On-shell amplitudes for Dark Matter

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(#2011.05339 with A. Falkowski and G. Isabella)

DESY.
מכון ויצמן למדע
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Amplitude for n-Gluon Scattering

Stephen J. Parke and T. R. Taylor

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

PACS numbers: 12.38.Bx

(362) Recursive Calculations for Processes with n Gluons - Berends, Frits A. *et al.* Nucl.Phys. B306 (1988) 759-808 Print-88-0100 (LEIDEN)
(358) Direct proof of tree-level recursion relation in Yang-Mills theory - Britto, Ruth *et al.* Phys.Rev.Lett. 94 (2005) 181602 hep-th/0501052
(350) Perturbative gauge theory as a string theory in twistor space - Witten, Edward Commun.Math.Phys. 252 (2004) 189-258 hep-th/0312171
(309) New recursion relations for tree amplitudes of gluons - Britto, Ruth *et al.* Nucl.Phys. B715 (2005) 499-522 hep-th/0412308
(284) Multiparton amplitudes in gauge theories - Mangano, Michelangelo L. *et al.* Phys.Rept. 200 (1991) 301-367 hep-th/0509223 FERMILAB-PUB-90-113-T
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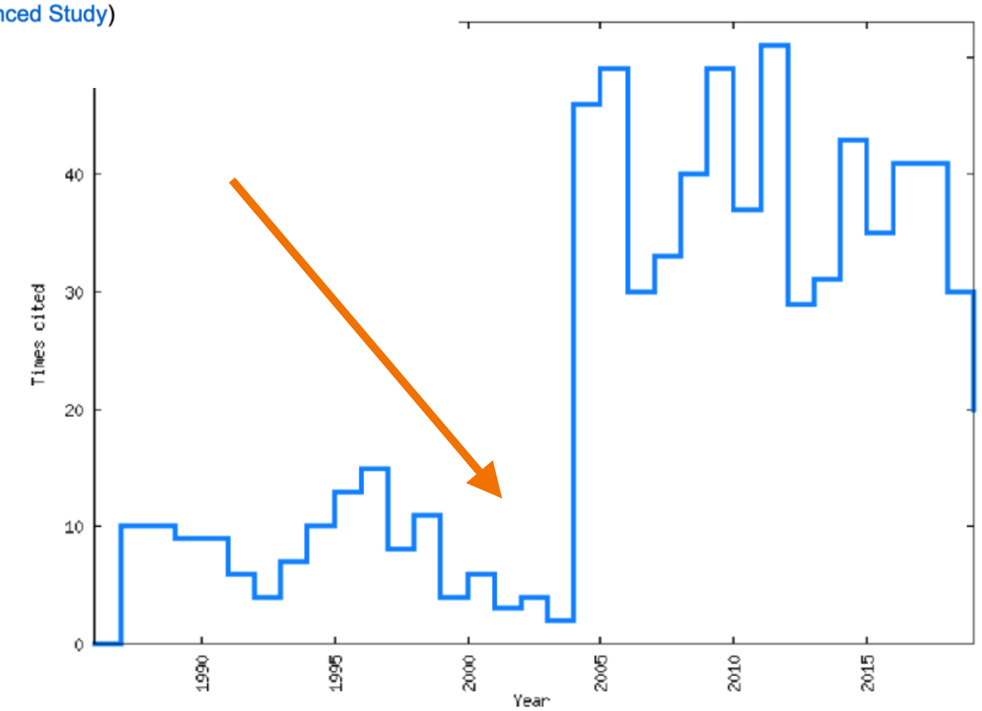
Perturbative gauge theory as a string theory in twistor space

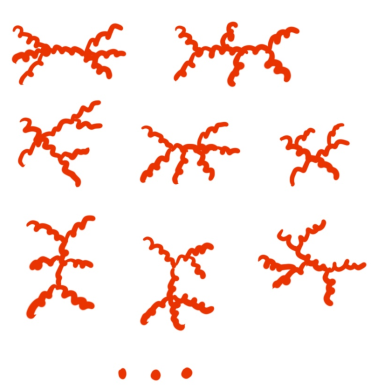
Edward Witten (Princeton, Inst. Advanced Study)

Dec 2003 - 97 pages



Amplitude program

Avoid Lagrangians



SM

(collider background)

Precision

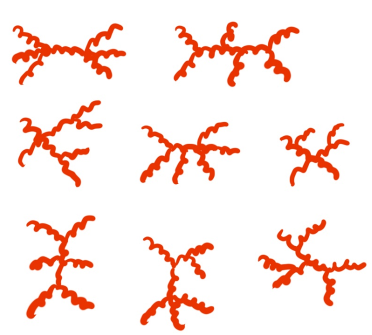
Many loops and legs

Insights

Hidden Mathematical structures

Amplitude program

Avoid Lagrangians



...

BSM

(collider predictions)

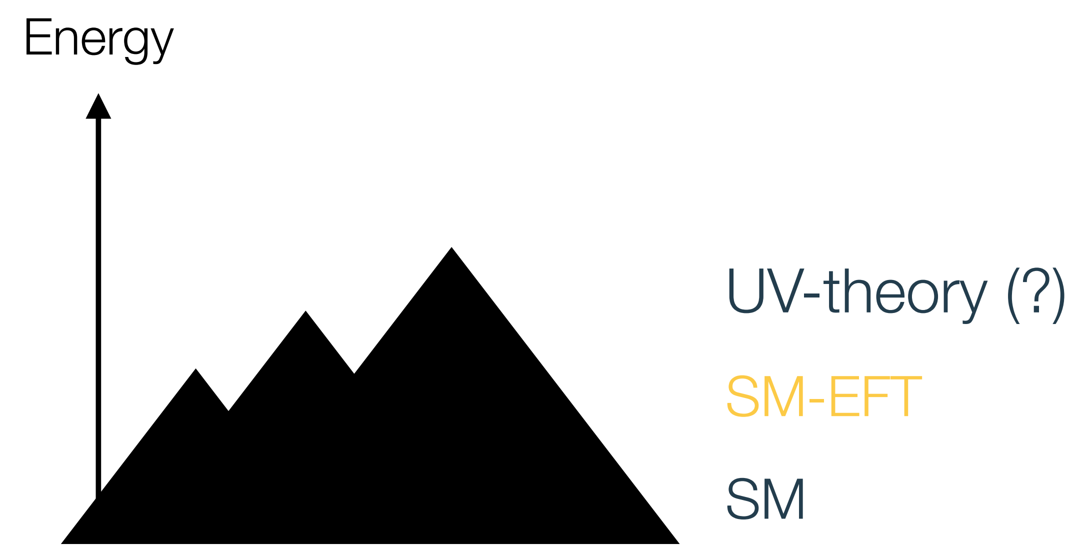
Precision

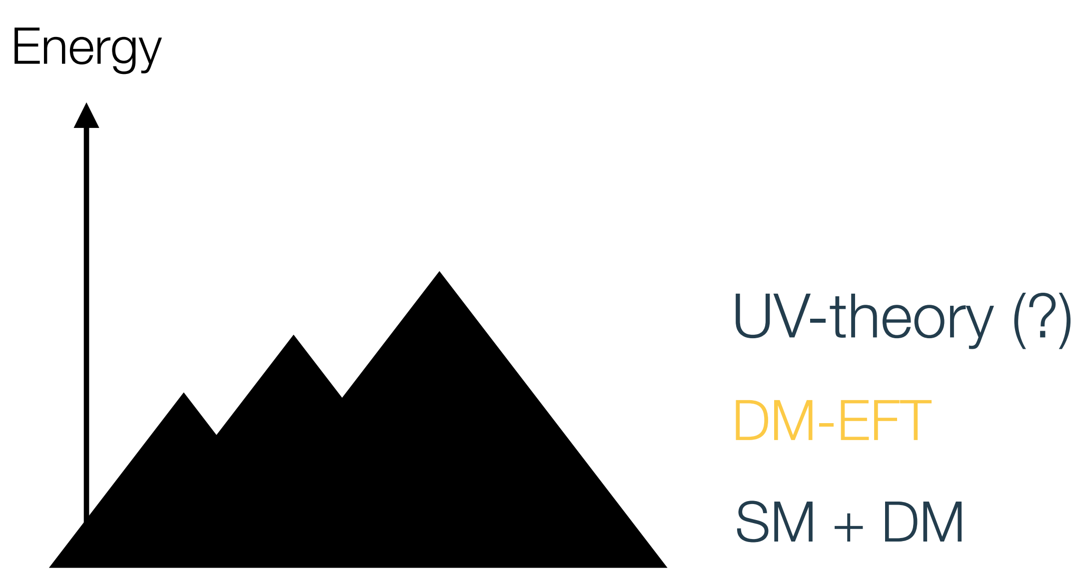
Many loops and legs

Insights

Hidden Mathematical structures

Effective Field Theories





Wilson coefficients

$$\mathscr{L}_{EFT} = \sum_{n,i} \frac{c_n^{(i)}}{\Lambda^n} \mathcal{O}_n^{(i)}$$

Validity of the EFT

Operators involving low energy fields

EFTs: the on-shell way

Wilson coefficients

$$\cancel{\mathscr{L}_{EFT} = \sum_{n,i} \frac{c_n^{(i)}}{\Lambda^n}\mathcal{O}_n^{(i)}}$$

Validity of the EFT

Operators involving
low energy fields

General ansatz

↓

Consistency conditions

↓

Physical amplitude

Scalar EFTs

$\phi^2(\partial\partial\phi)^2$

$(\partial\phi)^2(\partial\phi)^2$

$\phi(\partial_\mu\partial_\nu\phi)\partial^\mu\partial^\nu\phi$

$\ldots$

$\Box\phi = 0$

Equations of Motion

$\longrightarrow$

Integration by parts

Field redefinition

$\phi^2(\partial\partial\phi)^2$

$\ldots$

$\mathscr{M}(s,t,u)$

$$\xrightarrow[\sum_i p_i = 0]{p_i^2 = 0}$$

$s^2 + t^2 + u^2$

$\ldots$

Easier to build operator basis and to study the EFT landscape!

