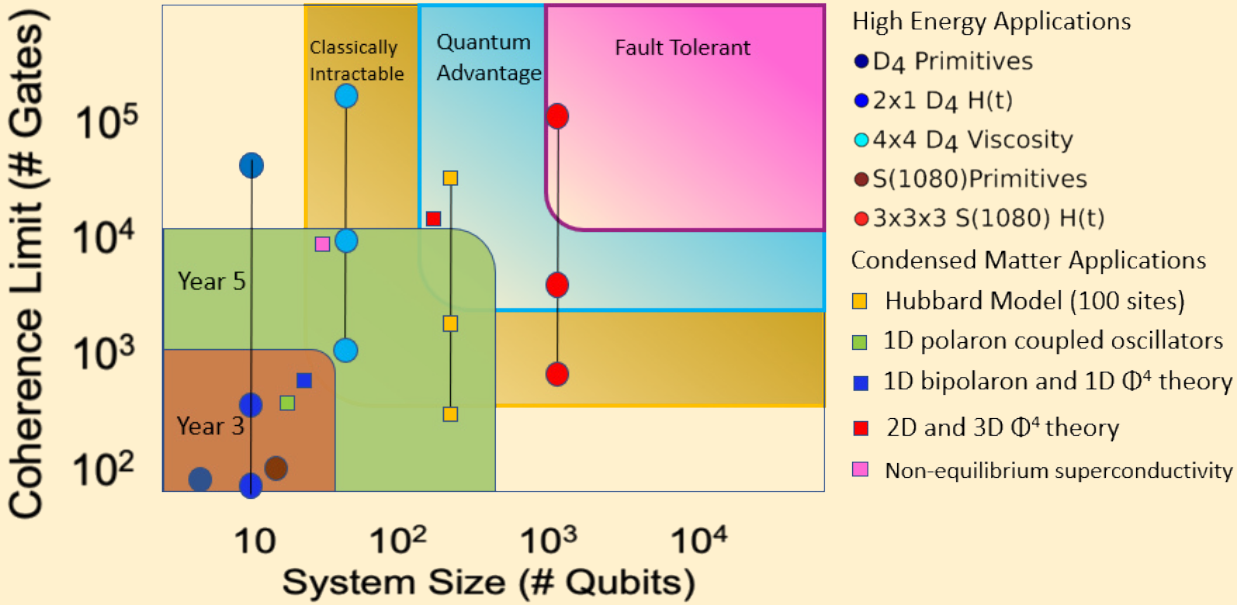
Laptop can do ~ 14 **logical** qubits and $\mathcal{O}(10^3)$ gates in minutes

Balancin' on breaking branches



IBM's Extrapolation

SQMS Center @ Fermilab



So ahead of the curve, the curve becomes a sphere

(1970s) Formulate th

(1980s) Reach

(1990s) cts

(ions

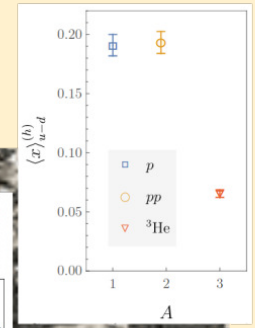
(ctors, QED

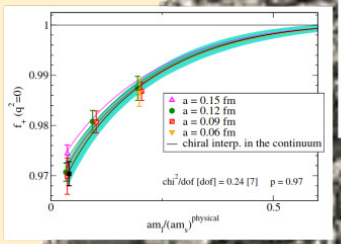
(20 DFs & Nuclei

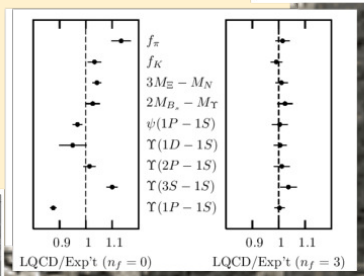
Hamiltonian formulation of Wilson's lattice gauge theories

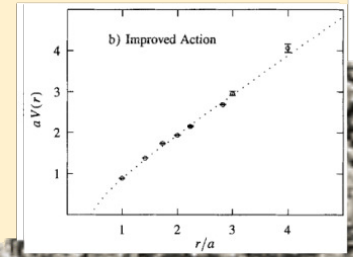
John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

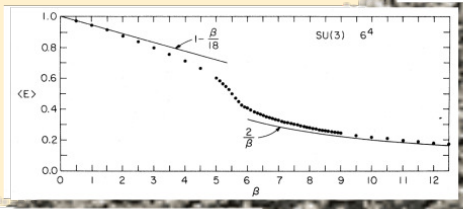
Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 9 July 1974)











Confinement of quarks*

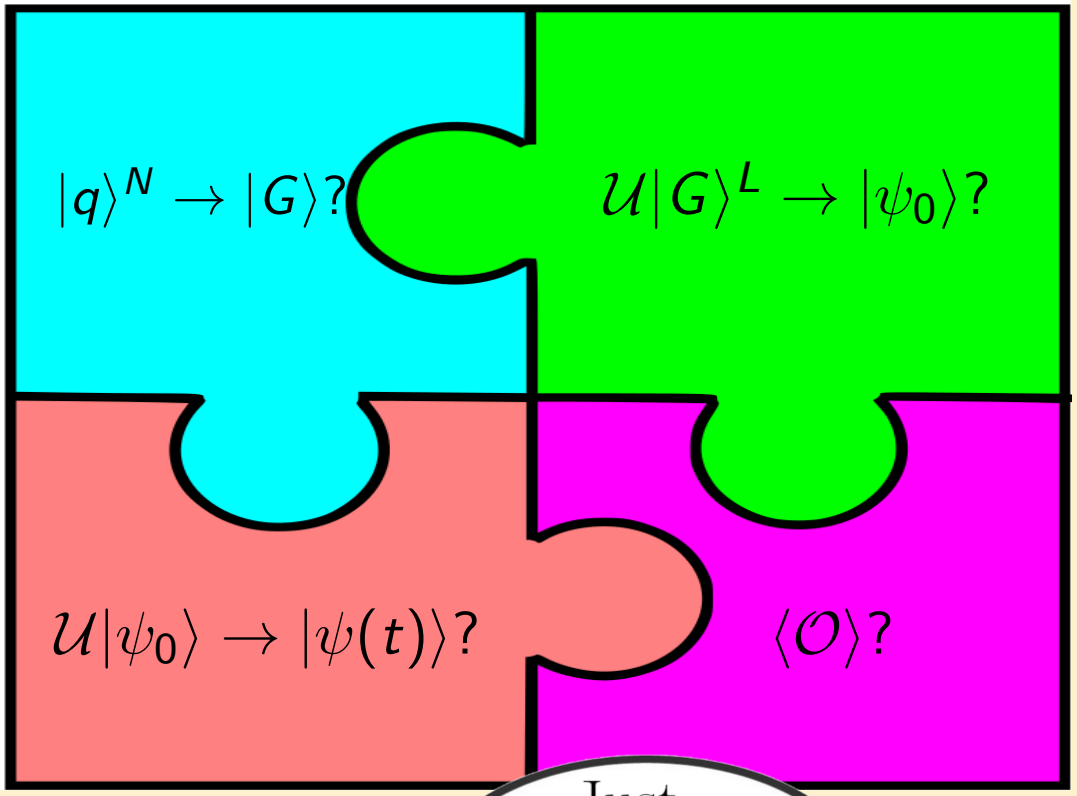
Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

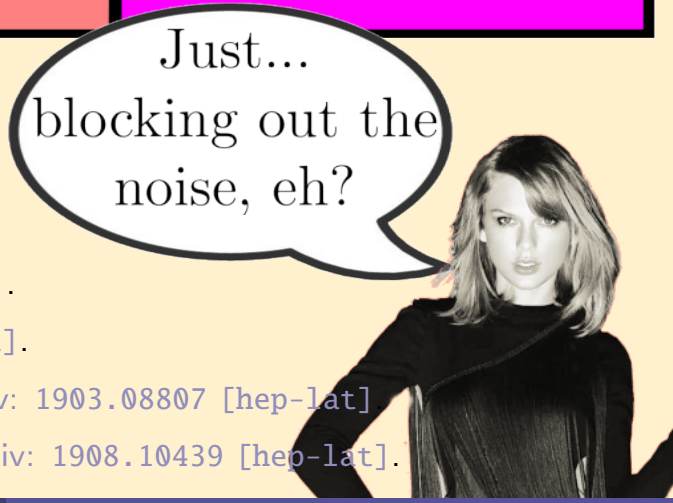
Bustling S or silent H, you're not sure which is worse

$$\langle x|e^{-iHt}|y\rangle = \int \mathcal{D}\phi e^{iS}$$

What "champange problems" need to be solved?

