Precise Predictions for the LHC

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PRISMA

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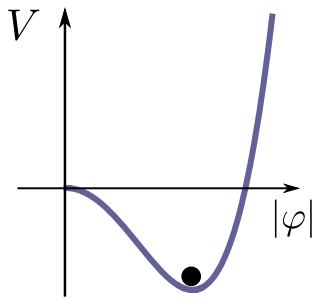
The Higgs mechanism

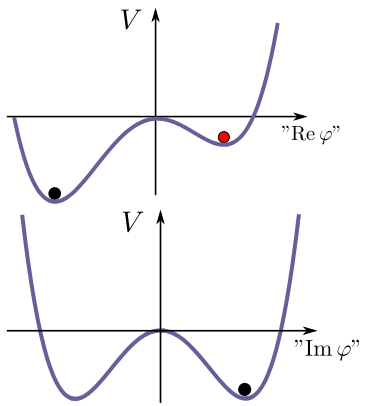
Higgs discovery by ATLAS & CMS (2012): a triumph for collider physics

Standard Model: complex Higgs doublet $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$



- $V = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2, \quad \mu^2 > 0, \ \lambda > 0$
- EWSB: $\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad v = \frac{\mu^2}{2\lambda} \approx 246$ GeV
- 1 neutral physical Higgs H
- [Higgs '64, '66; Englert, Brout '64; Guralnik, Hagen, Kibble '64]

New Physics models:

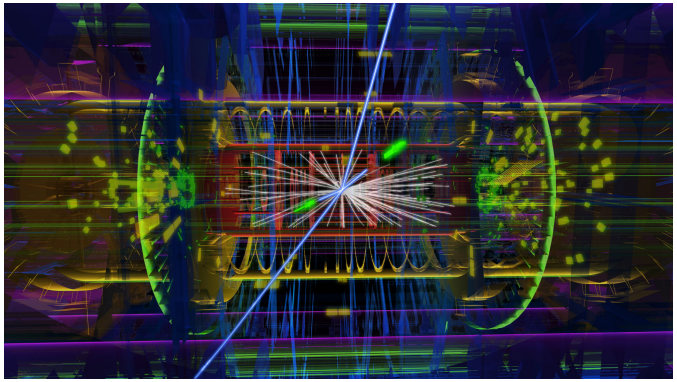


- solutions of SM problems often with modified Higgs sector: supersymmetry, multi-Higgs, ...
- new phenomena: scalar CP violation, instable vacua, ...
- relations to: matter asymmetry, dark matter, ...

LHC:

- direct searches for new resonances
- precision tests

collider experiment



$\xleftrightarrow{\text{non-trivial link}}$

theoretical foundation

$$\mathscr{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (\mathcal{D}_\mu \varphi)^\dagger (\mathcal{D}^\mu \varphi) - V + \ldots$$

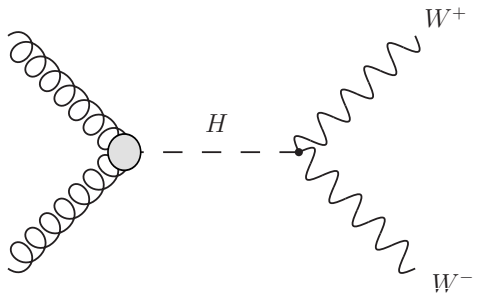
prediction for observable via perturbative expansion in coupling constant:

$$\sigma \approx \sigma^{(0)} + \alpha_s \Delta\sigma^{(1)} + \alpha_s^2 \Delta\sigma^{(2)} + \ldots$$

- $\sigma^{(0)}$ leading order (LO)
- $\Delta\sigma^{(1)}$ next-to-leading order (NLO) correction
- $\Delta\sigma^{(2)}$ next-to-next-to-leading order (NNLO)
- ...

Higgs precision channels at LHC

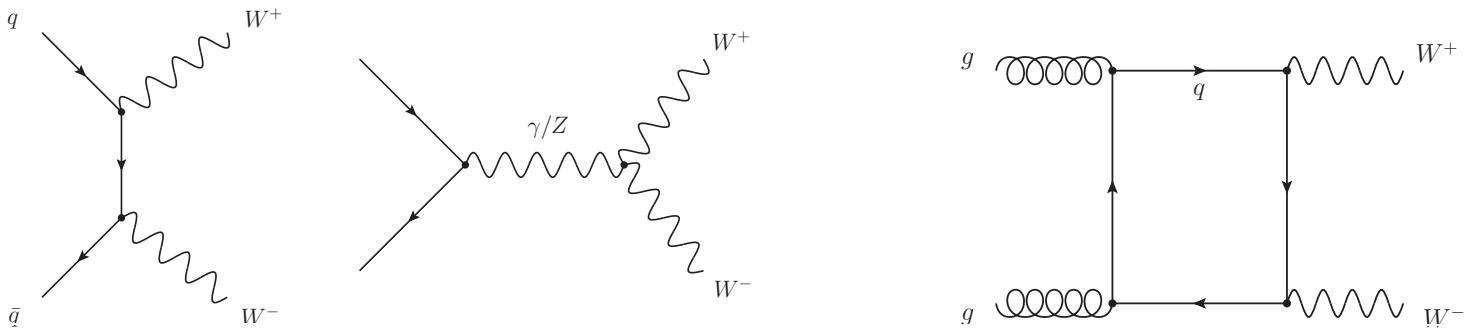
Higgs decays to boson pairs: $H \to \gamma\gamma$, $H \to ZZ^*$, $H \to WW^*$



