Hadronic light-by-light and vacuum polarization contributions to the muon anomalous magnetic moment from lattice QCD

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Fermilab Theory Seminar

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Collaborators

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and the RBC/UKQCD collaborations

Outline I

1. Introduction
2. Hadronic light-by-light (HLbL) scattering contribution
   - towards the continuum limit in finite volume
   - towards the infinite volume limit
3. Hadronic vacuum polarization (HVP) contribution
4. Beyond the Standard Model
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Muon g-2 experimental measurement [Bennett et al., 2006]

E821 at BNL measured relative precession of muon spin to it's momentum
$\omega_a = \frac{g-2}{2}\frac{eB}{m} = a_\mu \frac{eB}{m}$, the muon anomaly

The rate of detected electrons oscillates with ω_a, fit to
$N(t) = Be^{-\lambda t}(1 + A\cos\omega_a t + \phi)$

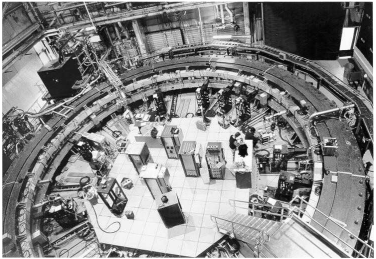


Figure 12. The storage-ring magnet. The cryostats for the inner-radius coils are clearly visible. The kickers have not yet been installed. The racks in the center are the quadrupole pulsers, and a few of the detector stations are installed, especially the quadrant of the ring closest to the person. The magnet power supply is in the upper left, above the plane of the ring. (Courtesy of Brookhaven National Laboratory)

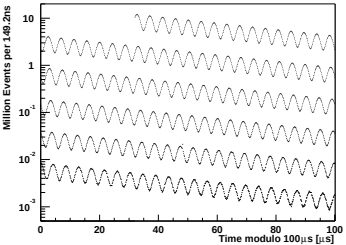
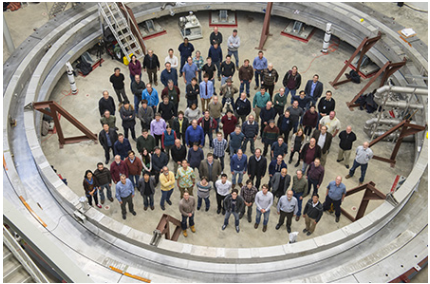


Figure 26. Histogram of the total number of electrons above 1.8 GeV versus time (modulo 100 μ s) from the 2001 μ^- data set. The bin size is the cyclotron period, $\approx$ 149.2 ns, and the total number of electrons is 3.6 billion.

$a_\mu(\mathrm{Expt}) = 11\,659\,208.0(5.4)(3.3) \times 10^{-10}$ 0.54 ppm!

New muon g-2 experiments

Storage ring moved to FNAL for E989, begun in 2017



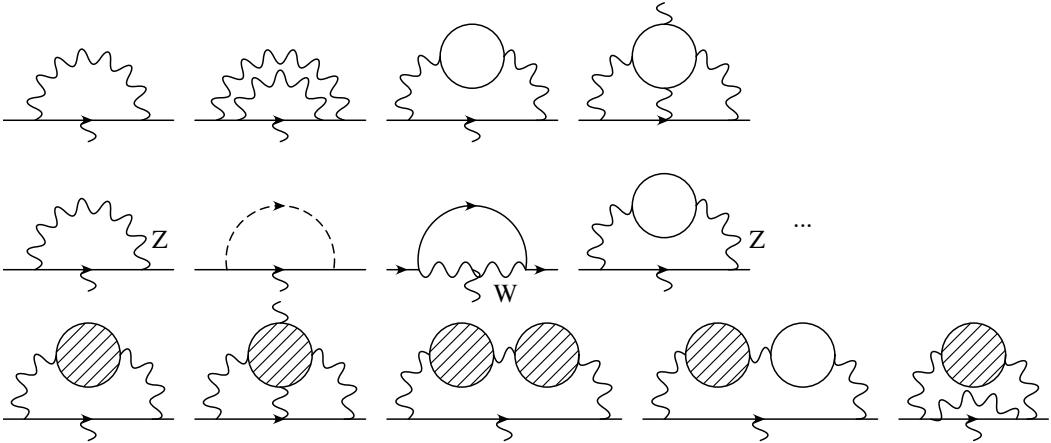
which is aiming for 0.14 ppm, 4$\times$ improvement!

In Japan at J-PARC, the E34 experiment will measure a_μ using ultra-cold muons, different systematics ($\sim$ 2020).

Standard Model Theory: QED+EW+QCD

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle = -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

$$a_\mu = F_2(0)$$



Experiment - Theory

SM Contribution	Value $\pm$ Error ($\times 10^{11}$)	Ref	Update
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]	
HVP LO	6923 ± 42	[Davier et al., 2011]	6926 (33) (Davier 16)
	6949 ± 43	[Hagiwara et al., 2011]	6922 (25) (KNT17 preliminary)
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]	
		[Kurz et al., 2014]	
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]	
HLbL	105 ± 26	[Prades et al., 2009]	
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014a]	
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]	
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]	
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]	
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]	
Exp (0.54 ppm)	116592080 ± 63	[Bennett et al., 2006]	
Diff (Exp $-$ SM)	287 ± 80	[Davier et al., 2011]	$\rightarrow 3.6\sigma$
	261 ± 78	[Hagiwara et al., 2011]	$\rightarrow 3.9\sigma$
	249 ± 87	[Aoyama et al., 2012]	

QCD errors largest, discrepancy large

New experiments+new theory=new physics?

- Fermilab E989 begins 2017, aims for 0.14 ppm

  J-PARC E34 ∼2020, aims for 0.3-0.4 ppm

  Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$ (possibly more)
- If both central values stay the same,

  E989 ($\sim 4\times$ smaller error) $\rightarrow$ $\sim 5\sigma$
  E989+new HLBL theory (models+lattice, 10%) $\rightarrow$ $\sim 6\sigma$
  E989+new HLBL +new HVP (50% reduction) $\rightarrow$ $\sim 8\sigma$
- Big discrepancy: new Physics $\sim 2\times$ Electroweak
- Lattice calculations important to trust theory errors, or lattice values may become (part of) the central values (muon g-2 theory initiative)
- Much progress, see many talks at Lattice 2017 (Granada) for latest results

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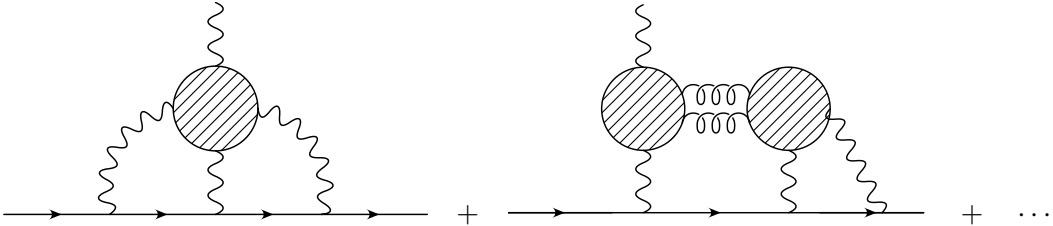
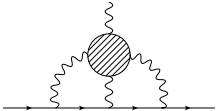
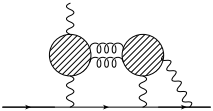


- Models: $(105 \pm 26) \times 10^{-11}$ [Prades et al., 2009, Benayoun et al., 2014]
  $(116 \pm 40) \times 10^{-11}$ [Jegerlehner and Nyffeler, 2009]
- Model errors difficult to quantify error now compatible with HVP error. see talk by A. Keshavarzi, 1st workshop muon g-2 theory initiative, June 2017, FNAL
- First lattice results promise reliable errors [Blum et al., 2015, Blum et al., 2016, Blum et al., 2017a] see also [Green et al., 2015, Asmussen et al., 2016]
- Dispersive/data approach also systematic