- **Digitize**: How are bosons represented as registers?
  - Discrete Subgroups[3]
- **Initalize**: How can registers be set to a state?
  - Stochastically?[4]
- **Propagate**: How can gates evolve states?[5]
- **Evaluate**: How can observables be computed?[6]





[3] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

[4] Gustafson, E. J. and **H. Lamm**. In: (Nov. 2020). arXiv: 2011.11677 [hep-lat].

[5] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev.* D100 (2019). arXiv: 1903.08807 [hep-lat].

[6] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev. Res.* 2 (2020). arXiv: 1908.10439 [hep-lat].

How do I digitize a gluon?

All Things Considered

Exploring Digitizations of Quantum Fields for Quantum Devices

Erik Gustafson,[1] Hiroki Kawai,[2,*] Henry Lamm,[3,†] Indrakshi Raychowdhury,[4,‡] Hersh Singh,[5,6,§] Jesse Stryker,[4,6,¶] and Judah Unmuth-Yockey[7]

[1] *University of Iowa, Iowa City, Iowa, 52242*[**]
[2] *Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA*
[3] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[4] *Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA*
[5] *Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA*
[6] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*
[7] *Syracuse University, Syracuse NY*[††]

In this LOI we undertake to enumerate promising digitization schemes for quantum fields that could allow near-term calculations on quantum devices. Further we discuss the outstanding questions that must be resolved in evaluating their potential, providing potential benchmarking on the way to practical quantum advantage in high energy physics.

Lots of choices for bosons[7]:

- Casimir Dynamics - Loop-String-Hadrons - Dual Variables - Light-Front - Quantum Link Models - Qubit Regularization - **Discrete Subgroups**

What qualities make a GOOD scheme?

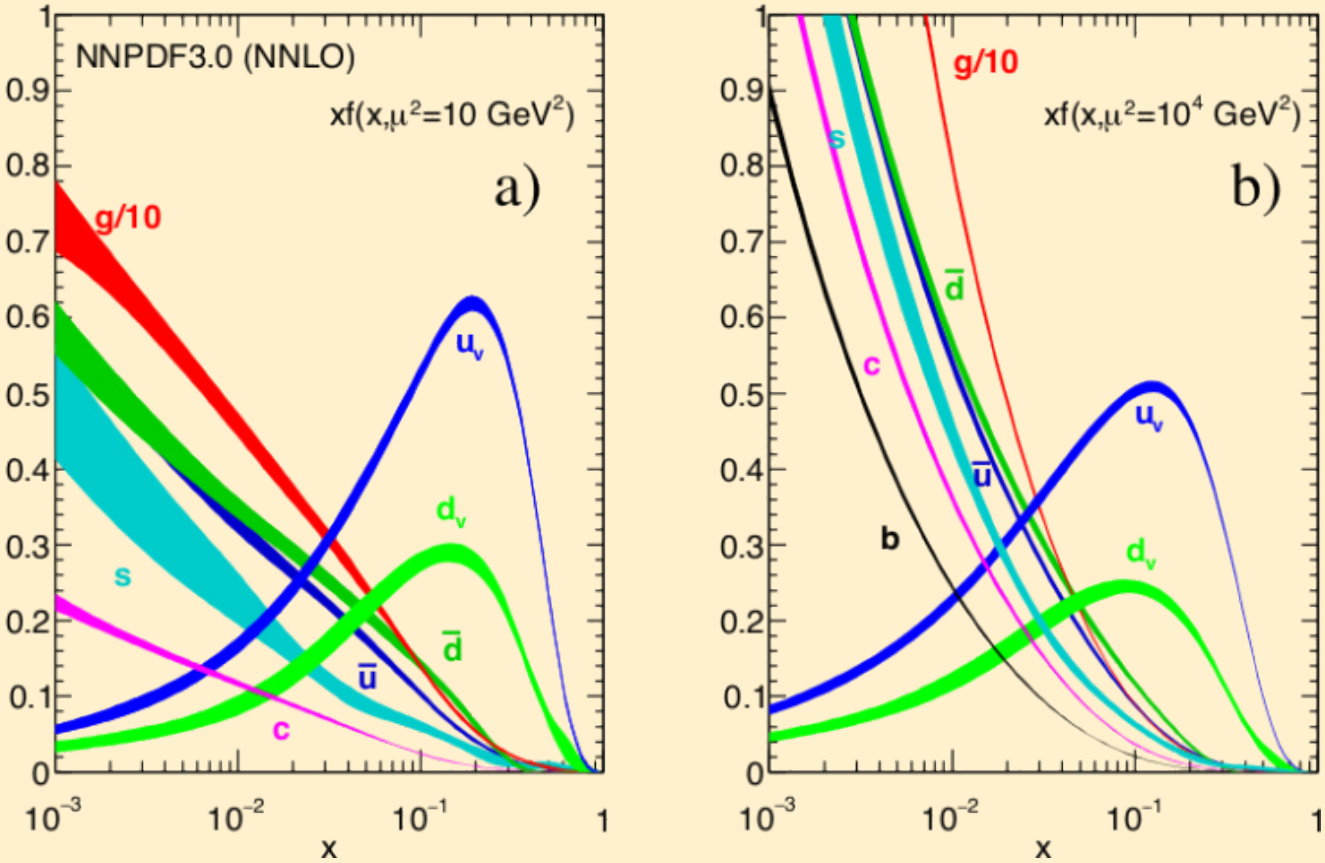
- What are the **quantum resources** required?
- What symmetries are being **broken** in truncation?
- What is the **rate of approach** to the physical point?
- Can the scheme be simulated **classically**?

[7] Gustafson, E. et al. In: *Snowmass 2021 LOI* TF10-97 (2020).

Consequences of your B(x) being a little too strong

$$H_{KS} \sim g^2 \sum_x E(x)^2 + \frac{1}{g^2} \sum_x B(x)^2$$

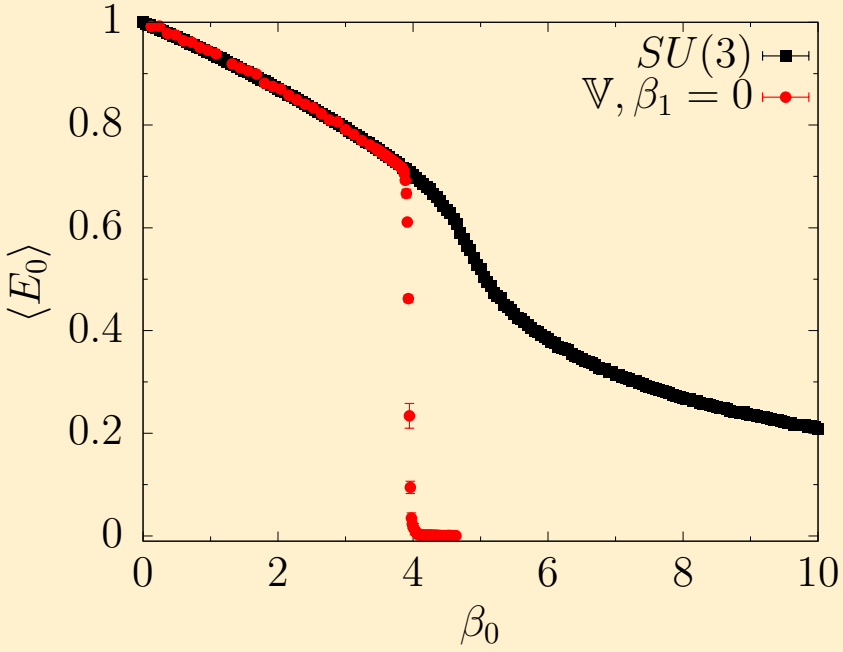
How good is the E **"free-particle"** basis in QCD?

