Higgs Production at Large Transverse Momentum

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In collaboration with ***Eric Braaten***

arXiv:1510.01686

Outline

- ✧ Motivation
- ✧ Factorization formula at large P_T
- ✧ Higgs Fragmentation functions and evolution
- ✧ Compare factorization formula with full LO in example $q\bar{q} \to H + t\bar{t}$
- ✧ Summary

Inclusive Higgs Production

$$\sigma_{AB\to H+X} = \sum_{a,b} \int \mathrm{d}x_1 f_{a/A}(x_1,\mu) \int \mathrm{d}x_2 f_{b/B}(x_2,\mu) \times \widehat{\sigma}_{ab\to H+X}(x_1,x_2,\hat{s},M_H;\mu)$$
$$+\mathcal{O}(\Lambda_{\mathrm{QCD}}^2/s)$$

- Factorize into soft parts and hard parts with an error.
- Partonic cross section is free of mass singularity, e.g. $\log(\hat{s}/m_q^2)$.

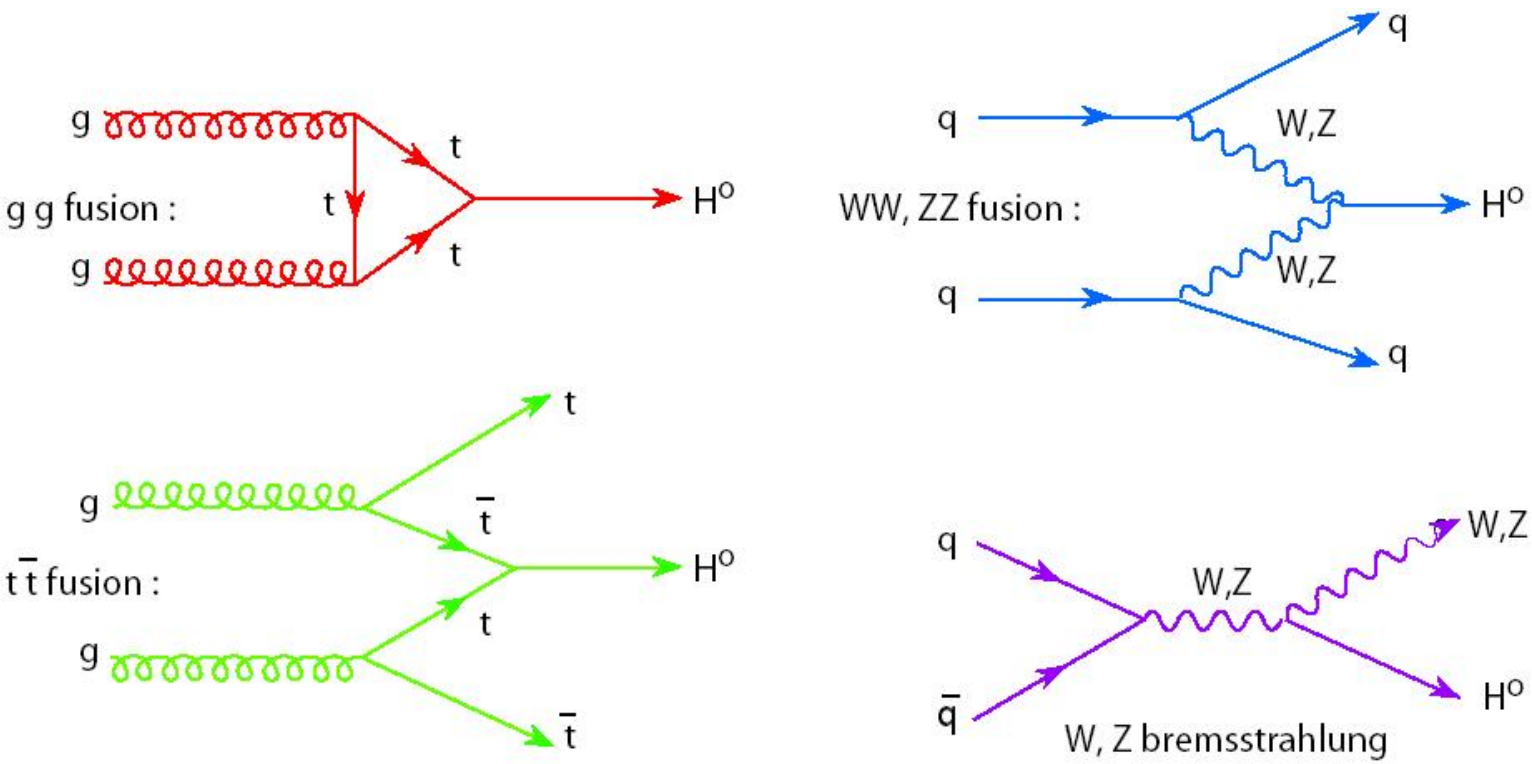
Predictive power relies on

- Universality of parton distribution function
- Convergence of the perturbative series of the partonic cross section.

$\log(\hat{s}/M_H^2)$ potentially ruins the perturbation at $\hat{s} \gg M_H^2$

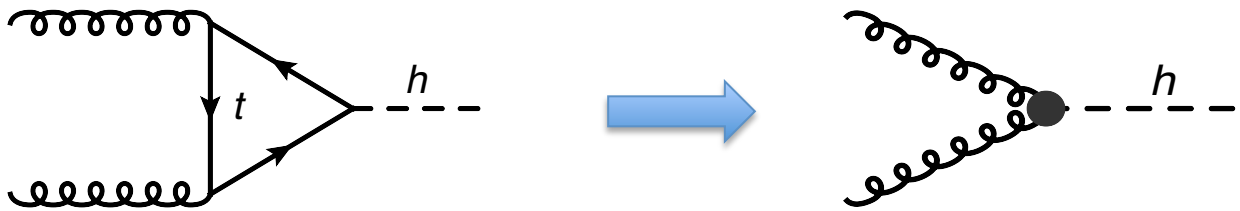
- Our ability to calculate the partonic cross section to high orders.

Dominant Partonic Processes



- Multi-scales: kinetic ($\hat{s}, P_T$) and masses ($M_H, M_t, M_{W/Z}, \ldots$)
- LO has two loop or 3-body phase-space integrals for most processes.

Higgs EFT



Standard Model $\mathcal{L} = -\frac{m_t}{v}\bar{t}th$

HEFT $\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi v} G^A_{\mu\nu} G^{\mu\nu,A} h$

- Eliminate the scale M_t
- Reduce the number of loops by 1.
- Inclusive Higgs production cross section from gluon fusion has been calculated to **N3LO** with Higgs EFT.

  Anastasiou, Duhr, Dulat, Herzog, Mistlberger, PRL 2015
- Higgs + 1 jet, **NNLO**

  Boughezal, Focke, Giele, Liu, Petriello, PLB 2015
- Higgs + ≥ 2 jet, **NLO**

  Campbell, Ellis, Williams, PRD 2010

Higgs EFT Beyond Standard Model

- New physics may change top quark Yukawa coupling and also introduce new heavy particles

Below scale of new heavy particle

$$\mathcal{L} = -\kappa_t \left(\frac{m_t}{v}\right) \bar{t}th + \kappa_g \left(\frac{\alpha_s}{12\pi v}\right) G^A_{\mu\nu} G^{\mu\nu,A} h$$

Standard Model: $\kappa_t = 1, \kappa_g = 0$

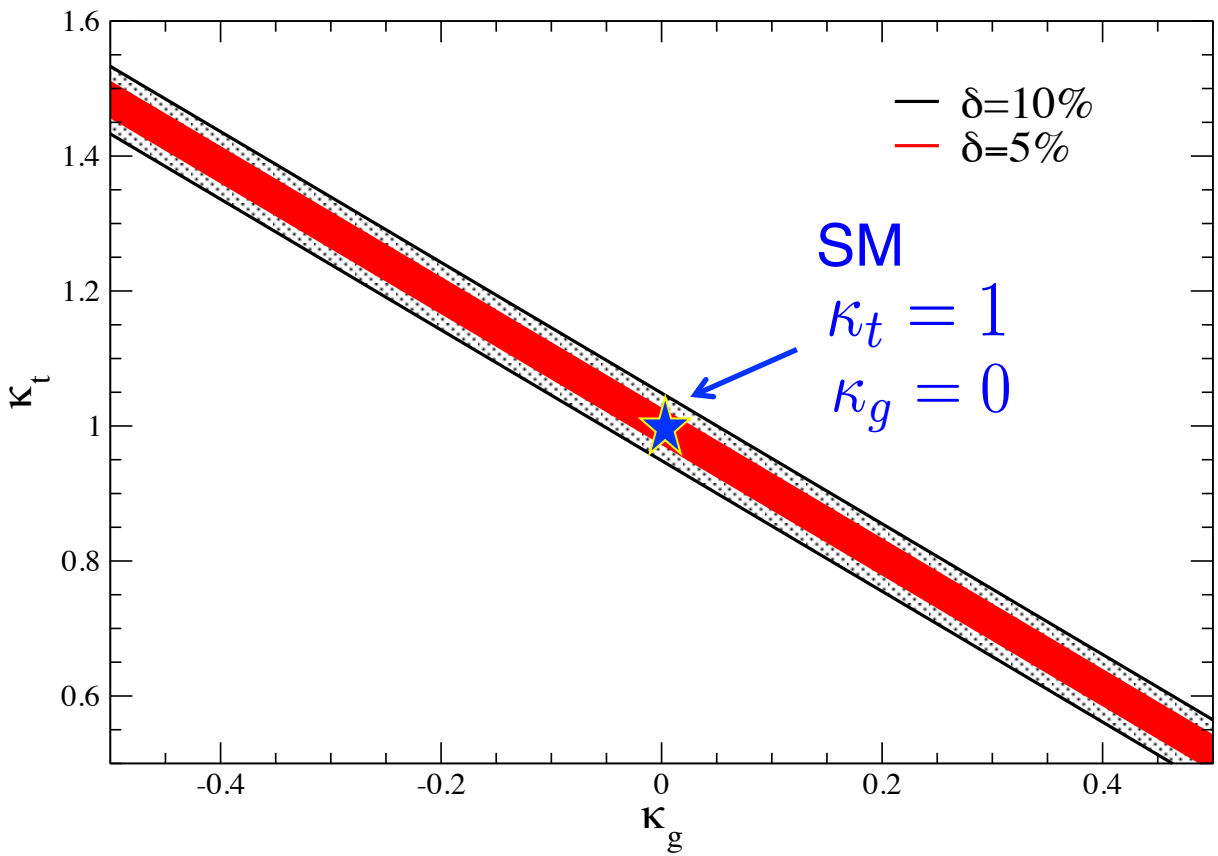
Below scale of top quark

$$\mathcal{L}_{\mathrm{HEFT}} = (\kappa_t + \kappa_g) \left(\frac{\alpha}{12\pi v}\right) G^A_{\mu\nu} G^{\mu\nu,A} h$$

Higgs cross section depends on $\kappa_t + \kappa_g$ only.

