

Higgs Alignment and the Top Quark

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Outline:

1. A long introduction.
2. A short getting to the point.
3. Comments on experimental consequences.

Higgs Alignment and the Top Quark: The Intro

There is a surprising connection between the top quark and Higgs alignment in Gildener-Weinberg multi-Higgs doublet models. Were it not for the top quark, its large mass, and the Glashow-Weinberg constraint on quark-Higgs couplings, the coupling of the 125 GeV Higgs to gauge bosons and fermions in such models would be indistinguishable from that of the Standard Model Higgs. The top quark's coupling to a single Higgs doublet breaks perfect alignment, but the effect is still small, $< O(1\%)$, and probably experimentally inaccessible. An experimental consequence of this alignment is that many popular searches for Beyond-Standard-Model Higgs bosons will remain fruitless.

Every LHC measurement of H(125)'s couplings to gauge bosons:

$$H \longleftrightarrow WW^*, ZZ^*, \gamma\gamma \text{ and } gg \rightarrow H$$

and to fermions:

$$H \rightarrow \bar{t}t, \bar{b}b, \tau^+\tau^-, \mu^+\mu^-$$

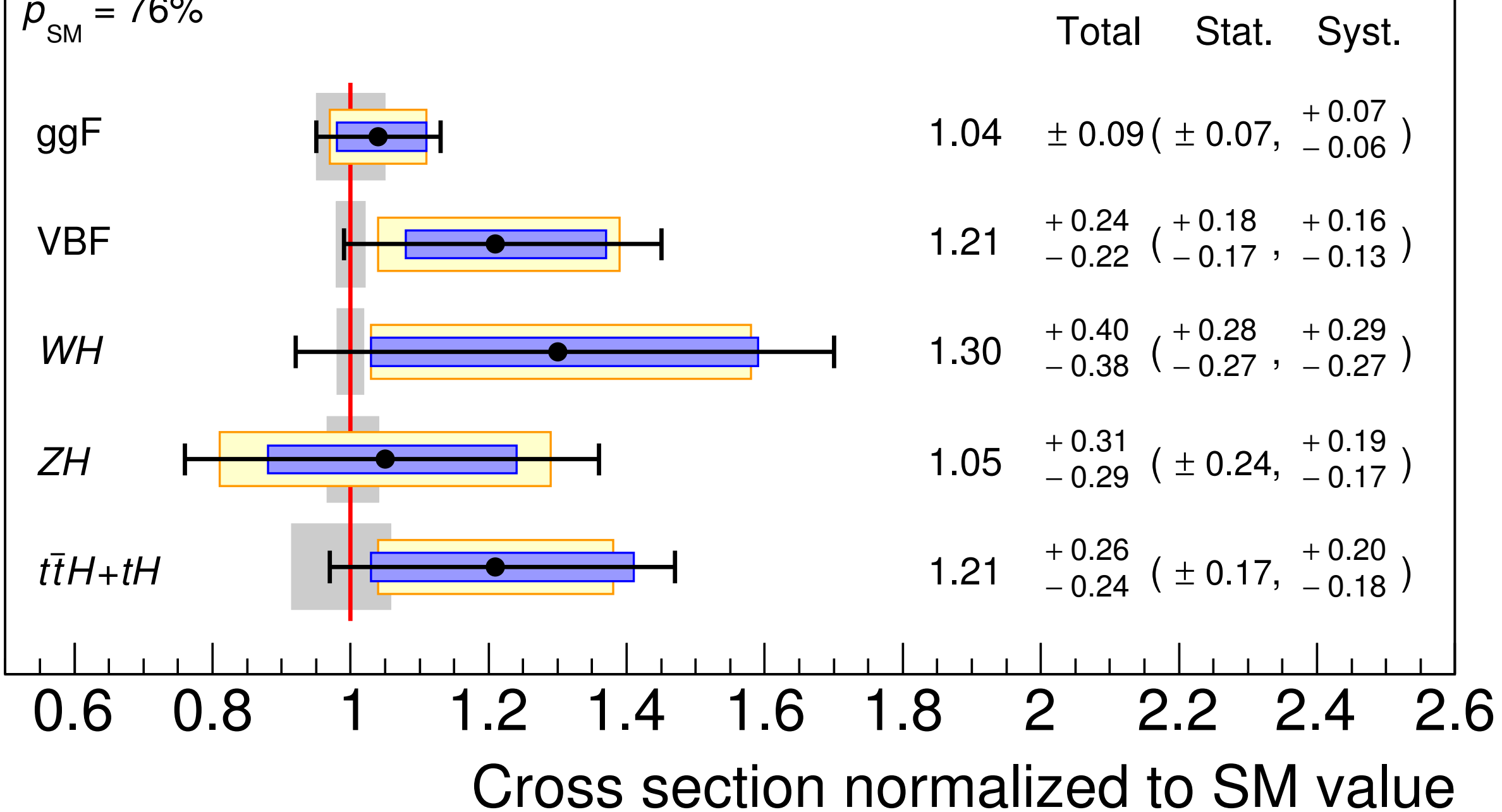
is consistent with its being the single Higgs boson of the SM:

ATLAS

$\sqrt{s} = 13 \text{ TeV}, 24.5 - 79.8 \text{ fb}^{-1}$

$m_H = 125.09 \text{ GeV}, |y_H| < 2.5$

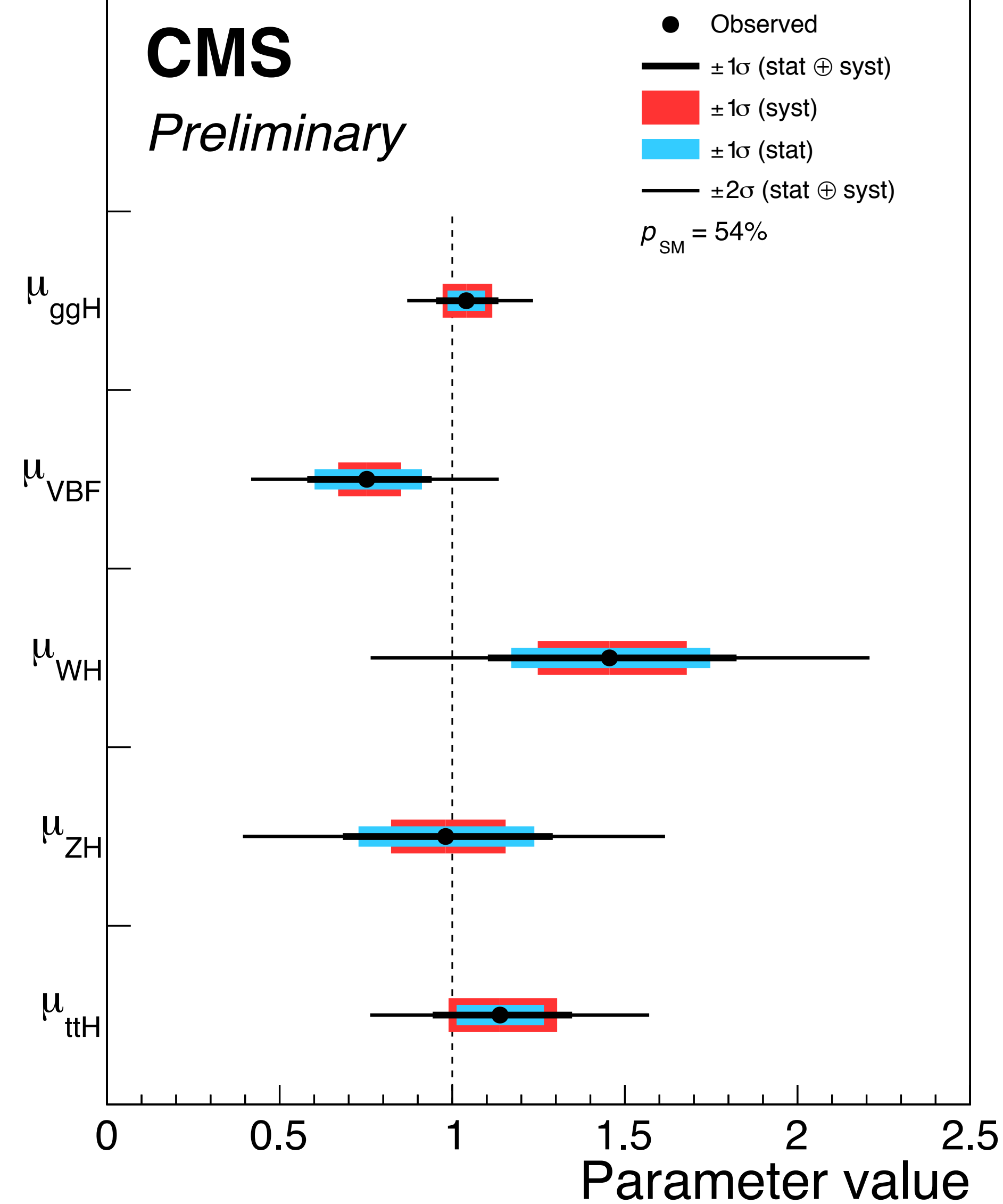
$p_{\text{SM}} = 76\%$

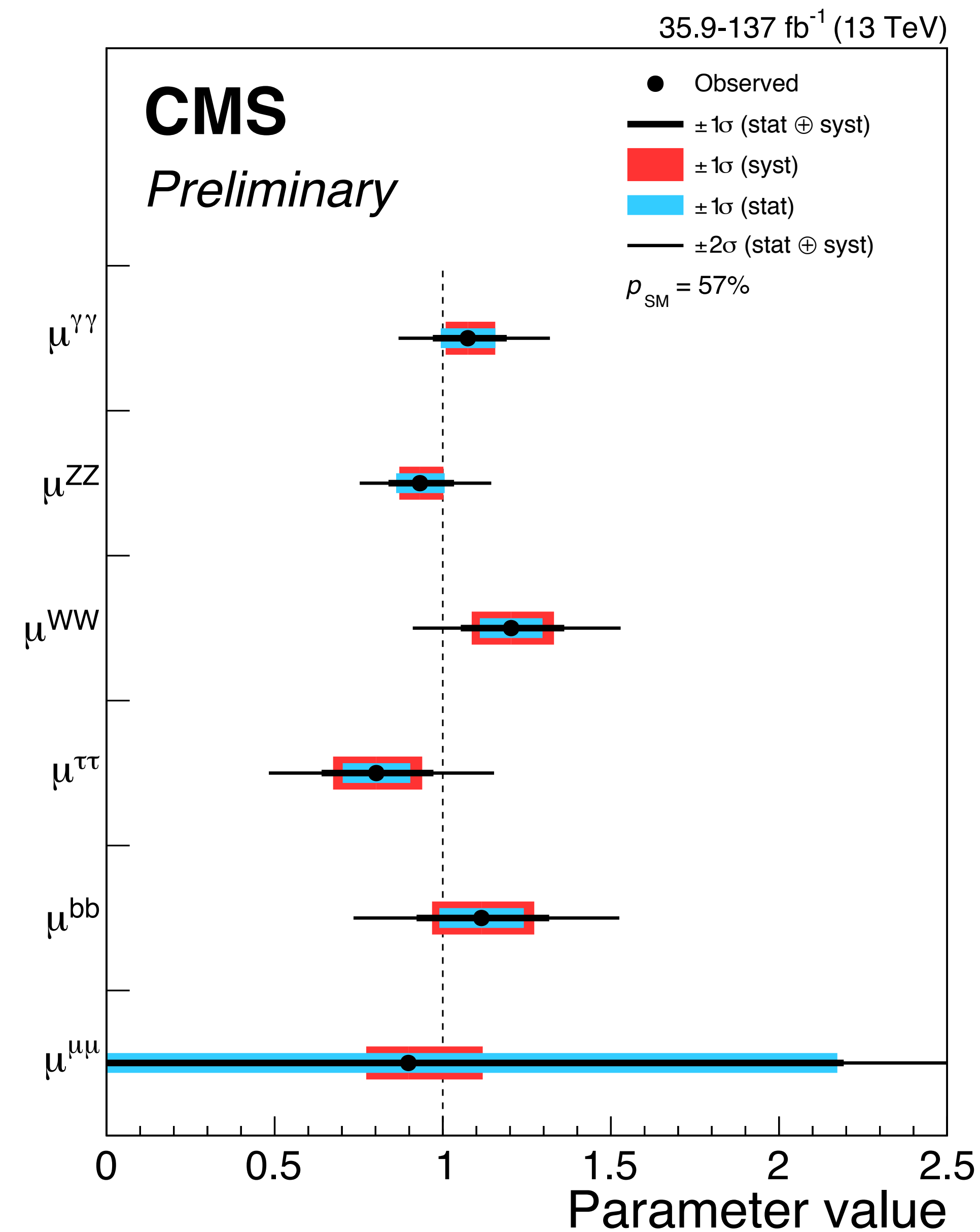
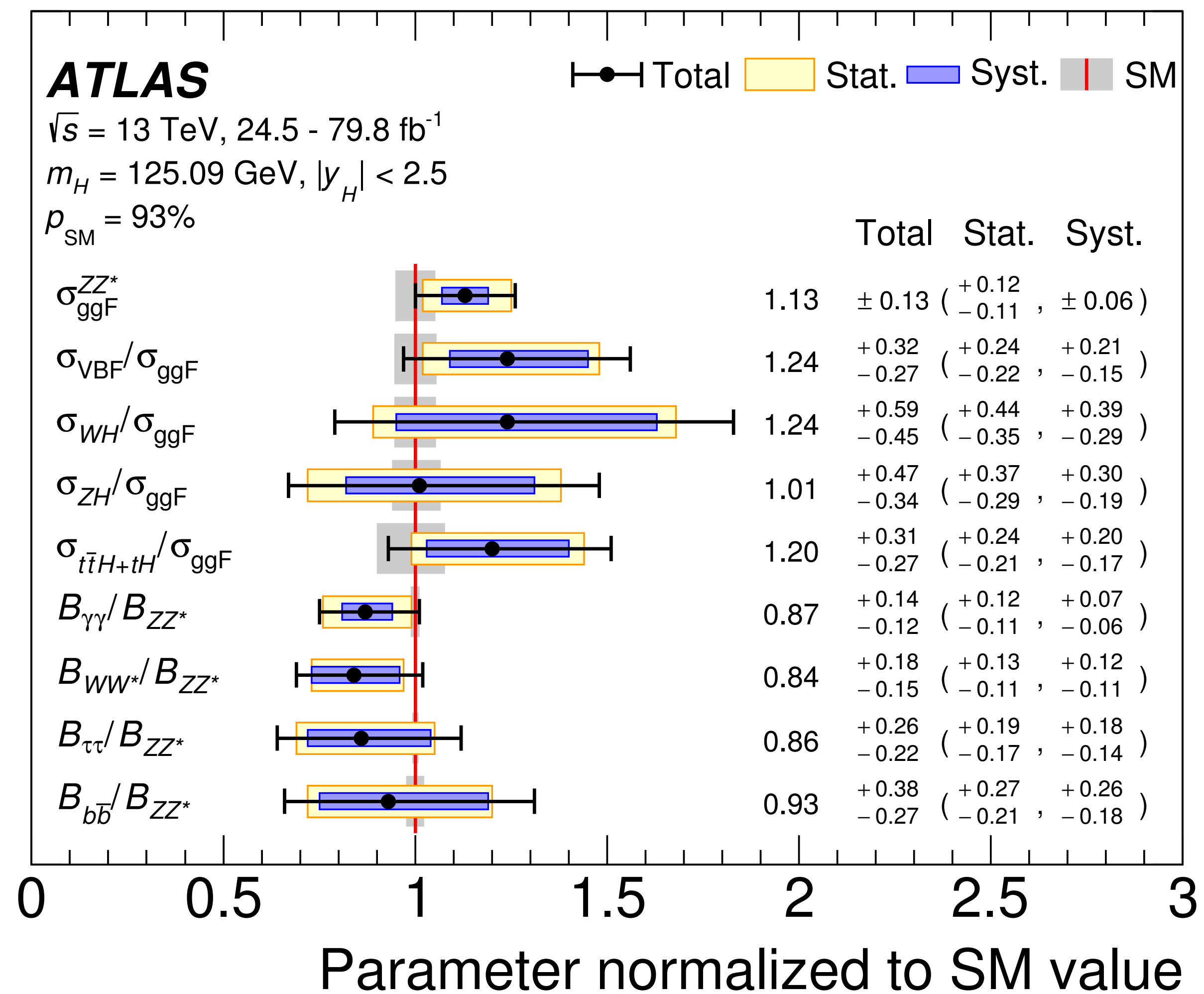


