NLO mixed QCD-electroweak corrections to Higgs boson production at the LHC

Marco Bonetti

Seminars of the Fermilab Theory Group

In collaboration with
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Topics

1. Motivations & Overview
2. ggH: two & three loops
3. $ggHg$: two loops
4. $PP \to H + j$: hadronic cross section
5. Conclusions & Outlook

Particle physics after the Higgs boson

2012 direct detection of the Higgs boson

- SM complete: all particles observed, all free parameters fixed

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- Direct observation: On-shell production and subsequent decay
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  - Accurate experimental results
  - Small theoretical uncertainties

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Higgs boson: good candidate

- Yukawa coupling
- Only spin-0 elementary particle in the SM
- Key ingredient of EW symmetry breaking

Higgs boson at the LHC

[1602.00695] [1610.07922] [1802.00833]

Higgs production modes

ggH	VVH	WH	ZH	$t\bar{t}H$	**Total**
$44.1^{+11\%}_{-11\%}$	$3.78^{+2\%}_{-2\%}$	$1.37^{+2\%}_{-2\%}$	$0.88^{+5\%}_{-5\%}$	$0.51^{+9\%}_{-13\%}$	**50.6**

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Theoretical uncertainties

δ(scale)	δ(PDF/TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$	δ(PDF)	$\delta(\alpha_S)$
$^{+0.10}_{-1.15}$ pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	~~± 0.49 pb~~	± 0.90 pb	$^{+1.27}_{-1.25}$ pb
$^{+0.21\%}_{-2.37\%}$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	~~$\pm 1\%$~~	$\pm 1.86\%$	$^{+2.61\%}_{-2.58\%}$

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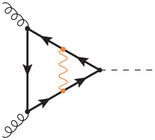
What is the form of QCD-EW contributions and of δ(EW)?

QCD-EW contributions

[ph0404071] [ph0407249] [ph0610033]

Exact LO Electroweak contributions

Yukawa coupling $\alpha_S \alpha \mathbf{Y_t}$



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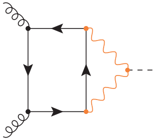

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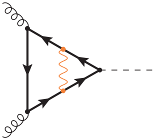

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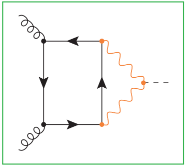

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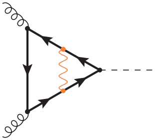

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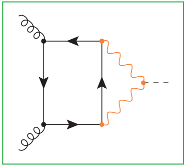

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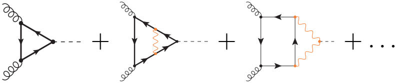
NLO QCD corrections in HEFT



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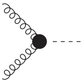

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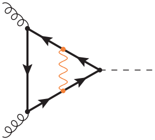

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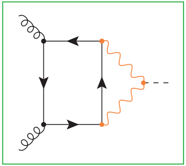

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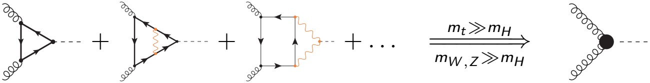

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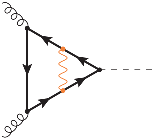

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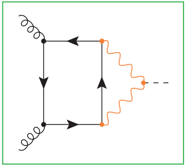

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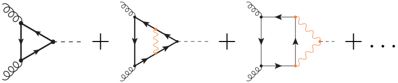

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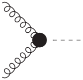

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Exact NLO computation required

QCD-EW contributions at the LHC

$$\sigma_{PP\to H+j}(\mu) = \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_{a/P}(x_1,\mu) f_{b/P}(x_2,\mu) \overline{\sigma}_{ab\to H+j}$$

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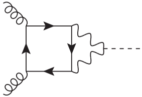
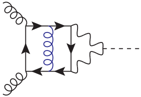
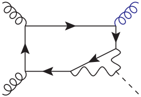
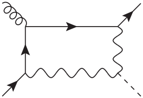
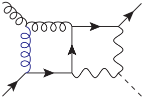
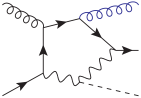
$\{g, g\}$	Primary			
$\{q, g\}$	Secondary			

