Nucleon Mass and Charges with Lattice QCD

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- James Simone
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- Fermilab Lattice and MILC collaborations

Nucleon Mass with Highly Improved Staggered Quarks

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(Dated: February 4, 2021)

We present the first computation in a program of lattice-QCD baryon physics using staggered fermions for sea *and* valence quarks. For this initial study, we present a calculation of the nucleon mass, obtaining 964 ± 16 MeV with all sources of statistical and systematic errors controlled and accounted for. This result is the most precise determination to date of the nucleon mass from first principles. We use the highly-improved staggered quark action, which is computationally efficient. Three gluon ensembles are employed, which have approximate lattice spacings $a \approx 0.09$ fm, 0.12 fm, and 0.15 fm, each with equal-mass u/d, s, and c quarks in the sea. Further, all ensembles have the light valence and sea u/d quarks tuned to reprod
complications from chiral extrapolations. Our work opens
of baryon properties, which are both feasible and relevant t
physics.

Computing Nucleon Charges with Highly Improved Staggered Quarks

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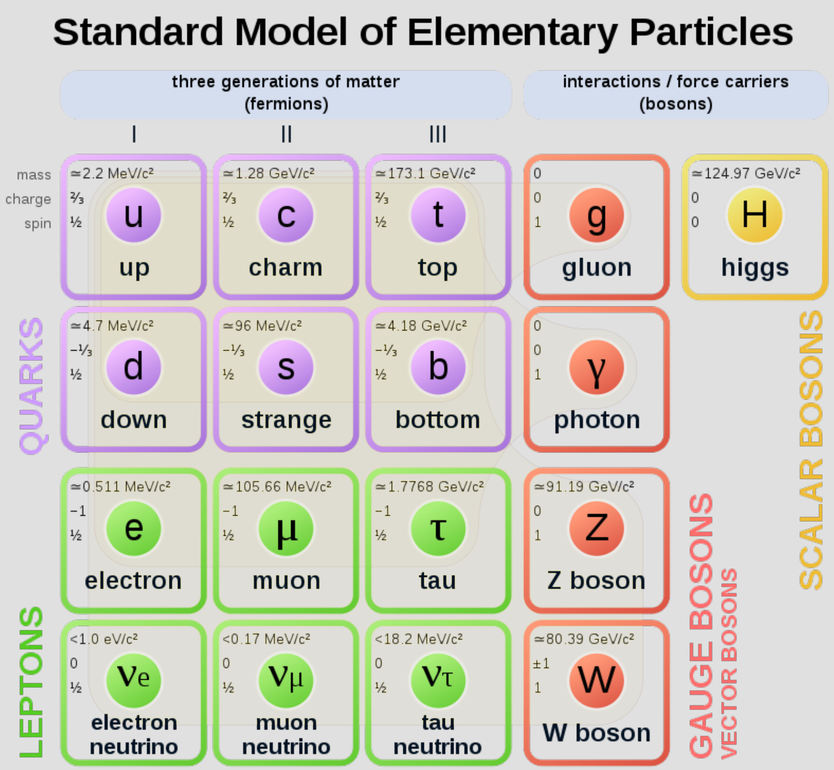
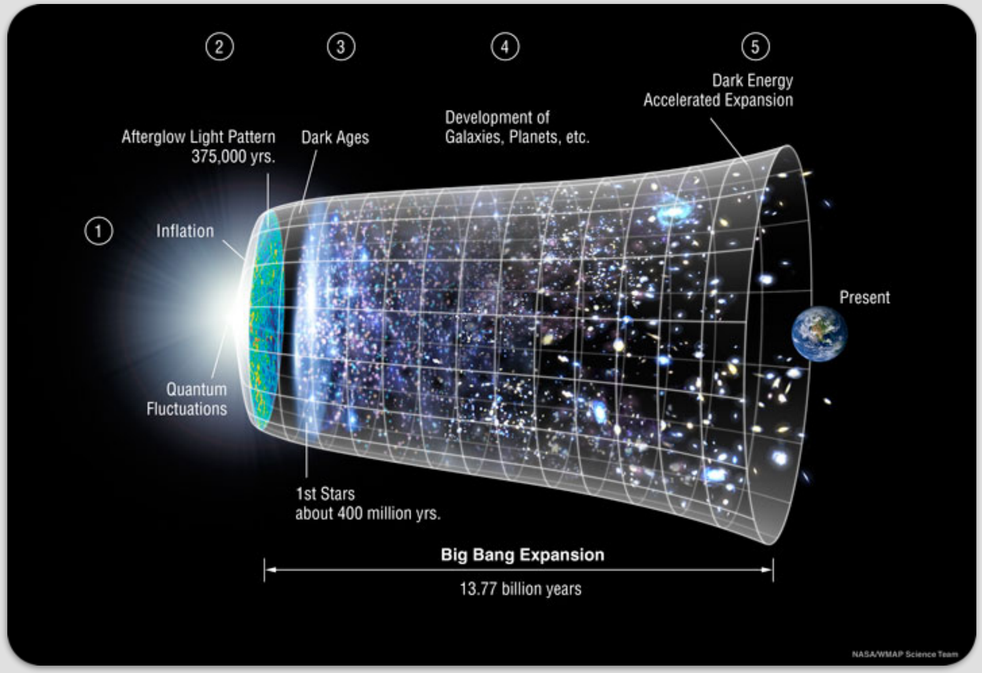
(Dated: October 21, 2020)

This work continues our program of lattice-QCD baryon physics using staggered fermions for both the sea and valence quarks. We present a proof-of-concept study that demonstrates, for the first time, how to calculate baryon matrix elements using staggered quarks for the valence sector. We show how to relate the representations of the continuum staggered flavor-taste group $\mathrm{SU}(8)_{FT}$ to those of the discrete lattice symmetry group. The resulting calculations yield the normalization factors relating staggered baryon matrix elements to their physical counterparts. We verify this methodology by calculating the isovector vector and axial-vector charges g_V and g_A. We use a single ensemble from the MILC Collaboration with 2+1+1 flavors of sea quark, lattice spacing $a \approx 0.12$ fm, and a pion mass $M_\pi \approx 305$ MeV. On this ensemble, we find results consistent with expectations from current conservation and neutron beta decay. Thus, this work demonstrates how highly-improved staggered quarks can be used for precision calculations of baryon properties, and, in particular, the isovector nucleon charges.

Neutrino: A Tale of Two Models



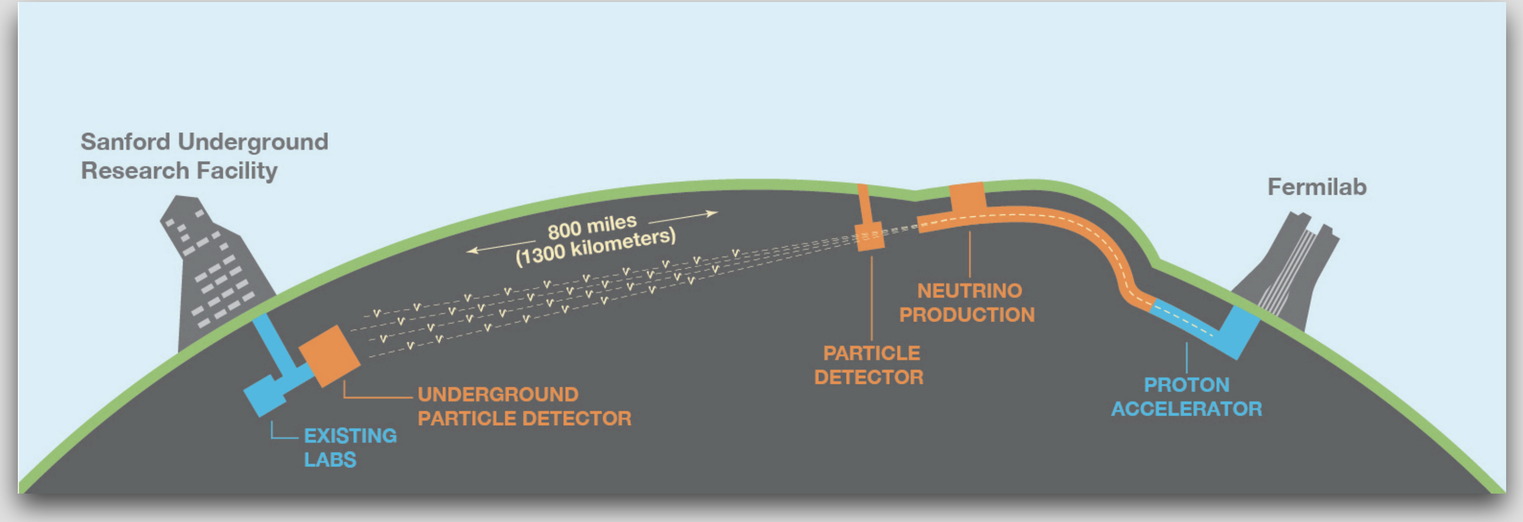
ΛCDM



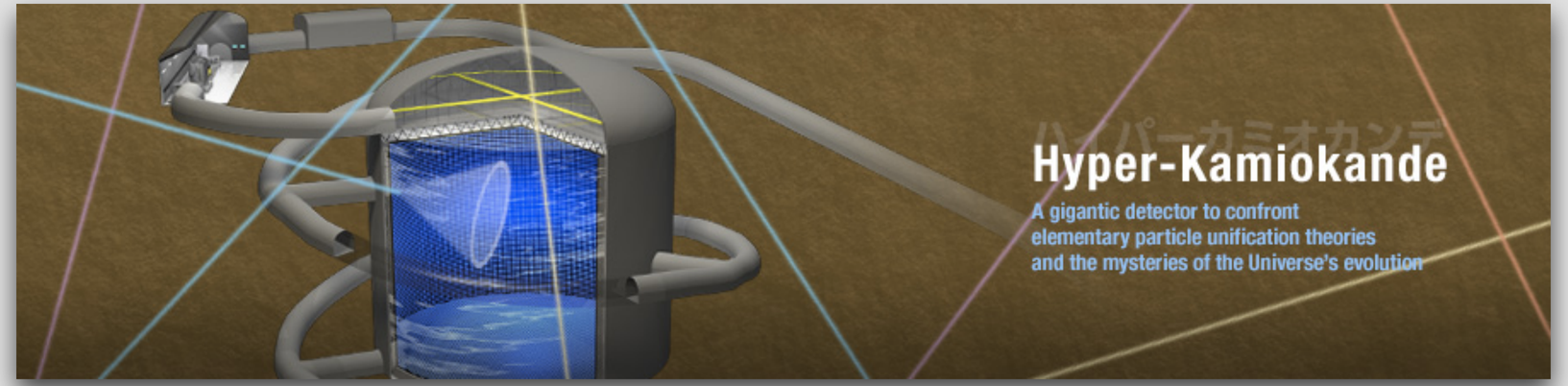
- Majorana vs Dirac
- ν parameters
- and more

