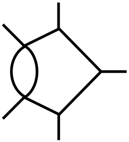
Multi-loop Scattering Amplitudes for the HL-LHC

Presenting the Caravel Framework



Caravel

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Department of Physics, Florida State University

Theoretical Physics Seminar, Fermilab, Batavia, May 2020

References

- S. Abreu, J. Dormans, FFC, H. Ita, B. Page, V. Sotnikov
  Analytic Form of the Planar 2-Loop 5-Parton Scattering Amplitudes in QCD
  JHEP 05 (2019) 084 [arxiv:1904.00945]
- S. Abreu, FFC, H. Ita, M. Jaquier, B. Page, M.S. Ruf, V. Sotnikov
  The Two-Loop Four-Graviton Scattering Amplitudes
  Phys. Rev. Lett. 124, 211601 [arxiv:2002.12374]
- S. Abreu, J. Dormans, FFC, H. Ita, M. Kraus, B. Page, E. Pascual, M.S. Ruf, V. Sotnikov
  Caravel: A C++ Framework for the Computation of Multi-Loop Amplitudes through Numerical Unitarity
  [arxiv:2006.xxxxx]

Outline

PERCENT-LEVEL QCD ERA

Precision @ (HL-)LHC, example p_T^{ll}, Theory uncertainties, NNLO QCD

GRAVITON-GRAVITON SCATTERING

Challenging EFT, 2-Loop Numerical Unitarity, QCD & Gravity Results

THE CARAVEL FRAMEWORK

Public release, Modules, Example programs, Outlook

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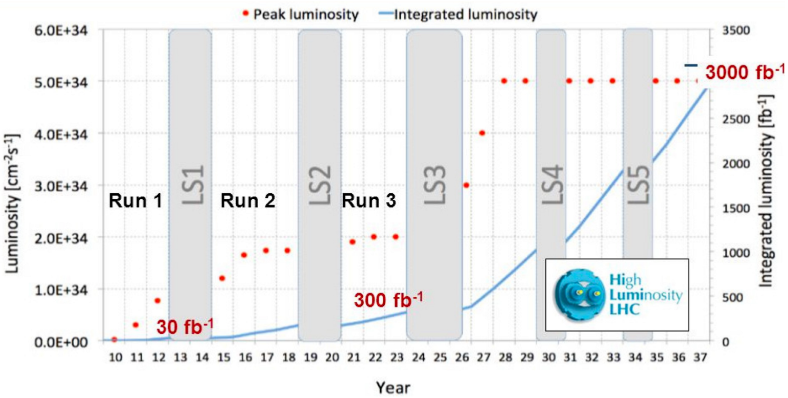
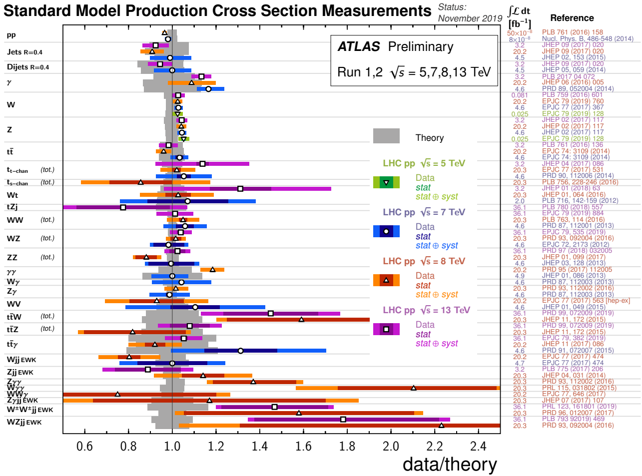
The *atto*barn Era



20-fold increase in data sets at the LHC experiments in the next decades

Reaching few-percent uncertainties in cross sections for processes with 3 (or more) objects in the final state

The *atto*barn Era



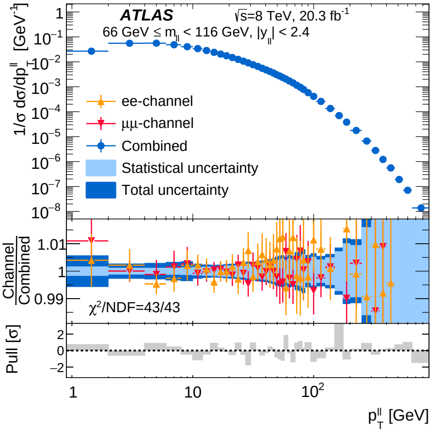
20-fold increase in data sets at the LHC experiments in the next decades

Reaching few-percent uncertainties in cross sections for processes with 3 (or more) objects in the final state

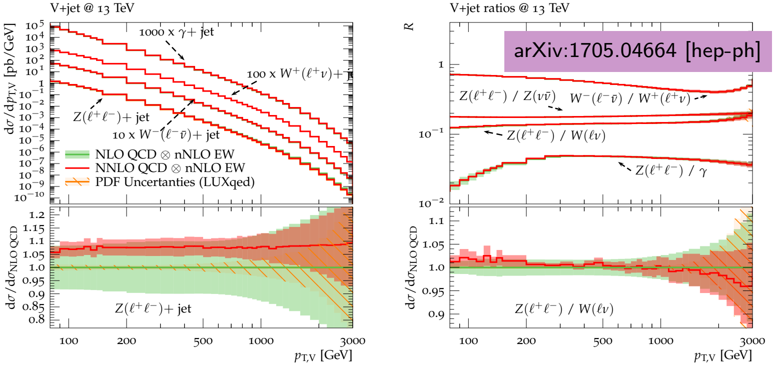
Few % Frontier at the LHC

- p_T^{ll} in Drell-Yan, an impressive example of precise differential measurements by ATLAS (8 TeV)
- By normalizing to inclusive Z cross section, improvement in uncertainties
- Total uncertainties below 1% for $p_T^{ll} < 200$ GeV

arXiv:1512.02192 [hep-ex]



