Three ways to weigh a quark

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- Quark masses – fundamental parameters of the Standard Model.

- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial widths from m_b, m_c, α_s. [1404.0319]

- Focus on precision results using three independent methods.

Outline

- Background
  - Lattice simulations
  - Mass determinations
- Quark mass methods
  - Current-current correlator moments
  - Regularisation Invariant (RI) methods
  - Minimal renormalon subtraction (MRS) masses
- Summary & Outlook

Lattice QCD simulations - I

Regulate QCD using a (Euclidean) spacetime lattice.

Integrate out fermionic degrees of freedom.

$$\begin{aligned}\mathcal{Z} &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\int (\mathcal{L}_{\mathrm{YM}} + \bar{\psi} D \psi)} \\ &= \int \mathcal{D}U \, (\det D) \, e^{-\int \mathcal{L}_{\mathrm{YM}}}\end{aligned}$$

Generate gluon configurations using Monte Carlo techniques.

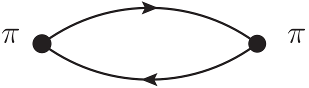
Effects of sea quarks are included in the determinant of the Dirac matrix.

Calculate valence quark propagators on gluon field configurations.

$$D^{-1} =$$

Tie together the quark propagators to create correlation functions.

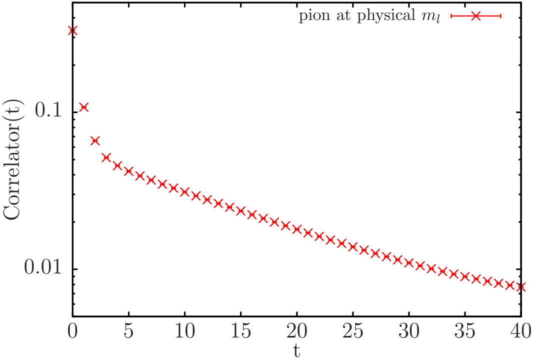
$$\langle \pi \ \pi^\dagger \rangle =$$



Lattice QCD simulations - III

Energies and matrix elements are determined by fitting (sums of) exponentials.

$$\langle \pi(t)\ \pi^\dagger(0)\rangle \underset{\text{large } t}{\longrightarrow} \frac{|\langle 0|\pi|\pi\rangle|^2}{2m_\pi} e^{-m_\pi t} \propto f_\pi^2\, e^{-m_\pi t}$$



Tuning mass input parameters

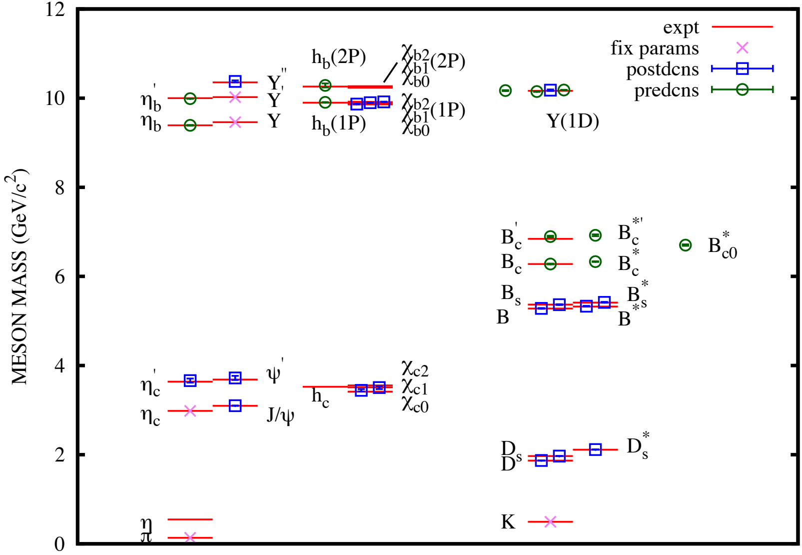
Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \to m_\pi^2$
- $m_{s0} \to m_K^2$
- $m_{c0} \to m_{\eta_c}$

