hadron resonances in coupled-channel scattering from lattice QCD

Jozef Dudek

WILLIAM & MARY
CHARTERED 1693

Jefferson Lab

hadron spectroscopy & QCD

features of the
QCD lagrangian

very light quarks
$m_{u,d}$ ~ few MeV

gluons
strongly coupled
to gluons

gluons
strongly coupled
to quarks

why not hadrons with
very many quarks ?

e.g. $\bar{u}\bar{u}dd$ Q=−2

why not
“glueballs”

why not
“hybrid mesons”

hadron spectroscopy & QCD

features of the
QCD lagrangian

very light quarks
$m_{u,d}$ ~ few MeV

gluons
strongly coupled
to gluons

gluons
strongly coupled
to quarks

why not hadrons with
very many quarks ?

e.g. $\bar{u}\bar{u}dd$ Q=−2

why not
“glueballs”

why not
“hybrid mesons”

features of the
empirical meson/baryon spectrum

mesons with “$q\bar{q}$”
quantum numbers

baryons with “qqq”
quantum numbers

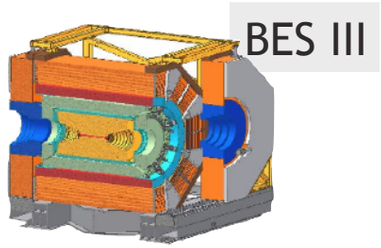
?

hadron spectroscopy & QCD

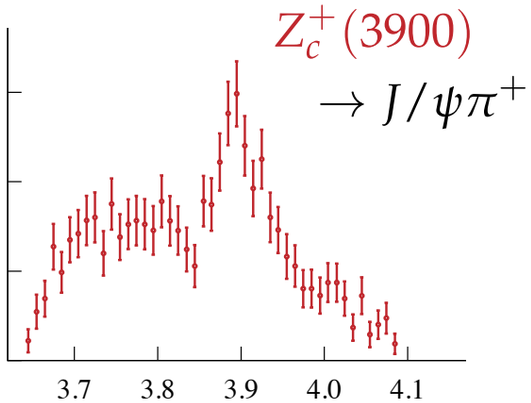
can't just read off properties from the lagrangian

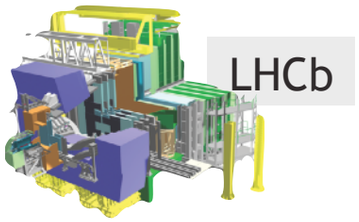
non-perturbative at low energies
- chiral symmetry breaking
- confinement
- ⋮

and the experimental situation is no longer so clear

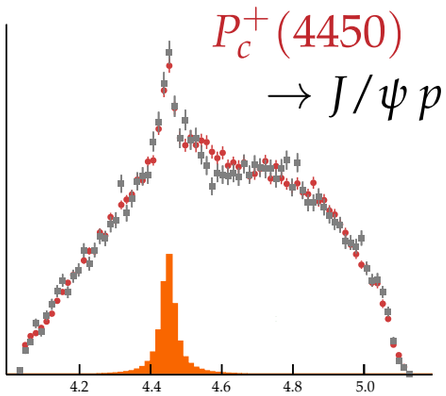


e^+e^-





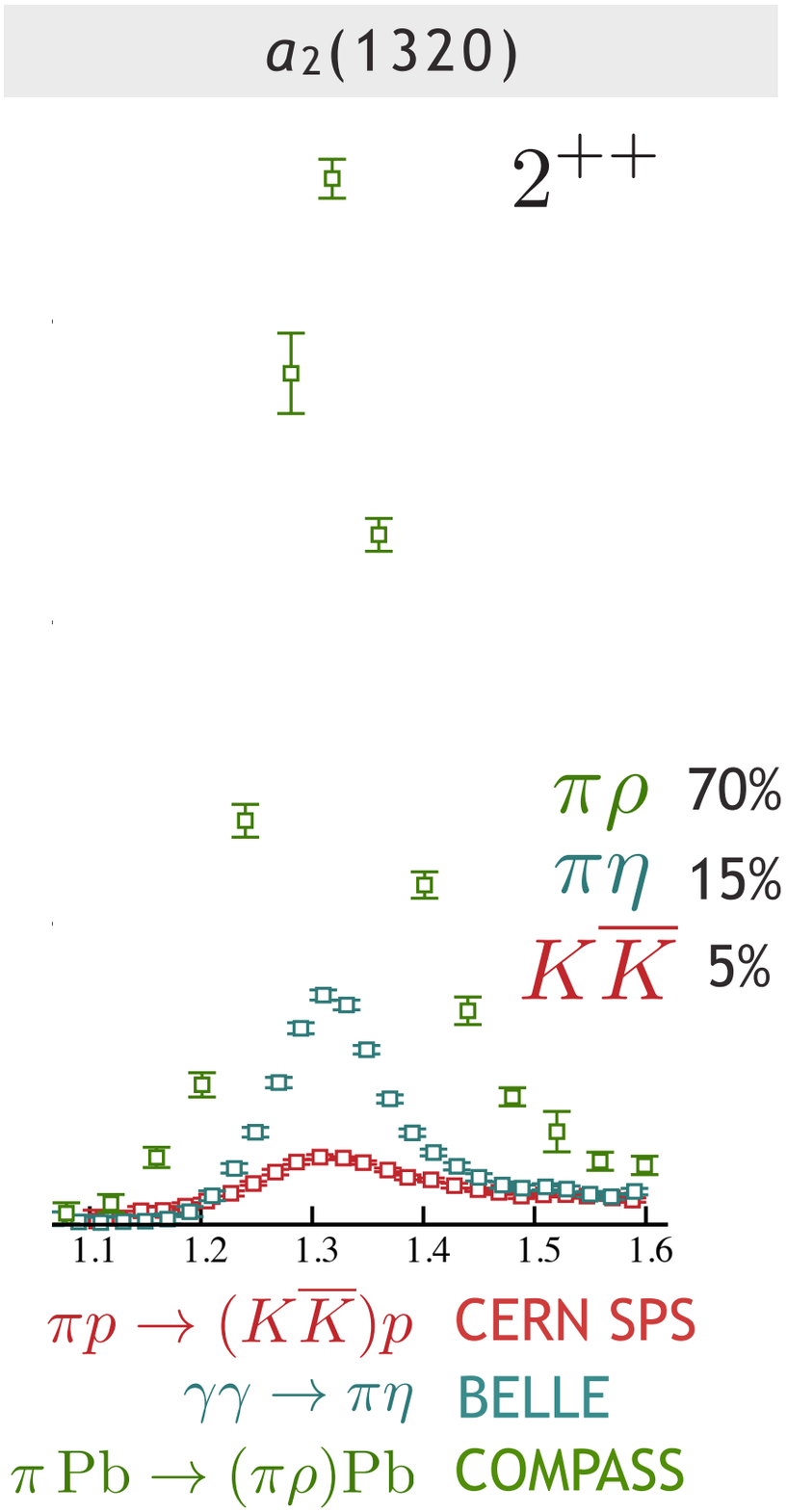
pp



the *XYZ* era ...

apply first-principles non-perturbative QCD methods to the hadron spectroscopy problem ...

multi-channel resonances



a classic 'straightforward' resonance

clear narrow 'bump' in each channel with no significant 'background'

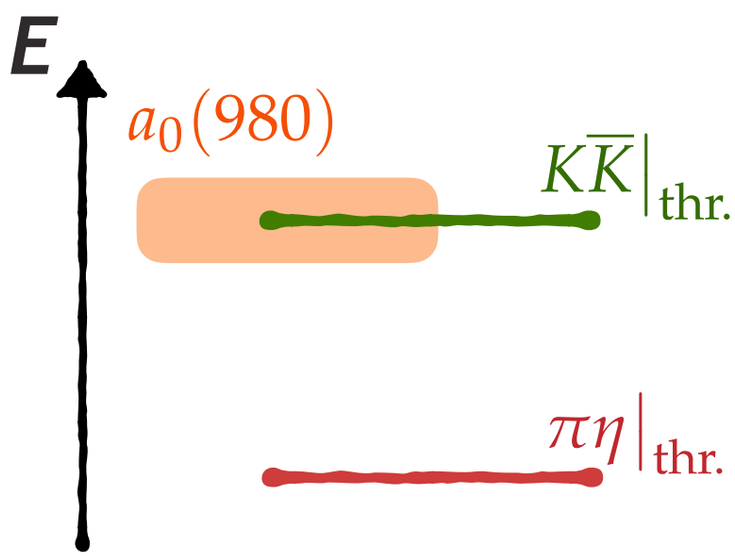
can be described by Breit-Wigner-like forms

$$t(E) \sim \frac{1}{E^2 - m^2 + i g_1^2\,\rho_1(E) + i g_2^2\,\rho_2(E) + \ldots}$$

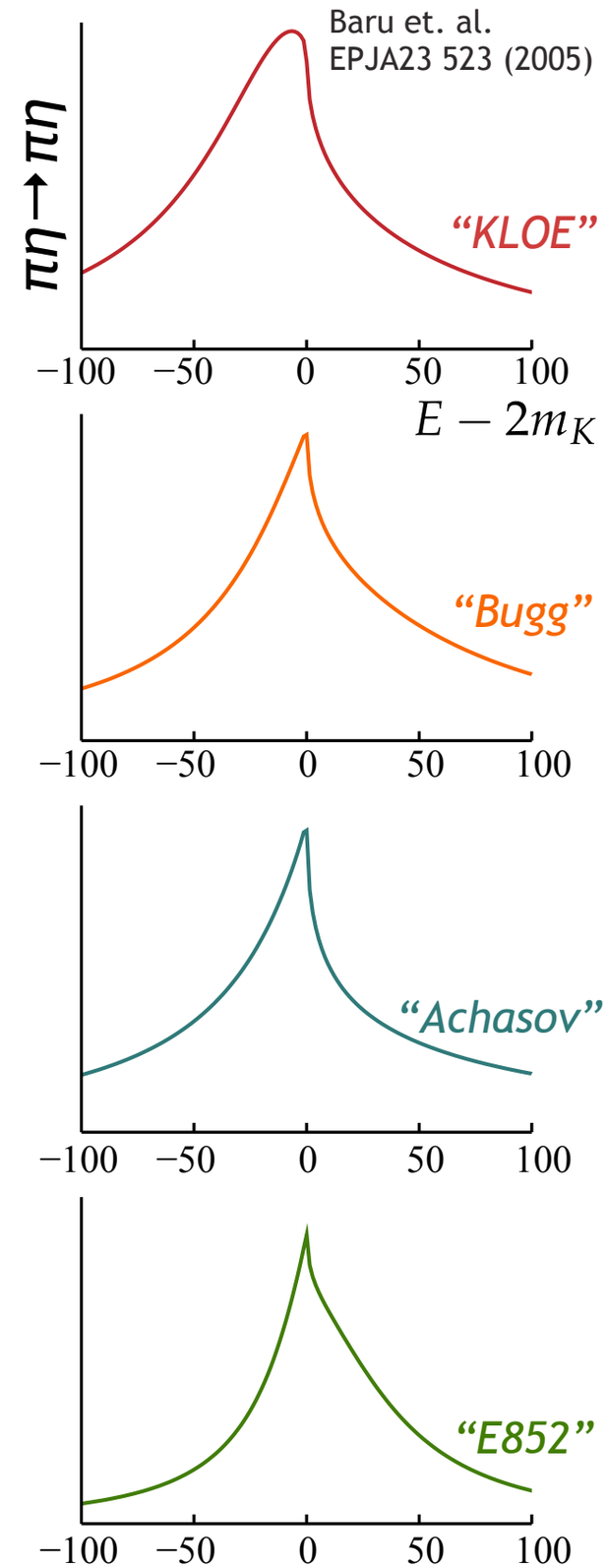
but resonance signatures aren't always so simple …

$a_0(980)$

sharp experimental enhancement at $K\bar{K}$ threshold decaying to $\pi\eta$



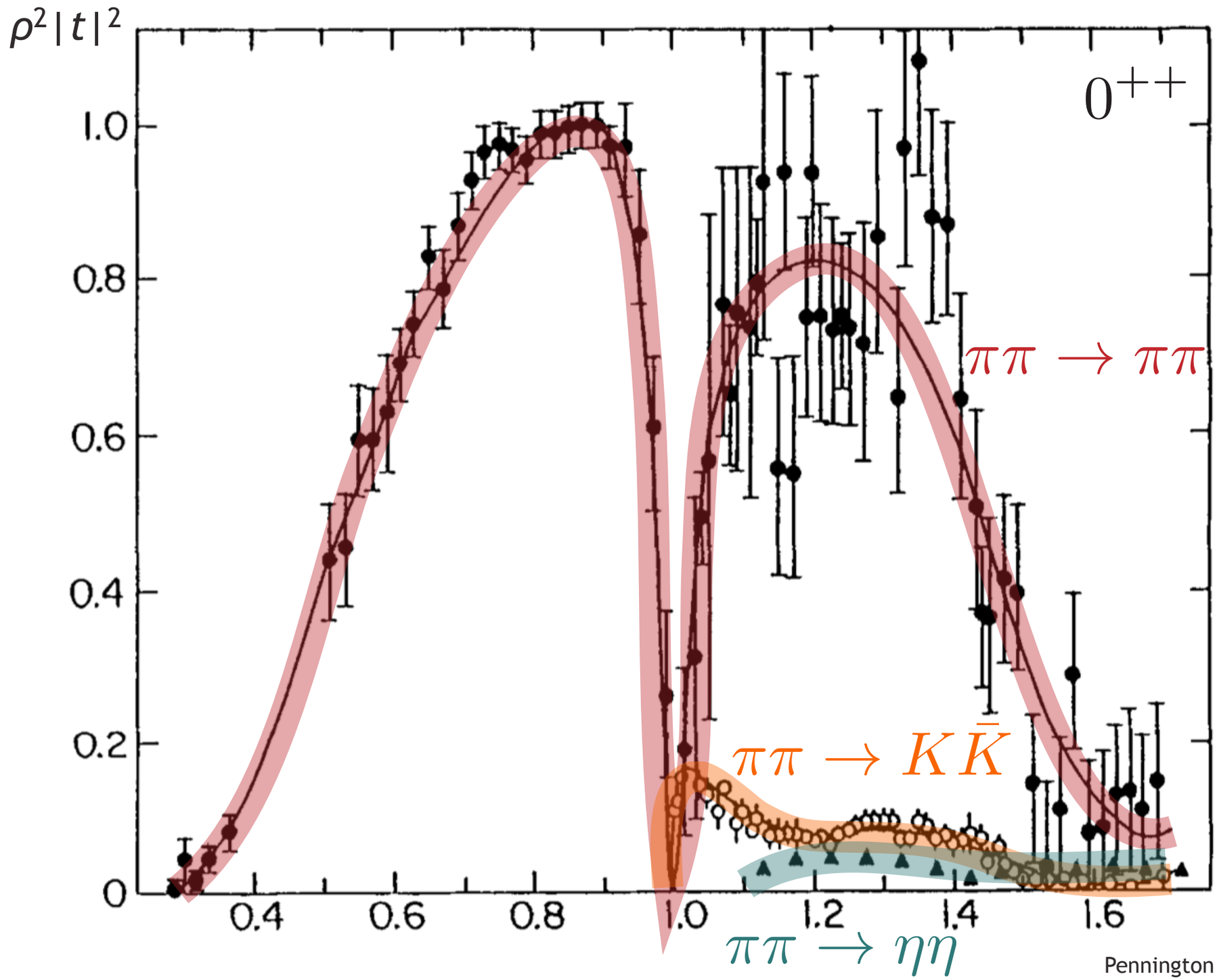
is this a resonance ?



$f_0(980)$ and σ

low-energy
broad structure
the "σ"

$f_0(980)$ as
dip in elastic
amplitude



Pennington
AIP Conf.Proc.
185 145 (1989)

… how many resonances here ?

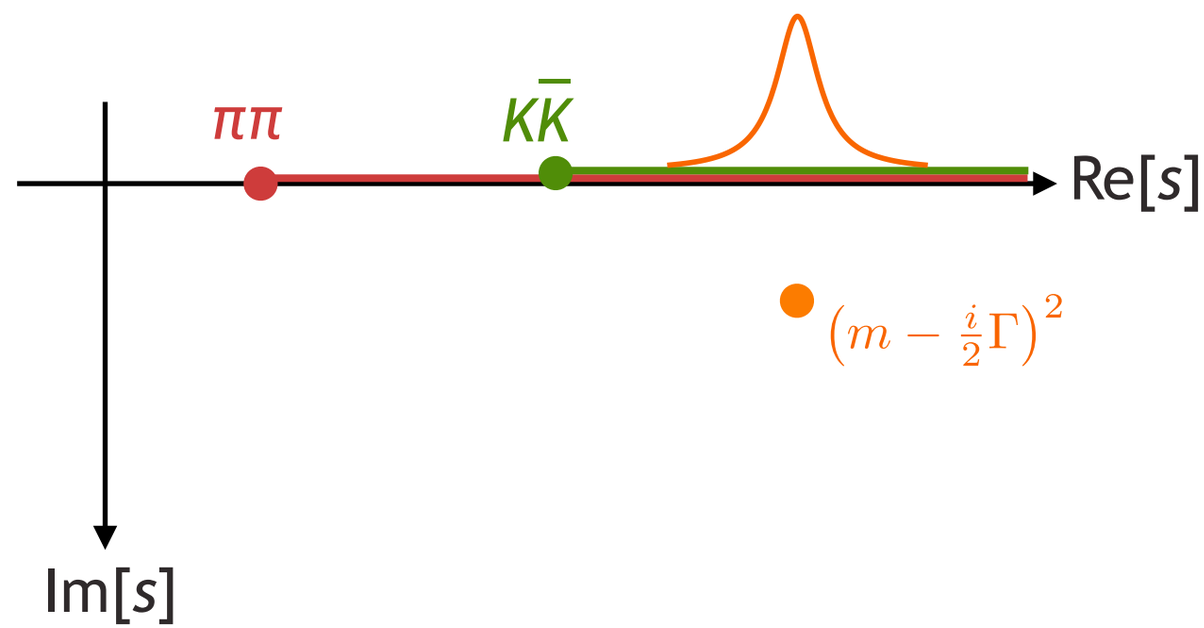
resonance poles

the rigorous way to characterize **resonances**
in terms of **scattering amplitudes** is as

pole singularities in the complex energy plane

near a pole $s \sim s_0$ $\quad t_{ij}(s) \sim \dfrac{c_i\, c_j}{s_0 - s}$

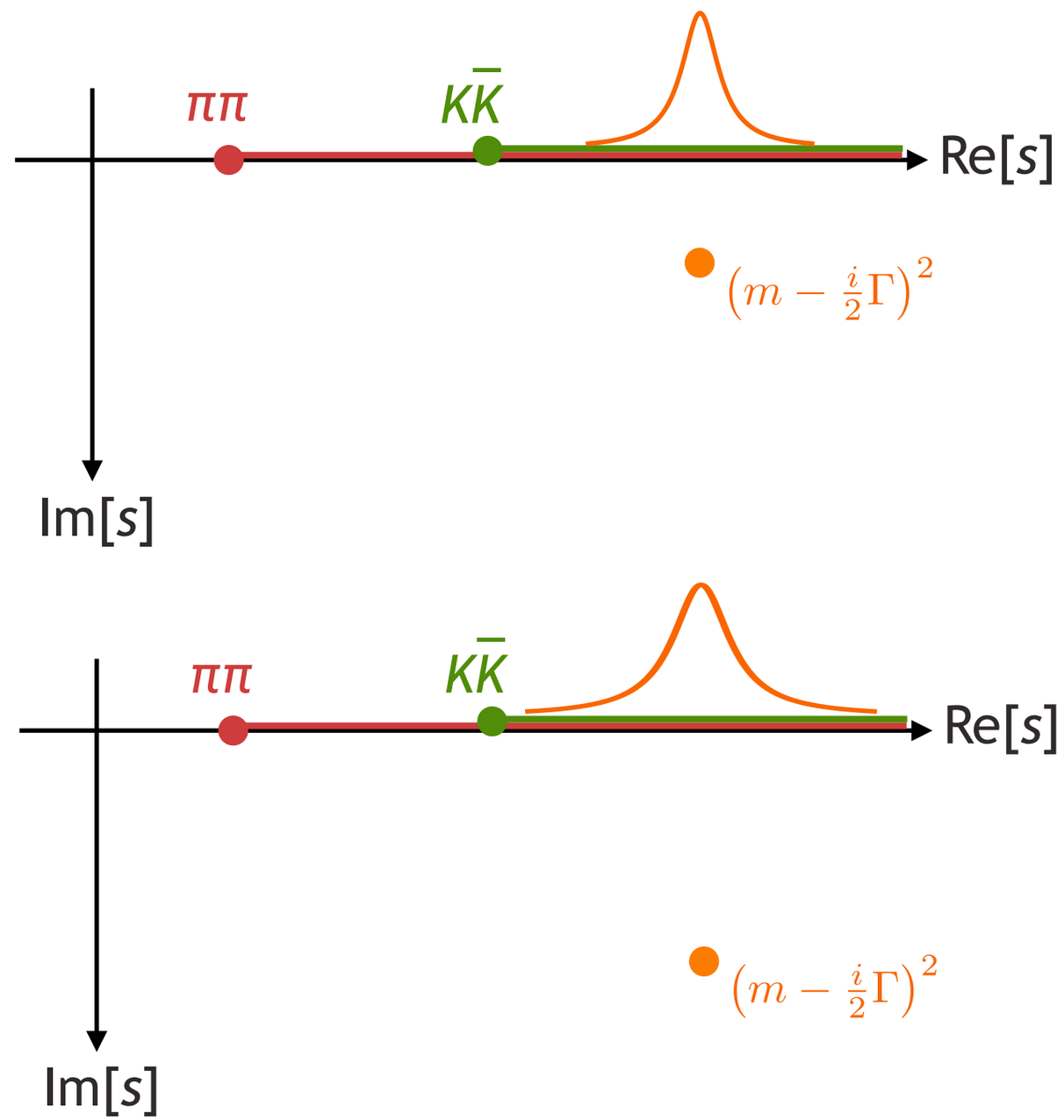
$$\sqrt{s_0} = m - i\tfrac{1}{2}\Gamma$$



resonance poles

the rigorous way to characterize **resonances** in terms of **scattering amplitudes** is as

pole singularities in the complex energy plane



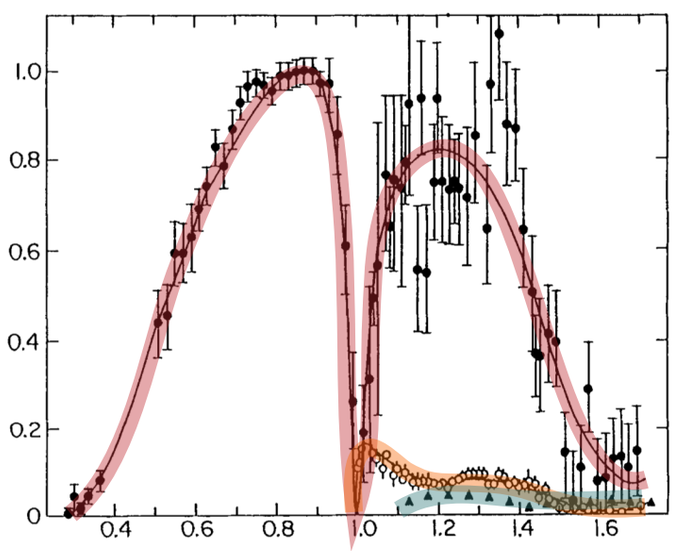
further from the real axis, the broader the resonance