  [Colangelo et al., 2014b, Pauk and Vanderhaeghen, 2014, Colangelo et al., 2015, Colangelo et al., 2017]

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

The desired amplitude  $+$  $+ \quad \cdots$ is obtained from a Euclidean space lattice calculation

$$\mathcal{M}_\nu(\vec{q}) \;=\; \lim_{\substack{t_{\rm src}\to-\infty \\ t_{\rm snk}\to\infty}} e^{E_{q/2}(t_{\rm snk}-t_{\rm src})} \sum_{\vec{x}_{\rm snk},\vec{x}_{\rm src}} e^{-i\frac{\vec{q}}{2}\cdot(\vec{x}_{\rm snk}+\vec{x}_{\rm src})} e^{i\vec{q}\cdot\vec{x}_{\rm op}} \mathcal{M}_\nu(x_{\rm snk}, x_{\rm op}, x_{\rm src}),$$

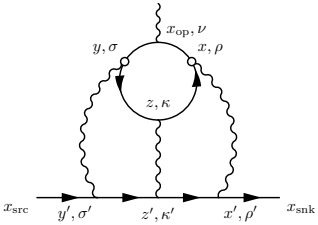
where

$$\begin{aligned} -e\mathcal{M}_\nu(x_{\rm src}, x_{\rm op}, x_{\rm snk}) &= \left\langle \mu(x_{\rm snk}) J_\nu(x_{\rm op}) \overline{\mu}(x_{\rm src}) \right\rangle \\ &= -e \sum_{x,y,z} \sum_{x',y',z'} \mathcal{F}_\nu(x,y,z,x',y',z',x_{\rm op},x_{\rm snk},x_{\rm src}). \end{aligned}$$

and

$$\left[\left(\frac{-i\not{q}^+ + m_\mu}{2E_{q/2}} \right) \left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho \right) \left(\frac{-i\not{q}^- + m_\mu}{2E_{q/2}} \right) \right]_{\alpha\beta} = \left(\mathcal{M}_\nu(\vec{q}) \right)_{\alpha\beta},$$

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]



$$\mathcal{F}^C_\nu\left(\vec{q}; x, y, z, x_{\rm op}\right) = (-ie)^6 \mathcal{G}_{\rho,\sigma,\kappa}(\vec{q}; x, y, z)\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op})$$

$$\begin{aligned} & i^4\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op}) \\ = & \sum_{q=u,d,s} \frac{(e_q/e)^4}{6} \left\langle \mathrm{tr}\left[-i \quad \gamma_\rho S_q(x,z) i\gamma_\kappa S_q(z,y) i\gamma_\sigma S_q\left(y, x_{\rm op}\right) i\gamma_\nu S_q\left(x_{\rm op}, x\right)\right]\right\rangle_{\rm QCD} + 5 \text{ permutations} \end{aligned}$$

$$\begin{aligned} & i^3\mathcal{G}_{\rho,\sigma,\kappa}(\vec{q}; x, y, z) \\ = & e^{\sqrt{m^2+\vec{q}^2/4}(t_{\rm snk}-t_{\rm src})} \sum_{x',y',z'} G_{\rho,\rho'}(x,x') G_{\sigma,\sigma'}(y,y') G_{\kappa,\kappa'}(z,z') \\ & \times \sum_{\vec{x}_{\rm snk},\vec{x}_{\rm src}} e^{-i\vec{q}/2\cdot(\vec{x}_{\rm snk}+\vec{x}_{\rm src})} S\left(x_{\rm snk}, x'\right) i\gamma_{\rho'} S(x',z') i\gamma_{\kappa'} S(z',y') i\gamma_{\sigma'} S\left(y', x_{\rm src}\right) + 5 \text{ permutations} \end{aligned}$$

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

- The initial and final muon states are plane waves
- Do all sums in the QED part exactly (using FFT's), QCD part done stochastically
- Key idea: contribution exponentially suppressed with $r = |x - y|$, so importance sample, concentrate on $r \lesssim \lambda_\pi^{\text{compton}}$
- space-time translational invariance allows coordinates relative to the hadronic loop

$$\mathcal{M}_\nu(\vec{q}) \quad = \quad \sum_r \left\{ \sum_{z, x_{\text{op}}} \mathcal{F}_\nu \left(\vec{q}, \frac{r}{2}, \frac{-r}{2}, z, x_{\text{op}} \right) e^{i\vec{q}\cdot\vec{x}_{\text{op}}} \right\}$$

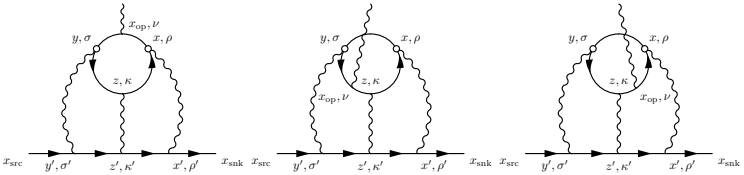
where $r = x - y$, $z \to z - w$, $x_{\text{op}} \to x_{\text{op}} - w$ and $w = (x + y)/2$

- We sum all the internal points over the entire space-time except we fix $x + y = 0$.
- (x, y) pairs stochastically sampled, z and x_{op} sums exact

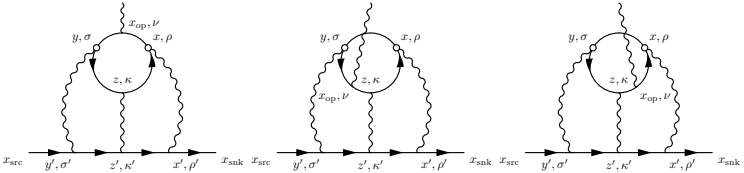
Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

$$\langle \mu(\vec{p}')|J_\nu(0)|\mu(\vec{p})\rangle \quad = \quad -e\bar{u}(\vec{p}')\left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m}[\gamma_\nu,\gamma_\rho]q_\rho\right)u(\vec{p})$$

- implies $F_2(0)$ only accessible by extrapolation $q \to 0$.
- Form is due to Ward Identity, or charge conservation
- need WI to be exact on each config, or error blows up as $\vec{q} \to 0$
- To enforce WI compute average of diagrams with all possible insertions of $J_\nu(x_{\rm op})$



Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]



- WI allows a moment method that projects directly to $q = 0$

$$
\begin{aligned}
\mathcal{M}_\nu(\vec{q}) &= \sum_{r,z,x_{\rm op}} \mathcal{F}^C_\nu\left(\vec{q}, \frac{r}{2}, -\frac{r}{2}, z, x_{\rm op}\right)\left(e^{i\vec{q}\cdot\vec{x}_{\rm op}} - 1\right) \\
&\approx \sum_{r,z,x_{\rm op}} \mathcal{F}^C_\nu\left(\vec{q}, \frac{r}{2}, -\frac{r}{2}, z, x_{\rm op}\right)\left(i\vec{q}\cdot\vec{x}_{\rm op}\right) \\
\frac{\partial}{\partial q_i}\mathcal{M}_\nu(\vec{q})|_{\vec{q}=0} &= i \sum_{r,z,x_{\rm op}} \mathcal{F}^C_\nu\left(\vec{q} = 0, r, -r, z, x_{\rm op}\right)(x_{\rm op})_i
\end{aligned}
$$

Point source method in QCD+pQED (L. Jin) [Blum et al., 2016]

