Remnants of a Chiral Universe

or

U(1) symmetries for Dark Matter and Dirac Neutrino Masses

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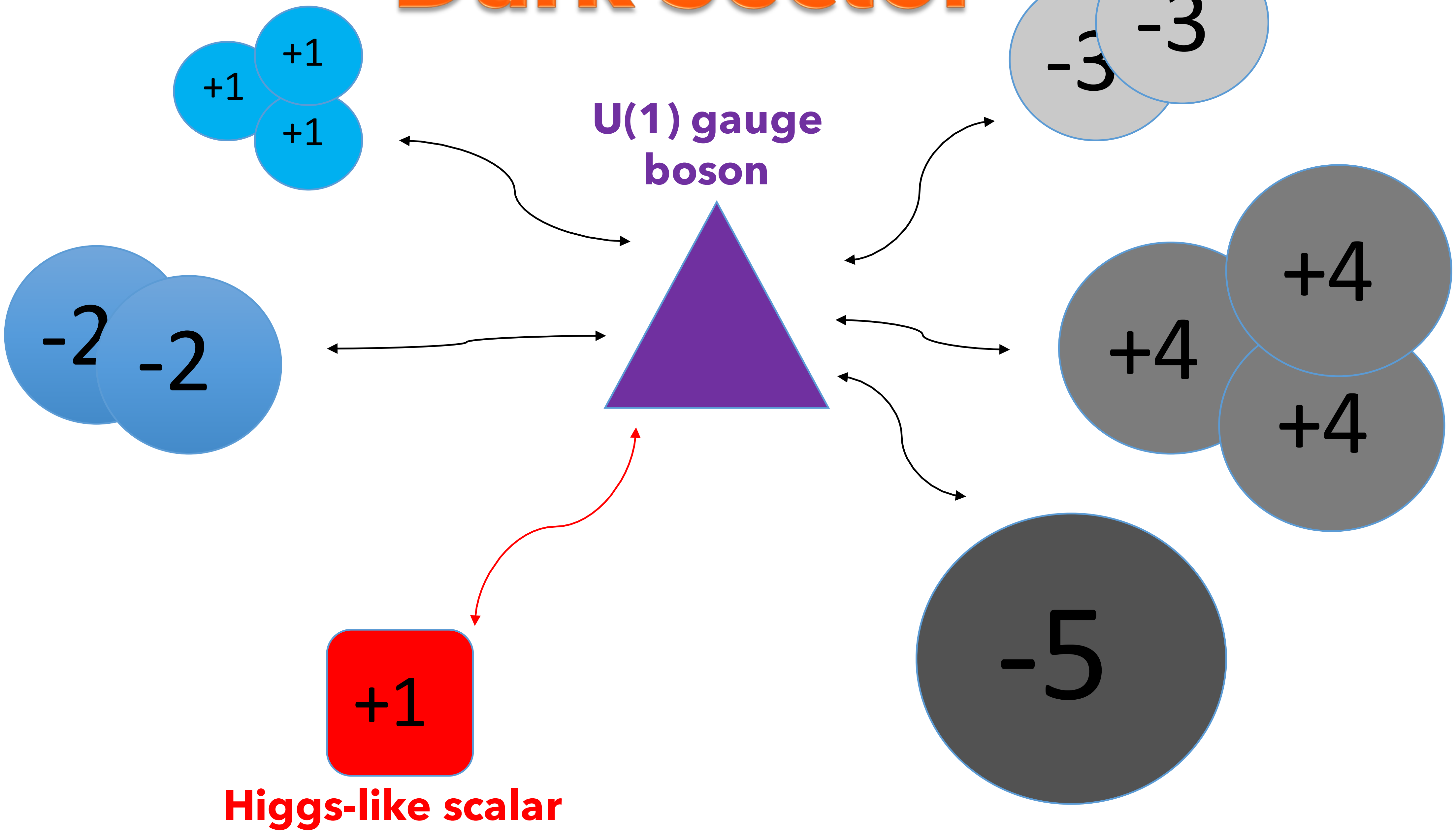
I have a model…

for a Dark Sector…

It is based on a U(1) gauge symmetry

Dark Sector



1-2 sector

- three fields with charge $+1$ – $\mathbf{1}_+^{0,1,2}$;
- two fields with charge -2 – $\mathbf{2}_-^{1,2}$;

3-4-5 sector

- two fields with charge -3 – $\mathbf{3}_-^{1,2}$;
- three fields with charge $+4$ – $\mathbf{4}_+^{0,1,2}$;
- one field with charge -5 – $\mathbf{5}_-^{0}$.

Plus minimal Higgs-like Sector

- **One scalar field φ with charge +1**

General features

Dark Sector Yukawas

$$-\mathscr{L}_{\mathrm{DS-Yuk}} = f_{ik}\, 1^i_+ 2^k_- \phi + h_{i0}\, 4^i_+ 5^0_- \phi + h_{ik}\, 4^i_+ 3^k_- \phi^* + \mathrm{h.c.}$$

Two Dirac fermions.
3 conserved symmetries
One massless particle
Neutrino connection?