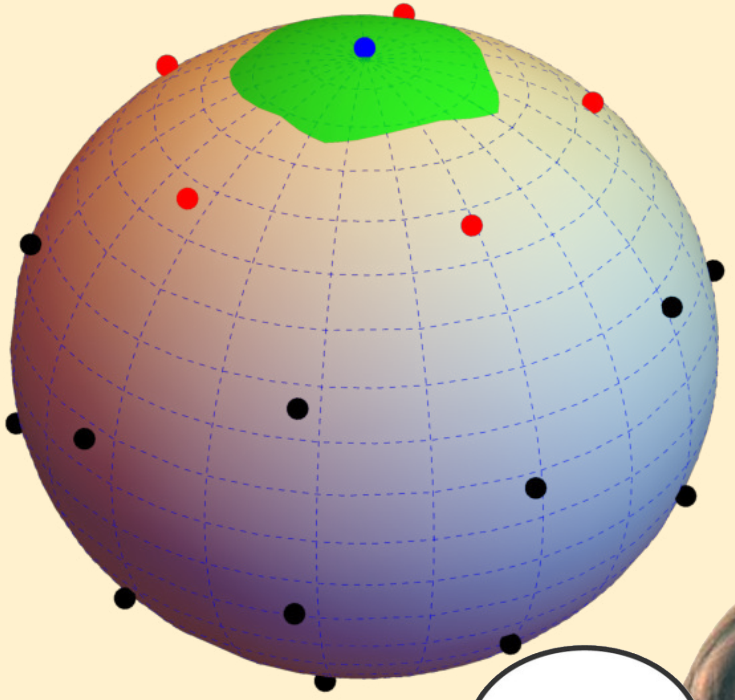


Naively truncating in either basis **breaks** gauge invariance

Discrete subgroups allow plug-and-play framework[8]

- Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$
- H has $\Delta S > 0$ so 1st PT at $\beta_f < \infty$...





- One $SU(3)$ link for **1152** qubit vs 4^3 lattice of $\mathbb{V}$ links

[8] Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019).

But all is not so dire

For discrete groups, the freezing is secretly the **Higgs mechanism**[9]:

$$S_W = \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - u_p] \approx -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - \sum_{\{h\}} \frac{c_h}{\Lambda^{n_h}} f(\phi^{n_h})$$

where $\{n_h\}$ **grows** with size of H[10]

There is a **close analogy** to lattice actions:

$$S_W = \beta \,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] \approx -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{12} a^2 D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}$$

...where the solution is **Symanzik improvement** (add lattice operators)

[9] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).

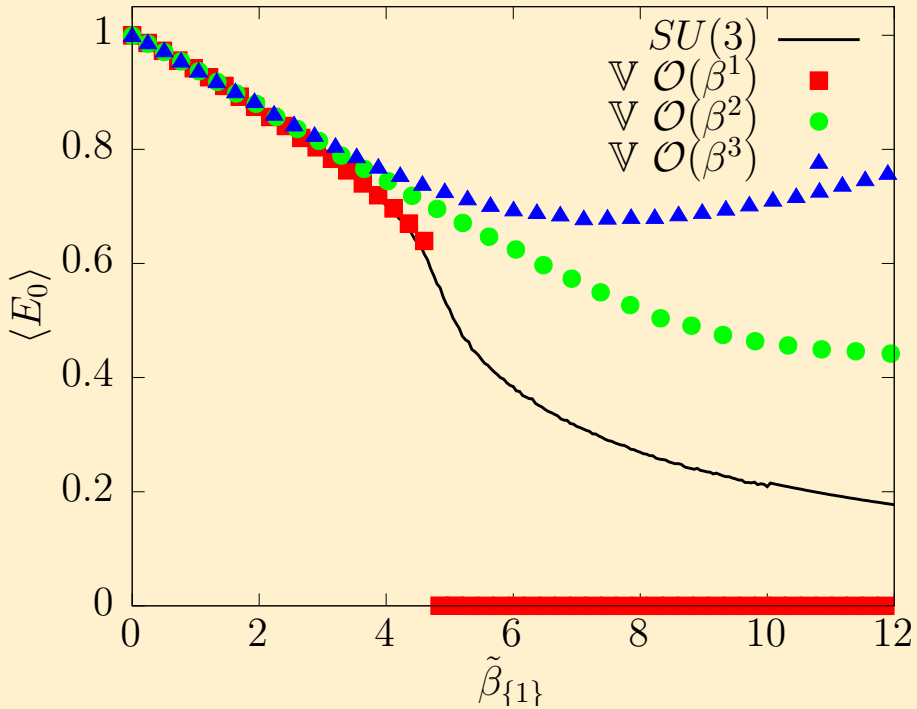
[10] Das, S. and A. Hook. In: *JHEP* 10 (2020). arXiv: 2006.10767 [hep-ph].

Systematics from Group Space Decimation[11]

Decimate via $U = u \cdot \epsilon$ in analogy to Wilsonian renormalization:

$$Z = \int_G DU\, e^{-S[U]} = \sum_{u \in H} \int_\Omega D\epsilon\, e^{-S[u,\epsilon]} = \sum_{u \in H} e^{-S[u]}\,,$$

$$S[u] = \sum_p \beta_{\{1\}} \frac{1}{3} \operatorname{Re} \chi_{\{1\}} + \beta_{\{2\}} \frac{1}{6} \operatorname{Re} \chi_{\{2\}} + \beta_{\{1,-1\}} \frac{1}{8} \chi_{\{1,-1\}} + \cdots$$

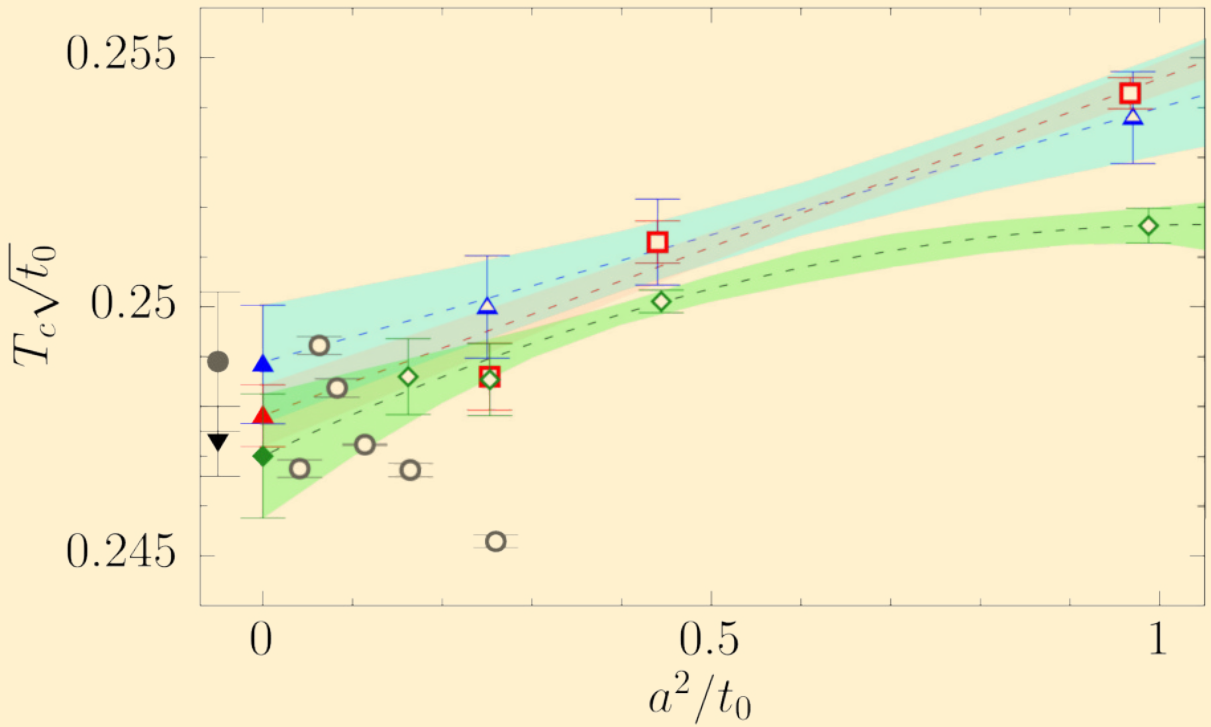


[11] Flyvbjerg, H. In: *Nucl. Phys.* B243 (1984), Ji, Y., **H. Lamm**, and S. Zhu. In: *Phys. Rev. D* 102 (2020). arXiv: 2005.14221 [hep-lat].