What can we do…

- Build operator basis efficiently (recent development also in the context of SMEFT)

  Elvang et al ’10 … Cheung et al ’16 … Low, Yin ’19 … Shadmi, Weiss ’18 Ma, Shu, Xiao ’19
  CSM, Durieux ’19 Durieux, Kitahara, CSM, Weiss ’19 Dong, Ma, Shu ’21 …

- Amplitudes with many external legs (recursion relations)

  Cohen, Elvang, Kiermaier ’10, Franken, Schwinn ’19, CSM, Falkowski ’20 …

- Explore EFT properties (helicity selection rules, soft theorems)

  Azatov, Contino, CSM, Riva ’16, Cheung , Shen ’15, Gu Lian-Tao Wang ’20 …

- Calculate loops efficiently (e.g. anomalous dimension matrix)

  Jiang, Shu, Xiao, Zheng ’21, Bern, Parra-Martinez, E-Sawyer ’20, Jiang, Ma, Shu ’20, Baratella, Fernandez, Pomarol ’20 …

- Computations with gravitons (double-copy, also applied for special EFTs)

  (massive graviton see Falkowski, Isabella ’20)

- Computations with higher-spin particles (avoid building a Lagrangian)

Moreover, development for massive particles boosted since Arkani-Hamed, Huang, Huang ‘17

Outline

1. Spinor basics

(and how to go from massive to massless spinors)

2. DM-EFT: Gravity mediated

3. DM-EFT: Higgs mediated

Conclusions

Spinor Basics

Why should we use spinors?

More transparent to see how the Lorentz symmetry can constrain the functional form of the amplitudes!

$$p_\mu \sigma^\mu_{\alpha\dot{\beta}} = \lambda_\alpha \tilde{\lambda}_{\dot{\beta}} \longrightarrow \quad \lambda \to w\lambda, \qquad \tilde{\lambda} \to w^{-1}\tilde{\lambda}.$$

$[\sigma^\mu]_{\alpha\dot{\beta}} = (1, \vec{\sigma})$

Helicity - Helicity +

Little group: transformations that leave the on-shell momentum invariant.

$$\mathcal{M}(\ldots; p_i, h_i; \ldots) \to w_i^{-2h_i} \mathcal{M}(\ldots; p_i, h_i; \ldots)$$

$$p_{\alpha\dot{\beta}} = \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix} \qquad \lambda^\alpha = \sqrt{2E} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{-i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

$$p_\mu = E(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \qquad \tilde{\lambda}^{\dot{\alpha}} = \sqrt{2E} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

Massless amplitudes

$$p_{\alpha\dot{\beta}} = \lambda_{p\,\alpha}\tilde{\lambda}_{p\,\dot{\beta}} \equiv |p\rangle_{\alpha}[p|_{\dot{\beta}}, \qquad p^{\dot{\alpha}\beta} = \tilde{\lambda}^{\dot{\alpha}}_{p}\lambda^{\beta}_{p} \equiv |p]^{\dot{\alpha}}\langle p|_{\beta}$$

$$\langle p\, q\rangle \equiv \lambda^{\alpha}_{p}\epsilon_{\alpha\beta}\lambda^{\beta}_{q} \qquad [p\, q] \equiv \tilde{\lambda}^{\dot{\alpha}}_{p}\epsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}}_{q}$$

Helicity - Helicity +

$$\langle 12\rangle[21] = 2p_1\cdot p_2 = s$$
$$\langle 13\rangle[31] = 2p_1\cdot p_3 = t$$
$$\langle 14\rangle[41] = 2p_1\cdot p_4 = u$$

$A_3 \sim [12]$ 1,2 helicity +1/2 ; 3 helicity 0

$A_4 \sim [12]^2\langle 34\rangle^2$ 1,2 helicity +1 ; 3,4 helicity -1

$A_4 \sim [12]^2\langle 23\rangle^2$ 1 helicity +1 ; 2 helicity 0 ; 3 helicity -1 ; 4 helicity 0

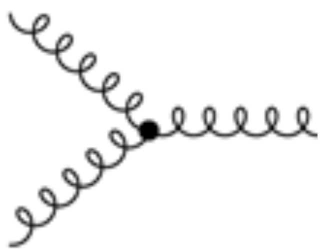
$$\mathcal{M}(\ldots; p_i, h_i; \ldots) \to w_i^{-2h_i}\mathcal{M}(\ldots; p_i, h_i; \ldots)$$

Massless amplitudes

little group + locality + 3pts kinematic

$$\mathcal{A}(1^{h_1}2^{h_2}3^{h_3}) = c\begin{cases}\langle 1\,2\rangle^{h_3-h_1-h_2}\langle 2\,3\rangle^{h_1-h_2-h_3}\langle 3\,1\rangle^{h_2-h_3-h_1} & \sum_i h_i \le 0\\ [1\,2]^{h_1+h_2-h_3}[2\,3]^{h_2+h_3-h_1}[3\,1]^{h_3+h_1-h_2} & \sum_i h_i \ge 0\end{cases}$$

Non-vanishing 3pts for complex momenta

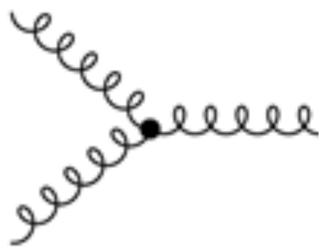


$$A_3[1\,2\,3] = -\sqrt{2}\Big[(\epsilon_1\epsilon_2)(\epsilon_3 p_1) + (\epsilon_2\epsilon_3)(\epsilon_1 p_2) + (\epsilon_3\epsilon_1)(\epsilon_2 p_3)\Big]$$

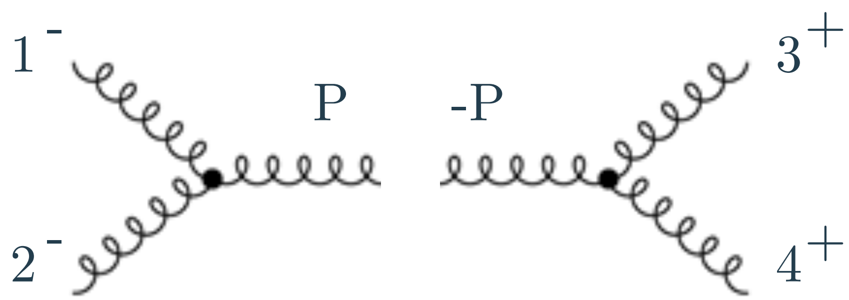
$$A_3[1^-2^-3^+] = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} \qquad A_3[1^+2^+3^-] = \frac{[12]^3}{[23][31]}$$

Inconsistent with Bose symmetry,
unless we multiply by an antisymmetric tensor!

Massless amplitudes



$$A_3[1_a^- 2_b^- 3_c^+] = f_{abc}\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}, \qquad A_3[1_a^+ 2_b^+ 3_c^-] = f_{abc}\frac{[12]^3}{[13][23]},$$



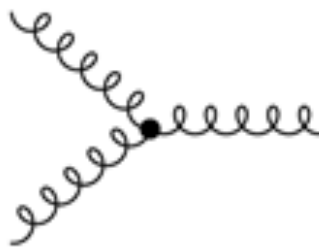
The amplitude with the correct t and u-residues is given by

$$A[1_a^- 2_b^- 3_c^+ 4_d^+] = \langle 12\rangle^2[34]^2\left(\frac{f_{ace}f_{bde}}{st} + \frac{f_{ade}f_{bce}}{su}\right)$$

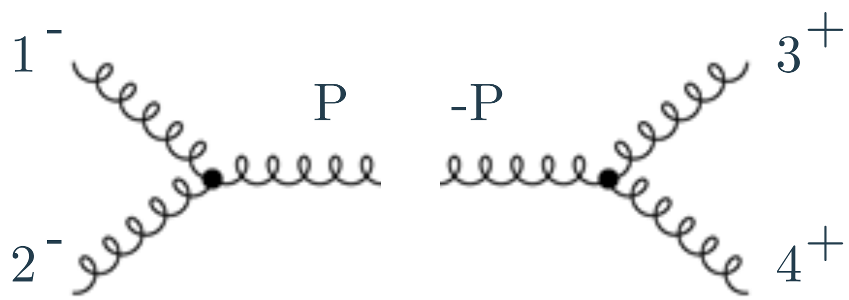
$$R_s = \lim_{s\to 0} sA[1_a^- 2_b^- 3_c^+ 4_d^+] = \langle 12\rangle^2[34]^2\left(\frac{f_{ace}f_{bde}}{t} + \frac{f_{ade}f_{bce}}{u}\right)$$

$$R_s = f_{abe}f_{cde}\frac{\langle 12\rangle^3}{\langle 1P\rangle\langle 2P\rangle}\frac{[34]^3}{[3P][4P]} = -f_{abe}f_{cde}\frac{\langle 12\rangle^2[34]^2}{u}$$

Massless amplitudes



$$A_3[1_a^- 2_b^- 3_c^+] = f_{abc}\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}, \quad A_3[1_a^+ 2_b^+ 3_c^-] = f_{abc}\frac{[12]^3}{[13][23]},$$



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$$A[1_a^- 2_b^- 3_c^+ 4_d^+] = \langle 12\rangle^2[34]^2\left(\frac{f_{ace}f_{bde}}{st} + \frac{f_{ade}f_{bce}}{su}\right)$$

Yang-Mills from unitarity+locality+Lorentz symmetry!

$$R_s = \lim_{s\to 0} sA[1_a^- 2_b^- 3_c^+ 4_d^+] = -f_{abe}f_{cde}\frac{\langle 12\rangle^2[34]^2}{u}$$