Three Dirac fermions.
Mix of fields of different charge (5 and 3).
One conserved symmetry
=> One stable particle (DM candidate)

Dark Matter

$$-\mathscr{L}_{\mathrm{DS-Yuk}} = f_{ik}\,\mathbf{1}^i_+\,\mathbf{2}^k_-\,\phi + h_{i0}\,\mathbf{4}^i_+\,\mathbf{5}^0_-\,\phi + h_{ik}\,\mathbf{4}^i_+\,\mathbf{3}^k_-\,\phi^* + \mathrm{h.c.}$$

- DM can be relatively heavy w.r.t. other Dark Sector states (like a proton in that sense)

Neutrino connection

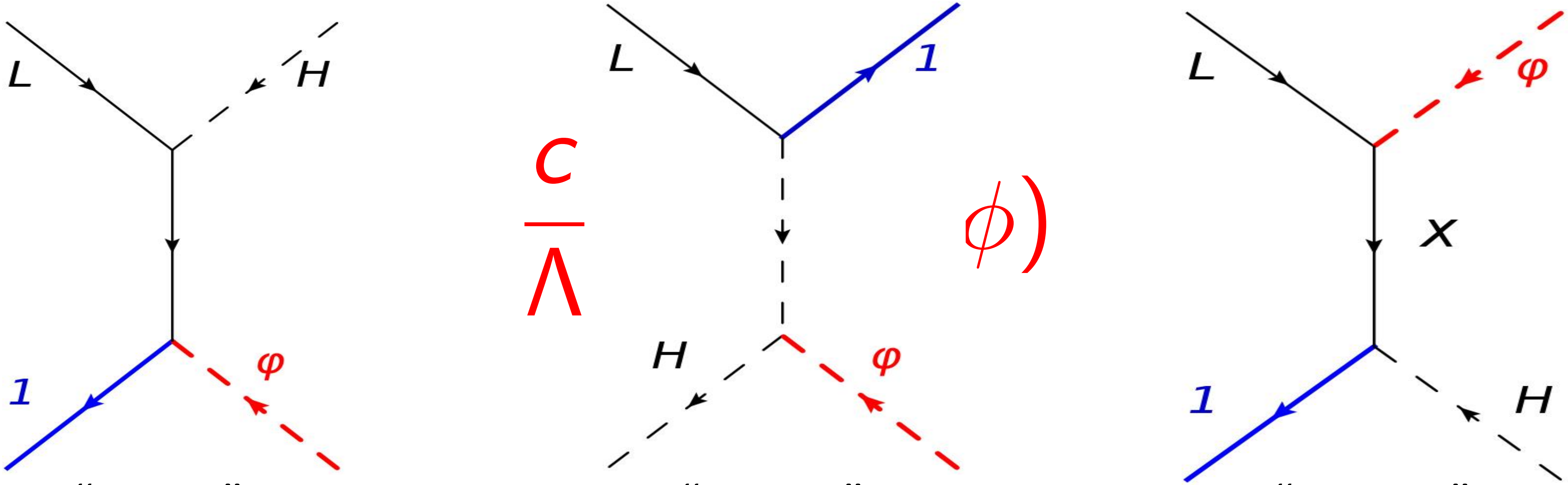
$$-\mathscr{L}_{\mathrm{DS-Yuk}} = f_{ik}\,\mathbf{1}^i_+\,\mathbf{2}^k_-\,\phi + h_{i0}\,\mathbf{4}^i_+\,\mathbf{5}^0_-\,\phi + h_{ik}\,\mathbf{4}^i_+\,\mathbf{3}^k_-\,\phi^* + \mathrm{h.c.}$$

Higher dimensional operators made out of the DS fields and the SM fields?

$$\frac{c}{\Lambda}(\mathbf{1}_+ L)(H\phi)$$

Dirac neutrino mass

Requirements: Mediator sector that ties the DS and the SM. It must break SM accidental LN symmetries and the 3 accidental DS 12-symmetries.



“Type I” “Type II” “Type III”

"Type III"

X is a Dirac particle charged under both SM SU(2)xU(1) and DS U(1).

$$\mathscr{L}_{\mathrm{Med}} = i\bar{X}\not{D}X + \Lambda\bar{X}X - \left(\kappa_L \bar{L}_L X_R \phi + \kappa_i \bar{\mathbf{1}}^{i}_{R} X_L \tilde{H} + \mathrm{h.c.}\right)$$

Dirac mass

Charged "portal"

But much is independent of the precise Lagrangian!

Neutrino mass

Dirac neutrino mass

$$\frac{\kappa_L \kappa_i}{\Lambda} \left(\mathbf{1}^i_+ L\right) \left(H\phi\right) + \mathrm{h.c.} \qquad m_i^D = \frac{\kappa_L \kappa_i v v_\phi}{\Lambda}$$

$$\begin{array}{cc} & \begin{array}{ccc} \nu_L & 2^1_L & 2^2_L \end{array} \\ M_\nu = & \begin{pmatrix} m_0^D & 0 & 0 \\ m_1^D & M_1 & 0 \\ m_2^D & 0 & M_2 \end{pmatrix} \begin{array}{c} 1^0_R \\ 1^1_R \\ 1^2_R \end{array} \end{array}$$

SM active neutrino mix with charge 2 states!