Higgs EFT Beyond Standard Model

- Any model with $\kappa_t + \kappa_g = 1$ reproduces the SM Higgs EFT.

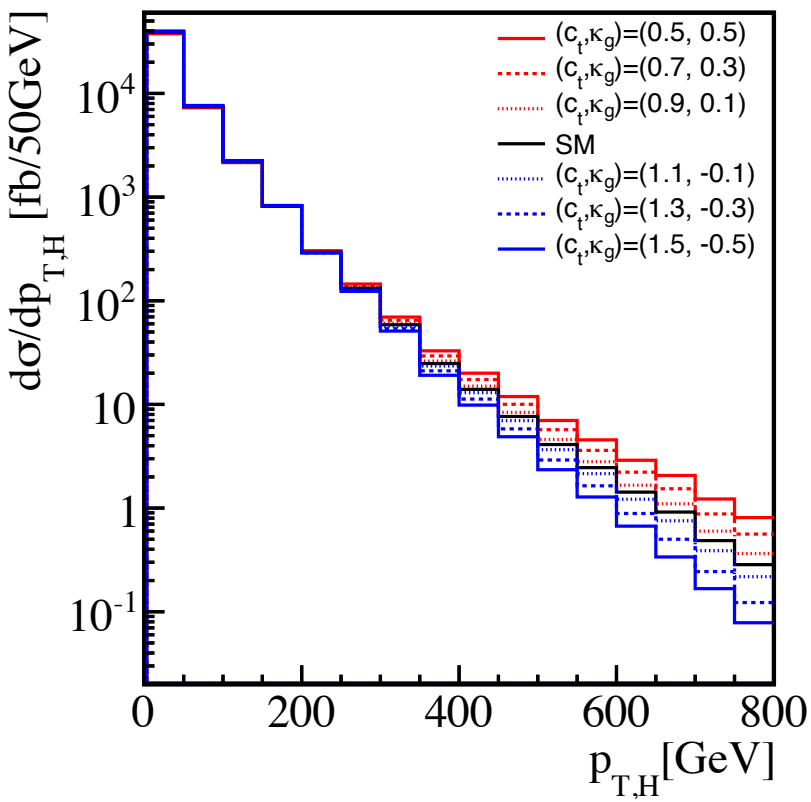


Dawson, Lewis and Zeng, PRD 2015

Higgs P_T distribution

$$\mathcal{L} = -\kappa_t \left(\frac{m_t}{v}\right) \bar{t}th + \kappa_g \left(\frac{\alpha_s}{12\pi v}\right) G^A_{\mu\nu} G^{\mu\nu,A} h$$

- Consistency with standard model requires $\kappa_t + \kappa_g = 1$.
- Differences of scenarios start at $P_T \geq 400\ \mathrm{GeV}$.



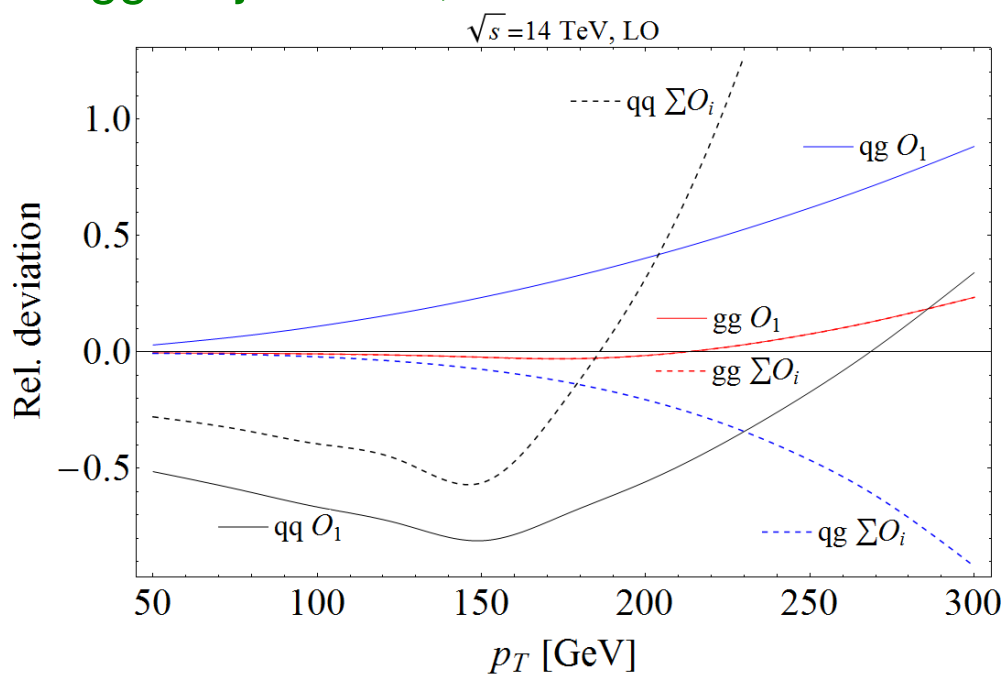
Schlaffer et al, Eur.Phys.J C 2014

HEFT Expansion Fails at Large P_T

- Higgs EFT with dim-7 operators for top quark

$$O_1 = G^A_{\mu\nu} G^{\mu\nu,A} h$$
$$O_3 = f_{ABC} G^{\mu,A}_{\nu} G^{\nu,B}_{\sigma} G^{\sigma,C}_{\mu} h$$
$$O_5 = g_s h \sum_{i,j=1}^{n_{lf}} G^A_{\mu\nu} D^\mu \bar{\psi}_i \gamma^\nu T^A \psi_i$$

- For Higgs + jet at LO, relative deviation from full LO "blows up" at large P_T



Dawson, Lewis and Zeng, PRD 2014

- Higgs produced at large pT is out of the radius of convergence of the Higgs EFT expansion.

Standard Factorization Formula

$$\frac{\mathrm{d}^2\sigma_{AB\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_{a,b}\int \mathrm{d}x_1 f_{a/A}(x_1,\mu)\int \mathrm{d}x_2 f_{b/B}(x_2,\mu)\times\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y}(x_1,x_2,P_T,y;\mu)$$
$$+\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_T^2)$$

- Scales in the partonic cross section

$$M_{W/Z}, M_H, M_t \qquad\qquad \sqrt{\hat{s}} \sim P_T$$

- At large P_T, can expand in M_i^2/P_T^2
  Guess: Choose $M_i = 0$ to get leading term with error $\mathcal{O}(M_i^2/P_T^2)$

- Example: $q\bar{q} \to Ht\bar{t}$ with $M_H \to 0$

$$\frac{\mathrm{d}^2\widehat{\sigma}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = F\left(\frac{M_H^2}{P_T^2}, \frac{M_t^2}{P_T^2}, \log\frac{M_H^2}{P_T^2}, \frac{M_H^2}{M_t^2}, \ldots\right)$$

Standard Factorization Formula

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Mass singularity: ill-defined in the $M_H, M_t\to 0$ limit.

Special treatment is necessary!

Mass Singularity

$$\frac{\mathrm{d}^2\widehat{\sigma}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = F\left(\frac{M_H^2}{\hat{s}}, \frac{M_t^2}{\hat{s}}, \boxed{\log\frac{M_H^2}{\hat{s}}, \frac{M_H^2}{M_t^2}, \ldots}\right)$$

- Mass singularity cannot be expanded in M_i^2/P_T^2
- These singular terms must be kept intact in the final result
- Strategy

  (1) Subtract the mass singularity,

  (2) Expand the non-singular terms in M_i^2/P_T^2 , keep the leading term (choose $M_i = 0$).

  (3) Add back the mass singularity.

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New Factorization Formula at high P_T

$$\frac{\mathrm{d}^2\sigma_{AB\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_{a,b}\int \mathrm{d}x_1 f_{a/A}(x_1,\mu)\int \mathrm{d}x_2 f_{b/B}(x_2,\mu)\times \boxed{\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y}(x_1,x_2,P_T,y;\mu)}$$

$$+\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_T^2)$$

Additional factorization

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i \int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i=\hat{P}/z,\mu)\,D_{i\to H}(z,\mu)+\mathcal{O}(M_H^2/P_T^2),$$

New Factorization Formula at high P_T

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$$+\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_T^2)$$

Additional factorization

Error

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i\int\frac{dz}{z}\boxed{\frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i=\hat{P}/z,\mu)}\boxed{D_{i\to H}(z,\mu)}+\boxed{\mathcal{O}(M_H^2/P_T^2)}$$

IR-safe cross sections Fragmentation functions

From factorization theorem

- IR-safe cross sections do not depend on softer scales: $M_{W/Z}, M_H$
- Fragmentation functions do not depend on harder scales: $P_T, \sqrt{\hat{s}}$
- Top mass can be considered either as soft scale or intermediate scale

New Factorization Formula at high P_T

Additional factorization of partonic cross section

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i \int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i = \hat{P}/z, \mu)\, D_{i\to H}(z,\mu) + \mathcal{O}(M_H^2/P_T^2),$$

New Factorization Formula at high P_T

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- **IR-safe cross section:** production of particle i in hard collision, with collinear regions subtracted.
- **Most important** i include $H, W^{\pm}, Z^0$, top quark and gluon.
- **Scale Separation:** softer mass scales $\lesssim M_H$ can be ignored

→ Easier to calculate.

New Factorization Formula at high P_T

Additional factorization of partonic cross section

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i \int \frac{dz}{z} \frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i = \hat{P}/z, \mu) \boxed{D_{i\to H}(z,\mu)} + \mathcal{O}(M_H^2/P_T^2),$$

- **Fragmentation functions:** the probability of finding a Higgs in a "jet" initiated by i, with the Higgs taking the longitudinal momentum fraction z.
- **Most important** i include $H, W^{\pm}, Z^0$, top quark and gluon.
- **Perturbative fragmentation functions** can be calculated order-by-order in perturbation theory, in contrast to the fragmentation into hadrons.