$T_c\sqrt{t_0}$ suggests $a \approx 0.07$ fm ≈ 2 GeV^{-1} possible[13]

$$S = \sum \frac{\beta_0}{3} \mathrm{Re}\,\mathrm{Tr}\, U + \beta_1 f(U) \text{ with } f(U) = \{\mathrm{Tr}^2\, U + \mathrm{Tr}\, U^2, |TrU|^2\}$$

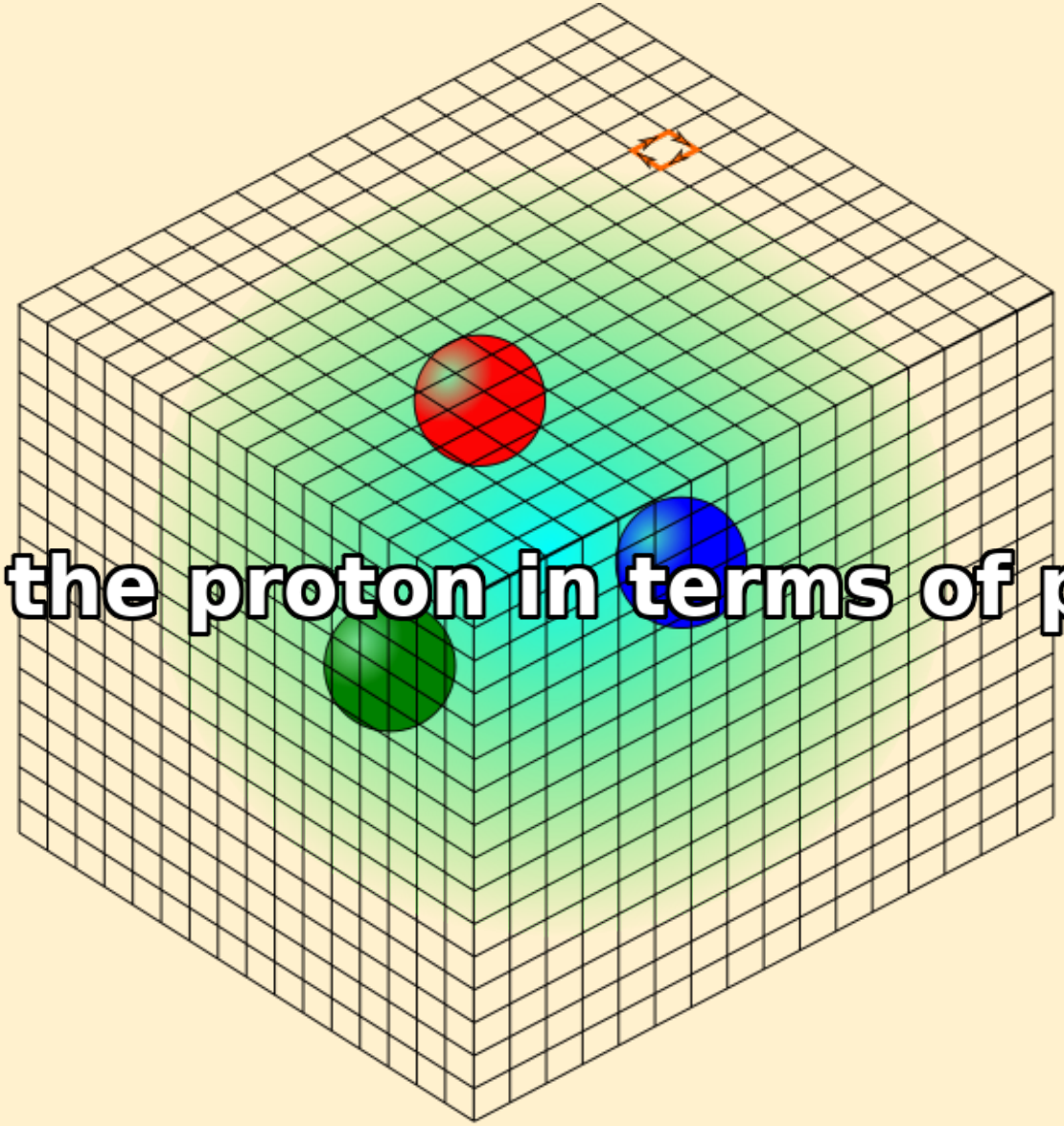
Compare to SU(3)[12]



On-going work to extract quenched spectroscopy

[12] Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev. D* 91 (2015).

[13] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

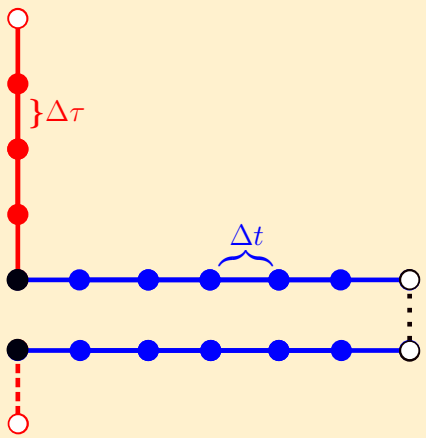


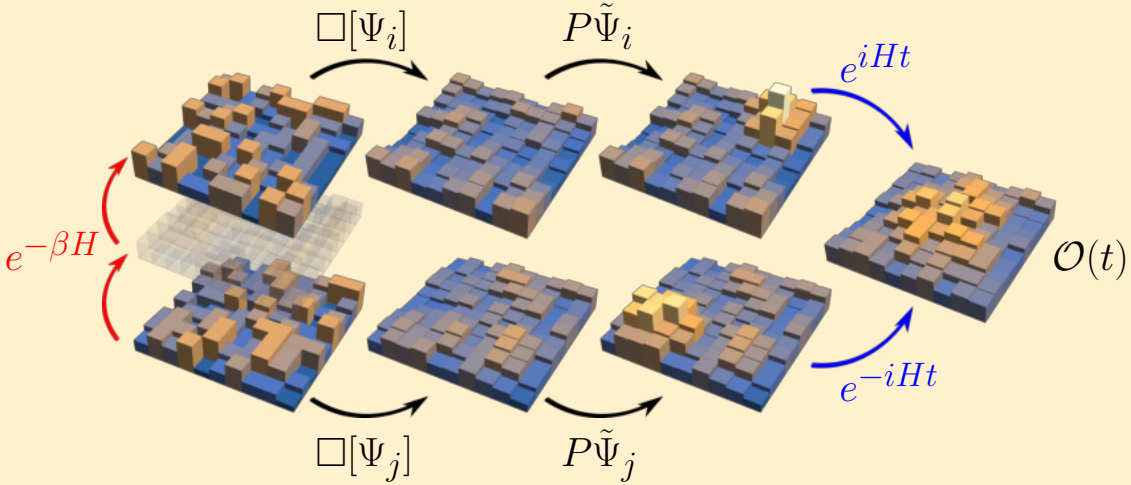
What is the proton in terms of partons?