"Sterile"-Gauge interactions

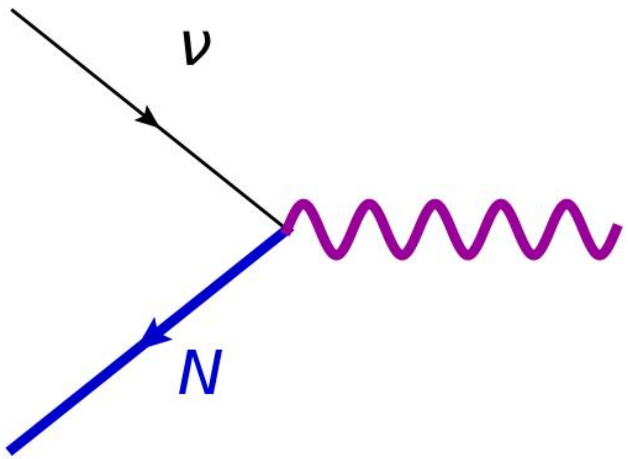
Neutral current

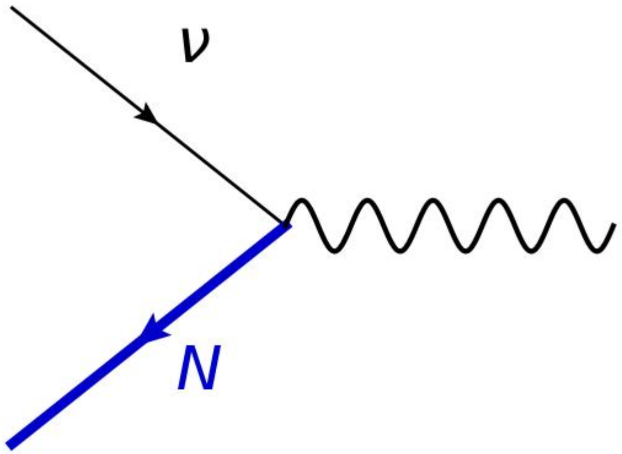
Mixing angle

Mass eigenstates

$$-\frac{m_1^D}{M_1}\bar{\nu}_L\left(\frac{g}{2c_W}\not{Z}-2g_\nu\tilde{\not{Z}}\right)N_{1L}-\frac{m_2^D}{M_2}\bar{\nu}_L\left(\frac{g}{2c_W}\not{Z}-2g_\nu\tilde{\not{Z}}\right)N_{2L}+\text{h.c.}\,,$$

Int. with both DS
and SM gauge boson

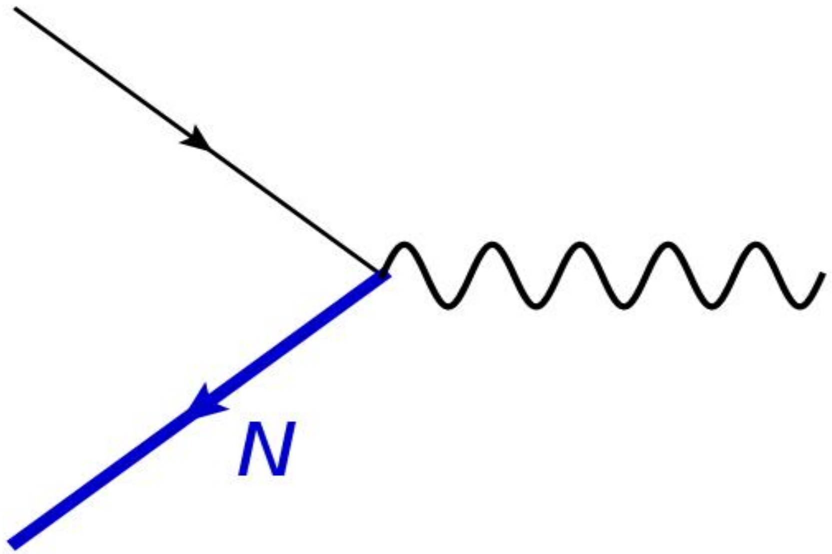




"Sterile"-Gauge interactions

Charged current

$$\frac{g}{\sqrt{2}}\left(-\frac{m_1^D}{M_1}\bar{\ell}_L \not{W}^- N_{1L} - \frac{m_2}{M_2^D}\bar{\ell}_L \not{W}^- N_{2L}\right) + \text{h.c.}\,,$$



"Sterile"-Gauge interactions

The Ns only decay into SM!

$$N_k \to Z\nu \qquad\qquad N_k \to \ell W$$

$$\Gamma_{N_1 \to SM} \sim N_{ch} \frac{G_F (m_1^D)^2 M_1}{8\pi\sqrt{2}} + \mathcal{O}\left(\frac{M_W^2}{M_1^2}\right)$$

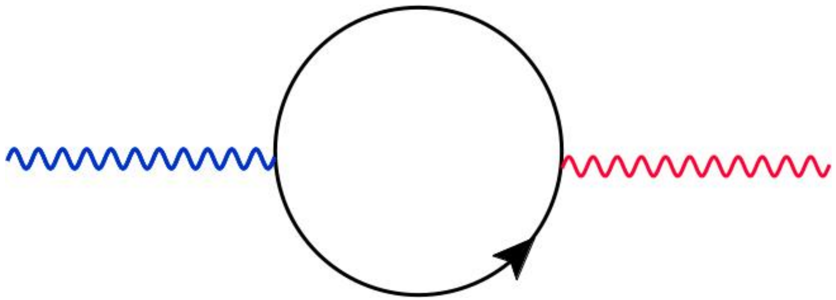
For $M_N \sim 1\ \mathrm{TeV} \quad \Rightarrow \Gamma_{N_1 \to \ell W} \sim 0.5\ \mathrm{s}^{-1}$

Important for cosmology. More on that later.

Kinetic mixing

$$\mathscr{L}_{\text{Kin-Mix}} = -\frac{\sin\eta}{2} B^{\mu\nu} \tilde{B}_{\mu\nu}$$

From quantum corrections:



$$\Delta \sin\eta \sim C_\eta \frac{g' g_\nu}{16\pi^2} \log\left(\frac{\Lambda}{\mu}\right)$$

$$\eta \sim 10^{-2} - 10^{-3}$$ ***is "natural"***

Babu, Kolda, March-Russell; 1998

$$M_Z^2 \ll M_{\tilde{Z}}^2$$

(DS – SM boson) interactions

$$g_\nu s_W \frac{M_Z^2}{M_{\tilde{Z}}^2} \sin\eta \left(\bar{\nu^c}\bar{\sigma}_\mu \nu^c - \bar{N}_R \gamma^\mu N_R - 2\bar{N}_L \gamma^\mu N_L - 4\bar{\chi}_R \gamma^\mu \chi_R + \bar{\chi}_L \mathcal{Q}_{35} \gamma^\mu \chi_L\right) Z_\mu$$

(SM – DS boson) interactions

$$-\frac{e \sin\eta}{c_W} \tilde{Z}_\mu \sum_f Y^{(f)} \bar{f} \gamma^\mu f$$

No DS – photon interaction!