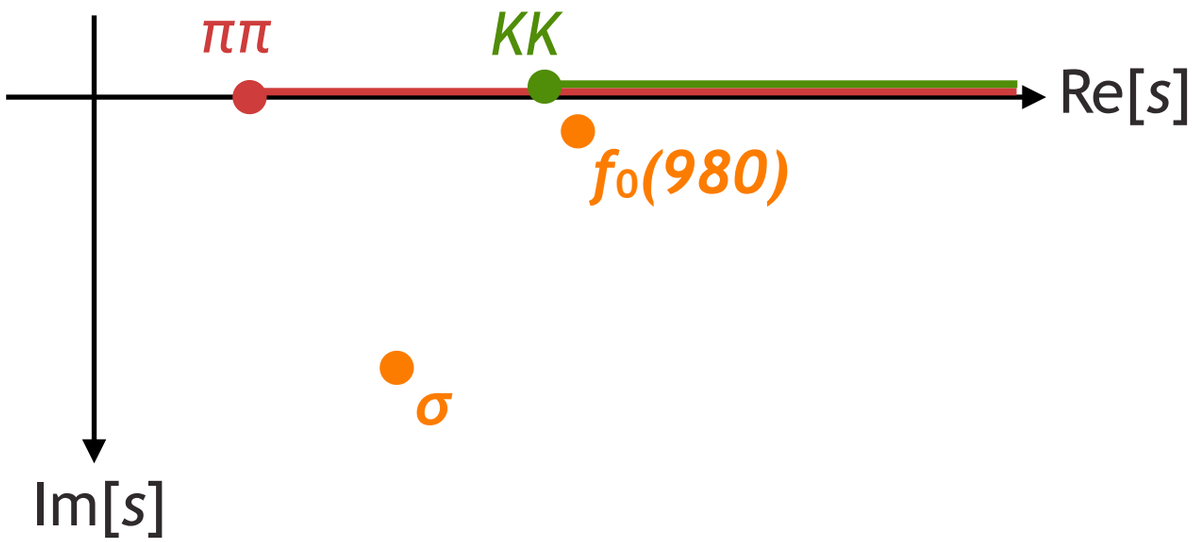
resonance poles

the rigorous way to characterize **resonances**
in terms of **scattering amplitudes** is as

pole singularities in the complex energy plane

multiple poles and/or poles near thresholds
can lead to less simple signatures



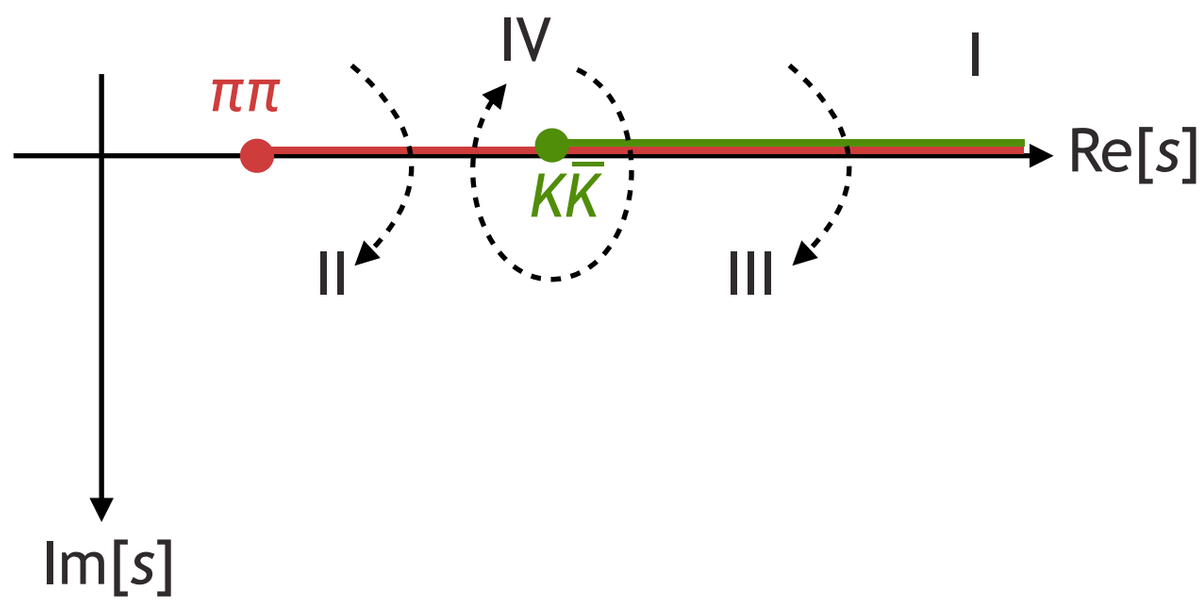


poles & sheets

multichannel scattering actually a bit more complicated: multiple Riemann sheets

unitarity: $\mathrm{Im}\, t = \rho\, |t|^2$

phase-space: $\rho(s) = \dfrac{2k}{\sqrt{s}} = \dfrac{\sqrt{s-4m^2}}{\sqrt{s}}$ branch cut at each threshold



which sheets the poles lie on does change
the scattering amplitude for real energies

how can any of this be related to QCD … ?

lattice QCD

- first-principles numerical approach to the field-theory
  - evaluate **correlation functions**

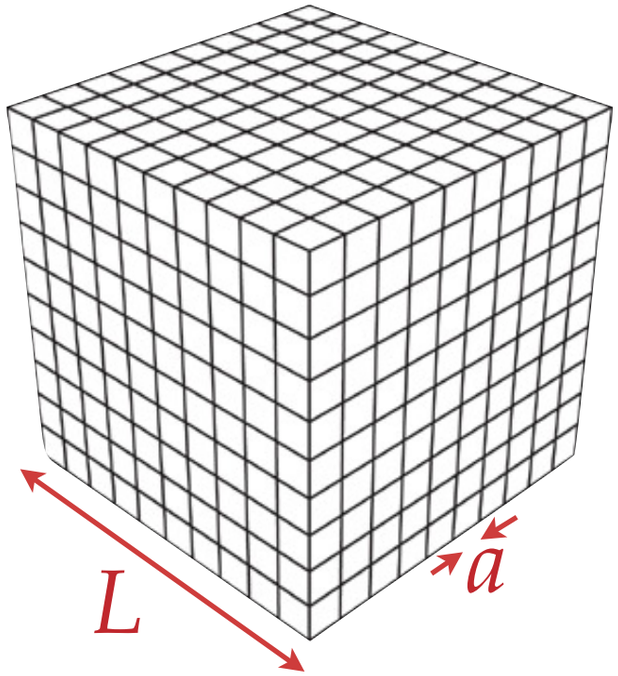
$$\int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}A_\mu \;\; f(\psi, \bar{\psi}, A_\mu)\; e^{i\int d^4x\, \mathcal{L}(\psi,\bar{\psi},A_\mu)}$$

via **Monte-Carlo** sampling of path-integral on a **finite cubic grid**

  - e.g. **discrete spectrum** from (euclidean) **two-point correlation functions**

$$\langle 0|\mathcal{O}(t)\,\mathcal{O}(0)|0\rangle = \sum_n e^{-E_n t}\left|\langle 0|\mathcal{O}|n\rangle\right|^2$$

CUBIC LATTICE



» in principle recover physical QCD as

$$a \to 0 \quad L \to \infty$$

» practical calculations often use

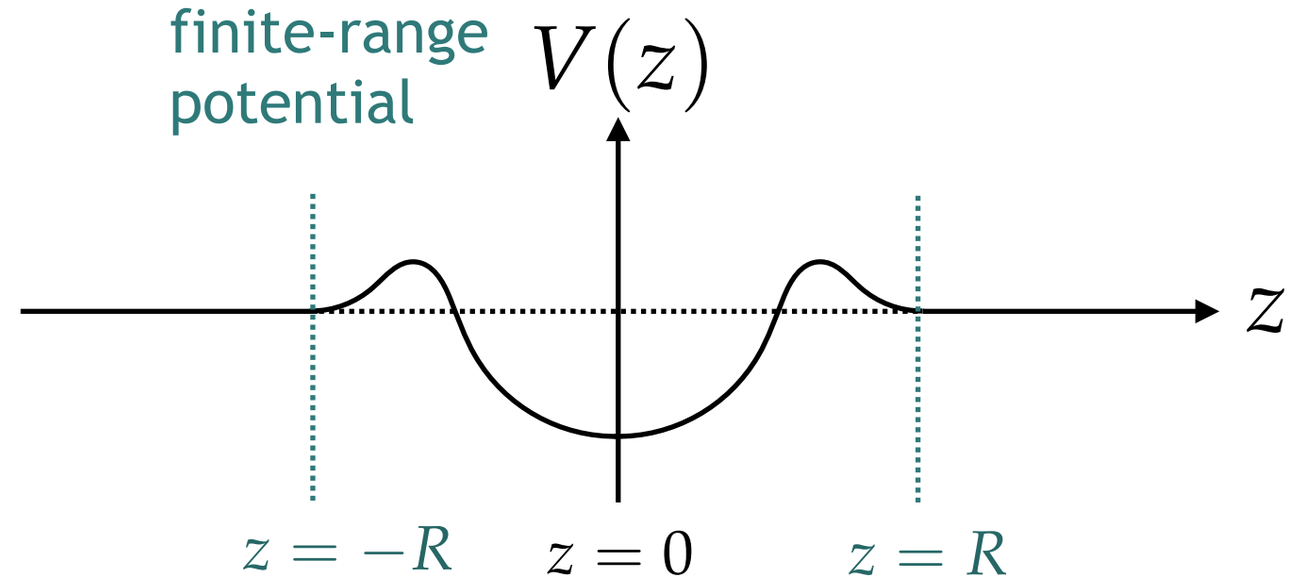
$$m_q^{\text{calc.}} > m_q^{\text{phys.}}$$

but how to get at scattering … ?

one-dim quantum mechanics

consider scattering of two identical bosons

finite-range potential

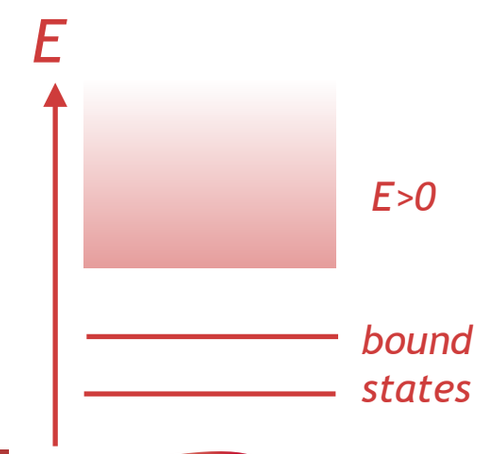


outside the well

$$\psi(|z| > R) \sim \cos\left(p|z| + \delta(p)\right)$$

scattering phase-shift $t = \frac{1}{\rho} e^{i\delta} \sin\delta$

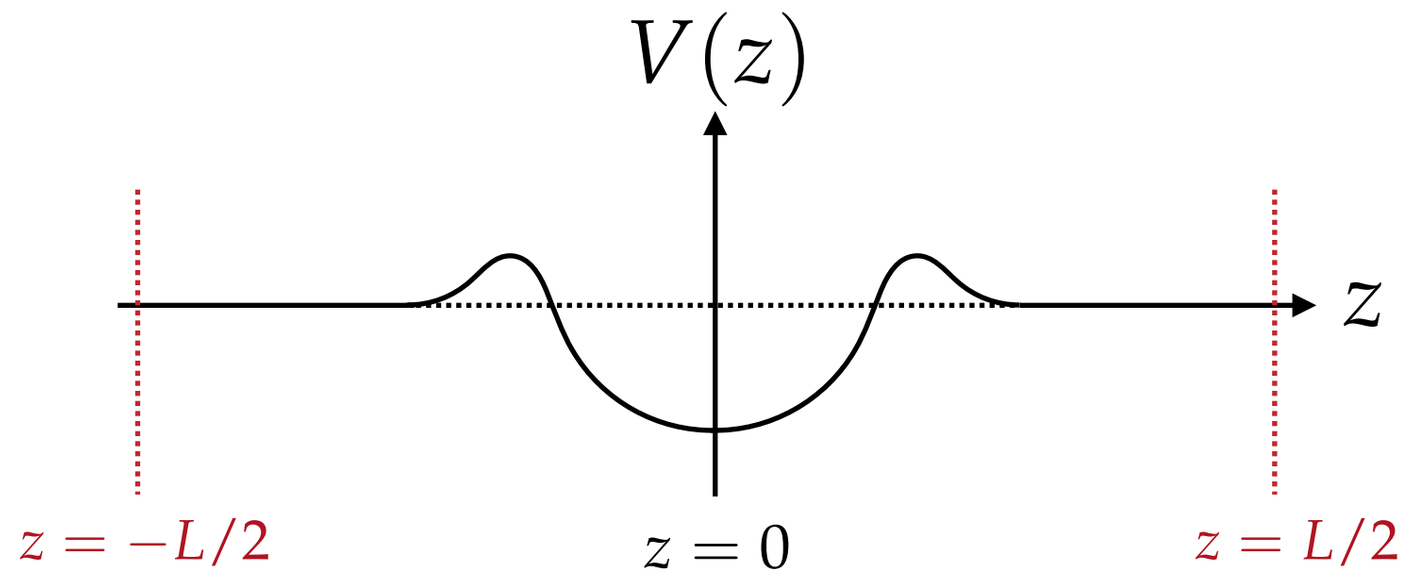
continuous momentum $\Rightarrow$ continuous spectrum of energy eigenstates



elastic 'scattering' in a finite-volume

outside the well

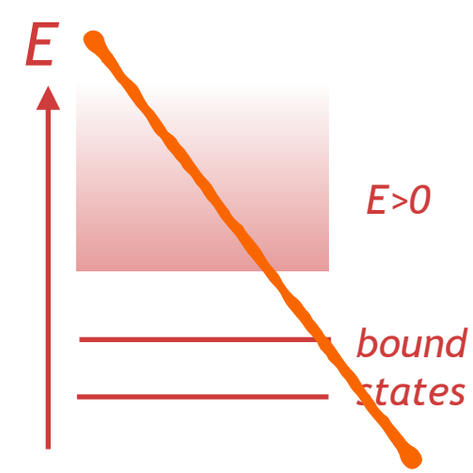
$$\psi(|z| > R) \sim \cos\left(p|z| + \delta(p)\right)$$

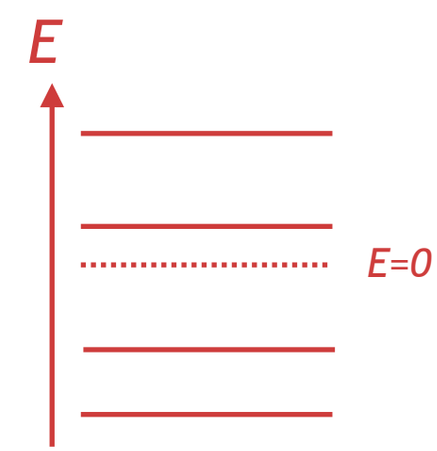


- apply periodic boundary conditions

$$\boxed{p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)}$$

discrete energy spectrum



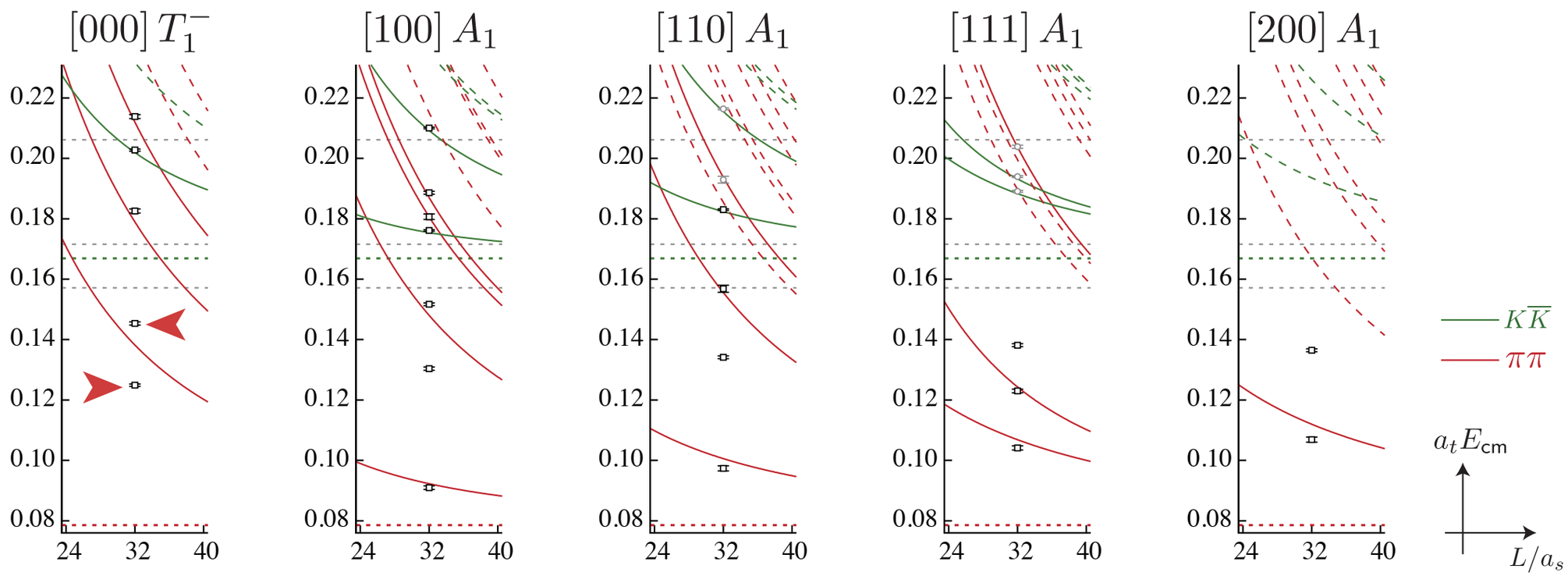


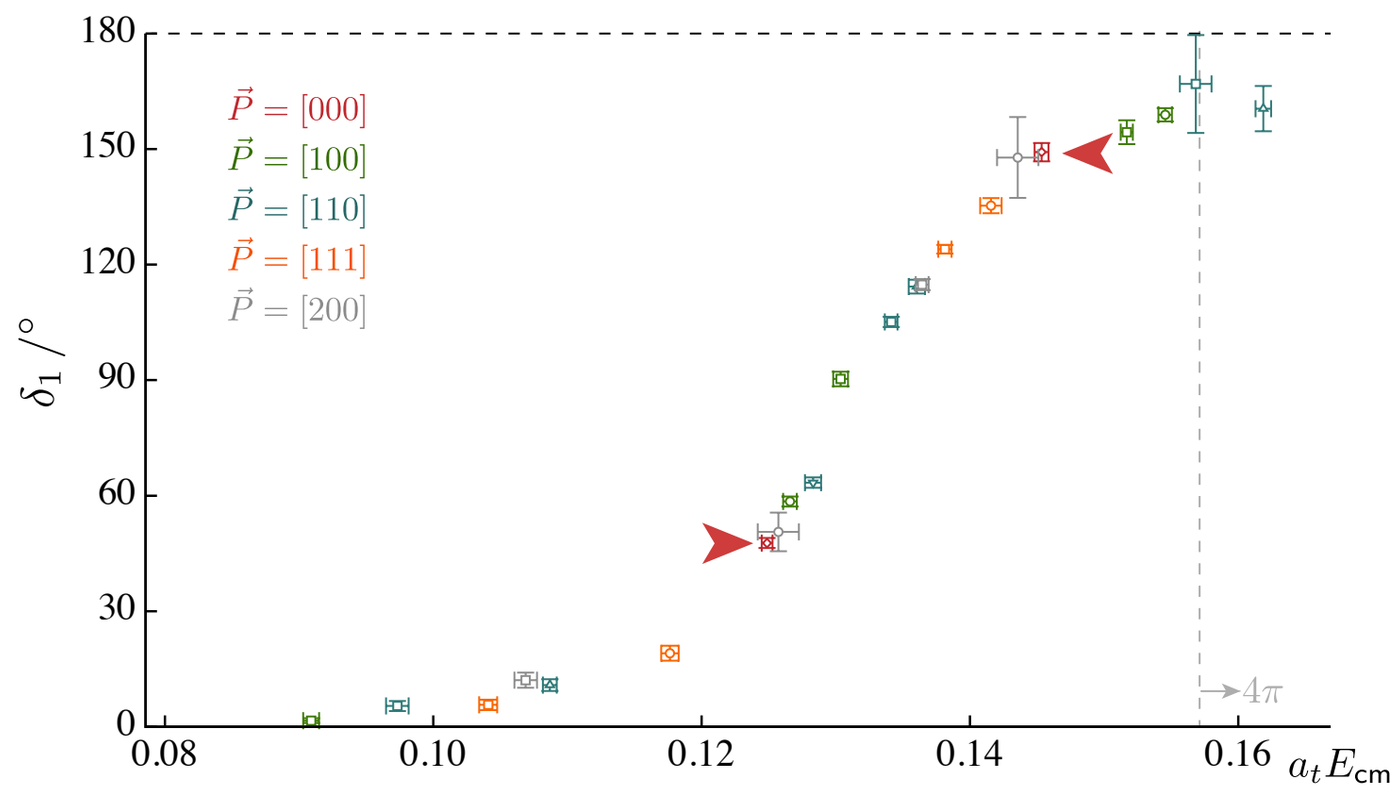
3+1 dim field theory version due to Lüscher

the ρ channel : elastic $\pi\pi$ scattering

m_π = 236 MeV

Wilson et al
PRD92 094502 (2015)