Few % Frontier in Theory



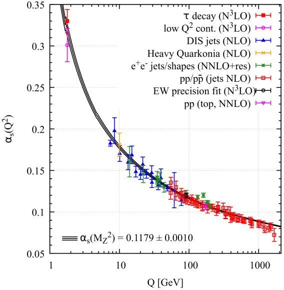
arXiv:1705.04664 [hep-ph]

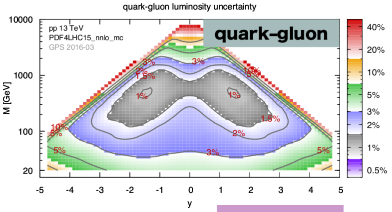
- $p_T^{ll'}$, an impressive example of precise differential predictions
- Uncertainty estimates from NNLO QCD, NLO EW including higher orders Sudakov logs and PDF uncertainties

Lindert, Pozzorini, Boughezal, Campbell, Denner, Dittmaier, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Kallweit, Maierhöfer, Mangano, Morgan, Mück, Petriello, Salam, Schönherr, Williams

Parametric Dependence of QCD Predictions

In order to compute quantum QCD corrections two fundamental inputs are required: the strong coupling α_s and the *Parton Distribution Functions*





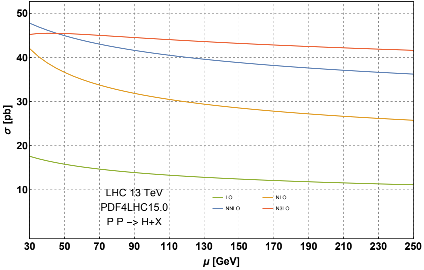
From G. Salam

- Perturbative calculations are also required for the partonic cross sections associated to the signal studied
- Naively at the LHC ($\alpha_s \sim 0.1$) one is to expect NLO QCD corrections to be of order $\sim 10\%$ and NNLO QCD at $\sim 1\%$

Perturbative Improvements for Predictions

- The smallness of α_s and α allows systematic improvements for SM predictions
- In particular hard-processes can be described with increasing precision by systematic addition of higher-order QCD corrections
- As an example Higgs production up to N^{3}LO QCD

B. Mistlberger, arXiv:1802.00833 [hep-ph]

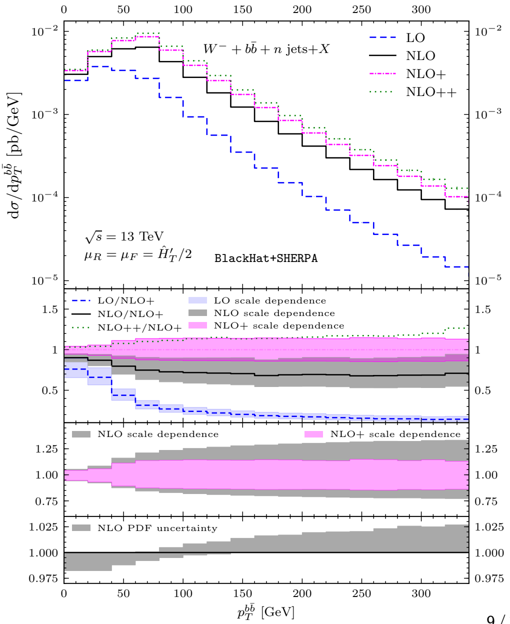


- But computations at NNLO QCD and beyond are challenging in particular for processes with many scales and colored partons
- Inherent need for automation to tackle these problems, even though judicious choices for studies will be mandatory (computationally intensive calculations)

A Higgs Boson Background

- A key irreducible background to $H(\to b\bar{b})W$ measurement are QCD production of $Wb\bar{b}$+jets
- This signature gives access to y_b
- NLO+ a exclusive sum: adds NLO corrections to hard contributions
- From unphysical scale sensitivity we expect important impact of NNLO QCD corrections

Anger, FFC, Ita, Sotnikov
arXiv:1712.05721 [hep-ph]



NNLO QCD for Multi-Scale Processes

- Great advances over the last several years on NNLO QCD studies for $2 \to 2$ processes, with up to four scales
  [Anastasiou, Angeles-Martinez, Asteriadis, Behring, Berger, Billis, Binoth, Bonciani, Boughezal, Brucherseifer, Buonocore, Cacciari, Campbell, Caola, Cascioli, Catani, Chen, Cieri, Cruz-Martinez, Currie, Czakon, de Florian, Del Duca, Delto, Devoto, Dreyer, Duhr, Ebert, Ellis, Ferrera, Fiedler, Focke, Frellesvig, Gao, Gauld, Gaunt, Gehrmann, Gehrmann-De Ridder, Giele, Glover, Grazzini, Hanga, Heinrich, Heymes, Huss, Höfer, Jaquier, Jones, Kallweit, Kardos, Karlberg, Kerner, Li, Lindert, Liu, Magnea, Maierhöfer, Maina, Majer, Mazzitelli, Melnikov, Michel, Mitov, Morgan, Neumann, Niehues, Pellicciolі, Petriello, Pires, Poncelet, Pozzorini, Rathlev, Rietkerk, Röntsch, Salam, Sapeta, Sargsyan, Schulze, Signorile-Signorile, Somogyi, Stahlhofen, Szőr, Tackmann, Tancredi, Torre, Torrielli, Tramontano, Trócsányi, Tulipánt, Uccirati, van Hameren, von Manteuffel, Walker, Walsh, Wang, Weihs, Wells, Wever, Wiesemann, Williams, Yuan, Zanderighi, Zhang, Zhu, $\cdots$]
  - First $2 \to 3$ NNLO QCD study completed!
    [Chawdhry, Czakon, Mitov, Poncelet, 2019]
- Physics cases make precision studies for more complex processes necessary, like $H + 2j$, $V + 2j$, $3j$, $t\bar{t} + H$, $VV'j$, among other (more than five scales!) [See e.g. *Les Houches Wish List*]
- About 15 years ago, $2 \to 3$ was the frontier for NLO QCD (one-loop) calculations, and the work beyond relied mainly on efficient numerical algorithms (now available through many powerful tools, e.g. *BlackHat, GoSam, HELAC-1Loop/CutTools, Madgraph, NJet, NLOX, OpenLoops, Recola,* $\cdots$)