Below DS U(1) breaking

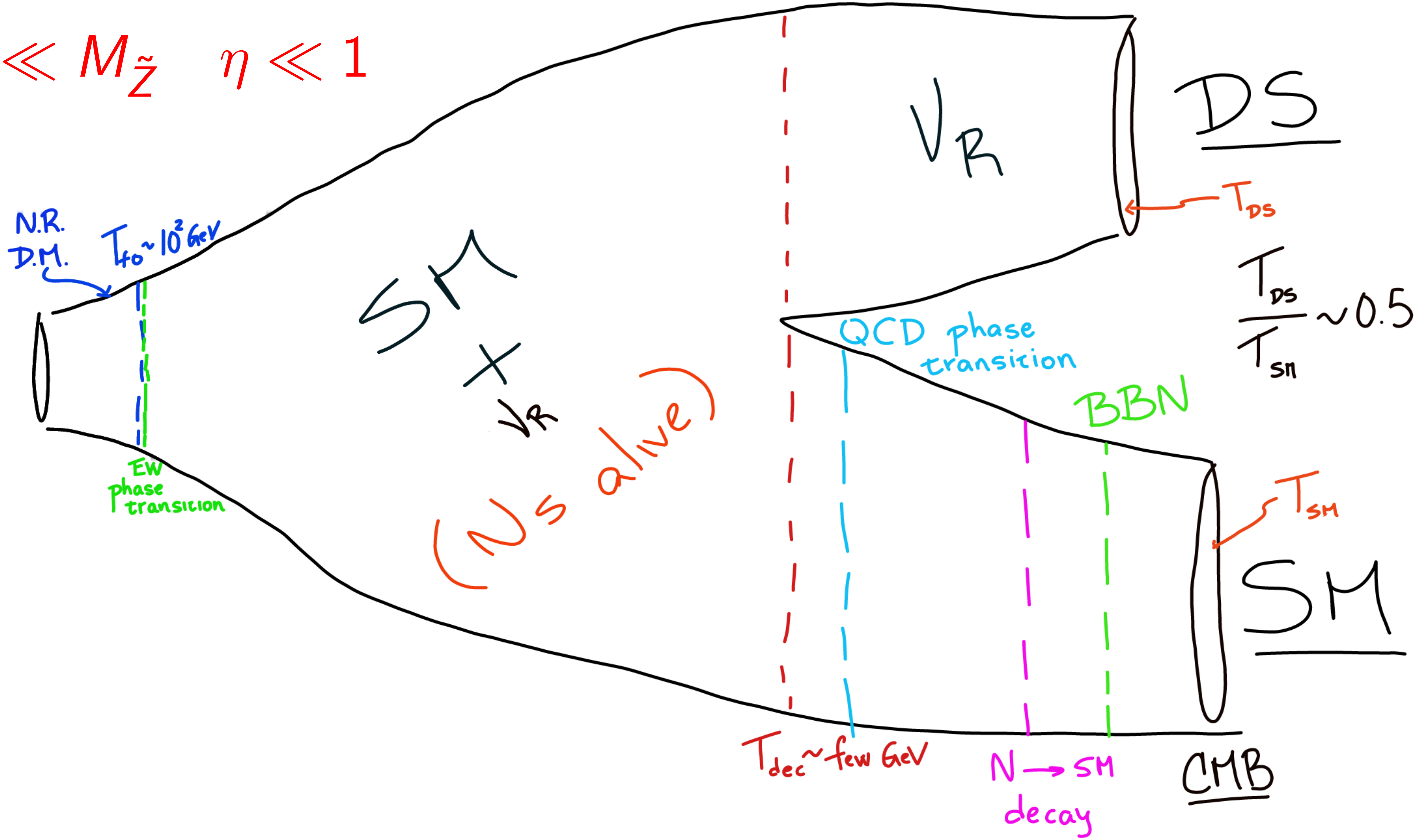
- **A stable Dirac particle that is a Dark Matter candidate** – lowest mass particle in the 345 sector:

- **Two heavy long-lived "neutrinos"** (w.r.t. the SM) that decay into SM particles: N_1, N_2

- **One light Dirac neutrino per generation**

Phenomenological bounds

$M_Z \ll M_{\tilde{Z}} \quad \eta \ll 1$



Cosmology

Dark matter abundance set by annihilation to Dark Sector RH neutrinos

$$\langle \sigma_{\rm ann} v \rangle \sim \sigma(\chi_1 + \bar{\chi}_1 \to \nu^c + \bar{\nu^c}) \sim N_f Q_\chi^2 \frac{g_\nu^4}{8\pi M_{\chi_1}^2}$$

$$\langle \sigma_{\rm ann} v \rangle \sim 3 \times 10^{-2}\ {\rm pb} \times \left(\frac{1\ {\rm TeV}}{M_{\chi_1}}\right)^2 \left(\frac{N_f Q_\chi^2}{20}\right) \left(\frac{g_\nu}{0.1}\right)^4$$

Relic density

$$\Omega_{\chi 0} = \frac{\rho_{\chi 0}}{\rho_{\rm cr}}$$

$$\Omega_{\chi 0} \sim \sqrt{\frac{4\pi^3 G \mathfrak{g}^*(x=1)}{45}} \frac{x_{\rm fo} T_0^3}{\langle \sigma_{\rm ann} v \rangle \rho_{\rm cr}} \frac{\mathfrak{g}^*_{\rm dec}}{\mathfrak{g}^*_{\infty}}$$

d.o.f. before decoupling

$$\Omega_{\chi 0} \sim \frac{10^{-2}}{\langle \sigma_{\rm ann} v \rangle}\,\text{pb.}$$