- CνB
- ν mass
- and more

Neutrino Oscillation Experiments

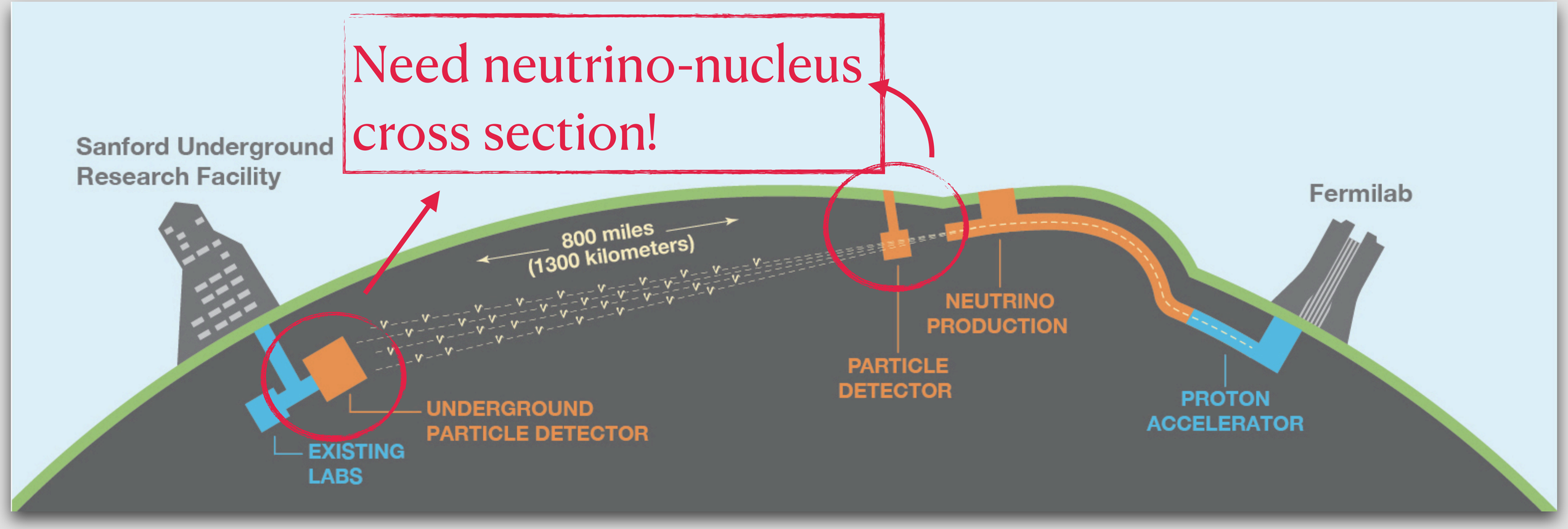


[DUNE Collaboration]



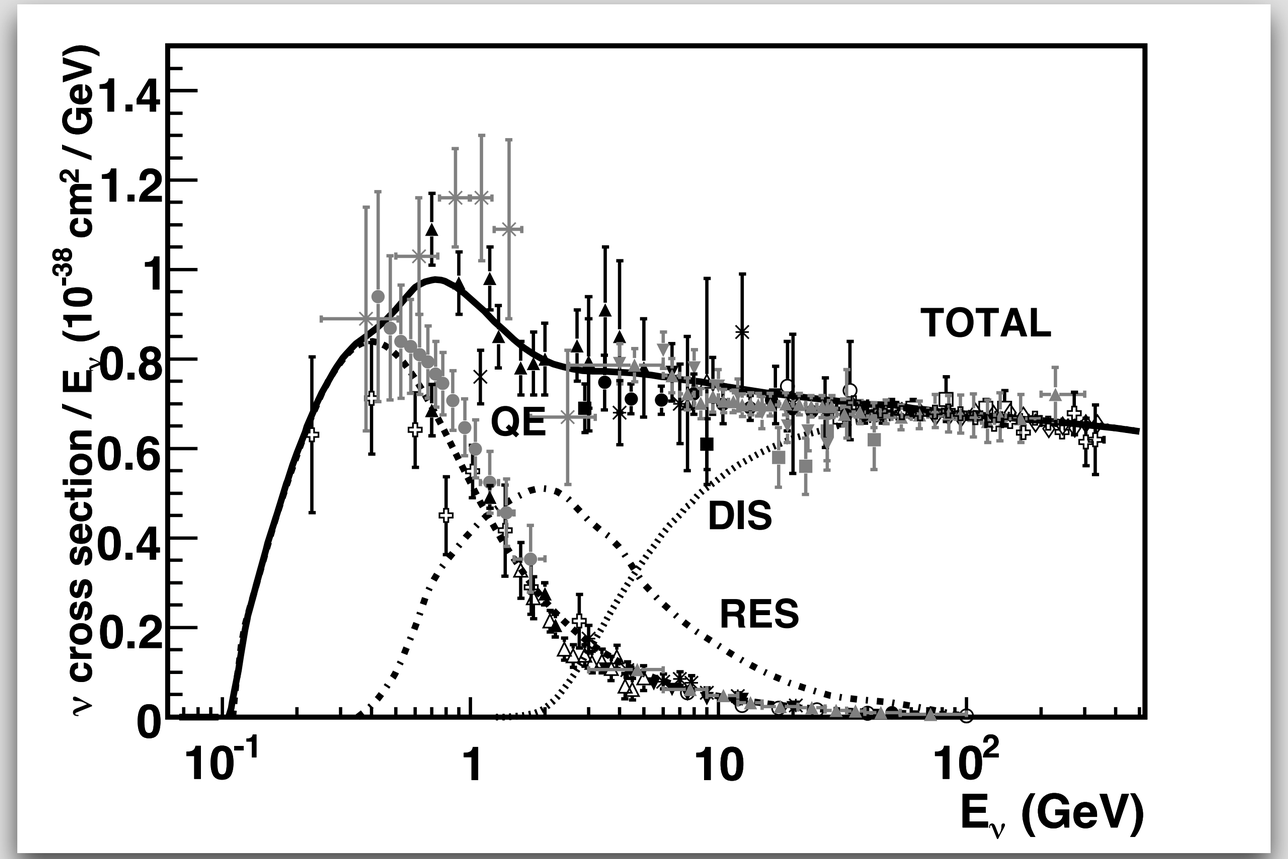
[Hyper-K Collaboration]

Neutrino Oscillation Experiments



[DUNE Collaboration]

Neutrino-Nucleus Cross Section



(cross section per nucleon, isoscalar targets)

[Formaggio, Zeller, arXiv: 1305.7513]

E_ν

Deep-inelastic (DIS)

$\langle N | J^\mu J^\nu | N \rangle$, PDF

Resonance Region (RES)

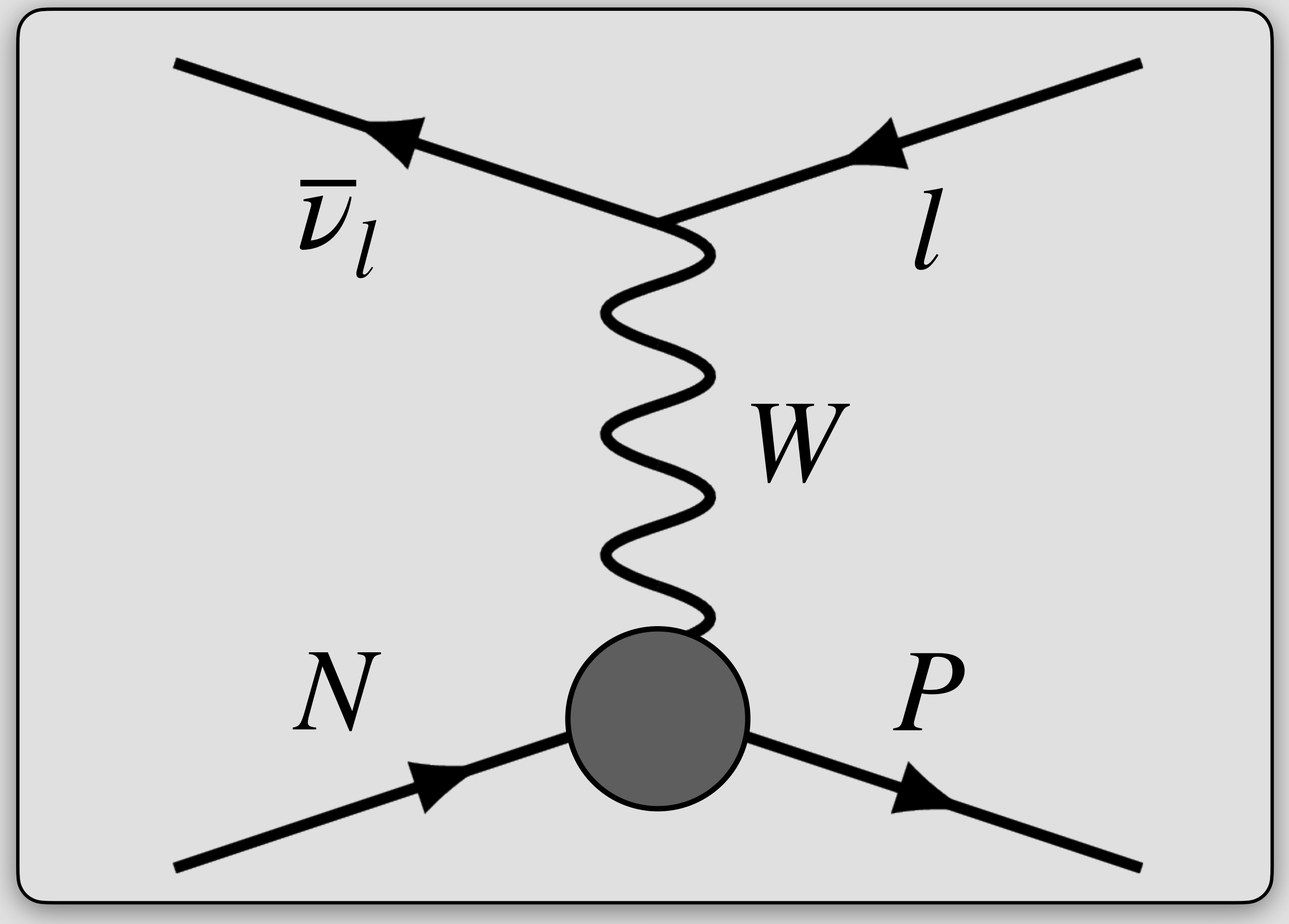
$\langle N | J^\mu | \text{res.} \rangle$

Quasi-elastic (QE)

$\langle N | J^\mu | N \rangle$

[USQCD white paper, arXiv: 1904.09931]

Charged Current Neutrino-Nucleon Cross Section



$\langle N | J_V^\mu(Q) | N \rangle$

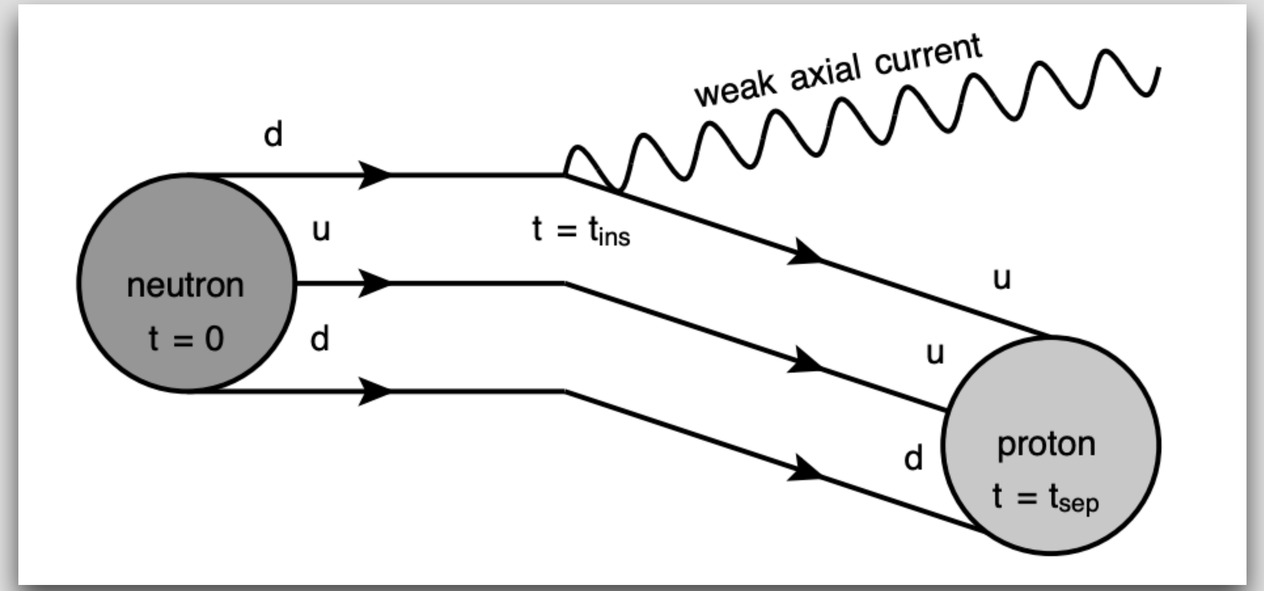
- Vector FF: High statistics measurements from e⁻ scattering