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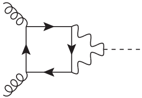
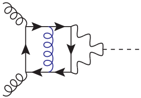
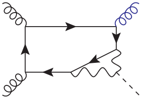
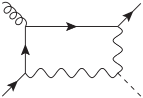
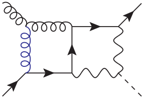
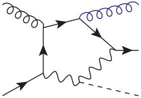
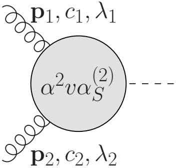
$\{g, g\}$	Primary			
$\{q, g\}$	Secondary			

- $\{q, \overline{q}\}$ suppressed by PDFs
- NNLO small (cfr. HEFT + QCD)

$gg \to H$: form factor decomposition



$$= \delta^{c_1 c_2} \epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2)\, \mathcal{F}\left(s, m_W^2, m_Z^2\right)$$

- LO: 3 two-loop diagrams
- vNLO: 47 three-loop diagrams

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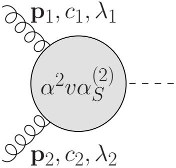

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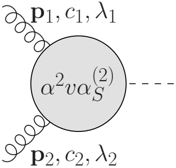

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- $C_W = 4$
  $\{u, d, c, s\}$
- $C_Z = \frac{2}{\cos^4\theta_w}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_w + \frac{22}{9}\sin^4\theta_w\right)$
  $\{u, d, s, c, b\}$

$$A(m^2/s, \mu^2/s) = A_{\text{LO}}(m^2/s) + \frac{\alpha_S(\mu)}{2\pi} A_{\text{vNLO}}(m^2/s, \mu^2/s) + \mathcal{O}(\alpha_S^2)$$

γ_5 vanishes summing over complete generations in quark loops

Master Integrals

[Chetyrkin. . . ,1981][Gehrmann. . . ,1999]

$A(m^2/s, \mu^2/s)$: sum of over 10 000 2 & 3-loop Feynman Integrals

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Integration-by-Parts Identities

$$\int \frac{\partial}{\partial k^\mu} \left(q^\mu \prod_{j=1}^{J} \frac{1}{\mathcal{D}_j^{a_j}} \right) \mathrm{d}^D k = 0, \qquad q^\mu = k^\mu, p^\mu$$

- Lorentz Invariance
- Symmetries
- Dim. Reg.

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System of linear relations among FIs

Master Integrals

Basis of loop integrals for the amplitude

- 2-loop: 12 MIs
- 3-loop: 95 MIs

Evaluation of Master Integrals

1. Change of variables
   - Only one dimensionful variable
   - Rationalization of square roots

$$y := \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1} \qquad s$$

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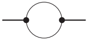
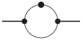
3. Evaluation of $\mathbf{J}(y, \epsilon)$ using Differential Equations

Differential Equations [Kotikov,1991][Remiddi,1997][Gehrmann...,1999]

1. Differentiate the MIs w.r.t. masses or scalar kinematic invariants

$$\frac{\partial}{\partial\,(m^2)}$$  $= -2$ 

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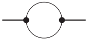
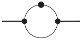
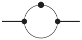
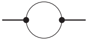
$$\frac{\partial}{\partial\left(m^2\right)}$$  $$= -2$$ 

2. Apply IBPs to recover the MIs (MI for subtopologies may arise)

 $$= -\frac{1-2\epsilon}{4m^2+s}$$  $$- \frac{1-\epsilon}{m^2\left(4m^2+s\right)}$$ 

Differential Equations

[Kotikov,1991][Remiddi,1997][Gehrmann. . . ,1999]

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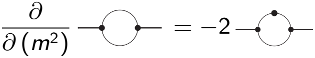