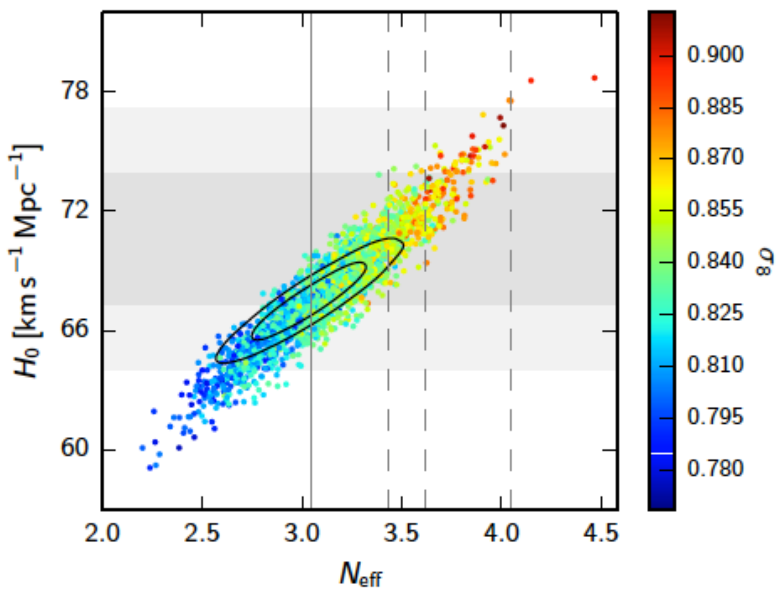
Decoupling temperature

DS should decouple before the QCD phase transition! (or else entropy deposited in the DS implies more relativistic d.o.f.s)

$$\frac{T_{\nu_L}^{\mathrm{dec}}}{T_{\mathrm{SM-DS}}^{\mathrm{dec}}} \sim \left(\frac{G_\nu G_F}{G_F^2}\right)^{1/3} \sim \left(\frac{M_Z g_\nu}{M_{\tilde{Z}} g} \sin\eta\right)^{2/3}$$

$$\sin\eta \lesssim 10^{-3}$$

Strongest bound on kinetic mixing



Neff

$$r_0 \equiv \frac{T_0}{\tilde{T}_0} \lesssim 0.56$$

$$\Delta N_{\rm eff} = \frac{4\tilde{g}^*}{7}\frac{\tilde{T}^4}{T^4} = \frac{4\tilde{g}^*}{7}\left(\frac{g^*\,\tilde{g}^*_{\rm dec}}{g^*_{\rm dec}\,\tilde{g}^*}\right)^{4/3}$$

d.o.f.s after decoupling

$$\Delta N_{\rm eff} = 0.33$$

PLANCK : $N_{\rm eff} = 3.15 \pm 0.46$

DM Direct Detection

DM – nucleus cross section

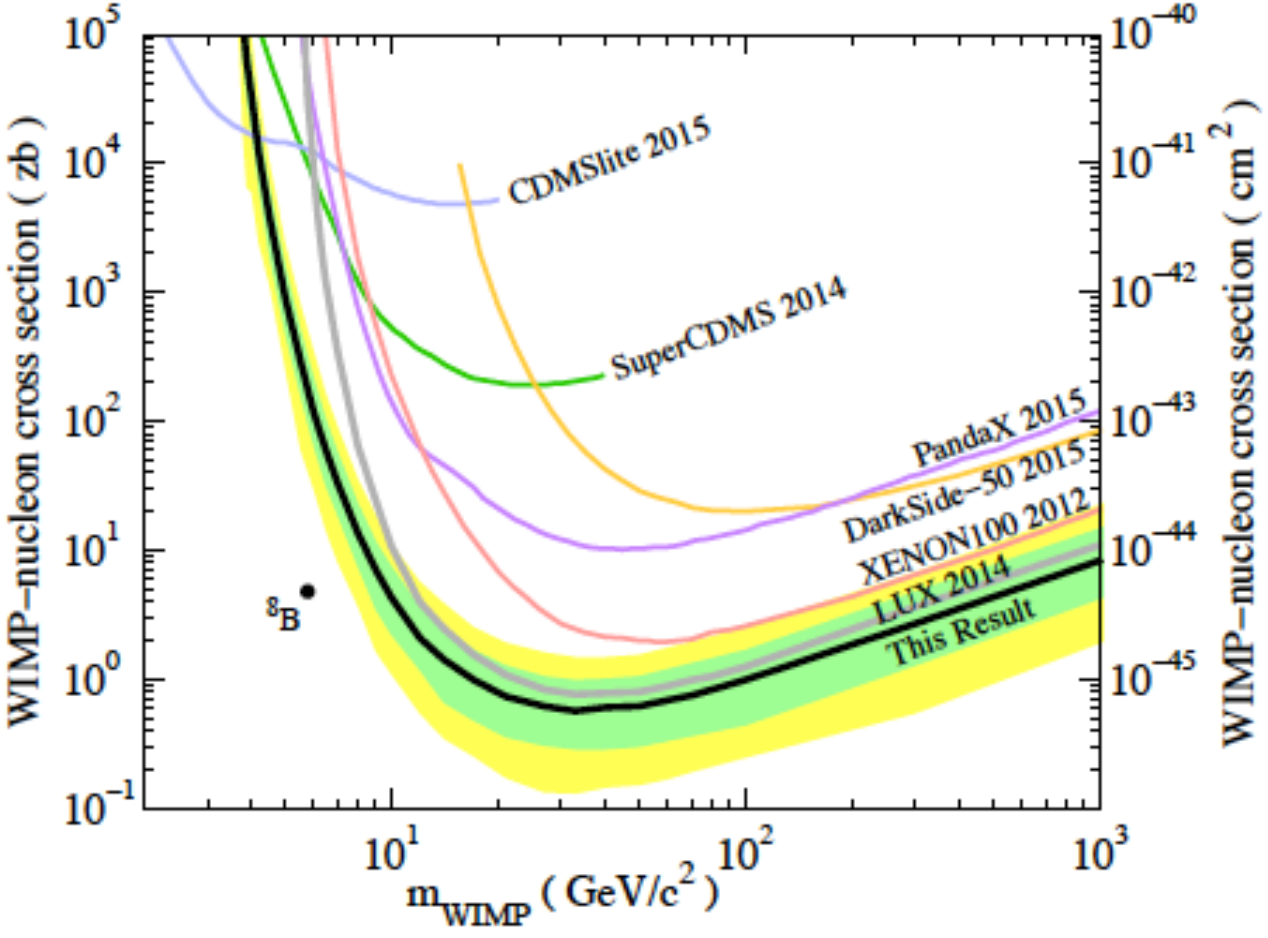
$$\sigma(\chi_1 + N \to \chi_1 + N) \sim \alpha \frac{m_N^2}{M_{\tilde{Z}}^4} \frac{g_\nu^2 \sin^2 \eta \, Q_V^2}{\cos^2 \theta_W} (1 - \sin^2 \theta_W)^2 Z^2$$