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

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Regarding IR structure

$\rightarrow$ real *hard*

$\rightarrow$ virtual *easy*

L. Magnea

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Full $\mathcal{O}(\epsilon^0)$ structure

$\rightarrow$ real *hard*

$\rightarrow$ virtual *hard*

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

- Many recent advances and complete calculations (e.g. $t\bar{t}$, $2j$, VV', Vj, HH, 3γ, etc)
- Several well-developed approaches
  - Antenna subtraction
  - ColorfulNNLO
  - Nested soft-collinear subtractions
  - N-Jettiness slicing
  - Projection to born
  - q_T slicing
  - SecToR Improved Phase sPacE for real Radiation
  - ...
- Different degrees of automation, handling many $2 \to 3$ processes maybe in sight

Key Building Blocks for NNLO QCD Corrections

- Strategy to handle and cancel IR divergences
- Two-loop matrix elements

- Great steps towards understanding mechanisms to compute multi-scale master Feynman integrals, including insights into functional forms and numerical procedures, over the last few years
- Also new efficient tools developed for multi-loop integral reduction
- Integrand reduction techniques have shown a lot of power to tackle complicated amplitudes. Here we focus on the numerical unitarity method

A Message...

This is the time for the **LHC***!*

(T. Binoth)

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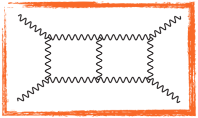
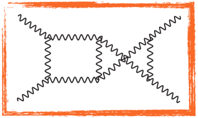
Stressing Computational Methodology





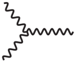
- Not related to collider phenomenology but, treated as an EFT, it can showcase the strengths and weaknesses of the multi-loop numerical unitarity method
- Of interest for classical gravitational applications, as already shown in the computation of classical deflection angles in Einstein gravity [Bern, Ita, Parra-Martinez, Ruf, 2020]
- Showing the robustness of our computational framework Caravel, testing non-planar, colorless calculations with different particle content (as compared to the SM)

Target Amplitudes

$$\mathcal{L} = \mathcal{L}_{\mathsf{EH}} + \mathcal{L}_{\mathsf{GB}} + \mathcal{L}_{\mathsf{R}^3} + \ldots$$

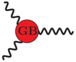
[Weinberg], ['t Hooft, Veltman], [Goroff, Sagnotti], [Donogue], ...

$$\mathcal{L}_{\rm EH} = -\frac{2}{\kappa^2}\sqrt{|g|}R$$



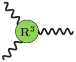
$\mathcal{O}(\kappa)$

$$\mathcal{L}_{\rm GB} = \frac{\mathcal{C}_{\rm GB}}{(4\pi)^2}\sqrt{|g|}(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})$$

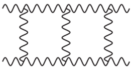


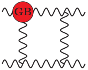
$\mathcal{O}(\kappa^3)$

$$\mathcal{L}_{\rm R^3} = \frac{\mathcal{C}_{\rm R^3}}{(4\pi)^4}\left(\frac{\kappa}{2}\right)^2\sqrt{|g|}R_{\alpha\beta}{}^{\mu\nu}R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}$$



$\mathcal{O}(\kappa^5)$

$\mathcal{A}^{(2)} =$  $+$ 

$+$ 

$+$ 

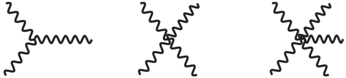
$\mathcal{O}(\kappa^6)$

Only three helicity configurations necessary:

$++++,\ -+++,\ ----$

Main Challenges

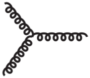
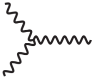
- EH Feynman rules are complicated



Terms: $\mathcal{O}(100)$ $\mathcal{O}(1000)$ $\mathcal{O}(10000)$

Feynman-diagram based calculation out of question
⇒ (Generalised) Unitarity

- EH interactions have high power-counting (QCD^2)

 $\sim \mathcal{O}(\ell)$ vs  $\sim \mathcal{O}(\ell^2)$

Complicated integrand
⇒ analytics from numerics

Plays to the strengths of
Two-loop numerical unitarity

Caravel

Two-Loop Numerical Unitarity

Decompose $\mathcal{A}$ in terms of *master* integrals:

$$\mathcal{A}^{(L)} = \sum_{\Gamma \in \Delta} \sum_{i \in M_\Gamma} c_{\Gamma,i} \, \mathcal{I}_{\Gamma,i}$$

All 4-point 2-loop integrals known [Anastasiou, Smirnov, Tausk, Tejeda-Yeomans, Veretin]

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Drop the integral symbol, introducing the integrand ansatz:

$$\mathcal{A}^{(L)}(\ell_l) = \sum_{\Gamma \in \Delta} \sum_{k \in Q_\Gamma} c_{\Gamma,k} \frac{m_{\Gamma,k}(\ell_l)}{\prod_{j \in P_\Gamma} \rho_j(\ell_l)}$$

Functions $\mathcal{Q}_\Gamma = \{m_{\Gamma,k}(\ell_l) | k \in Q_\Gamma\}$ parametrize every possible integrand (up to a given power of loop momenta).