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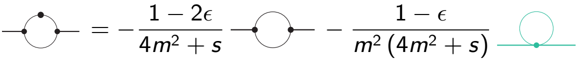
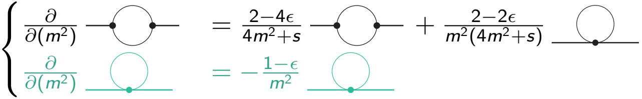


3. Include in cascade DEs for all the MIs in the subgraphs



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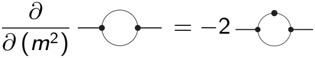

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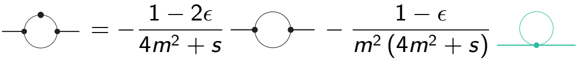

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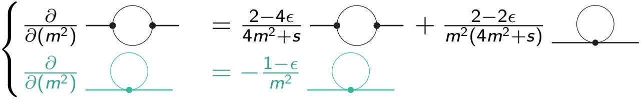
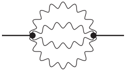


Closed system of linear Partial Differential Equations

$$\frac{\partial \mathbf{J}(y,\epsilon)}{\partial y} = A(y,\epsilon)\mathbf{J}(y,\epsilon)$$

A simple form for FIs

[Henn,2013][Argeri...,2014]



$$= \epsilon^{-1} \frac{s^2}{36} + \epsilon^0 s^2 \frac{71 - 18\log(-s)}{216} +$$
$$+ \epsilon^1 s^2 \left[\frac{\log^2(-s)}{8} - \frac{71\log(-s) + \pi^2}{72} + \frac{3115}{1296} \right] +$$
$$+ \epsilon^2 s^2 \left[-\frac{\log^3(-s)}{8} + \frac{71\log^2(-s)}{48} + \frac{\pi^2\log(-s)}{24} + \right.$$
$$\left. - \frac{3115\log(-s)}{432} - \frac{7\zeta(3)}{9} - \frac{71\pi^2}{432} + \frac{109403}{7776} \right] +$$
$$+ O\left(\epsilon^3\right)$$

A simple form for FIs

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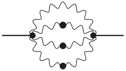
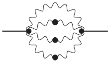
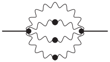
$\epsilon^3(-s)$ 

$$= \epsilon^0 \quad 1 + \epsilon^1 \quad [-3\log(-s)] + \epsilon^2 \quad \frac{9\log^2(-s) - \pi^2}{2} + \epsilon^3 \quad \left[\frac{-9\log^3(-s) - 3\pi^2\log(-s)}{2} - 28\zeta(3)\right] + \quad O\left(\epsilon^4\right)$$

- Constants from logs: $\pi \rightsquigarrow \log(-1), \quad \zeta(2k) \rightsquigarrow \pi^{2k} \rightsquigarrow \log^{2k}(-1)$
- ϵ^n coefficients are related to $\log^n x \rightsquigarrow \int \frac{1}{\xi_1} \cdots \int \frac{1}{\xi_n} \mathrm{d}\xi_n \ldots \mathrm{d}\xi_1$
- $\frac{\partial}{\partial s}\Big[\epsilon^3(-s)$  $\Big] = \frac{-3\epsilon}{s}\Big[\epsilon^3(-s)$  $\Big]$

Uniformly Transcendental functions [Henn,2013][Di Vita. . . ,2014]

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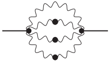
$\epsilon^3(-s)$  is a Uniformly Transcendental function

Weight W

Number of nested integrations over $\mathrm{d}\log R(\xi)$, $R(\xi)$ rational functions

$$F_n(y) = \int_0^y \cdots \int_0^{\xi_n} \mathrm{d}\log R_n(\xi) \ldots \mathrm{d}\log R_1(\xi) \quad \Rightarrow \quad W(F_n) := n$$

- Weight w functions in rational points give weight w constants
  $$W(\mathbb{Q}) = 0, \quad W(\pi) = 1, \quad W(\zeta(n)) = n$$
- $W(F_a\,F_b) = W(F_a) + W(F_b)$