Only if $\boxed{f_{abe}f_{cde} - f_{ace}f_{bde} + f_{ade}f_{bce} = 0}$

Jacobi identity

Massive amplitudes

$\mathrm{LG} = \mathrm{U}(1)$, helicity

$$p_{\alpha\dot\beta} = \lambda_\alpha \tilde\lambda_{\dot\beta} \equiv |p\rangle_\alpha [p|_{\dot\beta}$$

$$\epsilon_+^\mu = \frac{\langle\zeta|\sigma^\mu|\lambda]}{\sqrt{2}\langle\lambda\zeta\rangle}, \qquad \epsilon_-^\mu = \frac{\langle\lambda|\sigma_\mu|\zeta]}{\sqrt{2}[\lambda\zeta]}$$

$\langle ij\rangle[ji] = s_{ij}$

$\mathrm{LG} = \mathrm{SU}(2)$, spins

$$\mathbf{p}_{\alpha\dot\beta} = \boldsymbol{\chi}_\alpha^I \epsilon_{IJ} \tilde{\boldsymbol{\chi}}_{\dot\beta}^J \equiv \epsilon_{IJ} |\mathbf{p}\rangle_\alpha^I [\mathbf{p}|_{\dot\beta}^J,$$

2s symmetrized spinors = spin-s

$$[\epsilon^\mu]^{JK} = \frac{1}{\sqrt{2}m} \langle \boldsymbol{\chi}^J|\sigma^\mu|\boldsymbol{\chi}^K]$$

bolding: $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \propto \langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + \langle 1^I 3^{J'}\rangle\langle 2^K 3^J\rangle$

[Arkani-Hamed, T.-C.Huang, Y.t-Huang ’17]

Massive amplitudes

$$\mathrm{LG} = \mathrm{U}(1),\ \text{helicity}$$

$$p_{\alpha\dot\beta} = \lambda_\alpha \tilde\lambda_{\dot\beta} \equiv |p\rangle_\alpha [p|_{\dot\beta}$$

$$\epsilon^\mu_+ = \frac{\langle\zeta|\sigma^\mu|\lambda]}{\sqrt{2}\langle\lambda\zeta\rangle}, \qquad \epsilon^\mu_- = \frac{\langle\lambda|\sigma_\mu|\zeta]}{\sqrt{2}[\lambda\zeta]}$$

$\langle ij\rangle[ji] = s_{ij}$

UV

IR

High-energy Limit

$$\mathrm{LG} = \mathrm{SU}(2),\ \text{spins}$$

$$\mathbf{p}_{\alpha\dot\beta} = \boldsymbol{\chi}^I_\alpha \epsilon_{IJ} \tilde{\boldsymbol{\chi}}^J_{\dot\beta} \equiv \epsilon_{IJ} |\mathbf{p}\rangle^I_\alpha [\mathbf{p}|^J_{\dot\beta},$$

$$\text{2s symmetrized spinors} = \text{spin-s}$$

$$[\epsilon^\mu]^{JK} = \frac{1}{\sqrt{2}m} \langle \boldsymbol{\chi}^J|\sigma^\mu|\boldsymbol{\chi}^K]$$

bolding: $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \propto \langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + \langle 1^I 3^{J'}\rangle\langle 2^K 3^J\rangle$

[Arkani-Hamed, T.-C.Huang, Y.t-Huang '17]

$$\mathbf{p}^\mu = (E, P s_\theta c_\phi, P s_\theta s_\phi, P c_\theta)$$

$$\boldsymbol{\chi}^I_\alpha = \begin{pmatrix} \sqrt{E-P}\,c & -\sqrt{E+P}\,s^* \\ \sqrt{E-P}\,s & \sqrt{E+P}\,c \end{pmatrix}$$

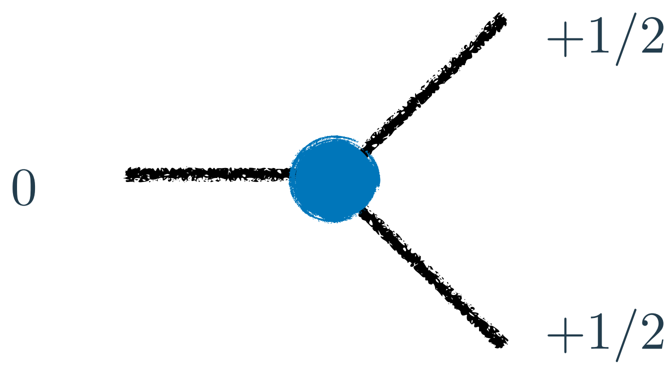
$$\tilde{\boldsymbol{\chi}}^J_{\dot\beta} = \begin{pmatrix} \sqrt{E-P}\,c & -\sqrt{E+P}\,s \\ \sqrt{E-P}\,s^* & \sqrt{E+P}\,c \end{pmatrix}$$

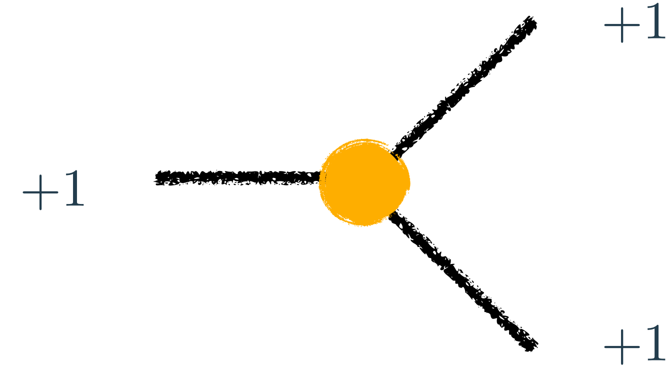
$$\xrightarrow{m/E \ll 1}$$

$$\langle\mathbf{12}\rangle^{\mathrm{IJ}} \approx \begin{pmatrix} \left(1 - \frac{m_1^2}{8E_1^2} - \frac{m_2^2}{8E_2^2}\right)\langle 12\rangle & -\frac{m_2}{\sqrt{2E_2}}\langle 1\zeta_2^+\rangle \\ -\frac{m_1}{\sqrt{2E_1}}\langle\zeta_1^+ 2\rangle & \frac{m_1 m_2}{4E_1E_2}[12] \end{pmatrix}$$

$$[\mathbf{12}]^{\mathrm{IJ}} \approx \begin{pmatrix} \frac{m_1 m_2}{4E_1E_2}\langle 12\rangle & -\frac{m_1}{\sqrt{2E_1}}[\zeta_1^+ 2] \\ -\frac{m_2}{\sqrt{2E_2}}[1\zeta_2^+] & \left(1 - \frac{m_1^2}{8E_1^2} - \frac{m_2^2}{8E_2^2}\right)[12] \end{pmatrix}$$