New Factorization Formula at high P_T

Additional factorization of partonic cross section

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i \int \frac{dz}{z} \frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i = \hat{P}/z, \mu)\, D_{i\to H}(z,\mu) + \boxed{\mathcal{O}(M_H^2/P_T^2)},$$

- **Error** from factorization approaches to zero as P_T increases.

New Factorization Formula at high P_T

Additional factorization of partonic cross section

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- **Factorization scale μ** separates softer and harder scales
- **Stable perturbation series:** large $\log(P_T^2/M_H^2)$ can be summed to all orders by solving the DGLAP evolution equation of $D_{i\to H}(z, \mu)$

Used to sum logarithms for W/Z production by
Berger, Gao, Kang, Qiu and Zhang, PRD 91, 113001 (2015)

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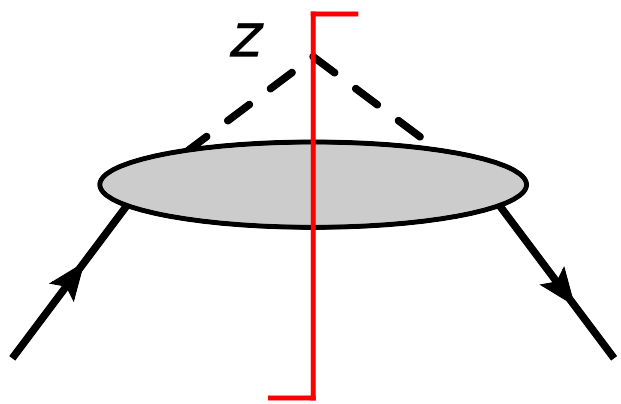
Fragmentation function $D_{i\to H}(z,\mu)$

- Probability of finding a Higgs in a "jet" initiated by i , with Higgs taking the longitudinal momentum fraction z.
- Most important i include $H, W^{\pm}, Z^0$, top quark and gluon.

Feynman diagram: top to Higgs fragmentation function



Evolution of FFs

<u>Factorization scale dependence</u> is determined by the DGLAP evolution equation.

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{i\to H}(z,\mu) = \boxed{\sum_j} \int_z^1 \frac{\mathrm{d}z'}{z'} P_{i\to j}(z/z',\mu) D_{j\to H}(z',\mu)$$

- Evolution comes from i splitting into j and then j fragmenting into Higgs.
- By solving the evolution equation, large $\log(P_T^2/M_H^2)$ can be summed to all orders.
- j can be $H, W^\pm, Z^0$, quarks and gluon.

Evolution of FFs

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- In the present work, we consider leading logarithm from QCD evolution with $j = t, g$.
- Different from QCD DGLAP, $j = H$ is also necessary here.
- Interesting consequence with Higgs emission kernel.

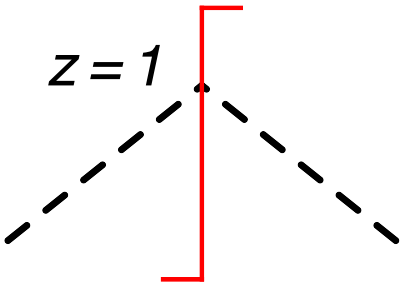
FFs at Initial Scale

- Need a set of initial conditions at some scale μ_0 to solve the evolution Eq.
- We choose initial scale at the mass threshold.
- FFs at initial scale can be calculated from the operator definitions directly.

  Collins and Soper, Nucl. Phys. B 194, 445 (1982)
- In the present work, we calculate the initial FFs at LO in strong, weak and Yukawa couplings.

Higgs to Higgs FF

- No counterpart in hadron production, since hadrons can not be produced directly in hard collision.



- At LO, $D_{H\to H}(z,\mu) = \delta(1-z)$.

- The LO result corresponds to Higgs produced isolated.

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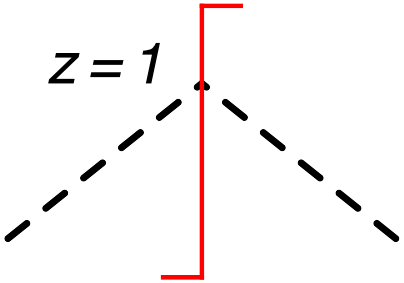

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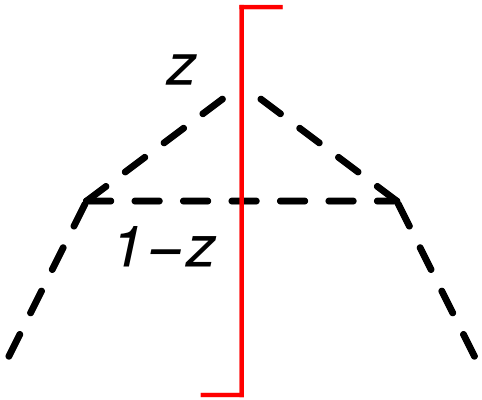
- At NLO, would need to consider $H \to H + H$



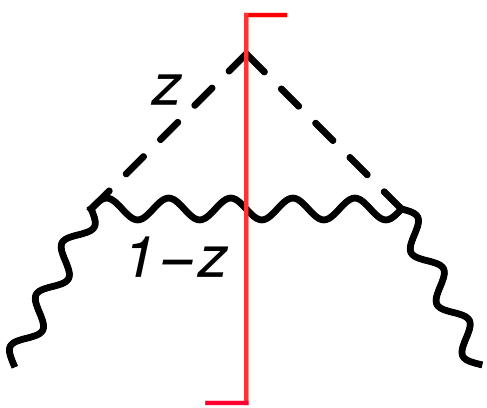
Weak Bosons to Higgs FF

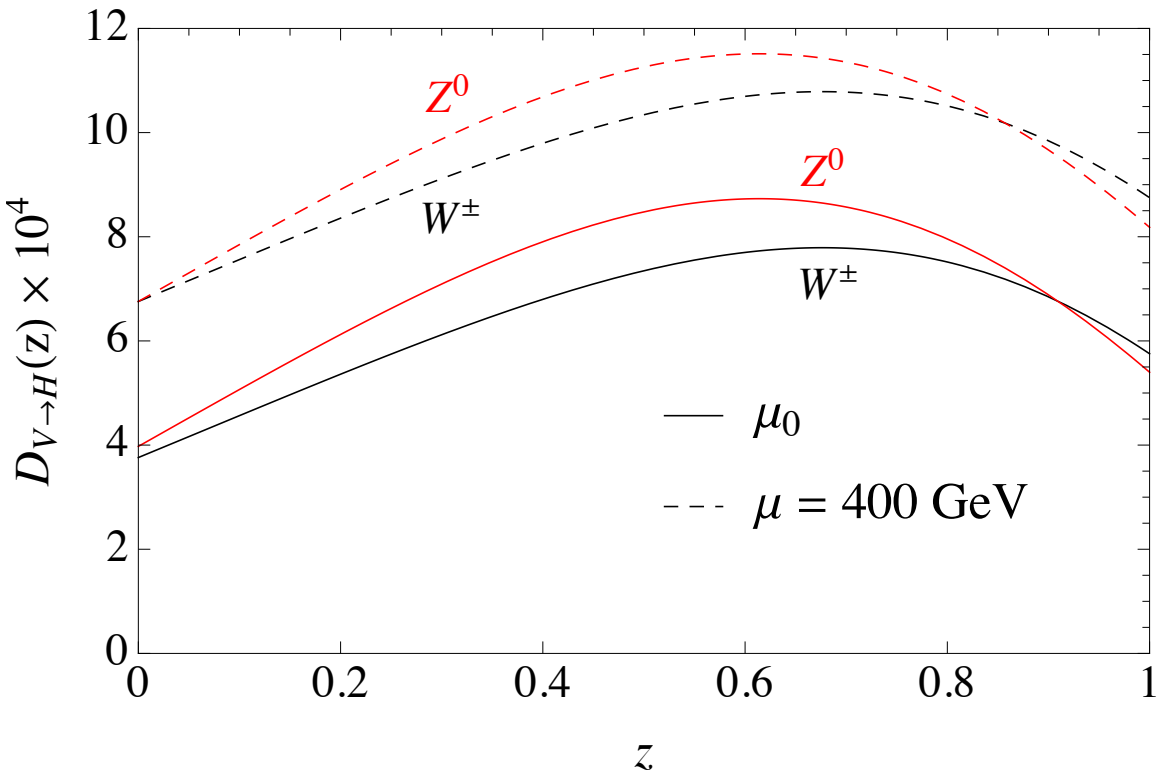
Choose initial scale at mass threshold

$$W^{\pm} : \mu_0 = M_W + M_H$$

$$Z^0 : \mu_0 = M_Z + M_H$$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{W/Z\to H} = P_{W/Z\to H}(z)$$