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UT function

Function having a finite ϵ-expansion with weight n coefficients at order ϵ^n

The UT Cauchy problem

[Remiddi. . . ,1999][Henn,2013][Lee,2014]

A UT function $\mathbf{F}(y,\epsilon)$ satisfies

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Differential Equations

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathbf{F}(y,\epsilon) = \epsilon \sum_{a=1}^{A} B_a \frac{\mathrm{d}\log R_a(y)}{\mathrm{d}y}\mathbf{F}(y,\epsilon)$$

Canonical form $\quad$ ϵ-homogeneous

Fuchsian system $\quad$ only simple poles in y

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Boundary Conditions

Integration constants — solutions equated at $y \to y_0$ to boundary functions

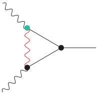
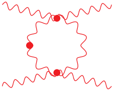
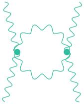
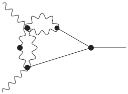
$$\lim_{y\to y_0}\left[\mathbf{F}(y,\epsilon) - \mathbf{L}(y,\epsilon)\right] = 0$$

y_0 rational point: $\mathbf{L}(y \to y_0,\epsilon)$ is also UT

A two-steps approach

[Argeri. . . ,2014][Gehrmann. . . ,2014]
[Lee,2014][Primo. . . ,2016][Gituliar. . . ,2017][Frellesvig. . . ,2017][Meyer,2017]

1. Study of $\mathbf{J}(y, \epsilon)$ $\qquad A(y,\epsilon) \rightsquigarrow A_0(y) + \epsilon A_1(y)\,[+\ldots]$
   - Building blocks

ϵ^2  $\otimes\ \epsilon$  $\otimes\ \epsilon\,(1-2\epsilon)$  $\Rightarrow\ \epsilon^4\,(1-2\epsilon)$ 

   - Maximal cut
     - All possible propagators are put on-shell
     - DEs: all terms not featuring cut propagators are put to 0
     - Requiring MIs with $\mathrm{d}\log$-form in all remaining integration variables

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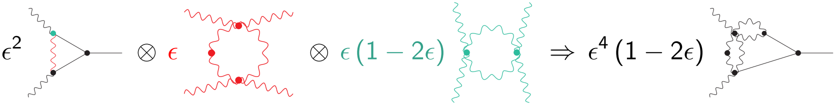

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   - Integrating away of $A_0(y)$

     Fuchsian structure can be spoiled: logs in $A_1(y)$
   - Algebraic techniques: `Fuchsia` & `CANONICA`

     BC can become non-UT: rescaling of lower UT MIs by ϵ-polynomials

Boundary Conditions

[Smirnov,2002]

Large-Mass Expansion

- MIs are functions of $p_1 \sim p_2 \sim \sqrt{s}$ and m

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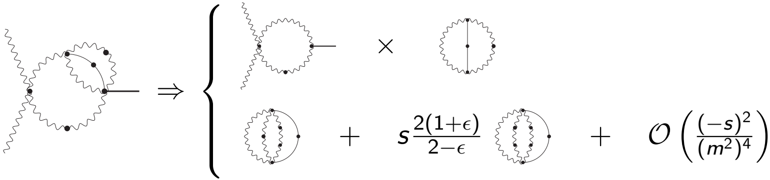
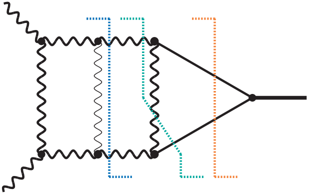


Form of the final result

Alphabet

$$\mathrm{d}\mathbf{F} = \epsilon \left[B_+ \mathrm{d}\log(1-y) + B_r \mathrm{d}\log(y^2 - y + 1) + B_- \mathrm{d}\log(y+1) + B_0 \mathrm{d}\log y \right] \mathbf{F}$$



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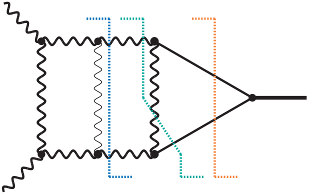

