NLO di-boson production by gluon fusion, in the high-energy limit

Fermilab Theoretical Physics Seminar

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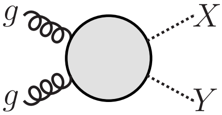
Introduction

Gluon fusion amplitudes are interesting at the LHC despite loop suppression: enhanced by large gluon PDF.



HH
- gives access to Higgs self-coupling λ
- $-5.0 < \lambda/\lambda_{SM} < 12.0$ [CERN-EP-2019-099]
- *gg* is the dominant production channel (10x)

ZZ
- significant background to $H \rightarrow 4l$
- constrains Higgs width via $H \rightarrow ZZ$ diagrams
- sub-leading cf. quark-induced, but ~60% of total NNLO

ZH
- $H \rightarrow b\bar{b}$ discovery
- sub-leading cf. quark-induced, but ~10% of total
- large scale uncertainties

Introduction

Amplitude structure:

$$\mathcal{M}^{\mu\nu\rho\sigma} = \sum_i \mathcal{A}_i^{\mu\nu\rho\sigma} F_i$$

$HH : i = \{1,2\}, \quad ZZ : i = \{1,\ldots,18\}, \quad ZH : i = \{1,\ldots,6\}.$

Two-loop computations of such form factors F_i are difficult:

- Depend on many scales, s, t, m_t, m_H, m_Z
- Feynman integrals are complicated

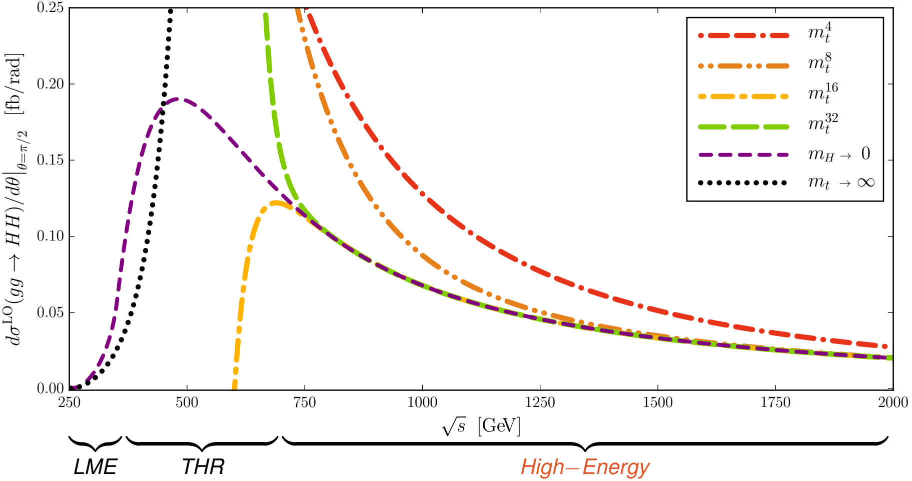
... not known analytically!

Numerical results exist, and expansions in various limits:

- large-m_t
- threshold
- small-p_T
- high-energy

Expansions

Seek an expansion in the region $s, t \gg m_t^2 > \{m_H^2, m_Z^2\}$:



(Incomplete) NLO Status

HH
- LO [Glover,van der Bij ‘88]
- NLO HEFT [Dawson,Dittmaier,Spira ‘98]
- NLO LME+THR Padé [Gröber,Maier,Rauh ‘17]
- NLO small-p_T [Bonciani,Degrassi,Giardino,Gröber ‘18]
- NLO numerical [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke ‘16] [Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher ‘18]

ZZ
- LO
- NLO (massless) [Caola,Melnikov,Röntsch,Tancredi ‘15]
- NLO LME [Dowling,Melnikov ‘15]
- NLO numerical [Agarwal,Jones,von Manteuffel ‘20] Brønnum-Hansen,Wang ‘21]

ZH
- LO [Dicus,Kao ‘88, Kniehl ‘90]
- NLO LME [Hasselhuhn,Luthe,Steinhauser ‘17]
- NLO small-p_T [Alasfar,Degrassi,Giardino,Gröber,Vitti ‘21]
- NLO numerical [Chen,Heinrich,Jones,Kerner,Klappert,Schlenk ‘20]

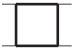
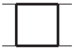
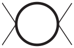
+ various higher-order efforts, mostly HEFT and LME

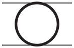
High-Energy Expansion: $\{m_h, m_z\}$

Seek an expansion in the region $s, t \gg m_t^2 > \{m_H^2, m_Z^2\}$:

- amplitude in terms of Feynman integrals: $I(\{m_H^2, m_Z^2\}, m_t^2, s, t, \epsilon)$
- expand around $\{m_H^2, m_Z^2\} \to 0$:

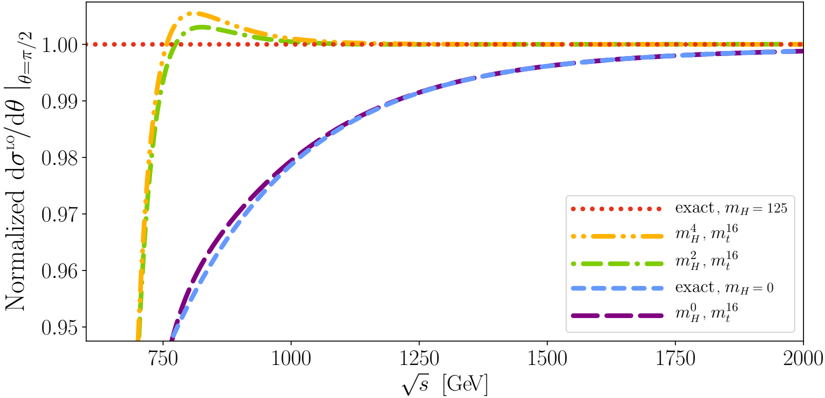
$$I(m_H^2, \ldots) = I(0, \ldots) + m_H^2 \frac{d}{dm_H^2} I(0, \ldots) + \cdots$$

 $\approx$  $+ m_H^2 \Bigg(\frac{2(2m_t^2 s - st - t^2)}{(s+t)(4m_t^2 s - st - t^2)}$  $- \frac{4s}{t(s+t)(4m_t^2 s - st - t^2)}$ 

$+ \frac{4}{(4m_t^2 + s + t)(4m_t^2 s - st - t^2)}$  $- \frac{4(4m_t^2 s + s^2 - t^2)}{m_t^2 t(s+t)(4m_t^2 + s + t)(4m_t^2 s - st - t^2)}$  $+ \mathcal{O}(\epsilon) \Bigg)$

High-Energy Expansion: $\{m_h, m_z\}$

Convergence of m_H expansion in LO $gg \to HH$:

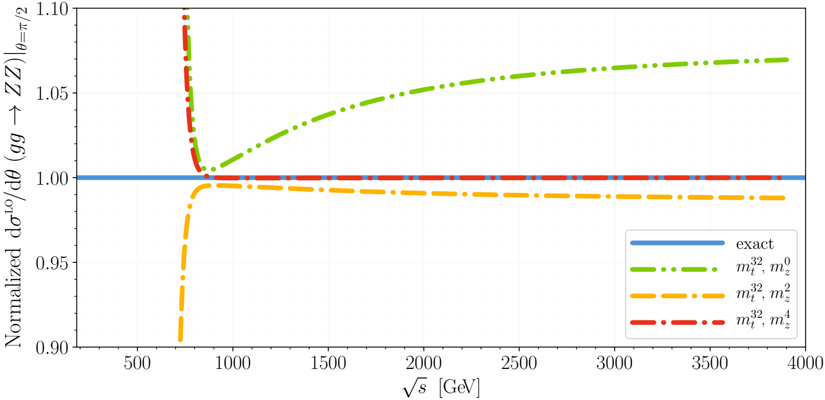


[Davies, Mishima, Steinhauser, Wellmann ‘18]

High-Energy Expansion: $\{m_h, m_z\}$

Convergence of m_Z expansion in LO $gg \to ZZ$:

[Davies, Mishima, Steinhauser, Wellmann '20]



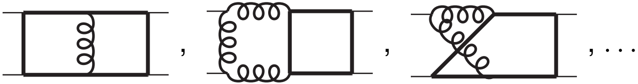
(Z pol. sums contain $p^\mu p^{\mu'} / m_Z^2$ term.)

IBP reduction

With this expansion in mind, we can perform IBP reduction (with FIRE) *only for the integrals with massless external legs*. [Smirnov '19]

- this removes scale(s) $\{m_H, m_Z\}$ from the problem – easier!
- (pySecDec-based numerical evaluations do not expand, but rather set $m_H^2/m_t^2 = 12/23$, $m_Z^2/m_t^2 = 23/83$ in the reduction)

At two loops, results in 131 planar MIs, 30 non-planar MIs.



The reduction and MIs do not depend on $\{m_H, m_Z\}$:

- we can use them for $gg \to HH$, $gg \to ZZ$, $gg \to ZH$, ...

High-Energy Expansion: m_t

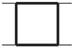
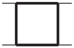
Seek an expansion in the region $s, t \gg m_t^2 > \{m_H^2, m_Z^2\}$:

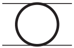
Master Integral coefficients: simple Taylor series in m_t.