$E\rho OQ$: A hybrid technique for real-time LQCD[14]

Preping $|\psi_0\rangle$ is expensive! Use classical resources via Schwinger-Keldysh

$$\langle \mathcal{O}(t) \rangle = \frac{\mathrm{Tr} P^\dagger e^{-\beta H} P e^{-iHt} \mathcal{O} e^{iHt}}{\mathrm{Tr}\, e^{-\beta H}}$$





- **Classical: open-BC LQCD yields $(e^{-\beta H})_{ij}$ then project with P**
- **Quantum: Time-evolve elements of simpler $(P^\dagger e^{-\beta H} P)_{ij}$**

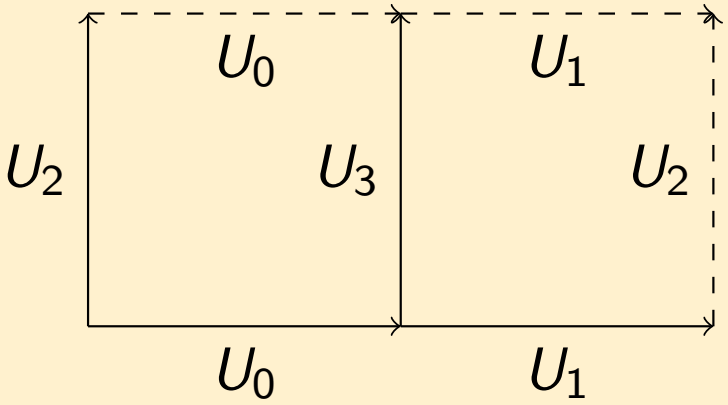
[14] Harmalkar, S., **H. Lamm**, and S. Lawrence. In: (). arXiv: 2001.11490 [hep-lat], Gustafson, E. J. and **H. Lamm**. In: (Nov. 2020). arXiv: 2011.11677 [hep-lat].

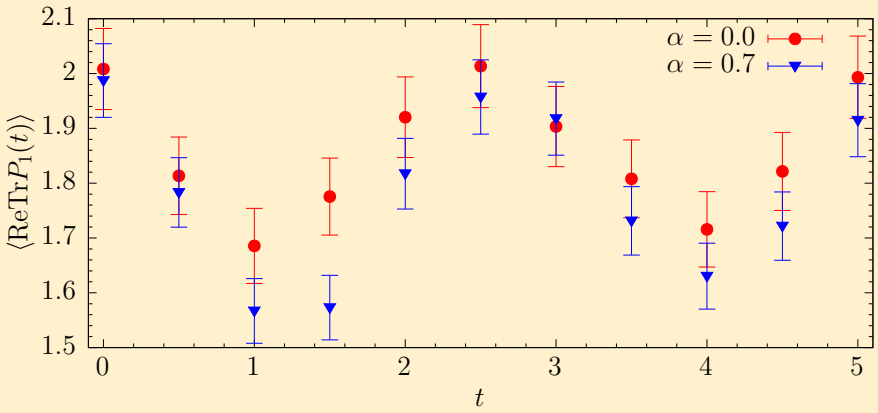
Simulator Results for 2+1D D_4 & $\mathbb{Z}_2$ gauge theories

14q D_4 gauge theory (NuQS)

✓ **Nonabelian** group with $N = 8$

✓ **Thermal** $|\psi\rangle$ with **Smearing**

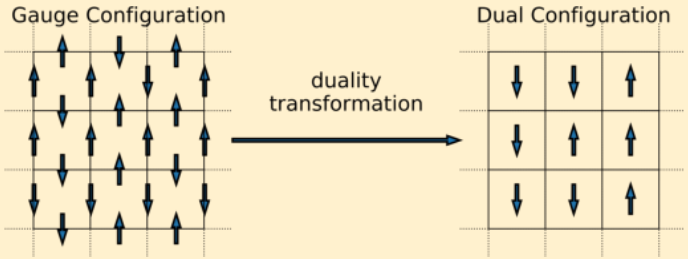


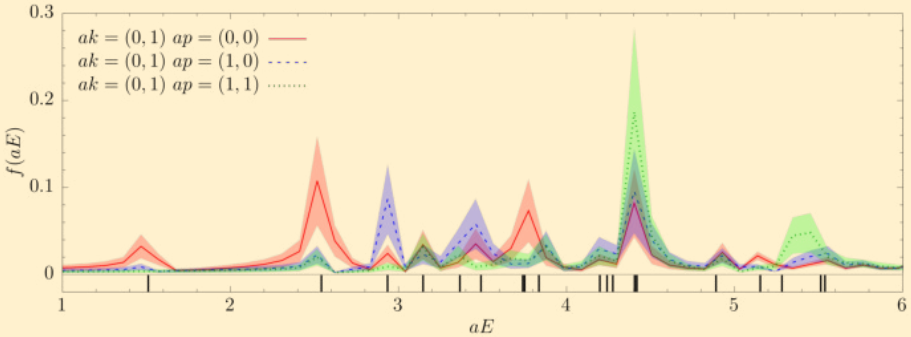


18q $\mathbb{Z}_2$ gauge theory (w/ Gustafson)