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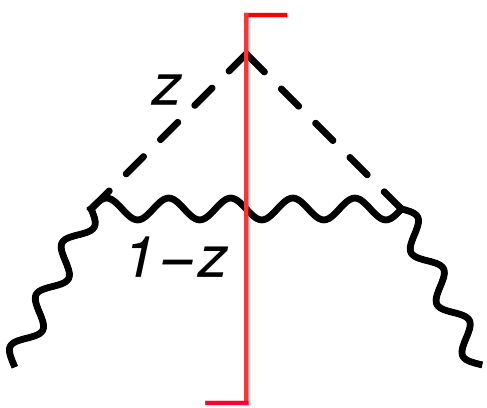
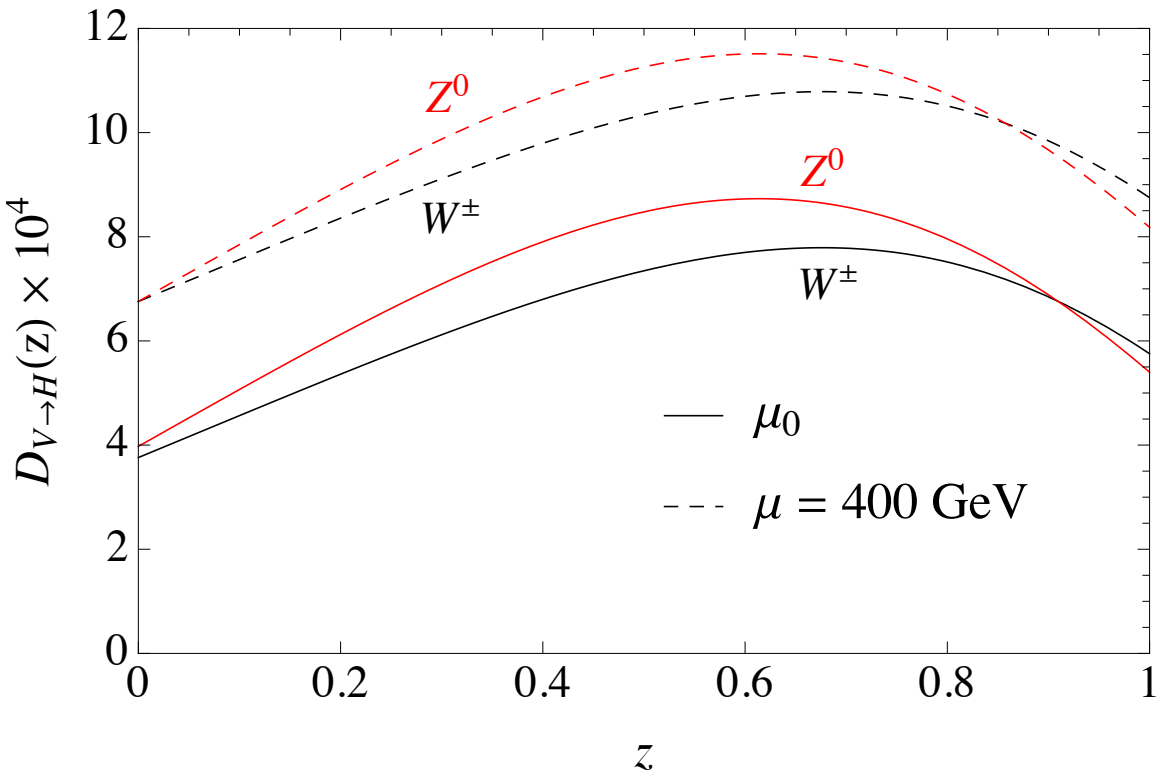




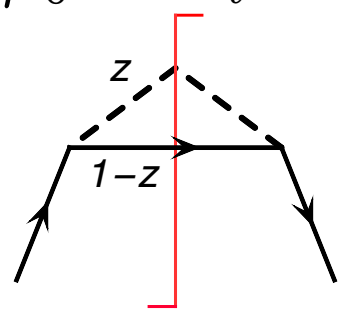
- At larger scale μ, larger probability to emit a Higgs.
- The increase of the probability is the same for all z.

Top and gluon FF at initial scale

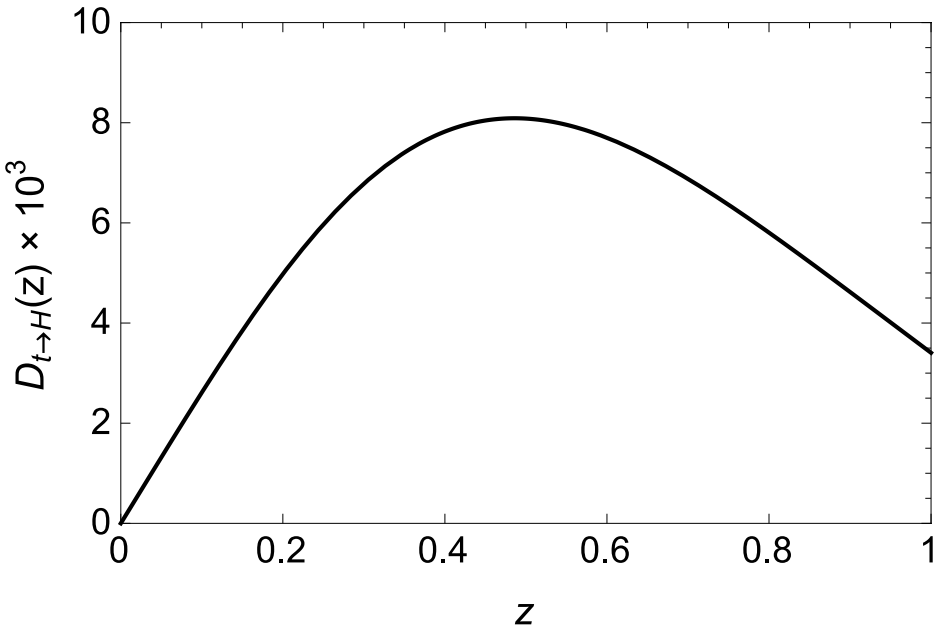
- Top quark and gluon FFs are mixed by QCD evolution.

- Top quark

$$\mu_0 = M_t + M_H$$



$$\sim y_t^2$$



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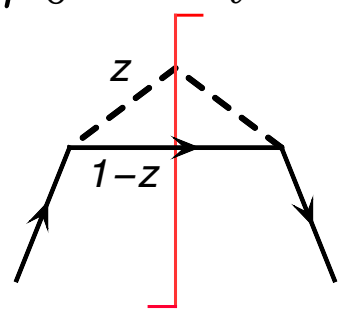

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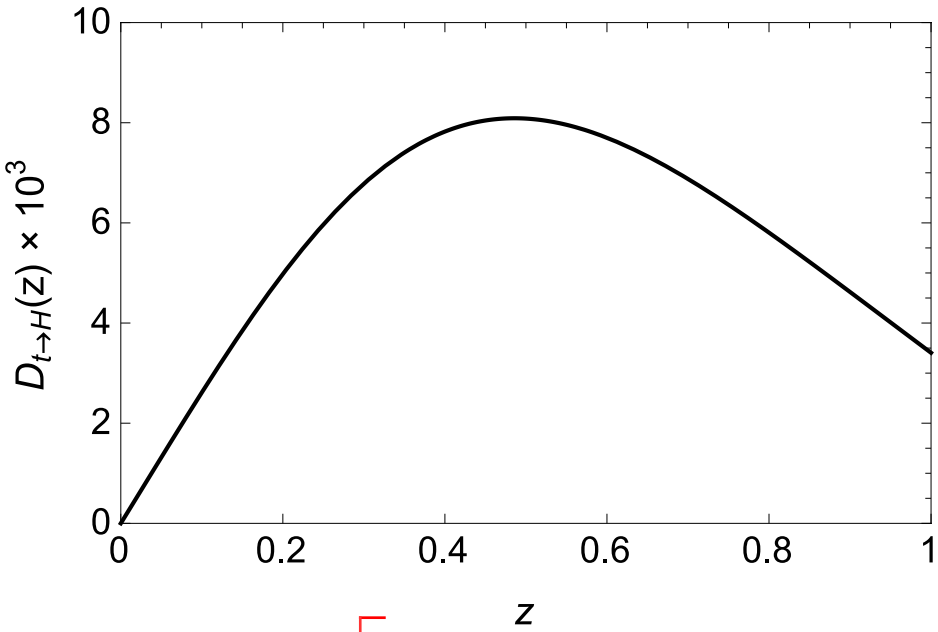
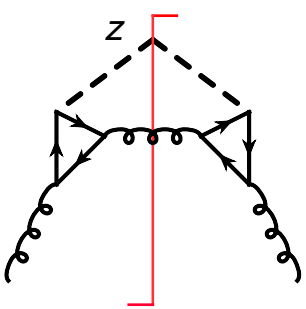
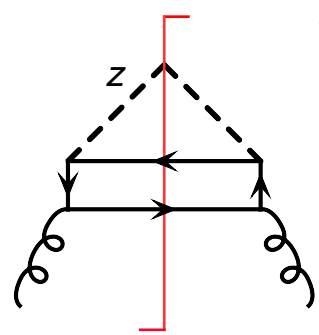


- Gluon



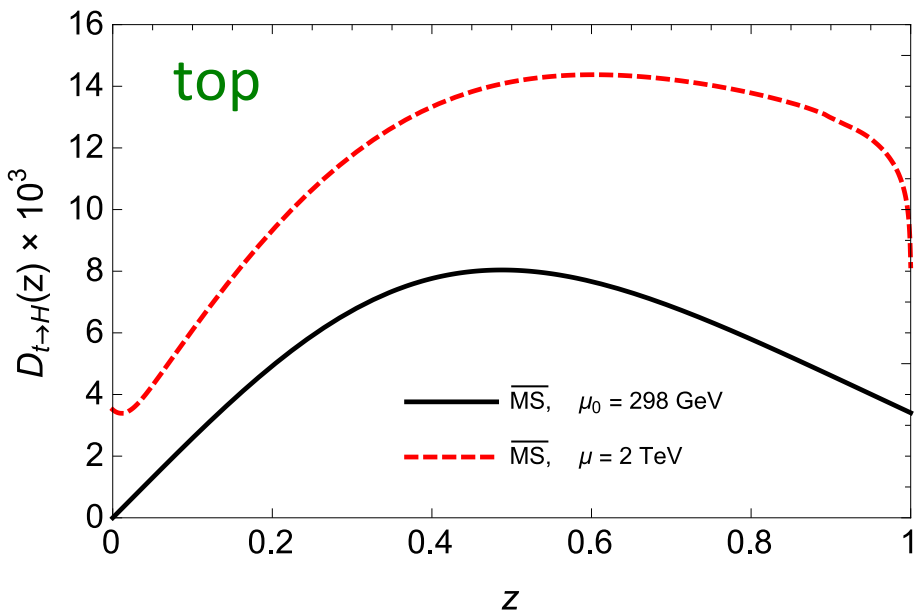
$$\sim \alpha_s^2 y_t^2$$

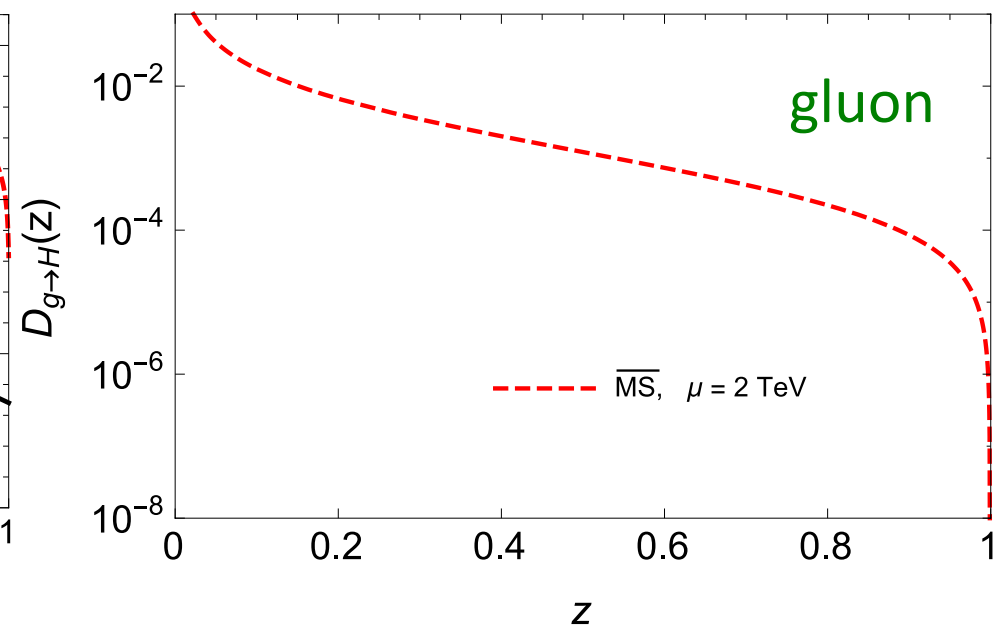


$$\sim \alpha_s y_t^2$$

Gluon to Higgs fragmentation is zero at LO.