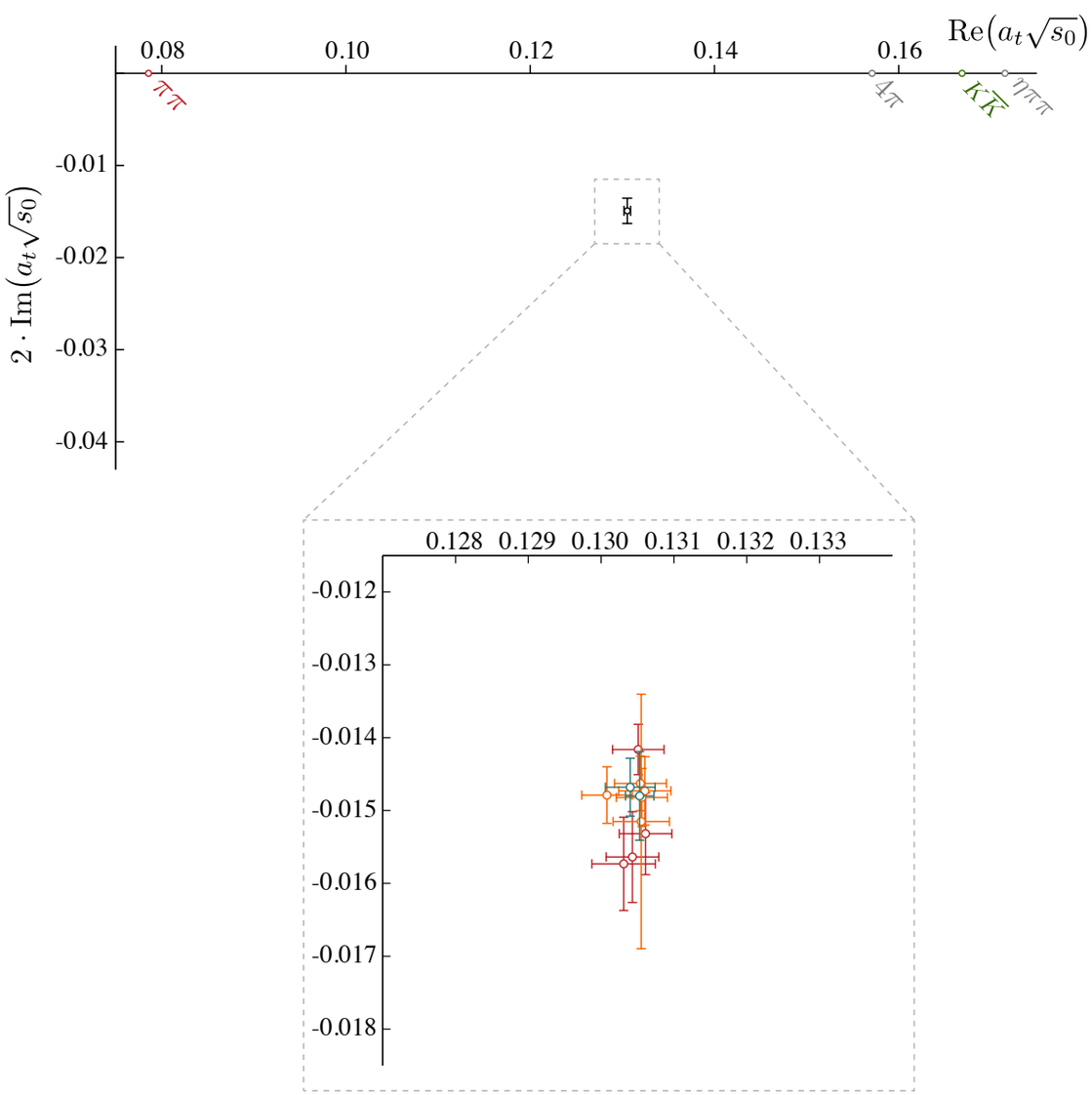


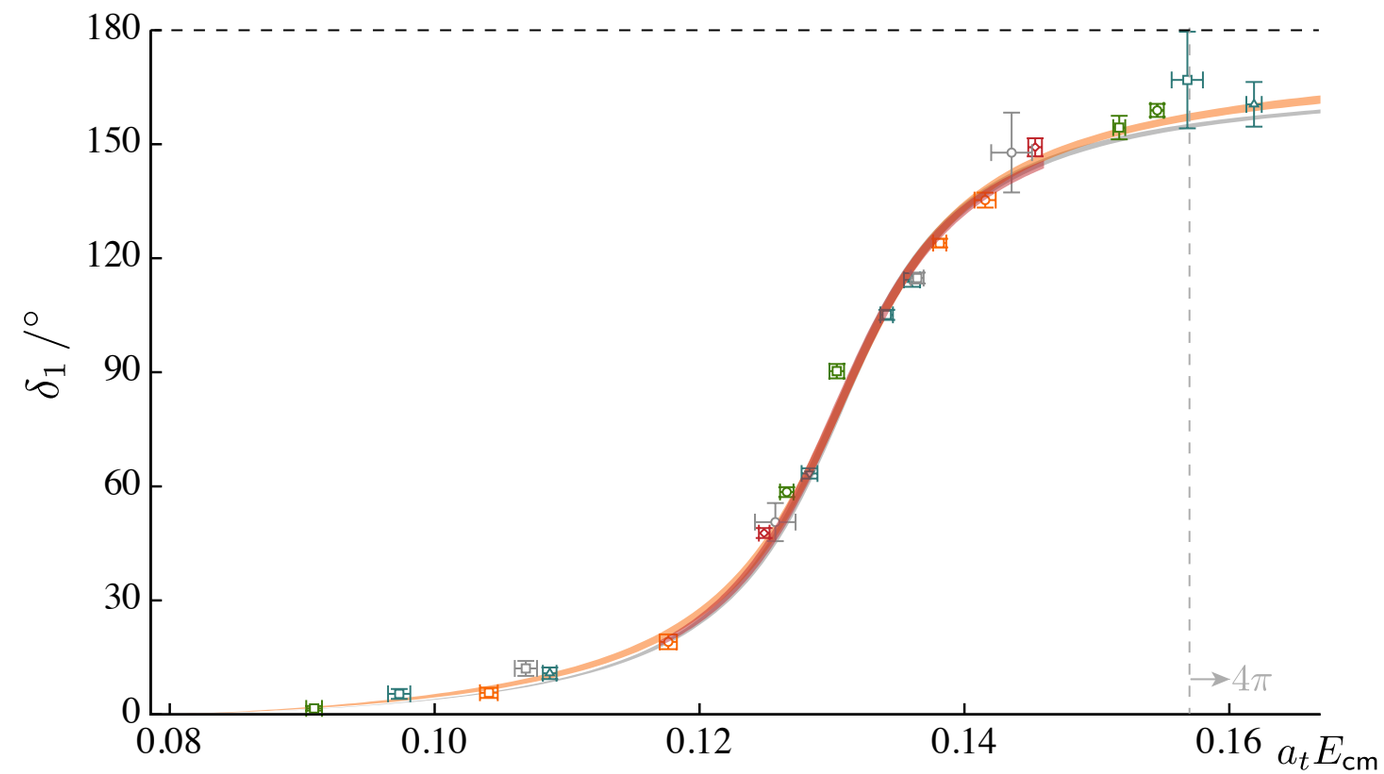
lots of extra points by considering moving frames …

the ρ channel : elastic $\pi\pi$ scattering

m_π = 236 MeV

Wilson et al
PRD92 094502 (2015)





coupled-channels in finite-volume

the discrete spectrum in a finite-volume given by solutions of

$$\det\left[\mathbf{t}^{-1} + i\boldsymbol{\rho} - \mathbf{M}\right] = 0$$

actually some additional complications from spin
can be handled systematically

scattering matrix $t_{ij}(E)$ matrix in channel space *i,j*

phase-space $\rho_{ij}(E) = \frac{2k_i}{E}\delta_{ij}$

depends on frame

finite-volume function $M_{ij} = \mathcal{M}^{(\mathbf{P})}(k_i L)\,\delta_{ij}$

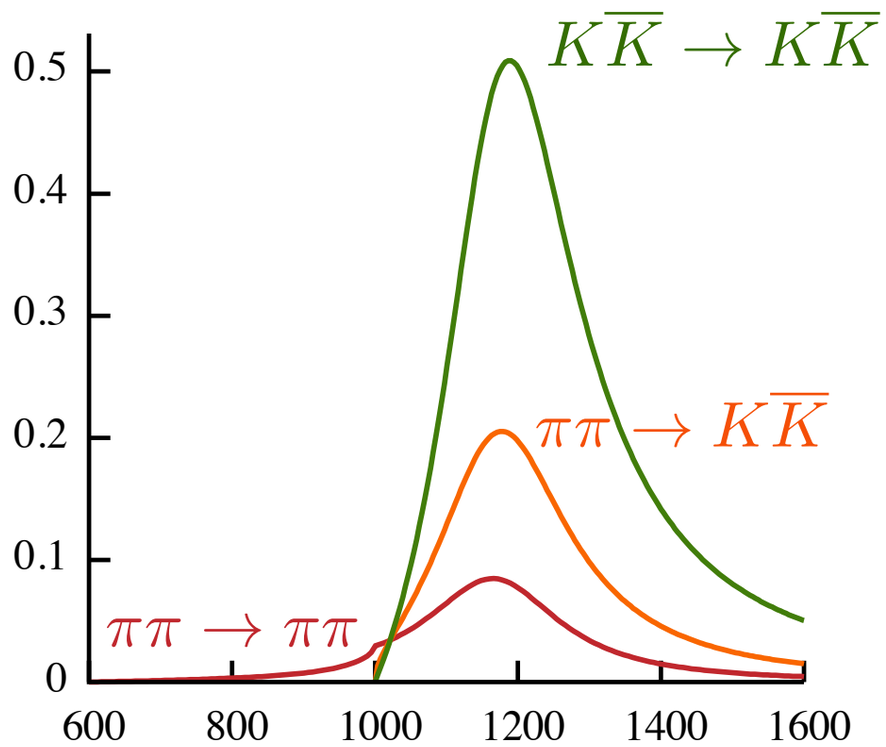
depends on volume

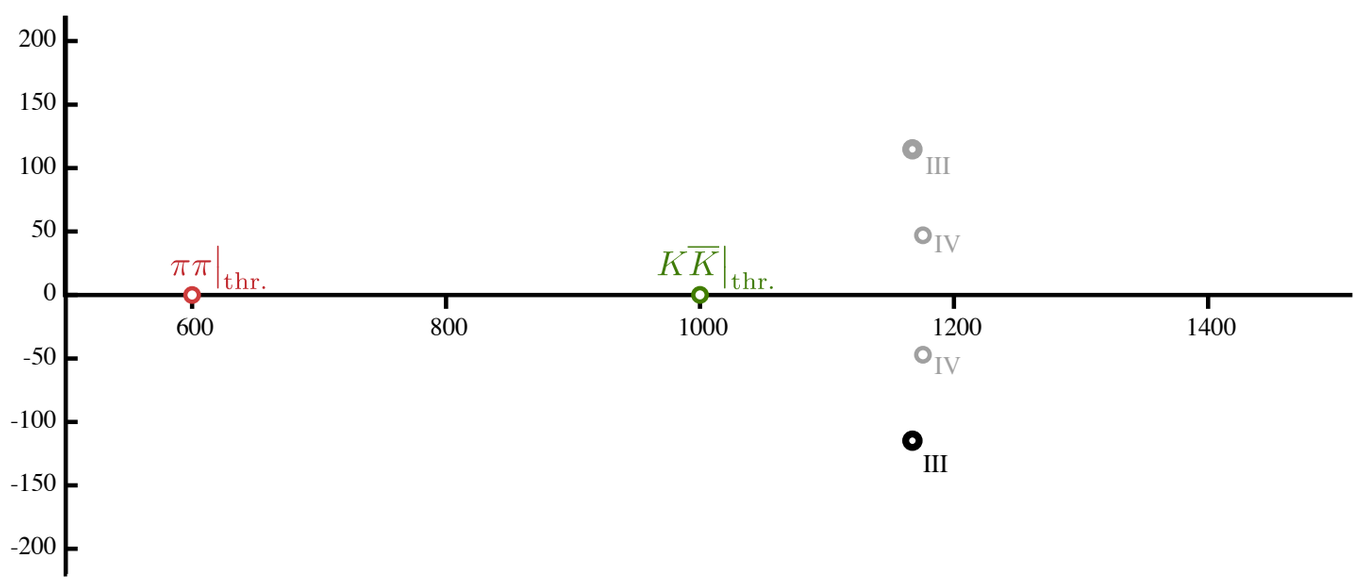
for a given $t_{ij}(E)$ find all *E* which solve the equation …