$\langle N | J_A^\mu(Q) | N \rangle$

[Borah, Hill, Lee, Tomalak, arXiv:2003.13640]

- Axial FF: Lacking new data, dominant uncertainty

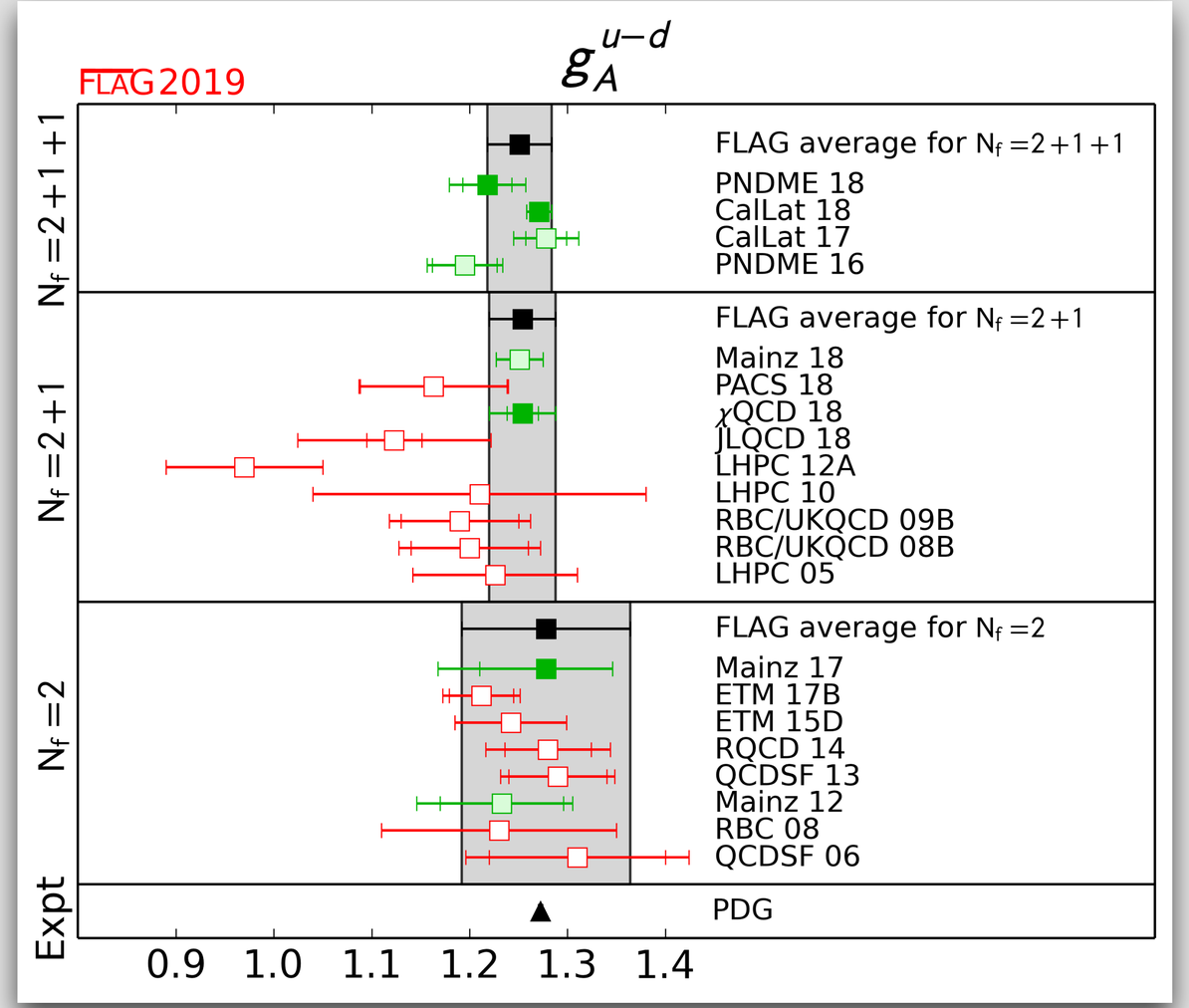
Nucleon Axial and Vector Charges



[C.C Chang et al, arXiv:1805.12130]

$$F_A(Q=0) = g_A = 1.2756 \pm 0.0013$$

$$F_V(Q=0) = g_V = 1$$



[FLAG review 2019, arXiv:1902.08191]

Lattice QCD is a Regularization of QCD

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\overline{\psi} e^{iS[A,\overline{\psi},\psi]} O$$

Wick rotate

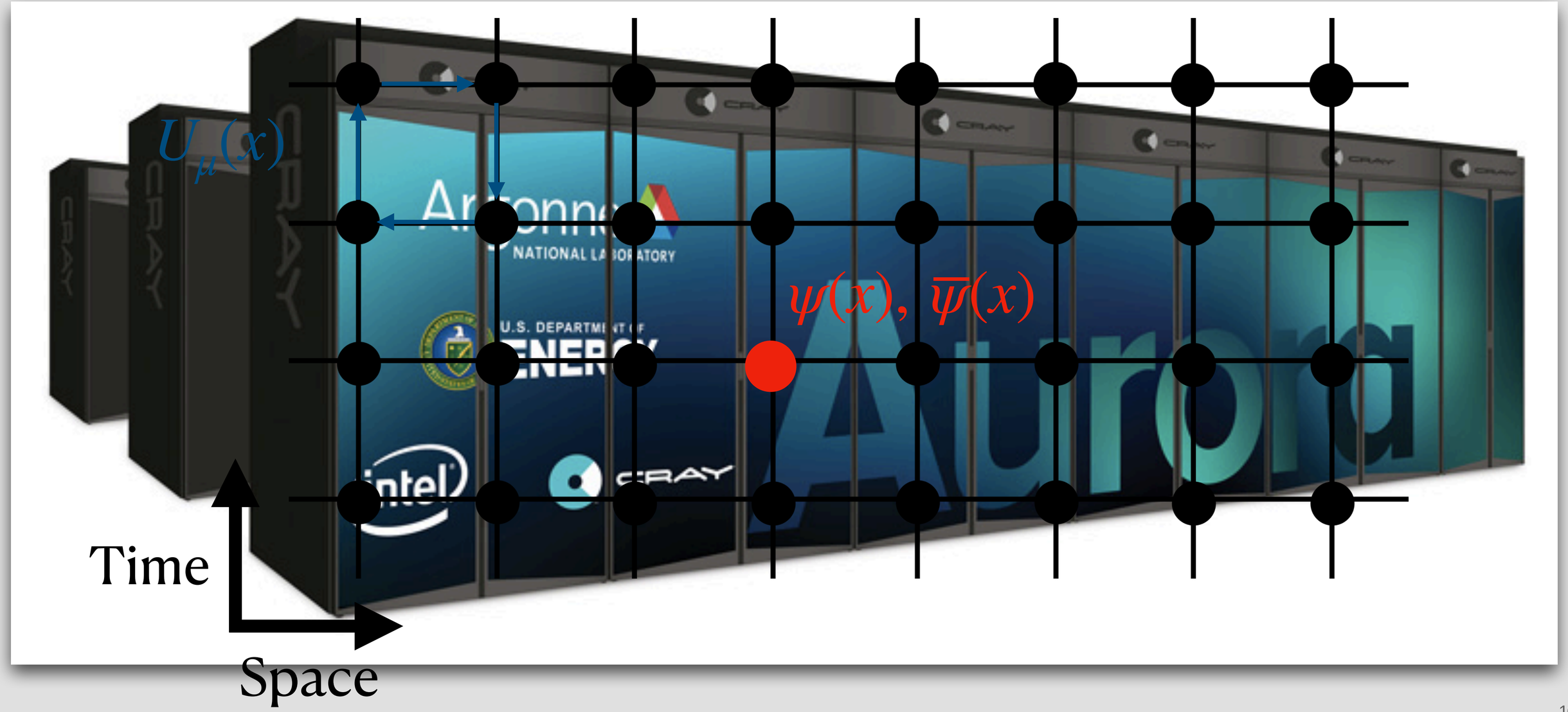
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\overline{\psi} e^{-S[A,\overline{\psi},\psi]} O$$

Put on lattice

$$\langle O_L \rangle = \int \prod_n dU(n) d\psi(n) d\overline{\psi}(n) \frac{e^{-S_L[U,\overline{\psi},\psi]}}{Z} O_L$$

probability density of "gauge configurations"

Visualizing Lattice QCD



Measuring Observables on Lattice

$$C_{2pt}(t) = \langle O(t)\overline{O}(0)\rangle$$

"sink" "source"

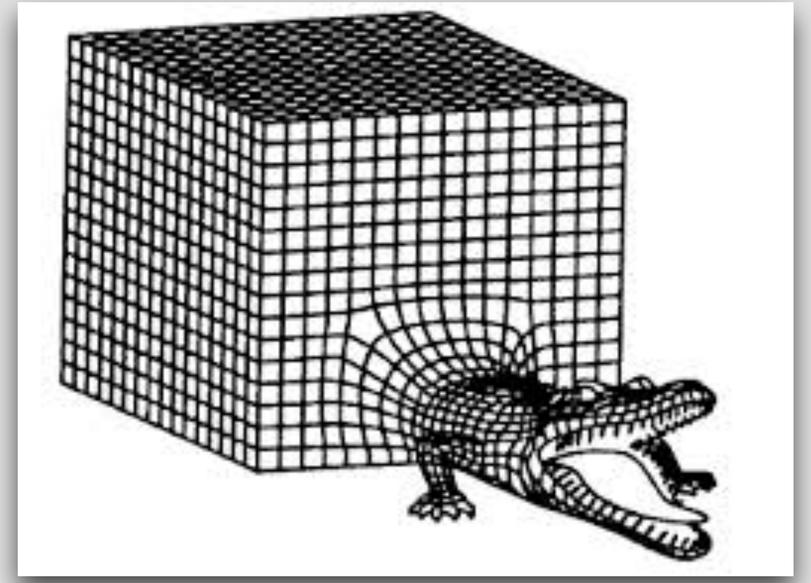
$$C_{3pt}(t,\tau) = \langle O(t)J(\tau)\overline{O}(0)\rangle$$

- **Mass** $C_{2pt}(t) \sim Ae^{-m_N t}$
- **Matrix element** $C_{3pt}(t,\tau)/C_{2pt}(t) \sim \langle N|J|N\rangle = g_{V,A}$

A Typical Lattice QCD Calculation

- Choose a lattice action
- Generate gauge configurations
- Measure observables on gauge configurations
- Analyze data

$$\langle O_L \rangle = \int \prod_n dU(n) d\psi(n) d\overline{\psi}(n) \frac{e^{-S_L[U,\overline{\psi},\psi]}}{Z} O_L$$



Free Market of Lattice Fermions

$$S_F[\psi, \overline{\psi}] = \int dx \overline{\psi}(\gamma^\mu \partial_\mu + m)\psi$$

$$S_F[\psi, \overline{\psi}] = \sum_{n,\mu} \left(\overline{\psi}(n)\gamma^\mu(\psi(n+\mu) - \psi(n-\mu) + m\psi(n) \right)$$

fermion doubling problem

Many alternative ways to discretize fermions:

Wilson fermion, overlap fermion, domain-wall fermion, twisted mass fermion, **staggered fermion**, ...