Sandwich $\mathcal{M}_\nu(\vec{q})$ between positive energy Dirac spinors $u(\vec{0}, s)$, $\bar{u}(\vec{0}, s)$

$$\overline{u}(\vec{0}, s')\left(\frac{F_2(q^2=0)}{2m_\mu}\frac{i}{2}[\gamma_i, \gamma_j]\right)u(\vec{0}, s) = \overline{u}(\vec{0}, s')\frac{\partial}{\partial q_j}\mathcal{M}_i(\vec{q})|_{\vec{q}=\vec{0}}u(\vec{0}, s)$$

multiply both sides by $\frac{1}{2}\epsilon_{ijk}$, sum over i and j,

$$\frac{F_2(0)}{m}\bar{u}_{s'}(\vec{0})\frac{\vec{\Sigma}}{2}u_s(\vec{0}) \quad = \quad \sum_r\left[\sum_{z,x_{\text{op}}}\frac{1}{2}\vec{x}_{\text{op}} \times \bar{u}_{s'}(\vec{0})i\vec{\mathcal{F}}^C\left(\vec{0}; x=-\frac{r}{2}, y=+\frac{r}{2}, z, x_{\text{op}}\right)u_s(\vec{0})\right]$$

where $\Sigma_i = \frac{1}{4i}\epsilon_{ijk}[\gamma_j, \gamma_k]$.

Lattice setup

- Photons: Feynman gauge, QED$_L$ [Hayakawa and Uno, 2008] (omit all modes with $\vec{q} = 0$)
- Gluons: Iwasaki gauge action (RG improved, plaquette+rectangle)
- muons: $L_s = \infty$ free domain-wall fermions (DWF)
- quarks: Möbius-DWF

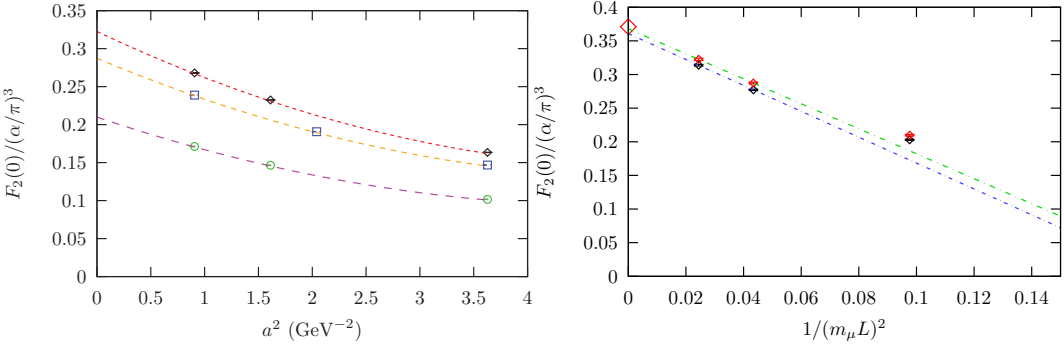
2+1f Möbius-DWF physical point ensembles (RBC/UKQCD) [Blum et al., 2014]

	$48^3 \times 96$	$64^3 \times 128$
a^{-1} (GeV)	1.73	2.36
a (fm)	0.114	0.084
L (fm)	5.47	5.38
L_s	24	12
m_π (MeV)	139	135
m_μ (MeV)	106	106

Continuum and ∞ volume limits in QED [Blum et al., 2016]

Test method in pure QED

QED systematics large, $O(a^4)$, $O(1/L^2)$, but under control



Limits quite consistent with well known PT result

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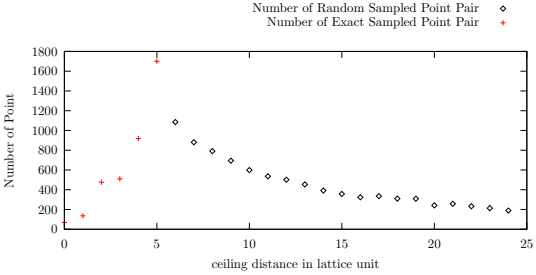
Physical point cHLbL contribution [Blum et al., 2017a]

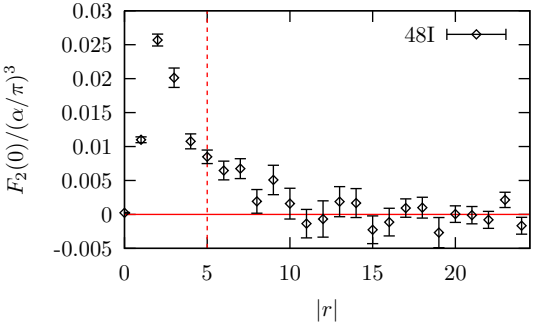
- Employ AMA with 2000 low-modes of the Dirac operator
- Compute 2 point, 4 sequential $\times$ (112+256) sloppy propagators per configuration
- On every configuration, do every distance $r \leq 5$ (and $r \leq 2$ twice), importance sample longer distances according to $\exp -0.01r/r^4$
- choose location of 1st point randomly, second point according to above

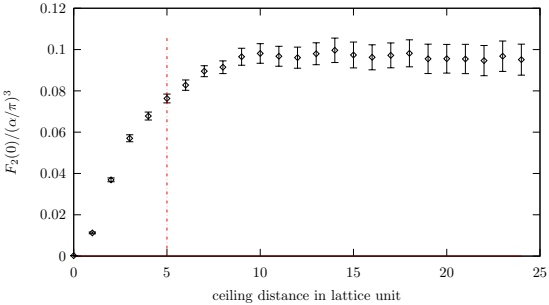


(numbers summed over ensemble)

Physical point cHLbL contribution, 48^3, 1.73 GeV lattice [Blum et al., 2017a]

- Measurements on 65 configurations, separated by 20 trajectories
- ignore strange quark contribution (down by 1/17 plus mass suppressed)
- exponentially suppressed with distance
- most of contribution by about 1 fm

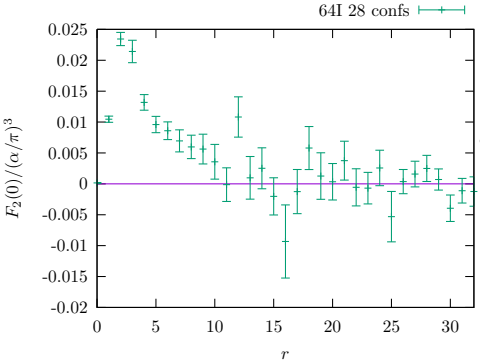


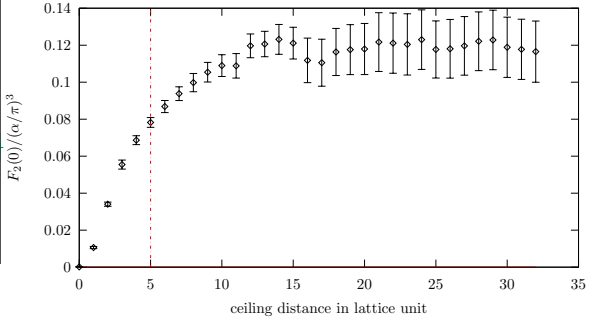


$$a_\mu^{\mathrm{cHLbL}} = 11.60 \pm 0.96 \times 10^{-10}$$

Physical point cHLbL contribution, 64^3, 2.36 GeV lattice (preliminary)

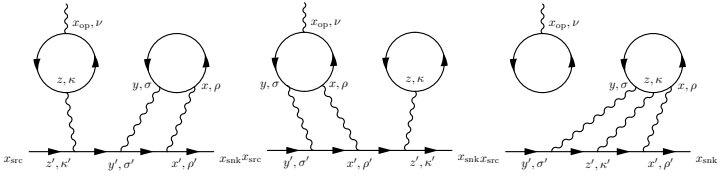
- Measurements as before, but only 28 configurations
- exponentially suppressed with distance
- most of contribution by about 1 fm