a ‘simple’ coupled-channel resonance

a two-channel *S*-wave resonance

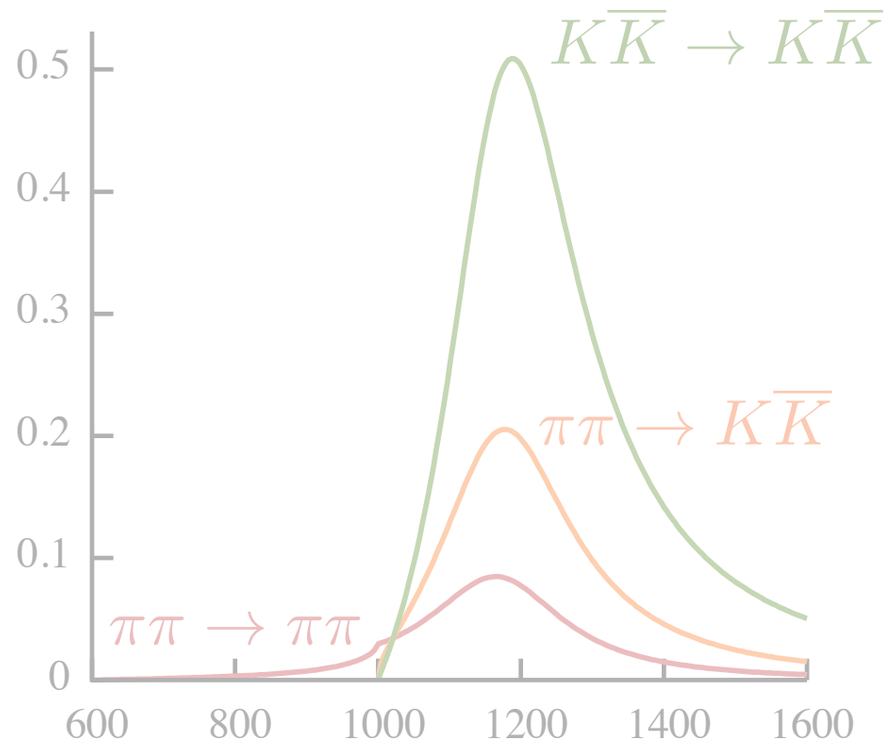
m_π = 300 MeV
m_K = 500 MeV

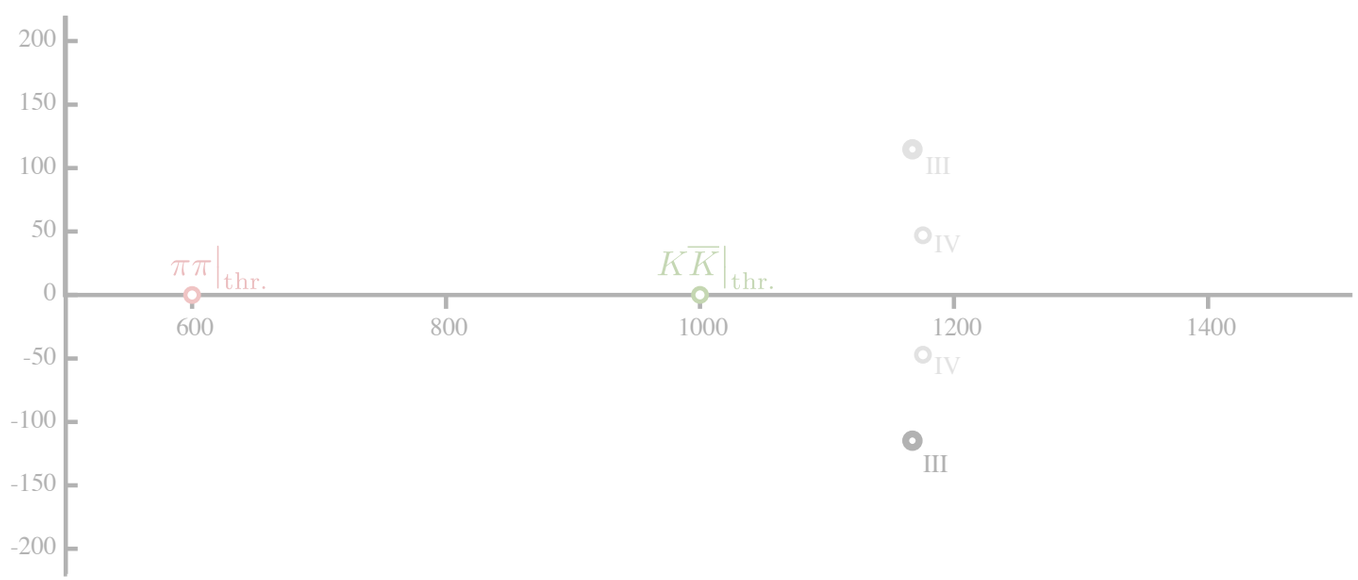


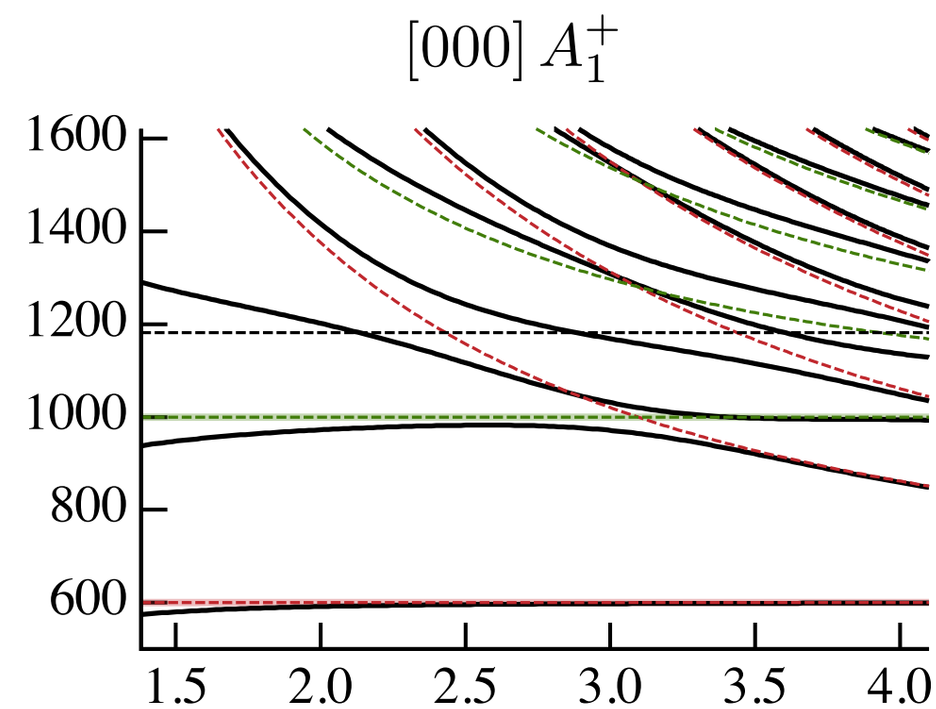


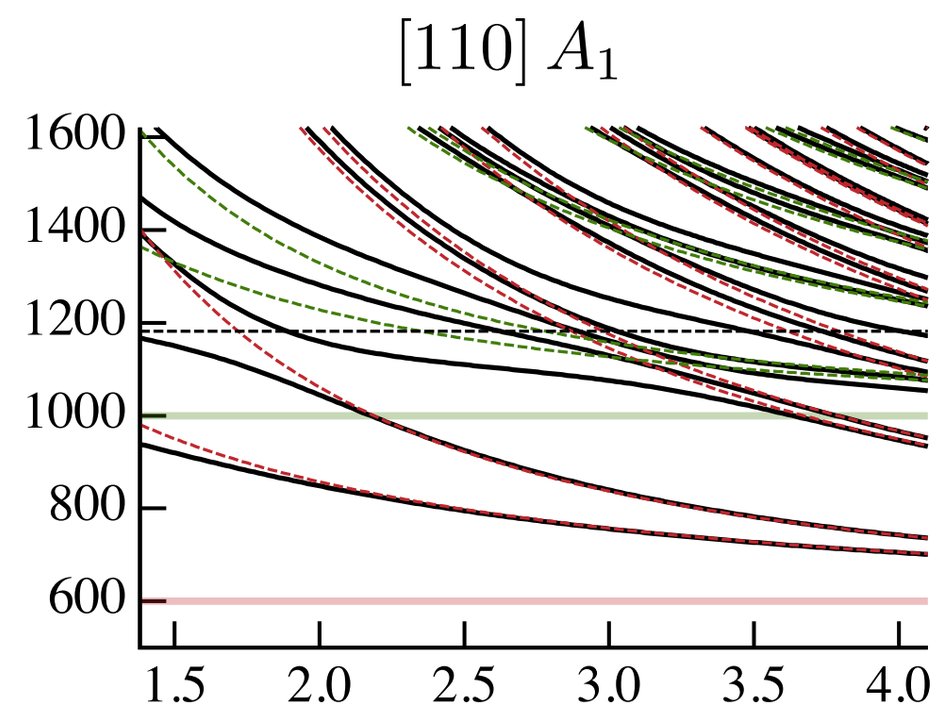
a ‘simple’ resonance in finite-volume

a two-channel *S*-wave resonance



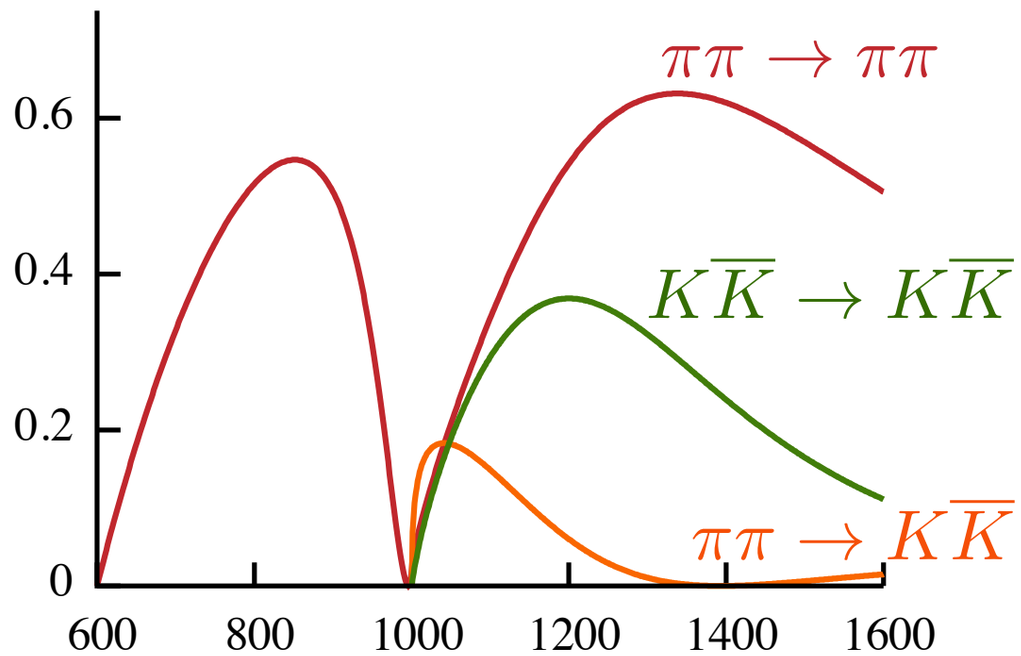


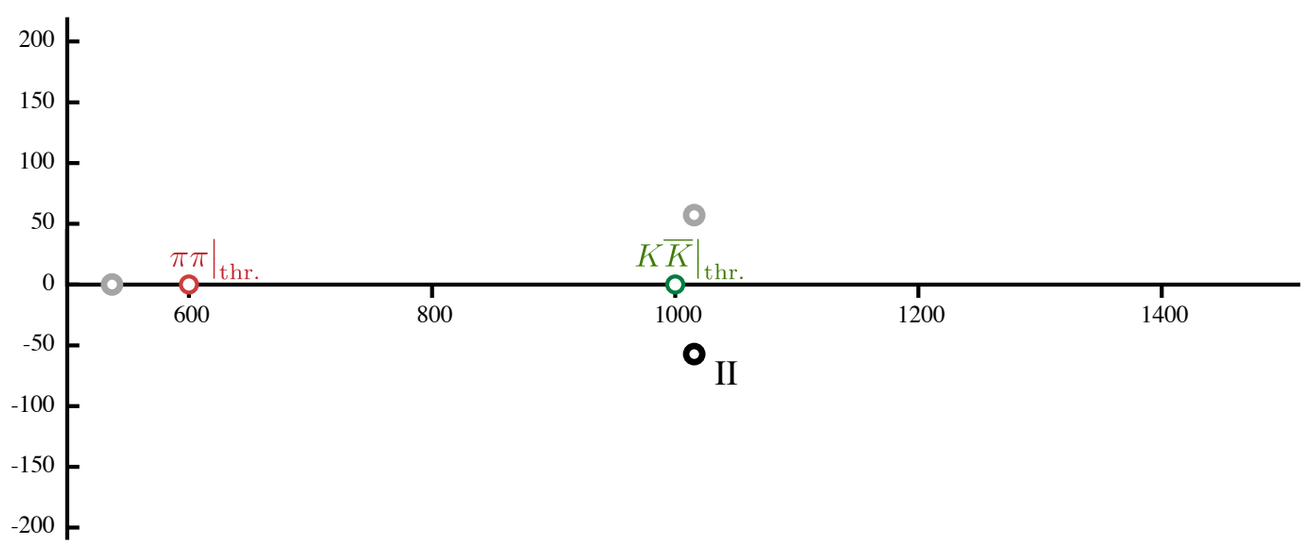




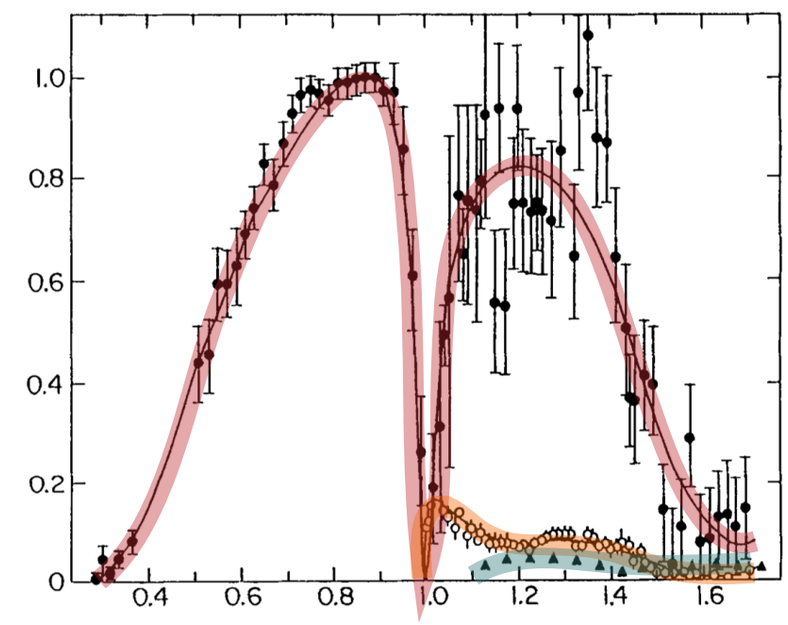
a less simple resonance

a two-channel *S*-wave amplitude with a sheet II pole



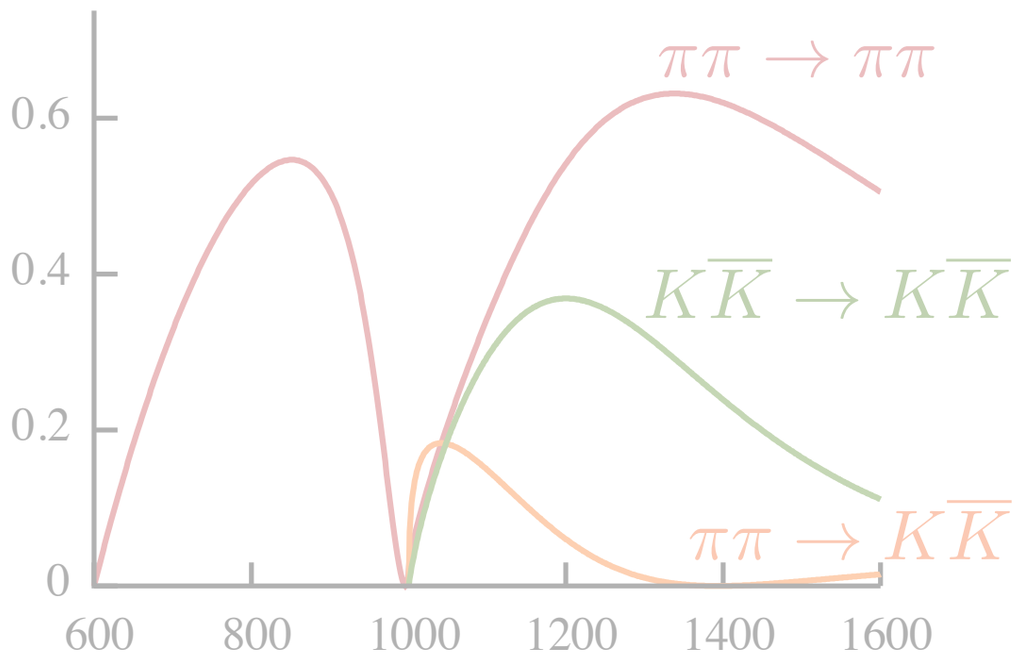


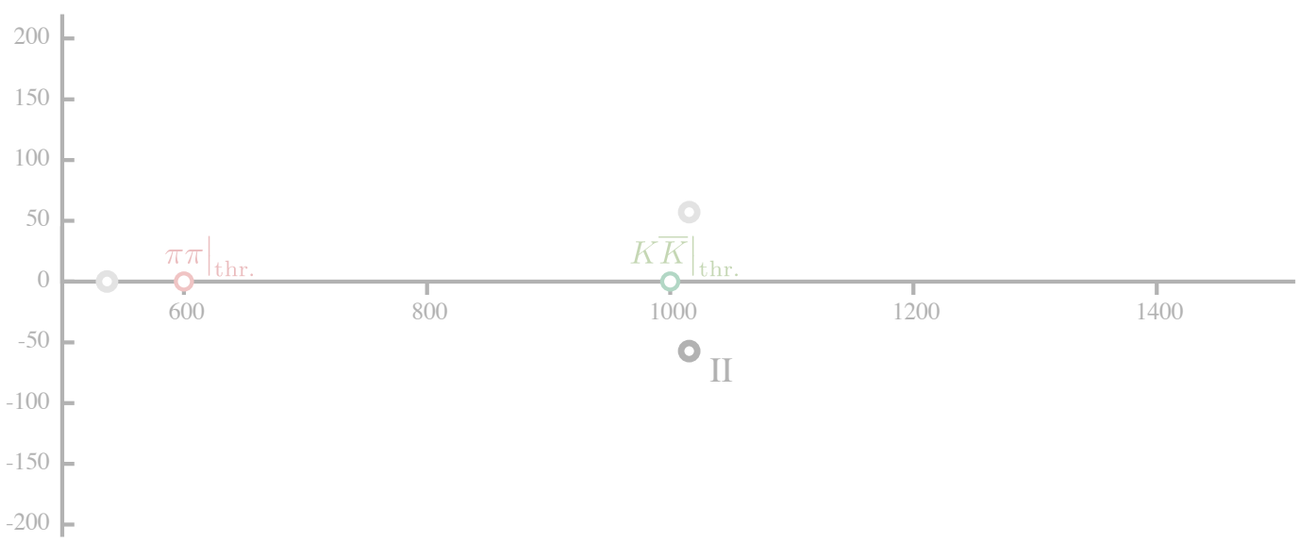
N.B. rather similar to the $f_0(980)$

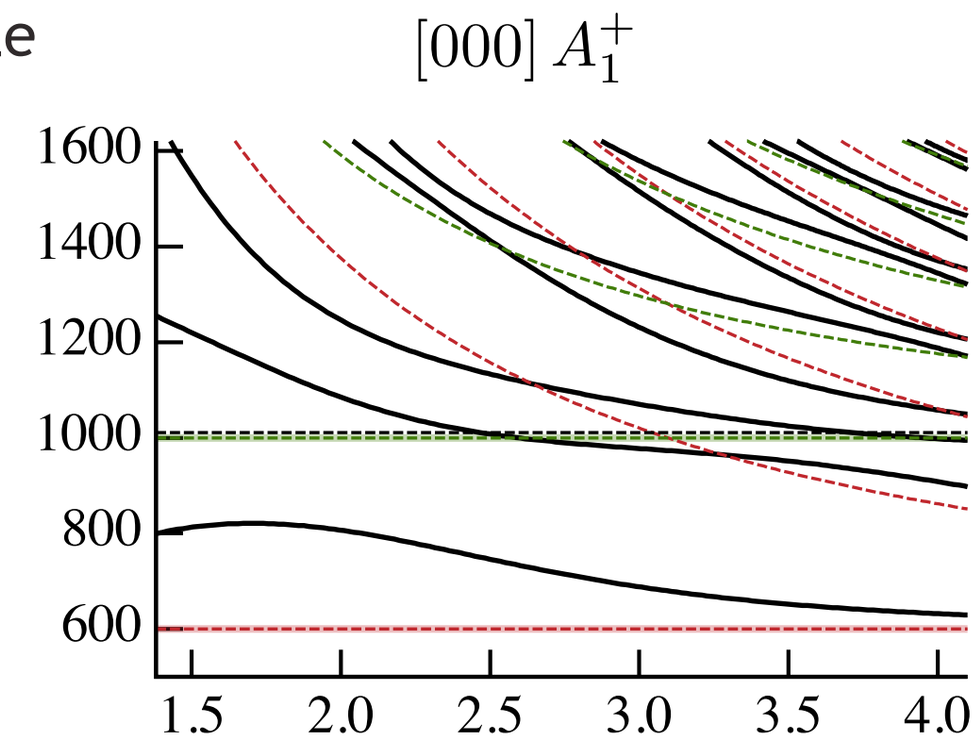


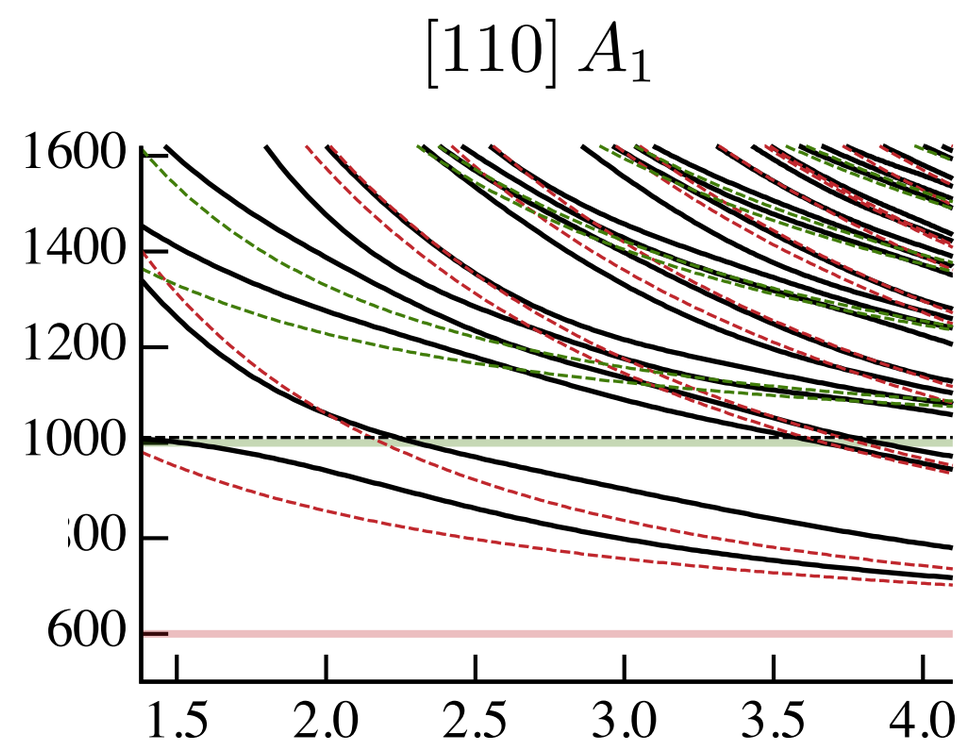
a less simple resonance in finite-volume