A First Look at Staggered Fermion

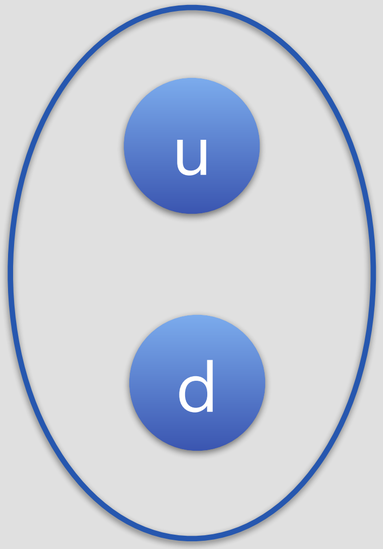
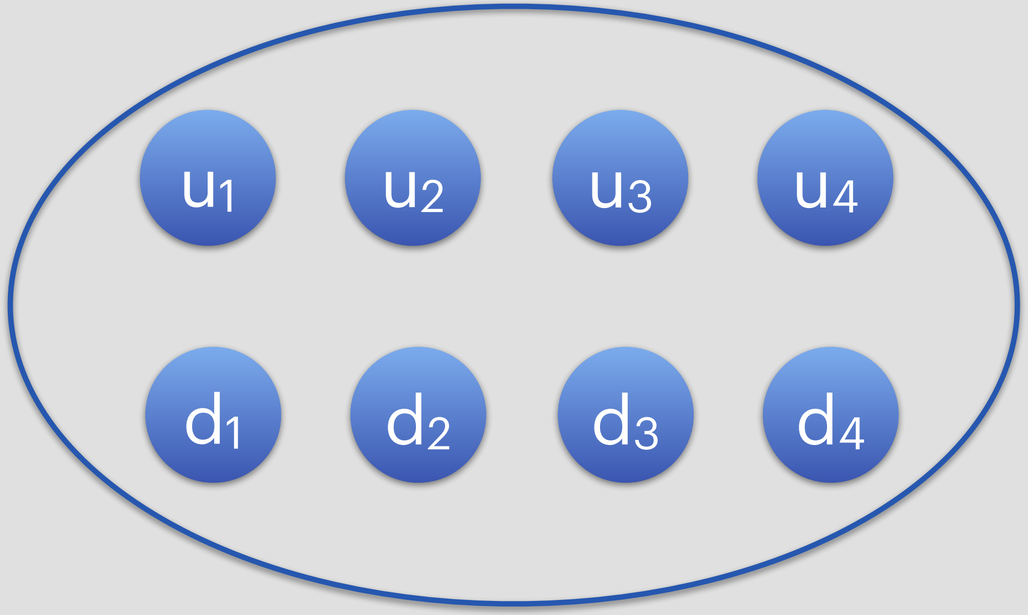
$$S_F = \sum_{x,y;a,b} \bar{\chi}^a(y) D^{ab}_{\text{stag}}(y,x) \chi^b(x)$$

one-component spinors

$$D^{ab}_{\text{stag}}(y,x) = \frac{1}{2} \sum_{\mu} \eta_\mu(y) \left(\delta_{x,y+\mu} - \delta_{x,y-\mu} \right) \delta_{a,b} + m \delta_{x,y} \delta_{a,b}$$

- Most efficient to simulate
- Remanent chiral symmetry
- Four degenerate fermion doublers (tastes) for each quark species in the continuum

Valence Symmetries of Continuum Staggered Fermions

Physical QCD	Staggered QCD
	
$SU(2)_F$	$SU(8)_{FT} \supset SU(2)_F \times SU(4)_T$

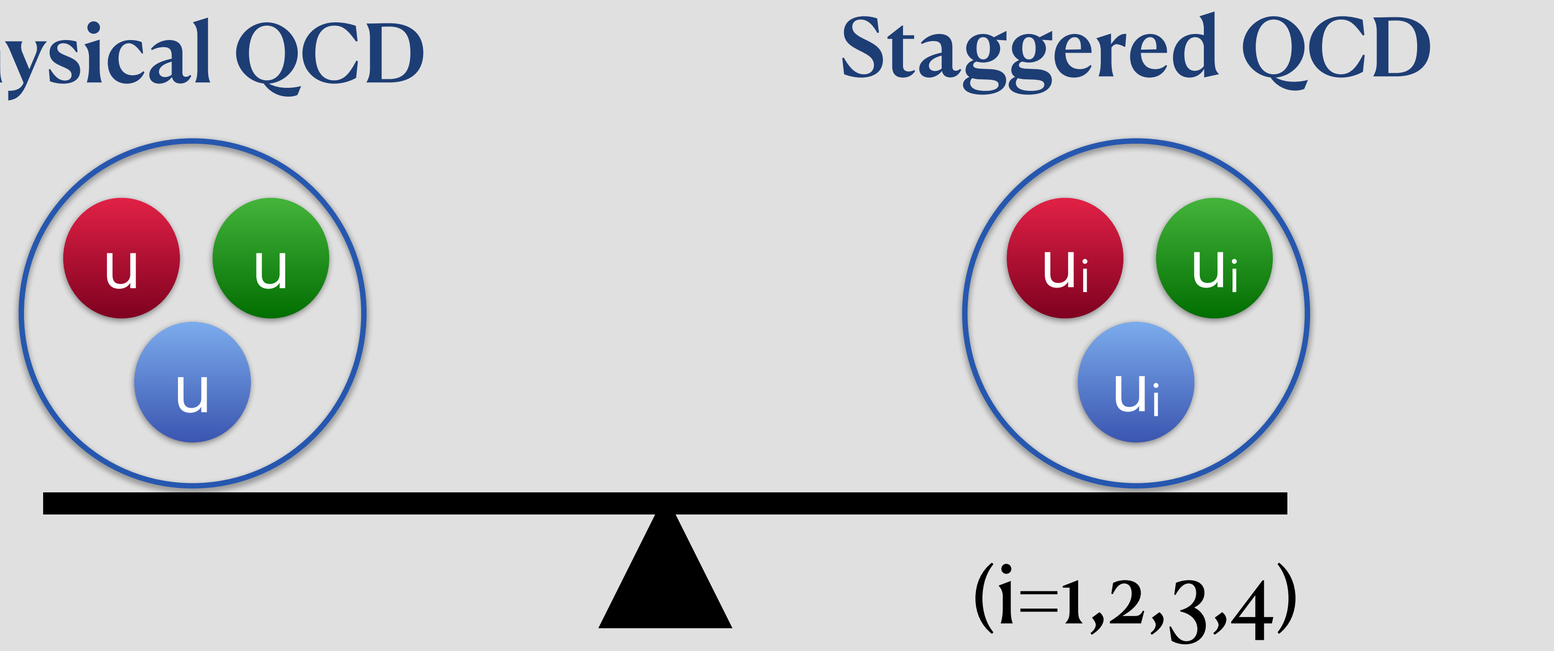
QCD $\subset$ Staggered QCD

Nucleon Mass

[YL, ASM, CH, ASK, JNS, AS, arXiv:1911.12256]

Physical States in Staggered QCD

Example: Δ^{++} resonance



Single-taste baryon

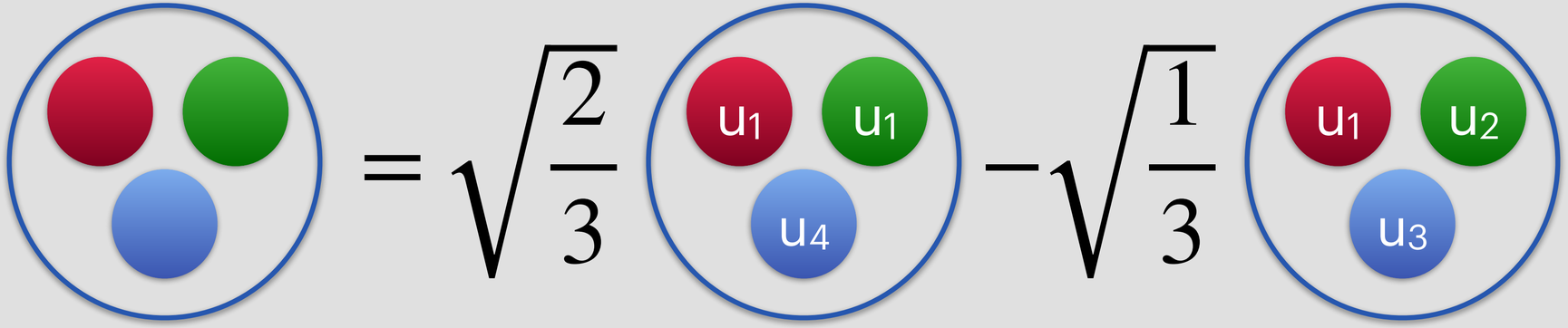
Single-taste baryons must have physical masses!

[Jon Bailey, arXiv:hep-lat/0611023]

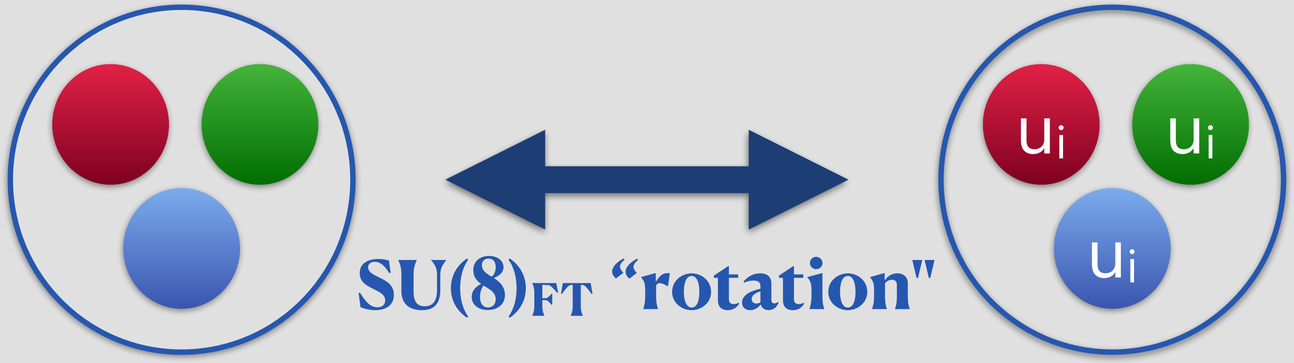
Physical States in Staggered QCD

Example: Δ^{++} resonance

There are more states in staggered QCD that have physical masses!



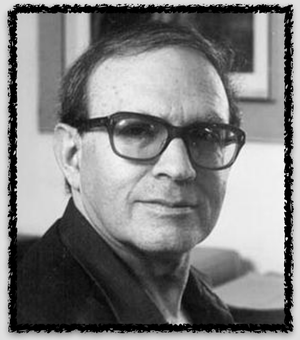
NOT a single-taste baryon

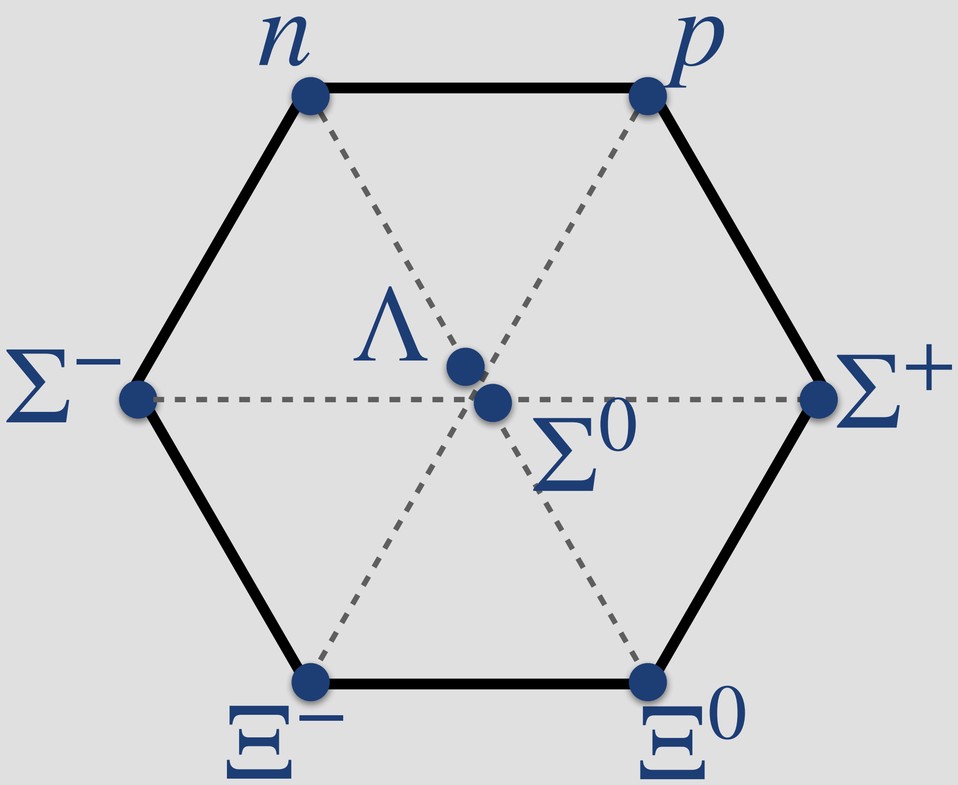


$SU(8)_{FT}$ can also rotate states to different isospin irrep

[Jon Bailey, arXiv:hep-lat/0611023]