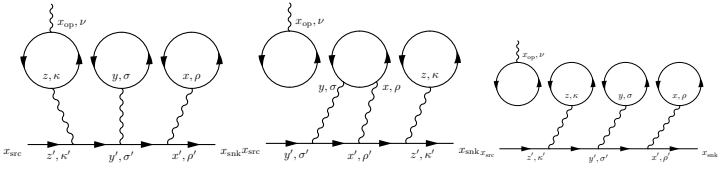


Disconnected contributions

SU(3) flavor:



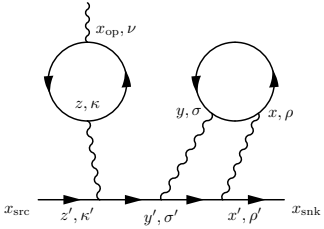
Leading $\qquad$ $O(m_s - m_{u,d})$



$O(m_s - m_{u,d})^2$ $\qquad$ and higher

- Gluons within and connecting quark loops have not been drawn
- To ensure loops are connected by gluons, explicit "vacuum" subtraction is required

Leading disconnected contribution



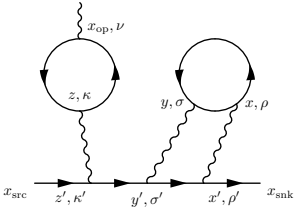
- We use two point sources at y and z, chosen randomly. The points sinks $x_{\rm op}$ and x are summed over exactly on lattice.
- Only point source quark propagators are needed. We compute M point source propagators and all M^2 combinations are used to perform the stochastic sum over $r = z - y$.

$$\mathcal{F}^D_\nu(x, y, z, x_{\rm op}) = (-ie)^6 \mathcal{G}_{\rho,\sigma,\kappa}(x, y, z) \mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op})$$

$$\mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op}) = \left\langle \frac{1}{2} \Pi_{\nu,\kappa}(x_{\rm op}, z) \left[\Pi_{\rho,\sigma}(x, y) - \Pi^{\rm avg}_{\rho,\sigma}(x - y) \right] \right\rangle_{\rm QCD}$$

$$\Pi_{\rho,\sigma}(x, y) = -\sum_q (e_q/e)^2 \, {\rm Tr}[\gamma_\rho S_q(x, y) \gamma_\sigma S_q(y, x)].$$

Leading disconnected contribution



$$\frac{F_2^{\rm dHLbL}(0)}{m}\frac{(\sigma_{s',s})_i}{2} = \sum_{r,x}\sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(x_{\rm op}\right)_j \cdot i\bar{u}_{s'}(\vec{0})\mathcal{F}_k^D\left(x, y=r, z=0, x_{\rm op}\right)u_s(\vec{0})$$

$$\mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op}) = \left\langle \frac{1}{2}\Pi_{\nu,\kappa}\left(x_{\rm op}, z\right)\left[\Pi_{\rho,\sigma}(x,y) - \Pi^{\rm avg}_{\rho,\sigma}(x-y)\right]\right\rangle_{\rm QCD}$$

$$\sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(x_{\rm op}\right)_j\left\langle\Pi_{\rho,\sigma}\left(x_{\rm op},0\right)\right\rangle_{\rm QCD} = \sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(-x_{\rm op}\right)_j\left\langle\Pi_{\rho,\sigma}\left(-x_{\rm op},0\right)\right\rangle_{\rm QCD} = 0$$

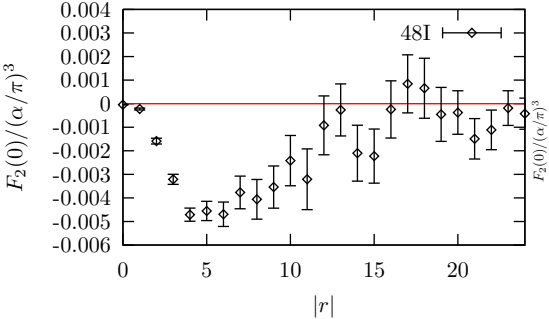
- Because of parity, the expectation value for the (moment of) left loop averages to zero.
- $\left[\Pi_{\rho,\sigma}(x,y) - \Pi^{\rm avg}_{\rho,\sigma}(x-y)\right]$ is only a noise reduction technique. $\Pi^{\rm avg}_{\rho,\sigma}(x-y)$ should remain constant through out the entire calculation.

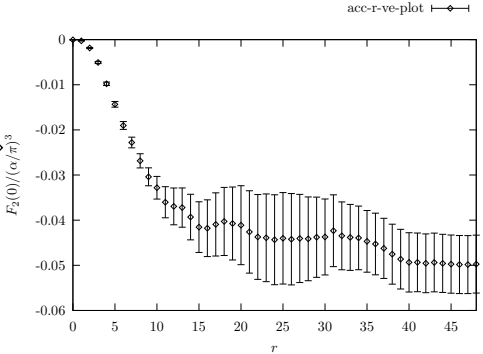
Physical point dHLbL contribution [Blum et al., 2017a]

- Use AMA with 2000 low-modes of the Dirac operator and
- randomly choose 256 "spheres" of radius 6 lattice units
- Uniformly sample 4 (unique) points in each
- do half as many strange quark props
- Construct $(1024 + 512)^2$ point-pairs per configuration

Physical point dHLbL contribution, 48^3, 1.73 GeV lattice [Blum et al., 2017a]

- strange contributes less than 5 %

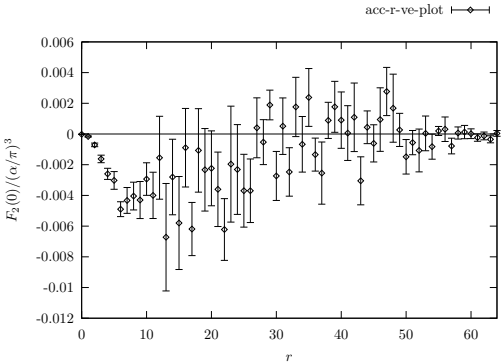


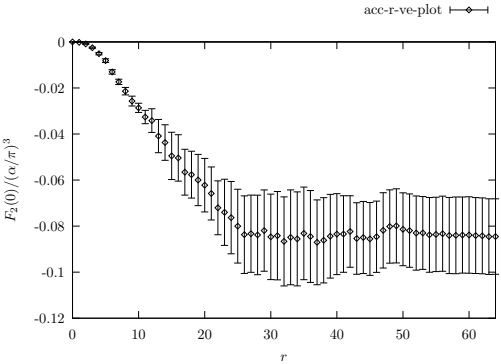


$$a_\mu^{\mathrm{dHLbL}} = -6.25 \pm 0.80 \times 10^{-10}$$

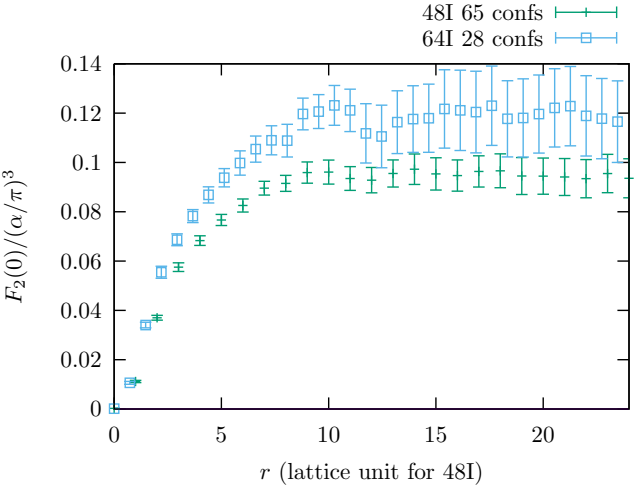
$$a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35 \pm 1.35 \times 10^{-10}$$