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Functions $\mathcal{Q}_\Gamma = \{m_{\Gamma,k}(\ell_l) | k \in Q_\Gamma\}$ parametrize every possible integrand (up to a given power of loop momenta). E.g.:

- **Tensor Basis**: construct $\mathcal{Q}$ from monomials of loop momenta (parameters). Easy to build for general integrands, tough to relate to master integrals. Easy to extract function-space dimension
- **Master-Surface Basis**: a clever choice of parametrization makes mapping to master integrals straightforward [Ita, 2015]. Break $Q_\Gamma = M_\Gamma \cup S_\Gamma$, where S_Γ integrate to zero and M_Γ correspond to master integrands

Master/surface decompositions

[Ita '15; Abreu, FFC, Ita, Page, Zeng '17]

Consider the integration by parts (IBP) relation on Γ

$$0 = \int \prod_i d^D \ell_i \ \frac{\partial}{\partial \ell_j^\nu} \left[\frac{u_j^\nu}{\prod_{k \in P_\Gamma} \rho_k} \right]$$

while controlling the propagator structure [Gluza, Kadja, Kosower '10; Schabinger '11]

$$u_j^\nu \frac{\partial}{\partial \ell_j^\nu} \rho_k = f_k \rho_k$$

Write ansatz for u_j^ν expanded in external and loop momenta, and find solution to the polynomial equations using SINGULAR

Build a full set of surface terms and fill the rest of the space with master integrands

Related [Boehm, Georgoudis, Larsen, Schulze, Zhang '16 - '19]
[Agarwal, von Manteuffel '19]

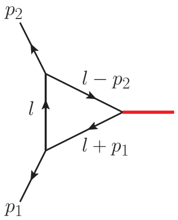
A simple example for surface terms: Part 1

Consider the 1-loop 1-mass triangle with

$$\rho_1 = (\ell + p_1)^2, \quad \rho_2 = \ell^2, \quad \rho_3 = (\ell - p_2)^2$$

and we construct $u^\nu \partial / \partial \ell^\nu$ by parametrizing

$$u^\nu = u_1^{\text{ext}} p_1^\nu + u_2^{\text{ext}} p_2^\nu + u^{\text{loop}} \ell^\nu$$



By constraining the propagator structure, we get the polynomial equation:

$$\left(u_1^{\text{ext}} p_1^\nu + u_2^{\text{ext}} p_2^\nu + u^{\text{loop}} \ell^\nu\right) \frac{\partial}{\partial \ell^\nu} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix} - \begin{pmatrix} f_1 \rho_1 \\ f_2 \rho_2 \\ f_3 \rho_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We can then show that we have an IBP-generating vector, with constrained propagator structure:

$$u^\nu \frac{\partial}{\partial \ell^\nu} = \left[(\rho_3 - \rho_2) p_1^\nu + (\rho_1 + \rho_2) p_2^\nu + (-s + 2\rho_3 - 2\rho_2) \ell^\nu\right] \frac{\partial}{\partial \ell^\nu}$$

A simple example for surface terms: Part 2

Now we have the surface term:

$$0 = \int d^D \ell \frac{\partial}{\partial l^\nu} \frac{u^\nu}{\rho_1 \rho_2 \rho_3} = \int d^D l \frac{1}{\rho_1 \rho_2 \rho_3} \left[-(D-4)s - 2(D-3)\rho_2 + 2(D-3)\rho_3 \right]$$

The scalar 1-loop triangle integrand on-shell could be replaced by a surface term, though commonly it is kept as a master integral.

The IBP relation between the triangle and the $s = (p_1 + p_2)^2$ bubble is:

$$-(D-4)sI_{\text{tri}} - 2(D-3)I_{\textsf{s-bub}} = 0$$

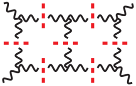
Similar manipulations can be carried out at two loops. More complicated polynomial relations (*syzygy* equations) need to be solved → SINGULAR. Surface terms appear as relatively compact

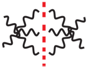
Computing Integrand Coefficients

[Bern, Dixon, Dunbar, Kosower] [Britto, Cachazo, Feng]

- In on-shell configurations of ℓ_l, the integrand factorizes

$$\sum_{\text{states}} \prod_{i \in T_\Gamma} \mathcal{A}_i^{\text{tree}}(\ell_l^\Gamma) = \sum_{\substack{\Gamma' \geq \Gamma \\ k \in Q_{\Gamma'}}} \frac{c_{\Gamma',k} \; m_{\Gamma',k}(\ell_l^\Gamma)}{\prod_{j \in (P_{\Gamma'}/P_\Gamma)} \rho_j(\ell_l^\Gamma)}$$