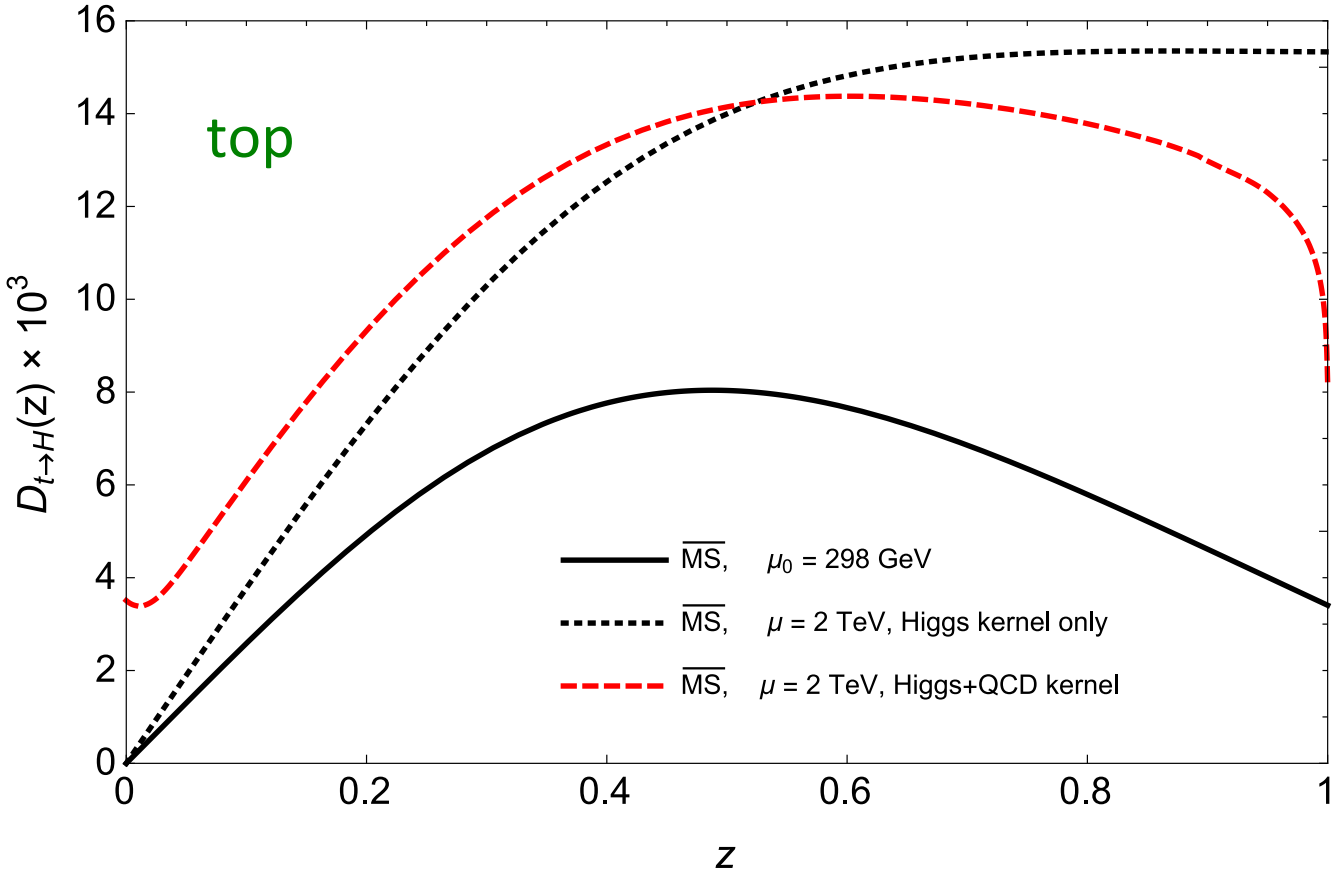
Evolved Top and gluon FF at 2 TeV





- Top quark FF is more important than gluon FF.
- Boosted top quark is more likely to emit a Higgs taking a large fraction of the longitudinal momentum.
- In contrast to gluon radiation, which is soft gluon dominant.

Higgs Emission Kernel Matters



- Higgs emission kernel greatly enhance the large z region.
- Gluon emission kernel reduces large z region, enhance small z region.
- Higgs emission is very effective for heavy quarks to lose energy.

Outline

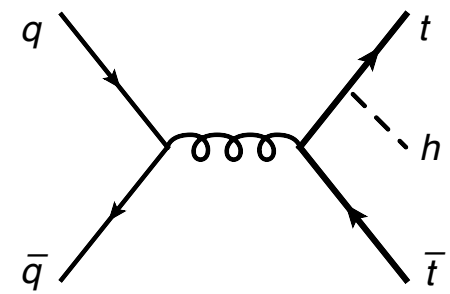
- ✧ Motivation
- ✧ Factorization formula at large P_T
- ✧ Higgs Fragmentation functions and evolution
- ✧ Compare factorization formula with full LO in example $q\bar{q} \to H + t\bar{t}$
- ✧ Summary

Scales in the example

- Example: $q\bar{q} \to H + t\bar{t}$
- Separation of scales

  Harder scale $Q_H \sim P_T, \sqrt{\hat{s}}$

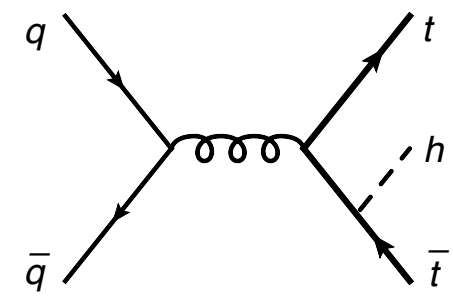
  Softer scale $Q_s \sim M_H$



Option 1: M_t is an intermediate scale

Expand in M_H^2/P_T^2

Error $\mathcal{O}(M_H^2/P_T^2)$



Option 2: M_t is a soft scale

Expand in M_H^2/P_T^2 and M_t^2/P_T^2

Error $\mathcal{O}(M_t^2/P_T^2)$

Question : how small are the errors numerically?

Mass Singularity in Full LO

- Full LO result has mass singularity terms.
- These terms cannot be expanded in (softer scales)/(harder scales) and need to be protected in the expansion.
  (choose rapidity $\hat{y} = 0$ for illustration)

$$\frac{\alpha_s^2\, y_t^2\, P_T}{36\,\pi\,\hat{s}^{\,2}} \left\{ \log \frac{4P_T^2}{4M_t^2\rho + M_H^2} + \frac{4M_t^2 - M_H^2}{4M_t^2\rho + M_H^2} \right\}$$

$$\rho = \frac{P_T^2}{\hat{s} - 2\sqrt{\hat{s}}P_T}$$

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$$\rho = \frac{P_T^2}{\hat{s} - 2\sqrt{\hat{s}}P_T}$$

Option 1: $Q_s \sim M_H$ $\quad Q_H \sim P_T, \sqrt{\hat{s}}$

$M_H \to 0$ → Error $\mathcal{O}\left(M_H^2/M_t^2\right)$

Instead of $\mathcal{O}(Q_s^2/Q_H^2)$

Mass Singularity in Full LO

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Option 1: $Q_s \sim M_H \qquad Q_H \sim P_T, \sqrt{\hat{s}}$

$M_H \to 0$ → Error $\mathcal{O}\left(M_H^2/M_t^2\right)$

Option 2: $Q_s \sim M_H, M_t \qquad Q_H \sim P_T, \sqrt{\hat{s}}$

$M_H \to 0, M_t \to 0$ → Divergent!

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Option 2: $Q_s \sim M_H, M_t \qquad Q_H \sim P_T, \sqrt{\hat{s}}$

$M_H \to 0, M_t \to 0$ → Divergent!

How the mass singular terms are protected
and kept intact in the factorization formula result?

Calculate IR-safe Cross Sections

- Example: $q\bar{q} \to H + t\bar{t}$

- Factorization formula at LO $i = H, t, \bar{t}$

  Index (m,n) $\to \alpha_s^m (y_t^2)^n$

$$\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = 2\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_{top} = \hat{P}/z, \mu)\, D^{(0,1)}_{t\to H}(z,\mu)$$
$$+\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H = \hat{P}/z, \mu) D^{(0,0)}_{H\to H}(z,\mu)$$

Calculate IR-safe Cross Sections

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- Factorization formula at LO $i = H, t, \bar{t}$

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$$+\int \frac{dz}{z}\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H = \hat{P}/z, \mu) D^{(0,0)}_{H\to H}(z,\mu)$$

Need to calculate
the IR-safe cross section

Already calculated.