Constant terms

$\mathbb{Q}$-linear combinations of weighted constants

w	0	1	2	3	4	5	6
Values	1		π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

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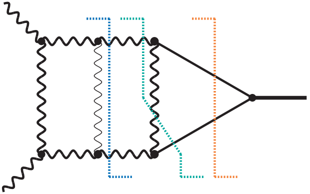

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- Weight drop: only up to $W = 5$ for the finite part of a 3-loop amplitude

Renormalized amplitude

[Catani,1998]

1. UV div. α_S renormalization only

$$\alpha_S^0 = \alpha_S(\mu^2)\left(\frac{\mu^2}{\mu_0^2}\right)^{\epsilon} S_\epsilon^{-1}\left[1+\alpha_S(\mu^2)\frac{\beta_0}{\epsilon}+O\left(\alpha_S^2(\mu^2)\right)\right]$$

2. IR div. described by Catani's formula, removed by real corrections

$$A_{\text{vNLO}} = \mathbf{I}_{gg}^{(1)} A_{\text{LO}} + A_{\text{vNLO}}^{\text{fin}}$$

$$\mathbf{I}_g^{(1)} = \left(-\frac{s}{\mu^2}\right)^{-\epsilon}\frac{\mathrm{e}^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left[-\frac{C_A}{\epsilon^2}-\frac{\beta_0}{\epsilon}\right]$$

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$$\begin{aligned}
A_{\mathrm{LO}}(m_Z^2/m_H^2,1) &= -6.880846 \;\; -\mathrm{i}\,0.5784119\\
A_{\mathrm{LO}}(m_W^2/m_H^2,1) &= -10.71693 \;\; -\mathrm{i}\,2.302953\\
A_{\mathrm{vNLO}}^{\mathrm{fin}}(m_Z^2/m_H^2,1) &= -2.975801 \;\; -\mathrm{i}\,41.19509\\
A_{\mathrm{vNLO}}^{\mathrm{fin}}(m_W^2/m_H^2,1) &= -11.31557 \;\; -\mathrm{i}\,54.02989
\end{aligned}$$

$s=\mu=m_H=125.09\,\mathrm{GeV},\ m_W=80.385\,\mathrm{GeV},\ m_Z=91.1876\,\mathrm{GeV},\ N_C=3,\ N_f=5$

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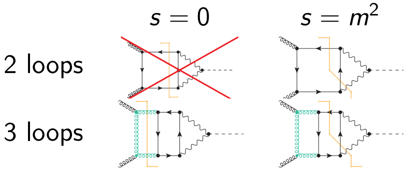
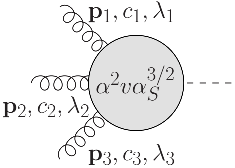
Cut analysis



$\Rightarrow$ $Hq\bar{q}=0$ for $m_q=0$

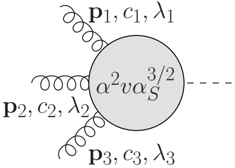
$gg \to Hg$: form factor decomposition



$$= f^{c_1 c_2 c_3} \epsilon^{\mu}_{\lambda_1}(\mathbf{p}_1)\ \epsilon^{\nu}_{\lambda_2}(\mathbf{p}_2)\ \epsilon^{\rho}_{\lambda_3}(\mathbf{p}_3) \times \left[\mathcal{F}_1 g_{\mu\nu} p_{2\rho} + \mathcal{F}_2 g_{\mu\rho} p_{2\nu} + \mathcal{F}_3 g_{\nu\rho} p_{2\mu} + \mathcal{F}_4 p_{3\mu} p_{1\nu} p_{2\rho}\right]$$