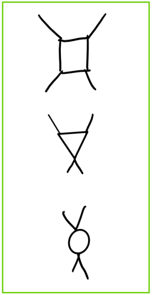




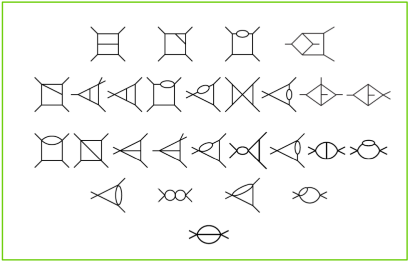
- Need efficient computation of (products of) tree-level amplitudes
  - Off-shell recursions [Berends, Giele, '88], [Cheung, Remmen '17]
  - D_s-dimensional state sum, $D_s = 6, \ldots, 10$
- Never construct analytic integrand, numerics for every phase-space point

Numerical Stability:

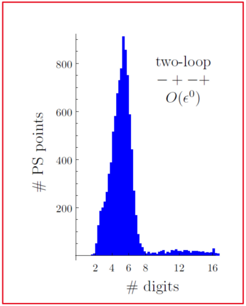
e.g. 4-gluon amplitudes



Function spaces with $\mathcal{O}(10/50)$ dimensions



Function spaces with $\mathcal{O}(100/1000)$ dimensions



* Relative precision of two-loop 4-gluon amp numerical calculation
* High-precision floating point arithmetic a remedy

[Abreu, FFC, Ita, Jaquier, Page, Zeng, '17]

MODULAR ALGEBRA:

A clever observation!
[von Manteuffel, Schabinger, 2014]

* Integral reduction can be performed exactly in CAS if kinematical info is RATIONAL ($x_i \in \mathbb{Q}^n$)
* Nevertheless, RATIONAL computer algebra reflects the numerical complexity of corresponding ANALYTIC STRUCTURE (COMPUTATIONAL ALGORITHM)

$$x_i \longrightarrow f(x_i) \longrightarrow \frac{p}{q}$$

"simple" input

"Complicated" function ⇒ "HEAVY" calculation → Even if result is "simple"

FINITE (NUMBER) FIELDS: [von Manteuffel, Schabinger, 2014]

* MAP $\mathbb{Q}^n$ into $\mathbb{F}_p^n$ and try to reconstruct result!
* If cardinality p is smaller than CPU's word size (2^{64}) operation will be very fast

$$x_i \xrightarrow{\frac{m}{n} = l \bmod p} y_i = I(x_i) \longrightarrow f(y_i) \longrightarrow t \xrightarrow{\text{"lift" back to } \mathbb{Q}} \frac{p}{q}$$

- x_i: "SIMPLE" $\mathbb{Q}^n$ input
- $y_i = I(x_i)$: $\mathbb{F}_p^n$ input
- $f(y_i)$: "complicated" function BUT "fast" calculation
- t: $\mathbb{F}_p$ result

* "Lift" back operation, or rational reconstruction works well if $\frac{p}{q}$ is "simple" enough (OR MORE $\mathbb{F}_p$'s needed!).

FINITE (NUMBER) FIELDS: [von Manteuffel, Schabinger, 2014]

* MAP $\mathbb{Q}^n$ into $\mathbb{F}_p^{\,n}$ and try to reconstruct result!
* If cardinality p is smaller than CPU's word size (2^{64}) operations will be very fast

$$\underbrace{x_i}_{\text{"SIMPLE" } \mathbb{Q}^n \text{ input}} \xrightarrow{\frac{n}{m} = l \bmod p} \underbrace{y_i = I(x_i)}_{\mathbb{F}_p^{\,n} \text{ input}} \longrightarrow \underbrace{f(y_i)}_{\text{"complicated" function BUT "fast" calculation}} \longrightarrow \underbrace{t}_{\mathbb{F}_p \text{ result}} \xrightarrow{\text{"lift" back to } \mathbb{Q}} \frac{p}{q}$$

* "Lift" back operation, or rational reconstruction works well if p/q is "simple" enough (OR MORE $\mathbb{F}_p$'s needed!).

Make your numerical evaluations in FF's & avoid all numerical-stability issues!

INTEGRAL COEFFS AS FUNCTIONS of ε:

(INTEGRAND'S ANSATZ)

$$A(\ell_e) = \sum_{\Gamma,i} c_{\Gamma,i} \frac{m_{\Gamma,i}(\ell_e)}{\prod_{k\in\Gamma} \rho_k(\ell_e)} \longrightarrow$$

$c_{\Gamma,i}$ are function of x_k & $D=4-2\varepsilon$

Indeed $c_{\Gamma,i}$ appear as rational function of ε

$$c_{\Gamma,i} = \frac{\sum_j f_j(x_k)\, \varepsilon^{j+N}}{\sum_j q_j\, \varepsilon^{j+M}}$$

STRUCTURE NOT KNOWN A PRIORI !

ε dependence comes from the structure of $m_{\Gamma,i}(\ell_e)$ and through linear algebra ("subtraction" procedure)

Thiele's interpolation formula:

Every rational function can be written as a continued fraction

$$f(x) = \frac{\sum_{r=0}^{R} n_r x^r}{\sum_{r'=0}^{R'} d_{r} x^{r'}} = a_0 + \cfrac{x - y_0}{a_1 + \cfrac{x - y_1}{a_2 + \cfrac{x - y_2}{\cdots + \cfrac{x - y_{N-1}}{a_N}}}}$$

* Determine a_i by evaluating $f(y_i)$ (y_i random)
* Stop when $f(y_{i+1})$ matches interpolated value (+ extra checks)
* Through only field operations recover rational function

(FF's result can be lifted to $\mathbb{Q}$)

See also [Peraro, '16] for multi-variate reconstruction algorithms!