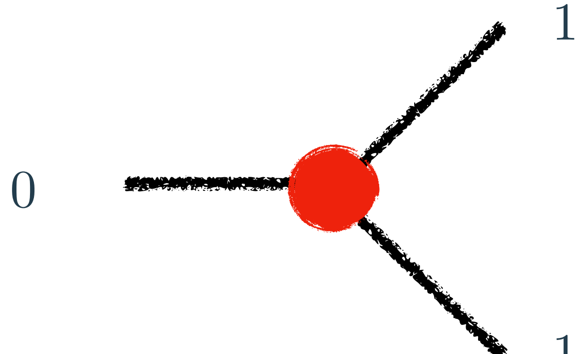
Massless/Massive amplitudes





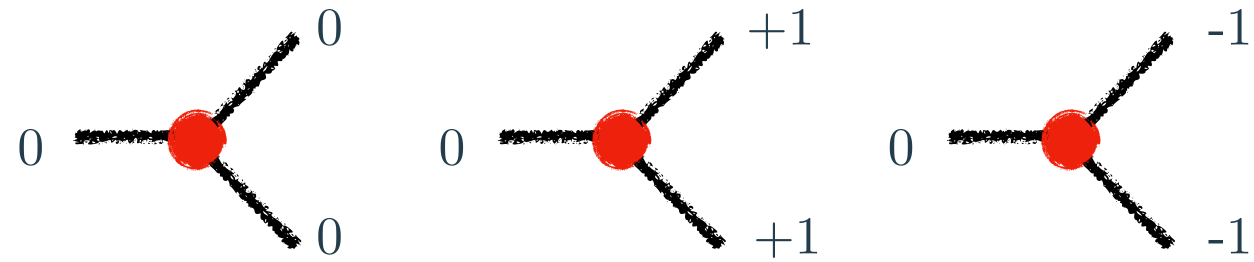
$$y\ [12]$$

$$c_{F^3}\ \frac{[12][23][31]}{\Lambda^2}$$



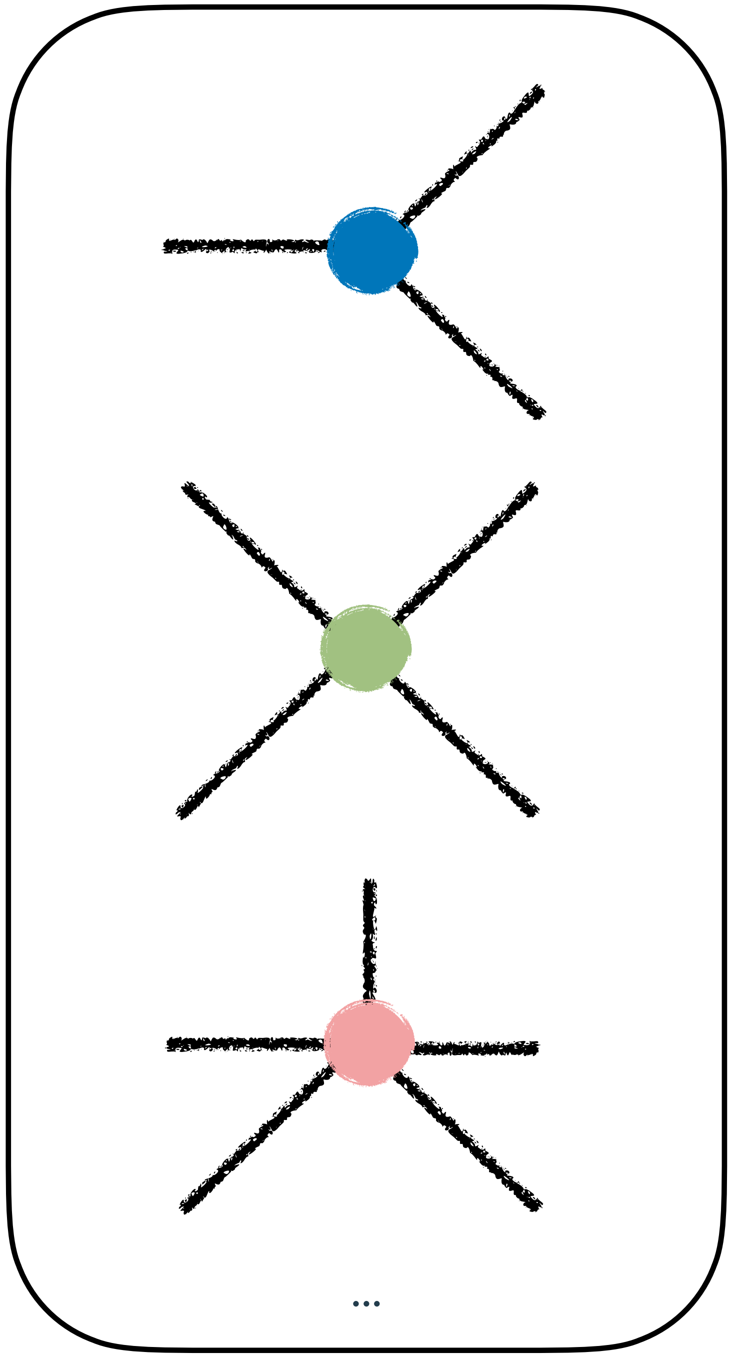
$$c_0\ \frac{\langle \mathbf{12}\rangle[\mathbf{12}]}{m} + c_+\ \frac{[\mathbf{12}]^2}{\Lambda} + c_-\ \frac{\langle \mathbf{12}\rangle^2}{\Lambda}$$

HE limit   "unbold"



$[A_n] = 4 - n$

EFT: the on-shell way



On-shell data

Locality, Unitarity, Lorentz

Particles Masses

(+ extra IR constrains,
e.g. soft-limit (Adler zero)
in the case of Goldstone bosons)

Outline

1. Spinor basics

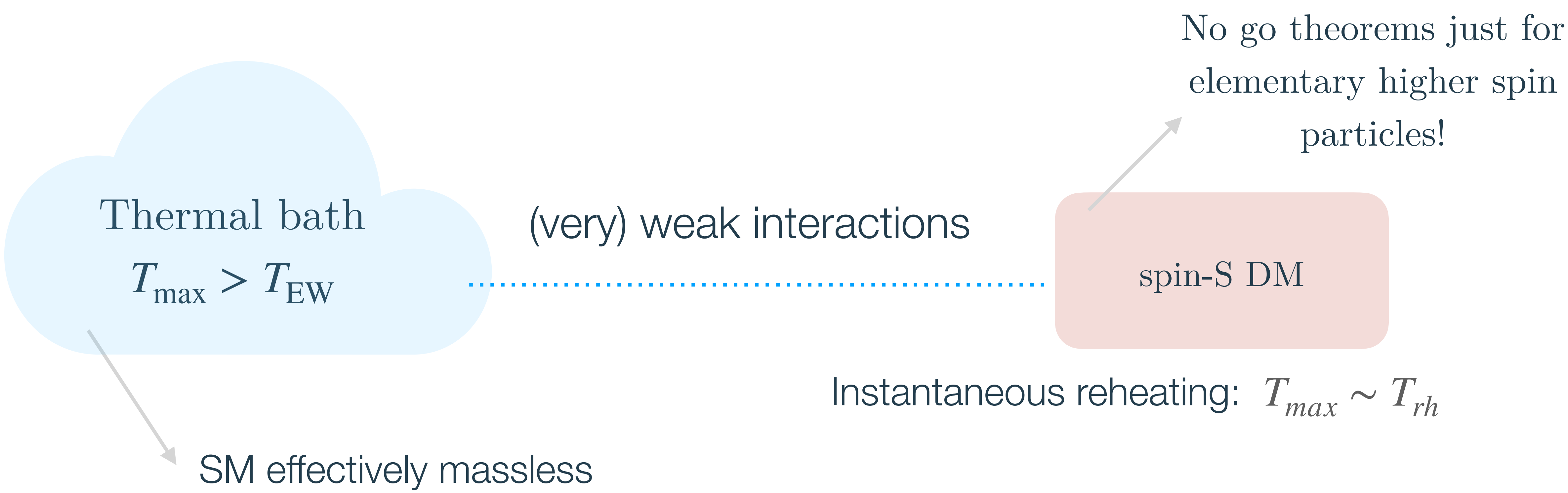
(and how to go from massive to massless spinors)

2. DM-EFT: Gravity mediated

3. DM-EFT: Higgs mediated

Conclusions

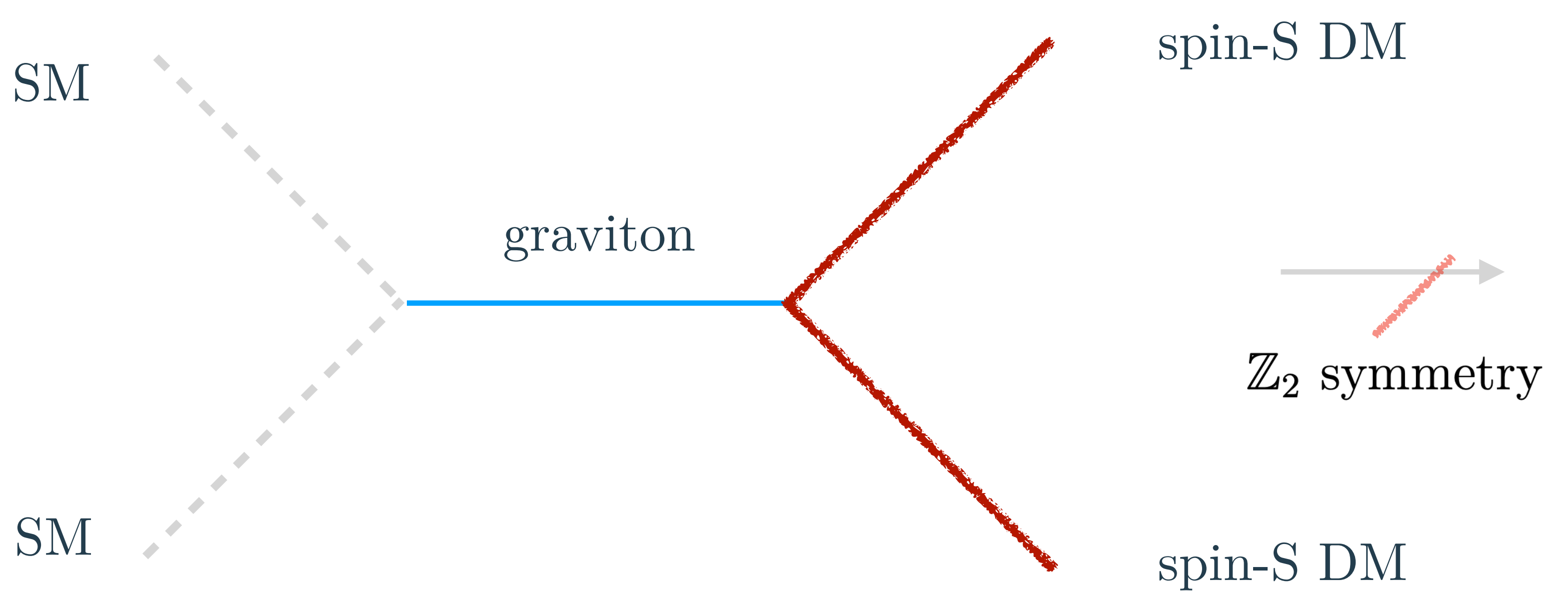
Dark Matter - EFT



Production via Freeze-in mechanism:

Particles in the thermal bath can annihilate into DM but the inverse process can be neglected due to feeble interactions and/or low number density of dark matter.

Gravity mediated



Lagrangian

↓

Feynman diagrams

↗

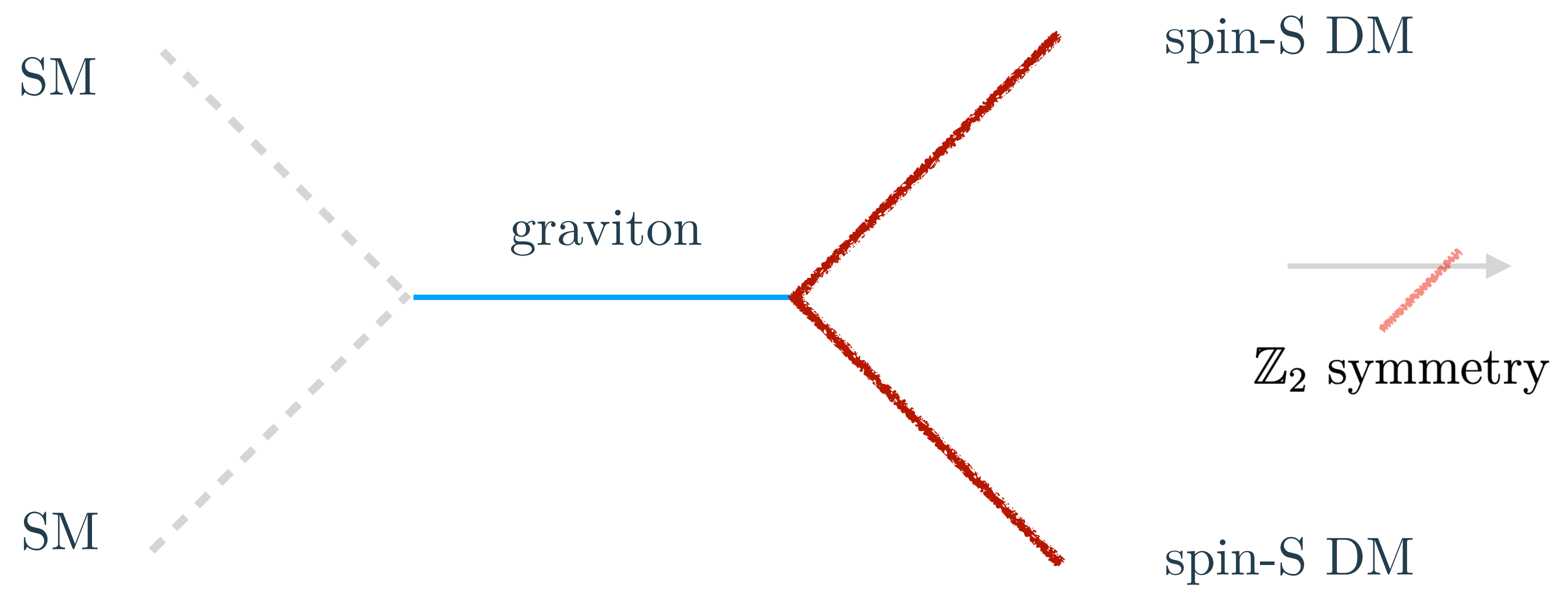
Scattering Amplitudes

↓

Observables

Difficult task for higher-spin particles!