- rNLO: 21 two-loop diagrams

$gg \to Hg$: form factor decomposition



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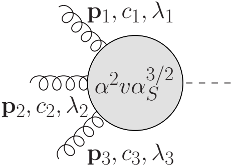
- rNLO: 21 two-loop diagrams

$$\mathcal{F}_j\left(s, t, m_H^2, m_W^2, m_Z^2\right) = -\frac{(\alpha \alpha_S)^{3/2}\, m_W}{16\pi \sin^3 \theta_w} \sum_{V=W,Z} C_V\, A_j(s, t, m_H^2, m_V^2)$$

$gg \to Hg$: form factor decomposition



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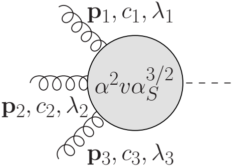
- Reduction to 61 MIs modulus permutations

$gg \to Hg$: form factor decomposition



$$= f^{c_1 c_2 c_3} \epsilon^{\mu}_{\lambda_1}(\mathbf{p}_1)\ \epsilon^{\nu}_{\lambda_2}(\mathbf{p}_2)\ \epsilon^{\rho}_{\lambda_3}(\mathbf{p}_3) \times \left[\mathcal{F}_1 g_{\mu\nu} p_{2\rho} + \mathcal{F}_2 g_{\mu\rho} p_{2\nu} + \mathcal{F}_3 g_{\nu\rho} p_{2\mu} + \mathcal{F}_4 p_{3\mu} p_{1\nu} p_{2\rho}\right]$$

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$$\mathcal{F}_j\left(s, t, m_H^2, m_W^2, m_Z^2\right) = -\frac{(\alpha\alpha_S)^{3/2}\, m_W}{16\pi \sin^3\theta_w} \sum_{V=W,Z} C_V\, A_j(s, t, m_H^2, m_V^2)$$

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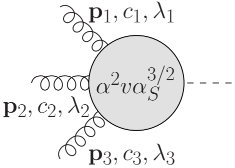

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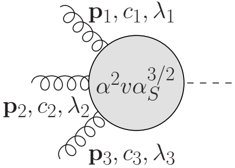

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DEs uneffective for *ggHg*

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Direct integration over Feynman parameters

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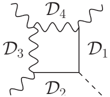

$$\propto \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_3 \int_0^1 \mathrm{d}x_4 \, \frac{\delta(1-X)}{\left[\sum x_j \mathcal{D}_j\right]^A}$$

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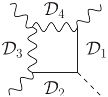

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Linear reducibility

There exists an integration order for the kernel f_0

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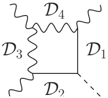

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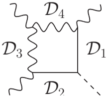

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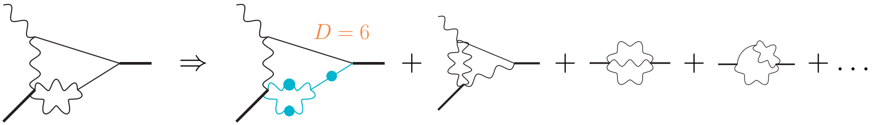

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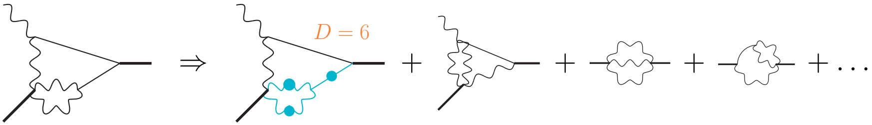

- MIs are shifted into finite integrals and divergent sub-graphs
- Good choices do not worsen the poles in the coefficients

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- GPL manipulation
  - $\mathcal{A}^{\text{rNLO}}_{+++} \ni \log, \text{Li}_2, \text{Li}_3$: fast, stable expressions
  - $\mathcal{A}^{\text{rNLO}}_{++-} \ni \log, \text{Li}_2, \text{Li}_3, G_4$ (to be done: $G_4 \to \text{Li}_4, \text{Li}_{2,2}$)

$PP \to H + j$: a first step

$$\sigma_{PP \to H+j} = \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 f_{a/P}(x_1, \mu) f_{b/P}(x_2, \mu) \overline{\sigma}_{ab \to H+j}$$