Calculate Cross Sections

Apply $D^{(0,0)}_{H\to H} = \delta(1-z)$

$$\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = 2\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_{top} = \hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu) + \frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H = \hat{P},\mu)$$

$t/\bar{t}$ to H Fragmentation contribution

Apply $D^{(0,0)}_{H\to H} = \delta(1-z)$

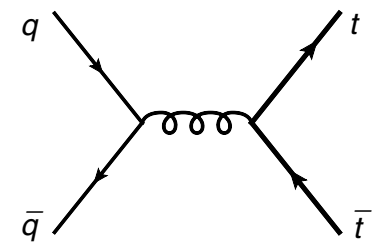
$$\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = \boxed{2\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_{top}=\hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu)} + \frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H=\hat{P},\mu)$$

$t/\bar{t}$ *fragmentation contribution*

- Mass singularity in this term
  (choose rapidity $\hat{y}=0$ for illustration)

$$\frac{\alpha_s^2\,y_t^2\,P_T}{36\,\pi\hat{s}^{\,2}}\left\{\log\frac{\mu^2}{4M_t^2\rho+M_H^2}+\frac{4M_t^2-M_H^2}{4M_t^2\rho+M_H^2}\right\}$$

$\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t\bar{t}}$



- Compare with the mass singular terms of full LO result

$$\frac{\alpha_s^2\,y_t^2\,P_T}{36\,\pi\hat{s}^{\,2}}\left\{\log\frac{4P_T^2}{4M_t^2\rho+M_H^2}+\frac{4M_t^2-M_H^2}{4M_t^2\rho+M_H^2}\right\}$$

$$\rho=\frac{P_T^2}{\hat{s}-2\sqrt{\hat{s}}P_T}$$

- All mass singularity is included in $t/\bar{t}$ to Higgs fragmentation contribution.

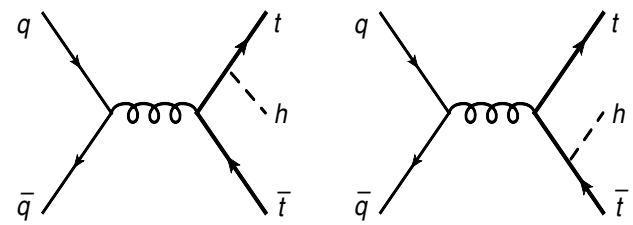
H to H Fragmentation contribution

Apply $D^{(0,0)}_{H\to H} = \delta(1-z)$

$$\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = 2\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_{top} = \hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu) + \boxed{\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H = \hat{P},\mu)}$$

H to H fragmentation contribution

- Rearrange the equation



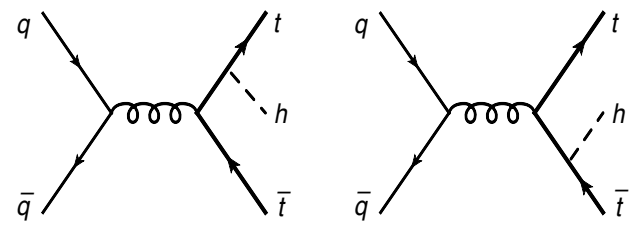
$$\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = \left[\boxed{\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y}} - 2\int \frac{\mathrm{d}z}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p = \hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu)\right]\Bigg|_{M_H=0}$$

H to H Fragmentation contribution

Apply $D^{(0,0)}_{H\to H} = \delta(1-z)$

$$\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to Ht\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = 2\int \frac{dz}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_{top}=\hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu) + \boxed{\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p_H=\hat{P},\mu)}$$

H to H fragmentation contribution



- Rearrange the equation

$$\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = \left[\boxed{\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y}} - 2\int \frac{\mathrm{d}z}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p=\hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu)\right]\Bigg|_{M_H=0}$$

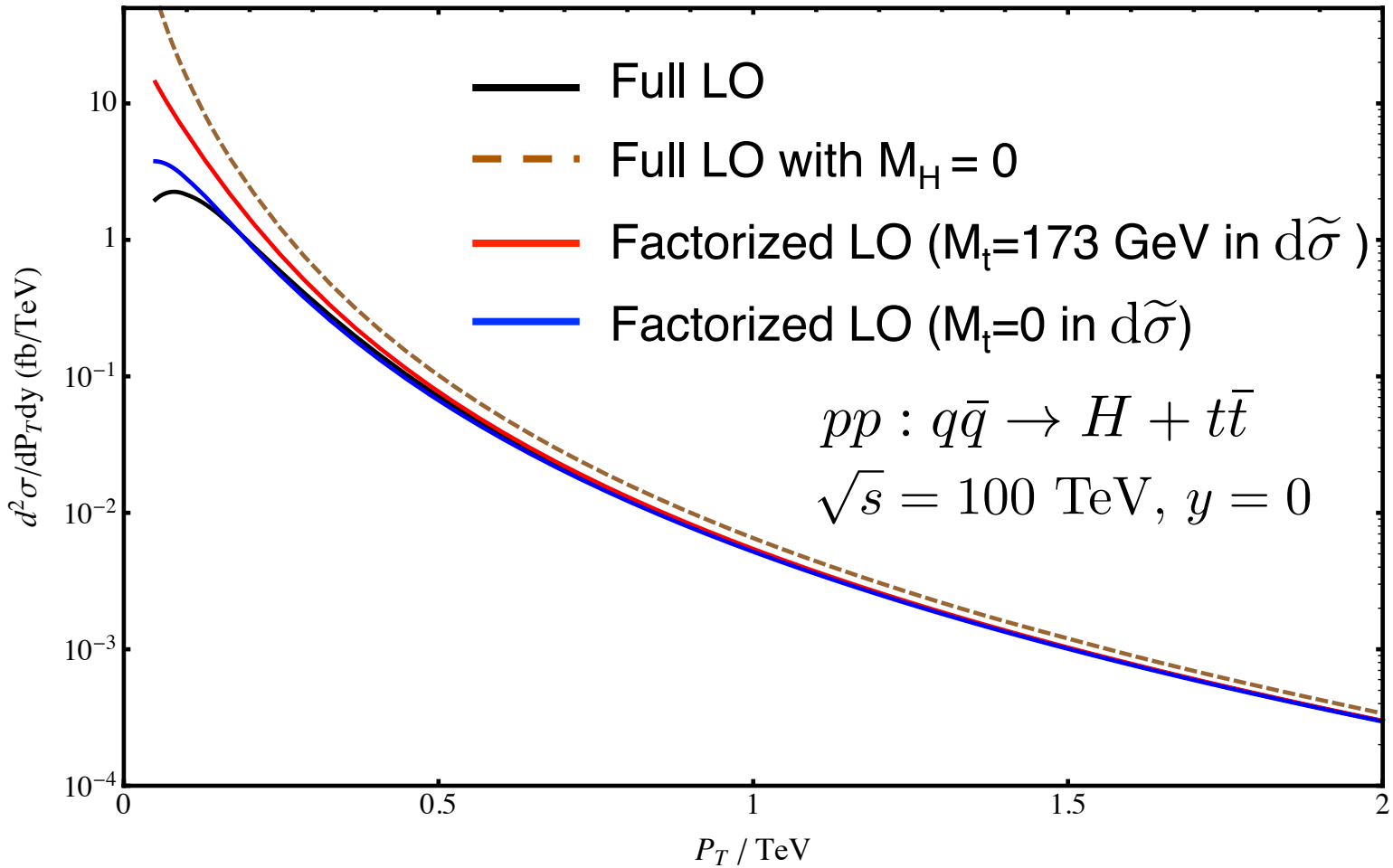
- Key observation: mass singularities are cancelled on the RHS

$$\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y} = \left[\frac{\mathrm{d}^2\widehat{\sigma}^{(2,1)}_{q\bar{q}\to H+t\bar{t}}}{\mathrm{d}P_T\mathrm{d}y}\right]\Bigg|_{M_H=0} - \left[2\int \frac{\mathrm{d}z}{z}\,\frac{\mathrm{d}^2\widetilde{\sigma}^{(2,0)}_{q\bar{q}\to t+\bar{t}}}{\mathrm{d}p_T\mathrm{d}y}(p=\hat{P}/z,\mu)\,D^{(0,1)}_{t\to H}(z,\mu)\right]\Bigg|_{M_H=0}$$

- Greatly simplify the calculation with $M_H = 0$ or even $M_t = 0$

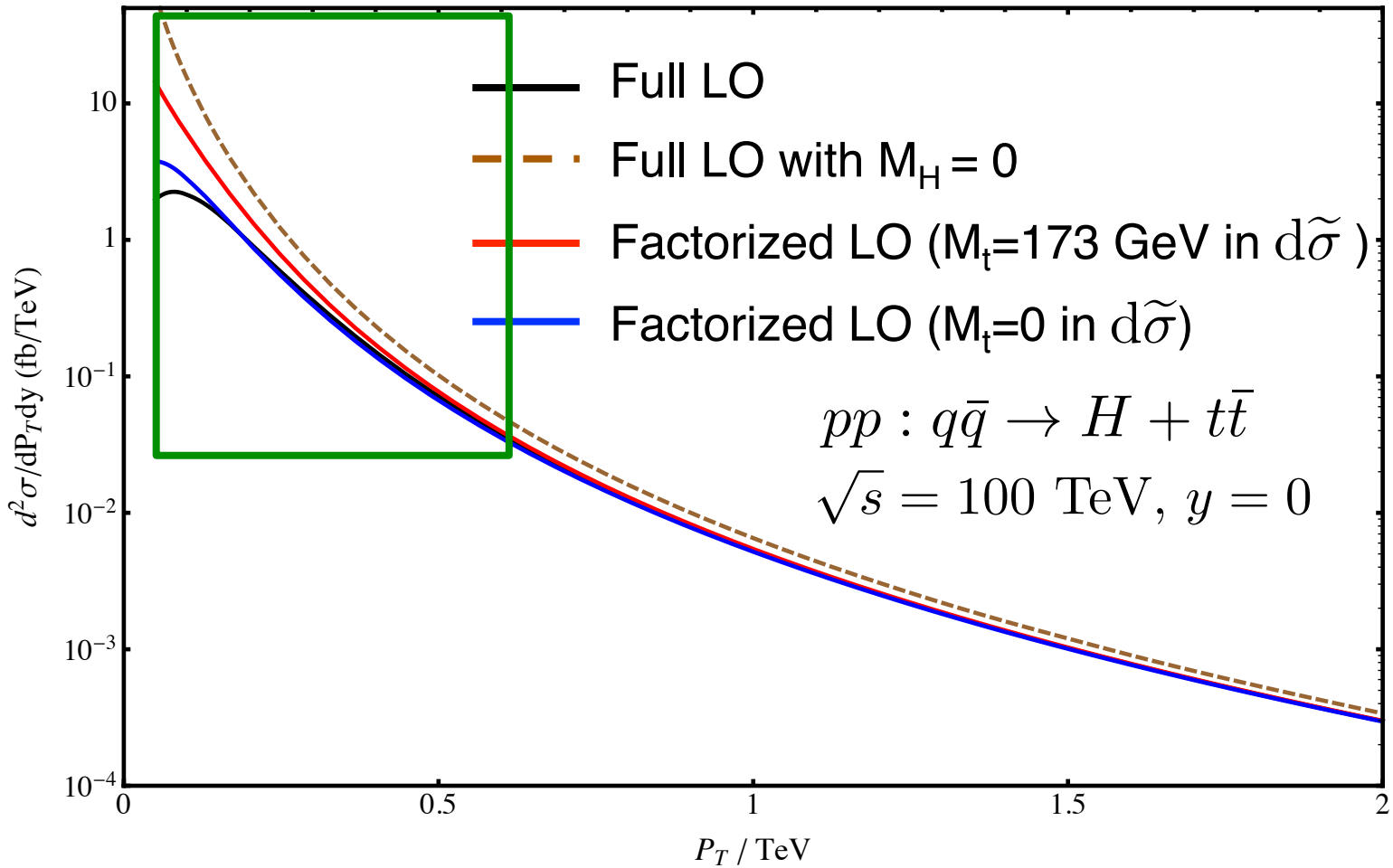
2 pages → A few lines

P_T Distribution



- Factorized LO converges to Full LO at large P_T .
- Setting $M_H = 0$ in Full LO introduces error that does not decrease with P_T .