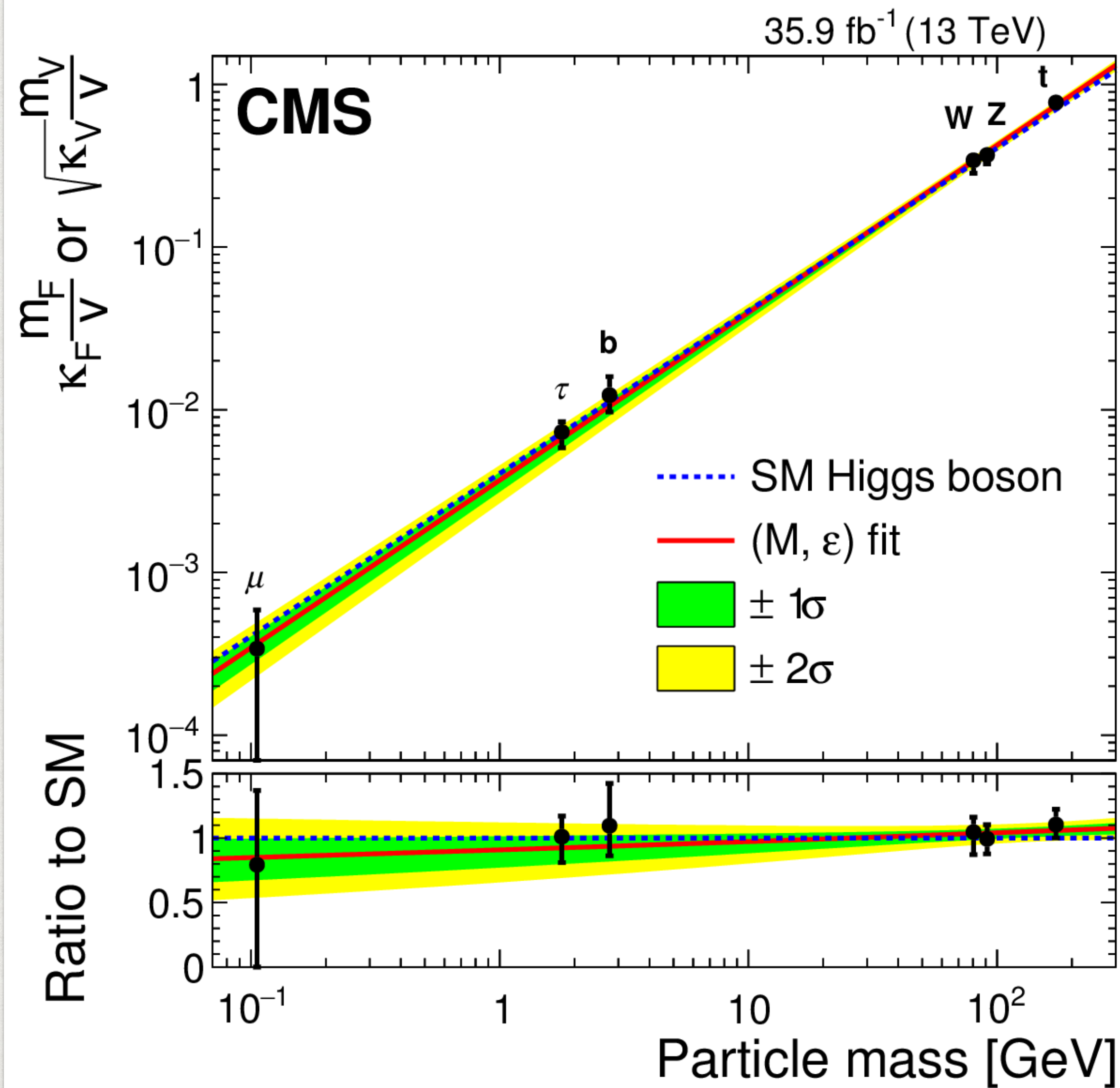
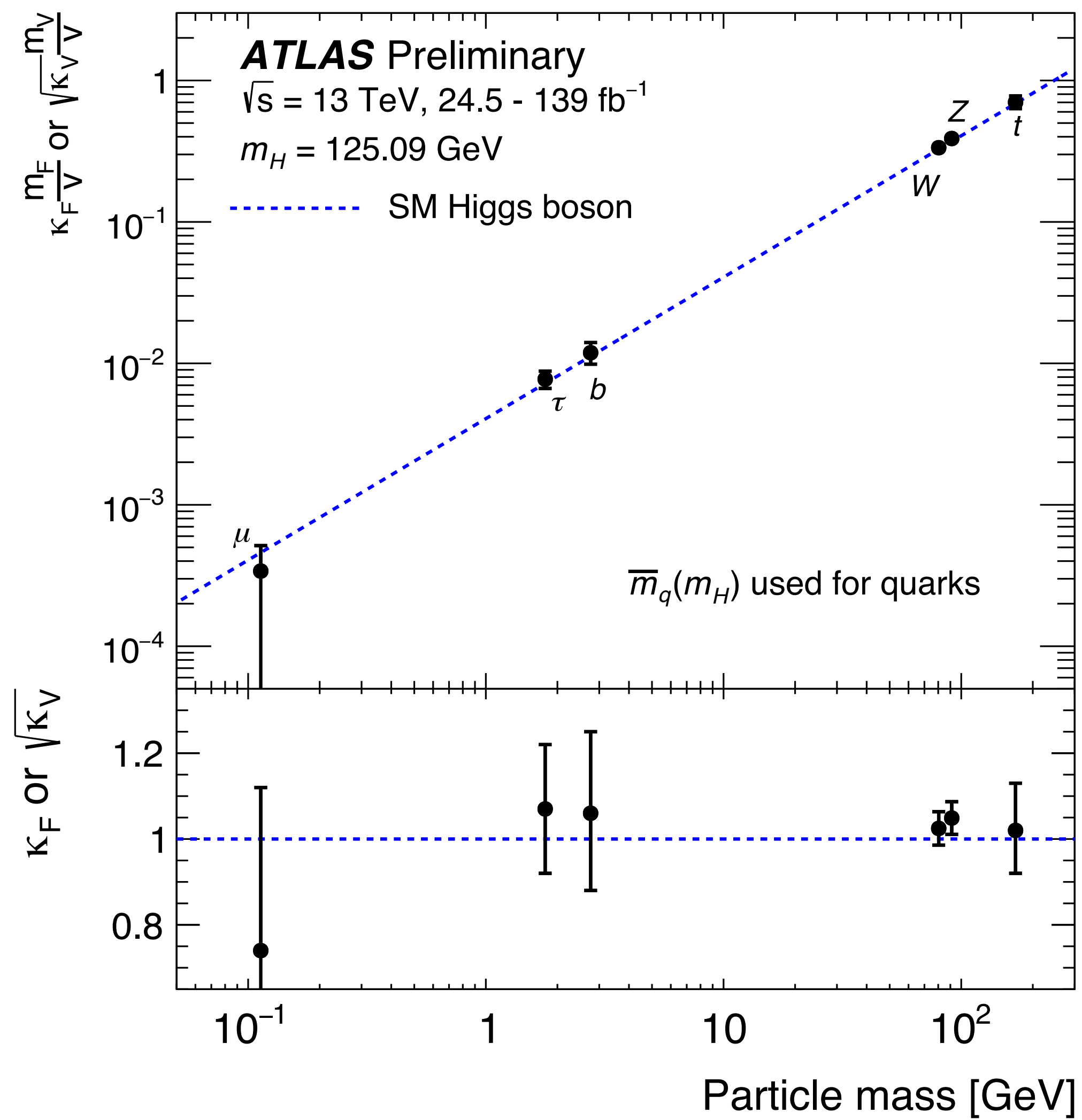
CMS

Preliminary

35.9-137 fb^{-1} (13 TeV)







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The usual answer is Higgs alignment: Something — originally decoupling of heavier Higgses (Boudjema & Semenov, PRD 66, 095007; Gunion & Haber, PRD 67, 075019) — causes the lightest CP-even H to be the linear combination

$$H \simeq \sum_i v_i \rho_i / v \quad \text{where} \quad v = \sqrt{\sum_i v_i^2}$$

(Note: I'm assuming N Higgs doublets b/c of the rho parameter.)