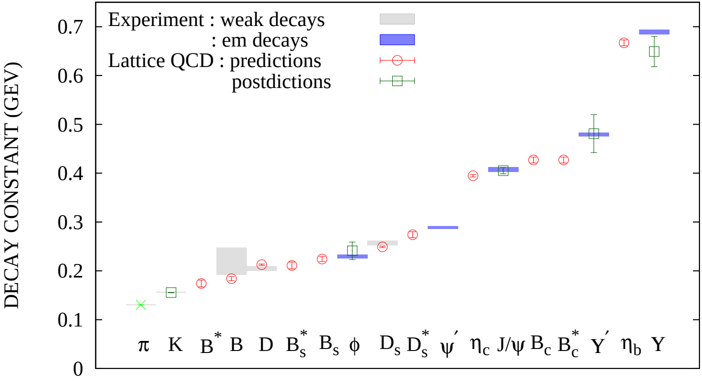
Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \to 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

Meson masses – summary plot



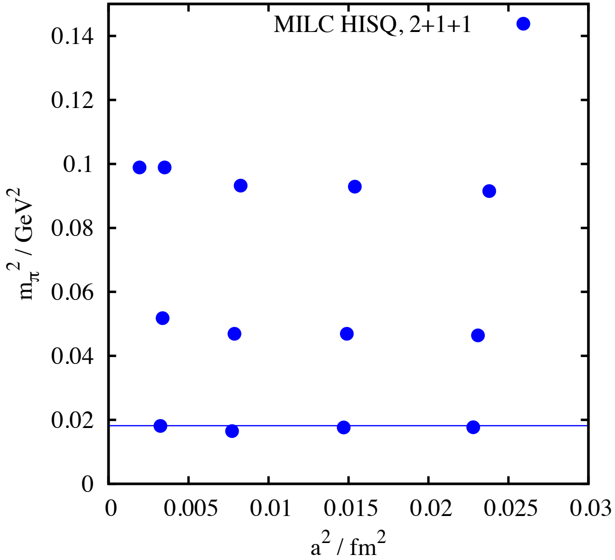
Decay constants – summary plot



MILC ensembles [1004.0342, 1212.4768]

- HISQ fermion action.
  - Discretization errors begin at $\mathcal{O}(\alpha_s a^2)$.
  - Designed for simulating heavy quarks (m_c and higher at current lattice spacings).
- Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]
- Multiple lattice spacings down to ~ 0.045 fm.
- Effects of u/d, s, and c quarks in the sea.
- Multiple light-quark input parameters down to physical pion mass.
  - Chiral fits.
  - Reduce statistical errors.

MILC ensemble parameters



Quark mass definitions

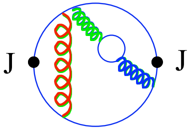
- Quarks are not asymptotic (physical) states due to confinement – mass cannot be measured directly.
- Quark masses are scheme and scale dependent, $m_q^{\text{scheme}}(\mu)$.
- Generally will quote results $m_q^{\overline{\text{MS}}}(\mu_{\text{ref}})$.
- Lattice input quark masses are non-universal (depend on discretisation), but can be connected to quark masses defined in a continuum scheme.

$\langle JJ \rangle$-correlator moments

Current-current correlators

Calculate time-moments of $J_5 \equiv \bar{\psi}_h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle J_5(t, \mathbf{x}) J_5(0, 0) \rangle$$



- Currents are absolutely normalized (no Zs required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\mathrm{cont}} = G(t)_{\mathrm{latt}} + \mathcal{O}(a^2)$.

Moments

The time-moments $G_n = \sum_t (t/a)^n G(t)$ can be computed in perturbation theory. For $n \geq 4$,

$$G_n = \frac{g_n(\alpha_{\overline{\rm MS}}, \mu)}{am_h(\mu)^{n-4}} \,.$$

Basic strategy:

1. Calculate $G_{n,\rm latt}$ for a variety of lattice spacings and m_{h0}.
2. Compare continuum limit $G_{n,\rm cont}$ with $G_{n,\rm pert}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\rm MS}}(m_h), m_h(m_h)$.