Master Integrals:

- Differentiate w.r.t. m_t^2, IBP reduce result. System of DEs:

$$\frac{d}{d\, m_t^2} \vec{J} = M(s, t, m_t^2, \epsilon) \cdot \vec{J}.$$

$\frac{d}{d\, m_t^2}$  $= \frac{2(s+t)}{st-4m_t^2(s+t)}$  $- \frac{2s}{m_t^2(s-4m_t^2)(4m_t^2(s+t)-st)}$ 

$- \frac{2t}{m_t^2(t-4m_t^2)(4m_t^2(s+t)-st)}$  $+ \frac{4(2m_t^2(s+t)-st)}{m_t^4(4m_t^2-s)(4m_t^2-t)(4m_t^2(s+t)-st)}$  $+\mathcal{O}(\epsilon)$

- Substitute series ansatz: $J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\epsilon^i (m_t^2)^j \log{(m_t^2)}^k$,
- Solve system of *linear* equations for C_{ijk}.
  - Requires some C_{ijk} as boundary conditions.

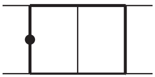
Boundary Conditions

Expansion-by-Regions yields Mellin-Barnes integrals for leading m_t^2 behaviour of the MIs, which depend on s, t. [asy.m Pak, Smirnov '11]

- Easiest way: set $s = 1$, expand MB integrals around $t = 0$,
- Fit expansions to basis of HPLs to obtain leading C_{ijk}.

Example:

$\epsilon^{\mathbf{0}}(m_t^2)^{\mathbf{0}} \log(m_t^2)^{\mathbf{0}}$



$$
\begin{aligned}
C_{\mathbf{0\,0\,0}} =& -(8\zeta_3 + 24 + 4\pi^2 + 7\pi^4/15) + (8\zeta_3 - 8 + 20\pi^2/3)\mathbf{t} \\
& - (5\pi^2 + 18)\mathbf{t^2} - (44/9 + 16\pi^2/9)\mathbf{t^3} - (41/18 + 11\pi^2/12)\mathbf{t^4} \\
& - (33/25 + 14\pi^2/25)\mathbf{t^5} - (194/225 + 17\pi^2/45)\mathbf{t^6} \\
& - (4/9 + 40\pi^2/147)\mathbf{t^7} + \mathcal{O}(\mathbf{t^8}) \\
=& -8(1-t)\zeta_3 - 24 - 4\pi^2 - 7\pi^4/15 + 8\pi^2 t/3 \\
& + 8\pi^2(1-t)H_1(t) - 4\pi^2 H_2(t) + 16(1-t)H_3(t) - 24H_4(t) \\
& + \cdots .
\end{aligned}
$$

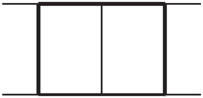
- Use BCs to determine C_{ijk} to desired depth in m_t, using DEs.

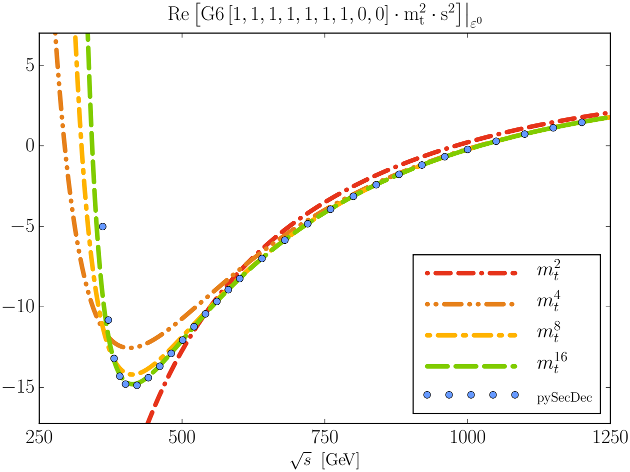
Master Integral Results

MI-level comparison: m_t expansion vs. `pySecDec` numerical values:

[Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '18]