Gravity mediated



CSM, Falkowski, Isabella, 2011.05339 [hep-ph]

Shortcut!

~~Lagrangian~~

↓

~~Feynman diagrams~~

↗

Scattering Amplitudes

↓

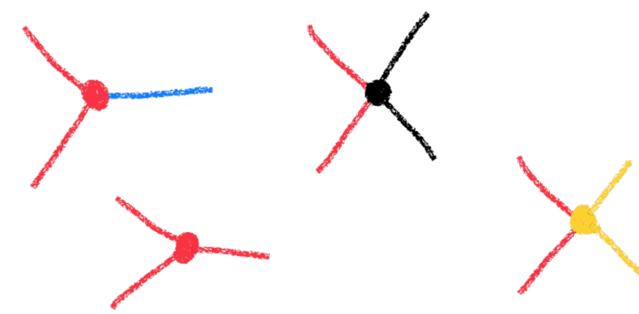
Observables

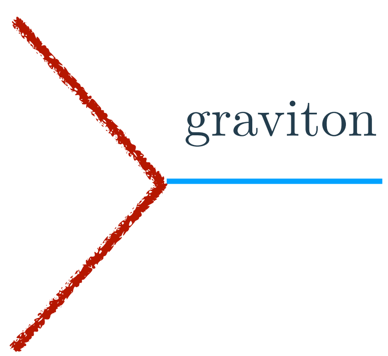
Gravity mediated

Basic rules:

Unitarity, Locality
Lorentz symmetry

3-point amplitudes and 4-point contact terms





(minimal coupling)

$$\mathcal{M}(\mathbf{1}_X \mathbf{2}_X 3_h^+) = -\frac{(A_{12}^+)^2}{M_{\rm Pl}} \frac{\langle \mathbf{21} \rangle^{2S}}{m^{2S}} \qquad \mathcal{M}(\mathbf{1}_X \mathbf{2}_X 3_h^-) = -\frac{(A_{12}^-)^2}{M_{\rm Pl}} \frac{[\mathbf{21}]^{2S}}{m^{2S}}$$

$$A_{12}^+ \equiv \langle \zeta p_1 3] / \langle 3 \zeta \rangle \qquad\qquad A_{12}^- \equiv [\zeta p_1 3\rangle / [3\zeta]$$

Gravity mediated

Basic rules:

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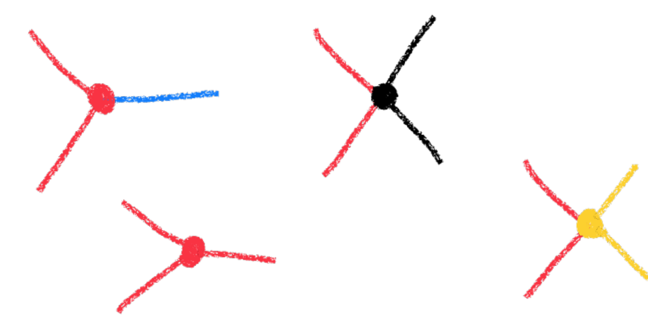
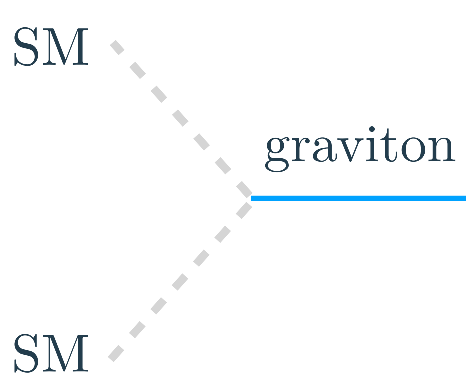




$$\text{Spin } 0: \quad \mathcal{M}(1_\phi 2_\phi 3_h^\pm) = -\frac{(A_{12}^\pm)^2}{M_{\text{Pl}}},$$

$$\text{Spin } 1/2: \quad \mathcal{M}(1_\psi^- 2_{\bar\psi}^+ 3_h^\pm) = -\frac{A_{12}^\pm B_{12}^\pm}{M_{\text{Pl}}},$$

$$\text{Spin } 1: \quad \mathcal{M}(1_v^- 2_v^+ 3_h^\pm) = -\frac{(B_{12}^\pm)^2}{M_{\text{Pl}}},$$

$$A_{12}^- \equiv [\zeta p_1 3\rangle / [3\zeta]$$

$$A_{12}^+ \equiv \langle \zeta p_1 3] / \langle 3\zeta\rangle$$

$$B_{12}^+ \equiv \langle 1\zeta\rangle[23] / \langle 3\zeta\rangle$$

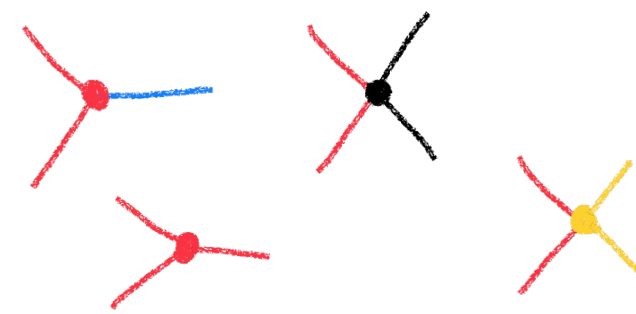
$$B_{12}^- \equiv \langle 13\rangle[2\zeta] / [3\zeta]$$

Gravity mediated

Basic rules:

Unitarity, Locality
Lorentz symmetry

3-point amplitudes and 4-point contact terms

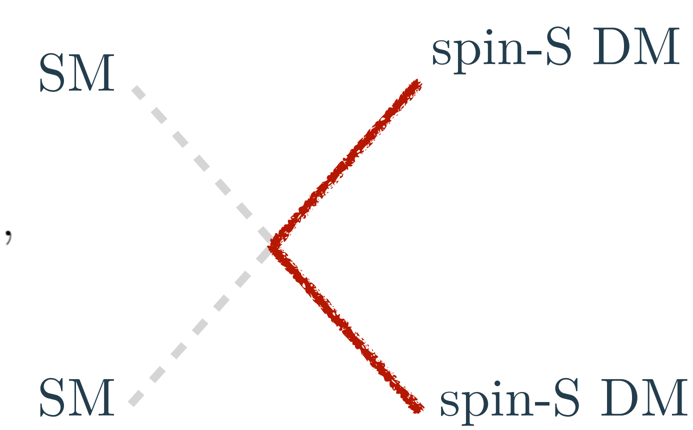


$$C_\phi = \frac{1}{M_{\rm Pl}^2 m^{2S-2}} \sum_{k=0}^{2S} \mathcal{C}_\phi^{(k+1)} [\mathbf{43}]^k \langle \mathbf{43} \rangle^{2S-k} + \cdots ,$$

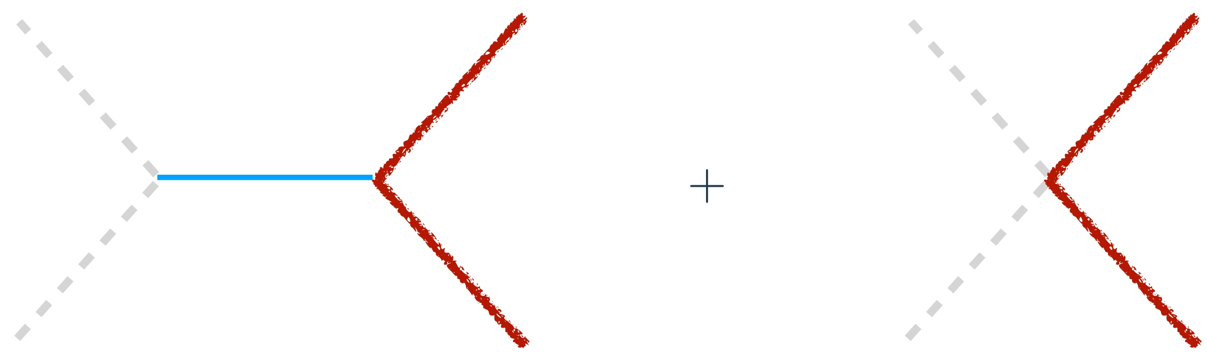
$$C_\psi = \frac{1}{M_{\rm Pl}^2 m^{2S-1}} (\langle \mathbf{3}1 \rangle [4\mathbf{2}] - \langle \mathbf{4}1 \rangle [\mathbf{3}2]) \sum_{k=0}^{2S-1} \mathcal{C}_\psi^{(k+1)} [\mathbf{43}]^k \langle \mathbf{43} \rangle^{2S-1-k} + \cdots ,$$

$$C_v = \frac{1}{M_{\rm Pl}^2 m^{2S}} \langle \mathbf{3}1 \rangle \langle \mathbf{4}1 \rangle [\mathbf{3}2][\mathbf{4}2] \sum_{k=0}^{2S-2} \mathcal{C}_v^{(k+1)} [\mathbf{43}]^k \langle \mathbf{43} \rangle^{2S-2-k} + \cdots .$$