Reduced moments

In practice comparison carried out using reduced moments.

$$R_4 = G_4/G_4^{(0)}$$
$$R_n = \frac{1}{m_{0c}}(G_n/G_n^{(0)})^{1/(n-4)} \quad (n \geq 6)\,.$$

On the perturbative side,

$$R_4 = r_4(\alpha_{\overline{\mathrm{MS}}}, \mu)$$
$$R_n = \frac{1}{m_c(\mu)}\, r_n(\alpha_{\overline{\mathrm{MS}}}, \mu) \quad (n \geq 6)\,.$$

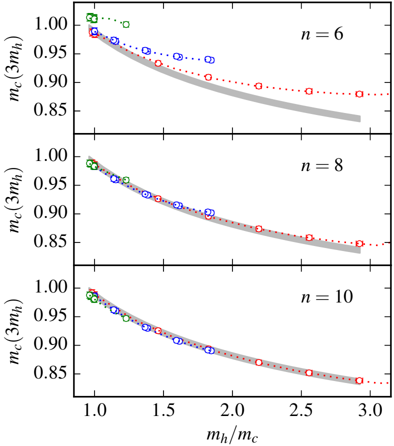
Reference scale is taken as $\mu = 3m_h (= m_c \frac{m_{h0}}{m_{c0}})$.

Some details

- Calculate moments for $n = 4, 6, 8, 10$.
- Three lattice spacings: $a \approx 0.12, 0.09, 0.06$ fm. (MILC)
- Seven input masses from $m_h = m_c - 0.7 m_b$.

All data points fit simultaneously with perturbative R_n expressions $\rightarrow m_c^{\overline{\mathrm{MS}}}(\mu)$, $\alpha_{\overline{\mathrm{MS}}}(\mu)$ for $\mu \approx 3 - 9$ GeV.

Results for $n_f = 4$ [1408.4169]



$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{\mathrm{MS}}}, \mu = 3m_h)}{R_n}$$

- Discretization effects grow with am_h and decrease with n.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

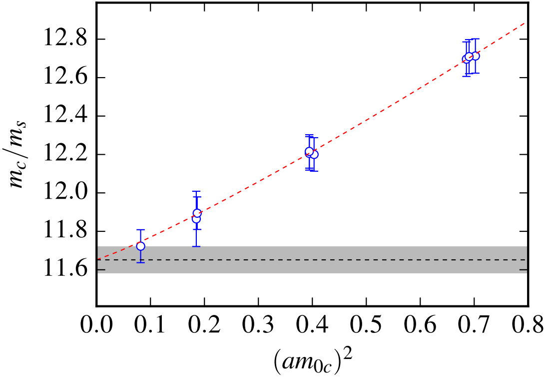
$$\boxed{m_c^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 0.9851(63)\ \mathrm{GeV}}$$

Mass ratios

$$\frac{m1_0}{m2_0} = \frac{m1^{\overline{\mathrm{MS}}}(\mu)}{m2^{\overline{\mathrm{MS}}}(\mu)} + \mathcal{O}(a^2)$$

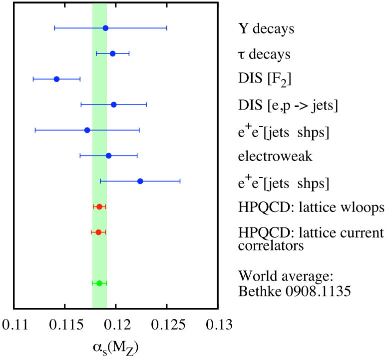
- Tuning of simulation $\rightarrow$ accurate determination of bare ratios.
- Precise determination of one renormalized mass can be translated to other masses.

$m_c/m_s\ (n_f = 4)$ [1408.4169]



$$m_c/m_s = 11.652(65) \to m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 93.6(8)\ \mathrm{MeV}.$$

$\alpha_s^{\overline{\mathrm{MS}}}(m_Z)$



HPQCD $\langle JJ \rangle$ result:

- $\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1182(7)$
- Agrees with $n_f = 3$ result.
- Agrees well with world average.