a two-channel *S*-wave amplitude with a sheet II pole







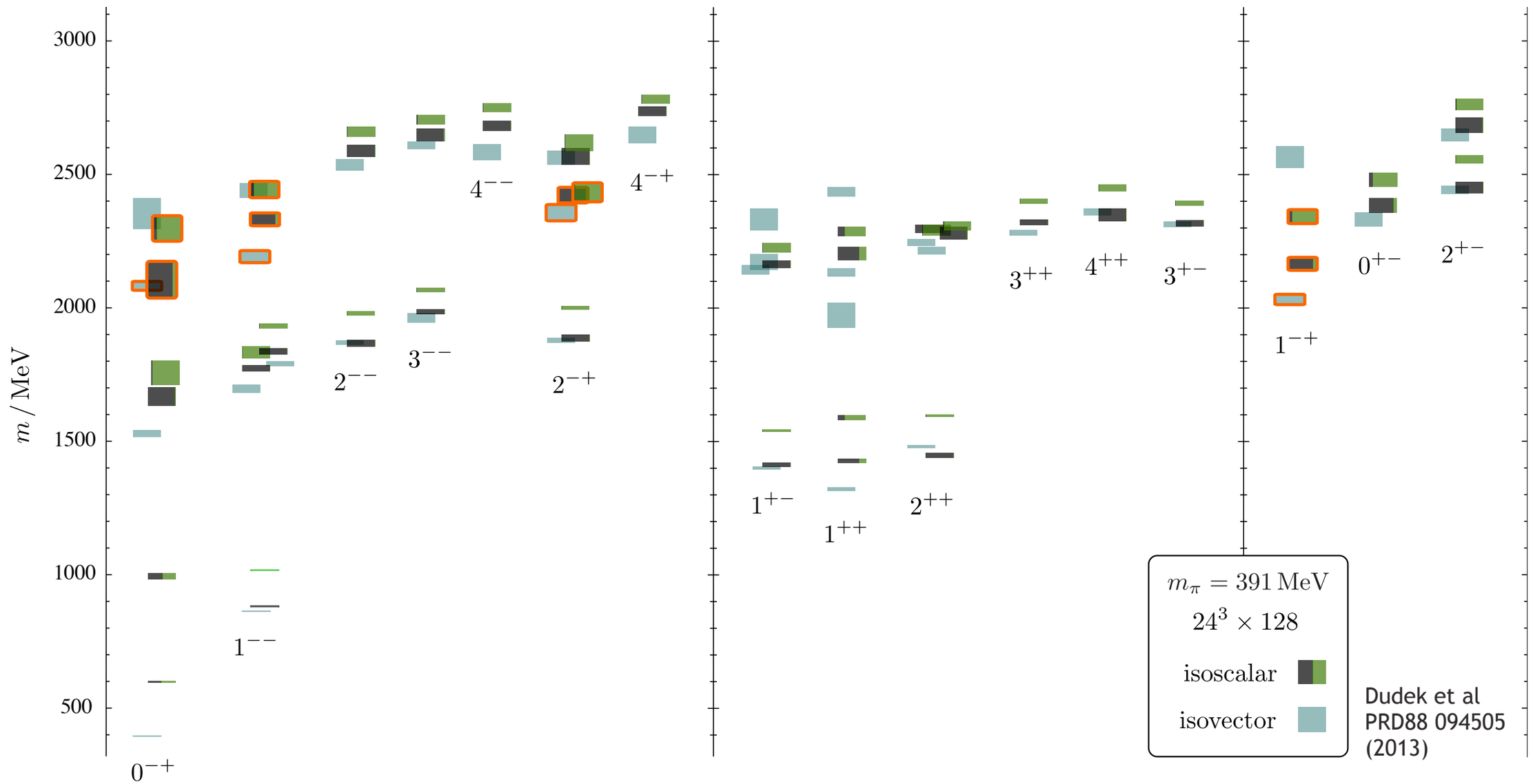


obtaining the finite-volume spectrum

in order to be able to do anything, need to obtain the complete spectrum

use a big basis of operators $\sim \bar{\psi}\Gamma D \ldots D\,\psi$

analyse variationally …



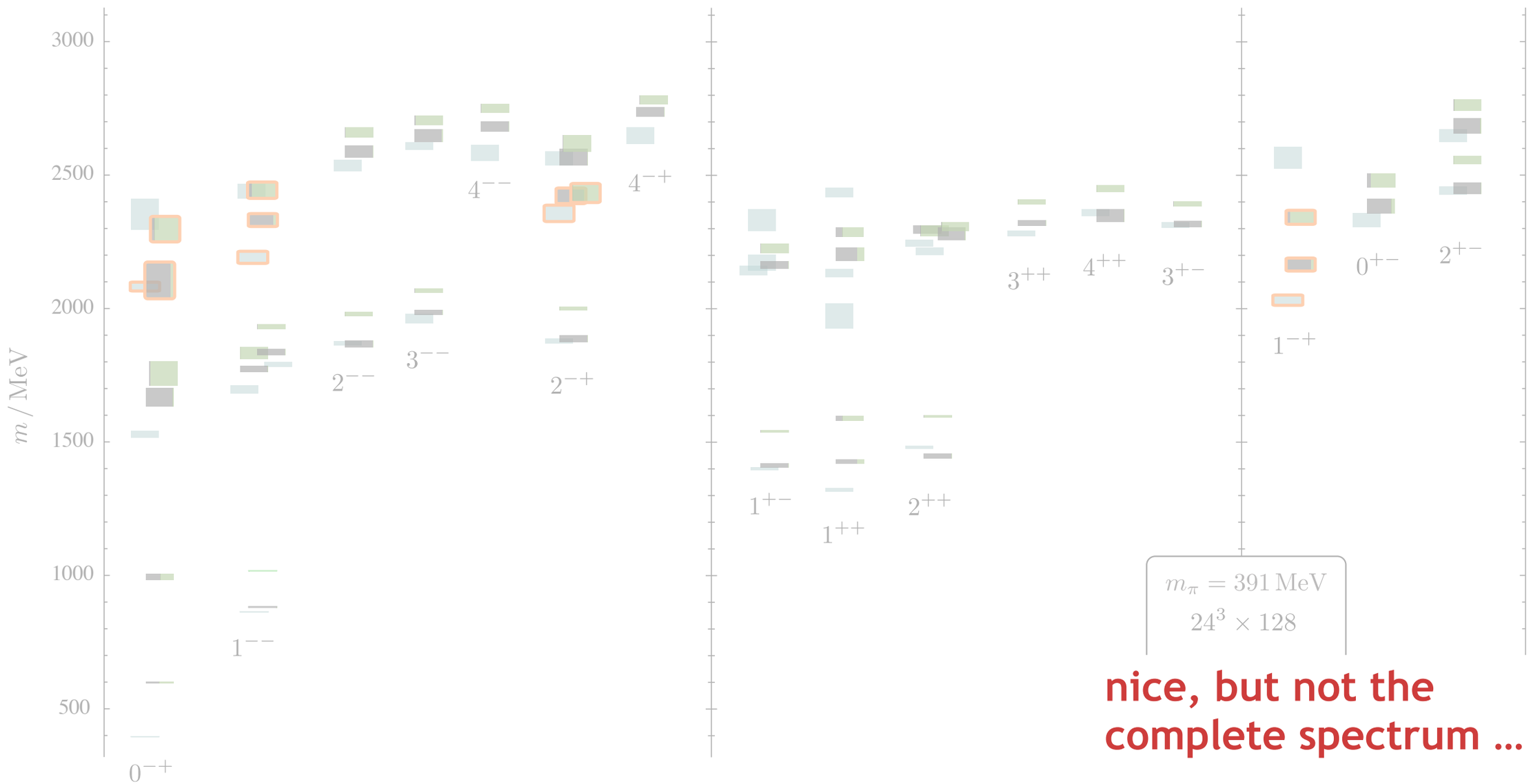
Dudek et al PRD88 094505 (2013)

obtaining the finite-volume spectrum

in order to be able to do anything, need to obtain the complete spectrum

use a big basis of operators $\sim \bar{\psi}\Gamma D \ldots D\,\psi$

analyse variationally …



nice, but not the complete spectrum …

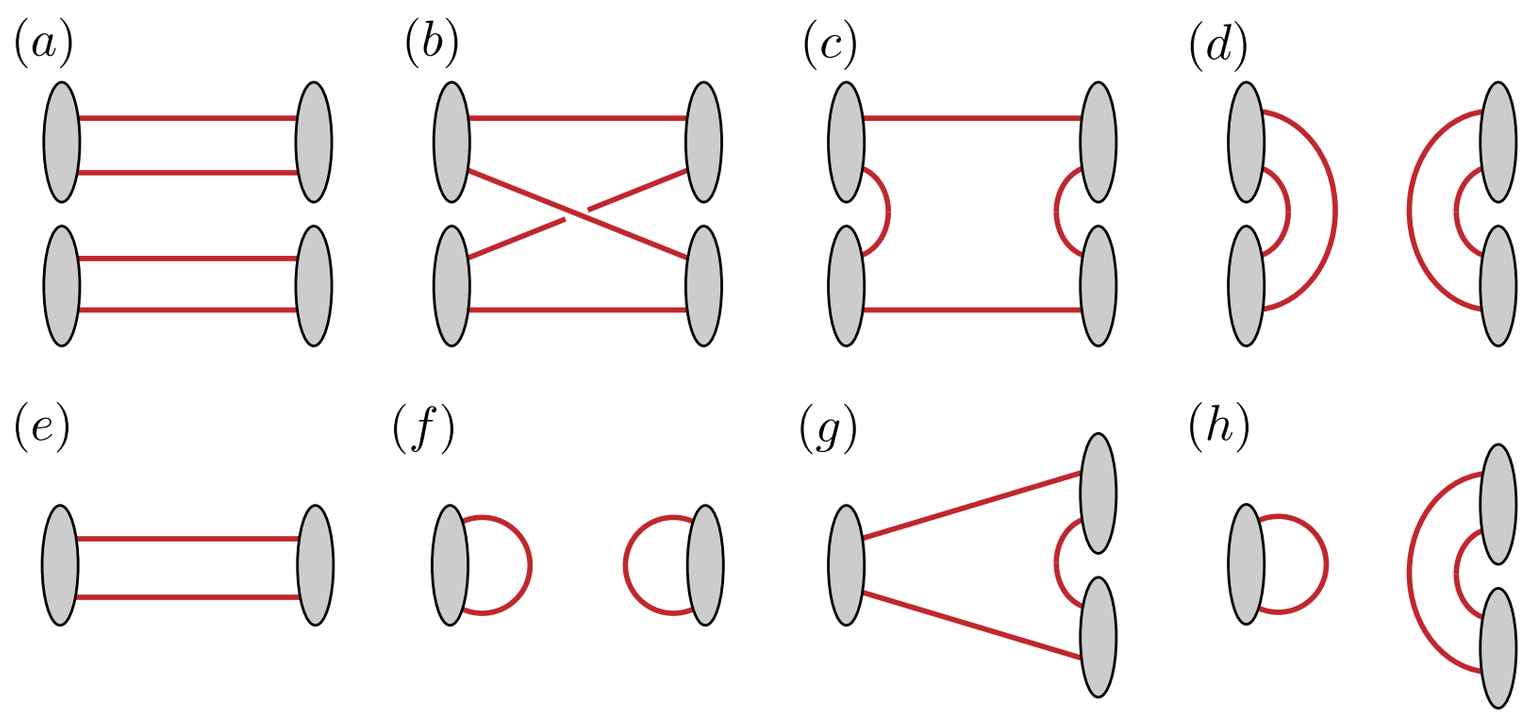
obtaining the finite-volume spectrum

important to include ‘meson-meson’-like operator constructions

$$\sum_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2} C(\mathbf{k}_1,\mathbf{k}_2)\,\pi(\mathbf{k}_1)\,\pi(\mathbf{k}_2) \qquad \mathbf{P} = \mathbf{k}_1 + \mathbf{k}_2$$

$$\pi(\mathbf{k}) \sim \sum_{\mathbf{x}} e^{i\mathbf{k}\cdot\mathbf{x}}\,\left(\bar{\psi}[\ldots]\psi\right)_{\mathbf{x}}$$

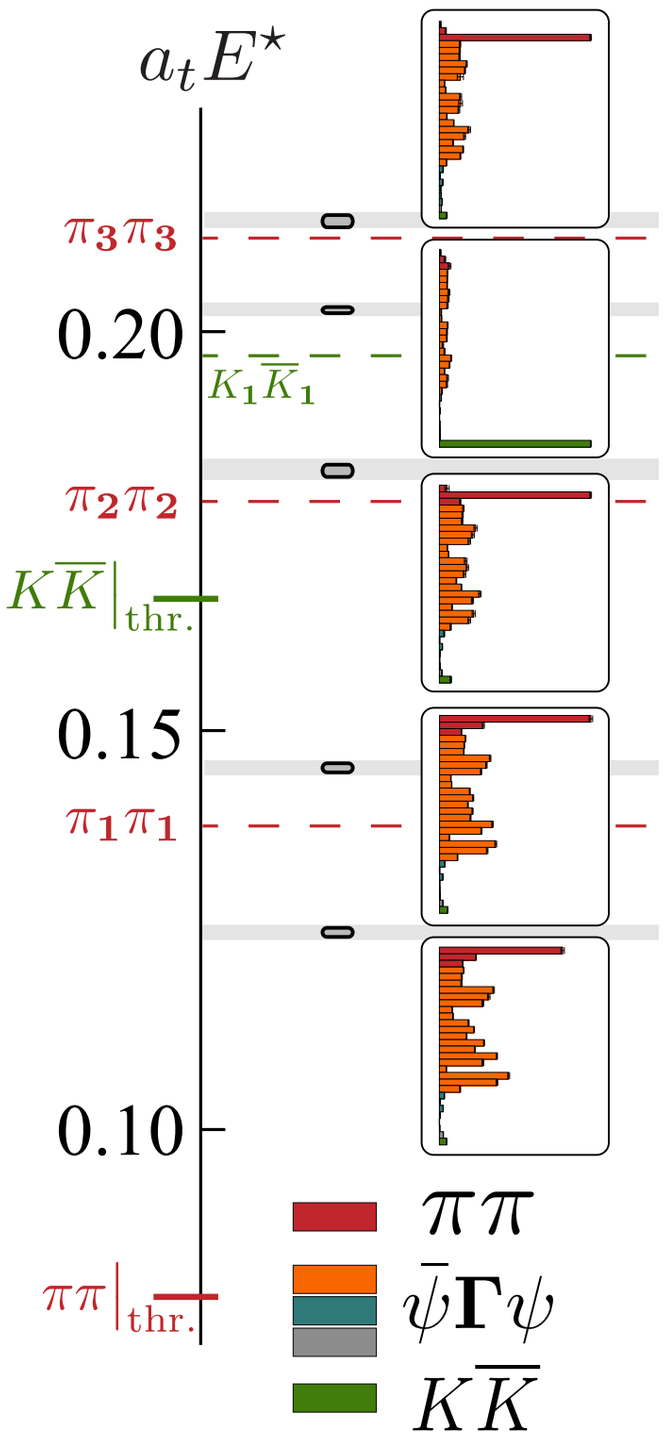
many Wick contractions: e.g. isospin=0 correlators (incl. $\pi\pi$)



the ρ channel

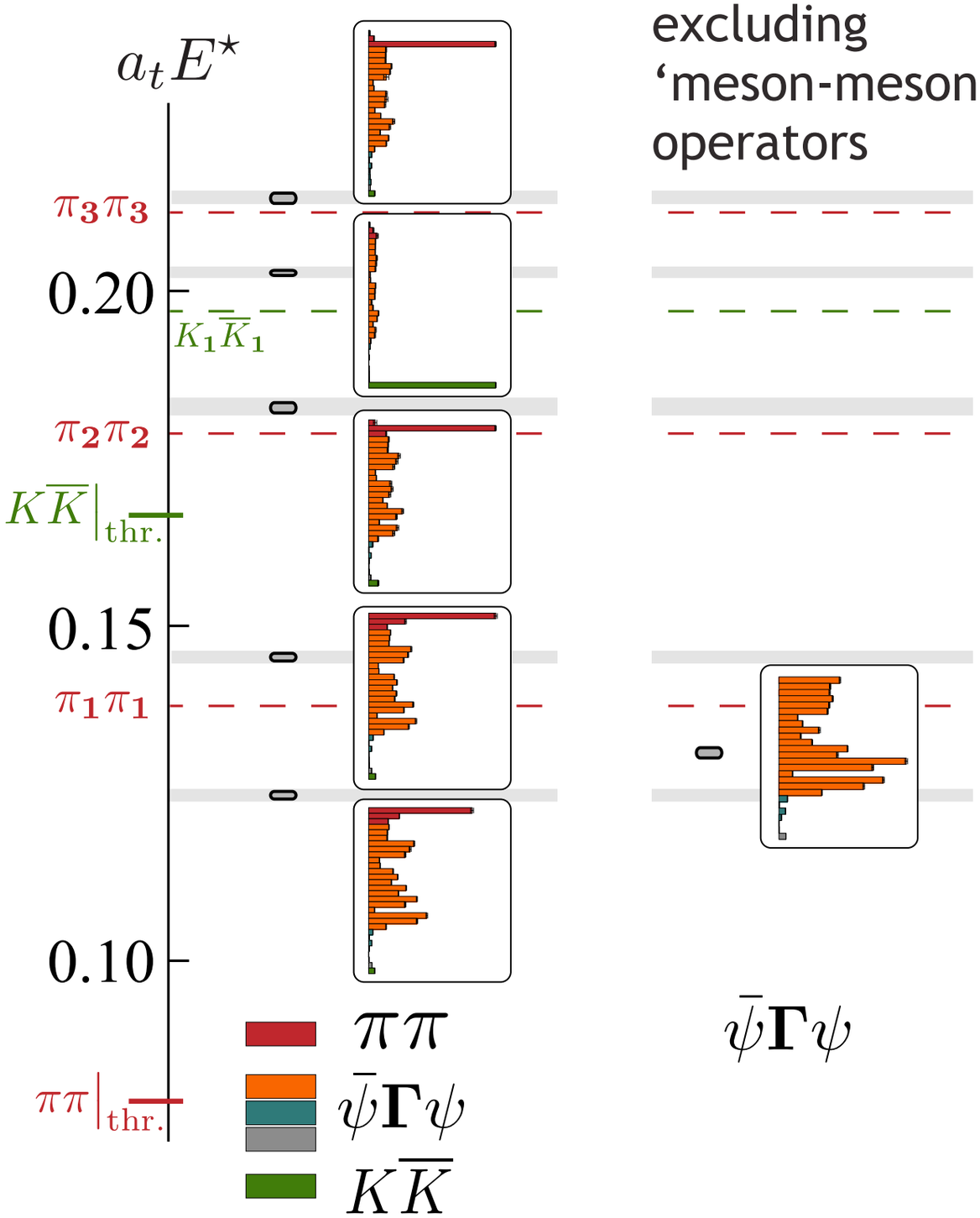
m_π = 236 MeV

Wilson et al
PRD92 094502 (2015)



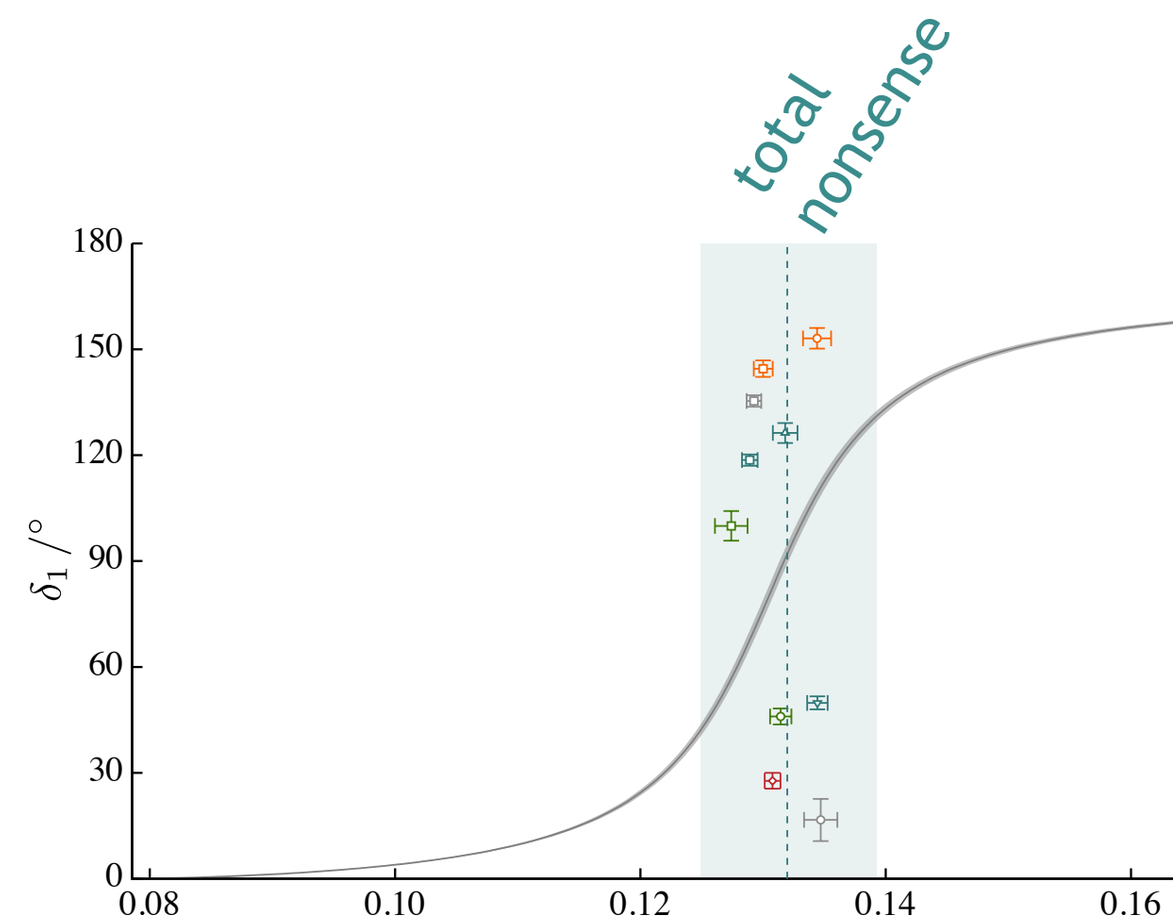
the ρ channel

m_π = 236 MeV



excluding 'meson-meson' operators

Wilson et al
PRD92 094502 (2015)



'meson-meson' operators appear to be required in practical calcs …

coupled-channels in finite-volume

the discrete spectrum in a finite-volume given by solutions of

$$\det\left[\mathbf{t}^{-1} + i\boldsymbol{\rho} - \mathbf{M}\right] = 0$$

if we calculate the discrete spectrum $E_n(\boldsymbol{P},L)$ using lattice QCD

how do we work backwards to find $t_{ij}(E)$?

a successful approach: **parameterize** $t_{ij}(E)$

needs to satisfy unitarity → *K*-matrix approach

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

$\mathbf{K}(s)$ real symmetric matrix

parameterize this

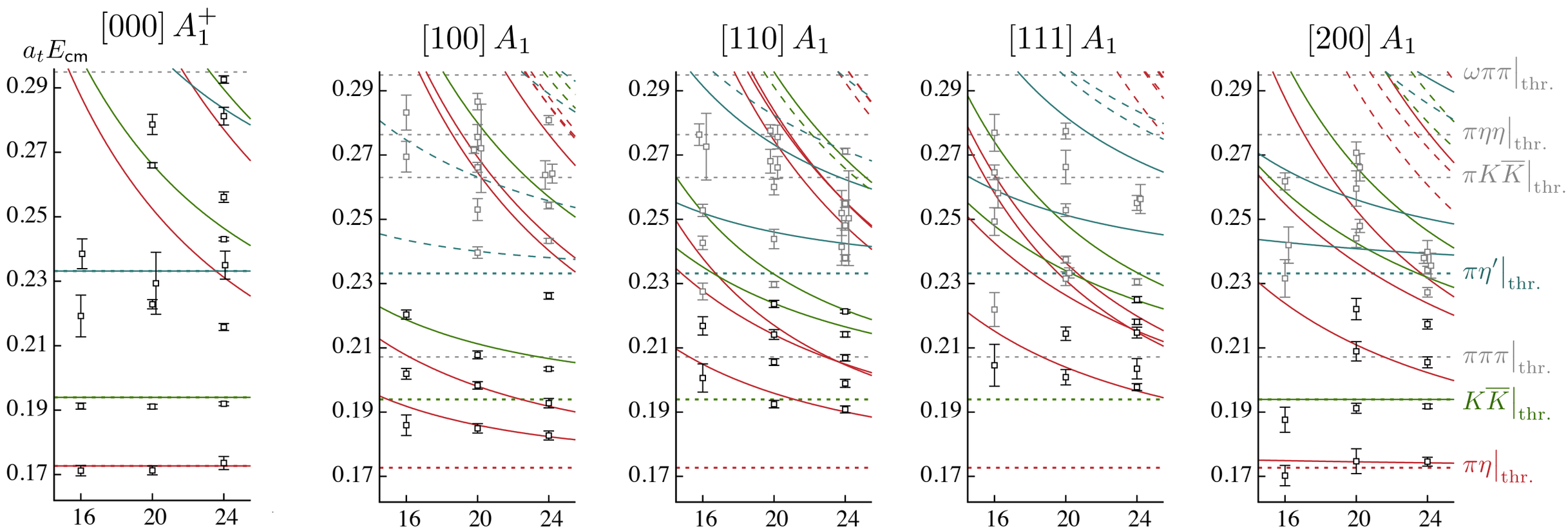
I=1, G=− : $\pi\eta$, $K\bar{K}$, $\pi\eta'$ …