(for opposite helicity fermions/vectors)



Gravity mediated



$$R_s = -\sum_{h'\pm} \mathcal{M}(1_f 2_f (-p_s)^{h'})\mathcal{M}((p_s)^{-h'}\mathbf{3}_X\mathbf{4}_X)|_{s\to 0},$$

$$C_i$$

$$\mathcal{M}(1_v^- 2_{\bar{v}}^+ \mathbf{3}_X \mathbf{4}_X) = \frac{1}{s\, M_{\rm Pl}^2 m^{2S-1}} \Big\{ \langle 1 p_3 2] \big(\langle \mathbf{31}\rangle [4\mathbf{2}] + \langle \mathbf{41}\rangle [\mathbf{32}]\big) \sum_{k=1}^{2S-2} [\mathbf{43}]^k \langle \mathbf{43}\rangle^{2S-1-k}$$

$$+ \quad m\big(\langle \mathbf{31}\rangle [\mathbf{42}] + \langle \mathbf{41}\rangle [\mathbf{32}]\big)^2 \sum_{k=0}^{2S-2} [\mathbf{43}]^k \langle \mathbf{43}\rangle^{2S-2-k} \Big\} + C_v,$$

$$\mathcal{M}_{\rm ann} \sim \frac{E^{2S+1}}{M_{\rm Pl}^2 m^{2S-1}}$$

$$C_v \sim \frac{E^{2S+2}}{M_{\rm Pl}^2 m^{2S}}$$

Phenomenology

Evolution equation

$$-THs\frac{dY_X}{dT} = R_X$$

DM density

$$Y_X = n_X/s$$

Freeze-in rate

$$R_X = \frac{T}{64\pi^4}\int_{s_0}^{\infty} ds\beta\sqrt{s}\,K_1\left(\frac{\sqrt{s}}{T}\right) I(s)$$

$$I(s) = \frac{g_\psi g_X}{16\pi}\int_{-1}^{1} d\cos\theta |\mathcal{M}_{\rm ann}|^2$$

$$\mathcal{M}_{\rm ann} \sim \frac{E^{2S+1}}{M_{\rm Pl}^2 m^{2S-1}}$$

$$C_v \sim \frac{E^{2S+2}}{M_{\rm Pl}^2 m^{2S}}$$

Phenomenology

(no contact term)

Evolution equation

$$-THs\frac{dY_X}{dT} = R_X$$

$$y_{\rm ref} = \rho_c \Omega_X / s_0 \approx 4.1 \cdot 10^{-10}\ \text{GeV}$$

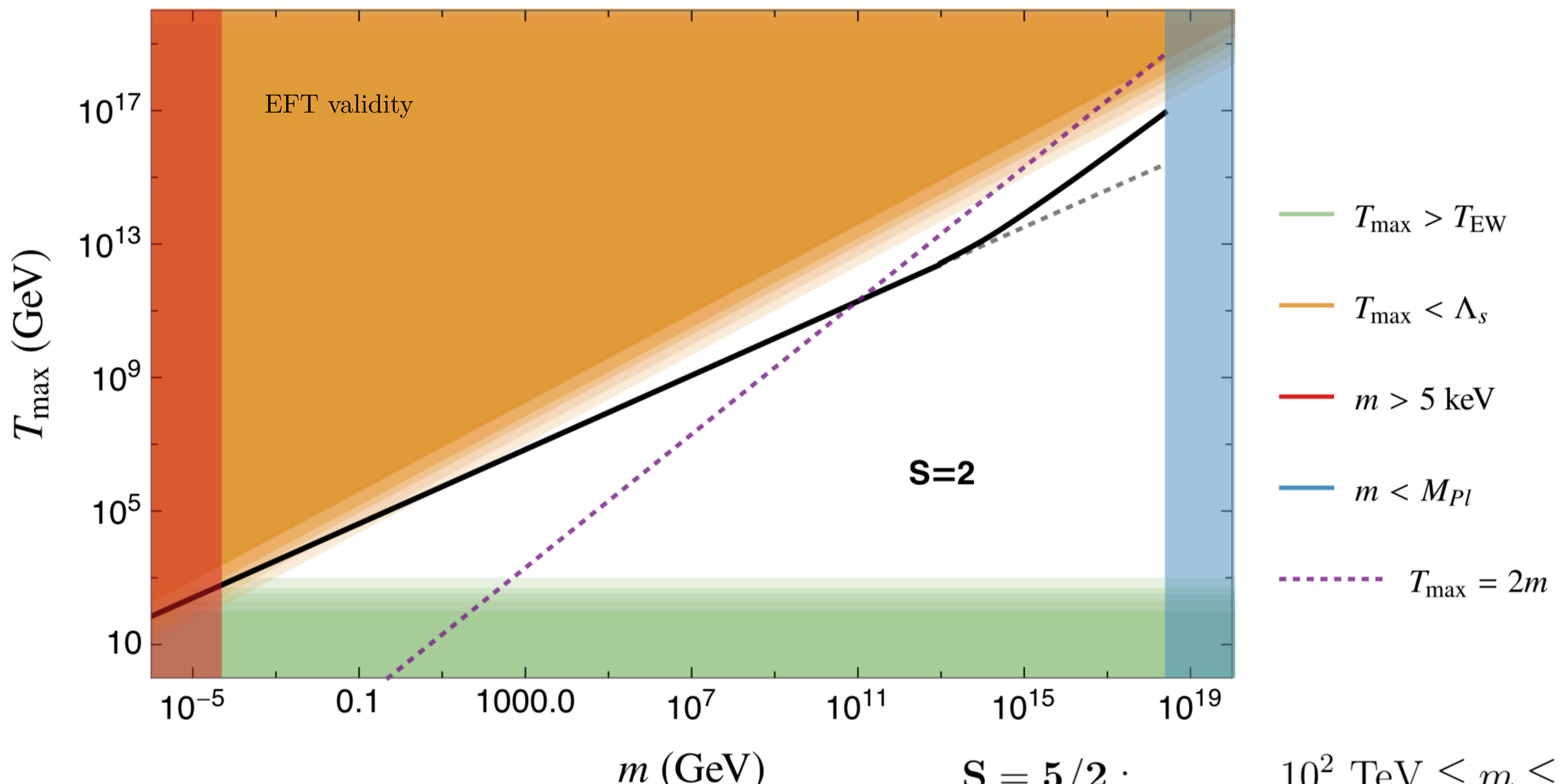
$$T_{\max} \gg m \quad \to \quad T_{\max} \sim \left(y_{\rm ref}\, M_{\rm Pl}^3 m^{4S-3}\right)^{\frac{1}{4S+1}}$$

$$T_{\max} \ll m \quad \to \quad T_{\max} \sim \frac{2m}{\log\left(\frac{m^4}{y_{\rm ref}\, M_{\rm Pl}^3}\right)}$$ (Boltzmann suppression)

(Maximal temperature of the universe required to match DM abundance)

Phenomenology

(no contact term)



$$\mathbf{S = 5/2}: \quad 10^2 \text{ TeV} \lesssim m \lesssim M_{\rm Pl},$$
$$\mathbf{S = 3}: \quad 10^4 \text{ TeV} \lesssim m \lesssim M_{\rm Pl}.$$

EFT cutoff determine by the process that first reach the strongly coupled regime: self-scattering, gravi-Compton, annihilation

Phenomenology

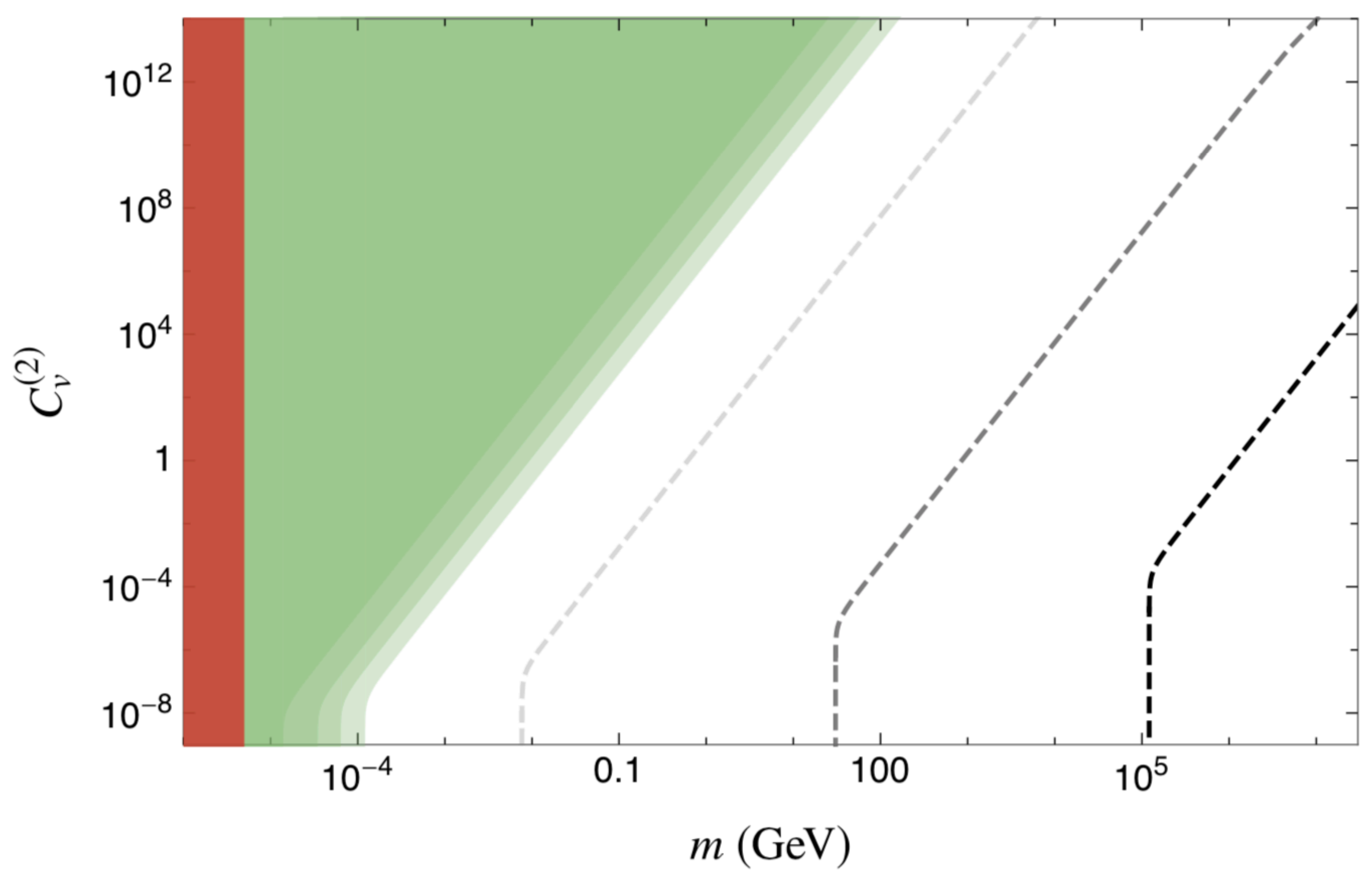
(with contact term)