P_T Distribution



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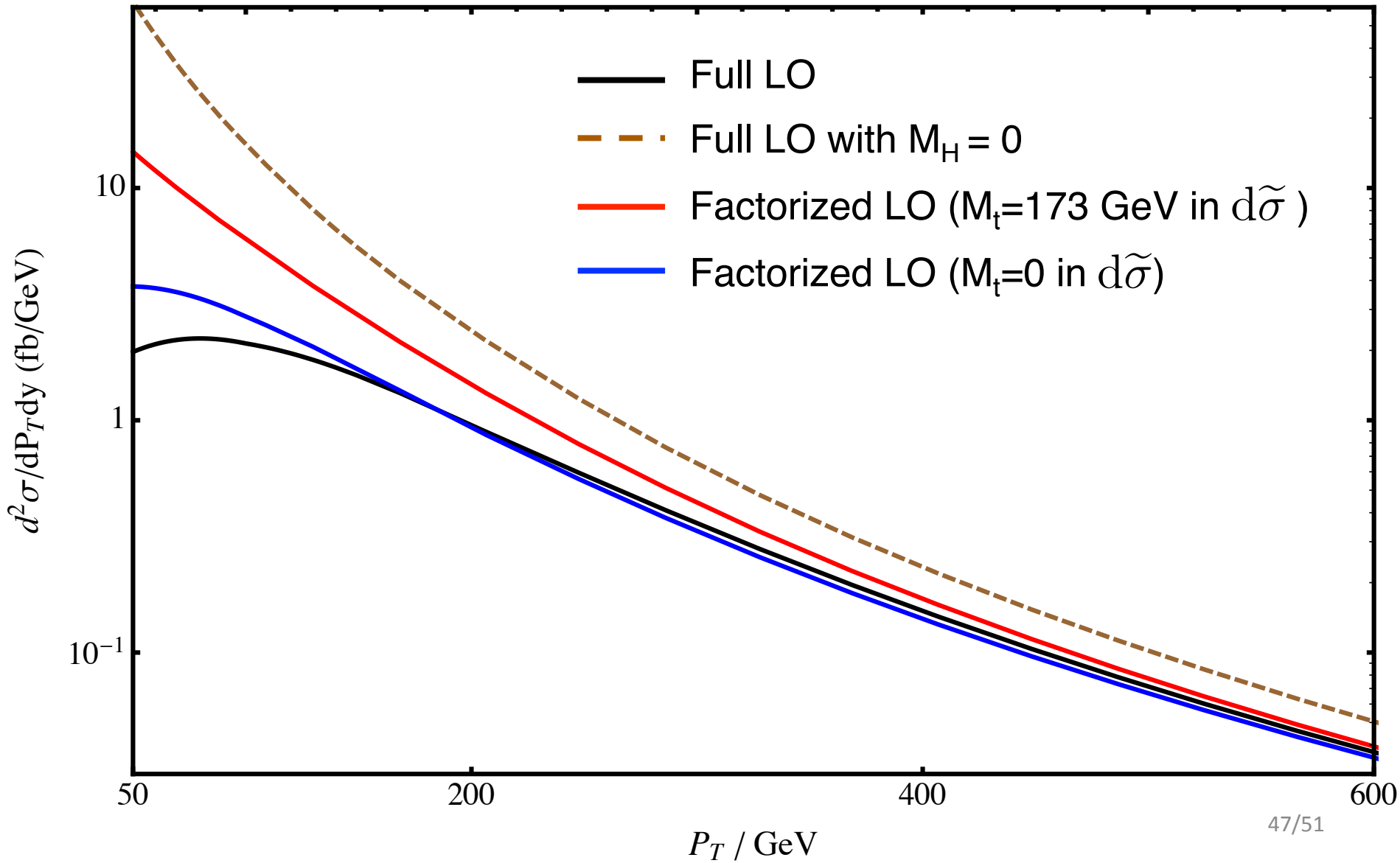
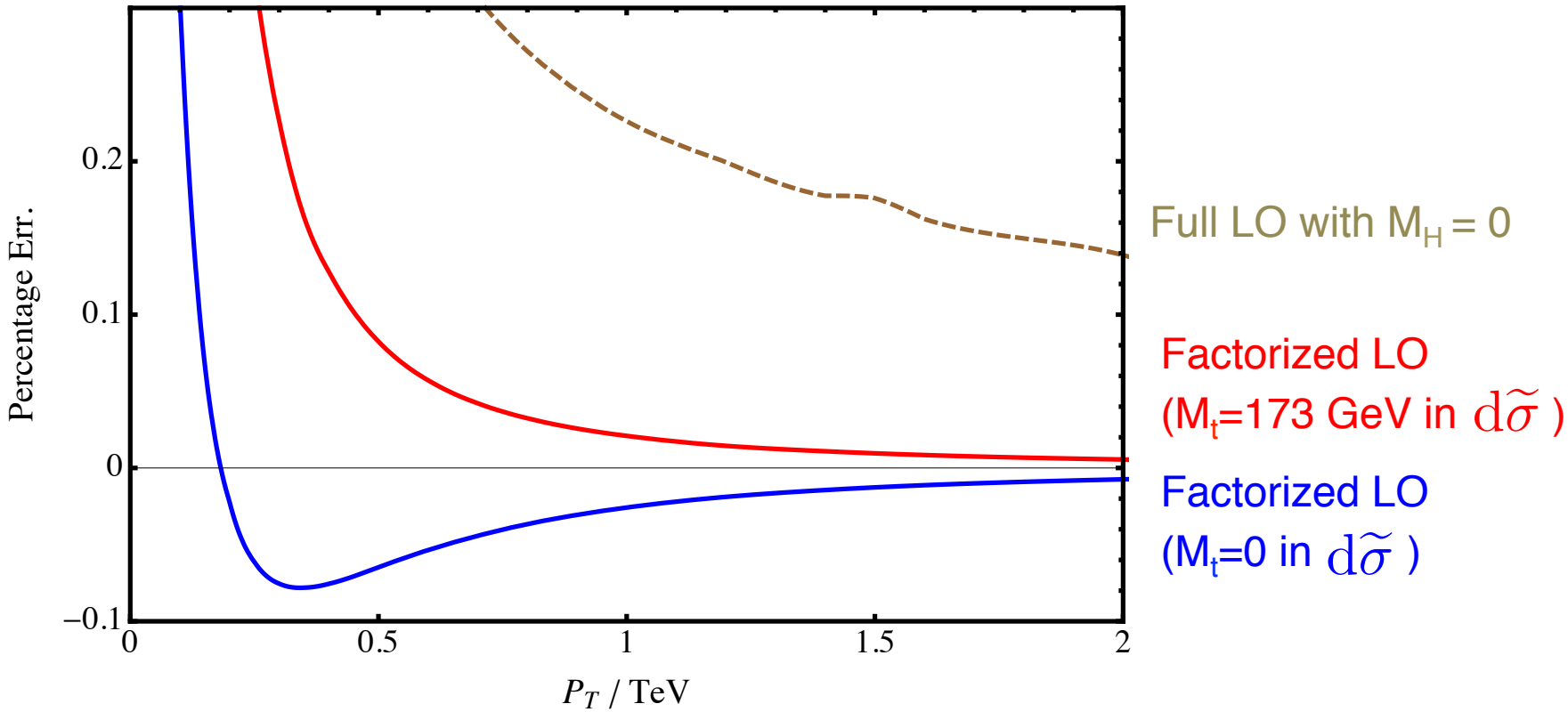
P_T Distribution



Error

$$Err = \frac{Approx. - Full\ LO}{Full\ LO}$$



Error is 5% at $P_T \sim 600\ \mathrm{GeV}$, agrees with naive estimate $\mathcal{O}(M_H^2/P_T^2)$

Keeping physical M_t in $d\widetilde{\sigma}$ has no significant benefit.

Leading Power Factorization

$$\frac{\mathrm{d}^2\widehat{\sigma}_{ab\to H+X}}{\mathrm{d}P_T\mathrm{d}y} = \sum_i \int \frac{dz}{z} \frac{\mathrm{d}^2\widetilde{\sigma}_{ab\to i+X}}{\mathrm{d}p_T\mathrm{d}y}(p_i = \hat{P}/z, \mu)\, D_{i\to H}(z,\mu) + \mathcal{O}(M_H^2/P_T^2),$$

Leading power of M_H^2/P_T^2 has been factorized into

- IR-safe cross section
- Single particle fragmentation function ($i \to H$)

Error is $\mathcal{O}(M_H^2/P_T^2)$

- Decreases with P_T
- Expected to be < 5% at $P_T \geq 600\ \mathrm{GeV}$

Next-to-leading Power Factorization

Next-to-leading power of M_H^2/P_T^2 can also be factorized into

- IR-safe cross section
- Double particle fragmentation function ($ij \to H$)

For quarkonium production, see

Kang, Qiu and Sterman, PRL (2011)
Fleming, Leibovich, Mehen and Rothstein PRD (2012)
Kang, Ma, Qiu and Sterman, PRD (2014)

Error is $\mathcal{O}(M_H^4/P_T^4)$

- Decreases more rapidly with P_T
- Expected to be < 5% at $P_T \geq 250\ \mathrm{GeV}$