THIELE'S INTERPOLATION FORMULA:

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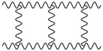
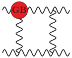
See also [Peraro, '16] for multi-variate reconstruction algorithms!

This same idea can be employed for the analytic reconstruction of the kinematic $x = t/s$ dependence of 4-pt amplitudes!

Gravity Results: 4-Graviton Amplitudes

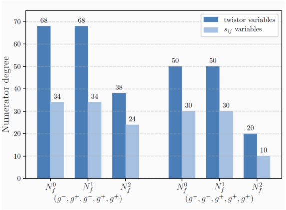
- Computed the three independent helicities for
  - ✓ EH gravity 
  - ✓ Tree-level R^3 

  - ✓ Tree-level GB-GB 

  - ✓ One-loop GB 

- Checks
  - ✓ Remainders are finite, correct symmetry, no spurious poles
  - ✓ 1-loop amplitudes [Dunbar, Norridge 95], [Bern, Cheung, Chi, Davies, Dixon, Nohle, unpub.]
  - ✓ GB tree and 1-loop: $++++$ and $--++$ [Bern, Cheung, Chi, Davies, Dixon, Nohle, 15, unpub.]
  - ✓ R^3 tree [Bern, Cheung, Chi, Davies, Dixon, Nohle, 15], [Dunbar, Jehu, Perkins, 17]
  - ✓ 2-loop: $++++$ [Bern, Cheung, Chi, Davies, Dixon, Nohle, unpub.], [Dunbar, Jehu, Perkins, 17]

QCD Results: 5-Parton Amplitudes



Analytic expressions [Abreu, JD, Febres Cordero, Ita, Page, Sotnikov '19]

All 5-parton 2-loop amplitudes for NNLO QCD 3-jet production at *leading-color*.

- ✓ 5g with N_f^0, N_f^1, N_f^2: 4 helicity configurations
- ✓ 2q3g with N_f^0, N_f^1, N_f^2: 4 helicity configurations
- ✓ 4q1g with N_f^0, N_f^1, N_f^2: 3 helicity configurations

33 different amplitudes obtained through analytical reconstruction
Most complex amplitude: ~ 95000 phase space points, ~ 4.5 min/point
Extremely compact: total size $\sim$ 10Mb (uncompressed)
Valid in euclidean region

* A series of simplification allow to compute 5-parton amplitudes with modest computer resources
  ~ 200k CPU hours
  for all 33 independent AMPs.

* 4-graviton amplitudes more complex numerically, but simpler analytic structure (UNIVARIATE)
  ~ 100k CPU hours
  up-to 40 PS points
  for all 3 independent AMPs

Outline

PERCENT-LEVEL QCD ERA

Precision @ (HL-)LHC, example p_T^{ll}, Theory uncertainties, NNLO QCD

GRAVITON-GRAVITON SCATTERING

Challenging EFT, 2-Loop Numerical Unitarity, QCD & Gravity Results

THE CARAVEL FRAMEWORK

Public release, Modules, Example programs, Outlook

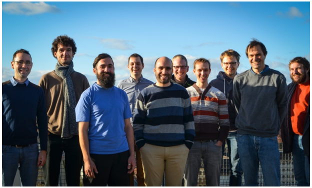
The Caravel Framework

A framework to explore multi-loop multi-leg scattering amplitudes in the SM and beyond

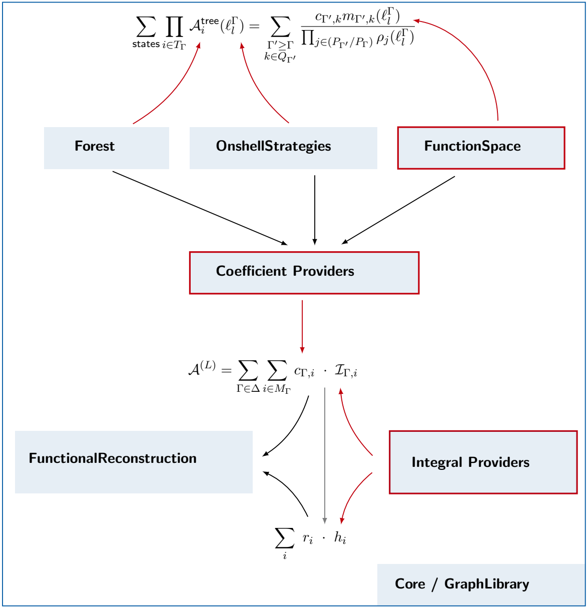
- A modular `C++17` library implementing the multi-loop numerical unitarity method

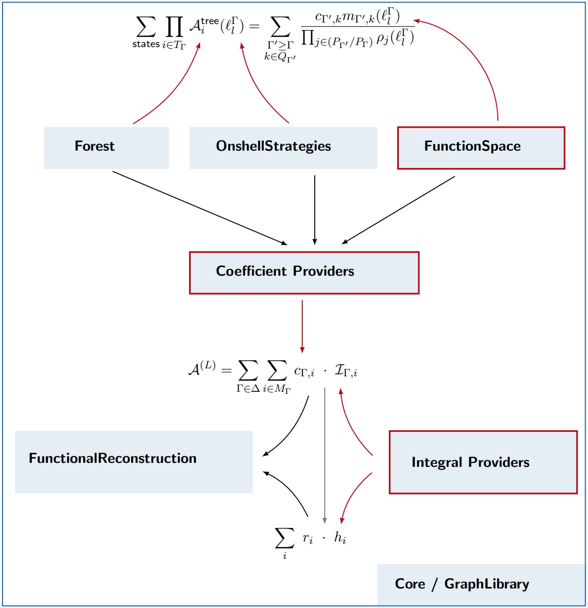
  [Abreu, Dormans, FFC, Ita, Kraus, Page, Pascual, Ruf, Sotnikov, arxiv:2006.xxxxx]