New lattice result from ALPHA collaboration using Schrodinger Functional and step-scaling:

$$\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1185(8) \text{ [1706.03821]}$$

RI intermediate schemes

NPR method

Trying to determine $Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)$ st

$$m^{\overline{\mathrm{MS}}}(\mu) = Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)\, m_0$$

Options:

- Lattice perturbation theory. – difficult!
- Alternatively, use two steps:
  latt $\leftrightarrow$ intermediate(continuum-like) $\leftrightarrow$ $\overline{\mathrm{MS}}$

NPR method

General idea is to renormalize operators using a scheme that is well-defined both in the continuum and on the lattice, e.g. the RI schemes:

Calculate off-shell Green's functions of operator-of-interest with external quark states.

$$G_{\Gamma}^{ij}(p) = \langle q^i(p) \left(\sum_x \bar{q}(x) \Gamma q(x) \right) \bar{q}^j(-p) \rangle_{\mathrm{amp}}$$

Require that the trace of the renormalized operator takes its tree-level value:

$$\Lambda_{\Gamma}(p) \equiv \frac{1}{12} \mathrm{Tr} \left[\Gamma \, G_{\Gamma}(p) \right] \simeq \frac{Z_q(p)}{Z_{\Gamma}(p)}$$

NPR method (cont.)

The RI (and $\overline{\mathrm{MS}}$) schemes satisfy $Z_m = Z_S^{-1} = Z_P^{-1}$. Z_m can be extracted from the scalar correlator provided

$$\Lambda_{\mathrm{QCD}} \ll |p| \ll \pi/a$$

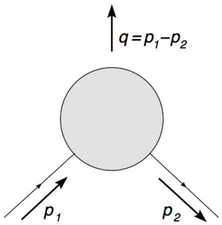
After determining $Z_m^{RI}(p)$, a perturbative calculation can be used to convert $Z^{\overline{\mathrm{MS}}}(p) = C^{\overline{\mathrm{MS}} \leftarrow RI}(p)\, Z_m^{RI}(p)$.

RI/SMOM scheme

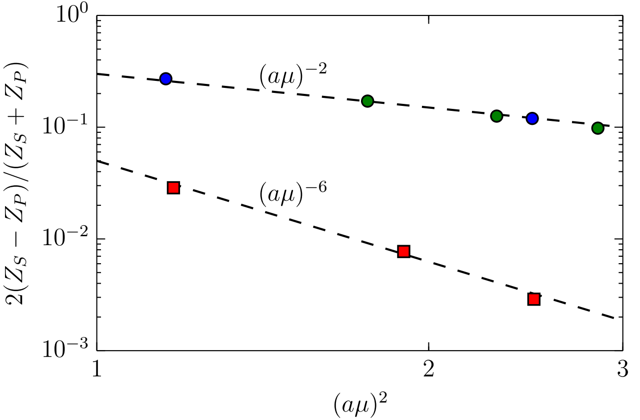
- Momentum flow suppresses infrared effects.
  $p_1^2 = p_2^2 = (p_1 - p_2)^2$

- $p_1 \sim (x, x, 0, 0)$,
  $p_2 \sim (0, x, x, 0)$ for
  $x = 2, 3, 4$

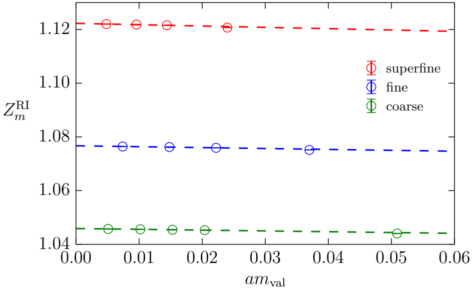
- Other advantages:
  - Reduced mass dependence.
  - SMOM $\to$ $\overline{\text{MS}}$ matching factors closer to 1.