BUT — is decoupling natural? Is there a global symmetry to prevent large radiative corrections to alignment? There have been a few proposals, but the symmetries are rather elaborate and artificial, or related to supersymmetry.

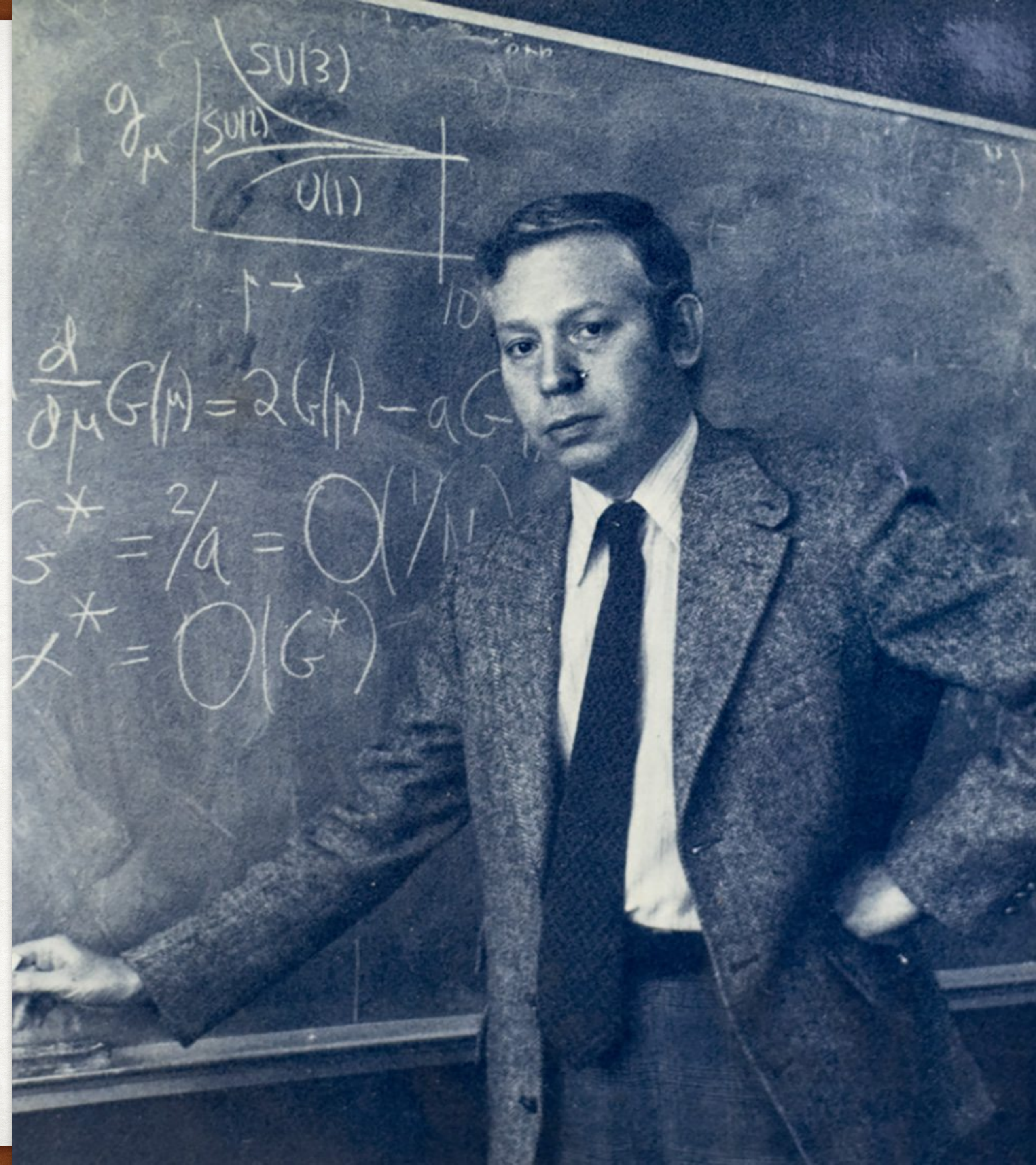
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There is one exception and, to me, it seems very attractive and very simple: The Higgs is a (pseudo-) Goldstone boson of spontaneously broken scale invariance. Then, Higgs alignment is automatic! (In tree approximation, it is exact.) And this has been in front of us since 1976:

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E. Gildener and S. Weinberg, PRD 13, 3333 (1976).



Consider an N-Higgs-doublet model (NHDM):

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_i^+ \\ \rho_i + ia_i \end{pmatrix}, \quad i = 1, 2, \dots, N$$

The Goldstone bosons eaten by W and Z are

$$w^\pm = \sum_{i=1}^N v_i \phi_i^\pm / v, \quad z = \sum_{i=1}^N v_i a_i / v \quad \text{where } v_i = \langle \rho_i \rangle \text{ and } v = \sqrt{\sum_i v_i^2}$$



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In GW-NHDM, H has the same form:

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It's aligned!

This has profound consequences
for BSM Higgs searches at LHC.
Discussed later.

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The GW-2HDM

The key assumption of GW models is that the classical Lagrangian is scale-invariant: the Higgs potential V_0 is purely quartic and all fermion hard masses arise from their dim'n-4 Yukawa couplings to Higgs bosons. (Hence the need for complex Higgs doublets (SW, PRL 19, 1264, 1967) and no vectorlike quarks or leptons with electroweak interactions!)

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We use a simple N=2 HDM with a CP-invariant quartic potential (Lee & Pilaftsis, PRD 86, 035004; also W. Shepherd & K.L., PRD 99, 055015):

$$V_0 = \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]$$

where $\lambda_i = \lambda_i^*$; $\lambda_1 > 0, \lambda_2 > 0$.

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V_0 is a homogeneous polynomial of degree 4:

$$\implies V_0 = \frac{1}{4} \sum_{i=1}^2 \left[\Phi_i^\dagger \frac{\partial V_0}{\partial \Phi_i^\dagger} + \frac{\partial V_0}{\partial \Phi_i} \Phi_i \right] \implies V_0 = 0 \text{ at } \underline{\text{all}} \text{ extrema.}$$

(E. Pilon & K.L., PRD **101**, 055032)

$\Phi_1 = \Phi_2 = 0$ is the trivial "minimum" of V_0 .

But a nontrivial minimum of V_0 can occur on the ray

$$\Phi_{1\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \cos \beta \end{pmatrix}, \quad \Phi_{2\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \sin \beta \end{pmatrix}$$

where $0 < \phi < \infty$ and $0 < \beta < \pi/2$.

The nontrivial extremal conditions are

$$\lambda_1 + \frac{1}{2} \lambda_{345} \tan^2 \beta = \lambda_2 + \frac{1}{2} \lambda_{345} \cot^2 \beta = 0;$$

$$\lambda_1, \lambda_2 > 0 \implies \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 < 0.$$

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N.B.: The zeroth-order extremal conditions $(\partial V_0 / \partial \rho_1)_{\Phi_{i\beta}} = (\partial V_0 / \partial \rho_2)_{\Phi_{i\beta}} = 0$ remain true in all orders in the loop expansion of the effective potential.