double box:
6 massive (m_t),
1 massless prop.
Real part, ϵ^0

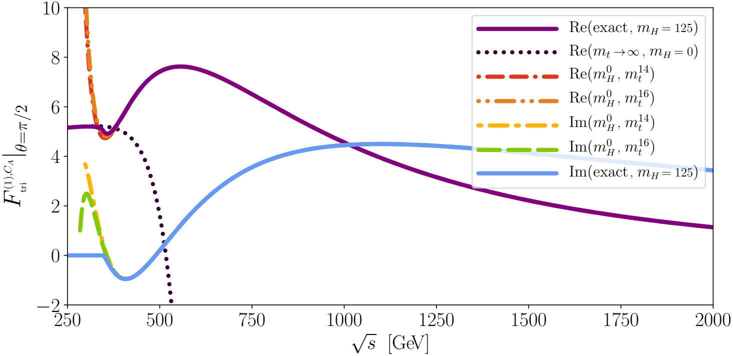




Form Factor Expansions

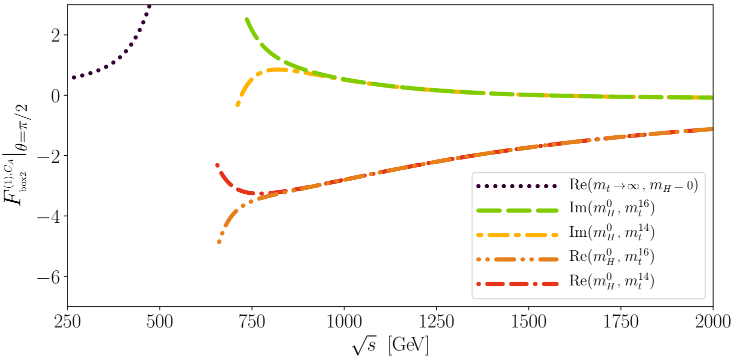
Renorm. and IR subtraction: $F_i^{(1),fin} = F_i^{(1),IR-div} - K_g^{(1)} F_i^{(0)}$.

In $gg \to HH$, NLO F_{tri} known analytically (from $gg \to H$):



Form Factor Expansions

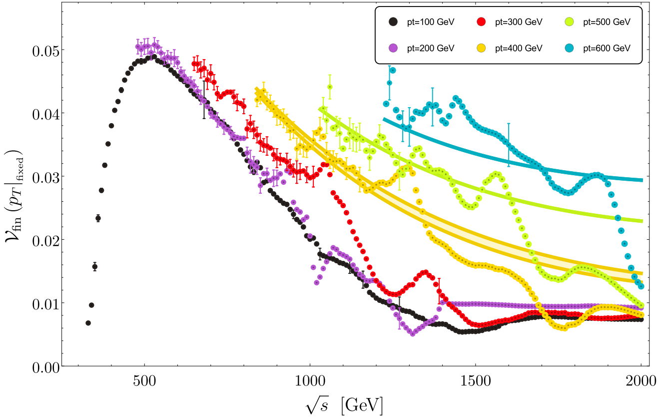
$gg \to HH$, F_{box} form factors are not known analytically for comparison:



$gg \to HH \; V_{fin}$

"Virtual finite cross-section" V_{fin}, m_t^{30}, m_t^{32} curves:

- compare with numerics, `hhgrid` [Heinrich,Jones,Kerner,Luisoni,Vryonidou '17]



For lower p_T values, high-energy expansion doesn't converge well.

Padé Approximants

Approximate a function $f(x)$ using a *rational polynomial*:

$$f(x) \approx [n/m](x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m}.$$

Use series coefficients of $f(x)$ to fix $a_0, \ldots, a_n, b_1, \ldots, b_m$.

- Agrees with series to order $n + m$ (but not beyond)
- Can be a better approximation of $f(x)$ than the truncated series.
- Even beyond the radius of convergence!

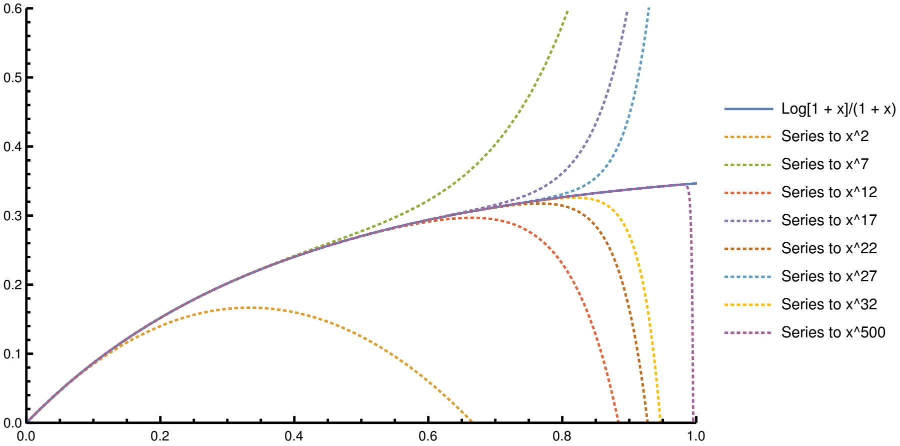
Proceed by example:

$$f(x) = \frac{\log(1+x)}{1+x},$$

approximate around $x = 0$, radius of convergence: $x < 1$.