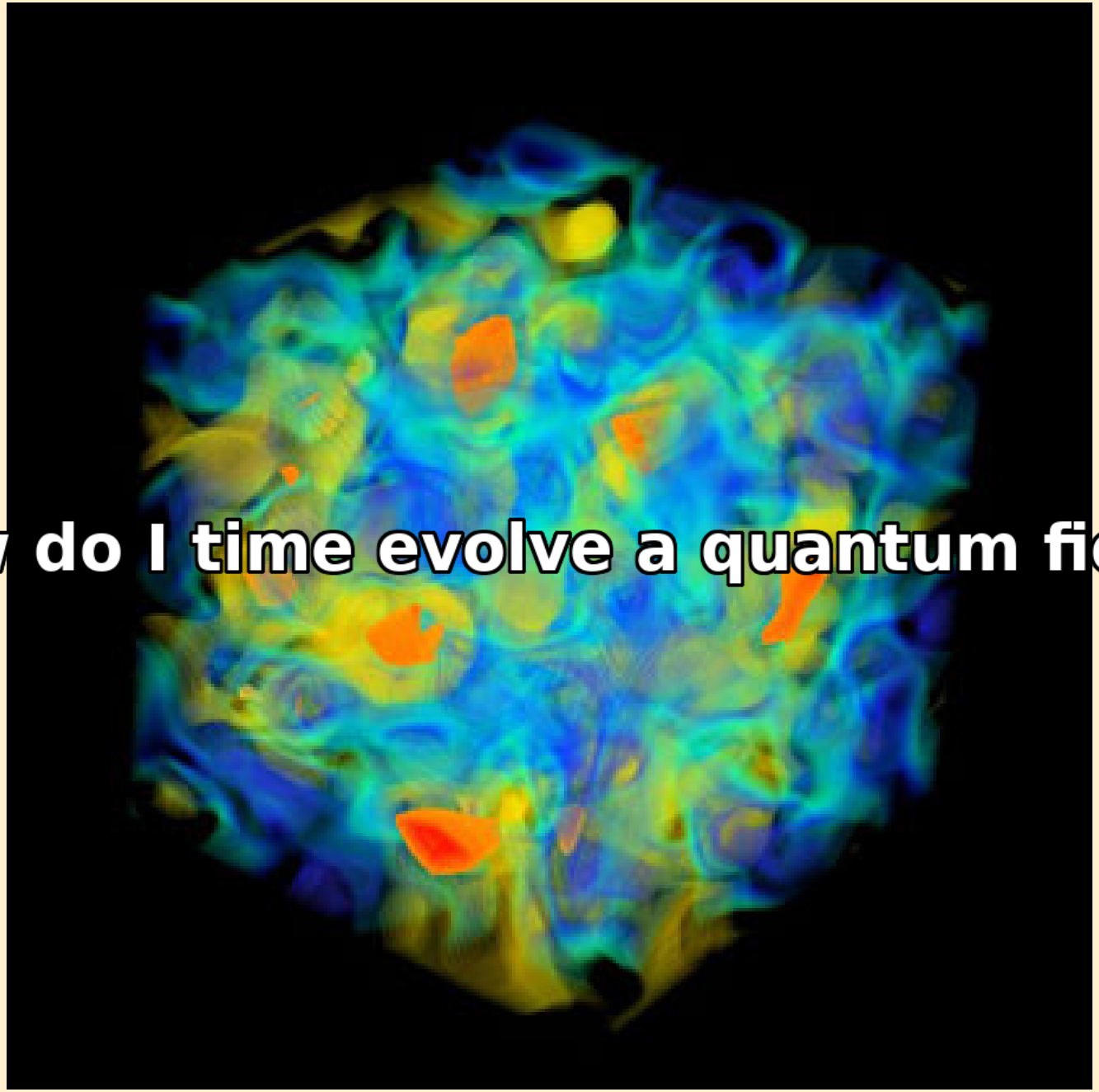
✓ **Abelian** group with $N = 2$

✓ ***n*-particle** $|\psi\rangle$





Proposals for these on **Rigetti & Google** on $\lesssim$ 5 year scale

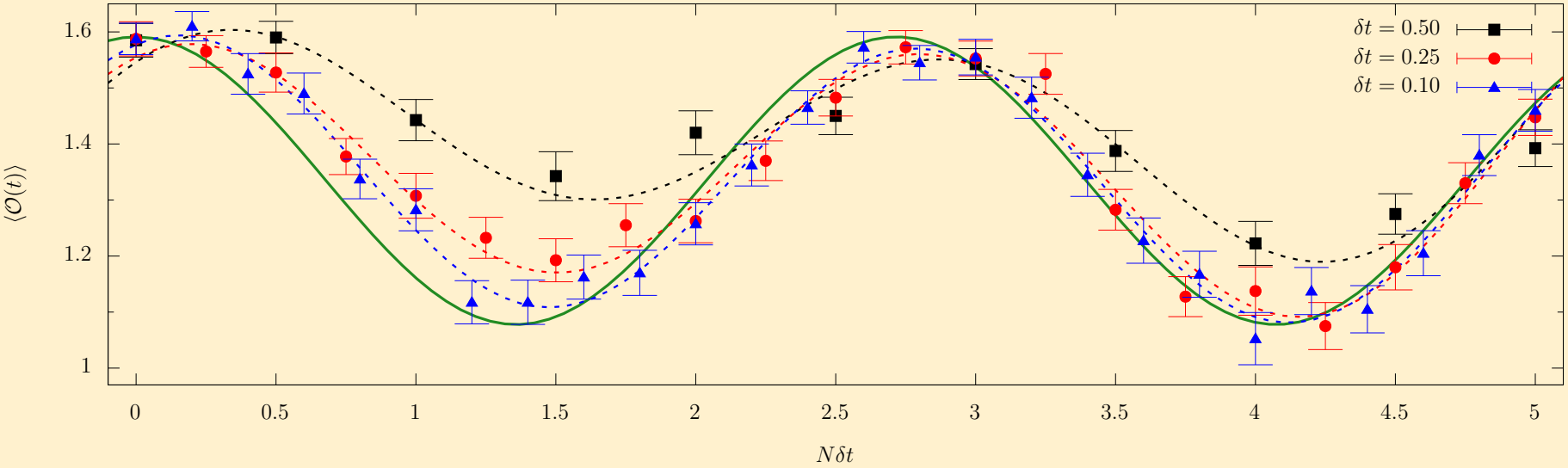


How do I time evolve a quantum field?

What is trotterization?

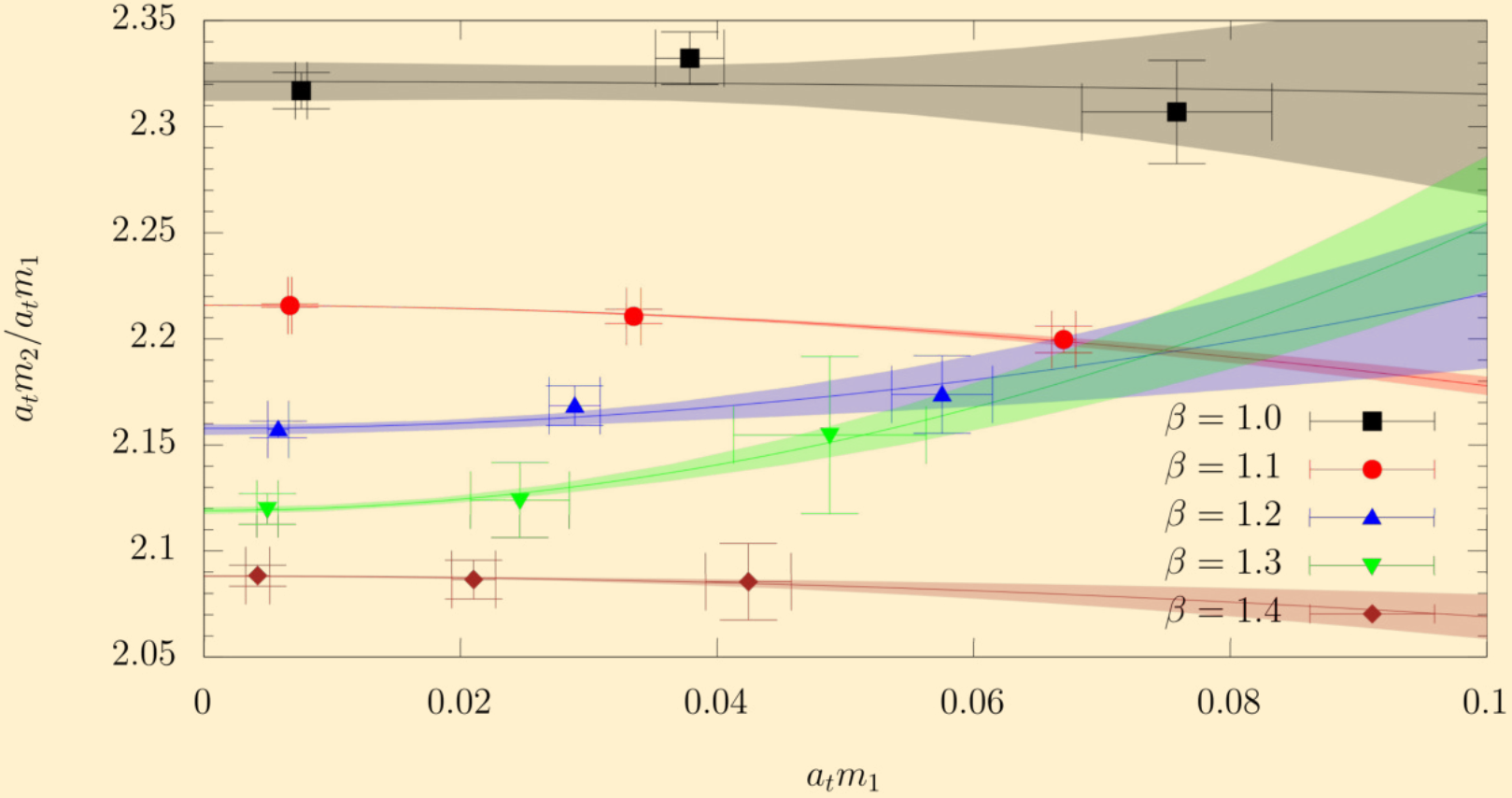
$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t \frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t \frac{H_V}{2}} \right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{ -it \left(H_K + H_V + \frac{\delta t^2}{24} (2[H_K, [H_K, H_V]] - [H_V, [H_V, H_K]]) \right) \right\}$$



- δt is bare $c(a, a_t)$ **not** physical a_t
- Introduces **higher dimension operators**
- Eigenstates of $H(a, a_t = 0)$ **mix** at finite $a_t \rightarrow$ **quantum smearing?**

Approaching the continuum



- *Hamiltonian* limit: $a_t \to 0$ (unnecessarily expensive)
- *Continuum* limit: $a_t, a \to 0$ (the one that I want)
- Fix $\xi = a/a_t$ to **efficiently** get QFT

Can **Euclidean** calculations be used to scale-set **Minkowski** ones?