Protons are the main contribution.
For the standard WIMP, the main contribution is off neutrons

Goodman, Witten

$49 \leq Q_V^2 \leq 64$

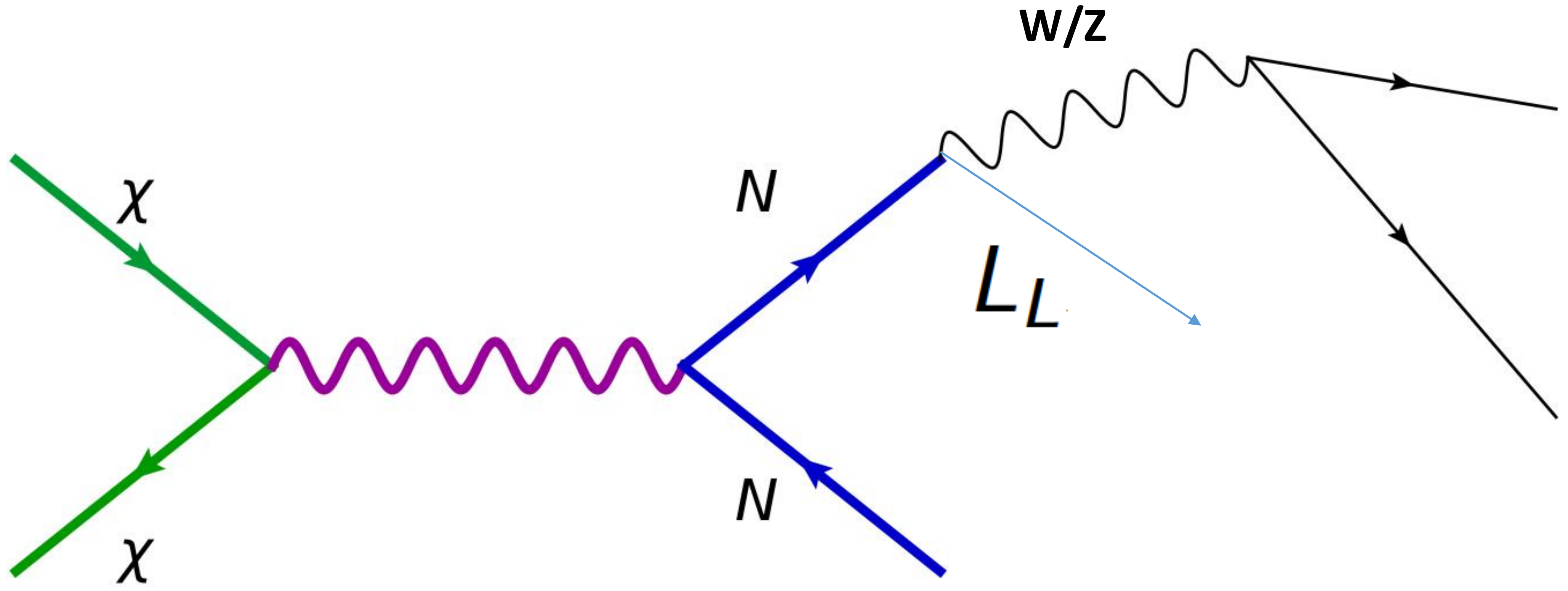
$$\sigma_{\chi p} = 1.2 \times \sin^2 \eta \left(\frac{g_\nu^2}{10^{-2}} \right) \frac{Q_V^2}{50} \times 10^{-41}\ \mathrm{cm}^2$$



LHC?

- Standard searches apply. (Missing energy, monojets, dijets and so on)
- Rates are very small
- Not to be discouraged! Remember freedom to play with the parameters.
- Besides, keep listening.

Indirect Detection?



Low rate due to small mixing angles

A reflection

Ok, so what?

I have a model…

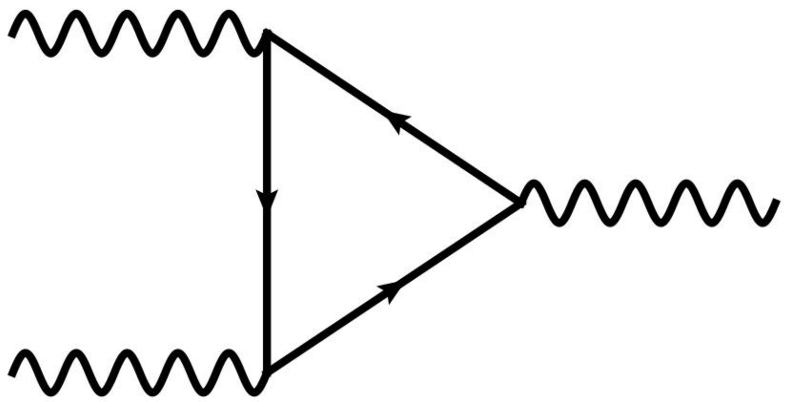
That is chiral.