  - Numerics in (high-precision) floating-point and modular arithmetic
  - Tested in the (analytic) computation of planar 2-loop 4- and 5-parton QCD amplitudes and 4-graviton amplitudes
  - Soon to be publicly released



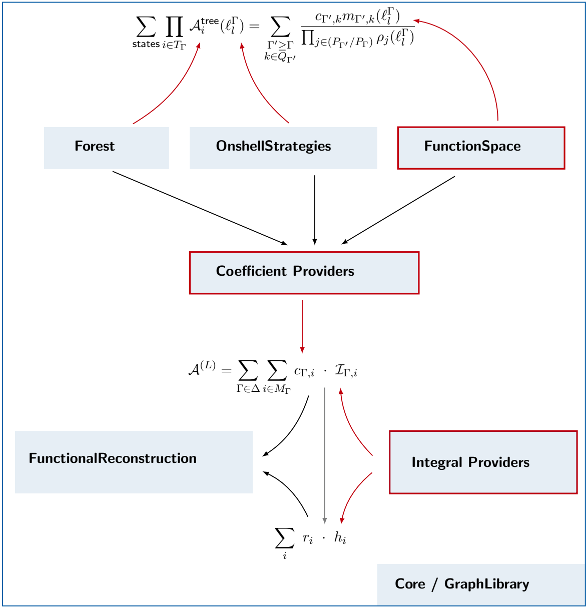
CARAVEL's Modules



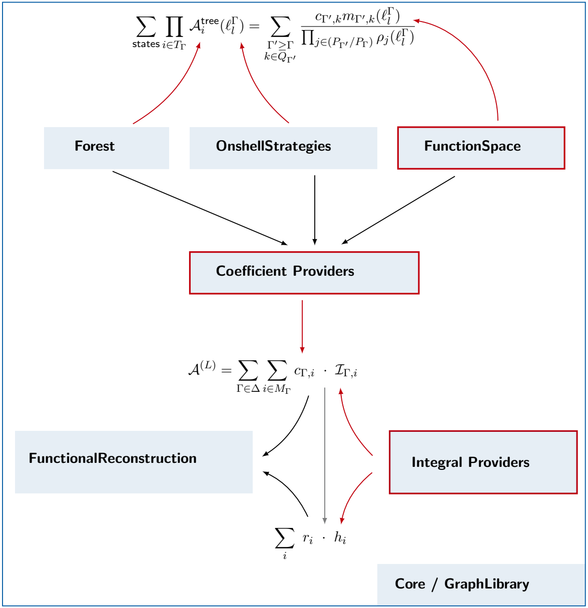


Core: Includes general tools for debugging, arithmetics, kinematics, as well as some utilities for linear algebra, rational reconstruction, type traits, and special algebra handling (like for example tools for Laurent expansions).
Optional dependencies: `QD`, `GMP`, `Eigen`, `Lapack`

Caravel's Modules



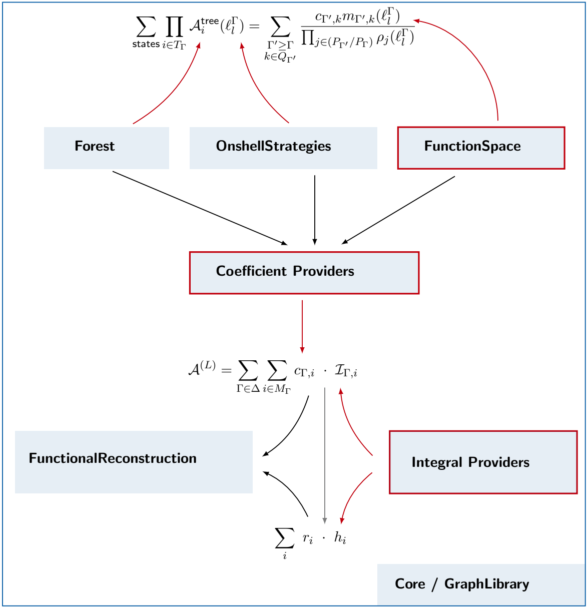
GraphLibrary: This module implements tools for the classification and canonicalization of multi-loop graphs. Graph isomorphism is implemented by building a partial order in the representation of the graph (which is ultimately based on the standard C++ function `std::lexicographical_compare`)



FunctionalReconstruction: Here we include algorithms for analytic reconstruction of univariate and multivariate rational functions from exact numerical evaluations. The reconstruction algorithms are parallelized, and can be run using native C++ threads or using `MPI`. The latter can be used for the runs on computer clusters.

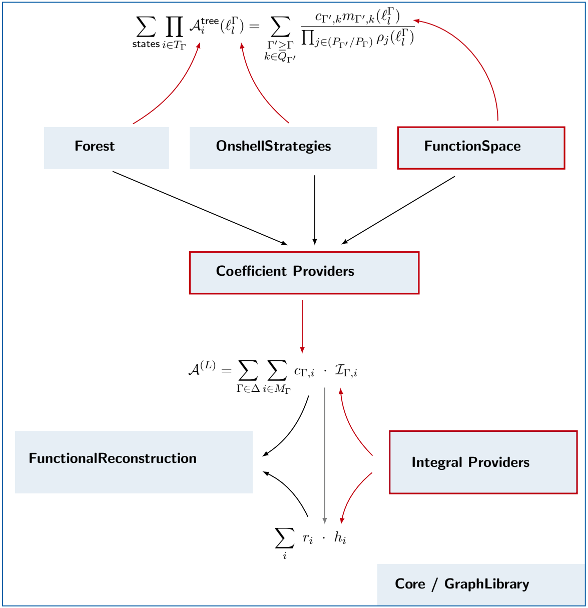
Checkout also `Firefly` [Klappert, Klein, Lange], and `FiniteFlow` [Peraro] !