$$T_{\rm max} \gg m \quad \rightarrow \quad T_{\rm max} \sim (y_{\rm ref}\, M_{\rm Pl}^3 m^{4S-1})^{\frac{1}{4S+3}}$$

(Lower than the previous scenario)

$$\frac{\mathcal{C}_v^{(2)}}{m^4\, M_{\rm Pl}^2}\langle \mathbf{31}\rangle\langle \mathbf{41}\rangle[\mathbf{3}2][\mathbf{4}2][\mathbf{43}]\langle \mathbf{43}\rangle.$$

$C_v \sim M_{\rm Pl}^2/\Lambda^2$



$$T_{\rm max} \lesssim \alpha \min\left(\Lambda_s, \Lambda_C\right)$$

$$\Lambda_C \sim (4\pi m^4\, M_{\rm Pl}^2/\mathcal{C}_v^{(2)})^{1/6}$$

$T_{\rm max} = 10^8$ GeV

$T_{\rm max} = 10^6$ GeV

$T_{\rm max} = 10^4$ GeV

$T_{\rm max} > T_{EW}$

$m > 5$ keV

Outline

1. **Spinor basics**

(and how to go from massive to massless spinors)

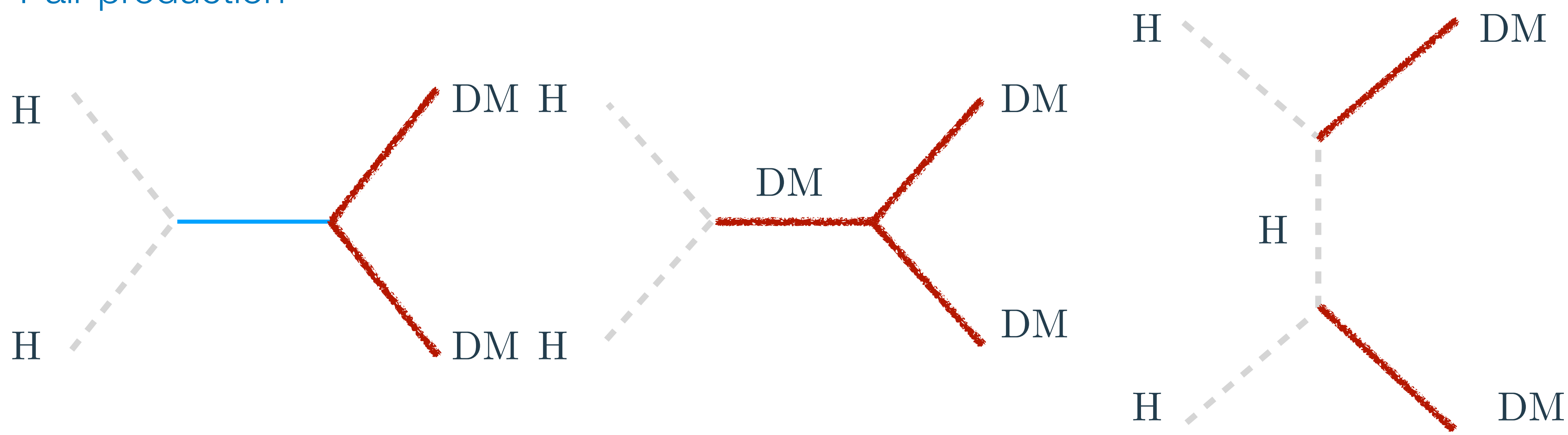
2. **DM-EFT: Gravity mediated**

3. **DM-EFT: Higgs mediated**

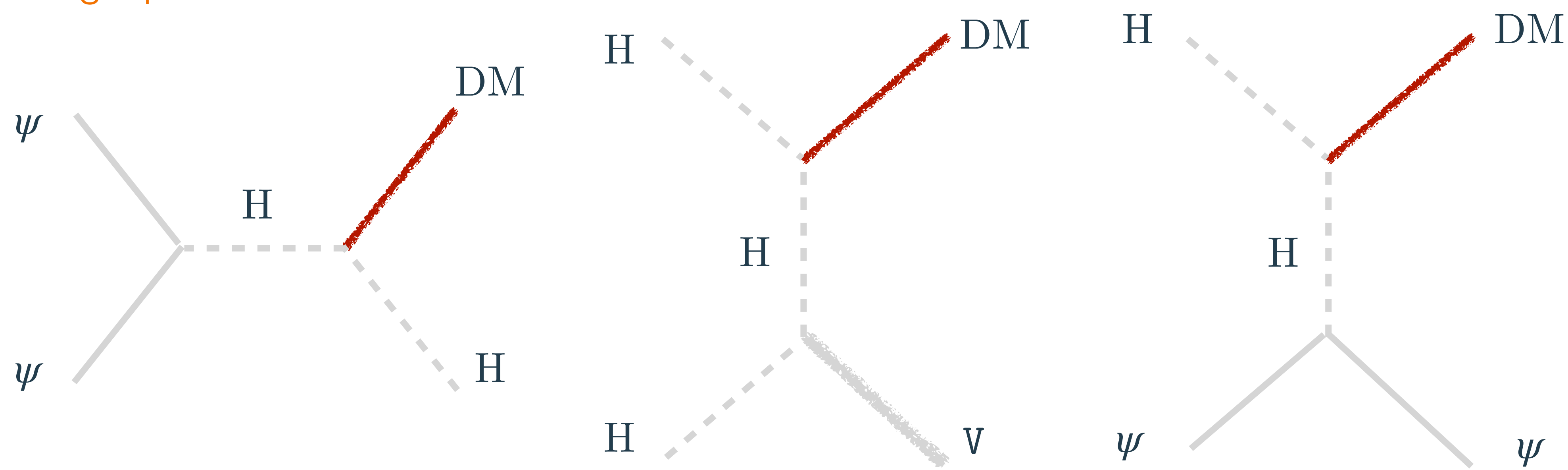
Conclusions

Higgs mediated

Pair production



Single production



Higgs mediated

DM S=2

$$\mathcal{M}(1_{H_a} 2_{\bar{H}_b}\ 3_X) = -\frac{c_H}{M_{\rm Pl} m^2}\delta_{ab}\langle \mathbf{3} p_1 \mathbf{3}]^2 \qquad c_H \sim \mathcal{O}(\,M_{\rm Pl}/\Lambda)$$

$$\mathcal{M}(1_{H_a} 2_{\bar{H}_b} 3^-_{v_c}) = -\sqrt{2} g_v\, T^c_{ab}\,\frac{\langle 13\rangle\langle 23\rangle}{\langle 12\rangle}$$

$$\mathcal{M}(1_{H_a} 2^-_{Q_b} 3^-_{\bar{t}_R}) = -y_t \epsilon_{ab}\langle 23\rangle$$

DM S=2

$$\begin{aligned}\mathcal{M}(\mathbf{1}_X \mathbf{2}_X \mathbf{3}_X) = -c_H \frac{[\mathbf{3}\sigma^\mu \mathbf{3}\rangle [\mathbf{3}\sigma^\nu \mathbf{3}\rangle}{4 M_{\rm Pl} m^6}\Big[& a_0 m^2 \langle \mathbf{12}\rangle [\mathbf{12}] \langle \mathbf{1}\sigma^\mu \mathbf{1}]\langle \mathbf{2}\sigma^\nu \mathbf{2}] \\ &+ a_2 \Big(\langle \mathbf{12}\rangle^2 p^\mu_{12} - (\langle \mathbf{12}\rangle + [\mathbf{12}]) Y^\mu\Big)\Big([\mathbf{12}]^2 p^\nu_{12} - (\langle \mathbf{12}\rangle + [\mathbf{12}])\tilde{Y}^\nu\Big)\Big]\end{aligned}$$

Higgs mediated

$$\mathcal{M}(1_{H_a}2_{\bar{H}_b}\mathbf{3}_X\mathbf{4}_X) = -\delta_{ab}\frac{c_H^2}{M_{\rm Pl}^2 m^3} \Bigg\{\langle\mathbf{3}p_1\mathbf{3}]\langle\mathbf{4}p_2\mathbf{4}]\frac{\langle\mathbf{34}\rangle\langle\mathbf{3}p_1\mathbf{4}]+[\mathbf{34}]\langle\mathbf{4}p_1\mathbf{3}]}{t} - \langle\mathbf{3}p_2\mathbf{3}]\langle\mathbf{4}p_1\mathbf{4}]\frac{\langle\mathbf{34}\rangle\langle\mathbf{4}p_1\mathbf{3}]+[\mathbf{34}]\langle\mathbf{3}p_1\mathbf{4}]}{u} + \mathcal{O}(m)\Bigg\}$$