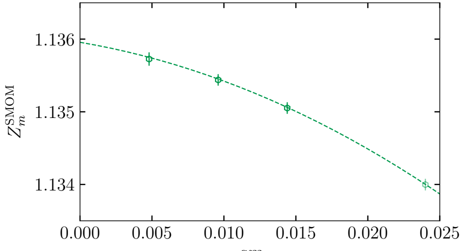


$Z_S - Z_P$



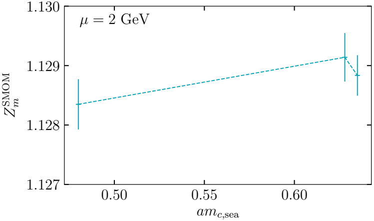
Z_m chiral extrapolation

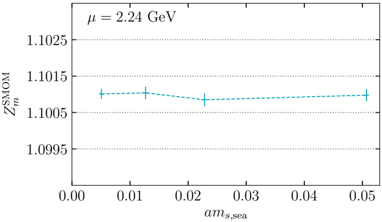




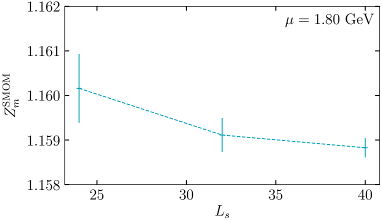
Checking systematics

Effect of varying charm, strange, and light sea masses:

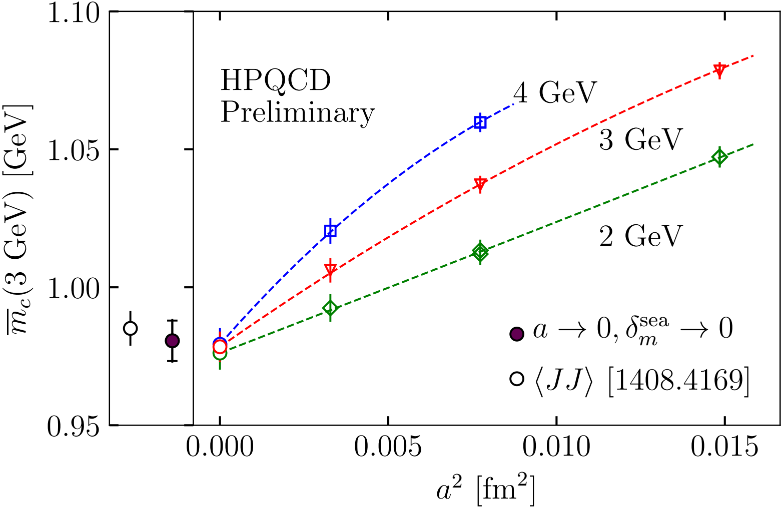




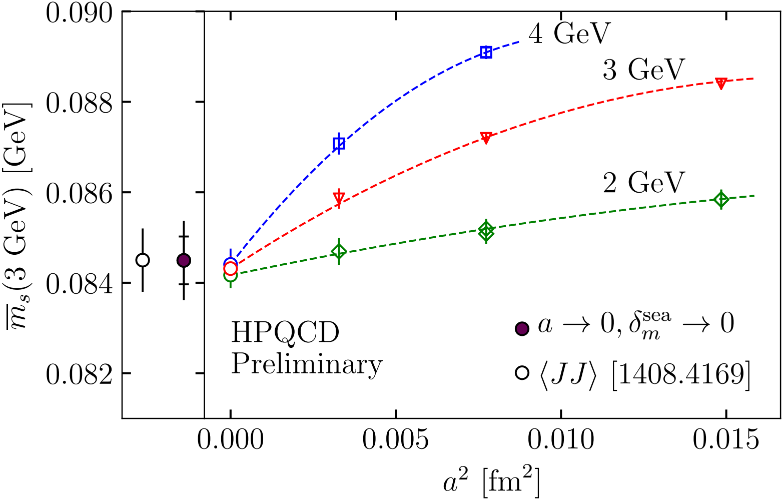
Finite volume effects:



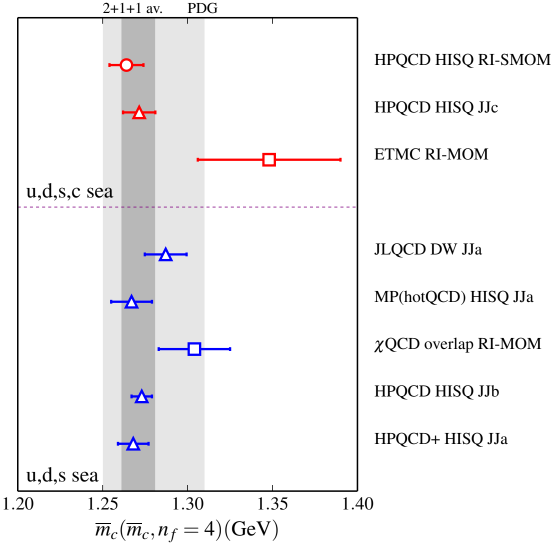
Continuum extrapolations



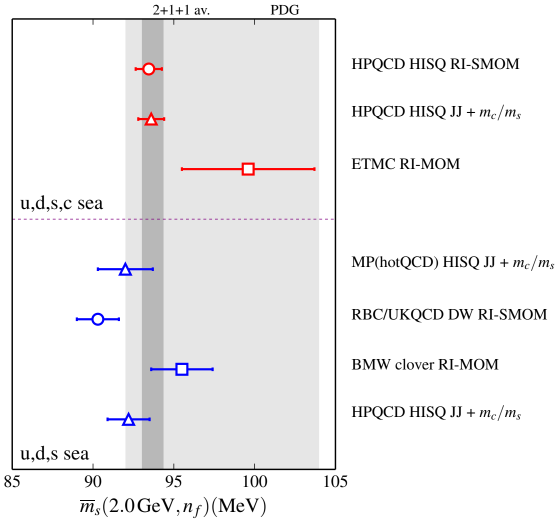
Continuum extrapolations



m_c comparison plot



m_s comparison plot



Renormalon subtracted masses

HQET masses

Mass of a heavy meson H in heavy quark effective theory (HQET)

$$M_H = m_Q + \overline{\Lambda} + \frac{\mu_\pi^2}{2m_Q} - \frac{\mu_G^2(m_Q)}{2m_Q} + \cdots ,$$

where

- m_Q: Pole mass of the heavy quark Q
- $\overline{\Lambda}$: Energy of light quarks and gluons
- $\frac{\mu_\pi^2}{2m_Q}$: Kinetic energy of heavy quark
- $\frac{\mu_G^2(m_Q)}{2m_Q}$: Hyperfine energy due to heavy quark spin

Want to relate pole mass to $\overline{\mathrm{MS}}$ mass,

$$\text{Meson mass} \leftrightarrow \text{quark pole mass} \leftrightarrow \text{quark } \overline{\mathrm{MS}} \text{ mass}$$

$m_Q \leftrightarrow m^{\overline{\text{MS}}}$ [1701.00347]

Perturbative series connecting the pole mass to the $\overline{\text{MS}}$ mass (known to four loops) diverges due to renormalons,

$$m_{\text{pole}} = \overline{m}\left(1 + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\overline{m})\right),$$

with

$$r_n \propto (2\beta_0)^n \Gamma(n+b+1) \text{ as } n \to \infty$$

but can be interpreted using Borel summation. After subtracting the (leading) renormalon from the pole mass, there is a well-behaved connection between the subtracted mass and the $\overline{\text{MS}}$ mass.

$$m_{\text{pole}} \to m_{\text{MRS}} + \mathcal{O}(\Lambda_{\text{QCD}})$$

$$
\begin{aligned}
m_{\text{pole}} + \overline{\Lambda} = \overline{m}\left(1 + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\overline{m})\right) + \overline{\Lambda} \to \\
\overline{m}\left(1 + \sum_{n=0}^{\infty} [r_n - R_n]\, \alpha_s^{n+1}(\overline{m})\right) + J_{\text{MRS}}(\overline{m}) + \left[\delta_m + \overline{\Lambda}\right] \\
= m_{\text{MRS}} + \overline{\Lambda}_{\text{MRS}}
\end{aligned}
$$

$$
\begin{aligned}
r_n &= (0.4244, 1.0351, 3.6932, 17.4358, \dots) \\
R_n &= (0.5350, 1.0691, 3.5966, 17.4195, \dots) \\
r_n - R_n &= (-0.1106, -0.0340, 0.0966, 0.0162, \dots)
\end{aligned}
$$