168-fold Way of Staggered Nucleons





spin flavor

$$\frac{SU(2)_S \times SU(3)_F}{\left(\frac{1}{2},\ 8_M\right)}$$

irrep notation:
SU(2) group - spin notation
other group - dimension + possible subscript

spin flavor-taste

$$\frac{SU(2)_S \times SU(8)_{FT}}{\left(\frac{1}{2},\ 168_M\right)}$$

$SU(3)_F$ QCD

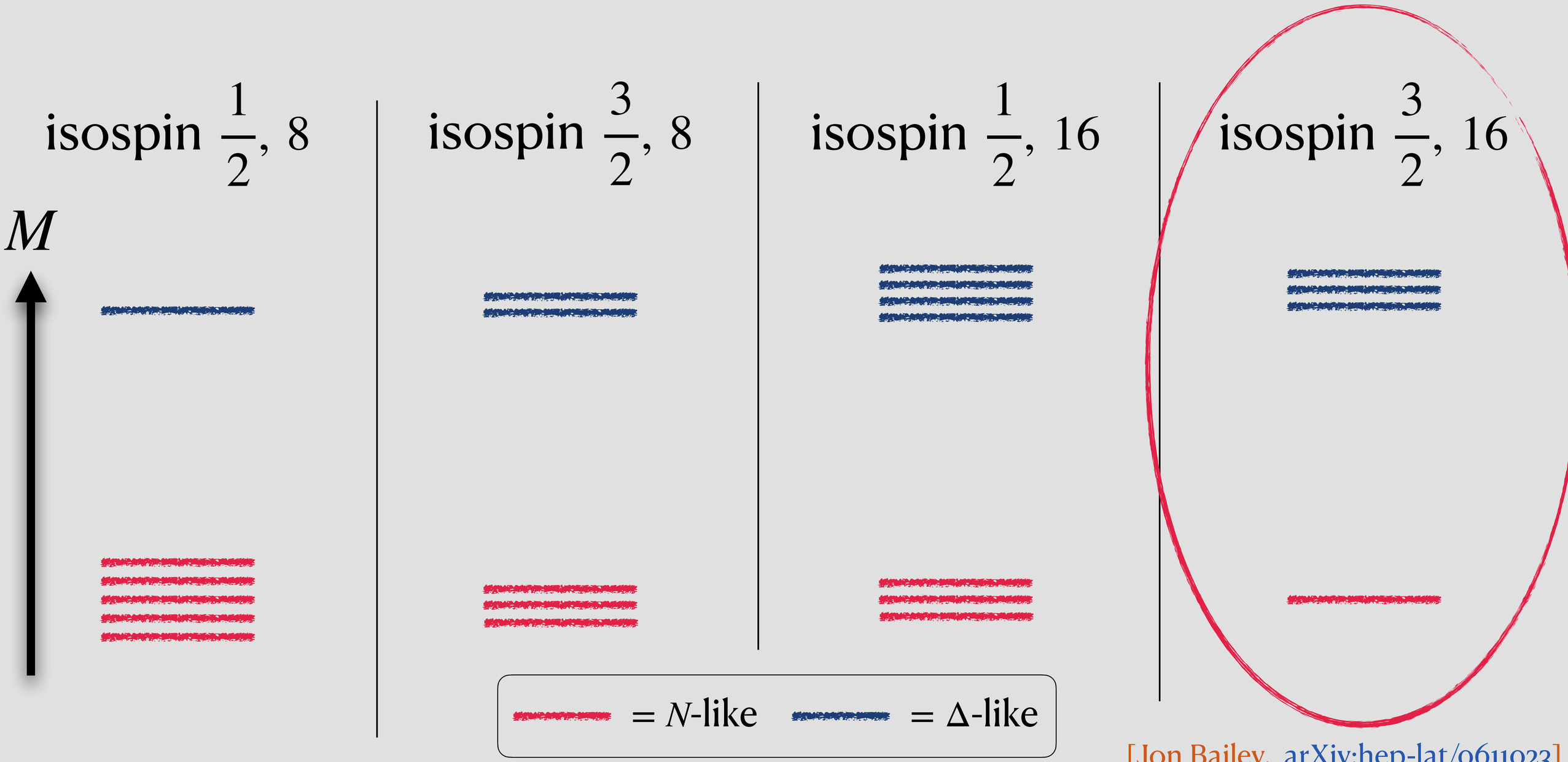
Baryons in the octet are degenerate

contains single-taste nucleons

Staggered QCD

Baryons in the 168-plet are degenerate

Spectrum of Staggered Nucleons with $SU(2)_F$

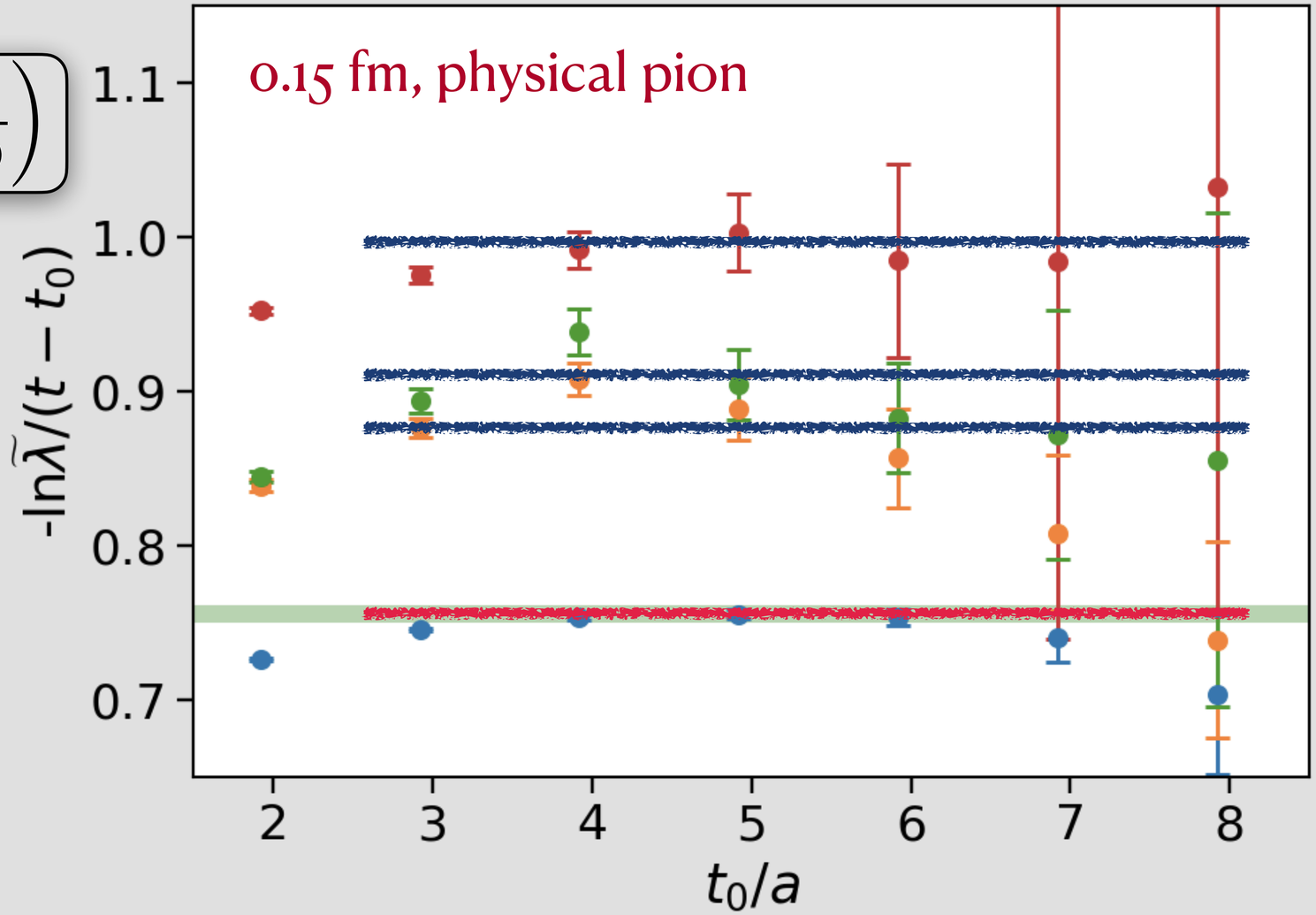


[Jon Bailey, arXiv:hep-lat/0611023]