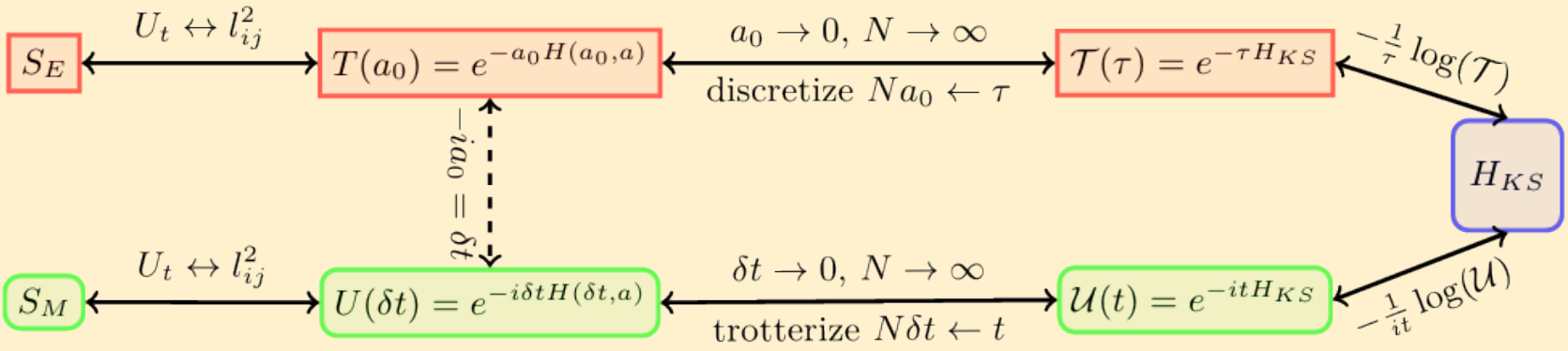
Relating H to S (Work with Carena, Li, Liu)

The **anisotropic Wilson** action is

$$S_{\mathrm{W}} = \frac{1}{g_t^2}\xi \sum_t \mathrm{Re}\,\mathrm{Tr}\, U_t + \frac{1}{g_s^2}\frac{1}{\xi}\sum_s \mathrm{Re}\,\mathrm{Tr}\, U_s \tag{1}$$

thru $T = \langle i|e^{-S}|j\rangle \approx \langle i|e^{-a_0 H}|j\rangle$ derives the **Kogut-Susskind** H[15]

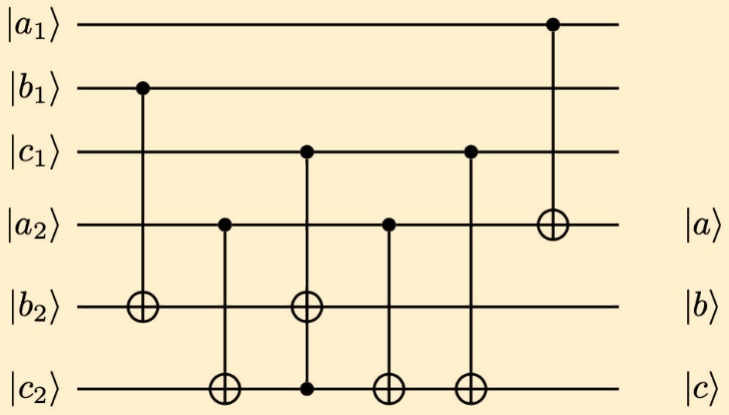
$$H_{KS} = \frac{1}{c(a,a_0)a}\left[\frac{g_H^2(a,a_0)}{2}\sum_l l_{ij}^2 + \frac{1}{g_H^2(a,a_0)}\sum_p \mathrm{Re}\,\mathrm{Tr}\, U_p\right] \tag{2}$$



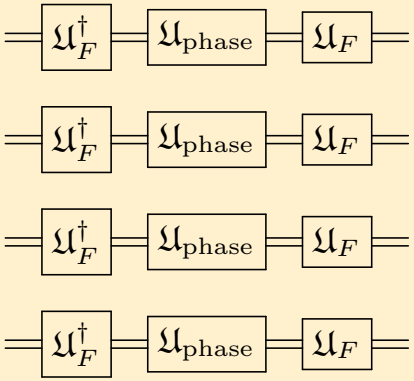
[15] Creutz, M. *Quarks, gluons and lattices*. Cambridge Monographs on Mathematical Physics. Cambridge, UK: Cambridge Univ. Press, June 1985.

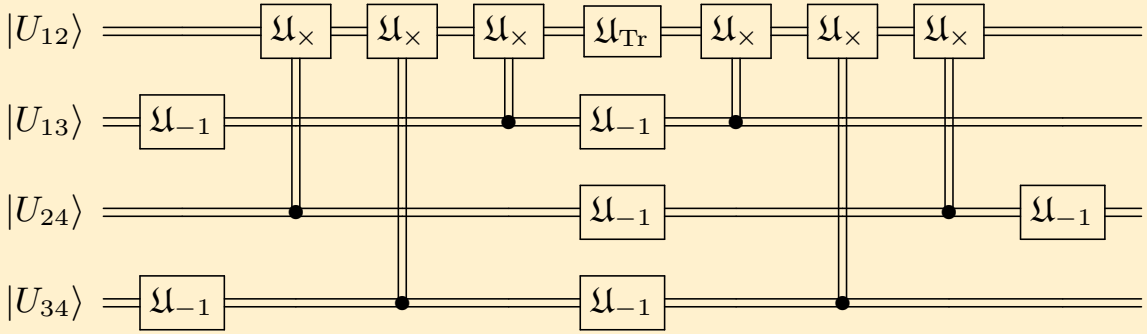
What low-level primatives are required for LGT?[16]

- Inversion gate: $\mathfrak{U}_{-1} |g\rangle = |g^{-1}\rangle$
- Multiplication gate: $\mathfrak{U}_{\times} |g\rangle |h\rangle = |g\rangle |gh\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta) |g\rangle = e^{i\theta \,\mathrm{Re}\,\mathrm{Tr}\, g} |g\rangle$



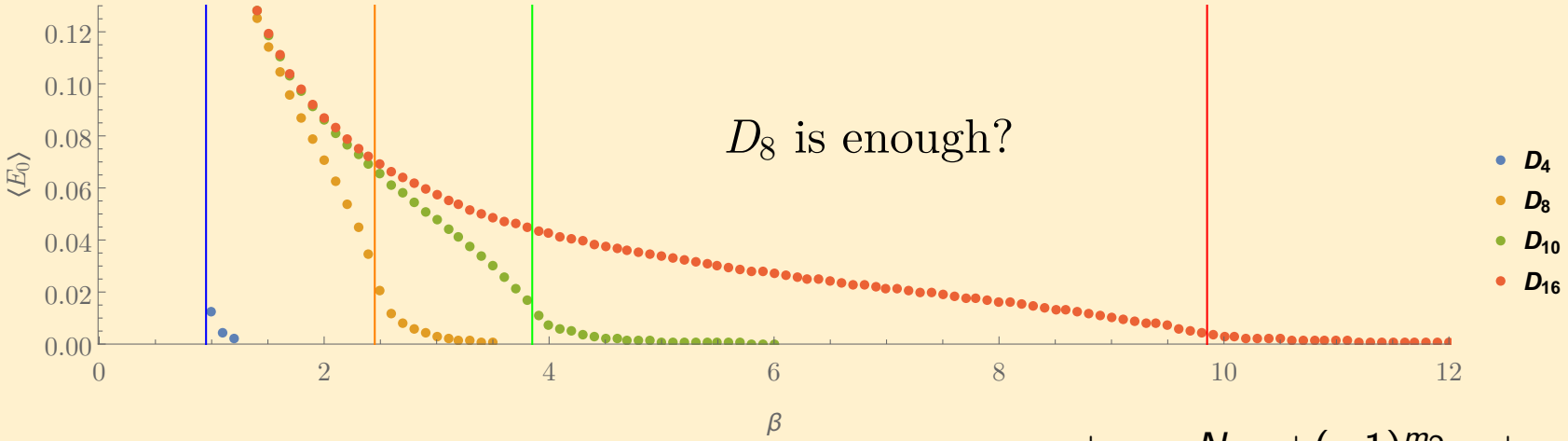
- Fourier Transform gate: $\mathfrak{U}_F \sum_{g\in G} f(g) |g\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij} |\rho, i, j\rangle$



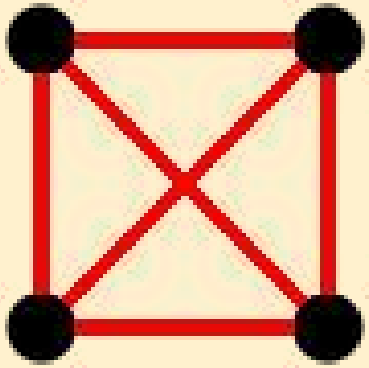


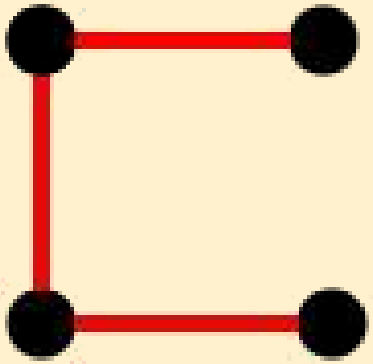
[16] **Lamm, H.**, S. Lawrence, and Y. Yamauchi. In: *Phys. Rev.* D100 (2019). arXiv: 1903.08807 [hep-lat].