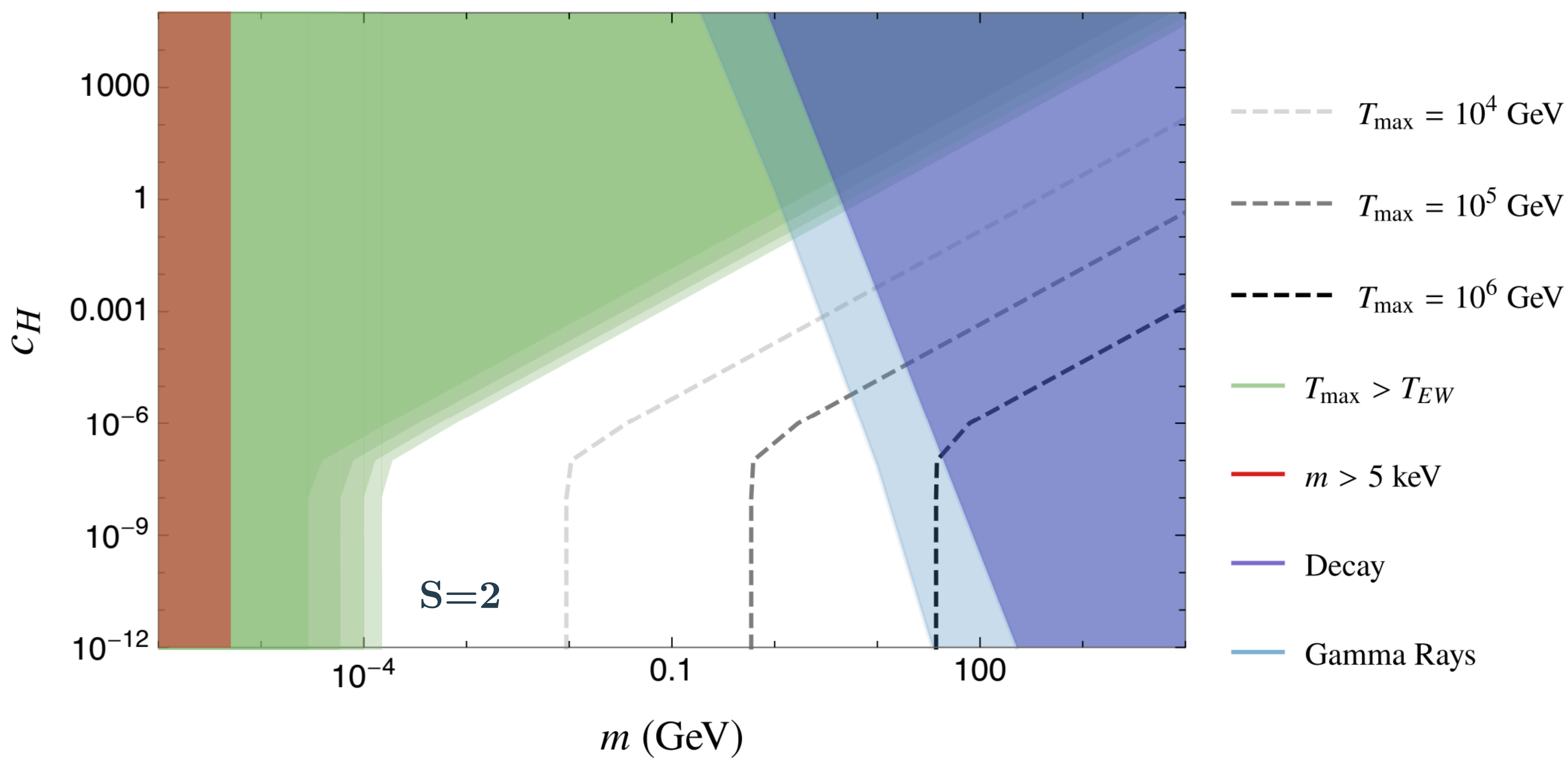
$$\mathcal{M}(1_{H_a}2_{\bar{H}_b}\mathbf{3}_X 4_v^-) = \frac{c_H g_v T^c_{ab}}{\sqrt{2}\,M_{\rm Pl}m^2}\left[\frac{\langle 4p_1p_2 4\rangle}{tu}\left([\mathbf{3}p_1\mathbf{3}\rangle^2+[\mathbf{3}p_2\mathbf{3}\rangle^2+[\mathbf{3}p_4\mathbf{3}\rangle^2\right)+ + 2\langle 4\mathbf{3}\rangle\left(\frac{\langle 4p_2\mathbf{3}][\mathbf{3}p_2\mathbf{3}\rangle}{t}-\frac{\langle 4p_1\mathbf{3}][\mathbf{3}p_1\mathbf{3}\rangle}{u}\right)\right],$$

$$\mathcal{M}(1^-_{Q_a}2^-_{\bar{t}_R}\mathbf{3}_X 4_{H_b}) = \epsilon_{ab}\frac{y_t\, c_H}{M_{\rm Pl}m^2 s}\langle 12\rangle\langle\mathbf{3}p_4\mathbf{3}]^2$$

DM S=2

Phenomenology

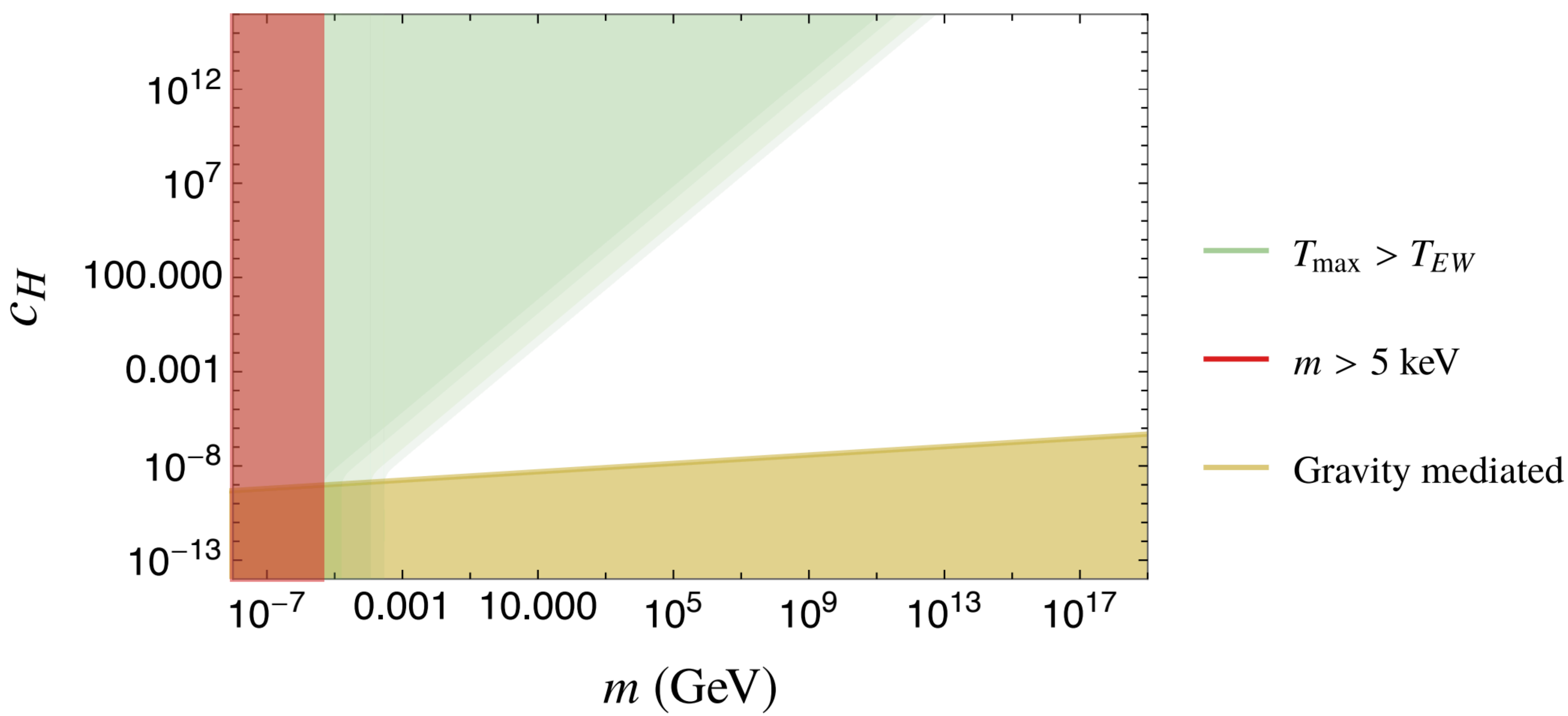
$c_H \sim \mathcal{O}(\,M_{\rm Pl}/\Lambda)$



$$T_{\rm max} \gg m \quad \to \quad Y_X^0 \sim \frac{T_{\rm max}^{4S-3}}{M_{\rm Pl} m^{4S-4}} \left[c_H^2 + \frac{T_{\rm max}^4}{M_{\rm Pl}^2 m^2} + c_H^4 \frac{T_{\rm max}^{4S-4}}{M_{\rm Pl}^2 m^{4S-6}} \right].$$

Phenomenology

$c_H \sim \mathcal{O}(M_{\rm Pl}/\Lambda)$



Single-production is dominant for most part of the parameter space

Double-production via Higgs just take place for values outside the EFT validity

Conclusions

On-shell methods can be applied to study EFTs!

Unitarity/locality/Lorentz symmetry are powerful tools and allowed us to study a higher-spin DM scenario without having to write a Lagrangian

I'd be happy to discuss more applications, please share your ideas :)

Thank you!