Physical point dHLbL contribution, 64^3, 2.36 GeV lattice (preliminary)



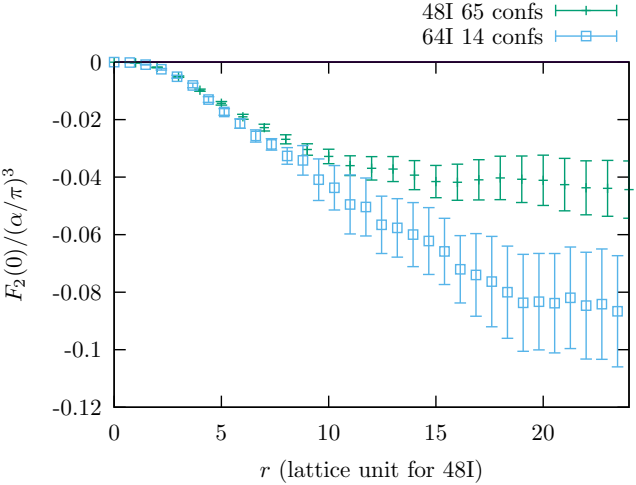


cHLbL contribution: lattice spacing effect (preliminary)



- Significant increase as $a \to 0$

dHLbL contribution: lattice spacing effect (preliminary)



- Large negative increase tends to cancel positive increase of connected piece
- Collecting more statistics, then $a \to 0$ limit

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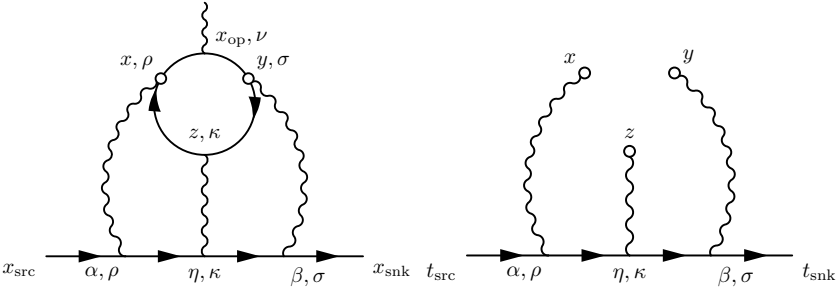
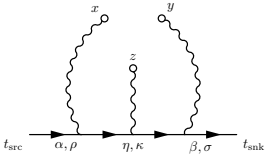
QED_∞ [Lehner and Izubuchi, 2015, Jin et al., 2015, Green et al., 2015, Asmussen et al., 2016, Blum et al., 2017b]



- Mainz group made first concrete proposal for QED_∞
- QED_∞: muon, photons computed in infinite volume ($c.f.$ HVP)
- QCD mass gap: $\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\mathrm{op}}) \sim \exp -m_\pi \times \mathrm{dist}(x, y, z, x_{\mathrm{op}})$
- QED weight function does not grow exponentially
- Then leading FV error is exponentially suppressed ($c.f.$ HVP) instead of $O(1/L^2)$

QED$_\infty$ weighting function [Blum et al., 2017b]



$$= \quad \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + 5 \text{ perms.}$$

- Note Hermitian part gives same F_2 but is infrared finite,

$$\mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) = \frac{1}{2}\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \frac{1}{2}\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z)^\dagger$$

- In units of the muon mass m_μ,

$$\begin{aligned}\mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) \quad &= \quad \frac{\gamma_0+1}{2} i\gamma_\sigma(-\partial\!\!\!/_y + \gamma_0 + 1) i\gamma_\kappa(\partial\!\!\!/_x + \gamma_0 + 1) i\gamma_\rho \frac{\gamma_0+1}{2} \\ &\quad \times \frac{1}{4\pi^2}\int d^4\eta \frac{1}{(\eta - z)^2} f(\eta - y) f(x - \eta)\end{aligned}$$

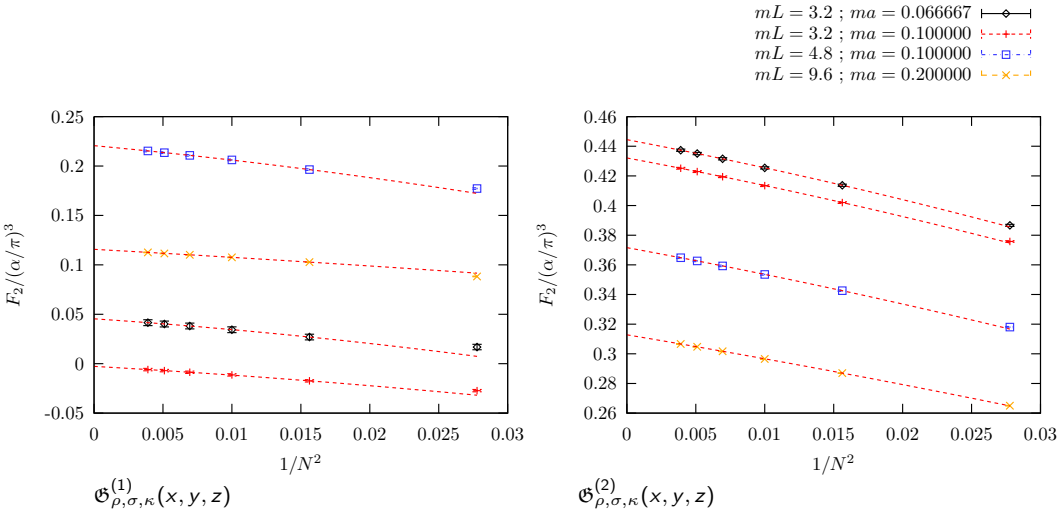
QED_∞ subtraction [Blum et al., 2017b]

- Current conservation implies $\sum_x \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\mathrm{op}}) = 0$ ($V \to \infty$ and $a \to 0$)
- Subtract terms that vanish as $a, V \to 0$
  $\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) = \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,y) + \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,y)$
- subtraction changes (may reduce) a and V systematic errors ($c.f.$ HVP)
- Further, $\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(z,z,x) = 0$ so short distance $O(a^2)$ effects suppressed.

- The 4-dim integral is (pre-)calculated numerically with CUBA library (cubature rules).
- Translation/rotation symmetry: parametrize (x,y,z) by 5 parameters on N^5 grid points (Mainz uses 3 params by averaging over muon time direction).
- (linearly) Interpolate grid in stochastic integral over (x,y)

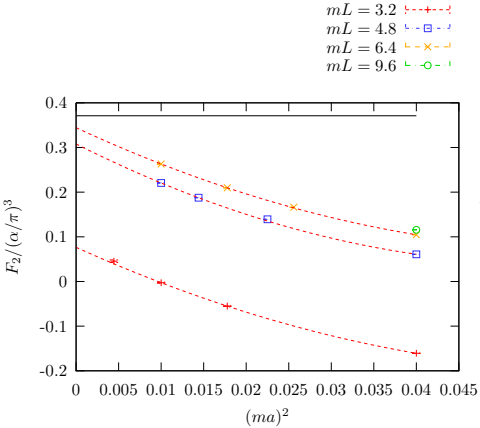
QED$_\infty$ results- pure QED, interpolation error [Blum et al., 2017b]

- $N = 6, 8, 10, 12, 14, 16 \to \infty$ (2nd order in $1/N^2$ fits)