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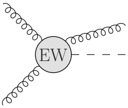
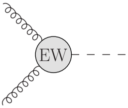
Soft limit

$\Big|$  $\Big|^2$

$$\xrightarrow{E_g\to 0} \underbrace{\frac{\alpha_S}{4\pi} C_A \frac{2\, p_1 \cdot p_2}{p_1 \cdot p_g \; p_2 \cdot p_g}}_{\text{Eikonal factor}}$$

$\Big|$  $\Big|^2$ $+ \, O\left(p_g^{-1}\right)$

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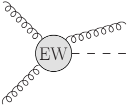
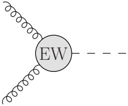
$\Bigg|$ 

$\Bigg|^2 \xRightarrow{E_g\to 0} \underbrace{\frac{\alpha_S}{4\pi} C_A \frac{2\, p_1\cdot p_2}{p_1\cdot p_g\; p_2\cdot p_g}}_{\text{Eikonal factor}}$ $\Bigg|$ 

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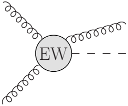

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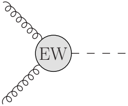

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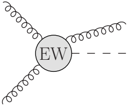

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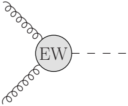

$$\sigma^{\text{QCD-EW}}_{\mathrm{LO}} = +5.3\,\%\ \sigma^{\text{QCD-EW}}_{\mathrm{LO}}$$

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$PP \to H + j$: full gluon channel [Becchetti. . . ,2020]

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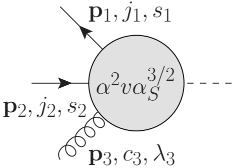
Best estimate

- Heavy-quark mass effects
- Top quark contributions
- Higher-order QCD corrections

$$\sigma_{gg\to H+X}^{(\text{EW,best})} = (7.11 \pm 0.6)\,\%\ \ \sigma_{gg\to H+X}^{(\text{HEFT},\alpha_S^2\alpha+\alpha_S^3\alpha)}$$

$qg \to Hq$ channel

[PRELIMINARY]



$$= T^{c_3}_{j_1 j_2} \bar{v}_{s_1}(\mathbf{p}_1) \left[\left(\not{p}_3 p_2^\mu - \gamma^\mu p_2 \cdot p_3 \right) (\mathcal{F}_1 + \gamma_5 \mathcal{F}_{51}) + \left(\not{p}_3 p_1^\mu - \gamma^\mu p_1 \cdot p_3 \right) (\mathcal{F}_2 + \gamma_5 \mathcal{F}_{52}) \right] u_{s_2}(\mathbf{p}_2) \epsilon^{\lambda_3}_\mu (\mathbf{p}_3)$$

- LO: 3 one-loop diagrams
- vNLO: 45 two-loop diagrams

$qg \to Hq$ channel

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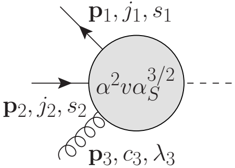

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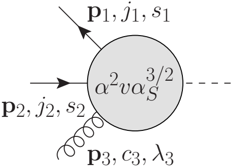
$qg \to Hq$ channel [PRELIMINARY]



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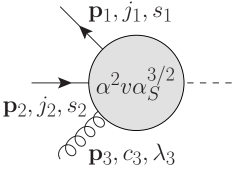
$qg \to Hq$ channel

[PRELIMINARY]



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- Huge expressions: work in progress!

Conclusions & Outlook

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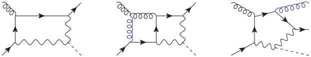
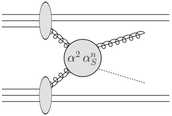
- Still work to do

The road ahead

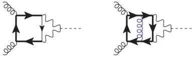
- Analytic computation of NLO $qg \to Hq(g)$



- Implementation of qg channel in the cross section



- Very long run: implementation of top quark effects



Thank you for your attention