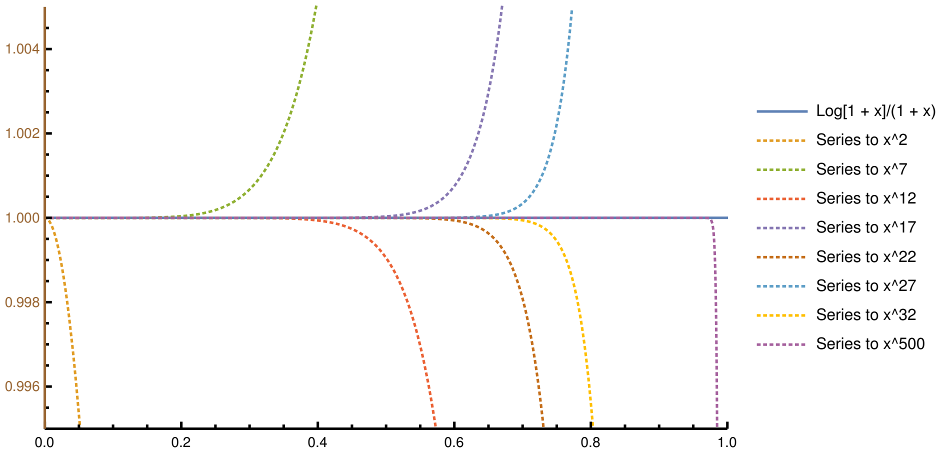
Padé Approximants: $\log(1+x)/(1+x)$

Start with Taylor series to various orders:



Padé Approximants: $\log(1+x)/(1+x)$

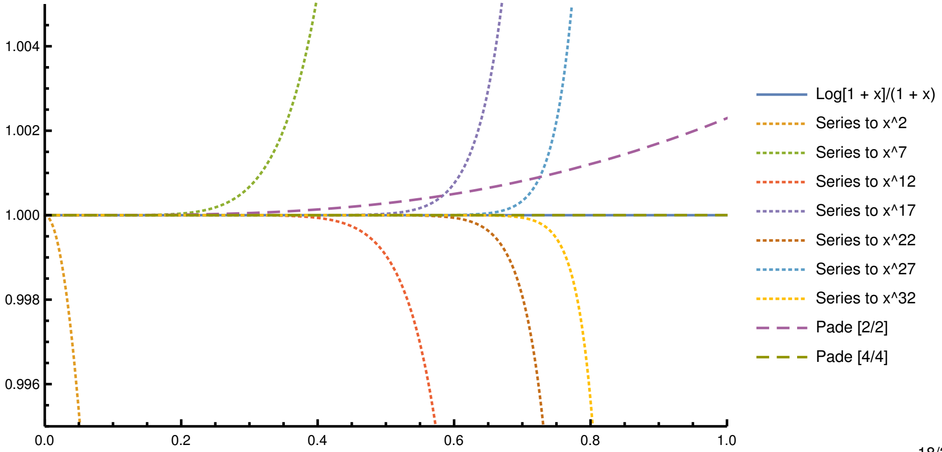
Ratio to exact function:



Padé Approximants: $\log(1+x)/(1+x)$

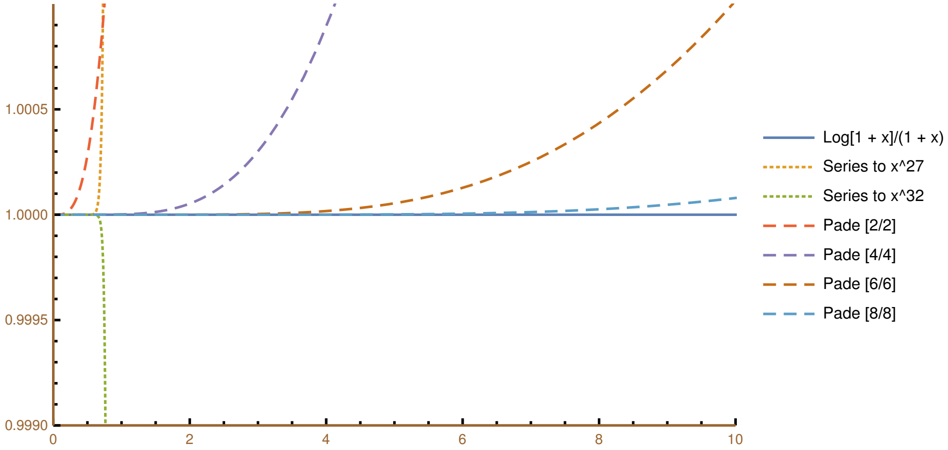
Add simple Padé approximants:

$$\text{Pade [2/2]} = \frac{x+\frac{1}{10}x^2}{1+\frac{8}{5}x+\frac{17}{30}x^2}, \quad \text{Pade [4/4]} = \frac{x+\frac{97}{94}x^2+\frac{230}{987}x^3+\frac{5}{1316}x^4}{1+\frac{119}{47}x+\frac{723}{329}x^2+\frac{244}{329}x^3+\frac{247}{3290}x^4}.$$