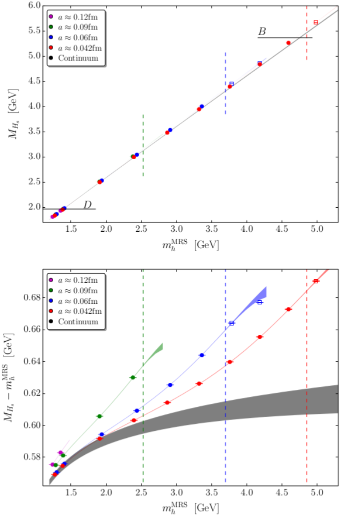
MRS - calculation

Measure meson mass M_{Hs} varying heavy input mass $am_{h,0}$.

$$m_h^{\overline{\rm MS}}(\mu) = m_r^{\overline{\rm MS}}(\mu)\frac{am_{h,0}}{am_{r,0}} + \mathcal{O}(a^2)\,,$$

with $m_r^{\overline{\rm MS}}(\mu)$ treated as a fit parameter.

- Fit data including discretization artifacts as as well as HQET parameters $\overline{\Lambda}_{\rm MRS}$, μ_π^2, $\mu_G^2(\mu)$.
- Evaluate fit at M_{D_s},M_{B_s} to obtain $\overline{m_c}$, $\overline{m_b}$.



MRS - preliminary results J. Komijani QWG 2017

$$m_s^{\overline{\text{MS}}}(2\text{ GeV}) = 92.66(28)_{\text{stat}}(40)_{\text{sys}}(48)_{\alpha_s}(11)_{f_{\pi,\text{PDG}}}\text{ MeV}$$
$$\overline{m_c} = 1274(3)_{\text{stat}}(3)_{\text{sys}}(9)_{\alpha_s}(0)_{f_{\pi,\text{PDG}}}\text{ MeV}$$
$$\overline{m_b} = 4206(8)_{\text{stat}}(8)_{\text{sys}}(6)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}}\text{ MeV}$$

These results can be compared e.g. with current-correlator results:

$$m_s^{\overline{\text{MS}}}(2\text{ GeV}) = 93.6\,(8)\text{ MeV} \qquad [1408.4169]$$
$$\overline{m_c} = 1271\,(10)\text{ MeV}$$
$$\overline{m_b} = 4196\,(23)\text{ MeV} \qquad [1408.5768]$$

Summary and Outlook - I

- Bare input mass parameters can be tuned to reproduce hadron masses measured in experiment, and can also be varied away from physical values.

- Now several independent and complementary techniques which establish strange, charm, and bottom quark masses at the (sub-)percent level.

- It is increasingly feasible to perform relativistic simulations with b quarks – currently some form of effective theory is used or an extrapolation to m_b is required – these techniques can then be applied in the same way as for charm [already the case for MRS].

Summary and Outlook - II

- RI/SMOM intermediate scheme
  - Perturbative and IR (condensate) uncertainties decrease with lattice spacing.
  - Main uncertainty comes from tuning uncertainties - need improved determinations of lattice spacings and input masses.
- Current-current correlators
  - Main uncertainty from perturbation theory.
  - Finer lattice means reference scale am_h can be increased.
  - See talk by A. Veernala (FNAL/MILC) Lattice 2017.
- MRS subtracted masses
  - Calculation already includes $a \sim 0.045$ fm lattices.
  - Uncertainty in α_s is a major source of error.

The main results presented here use MILC lattice ensembles – important to calculate with additional fermion formulations!

Thank you!