It is convenient to use the "aligned basis":

$$\Phi = \Phi_1 c_\beta + \Phi_2 s_\beta \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} w^+ \\ H + iz \end{pmatrix} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

$$\Phi' = -\Phi_1 s_\beta + \Phi_2 c_\beta \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H' + iA \end{pmatrix} \implies \langle \Phi' \rangle = 0$$

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N.B.: $H = \rho_1 c_\beta + \rho_2 s_\beta \equiv (\rho_1 \phi_1 + \rho_2 \phi_2)/\phi$ is aligned!

Tree-level Higgs “mass” matrices are diagonal in this basis —

$$\mathcal{M}_{0-}^2 = \begin{pmatrix} M_z^2 & 0 \\ 0 & M_A^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_5 \phi^2 \end{pmatrix}$$

$$\mathcal{M}_{\pm}^2 = \begin{pmatrix} M_{w^\pm}^2 & 0 \\ 0 & M_{H^\pm}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \lambda_{45} \phi^2 \end{pmatrix}$$

$$\mathcal{M}_{0+}^2 = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_{H'}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345} \phi^2 \end{pmatrix}$$

The ray is a flat minimum if, like $\lambda_{345} = -2\lambda_1 \cot^2 \beta < 0$, $\lambda_5 < 0$ and $\lambda_{45} = \lambda_4 + \lambda_5 < 0$. And H is the dilaton of spontaneously broken scale invariance.

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This means that H keeps its very nearly SM couplings thru $O(V_2)$!

(E. Eichten and K.L., in preparation)

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Spoiler alert: H acquires +ve mass-squared in $O(V_1)$ — next.

Higgs Alignment and the Top Quark: The Result

To establish the top quark's role in Higgs alignment, it suffices to look at the one-loop effective potential of the GW -2HDM:

$$V_1 = \frac{1}{64\pi^2} \sum_n \alpha_n \overline{M}_n^4 \left(\ln \frac{\overline{M}_n^2}{\Lambda_{GW}^2} - k_n \right) \quad (\text{S. Martin, PRD 65,116003})$$

$$(\alpha_n, k_n) = (6, \frac{5}{6}), (3, \frac{5}{6}), (-12, \frac{3}{2}), (1, \frac{3}{2}), (1, \frac{3}{2}), (2, \frac{3}{2})$$

for $n = W^\pm, Z, t, H', A, H^\pm$.

N.B.: The renormalization scale Λ_{GW} explicitly breaks scale invariance.

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The field-dependent masses are (R. Jackiw, PRD 9, 1686; L.-P., ibid):

$$\overline{M}_n^2 = \begin{cases} M_n^2 (2 (\Phi^\dagger \Phi + \Phi'^\dagger \Phi') / \phi^2) = M_n^2 ((H^2 + H'^2 + \dots) / \phi^2), & n \neq t \\ M_t^2 (2 \Phi_1^\dagger \Phi_1 / (\phi^2 c_\beta^2)) = M_t^2 ((H c_\beta - H' s_\beta)^2 + \dots) / (\phi^2 c_\beta^2), \end{cases}$$

where $M_W^2 = \frac{1}{4} g^2 \phi^2$, $M_{H'}^2 = -\lambda_{345} \phi^2$, $M_t^2 = \frac{1}{2} \Gamma_t^2 \phi^2 c_\beta^2$, etc.

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The different top-mass term ("Type 1") is dictated by the Glashow-Weinberg (PRD 15 1958 (1977)) criterion for avoiding FCNC via neutral Higgs exchange. It makes all the difference between experimentally perfect and approximate alignment.

Following Gildener-Weinberg, the one-loop extremal conditions are (recall that tree-level extremal conditions remain in force):

$$0 = \left. \frac{\partial(V_0 + V_1)}{\partial H} \right|_{\langle \rangle + \delta_1 H + \delta_1 H'} = \left. \frac{\partial V_1}{\partial H} \right|_{\langle \rangle} + \mathcal{O}(V_2),$$

$$0 = \left. \frac{\partial(V_0 + V_1)}{\partial H'} \right|_{\langle \rangle + \delta_1 H + \delta_1 H'} = \left. \frac{\partial^2 V_0}{\partial H'^2} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial V_1}{\partial H'} \right|_{\langle \rangle} + \mathcal{O}(V_2).$$

where $\langle \rangle$ means $\langle H \rangle = \phi$, $\langle H' \rangle = 0$.

$$\mathbf{H} : 0 = \left. \frac{\partial V_1}{\partial H} \right|_{\langle \rangle} = \frac{1}{64\pi^2} \left[\sum_n \alpha_n \frac{\partial \overline{M}_n^2}{\partial H} \overline{M}_n^2 \left(\ln \frac{\overline{M}_n^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_n \right) \right]_{\langle \rangle}$$

$$\left. \frac{\partial \overline{M}_n^2}{\partial H} \right|_{\langle \rangle} = \begin{cases} 2M_n^2 H / \phi^2 \big|_{\langle \rangle} = 2M_n^2 / \phi & (n \neq t) \\ 2M_t^2 (H c_\beta - H' s_\beta) / \phi^2 c_\beta \big|_{\langle \rangle} = 2M_t^2 / \phi \end{cases}$$

$$\Rightarrow 0 = \left. \frac{\partial V_1}{\partial H} \right|_{\langle \rangle} \propto \sum_n \alpha_n M_n^4 \left(\ln \frac{M_n^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_n \right) = A + B \left(\ln \frac{v^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} \right)$$

$$\text{where } A = \sum_n \alpha_n M_n^4 \left(\ln \frac{M_n^2}{\underset{\uparrow}{v^2}} - k_n \right) \text{ and } B = \sum_n \alpha_n M_n^4.$$

This fixes the scale $\Lambda_{\text{GW}} = v \exp [(A + \frac{1}{2}B)/2B]$ in terms of $\phi = v$ at which V_1 has a minimum:

Identify $\langle H \rangle = v = 2^{-1/4} G_F^{-1/2} = 246 \text{ GeV}$, and $M_W^2 = \frac{1}{4} g^2 v^2$, etc.

$$V_1|_{\langle \rangle} = \frac{1}{64\pi^2} \sum_n \alpha_n M_n^4 \left[\left(\ln \frac{M_n^2}{v^2} - k_n \right) + \ln \frac{v^2}{\Lambda_{\text{GW}}^2} \right]$$

$$= \frac{1}{64\pi^2} \left(A + B \ln \frac{v^2}{\Lambda_{\text{GW}}^2} \right) = -\frac{B}{128\pi^2}$$

This is a deeper minimum (< 0) than $V_0 = 0$ iff

$$B = \sum_n \alpha_n M_n^4 = 6M_W^4 + 3M_Z^4 + 2M_{H^\pm}^4 + M_A^4 + M_{H'}^4 - 12M_t^4 > 0.$$

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We'll see that it is.

$$\begin{aligned}
H' : 0 &= \left. \frac{\partial(V_0 + V_1)}{\partial H'} \right|_{\langle \rangle + \delta_1 H + \delta_1 H'} \\
&= \underbrace{\left. \frac{\partial^2 V_0}{\partial H \partial H'} \right|_{\langle \rangle}}_{\mathcal{M}_{HH'}^2 = 0 \text{ } (\mathcal{O}(V_0))} \delta_1 H + \underbrace{\left. \frac{\partial^2 V_0}{\partial H'^2} \right|_{\langle \rangle}}_{\mathcal{M}_{H'H'}^2 = M_{H'}^2} \delta_1 H' + \left. \frac{\partial V_1}{\partial H'} \right|_{\langle \rangle} \\
&\mathcal{M}_{HH'}^2 = 0 \text{ } (\mathcal{O}(V_0)) \quad \mathcal{M}_{H'H'}^2 = M_{H'}^2 = -\lambda_{345} \phi^2
\end{aligned}$$

$$\Rightarrow \text{the shift of } \langle H' \rangle \text{ from zero in } \mathcal{O}(V_1) \text{ is } \delta_1 H' = -\frac{1}{M_{H'}^2} \left. \frac{\partial V_1}{\partial H'} \right|_{\langle \rangle}$$

$$\left. \frac{\partial \overline{M}_n^2}{\partial H'} \right|_{\langle \rangle} = \begin{cases} 2M_n^2 H' / \phi^2 \big|_{\langle \rangle} = 0 & (n \neq t) \\ 2M_t^2 (H' s_\beta - H c_\beta) s_\beta / (\phi c_\beta)^2 \big|_{\langle \rangle} = -2M_t^2 \tan \beta / \phi \end{cases}$$

$$\Rightarrow \delta_1 H' = \frac{\alpha_t M_t^4 \tan \beta}{16\pi^2 M_{H'}^2 v} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_t \right) \quad (\text{at } \phi = v)$$

Typically, $\delta_1 H' = 1\text{-}3 \text{ GeV}$, a tiny increase in $\sqrt{v^2 + (\delta_1 H')^2}$.