Anomalies.

This is as complicated as the SM

ANOMALIES

Gauge anomalies signal a breaking of the gauge symmetry at the quantum level. Not acceptable!



For purely *U(1)* gauge theories

Gauge-gravitational anomaly $\sum_{i=1} q_i = 0\,,$ $\sum_{i=1} q_i^3 = 0$ Cubic U(1) anomaly

Fermat's Last Theorem?

- No solution for ***n* = 3**
- For ***n* = 4**, only trivial, solutions ***q1* = -*q2*, *q3* = -*q4***
- At ***n* = 5**, things get interesting

$$\mathbf{10}\times 1\,,\quad -\mathbf{9}\times 1\,,\quad -\mathbf{7}\times 1\,,\quad \mathbf{4}\times 1\,,\quad \mathbf{2}\times 1$$

- The Standard Model

$$\sum q_i = 0\,,\qquad \sum q_i^3 = 0$$

$$-\mathbf{6}\times 1,\quad \mathbf{4}\times 3,\quad \mathbf{3}\times 2,\quad -\mathbf{2}\times 3,\quad \mathbf{1}\times 6$$

$$\sum_{i=1} q_i = 0\,, \qquad \sum_{i=1} q_i^3 = 0$$

How to build an anomaly-free chiral model?

Other model builders have their recipes

Babu, Seidl; 2004
Batra, Dobrescu, Spivak; 2006

Here's a different way

Take 5 numbers

$a \quad b \quad c \quad d$

Combine them with plus and minus signs

$\pm\, a \;\pm\; \pm\; \pm\, c \;\pm\; d$

32 ways to do so

If *X* is present, strike off -*X*

$a + b + c - d + e$ ~~$-a - b - c + d - e$~~

…

…

Only 16 are left!

$$q_i = \left[\begin{array}{l} a+b+c+d+e, \ -a+b+c+d-e \\ a-b+c+d-e, \ -a-b+c+d+e \\ a+b-c+d-e, \ -a+b-c+d+e \\ a-b-c+d+e, \ -a-b-c+d-e \\ a+b+c-d-e, \ -a+b+c-d+e \\ a-b+c-d+e, \ -a-b+c-d-e \\ a+b-c-d+e, \ -a+b-c-d-e \\ a-b-c-d-e, \ -a-b-c-d+e \end{array} \right\}$$

$$\sum_{i=1}^{16} q_i = 0\,, \qquad \sum_{i=1}^{16} q_i^3 = 0$$

Why this works?

Some group theory

For a group $\mathcal{G}$

$$g \in \mathcal{G} \qquad g = e^{i\alpha \cdot T}$$

T^i are the generators

$$Tr[\{T^i, T^j\}T^k] = 0$$ → ***The group is anomaly-safe***

Georgi, Glashow, 1972

Some group theory

$SU(2),\ SO(N \neq 6),\ G2,\ E_7,\ E_8$ are anomaly free

- Complex conjugate representation

  If {T} forms a representation, then {-T*} is a representation as well.

- A representation is **real** if

$$T = U(-T^*)U^\dagger$$

- <u>Chirality</u> → $T \neq U(-T^*)U^\dagger$ <u>Complex representations</u>

Some group theory

$SU(2),\ SO(N \neq 6),\ G2,\ E_7,\ E_8$ are anomaly free

+

Must have complex representations

⬇

$$SO(4n+2),\ n \geq 2$$

Some group theory

Smallest group: $SO(10)$

Cartan subalgebra: Maximal set of generators that can be diagonalized simultaneously

1. *Pick a complex representation*

2. *Write a generic element of the Cartan subalgebra for that rep.*

3. *Find the eigenvalues. These are a solution to the U(1) anomaly equations*

In the 16 representation

$$
\begin{aligned}
H(a,b,c,d,e) = \frac{1}{N}\mathrm{diag}\{&a+b+c+d+e,\ -a+b+c+d-e,\\
&a-b+c+d-e,\ -a-b+c+d+e,\\
&a+b-c+d-e,\ -a+b-c+d+e,\\
&a-b-c+d+e,\ -a-b-c+d-e,\\
&a+b+c-d-e,\ -a+b+c-d+e,\\
&a-b+c-d+e,\ -a-b+c-d-e,\\
&a+b-c-d+e,\ -a+b-c-d-e,\\
&a-b-c-d-e,\ -a-b-c-d+e\}
\end{aligned}
$$

$$
\begin{aligned}
H(a,b,c,d,e) = \frac{1}{N}\mathrm{diag}\{ & a+b+c+d+e,\ -a+b+c+d-e, \\
& a-b+c+d-e,\ -a-b+c+d+e, \\
& a+b-c+d-e,\ -a+b-c+d+e, \\
& a-b-c+d+e,\ -a-b-c+d-e, \\
& a+b+c-d-e,\ -a+b+c-d+e, \\
& a-b+c-d+e,\ -a-b+c-d-e, \\
& a+b-c-d+e,\ -a+b-c-d-e, \\
& a-b-c-d-e,\ -a-b-c-d+e\}
\end{aligned}
$$

Integer charges → 0, 2 or 4 half integers among *a, b, c, d, e*

Some cute things

- If *a, b, c, d* or *e* vanish, *H(a,b,c,d,e) = -H(a,b,c,d,e)*. **NOT CHIRAL!**

Hence we MUST take all of them nonvanishing

Moreover

$$H(-a,b,c,d,e) = H(a,-b,c,d,e) = H(a,b,-c,d,e)$$
$$= H(a,b,c,-d,e) = H(a,b,c,d,-e) = -H(a,b,c,d,e)$$