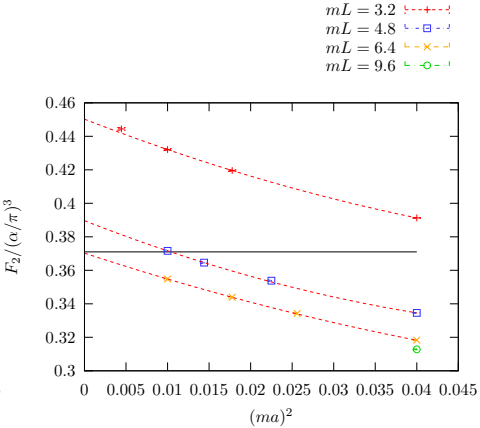


QED_∞ results- pure QED, lattice-spacing error [Blum et al., 2017b]

- error $\to\sim$const for $mL \gtrsim 4.8$
- FV effect $\lesssim 1\%$ for $mL = 9.6$
- fit: $F_2(L,a) = F_2(L) + k_1 a^2 + k_2 a^4$



$\mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z)$



$\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z)$

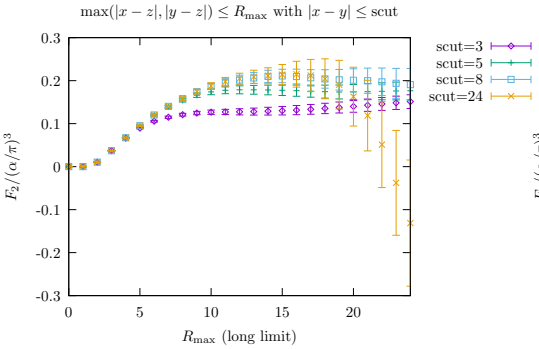
QED_∞ results- pure QED, finite volume error [Blum et al., 2017b]

- Take $F_2(\infty) \approx F_2(mL = 9.6)$
- results for $m_{\mathrm{loop}} = m_{\mathrm{line}}$ (a_e)
- Similar results hold for $m_{\mathrm{loop}} = 2m_{\mathrm{line}}$
- $F_2/(\alpha/\pi)^3 = 0.3686(37)(35)$ and $0.1232(30)(28)$ compared to
- QED perturbation theory results : 0.371 and 0.120

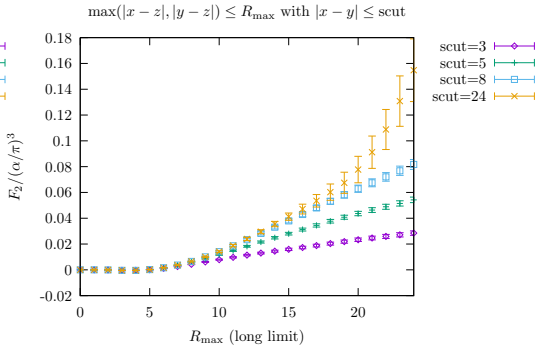
QED$_\infty$+QCD preliminary results: cHLbL

48^3, $a^{-1} = 1.73$ GeV, physical mass ensemble (48 configs)

no subtraction

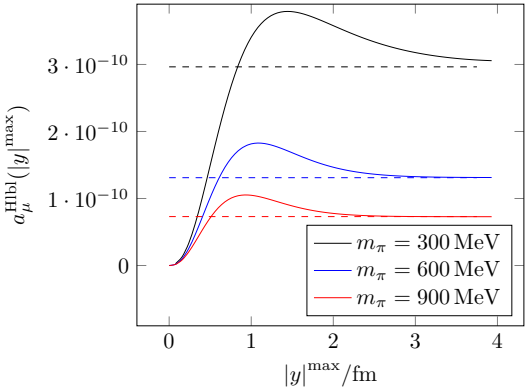


with subtraction



- different QED weights, agree if summed over all points (up to $O(a^2, e^{-L})$)
- same stochastic sampling as before for hadronic loop
- 5 fm box may be too small. Need more statistics

QED$_\infty$+π^0-pole; Mainz group talk by N. Asmussen, Lattice 2017



- ∞-volume model, cut off at finite distance $|y|^{\max}$
- similar QED$_\infty$, different FV effects (no sub, ...)

QED_∞+QCD Finite Volume Study

$a^{-1} = 1.0$ GeV, physical mass ensembles (I-DSDR gauge field action)

- 24^3, 32^3 and 48^3 lattices
- $a \sim 0.2$ fm, but small lattice artifacts ($e.g.$, $af_\pi = 0.13055(11)$)
- physical sizes of 4.8, 6.4, and 9.6 fm
- use QED_∞ to extrapolate HLbL g-2 to QCD_∞

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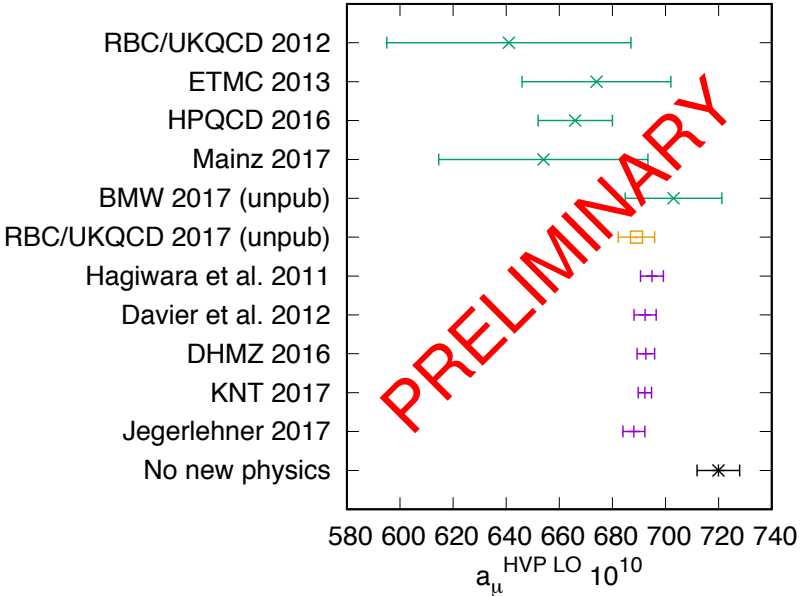
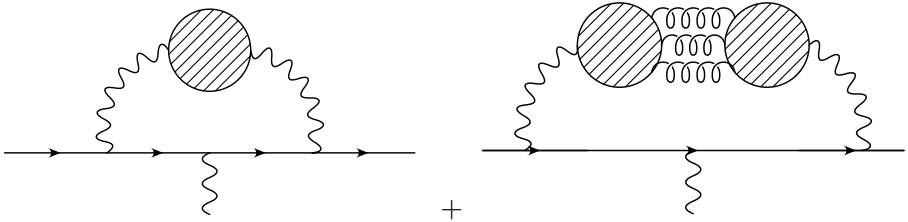
Summary of HVP theory results

Plenary talk by C. Lehner at Lattice 2017, ...