Small steps with D_{2^N} for quantum leaps



D_{2^N} multiply: $h_1 \times h_2 = v^{m_1} u^{n_1} v^{m_2} u^{n_2} = v^{m_1+m_2} u^{Nm_2+(-1)^{m_2} n_1+n_2}$





All-to-all connectivity

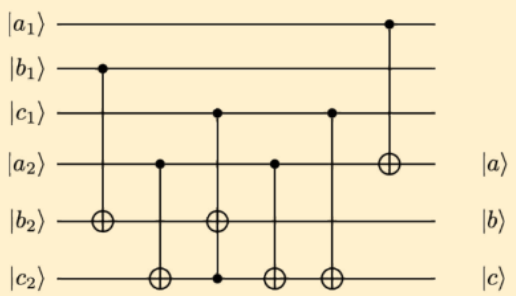
$N_{CNOT} = 21N_q - 31$

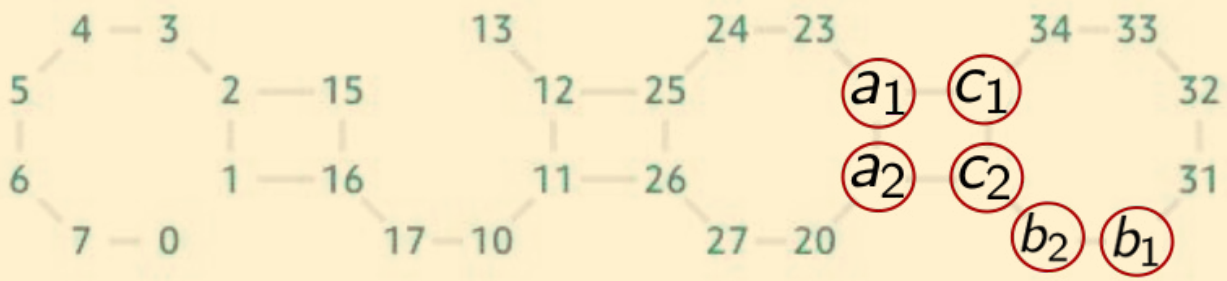
Linear connectivity

$N_{CNOT} \leq 126N_q^2 - 438N_q + 372$

Ancilla qubits: $N_q - 1$

Primative gates on Rigetti (w/ Alam)



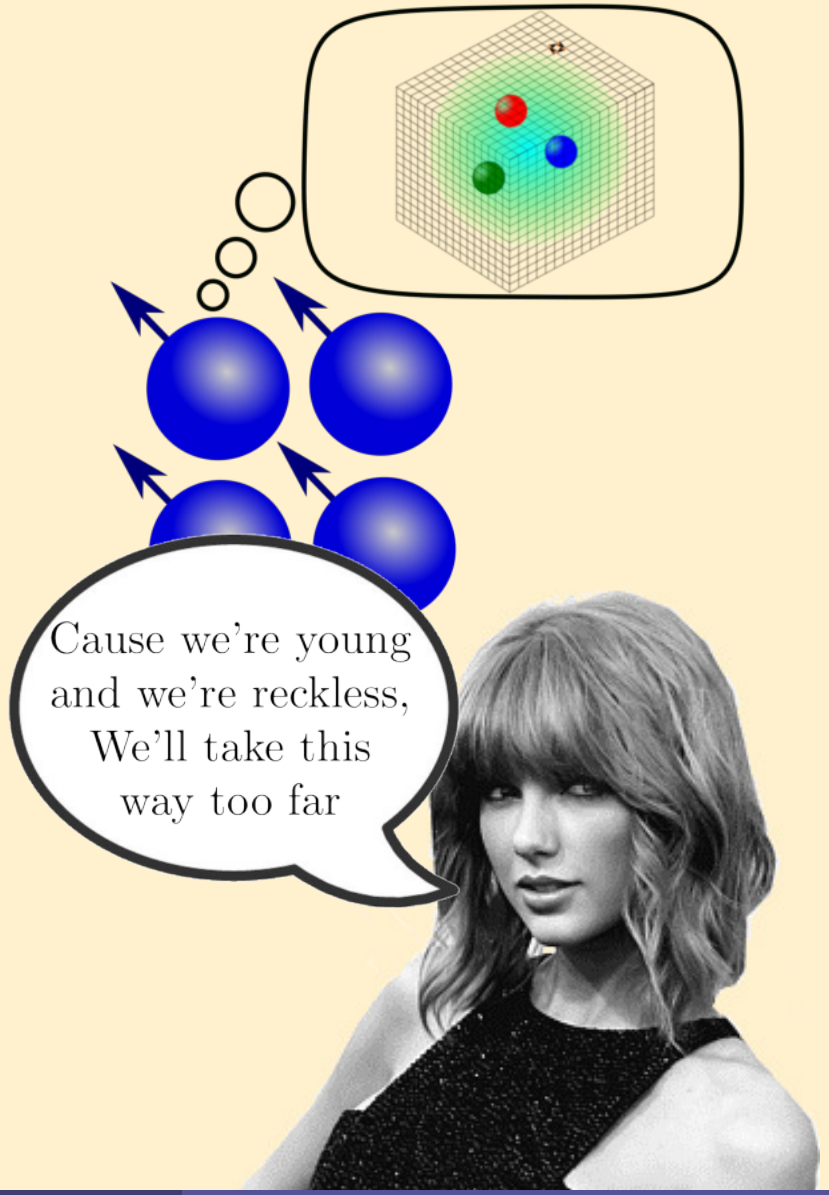


Prelim: $\mathfrak{U}_\times$ for D_4 has $\approx 30\%$ fidelity $\implies$ $\mathfrak{U}_V$ has $< 10^{-3}$ fidelity

```
(0, 0, 0) x (0, 0, 0)
['001', '001', '001', '101', '001']
000
0.3577
**********************************************************************
(0, 0, 0) x (0, 0, 1)
['001', '101', '100', '001', '001']
001
0.2654
**********************************************************************
(0, 0, 0) x (0, 1, 0)
['010', '110', '011', '010', '010']
010
0.3339
**********************************************************************
(0, 0, 0) x (0, 1, 1)
['001', '001', '000', '001', '000']
011
0.2494
**********************************************************************
(0, 0, 0) x (1, 0, 0)
['100', '100', '101', '101', '110']
100
0.3613
**********************************************************************
```

```
**********************************************************************
(1, 0, 1) x (1, 1, 1)
['000', '011', '000', '010', '001']
010
0.2594
**********************************************************************
(1, 1, 0) x (0, 0, 0)
['011', '100', '011', '100', '110']
110
0.2698
**********************************************************************
(1, 1, 0) x (0, 0, 1)
['101', '001', '110', '100', '111']
111
0.2426
**********************************************************************
(1, 1, 0) x (0, 1, 0)
['100', '100', '111', '110', '101']
100
0.3399
**********************************************************************
(1, 1, 0) x (0, 1, 1)
['110', '100', '111', '111', '101']
101
0.2953
**********************************************************************
```

It's time to go

So many things to do!...and lots can be done before the machine exists

- Digitizing SU(3)
  - **Spectroscopy** for $\mathbb{V}$
  - $\mathbb{V}$ **circuits**
- Reducing the errors
  - e.g. Finite volume, finite a, a_t, decimation errors, fidelity to obtain **realisitic** resource estimates
- Algorithms for **state prep, smearing**
- Investigate desirable properties
  - **PDF?, Viscosity?, Cosmology?**
- **Actual** simulations of toy models
  - $\mathbb{Z}_2$ & $\mathbb{D}_4$