Dudek et al
PRD93 094506 (2016)

$m_\pi \sim 391\,\mathrm{MeV}$

16^3, 20^3, 24^3

spectra obtained from variational analysis



(18) (11) (21) (12) (16) (19) (12) (26) (17) (11) (12) (13) (13) (14) (20)

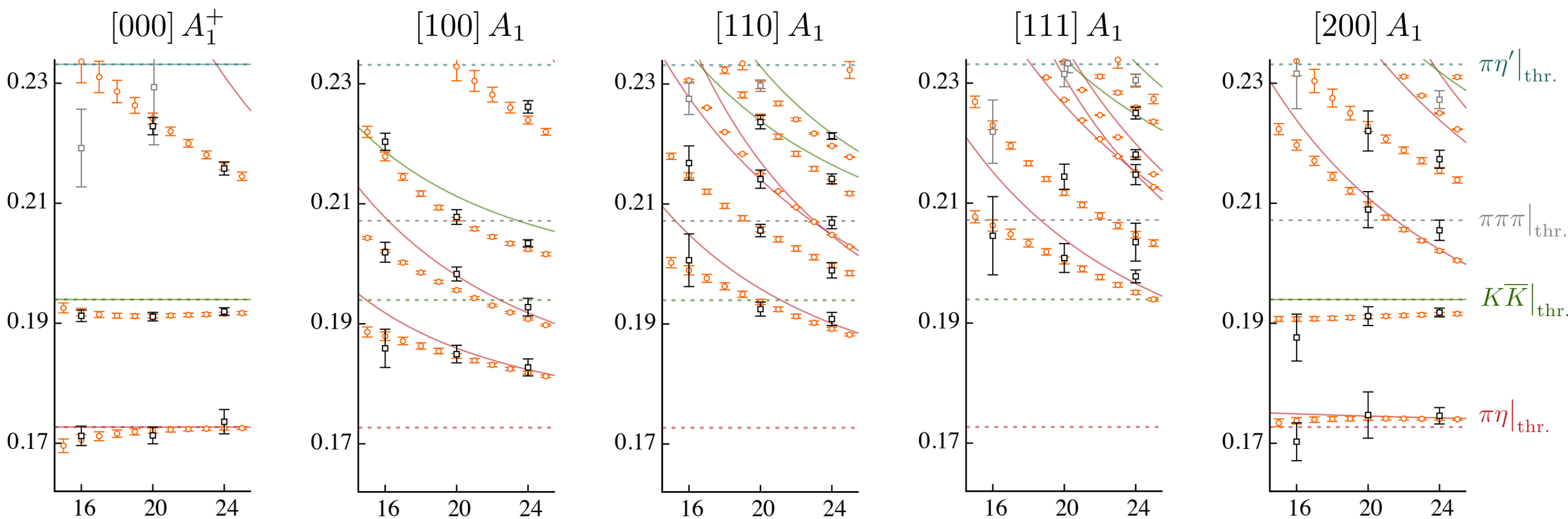
(*conservatively*) **47** levels
in the relevant energy region

describing the spectrum

$m_\pi \sim 391\,\mathrm{MeV}$

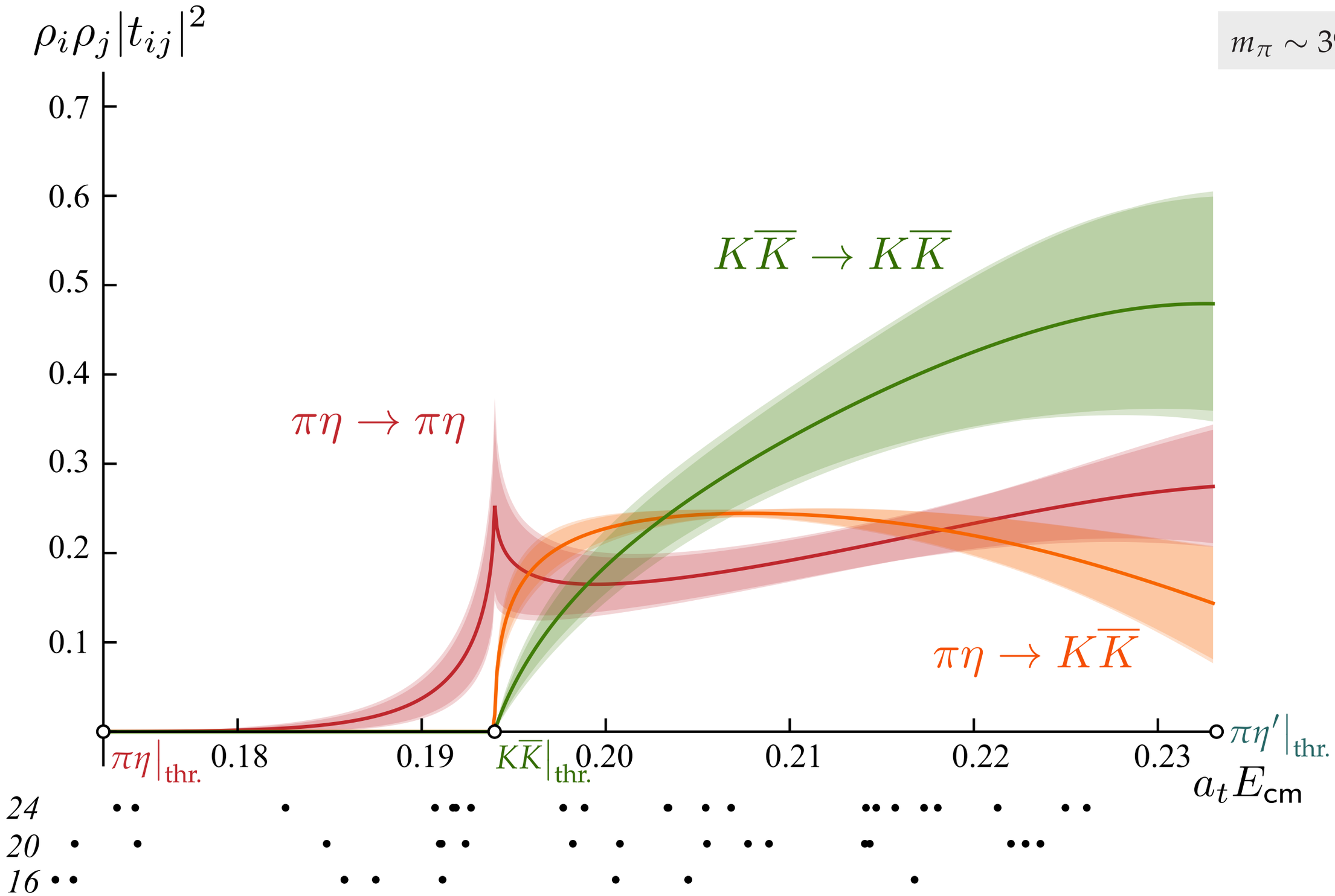
K-matrix parameterization: one pole plus constant matrix

$$K_{ij}(s) = \frac{g_i\, g_j}{m^2 - s} + c_{ij}$$ (6 free parameters)

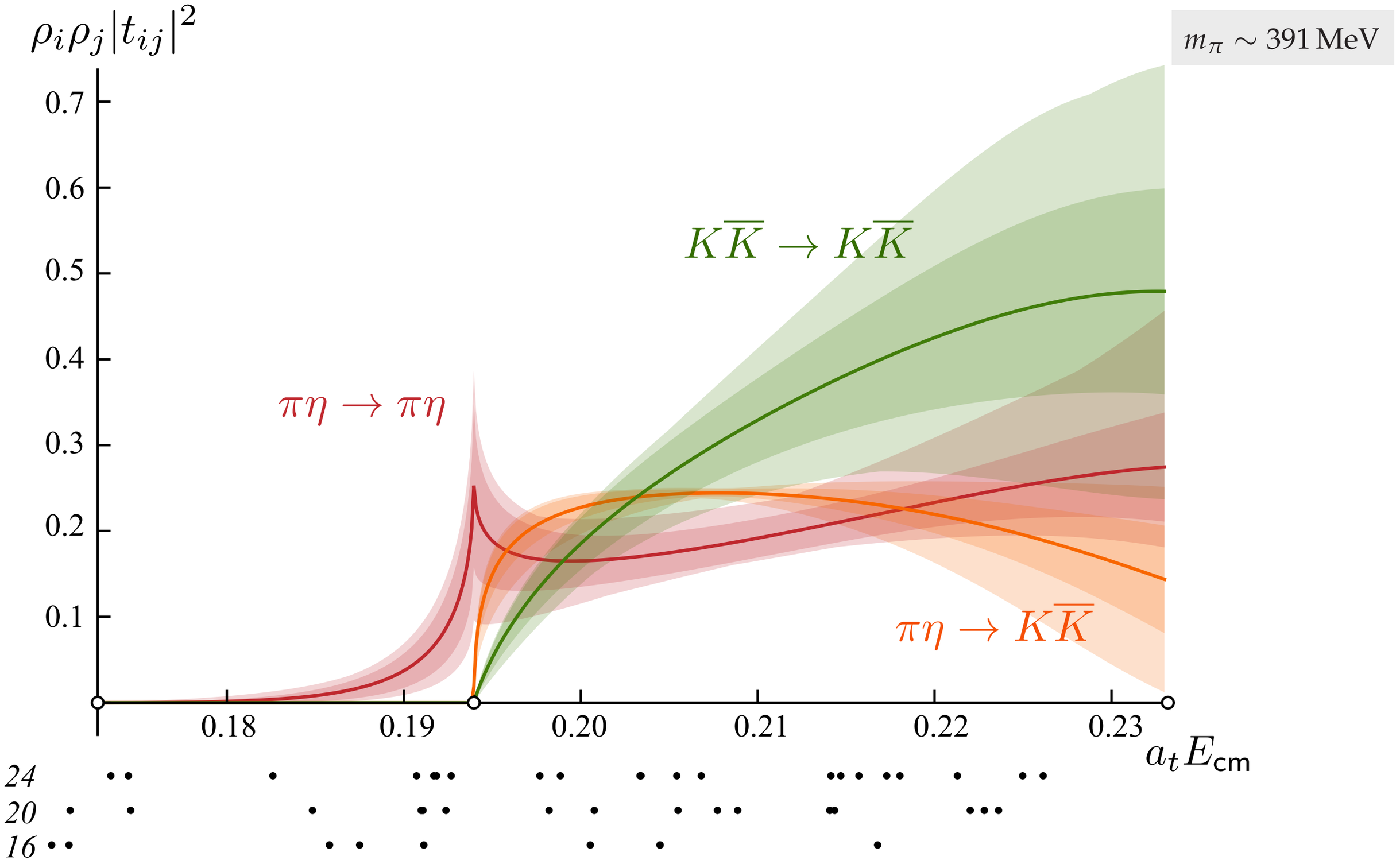


$$\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.4$$

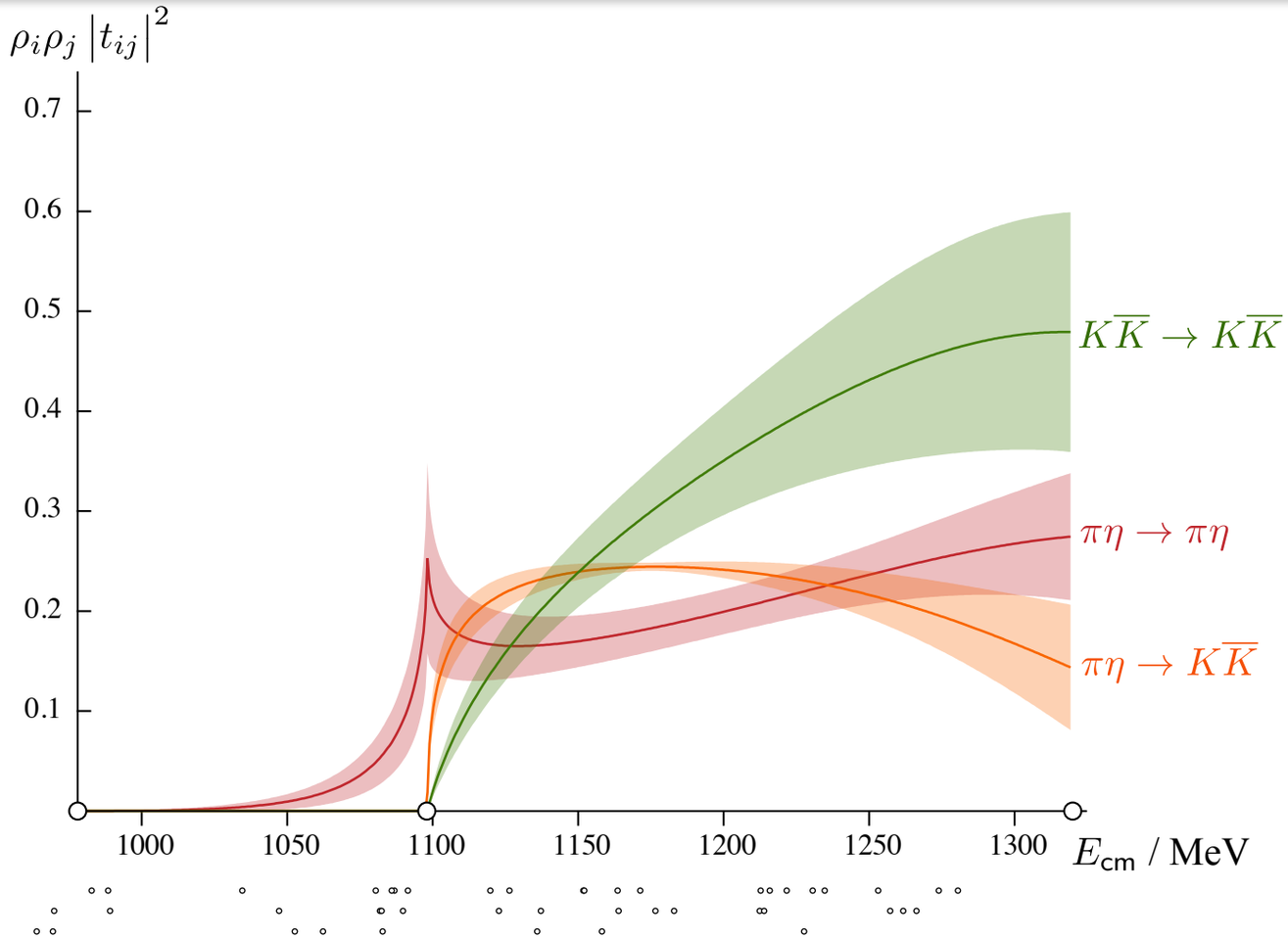
scattering amplitudes



variation with changing parameterization



scattering amplitudes



$m_\pi \sim 391\,\mathrm{MeV}$

strong cusp in $\pi\eta$ at $K\overline{K}$ threshold

rapid turn-on of $K\overline{K}$ amplitudes

indicative of a nearby **resonance** ?

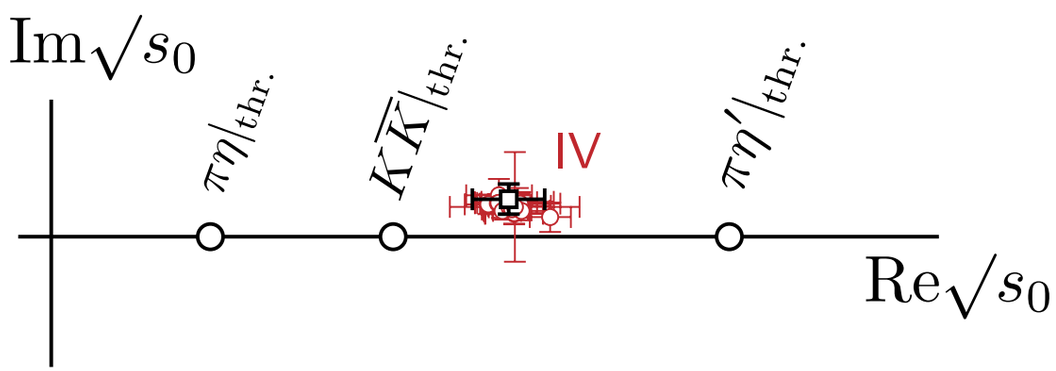
what's the pole structure of the amplitude … ?

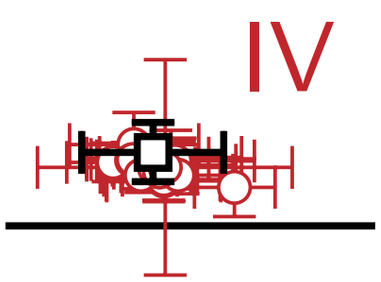
pole structure

$m_\pi \sim 391\,\mathrm{MeV}$

- we find a single dominant (nearby) pole

COMPLEX ENERGY PLANE

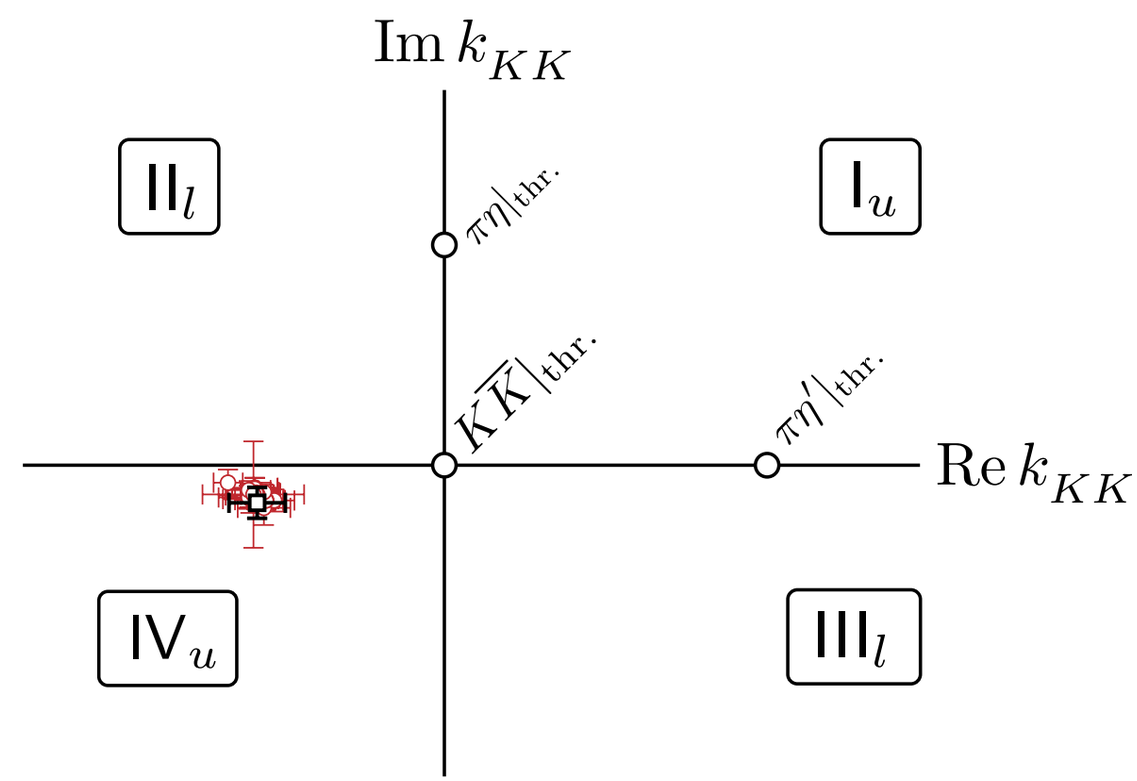




variation with parameterization choice

Sheet	$\mathrm{Im}k_{\pi\eta}$	$\mathrm{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