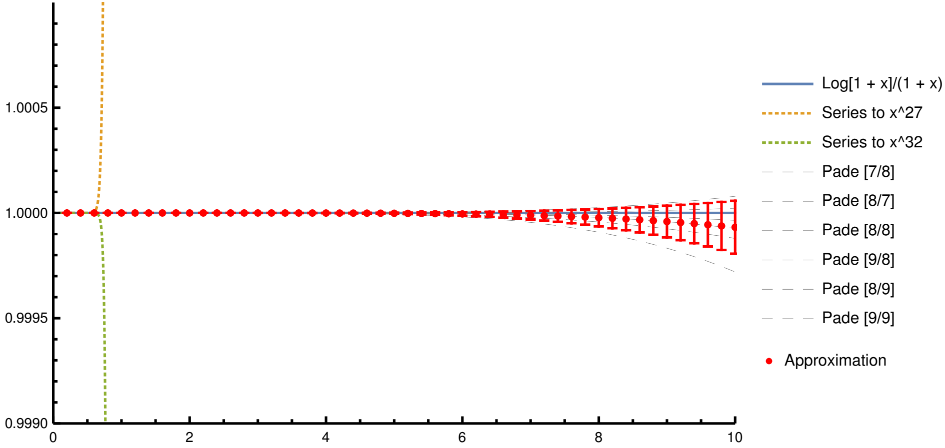
Padé Approximants: $\log(1+x)/(1+x)$

Higher-order Padé approximants approximate $f(x)$ well, far beyond radius of convergence of the series.



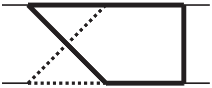
Padé Approximants: $\log(1+x)/(1+x)$

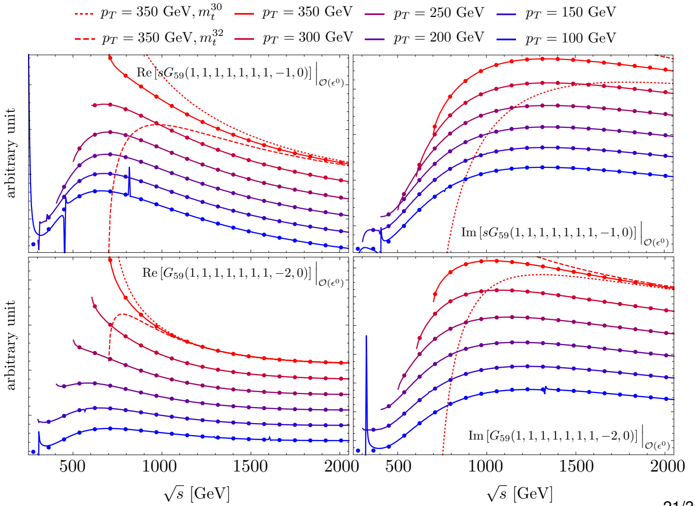
Use a set of Padé approximants to provide a central value and error estimate.



Padé Approximants: Master Integral Level

Dashed lines: m_t^{30}, m_t^{32}. Smaller p_T values: series doesn't converge.
Solid lines: Padé approximant. Points: `pySecDec`.





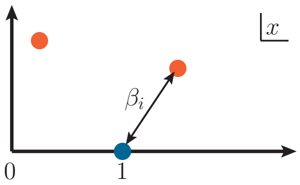
Padé Approximants: applied to m_t expansion

In small-m_t expansions:

- Replace $m_t^{2k} \to m_t^{2k} x^k$ and $m_t^{2k-1} \to m_t^{2k-1} x^k$,
- Set variables (m_t, s, t, etc) to numerical values,
- Padé approximants at $x = 0$, then evaluate at $x = 1$. $[7/8], [8/7], [7/9], [8/8], [9/7]$

Central value and error:

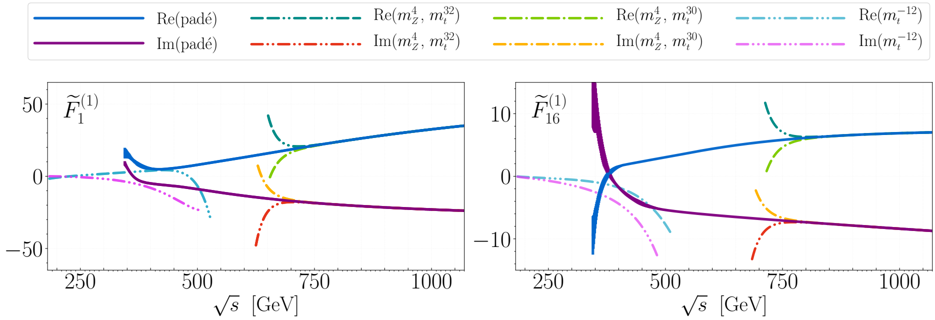
- Padé approximants have poles in the complex x plane, which can lead to poor behaviour if close to $x = 1$.
- Compute a weighted average and deviation, with $\omega_i = \beta_i^2 / \sum_j \beta_j^2$. β_i is the distance from $x = 1$ to the nearest pole.



Form Factor Approximations

Two example Form Factors for $gg \to ZZ$:

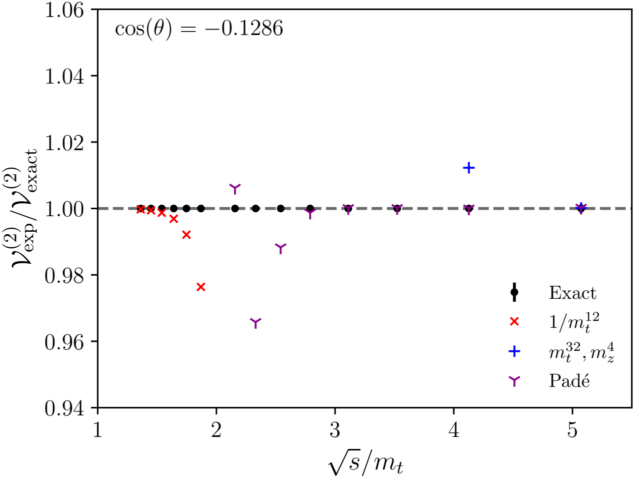
- the high-energy exp. diverges around $\sqrt{s} \approx 750$GeV as usual
- the Padé-based approximation continues to lower $\sqrt{s}$ values



[Davies,Mishima,Steinhauser,Wellmann '20]

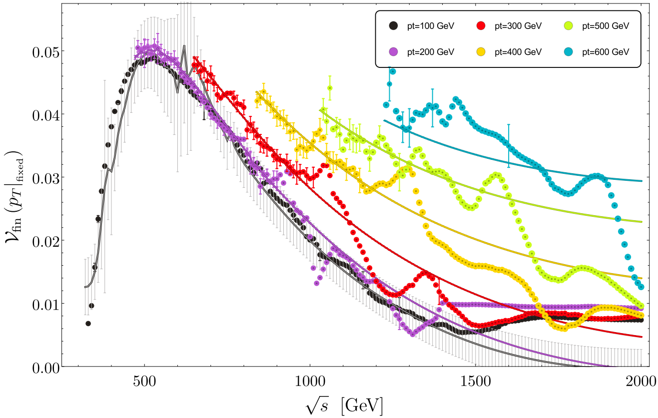
Comparison with $gg \to ZZ$ numerical V_{fin}

Padé-improved V_{fin} shows excellent agreement with `pySecDec`-based evaluation above $\sqrt{s} \approx 3m_t$. [Agarwal,Jones,von Manteuffel '20]



Return to $gg \to HH$ V_{fin}

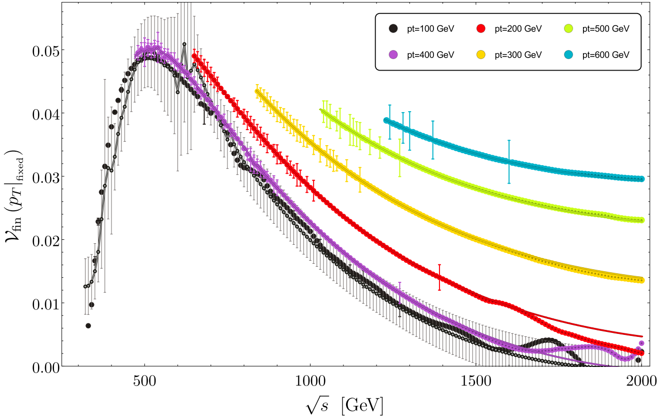
The Padé curves allow a much better comparison with numerical results.



Augment `hhgrid` with high-energy input points?