HVP contribution to muon g-2 [Blum, 2003, Lautrup et al., 1971]



Using lattice QCD and continuum, ∞-volume pQED

$$a_\mu(\mathrm{HVP}) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$f(q^2)$ is known, $\hat{\Pi}(q^2)$ is subtracted HVP, $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$, computed directly on Euclidean space-time lattice

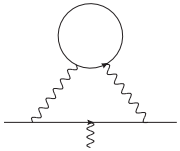
$$\begin{aligned} \Pi^{\mu\nu}(q) &= \int e^{iqx} \langle j^\mu(x) j^\nu(0) \rangle \qquad j^\mu(x) = \sum_i Q_i \bar{\psi}(x) \gamma^\mu \psi(x) \\ &= \Pi(q^2)(q^\mu q^\nu - q^2 \delta^{\mu\nu}) \end{aligned}$$

Double subtraction Bernecker-Meyer 2011

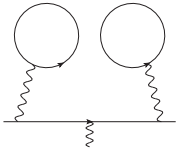
$$
\begin{aligned}
\Pi(q^2) - \Pi(0) &= \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2} t^2 \right) C(t) \\
C(t) &= \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle \\
a_\mu^{\mathrm{HVP}} &= \sum_t w(t) C(t) \\
w(t) &= 2 \int_0^\infty \frac{d\omega}{\omega} f(\omega^2) \left[\frac{\cos \omega t - 1}{(2 \sin \omega t/2)^2} + \frac{t^2}{2} \right]
\end{aligned}
$$

$w(t)$ includes the continuum QED part of the diagram

HVP contributions computed on lattice (u,d,s,c quarks)



Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$



Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

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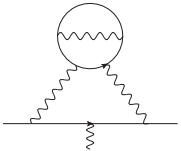
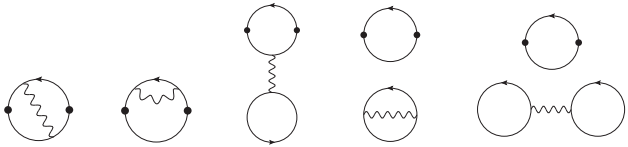


QED corrections, $\mathcal{O}(10 \times 10^{-10})$

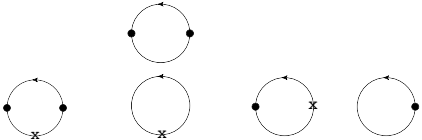
QED and strong isospin corrections



(a) V (b) S (c) T (d) D1 (e) D2

(f) F (g) D3

QED



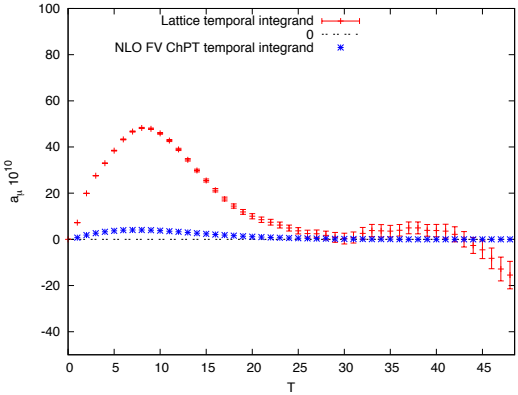
(a) M (b) R (c) O

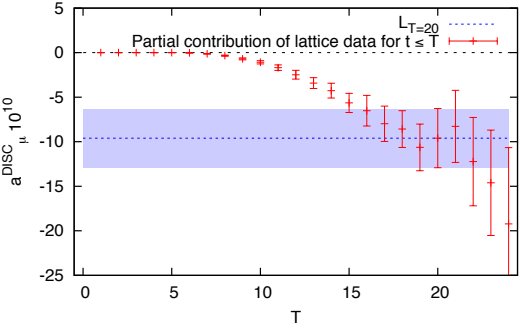
Strong Isospin

- Focus on V, S, F, and M so far

cHVP and dHVP– results at physical pion mass

RBC/UKQCD

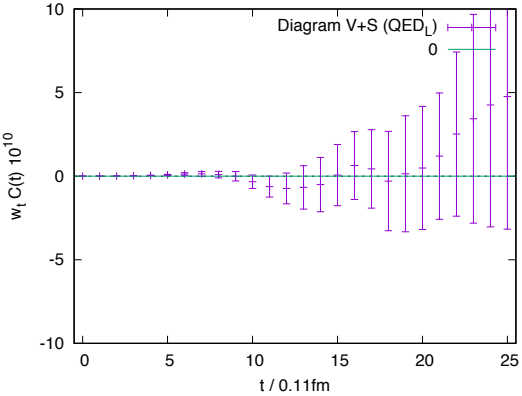


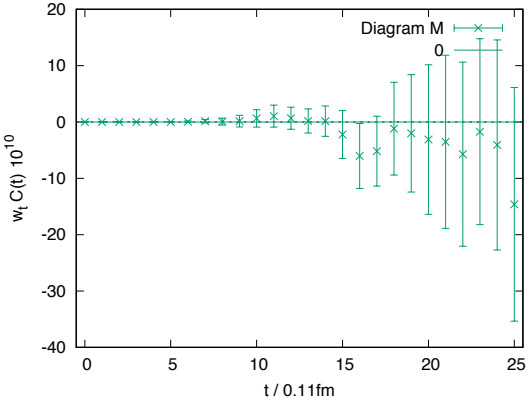


- small statistical errors from full volume 2000 low-mode average and improved stochastic method (disc)
- BMW confirms our a_μ^{DISC} result
- statistical errors grow at long distance

QED and strong isospin corrections- preliminary results

RBC/UKQCD





- Importance sampling a'la HLbL calc
- physical pion mass, $a^{-1} = 1.73$ GeV lattice
- increased statistics in progress

Connection to the R-ratio Bernecker-Meyer 2011

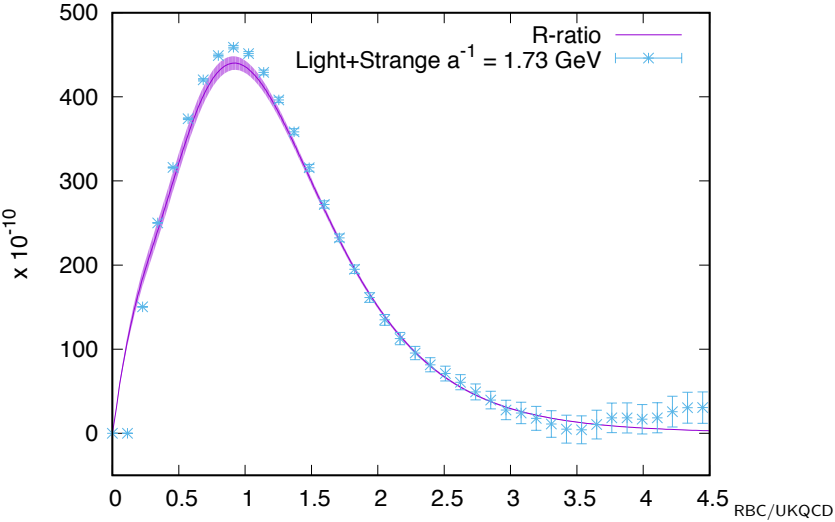
$$\Pi(-Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{s}{s+Q^2}\sigma(s, e^+e^- \to \text{ had})$$

$$R(s) = \frac{\sigma(s, e^+e^- \to \text{ had})}{\sigma(s, e^+e^- \to \mu^+\mu^-, \text{ tree})} = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \text{ had})$$

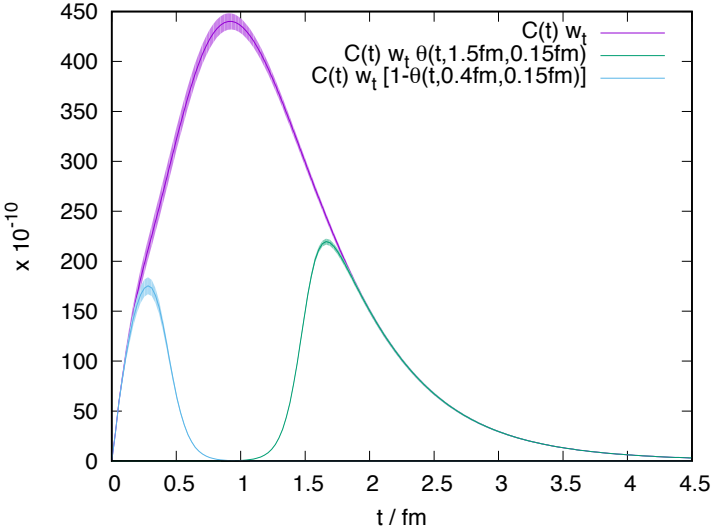
Fourier transform gives

$$C(t) \propto \int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

Lattice - R-ratio Jegerlehner 2016 comparison Bernecker-Meyer 2011



Select window in t (or, $\equiv \sqrt{s}$)