COMPLEX MOMENTUM PLANE



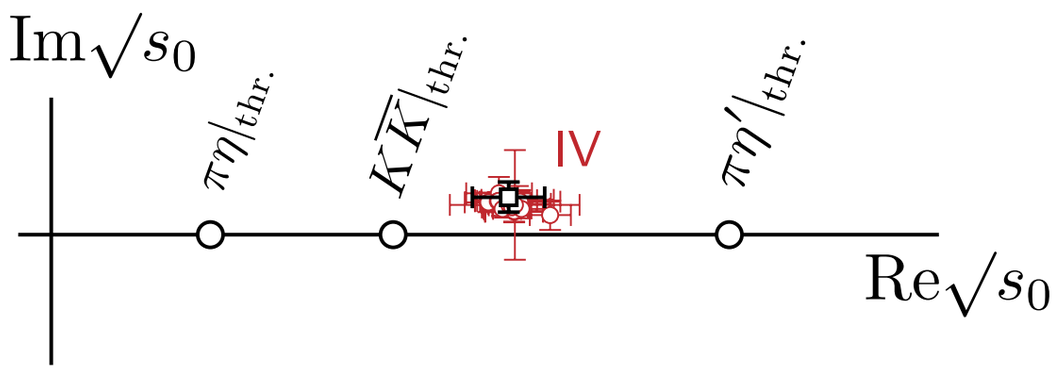
pole couplings

- we find a single dominant (nearby) pole

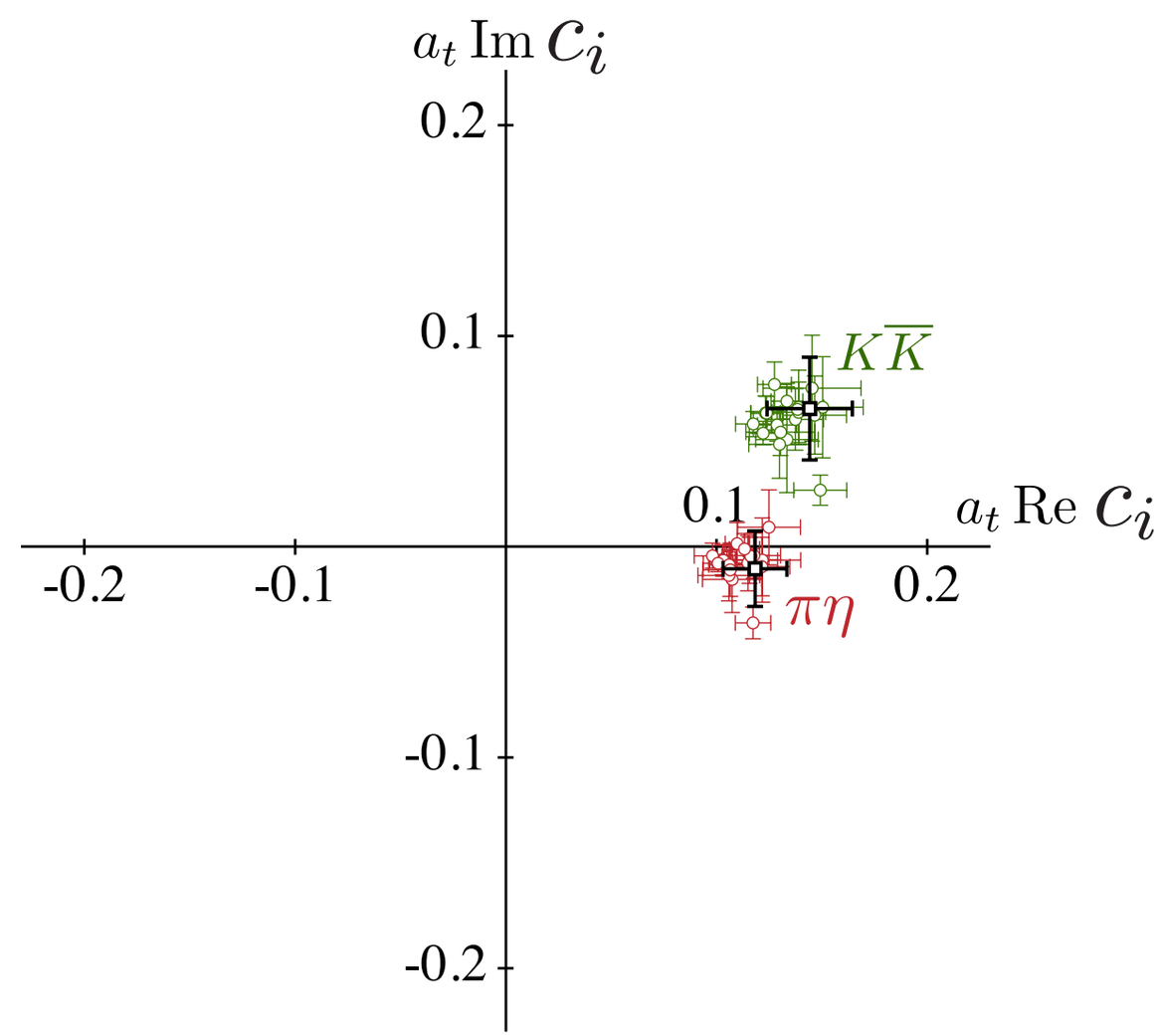
$m_\pi \sim 391\,\mathrm{MeV}$

$$t_{ij} \sim \frac{c_i\, c_j}{s_0 - s}$$

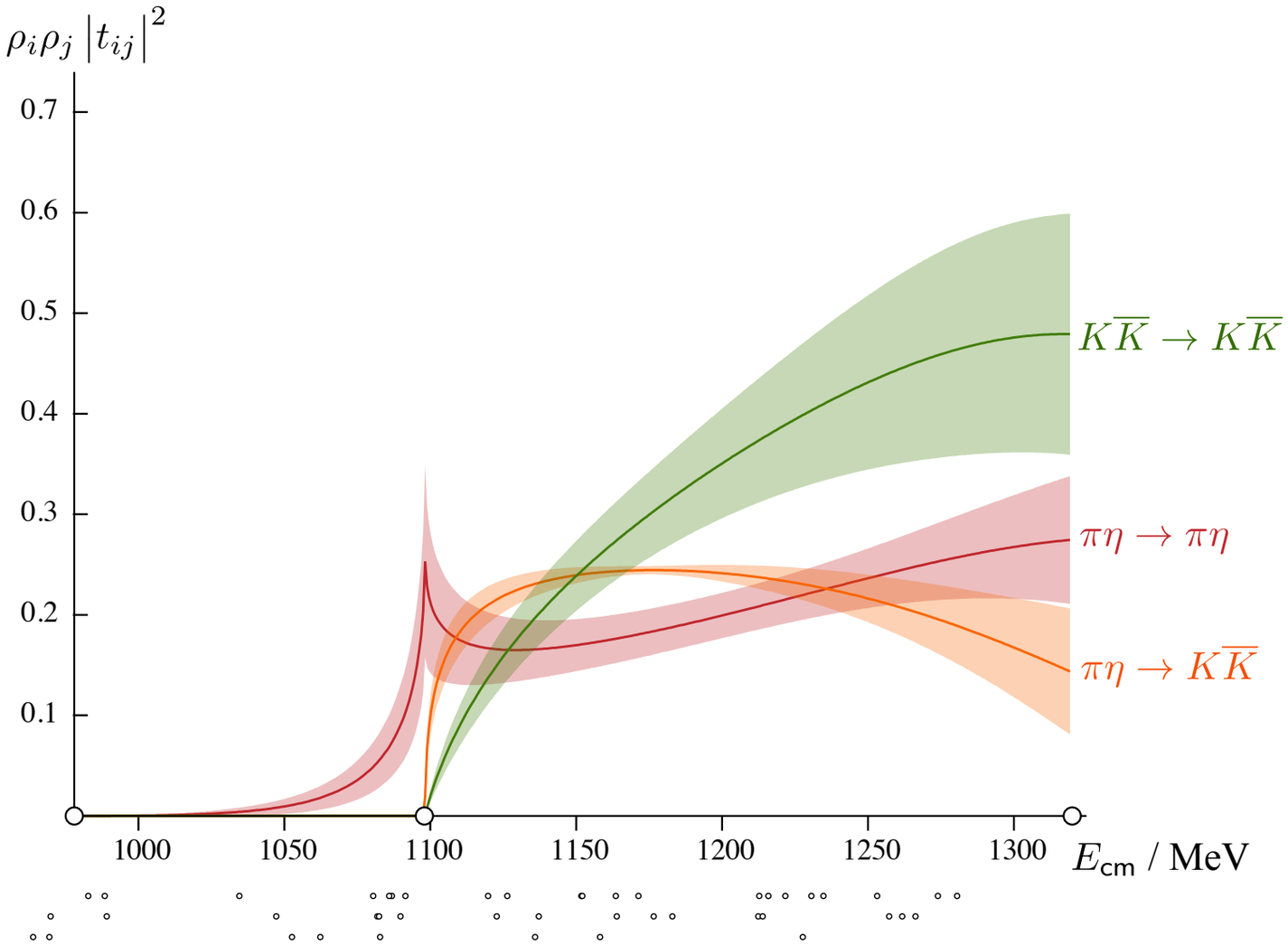
COMPLEX ENERGY PLANE



RESONANCE POLE COUPLINGS



an a_0 resonance from QCD



$m_\pi \sim 391\,\mathrm{MeV}$

- resonance found near $K\bar{K}$ threshold

pole position

$$\sqrt{s_0} = \left((1177 \pm 27) + \tfrac{i}{2}(49 \pm 33)\right)\,\mathrm{MeV}$$

$$\mathrm{Re}\sqrt{s_0} - 2m_K = 79 \pm 27$$

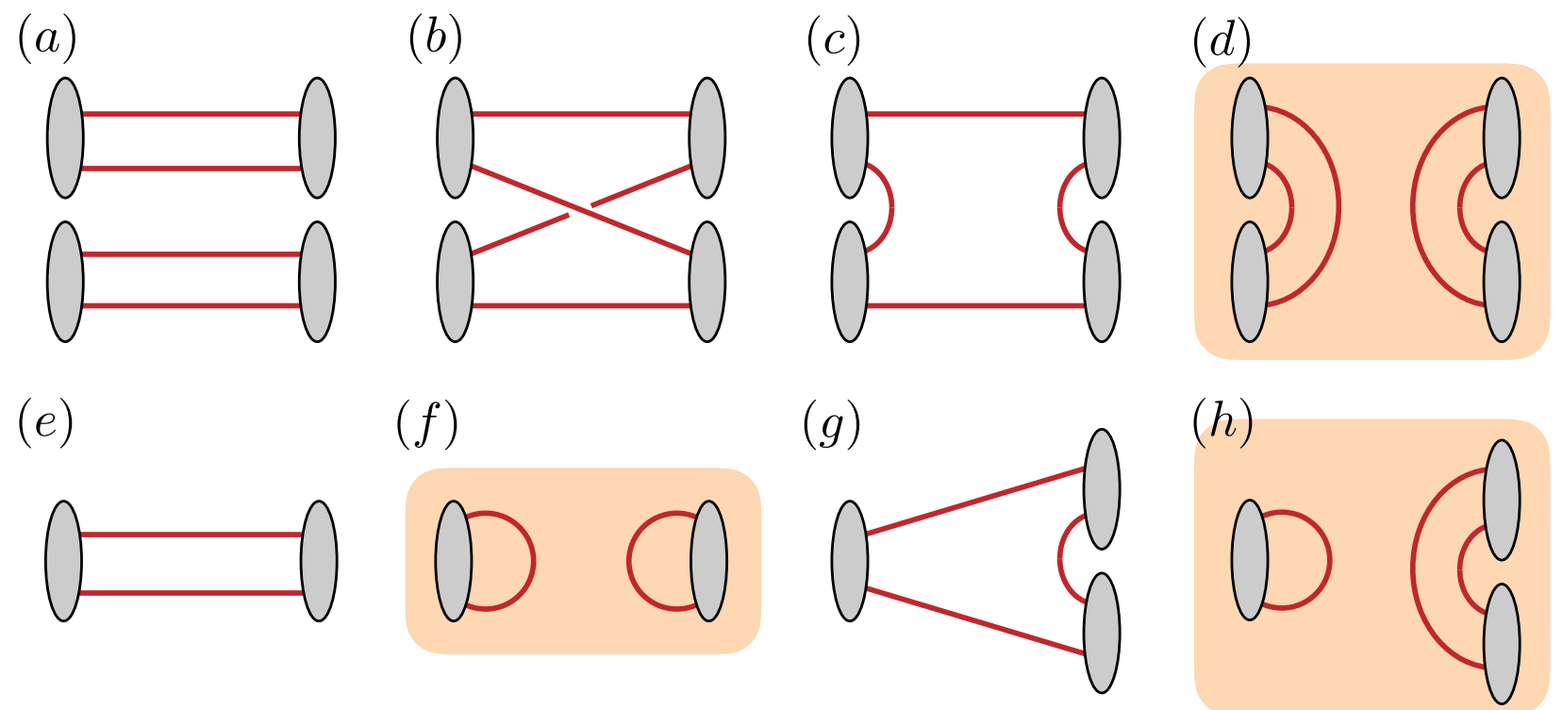
pole coupling ratio

$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right| = 1.3(4)$$

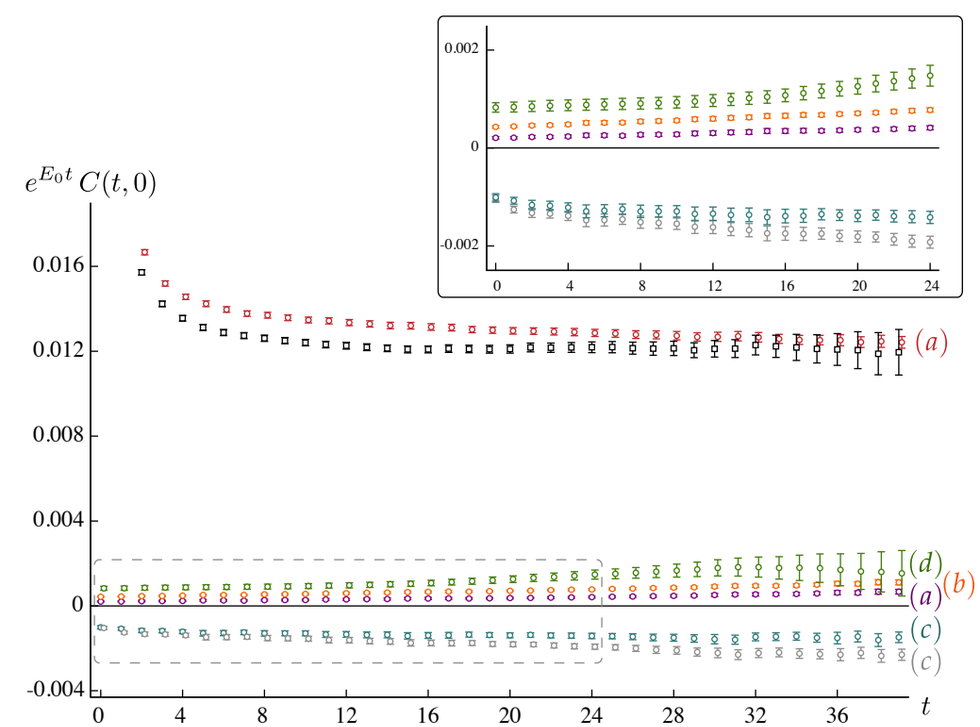
$\pi\pi$ isospin=0 S-wave

use basis of $\bar{\psi}[\ldots]\psi$ and $\pi\pi$ operators

actually also $K\bar{K}$,$\eta\eta$ operators



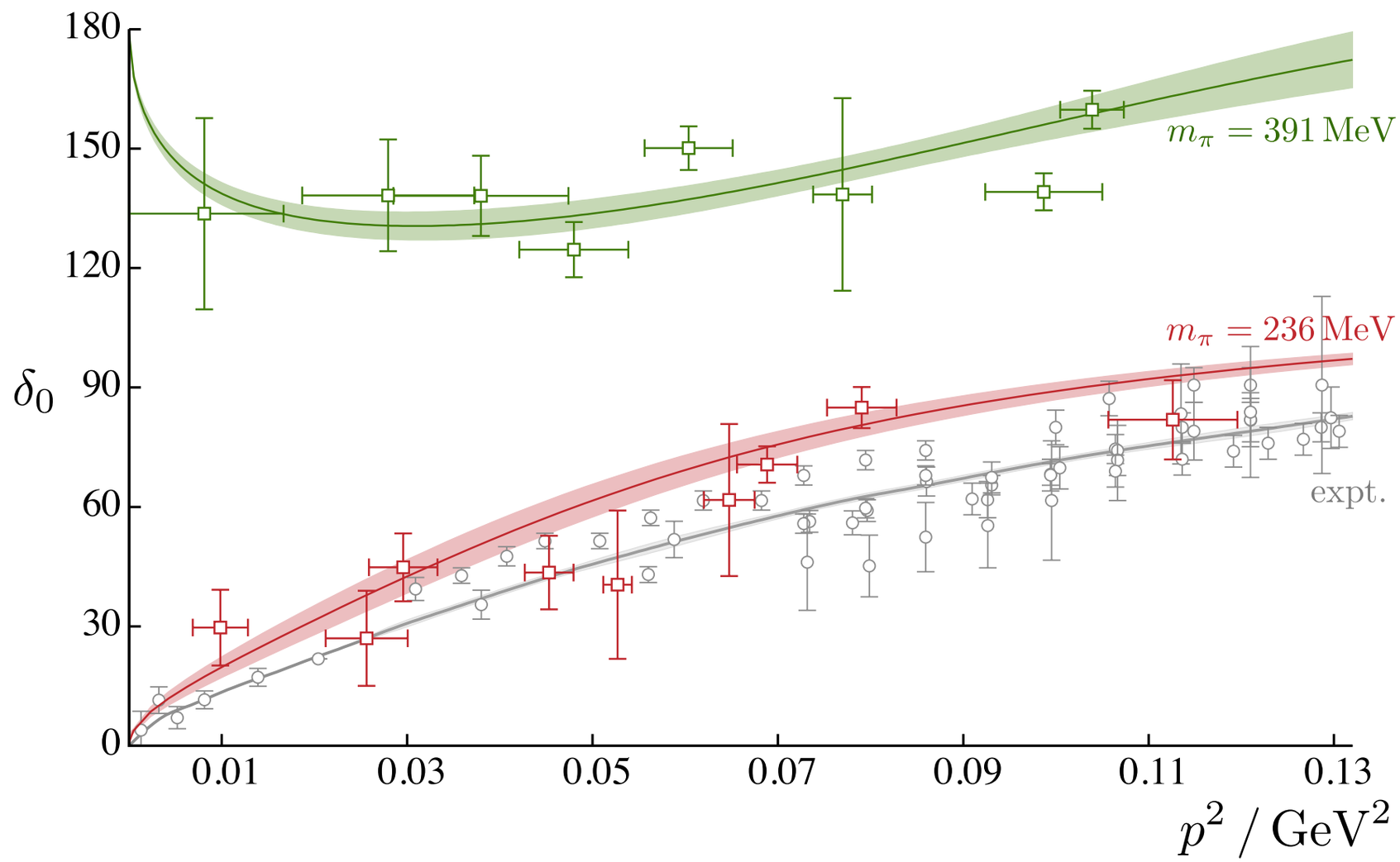
(historically) challenging to get good signals



Briceno et al PRL118 022002 (2017)

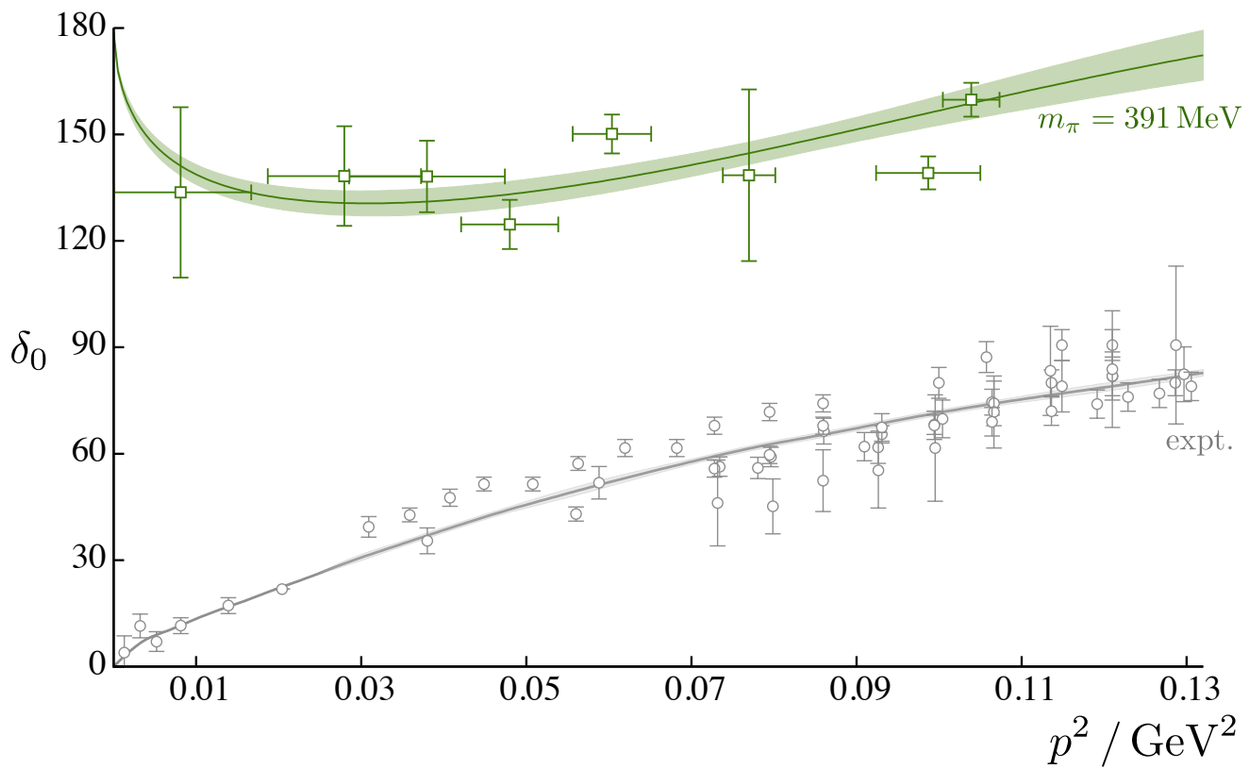
$\pi\pi$ isospin=0 S-wave

elastic scattering region



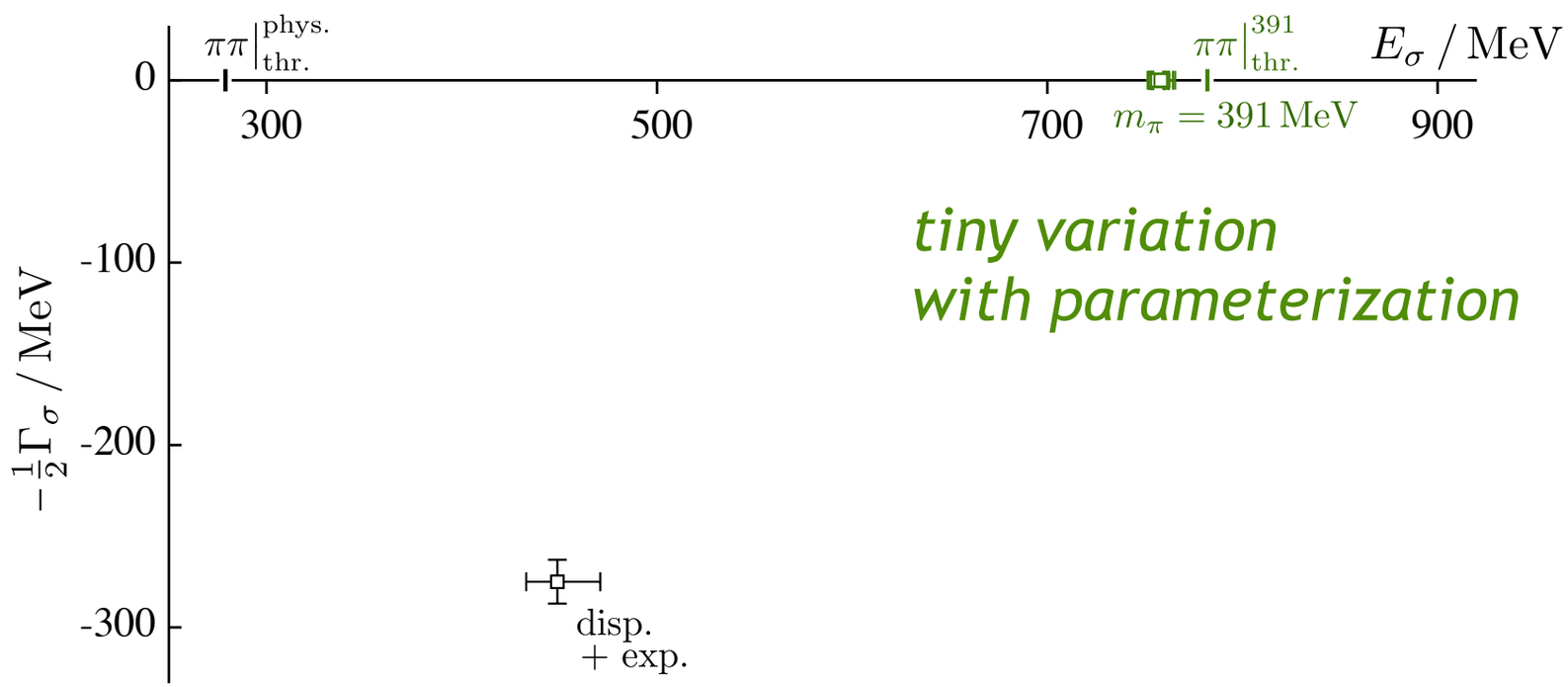
notice the change in behavior
as the quark mass is reduced …