CARAVEL's Modules



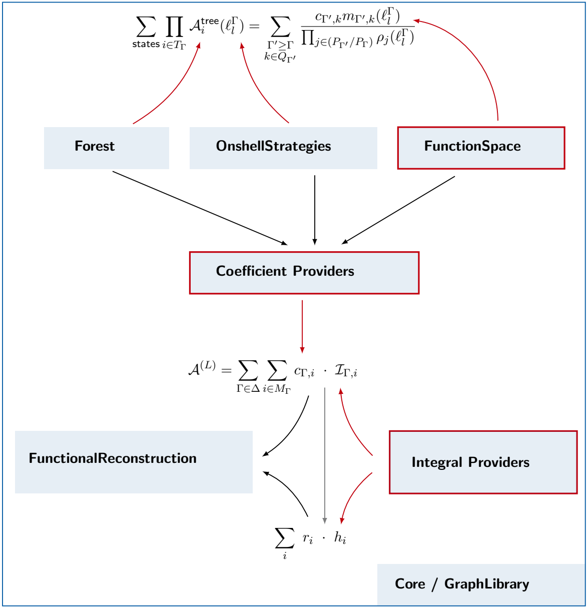
Forest: This module supplies the tools required for the computation of general tree-level amplitudes and *cuts* (the products of trees on the left-hand side of the top equation) in general D_s dimensions. Calculations are performed through off-shell recursion relations. The recursions can be constructed from any given set of Feynman rules and can be evaluated over an arbitrary numerical type.

CARAVEL's Modules

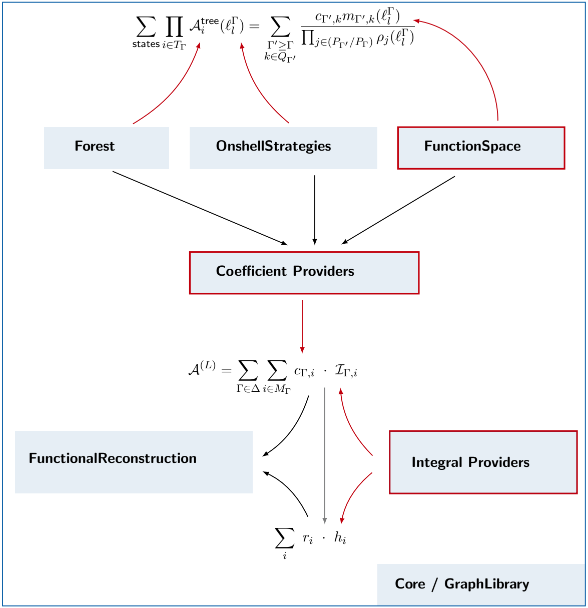


FunctionSpace: Takes care of constructing the integrand ansaetze, both for *tensor bases* and *master-surface bases*. The former can be constructed for general two-loop diagrams Γ while the latter are provided for general one-loop diagrams and for those two-loop diagrams required for completed calculations. Those master-surface bases have been produced with the usage of several in-house computer-algebra programs, and finally collected as `Mathematica` expressions. The latter can be transformed in an automated fashion into C++ code to be handled by this module.

CARAVEL's Modules



Integral Providers: Two distinct modules are included, one for general one-loop master integrals (up to $\mathcal{O}(\epsilon^0)$) and one for the (semi-analytical) evaluation of 1- and 2-loop master integrals for the evaluation of 4- and 5-parton two-loop amplitudes. For the latter, analytic expressions in `Mathematica` format are automatically mapped into C++ code by in-house tools



Coefficient Providers: These modules can handle the hierarchical extraction of master-integrand coefficients through the usage of cut equations. For a given 2-loop amplitude, it requires an input data file (the *process library*). These process libraries contain all hierarchical kinematical relations between the included diagrams (propagator structures) in the amplitude, as well as information about color decomposition

Example Programs [PRELIMINARY]

Other than an extensive suite of unit tests and integration tests, which continuosly check that libraries work as expected, we provide a series of example programs to showcase the following functionalities:

- Analytic reconstruction of a 4-point amplitude's master integral coefficients, employing Thiele's formula
- Numerical evaluation of 4- and 5-parton one-loop amplitudes to $\mathcal{O}(\epsilon^2)$ as required for 2-loop *finite remainder* computations
- Numerical evaluation of planar 4- and 5-parton two-loop amplitudes to $\mathcal{O}(\epsilon^0)$ and also for the corresponding finite remainder
- Example of numerical reduction of two-loop integrals within the numerical unitarity method
- Multi-variate polynomial reconstruction through a nested Newton decomposition

Outlook

- We have numerically computed the planar two-loop five-parton QCD amplitudes, as well as the two-loop four-graviton amplitudes in Einstein gravity.
- Exploiting modular arithmetic, we have also extracted the analytic form of those amplitudes
- We expect these and future results to contribute to the coming precision program at the HL-LHC
- Multi-loop numerical unitarity appears as a robust method to explore multi-loop multi-leg amplitudes
- We presented the Caravel framework which will be released soon! We hope that this will benefit the larger HEP theory community, by giving access to related implementations

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Thanks!