(We use $\tan \beta = 0.5$. See W. Shepherd & K.L., PRD **99**, 055015.)

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Typically, $\delta_1 H' = 1\text{-}3 \text{ GeV}$, a tiny increase in $\sqrt{v^2 + (\delta_1 H')^2}$.

$\delta_1 H'$ establishes the connection of the top quark to Higgs alignment:

If $\delta_1 H' = 0$, $H = \rho_1 c_\beta + \rho_2 s_\beta$ is still a mass eigenstate.

→ Large $M_t \Rightarrow$ its appearance in V_1 .

→ The Glashow-Weinberg no-FCNC $\Rightarrow \delta_1 H' \neq 0$ in $\mathcal{O}(V_1)$.

The $\mathcal{O}(V_1)$ elements of \mathcal{M}_{0+}^2 further emphasize this connection:

$$\mathcal{M}_{HH}^2 = \left. \frac{\partial^2 V_1}{\partial H^2} \right|_{\langle \rangle} = \frac{\sum_n \alpha_n M_n^4}{8\pi^2 v^2} \equiv \frac{B}{8\pi^2 v^2},$$

$$\begin{aligned} \mathcal{M}_{HH'}^2 &= \left. \frac{\partial^3 V_0}{\partial H \partial H'^2} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial^2 V_1}{\partial H \partial H'} \right|_{\langle \rangle} \\ &= -\frac{\alpha_t M_t^4 \tan \beta}{16\pi^2 v^2} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{5}{2} - k_t \right), \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{H'H'}^2 &= \left. \frac{\partial^2 V_0}{\partial H'^2} \right|_{\langle \rangle} + \left. \frac{\partial^3 V_0}{\partial H'^3} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial^2 V_1}{\partial H'^2} \right|_{\langle \rangle} \\ &= M_{H'}^2 + \frac{\alpha_t M_t^4 \tan \beta}{8\pi^2 v^2} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_t + \tan^2 \beta \right). \end{aligned}$$

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← Small and, strictly speaking,
does not enter M_H^2 until $\mathcal{O}(V_2)$.
($\Rightarrow H$ - H' mixing angle $\delta < 1\%$)

$$\mathcal{M}_{H'H'}^2 = \left. \frac{\partial^2 V_0}{\partial H'^2} \right|_{\langle \rangle} + \left. \frac{\partial^3 V_0}{\partial H'^3} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial^2 V_1}{\partial H'^2} \right|_{\langle \rangle}$$

$$= M_{H'}^2 + \frac{\alpha_t M_t^4 \tan \beta}{8\pi^2 v^2} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_t + \tan^2 \beta \right).$$

The $\mathcal{O}(V_1)$ elements of \mathcal{M}_{0+}^2 further emphasize this connection:

$$\mathcal{M}_{HH}^2 = \left. \frac{\partial^2 V_1}{\partial H^2} \right|_{\langle \rangle} = \frac{\sum_n \alpha_n M_n^4}{8\pi^2 v^2} \equiv \frac{B}{8\pi^2 v^2}, \quad \begin{aligned} \therefore, \mathcal{M}_{HH}^2 &\cong M_H^2 = (125 \text{ GeV})^2 \\ \Rightarrow V_1|_{\langle \rangle} &= -\frac{v^2 M_H^2}{16} < V_0|_{\langle \rangle} = 0 \\ &\text{(as promised)} \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{HH'}^2 &= \left. \frac{\partial^3 V_0}{\partial H \partial H'^2} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial^2 V_1}{\partial H \partial H'} \right|_{\langle \rangle} \\ &= -\frac{\alpha_t M_t^4 \tan \beta}{16\pi^2 v^2} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{5}{2} - k_t \right), \end{aligned} \quad \begin{aligned} \leftarrow \text{Small and, strictly speaking,} \\ \text{does not enter } M_H^2 \text{ until } \mathcal{O}(V_2). \\ (\Rightarrow H\text{-}H' \text{ mixing angle } \delta < 1\%) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{H'H'}^2 &= \left. \frac{\partial^2 V_0}{\partial H'^2} \right|_{\langle \rangle} + \left. \frac{\partial^3 V_0}{\partial H'^3} \right|_{\langle \rangle} \delta_1 H' + \left. \frac{\partial^2 V_1}{\partial H'^2} \right|_{\langle \rangle} \\ &= M_{H'}^2 + \frac{\alpha_t M_t^4 \tan \beta}{8\pi^2 v^2} \left(\ln \frac{M_t^2}{\Lambda_{\text{GW}}^2} + \frac{1}{2} - k_t + \tan^2 \beta \right). \end{aligned}$$

Higgs Alignment: Experimental consequences

ATLAS & CMS discovered $H(125)$ relatively easily because of its rather strong coupling to WW and ZZ : production via WW and ZZ fusion and decay to WW^* and ZZ^* , ($ZZ^* \rightarrow 4$ leptons was much more convincing than $\gamma\gamma$). gg fusion via the top quark loop is important because the Htt coupling is "full strength" M_t/v

Thus, many searches for new Beyond-Standard-Model (BSM) Higgs bosons rely on the lamp-post strategy: Assume $VV \rightarrow H'$ and $H' \rightarrow VV$ are important modes of the H' and that the $H'tt$ coupling is full strength.

For example (quickly) —

ATLAS: Search BSM $H \rightarrow ZZ \rightarrow 4l + ll\nu\nu$ Search BSM $H \rightarrow WW \rightarrow l\nu l\nu$
VH all hadronic resonance search Search BSM $H \rightarrow ZZ \rightarrow 4l$ and $ll\nu\nu$
Search BSM $h \rightarrow 2a \rightarrow 4b$ Search BSM $H \rightarrow ZZ$
Search for heavy resonances decaying to VV in the semileptonic final states
Search BSM $h(125) \rightarrow Za, a \rightarrow \text{jet}$ VV/VH and $ll/l\nu$ search combination 13 TeV 2016

CMS: Search for a heavy Higgs boson decaying to a pair of W bosons
in proton-proton collisions at $\sqrt{s} = 13$ TeV

Search for a heavy pseudoscalar Higgs boson decaying into a 125 GeV Higgs boson
and a Z boson in final states with two tau and two light leptons at $\sqrt{s} = 13$ TeV

Search for a new scalar resonance decaying to a pair of
 Z bosons in proton-proton collisions at $\sqrt{s} = 13$ TeV

Search for charged Higgs bosons produced via vector boson fusion and decaying
into a pair of W and Z bosons using proton-proton collisions at $\sqrt{s} = 13$ TeV