$\pi\pi$ isospin=0 S-wave



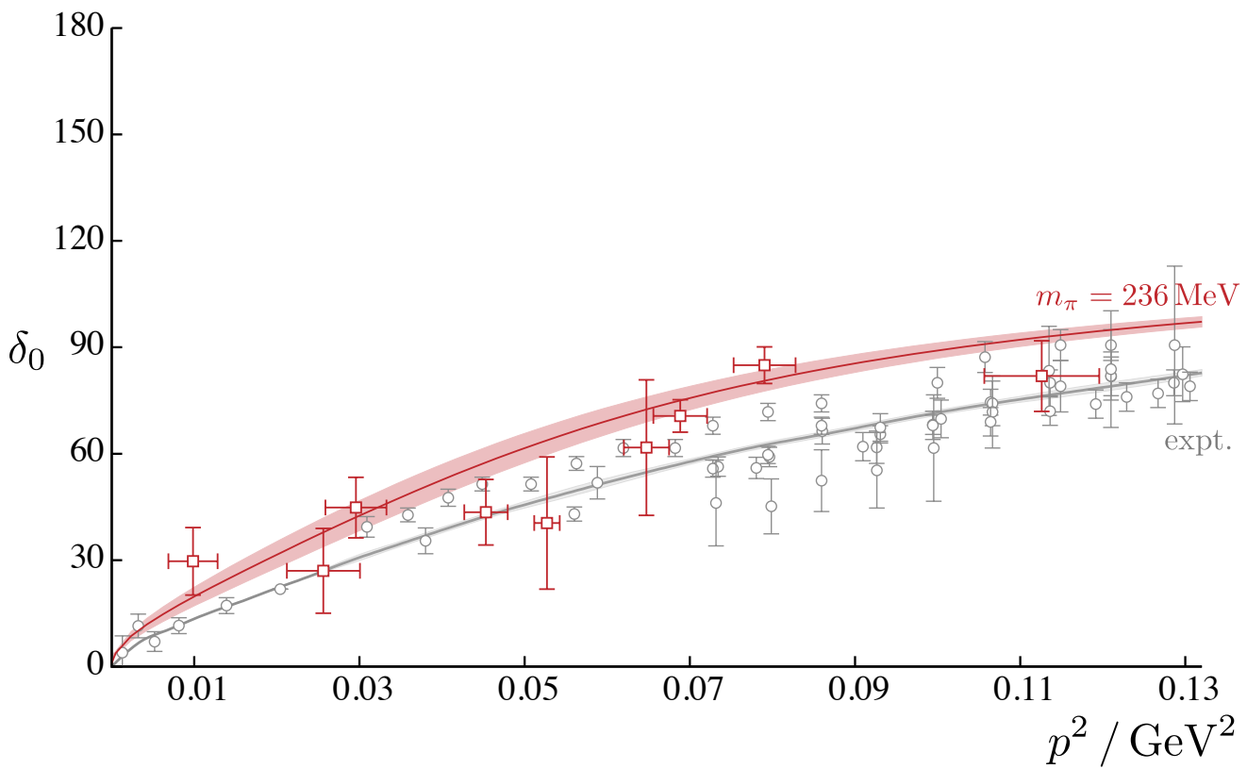
at m_π=391 MeV

σ as bound-state

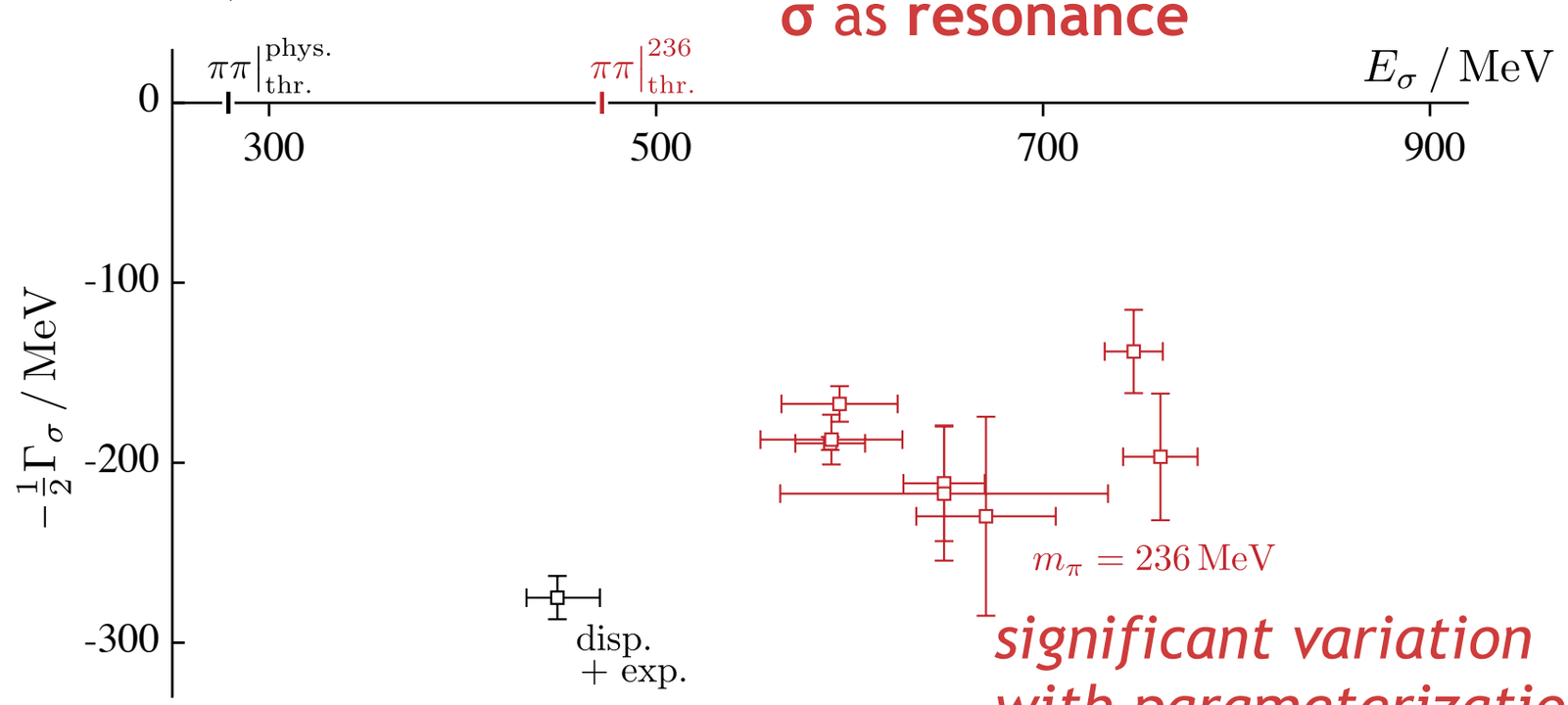


tiny variation
with parameterization

$\pi\pi$ isospin=0 S-wave



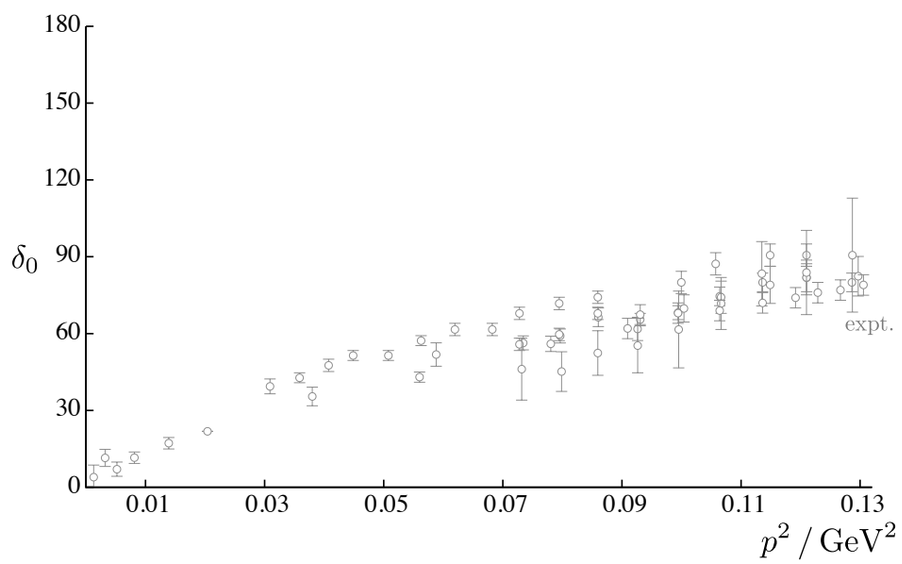
at m_π=236 MeV
σ as resonance

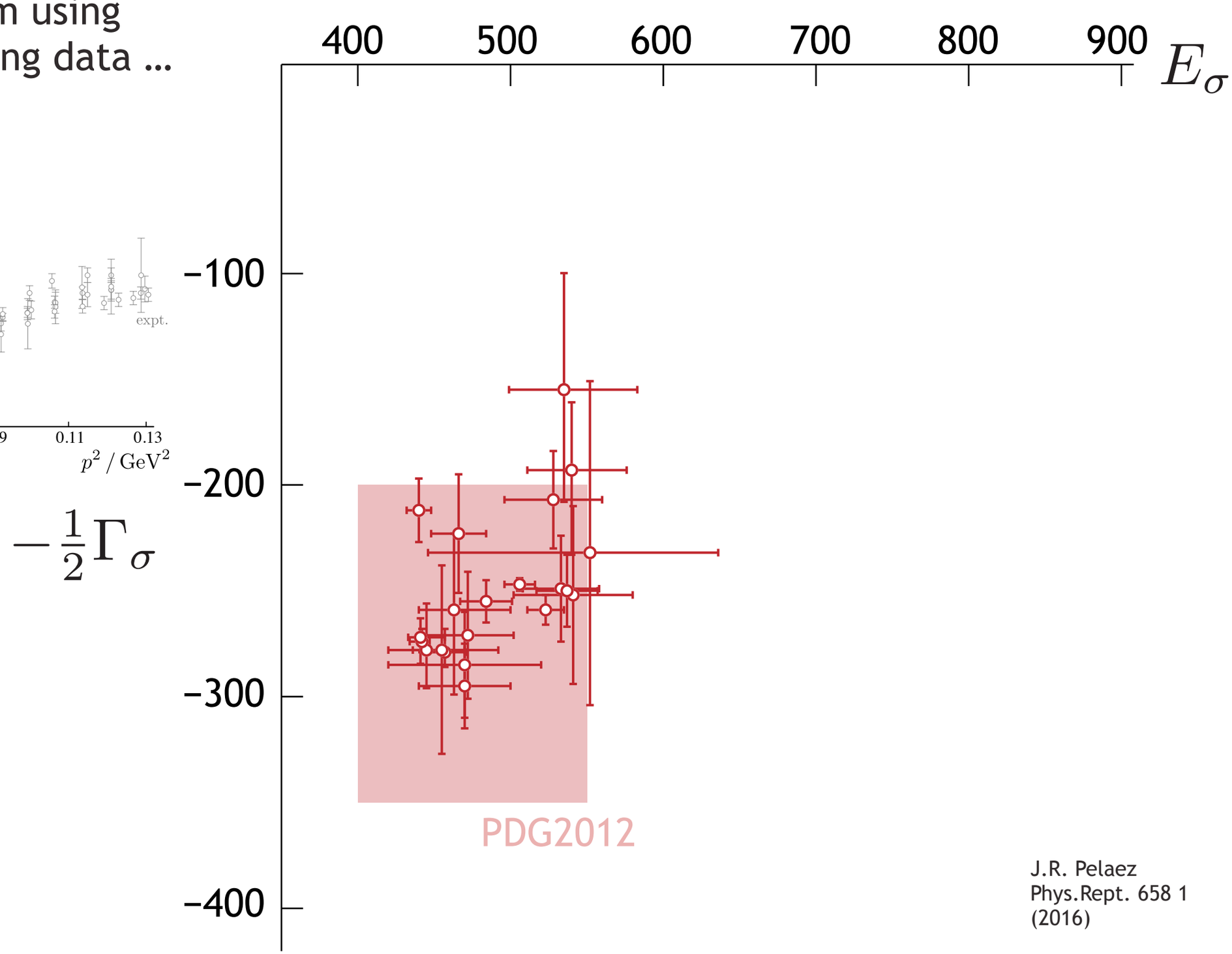


significant variation with parameterization

σ pole position variation

actually same problem using experimental scattering data …

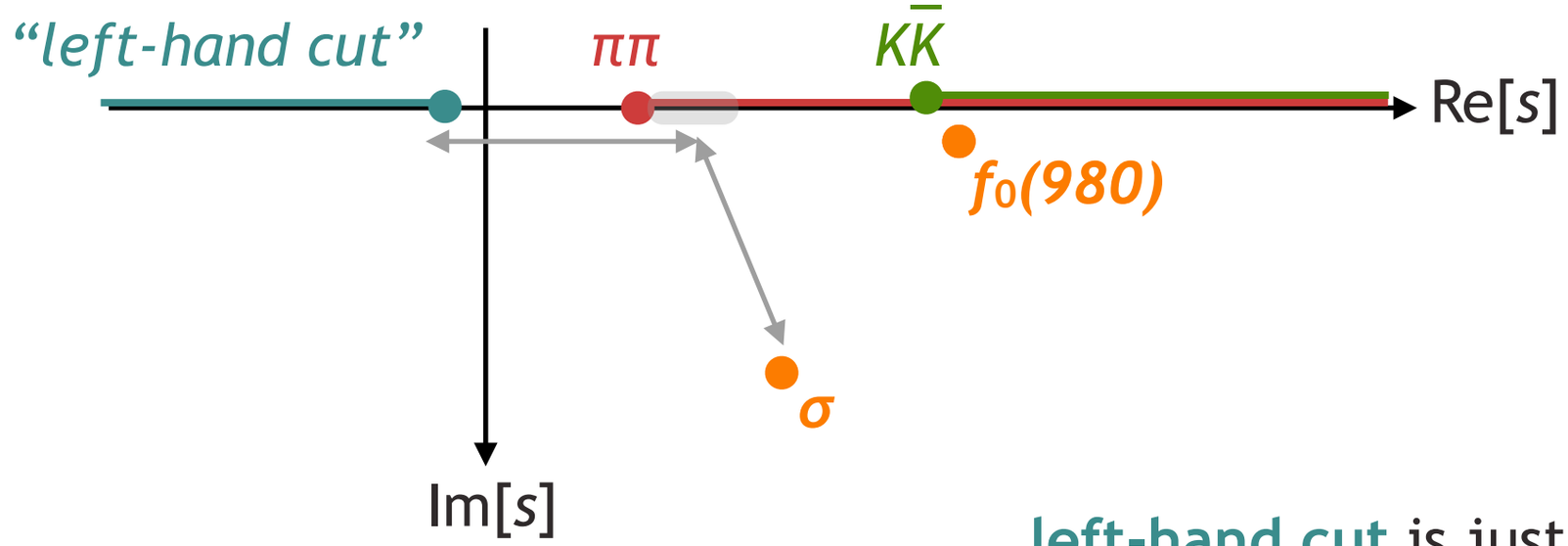




J.R. Pelaez
Phys.Rept. 658 1
(2016)

why is this case different ?

the real energy axis behavior isn't just dominated by a single (nearby) pole

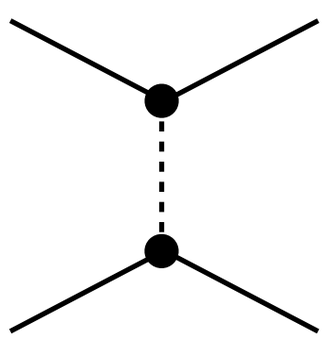


left-hand cut is just as close as the σ pole

but our amplitude parameterizations don't include it !

left-hand cut related to crossing symmetry

e.g.



$$t(s,t) \sim \frac{1}{t - M^2}$$

$$t_\ell(s) \sim \int d\cos\theta \, P_\ell(\cos\theta) \, t\big(s, t(\cos\theta)\big)$$

$$t_\ell(s) \sim \log\left(s - (4m_\pi^2 - M^2)\right)$$

branch point at $s = -M^2 + 4m_\pi^2$

where do we stand

finite-volume formalism in place to deal with

any number of coupled two-body channels

tested in (πK,ηK), ($\pi\eta$, $K\bar{K}$), (πD, ηD, $\bar{K}D_s$)

and more calculations underway

finite-volume formalism not yet mature for

three-body scattering channels

currently restricts which resonances can be computed

hadspec

JEFFERSON LAB

Jozef Dudek
Robert Edwards
Balint Joo
David Richards
Raul Briceno

TRINITY, DUBLIN

Michael Peardon
Sinead Ryan
David Wilson

CAMBRIDGE

Christopher Thomas
Graham Moir

TATA, MUMBAI

Nilmani Mathur

MESON SPECTRUM

PRL 103 262001 (2009)	$I = 1$
PRD82 034508 (2010)	$I = 1$, $K^\star$
PRD83 111502 (2011)	$I = 0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I = 0$
JHEP05 021 (2013)	D, D_s
JHEP 1612 089 (2016)	$c\bar{c}$ D, D_s

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi$ $I = 2$
PRD86 034031 (2012)	$\pi\pi$ $I = 2$
PRD87 034505 (2013)	$\pi\pi$ $I = 1$, ρ
PRL 113 182001 (2014)	$\pi K, \eta K : K^\star$
PRD91 054008 (2015)	$\pi K, \eta K : K^\star$
PRD92 094502 (2015)	$\pi\pi, K\bar{K} : \rho$
PRD93 094506 (2016)	$\pi\eta, K\bar{K} : a_0$
JHEP 1610 011 (2016)	$D\pi, D\eta, D_s\bar{K}$
PRL 118 022002 (2017)	$\pi\pi$ $I = 0$, σ

BARYON SPECTRUM

PRD84 074508 (2011)	$(N, \Delta)^\star$
PRD85 054016 (2012)	$(N, \Delta)_{\text{hyb}}$
PRD87 054506 (2013)	$(N \ldots \Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
PRD91 094502 (2015)	$\Xi^\star_{cc}$

MATRIX ELEMENTS

PRD90 014511 (2014)	$f_{\pi^\star}$
PRD91 114501 (2015)	$M' \to \gamma M$
PRL 115 242001 (2015)	$\gamma^\star \pi \to \pi\pi$
PRD93 114508 (2016)	$\gamma^\star \pi \to \pi\pi$

LATTICE TECH.

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p} > 0$