Two-point Correlators of Isospin-3/2, 16 Irrep

$$aM_{eff} = \ln\left(\frac{C_{2pt}(t)}{C_{2pt}(t+a)}\right)$$

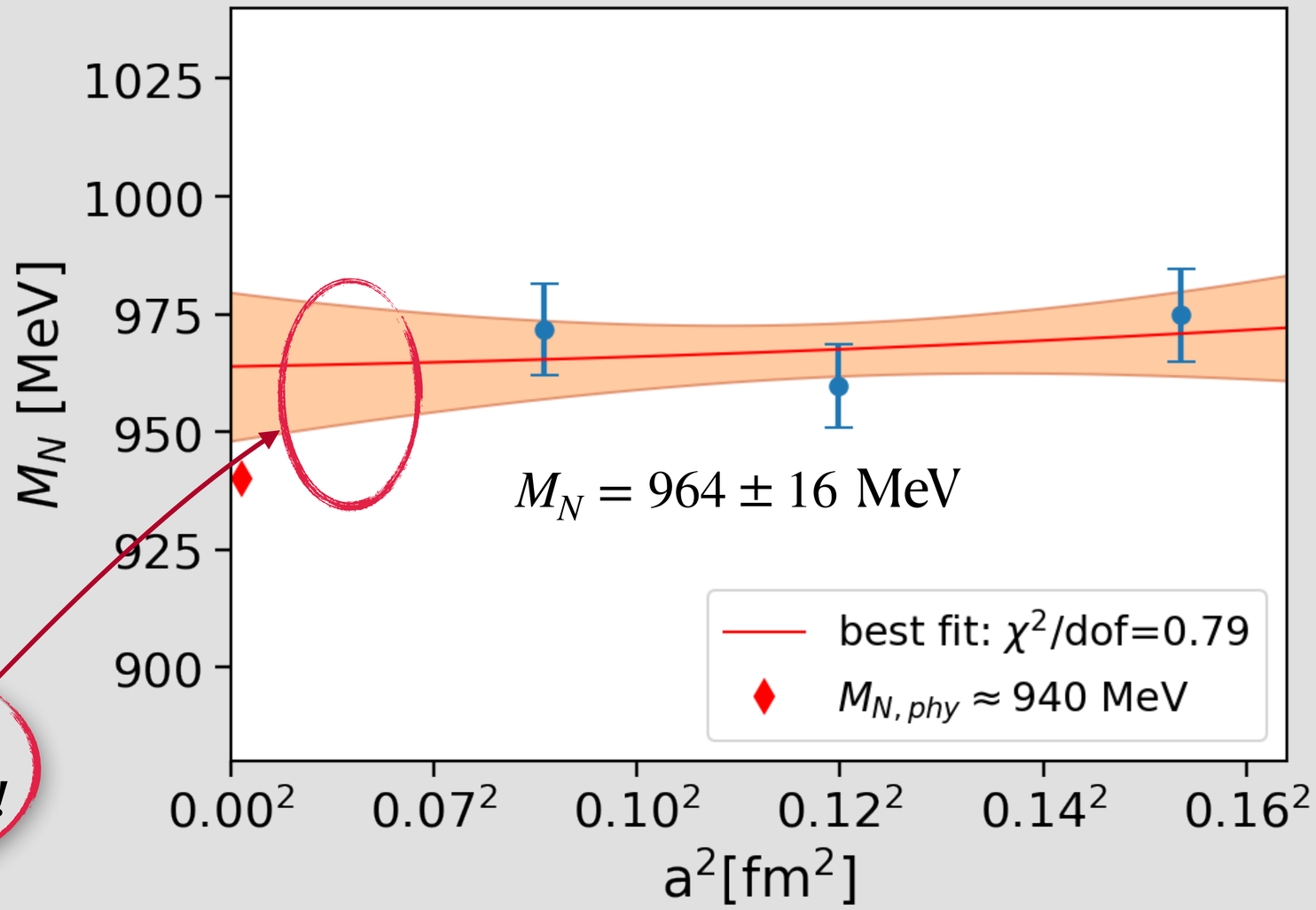
$$aM_{eff} \approx$$



Δ-like

N-like

Continuum Extrapolation of Nucleon Masses



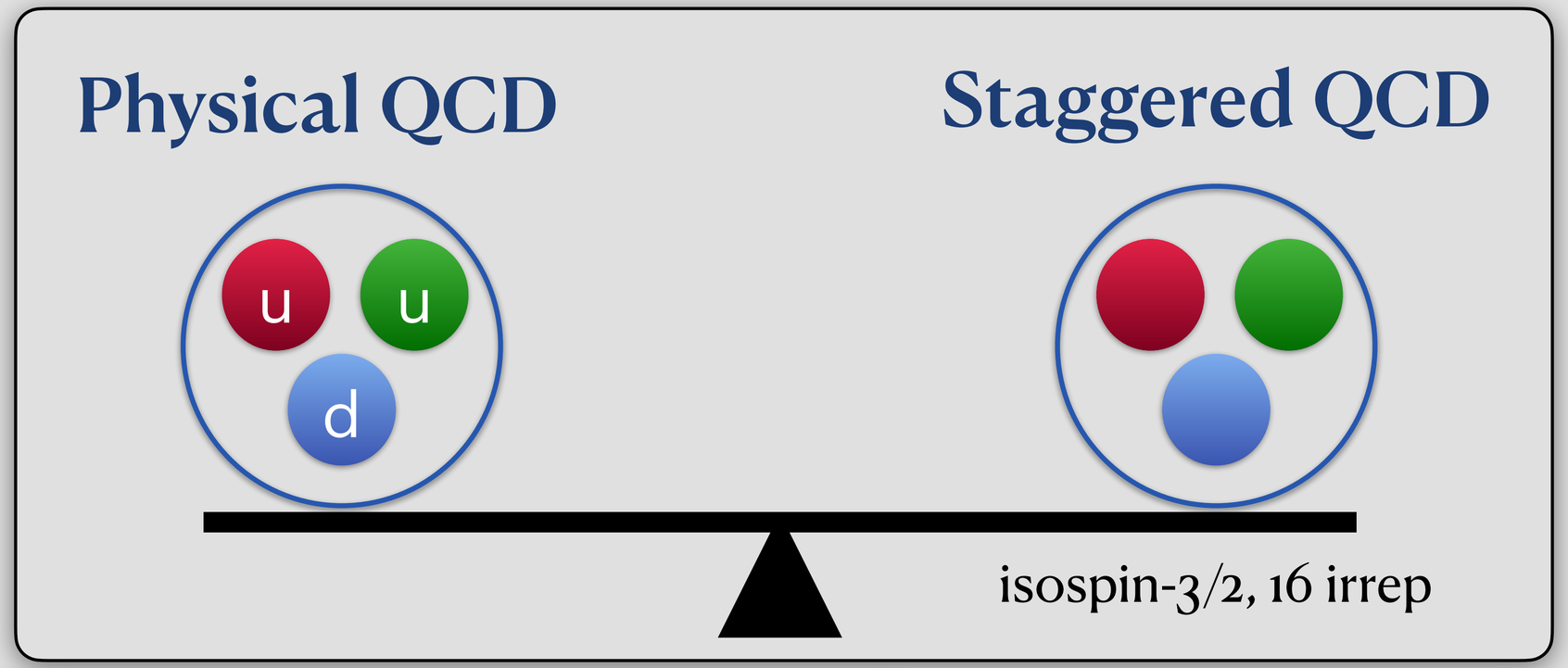
a=0.056 fm result soon!

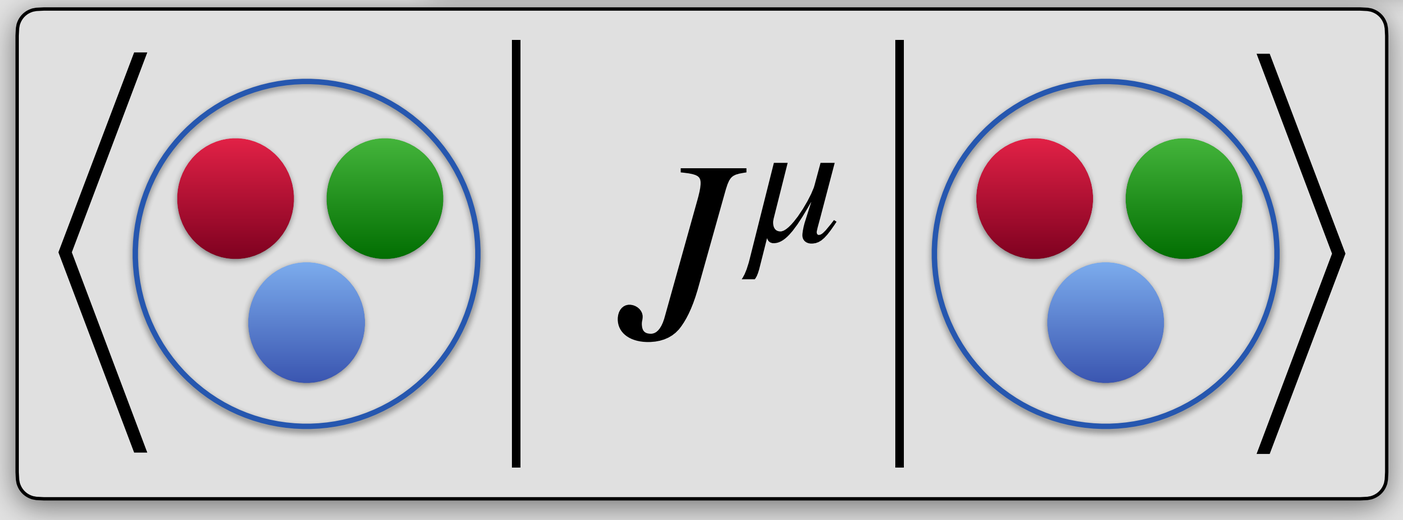
[YL, ASM, CH, ASK, JNS, AS, arXiv:1911.12256]

Nucleon Charges

[YL, ASM, SG, CH, ASK, JNS, AS, arXiv:2010.10455]

Matrix Elements are More Complicated



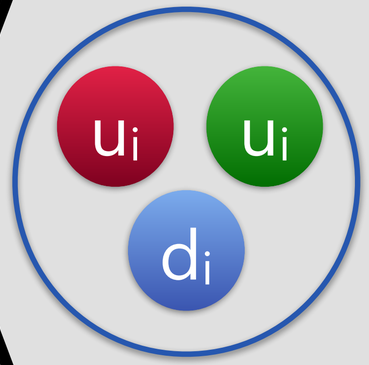


Q: How to extract physical charges $g_{A,V}$ from staggered matrix elements?

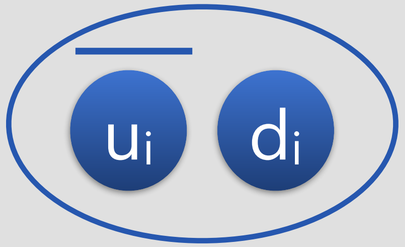
Physical ME in continuum Staggered QCD

$(i = 1,2,3,4)$

$$g_{V,A} = \Big\langle$$



$$\Big|$$



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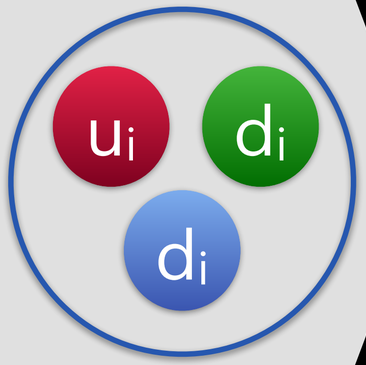


$$\Big\rangle$$

isospin-1/2 single-taste nucleon

isospin-1 single-taste current

isospin-1/2 single-taste nucleon

But there are no single-taste currents due to mixing

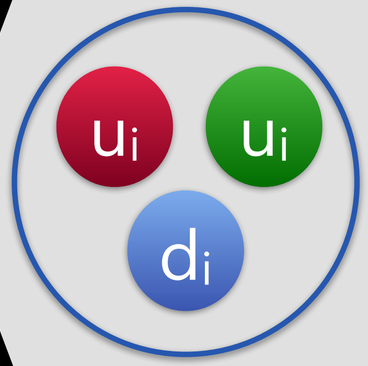
$SU(2)_F$ analogy

Δ^{++}, Δ^{-} : Single-flavor baryons $\qquad \pi^0 : \sqrt{\frac{1}{2}}\left(u\bar{u} - d\bar{d}\right)$

Physical ME in Staggered QCD

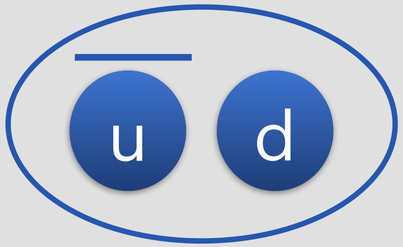
$(i = 1,2,3,4)$

$$g_{V,A} = \Big\langle$$



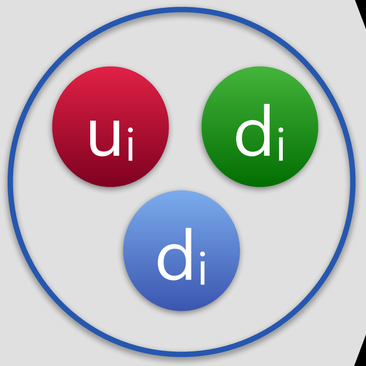
isospin-1/2 single-taste nucleon

$$\Big|$$



isospin-1 taste-diagonal current

$$\Big|$$



isospin-1/2 single-taste nucleon

$$\Big\rangle$$

taste-diagonal currents:

$$\sum_{i=1}^{4} \bar{u}_i d_i \ , \quad \bar{u}_1 d_1 - \bar{u}_2 d_2 + \bar{u}_3 d_3 - \bar{u}_4 d_4 \ , \quad \cdots$$

taste-diagonal currents are irreps of $SU(4)_T$ and lattice symmetry group

Physical ME in Staggered QCD

What we want

$$g_{V,A} = \left\langle \; \text{u}_i \; \text{u}_i \; \text{d}_i \;\middle|\; \bar{\text{u}} \; \text{d} \;\middle|\; \text{u}_i \; \text{d}_i \; \text{d}_i \; \right\rangle \qquad (i = 1,2,3,4)$$

isospin-1/2 single-taste nucleon

isospin-1 taste-diagonal current

isospin-1/2 single-taste nucleon

What we simulate on the lattice

$$M^{V,A}_{16,\pm 0} \equiv \left\langle N_{16,\pm 0} \middle| J^{V,A} \middle| N_{16,\pm 0} \right\rangle_{\text{cont.}}$$

isospin-3/2 NOT single-taste

On the Generalized Wigner-Eckart Theorem

SU(2) Wigner-Eckart Theorem

$$\left\langle j,\, m \middle| T_q^{(k)} \middle| j',\, m' \right\rangle = \langle j',\, m', k, q \,|\, j,\, m \rangle \left\langle j \,\|T^{(k)}\|\, j' \right\rangle$$