Drell-Yan and VV-fusion processes:

$$\begin{aligned}
 \mathcal{L}_{EW} = & ieH^- \overleftrightarrow{\partial}_\mu H^+ (A^\mu + Z^\mu \cot 2\theta_W) \\
 & + \frac{e}{\sin 2\theta_W} A \overleftrightarrow{\partial}_\mu (\overset{\uparrow}{H_1} \overset{\uparrow}{\sin} \delta - H_2 \cos \delta) Z^\mu \\
 & + \frac{ie}{2\sin \theta_W} \left(H^- \overleftrightarrow{\partial}_\mu (\overset{\uparrow}{H_1} \overset{\uparrow}{\sin} \delta - H_2 \cos \delta + iA) W^{+,\mu} - \text{h.c.} \right) \\
 & + (H_1 \cos \delta + \overset{\uparrow}{H_2} \overset{\uparrow}{\sin} \delta) \left(\frac{eM_W}{\sin \theta_W} W^{+,\mu} W_\mu^- + \frac{eM_Z}{\sin 2\theta_W} Z^\mu Z_\mu \right)
 \end{aligned}$$

$$|\delta| \lesssim 1\%$$

$$H_1 = Hc_\delta - H's_\delta, \quad H_2 = Hs_\delta + H'c_\delta, \quad \text{where} \quad \tan 2\delta = \frac{2\mathcal{M}_{HH'}^2}{\mathcal{M}_{H'H'}^2 - \mathcal{M}_{HH}^2}$$

$$M_{H_1}^2 = \mathcal{M}_{HH}^2 c_\delta^2 + \mathcal{M}_{H'H'}^2 s_\delta^2 - 2\mathcal{M}_{HH'}^2 s_\delta c_\delta$$

$$M_{H_2}^2 = \mathcal{M}_{HH}^2 s_\delta^2 + \mathcal{M}_{H'H'}^2 c_\delta^2 + 2\mathcal{M}_{HH'}^2 s_\delta c_\delta$$

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 & + (H_1 \cos \delta + \overset{\uparrow}{H_2} \overset{\uparrow}{\sin} \delta) \left(\frac{eM_W}{\sin \theta_W} W^{+,\mu} W_\mu^- + \frac{eM_Z}{\sin 2\theta_W} Z^\mu Z_\mu \right)
 \end{aligned}$$

$$|\delta| \lesssim 1\%$$

gb and gg-fusion processes:

$$\begin{aligned}
 \mathcal{L}_Y = & \frac{\sqrt{2} \tan \beta}{v} \sum_{k,l=1}^3 \left[\overset{\uparrow}{H^+} (\bar{u}_{kL} V_{kl} m_{d_l} d_{lR} - \bar{u}_{kR} \overset{\uparrow}{m_{u_k}} V_{kl} d_{lL} + m_{\ell_k} \bar{\nu}_{kL} \ell_{kR} \delta_{kl}) + \text{h.c.} \right] \\
 & - \left(\frac{v \cos \beta + \overset{\uparrow}{H_1} \overset{\uparrow}{\cos} \beta' - H_2 \overset{\uparrow}{\sin} \beta'}{v \cos \beta} \right) \sum_{k=1}^3 (\overset{\uparrow}{m_{u_k}} \bar{u}_k u_k + m_{d_k} \bar{d}_k d_k + m_{\ell_k} \bar{\ell}_k \ell_k) \\
 & - \frac{\overset{\uparrow}{iA} \overset{\uparrow}{\tan} \beta}{\overset{\uparrow}{v}} \sum_{k=1}^3 (\overset{\uparrow}{m_{u_k}} \bar{u}_k \gamma_5 u_k - m_{d_k} \bar{d}_k \gamma_5 d_k - m_{\ell_k} \bar{\ell}_k \gamma_5 \ell_k) .
 \end{aligned}$$

$$\tan \beta \lesssim 0.50$$

$$\beta' = \beta - \delta \cong \beta$$

The limit $\tan \beta \lesssim 0.50$ comes from the search for $gg \rightarrow \bar{t}H^+ + c.c. \rightarrow t\bar{t}b\bar{b}$

It has not been improved upon since 2018 (Run 1!).

The reason is the large top-quark background at low masses.

gb and gg-fusion processes:

$$\tan \beta \lesssim 0.50$$

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Is it all so bleak?

NO! There is another result of the GW -2HDM (in fact, all such models) that has very important consequences —

The one-loop sum rule for the new Higgs boson masses

$$M_H^2 = \frac{B}{8\pi^2 v^2} = \frac{1}{8\pi^2 v^2} (6M_W^4 + 3M_Z^4 + 2M_{H^\pm}^4 + M_A^4 + M_{H'}^4 - 12M_t^4)$$

$$\implies (M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4)^{1/4} = 540 \text{ GeV} \quad (\text{W. Shepherd \& K.L., ibid; E. Pilon \& K.L., ibid})$$

The significance of this sum rule is obvious: All the BSM Higgs bosons in the GW-2HDM must lie below about 400 GeV! (N.B.: $M_A = M_{H^\pm}$ is assumed to eliminate the BSM Higgs contribution to the T-parameter.)

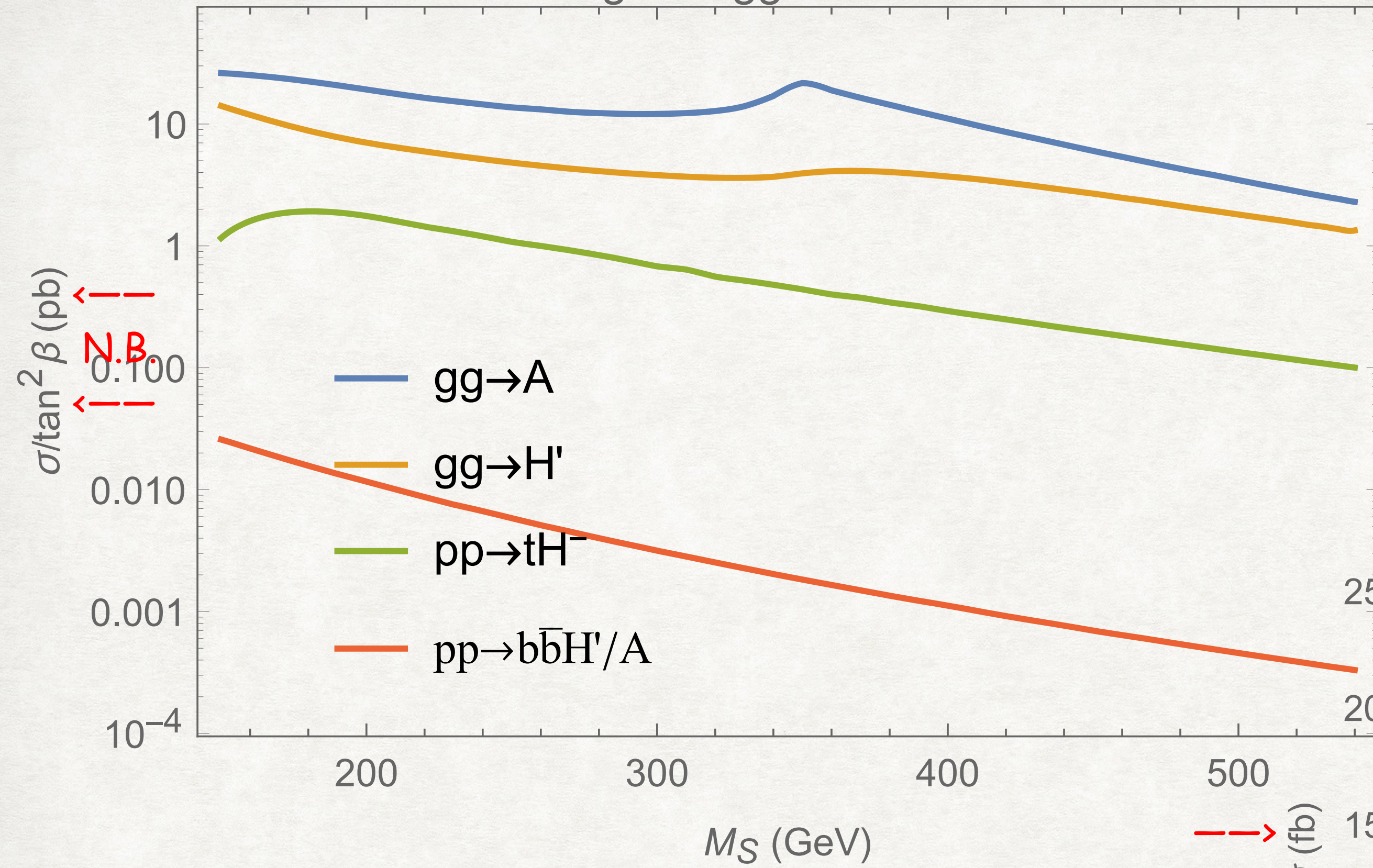
The targets of opportunity: (see E. Pilon and K.L., PRD 101, 055032)

$$H^{\pm} \rightarrow t\bar{b} \text{ or } W^{\pm} H' \rightarrow \ell^{\pm} b\bar{b} \quad \begin{array}{l} \text{(Note the same final state,} \\ \text{but resonant } H' \rightarrow b\bar{b}. \\ \text{No searches yet!)} \end{array}$$

$$A \rightarrow t\bar{t} \text{ or } Z H' \rightarrow \ell^{+} \ell^{-} b\bar{b} \quad (\text{CMS } A \rightarrow t\bar{t} \text{ excess at 400 GeV?})$$