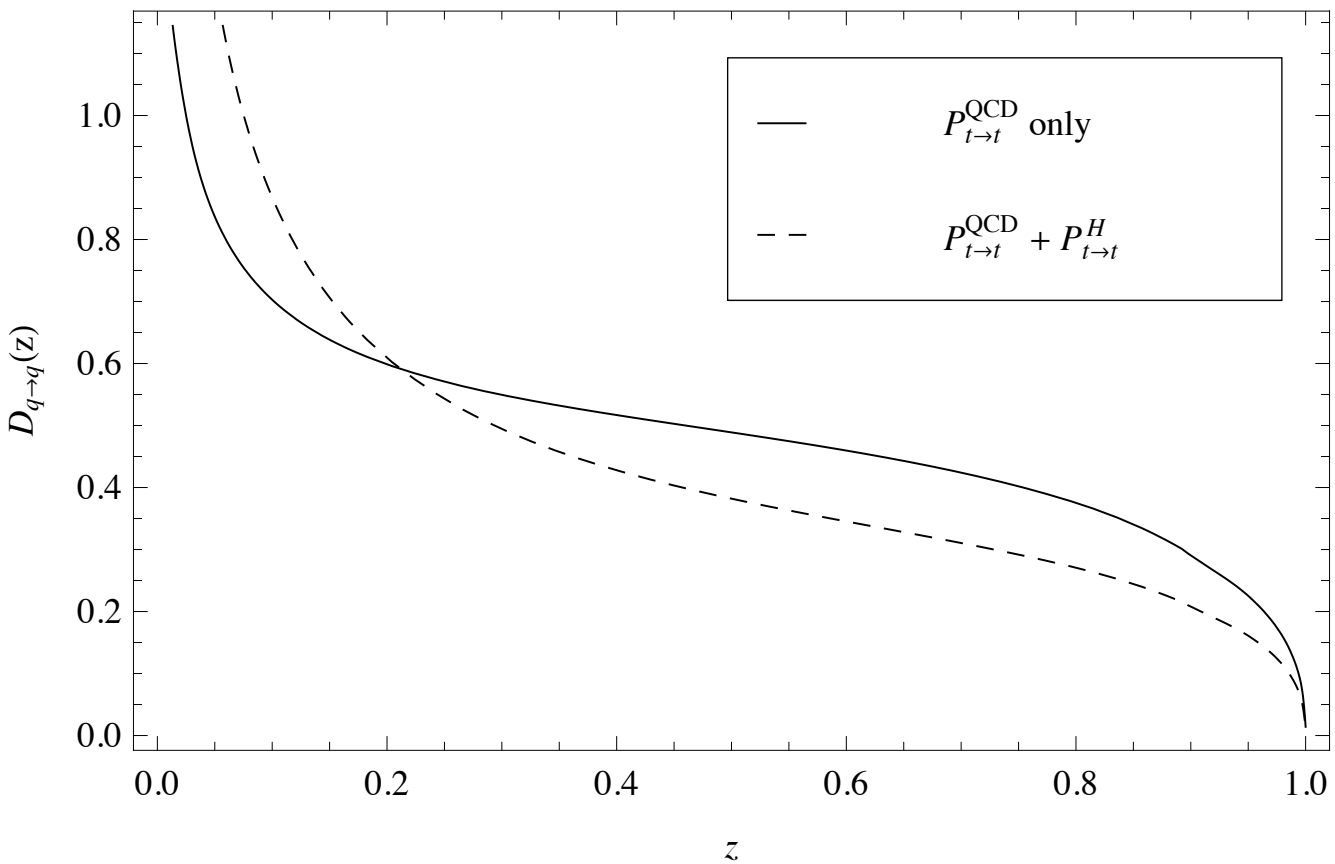
Summary

- Factorization formula is a systematic framework to expand the Higgs P_T distribution by M_H^2/P_T^2 , in spite of mass singularity.
- Hard scales and soft scales are separated into IR-safe cross sections and fragmentation functions, which are much easier to be calculated to higher orders.
- Large logarithms can be summed to all orders. Higgs emission is very effective for heavy quark to lose energy.
- LO calculation of $q\bar{q} \to H + t\bar{t}$ shows the error of leading power factorization formula has error < 5% for $p_T \geq 600\ \mathrm{GeV}$.
- The next-to-leading power factorization formula is expected to have error < 5% for $p_T \geq 250\ \mathrm{GeV}$.
- Same factorization formula can be applied to other processes.

Backup slides

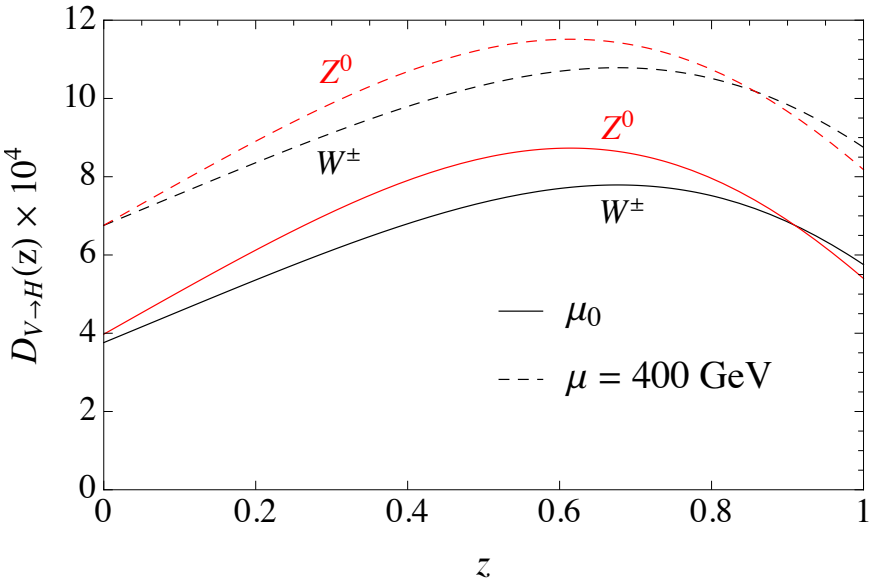
Higgs Emission Effect

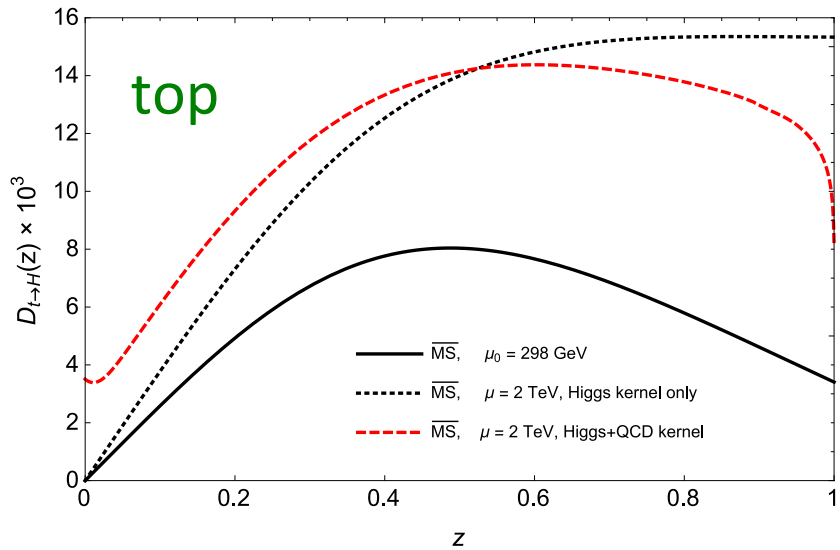
- Higgs emission effect for heavy quark FF $D_{Q\to Q}(z,\mu)$
- Quark mass 1TeV, evolve to $\mu = 5$ TeV with only gluon radiation, and with both gluon and Higgs radiation

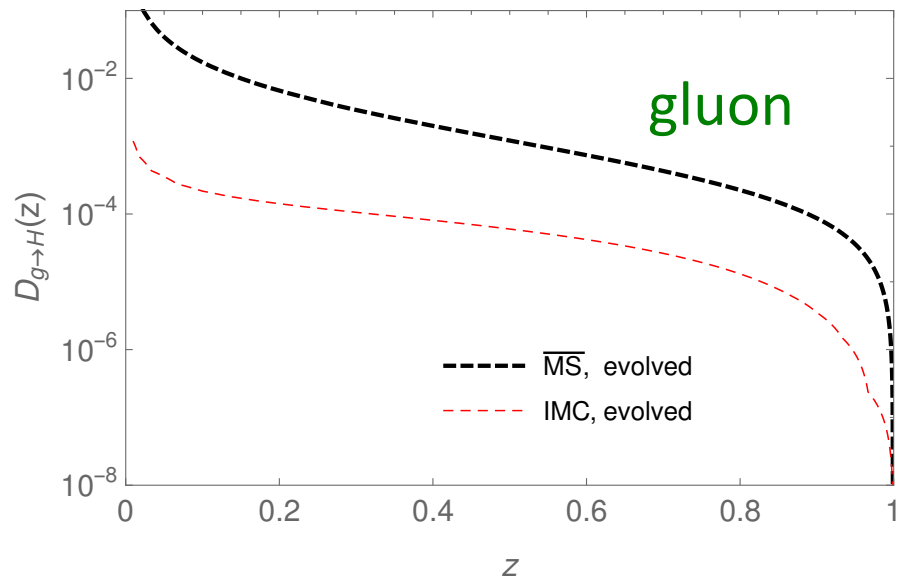


All FFs

- $D_{H\to H}(z,\mu) = \delta(1-z)$







Evolution of top and gluon FF

- Region 1: $M_H + M_t < \mu < M_H + 2M_t$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{t\to H}(z,\mu) = P_{t\to H}(z,\mu) + \int_z^1 \frac{\mathrm{d}y}{y} P_{t\to t}(z/y,\mu) D_{t\to H}(y,\mu).$$

$$D_{g\to H}(z,\mu) = 0$$

- Region 2: $\mu > M_H + 2M_t$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{t\to H}(z,\mu) = P_{t\to H}(z,\mu) + \int_z^1 \frac{\mathrm{d}y}{y} P_{t\to t}(z/y,\mu) D_{t\to H}(y,\mu) + \int_z^1 \frac{\mathrm{d}y}{y} P_{t\to g}(z/y,\mu) D_{g\to H}(y,\mu),$$

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{g\to H}(z,\mu) = \int_z^1 \frac{\mathrm{d}y}{y} P_{g\to g}(z/y,\mu) D_{g\to H}(y,\mu) + 2\int_z^1 \frac{\mathrm{d}y}{y} P_{g\to t}(z/y,\mu) D_{t\to H}(y,\mu).$$