Matrix Element

CG coeff.
(group theory)

reduced ME
(physics)

SU(N) Wigner-Eckart Theorem

$$\left\langle \lambda,\, \sigma \middle| T_\tau^{(\alpha)} \middle| \lambda',\, \sigma' \right\rangle = G(\lambda, \sigma, \alpha, \tau, \lambda', \sigma') \times P(\lambda, T^{(\alpha)}, \lambda')$$

Matrix Element

Group theory

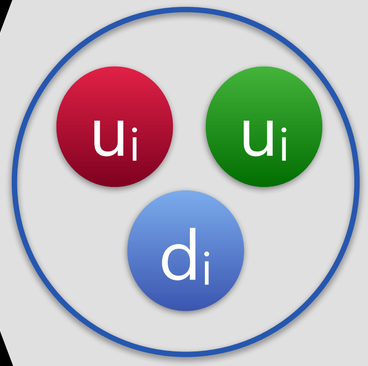
physics

$(\lambda, \lambda', \alpha \rightarrow$ irrep labels, $\sigma, \sigma', \tau \rightarrow$ component labels$)$

Physical ME in Staggered QCD

What we want

$$g_{V,A} = \Big\langle$$



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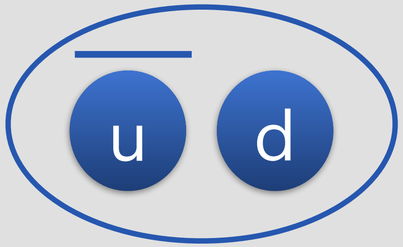
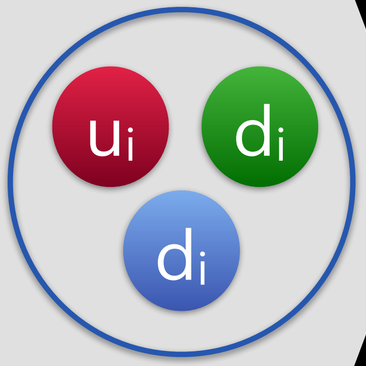


$$\Big|$$

$(i = 1,2,3,4)$



$$\Big\rangle$$

isospin-1/2 single-taste nucleon

isospin-1 taste-diagonal current

isospin-1/2 single-taste nucleon

What we simulate on the lattice

$$M^{V,A}_{16,\pm 0} = \Big\langle N_{16,\pm 0} \Big| J^{V,A} \Big| N_{16,\pm 0} \Big\rangle_{\text{cont.}}$$

isospin-3/2 NOT single-taste

Physical ME in Staggered QCD

What we want

$$g_{V,A} = G_1 \times \text{Physics}$$

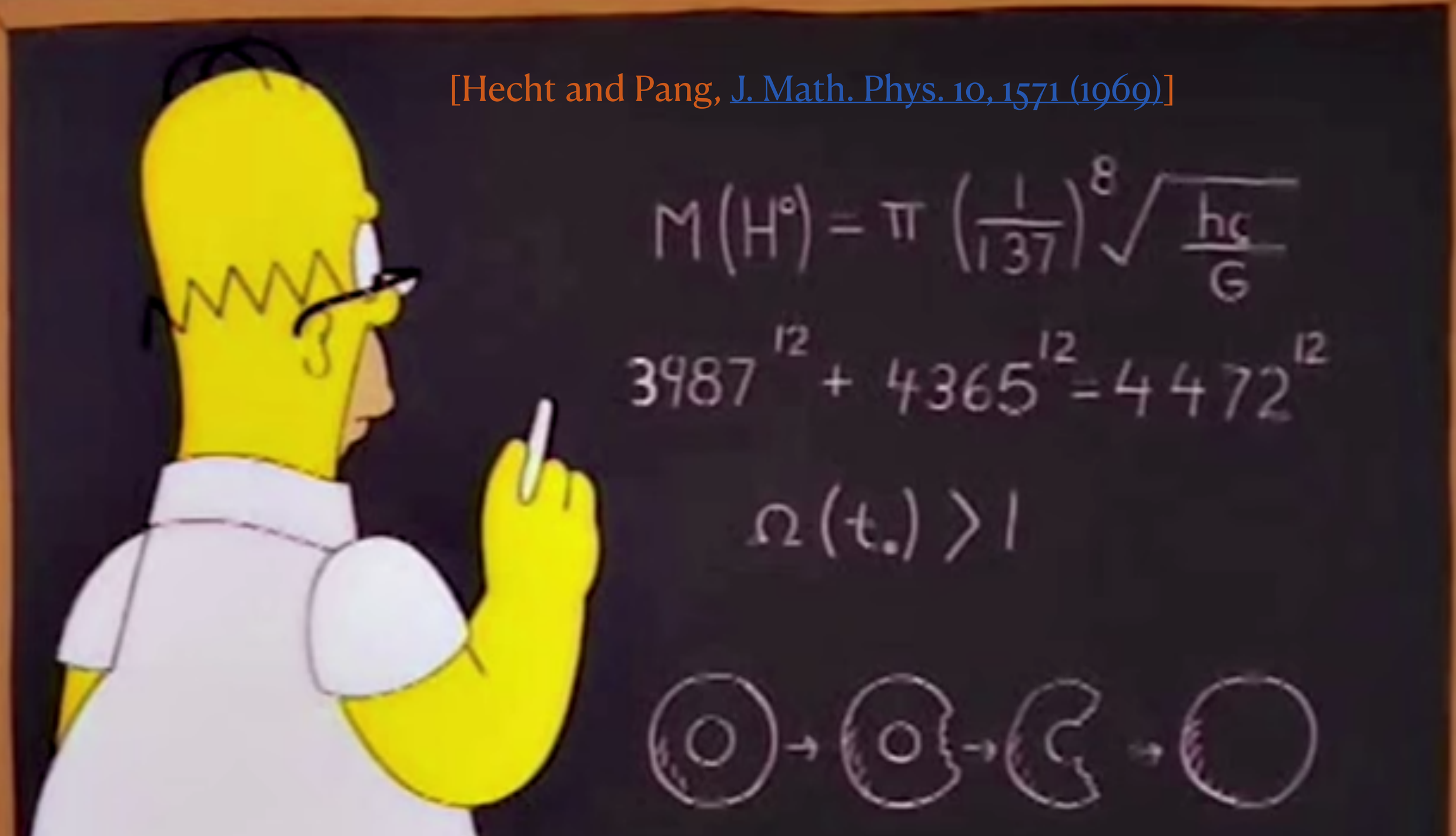
$$\frac{g_{V,A}}{M^{V,A}_{16,\pm 0}} = \text{Group theory}$$

ctors

What we

$$M^{V,A}_{16,\pm 0} = G'_{2,\pm 0} \times \text{Physics}$$

[Hecht and Pang, J. Math. Phys. 10, 1571 (1969)]



Physical ME in Staggered QCD

$$M^V_{16,+0} = \left\langle N_{16,+0} \middle| J^V \middle| N_{16,+0} \right\rangle_{\text{cont.}} = -g_V$$

$$M^V_{16,-0} = \left\langle N_{16,-0} \middle| J^V \middle| N_{16,-0} \right\rangle_{\text{cont.}} = -g_V$$

$$M^A_{16,+0} = \left\langle N_{16,+0} \middle| J^A \middle| N_{16,+0} \right\rangle_{\text{cont.}} = -\frac{g_A}{3}$$

$$M^A_{16,-0} = \left\langle N_{16,-0} \middle| J^A \middle| N_{16,-0} \right\rangle_{\text{cont.}} = g_A$$

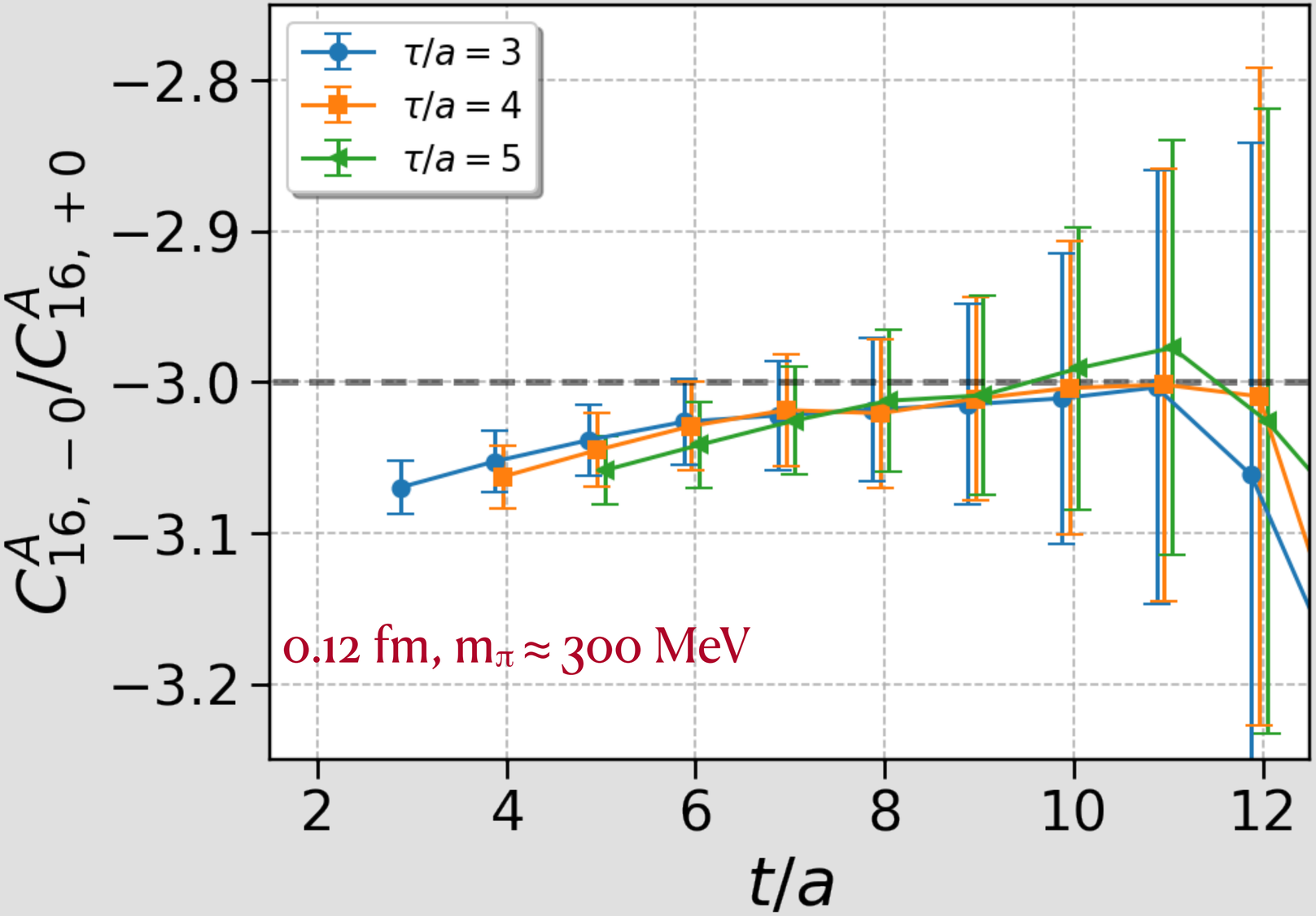
Results

[YL, ASM, SG, CH, ASK, JNS, AS, arXiv:2010.10455]