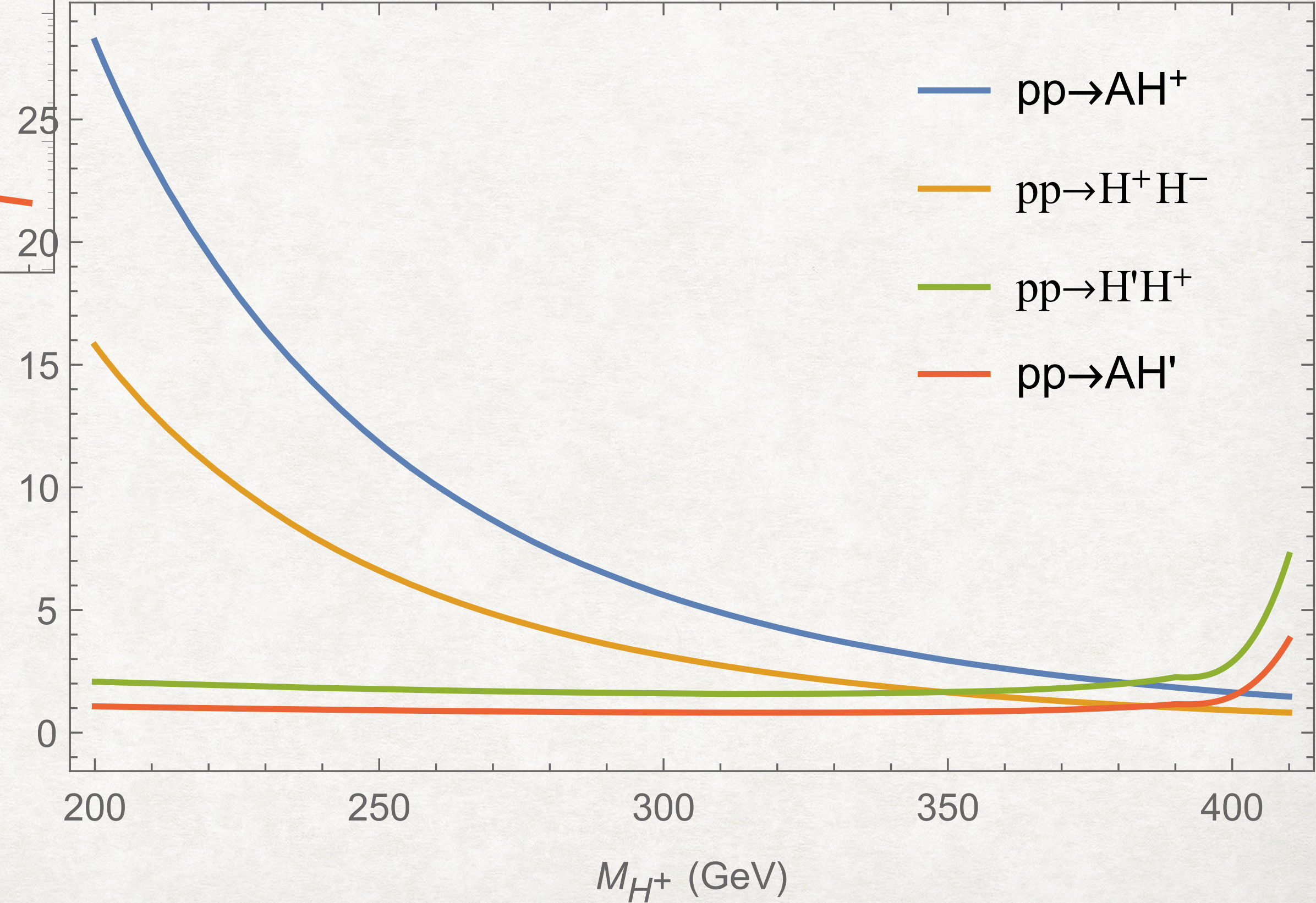
$$H' \rightarrow t\bar{t} \text{ or } Z A \rightarrow \ell^{+} \ell^{-} b\bar{b} \text{ and } W^{\pm} H^{\mp} \rightarrow \ell^{\pm} t\bar{b} \\ \text{(No searches yet)}$$

Single-Higgs Production



(W. Shepherd & K.L.,
PRD99, 055015)

Drell-Yan Production



$$\underline{gg \rightarrow H^\pm \bar{t}b \rightarrow t\bar{t}b\bar{b}}$$

four relevant searches so far:

1. CMS in JHEP 11 (2015) 018 (at 8 TeV).
2. ATLAS in JHEP 11 (2018) 085 (at 13 TeV).
3. CMS in JHEP 01 (2020) 096 (at 13 TeV).
4. ATLAS in arXiv:2102.10076 (update of #2 using full Run 2 data).

#1 set $\tan \beta \lesssim 0.50$ for $180 \lesssim M_{H^\pm} \lesssim 500$ GeV.

The sensitivities of #2,3,4 at these low masses have been no greater than #1's.

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The sensitivities of #2,3,4 at these low masses have been no greater than #1's. **A challenge!**

$$\underline{gg \rightarrow A \text{ or } H' \rightarrow t\bar{t}}$$

Only one search at low masses: CMS JHEP 04 (2020) 171 (at 13 TeV).
For the CP-odd case with $400 < M_A < 500 \text{ GeV}$, $\tan \beta < 0.5$ is not excluded. This is possibly due to:

“The largest deviation from the SM background is observed for a pseudoscalar Higgs boson with a mass of 400 GeV and a total relative width of 4%, with a local significance of 3.5 ± 0.3 standard deviations.”

However, this may be due to $t\bar{t}$ threshold effects.

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However, this may be due to $t\bar{t}$ threshold effects. **Another challenge!**

$$\underline{gg \rightarrow A(H') \rightarrow ZH'(A) \rightarrow \ell^+ \ell^- b\bar{b}}$$

Three searches so far, all at 13 TeV:

1. ATLAS in PL B283 (2018) 392.
2. CMS in JHEP 03 (2020) 055.
3. ATLAS in arXiv:2011.05639 (update of #1 using full Run 2 data).

(A comment on these $A \rightarrow ZH'$, etc. decays:
the sum rule limits the likely phase space.)

Results of #2 & 3 for two mass choices:

$M_A = M_{H^\pm}$	M_{H_2}	ATLAS	CMS	GW-2HDM
400	300	90	75	65
300	500	51	50	100

Table 2: 95% CL upper limits on $\sigma(pp \rightarrow A(H')) B(A(H') \rightarrow ZH'(A)) B(H'(A) \rightarrow \bar{b}b)$ via gluon fusion at $\sqrt{s} = 13$ TeV from ATLAS [21] (for 139 fb^{-1}), CMS [22] (for 36 fb^{-1}) and GW-2HDM calculations for two cases of large M_A and $M_{H'}$. The CMS limits include $B(Z \rightarrow e^+e^-, \mu^+\mu^-)$; the ATLAS limits and GW-2HDM predictions do not. Masses are in GeV and σB in femtobarns. $M_A = M_{H^\pm}$ is assumed and $M_{H'}$ is taken from the one-loop sum rule $(M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4)^{1/4} = 540 \text{ GeV}$. Model cross sections are taken from the accompanying figure multiplied by $\tan^2 \beta = 0.25$.

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New from ATLAS

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That's it, folks. I hope you've enjoyed the talk or, at least, been intrigued by the results of the oh-so-simple (but oh-so-mysterious) Gildener-Weinberg scale-invariance hypothesis.