Hence we can take all of them positive

- **Highest charged particle is ALWAYS a singlet of everything else (This would be the RH electron in the SM)**

$$
\begin{aligned}
H(a,b,c,d,e) = \frac{1}{N}\mathrm{diag}\{&a+b+c+d+e,\ -a+b+c+d-e,\\
&a-b+c+d-e,\ -a-b+c+d+e,\\
&a+b-c+d-e,\ -a+b-c+d+e,\\
&a-b-c+d+e,\ -a-b-c+d-e,\\
&a+b+c-d-e,\ -a+b+c-d+e,\\
&a-b+c-d+e,\ -a-b+c-d-e,\\
&a+b-c-d+e,\ -a+b-c-d-e,\\
&a-b-c-d-e,\ -a-b-c-d+e\}
\end{aligned}
$$

Some cute things

- Model with minimal highest charge:

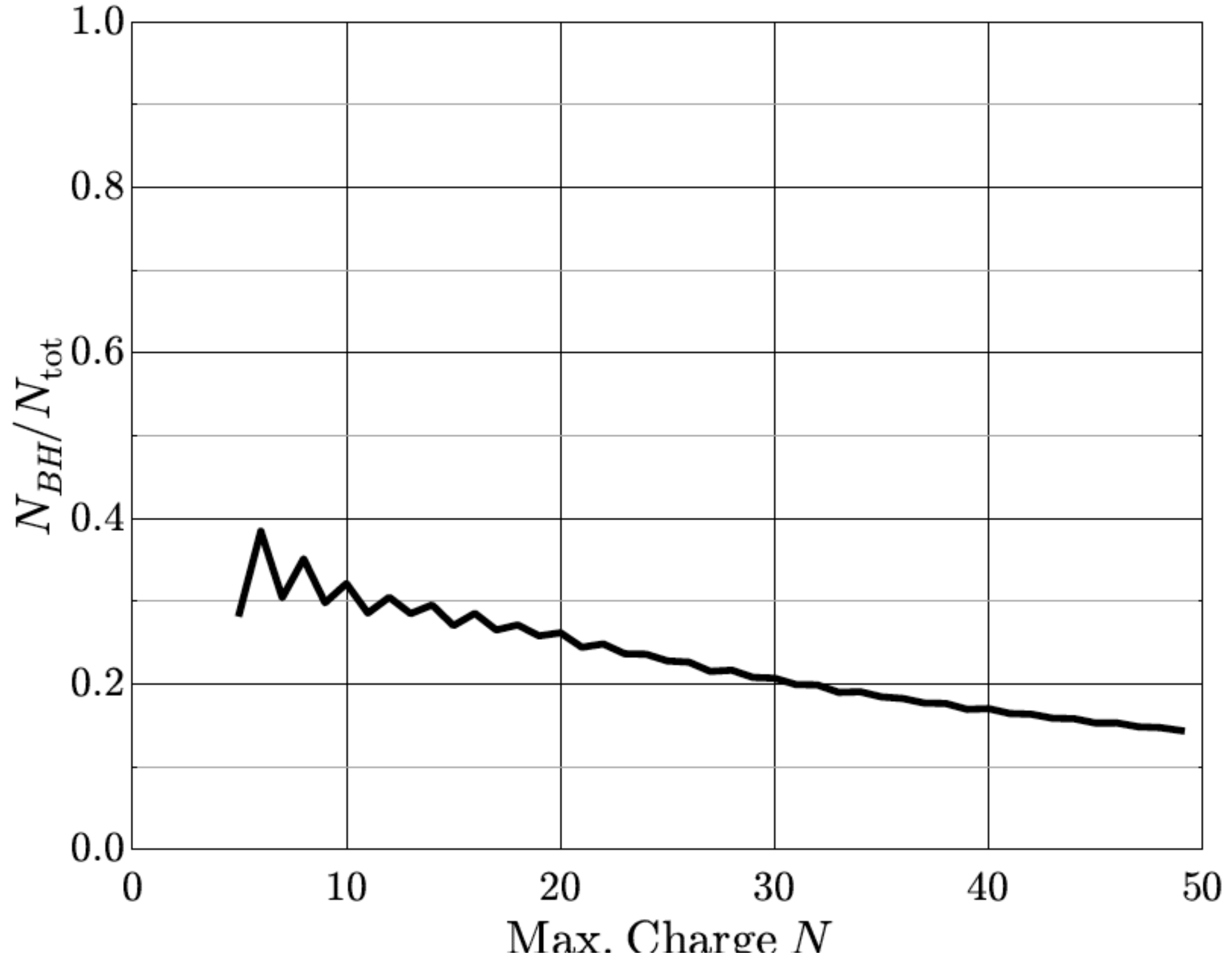
$$\mathbf{3}\times 1\,,\quad -\mathbf{2}\times 4\,,\quad \mathbf{1}\times 5$$

- Minimal number of particles: 5

$$\mathbf{10}\times 1\,,\quad -\mathbf{9}\times 1\,,\quad -\mathbf{7}\times 1\,,\quad \mathbf{4}\times 1\,,\quad \mathbf{2}\times 1$$

- Does this method generates all possible solutions to the anomaly equations under some conditions?

One Higgs -> mass to all?



What is special about the SM?

SUMMARY

- Light fermions are chiral (hypothesis) and gauge charged
- A recipe (not unique) to build chiral models
- Very generally, Dark Matter candidate present
- Very generally, Dirac neutrinos with naturally small masses are present
- Cosmology bounds, direct detection, etc, not hard to pass.

AVENUES to EXPLORE

- Other models? Simplicity as a paradigm?
- Phenomenology
- Dark Matter asymmetry
- Higgs sector?
- Non abelian symmetries?