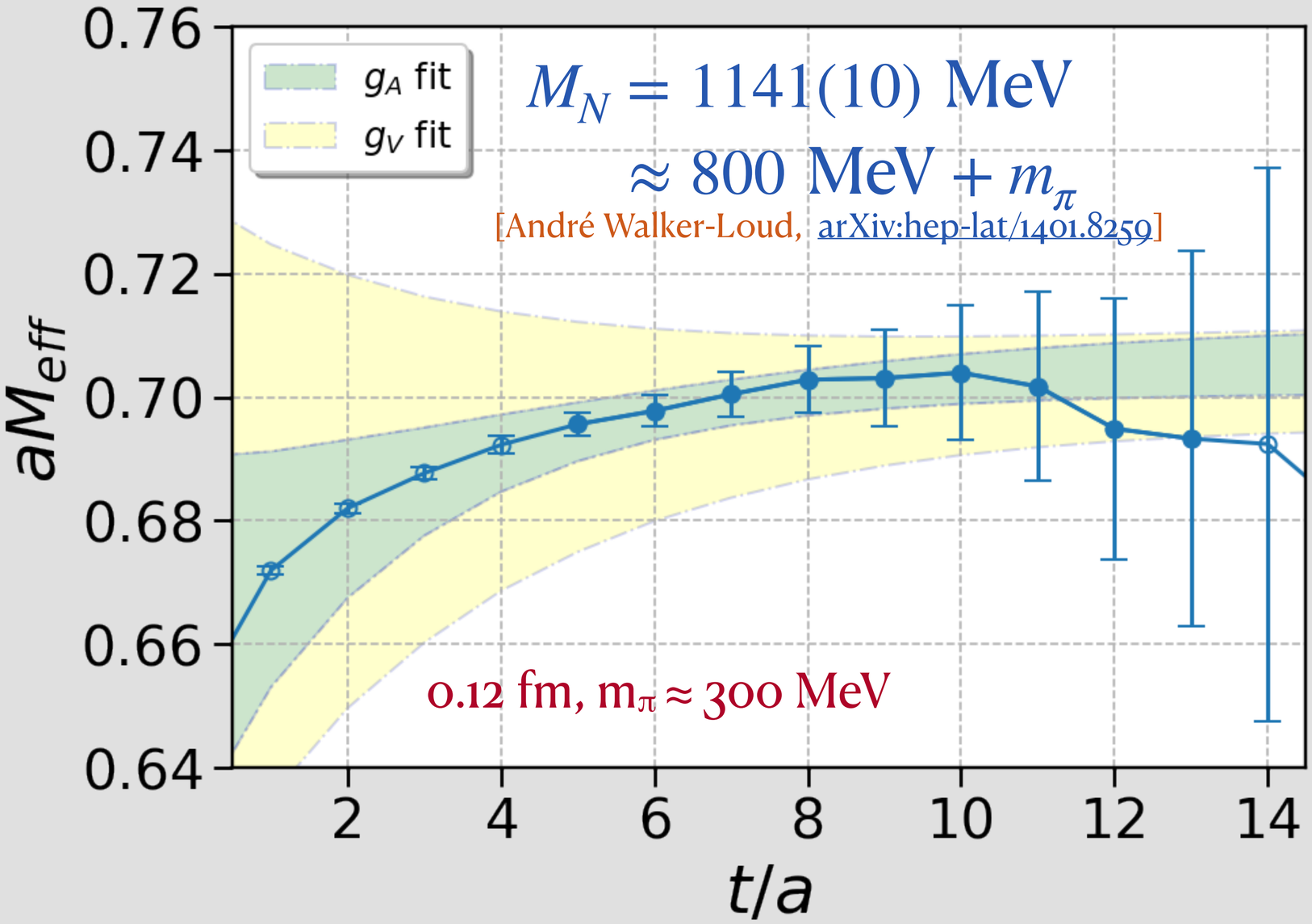
Three-point Sanity Check: g_A

$$\frac{M^{V,A}_{16,-0}}{M^{V,A}_{16,+0}} = -3$$

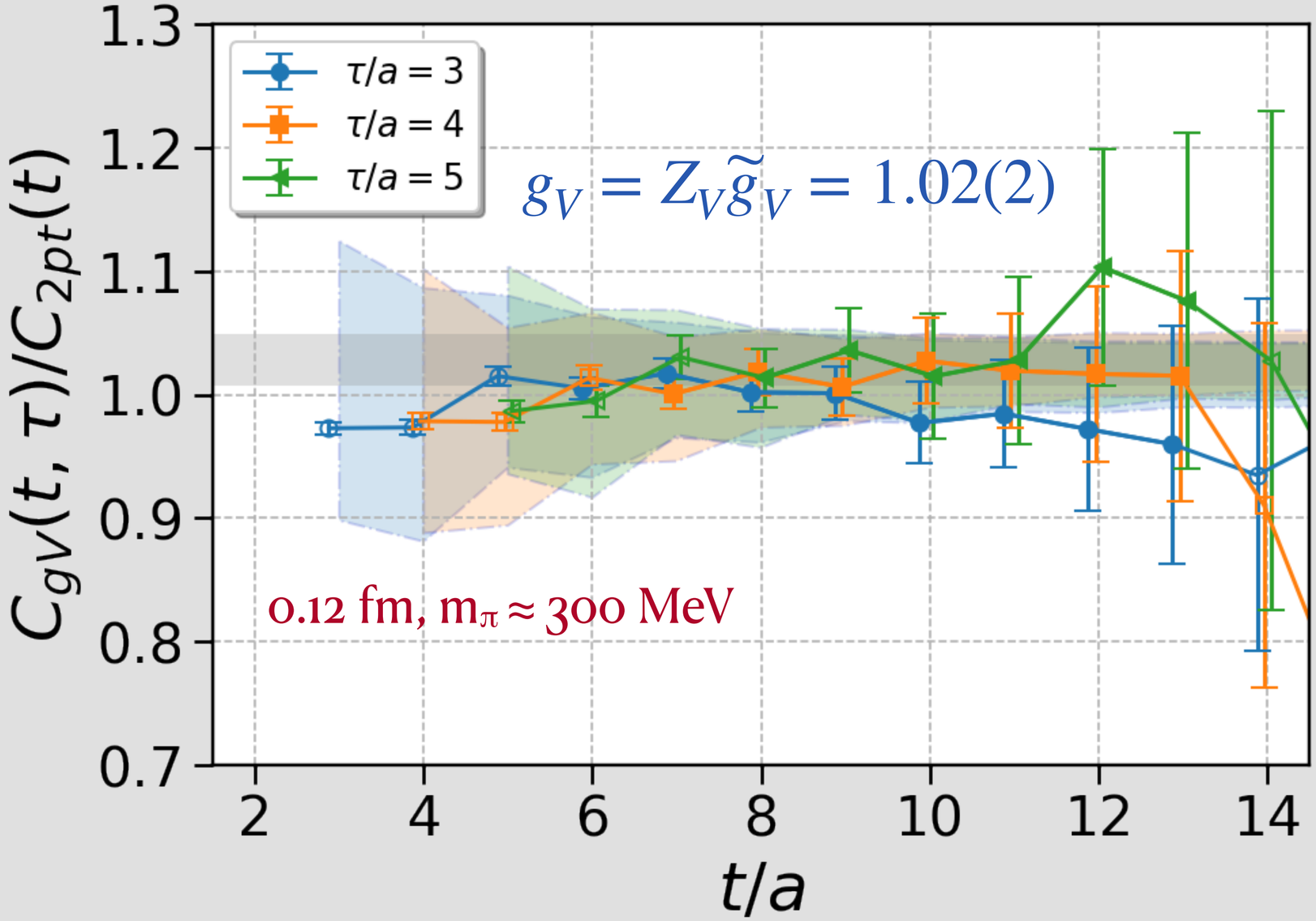
$$\frac{M^{A}_{16,-0}}{M^{A}_{16,+0}} \approx C^{A}_{16,-0}/C^{A}_{16,+0}$$



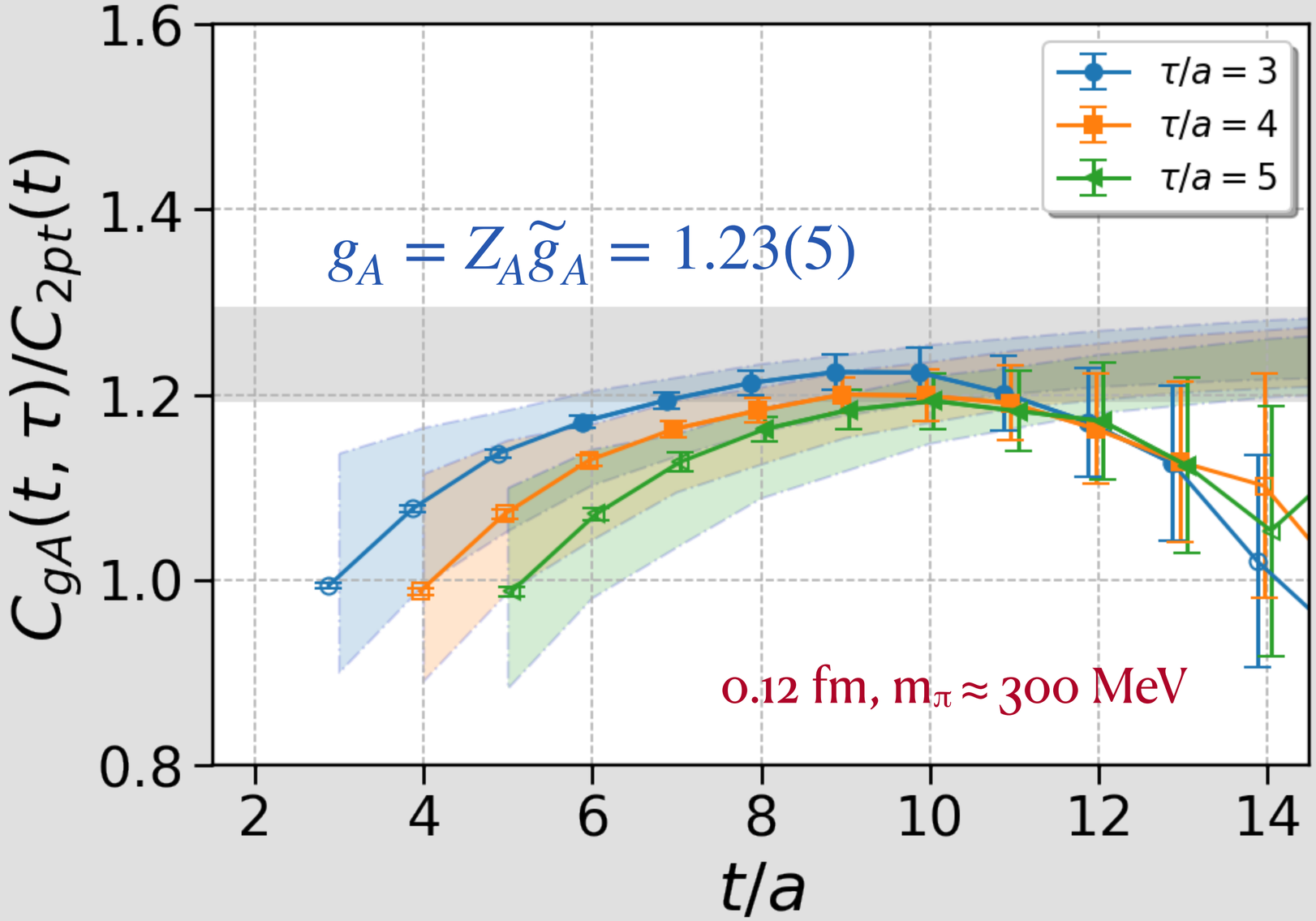
Two-point Correlators



Three-point Correlators: g_V



Three-point Correlators: g_A



Summary and Outlook

- Feasible to calculate matrix elements with staggered fermions
- Extending g_A to multiple lattice spacings at physical pion masses
- Extending to nonzero momentum transfer for the form factors (ongoing!)