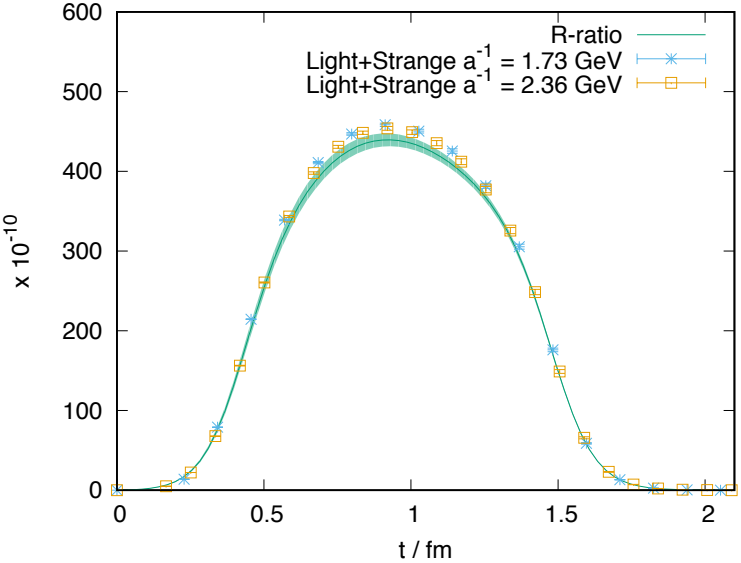
$$\Theta(t, \mu, \sigma) \equiv [1 + \tanh[(t - \mu)/\sigma]]/2$$

Window method

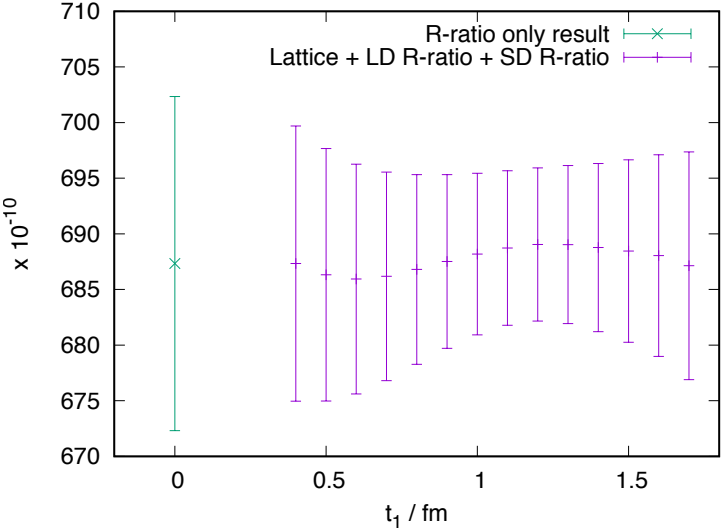
$$
\begin{aligned}
a_\mu &= \sum_t w_t C(t) = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}} \\
a_\mu^{\mathrm{SD}} &= \sum_t w_t C(t)[1 - \Theta(t, t_0, \Delta)] \\
a_\mu^{\mathrm{W}} &= \sum_t w_t C(t)[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \\
a_\mu^{\mathrm{LD}} &= \sum_t w_t C(t)\Theta(t, t_t, \Delta)
\end{aligned}
$$

Take a_μ^W from lattice, rest from R-ratio

a_μ^W $t_0 = 0.4$ fm, $t_1 = 1.5$ fm, $\Delta = 0.15$ fm



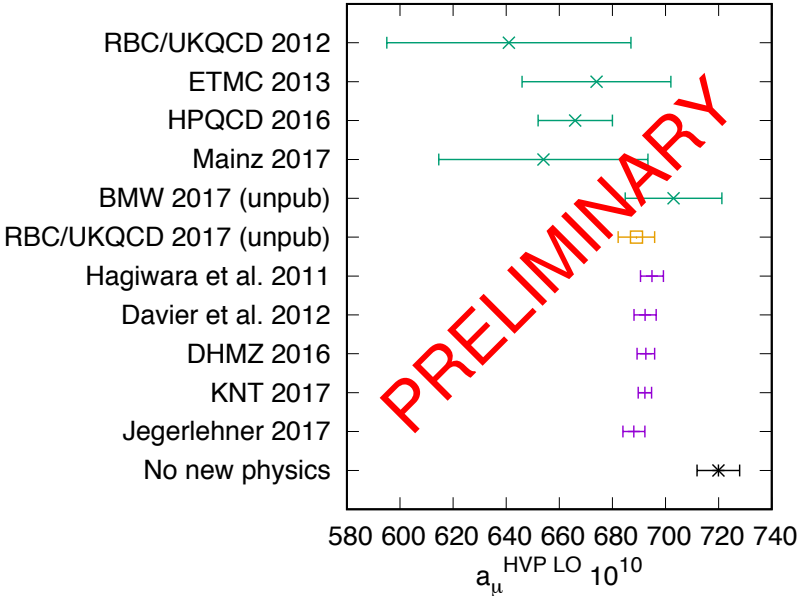
Combined lattice (u,d,s,c) and R-ratio result for a_μ



Lattice:

sQED FV correction

Summary of HVP theory results



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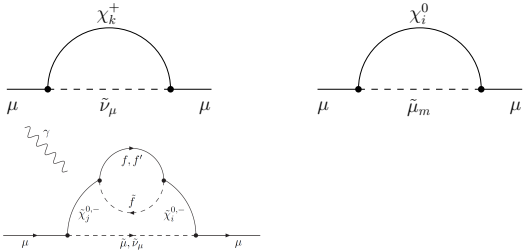
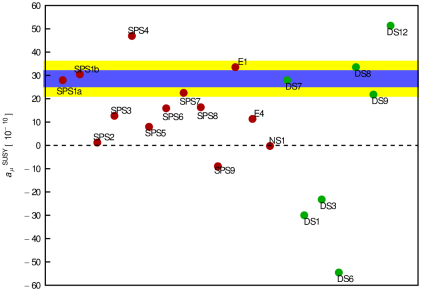
Explaining g-2 beyond the SM

If there really is a discrepancy, where does it come from?

Most likely scenario is still SUSY (?) [Bach et al., 2015, Athron et al., 2016, Belyaev et al., 2016], . . .





SUSY signatures at LHC

But there are other models too: 2HDM [Crivellin et al., 2016, Cherchiglia et al., 2016], Dark Matter [Kobakhidze et al., 2016], . . . , LFV [Altmannshofer et al., 2016], light scalars [Batell et al., 2017], . . .

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Hadronic Light-by-Light

- Lattice QCD(+QED) calculations done with physical masses, large boxes + improved measurement algorithms
- Physical point calculations complete at $a = 0.114$ fm [Blum et al., 2017a]
- Physical point nearly complete at $a = 0.084$ fm (increasing statistics)
- together, good control of non-zero a systematic error.
- FV corrections: QED_∞ + large 9.5 fm QCD box (underway)
- Need non-leading disconnected diagrams
- Lattice: unlikely that HLbL contribution will rescue standard model

Hadronic vacuum polarization

- Many groups working on it (see Lattice 2017 talks– world-wide effort!)
- Lattice QCD(+QED) calculations done with physical masses, large boxes + improved measurement algorithms
- disconnected contributions computed by 2 groups
- Included NLO QED and Strong Isospin breaking corrections
- Lattice and R-ratio results cross-checked and combined with window method
- Window method allows further error reduction over R-ratio alone

On track for solid SM result in time for E989 result

Acknowledgments

- Thanks to Taku Izubuchi, Luchang Jin, and Christoph Lehner for help in preparing this talk
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