Return to $gg \to HH$ V_{fin}

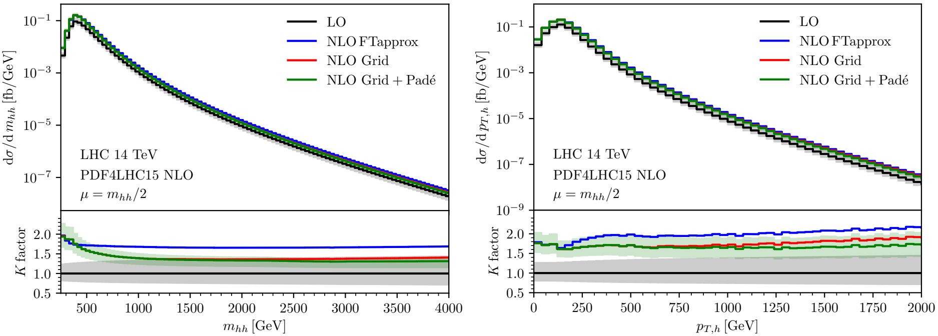
∼6300 points from `pySecDec`, ∼5100 high-energy Padé points



$gg \to HH$ Distributions

Real radiation added, convoluted with PDF:

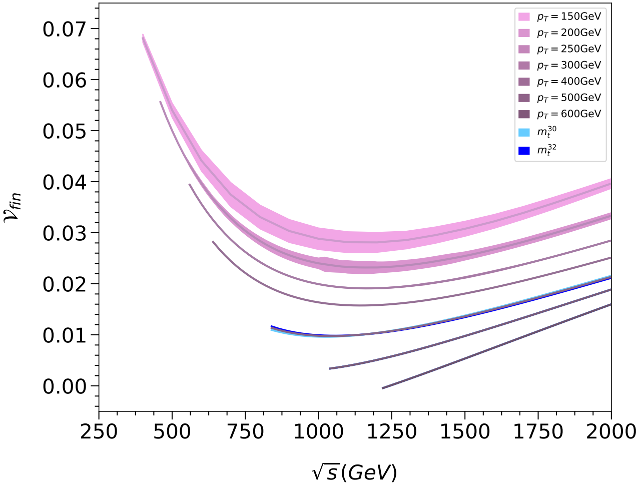
[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann '19]



$gg \to ZH$ V_{fin}

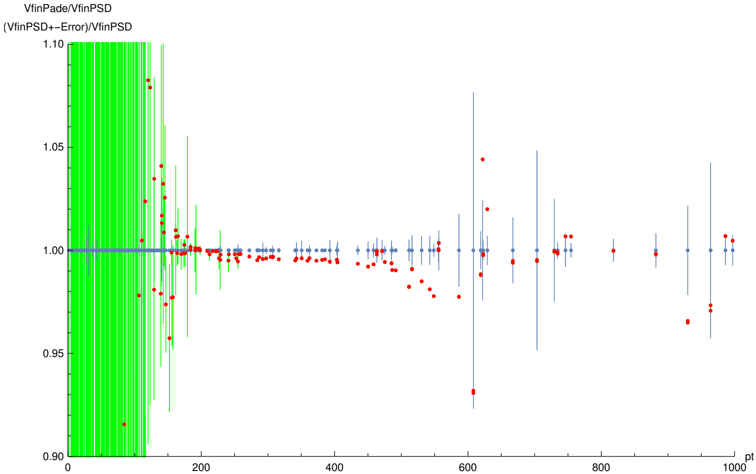
Similarly, we construct a V_{fin} for ZH. [Davies,Mishima,Steinhauser '20]

Blue curves show high-energy exp.; does not converge for $p_T <= 300$.



Comparison with $gg \to ZH$ numerical V_{fin}

In progress: $gg \to ZH$ comparison. Blue: `pySecDec`, Red+Green: Padé



Data points: [Chen,Heinrich,Jones,Kerner,Klappert,Schlenk]

Conclusions

Two-loop $2 \to 2$ amplitudes which depend on extra mass scales are not known analytically.

Nonetheless we can learn about their behaviour through expansions in various limits, and direct numerical evaluations.

High-energy expansions give the behaviour of the amplitudes in a region which is difficult to describe precisely by numerical evaluation.

- Padé approximants significantly improve the high-energy description

Backup: V_{fin}

For $gg \to HH$,

$$V_{fin} = \frac{\alpha_s^2(\mu)}{16\pi^2} \frac{G_F^2 s^2}{64} \left[C + 2\left(F_1^{(0)*} F_1^{(1)} + F_2^{(0)*} F_2^{(1)} + c.c. \right) \right]$$

with

$$C = \left[\left| F_1^{(0)} \right|^2 + \left| F_2^{(0)} \right|^2 \right] \left(C_A \pi^2 - C_A \log^2 \frac{\mu^2}{s} \right).$$

We use: $F_i = \underbrace{F_i^{(0)}}_{exact} + \underbrace{\alpha_s F_i^{(1)}}_{expanded}$ to construct V_{fin}.