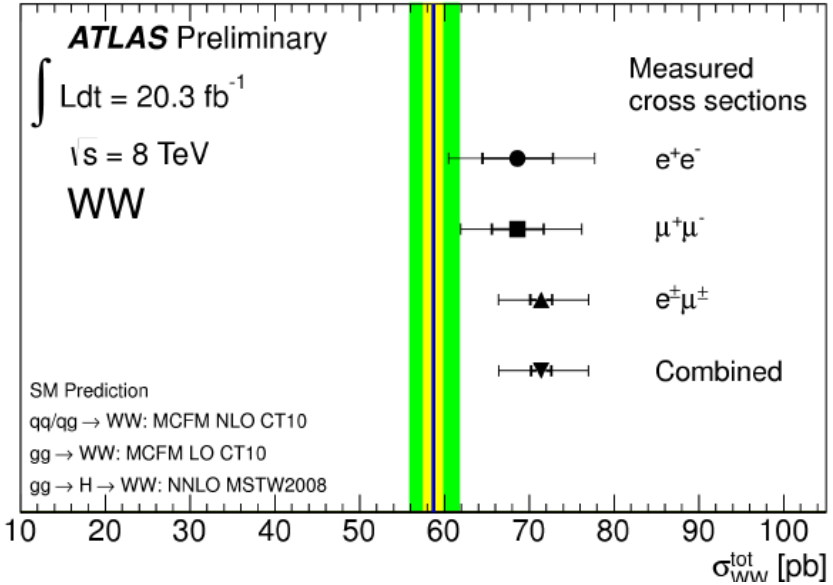
Vector boson pair production at LHC

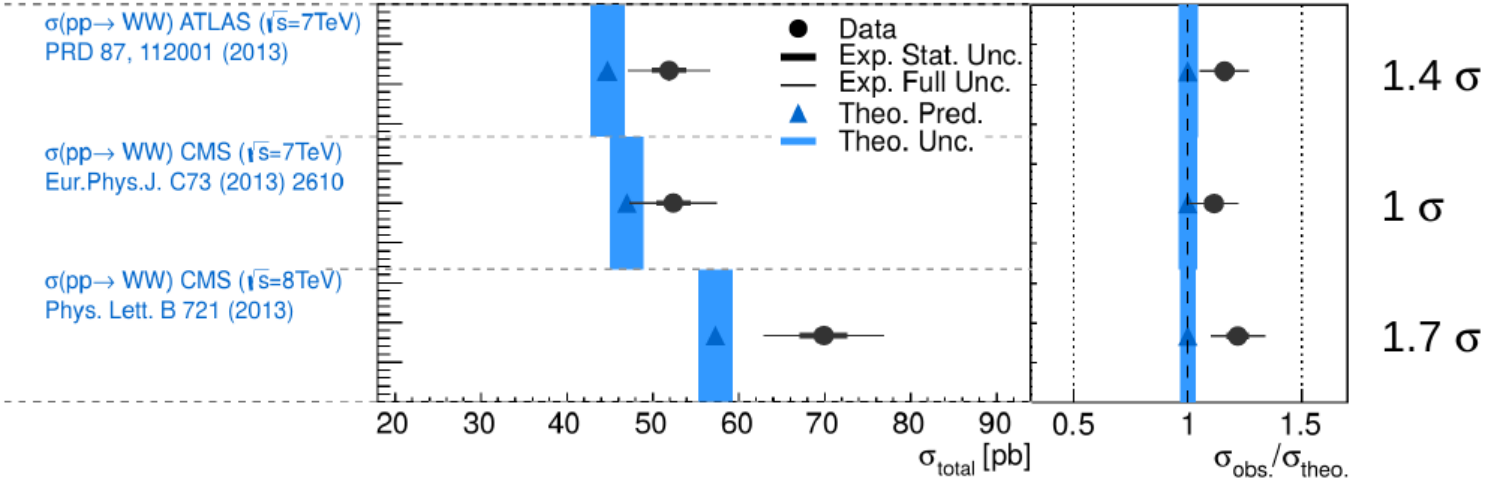
$pp \to VV' + X \to 4$ leptons $+ X$, where VV'=ZZ, W^+W^-, $\gamma^*\gamma^*$, $ZW^\pm$, $Z\gamma^*$, $W^\pm\gamma^*$

- important background to Higgs production + precision tests of SM
- sensitive to details of EWSB
- possible NP contributions at tree or loop level



2014 excess in *WW* production at LHC



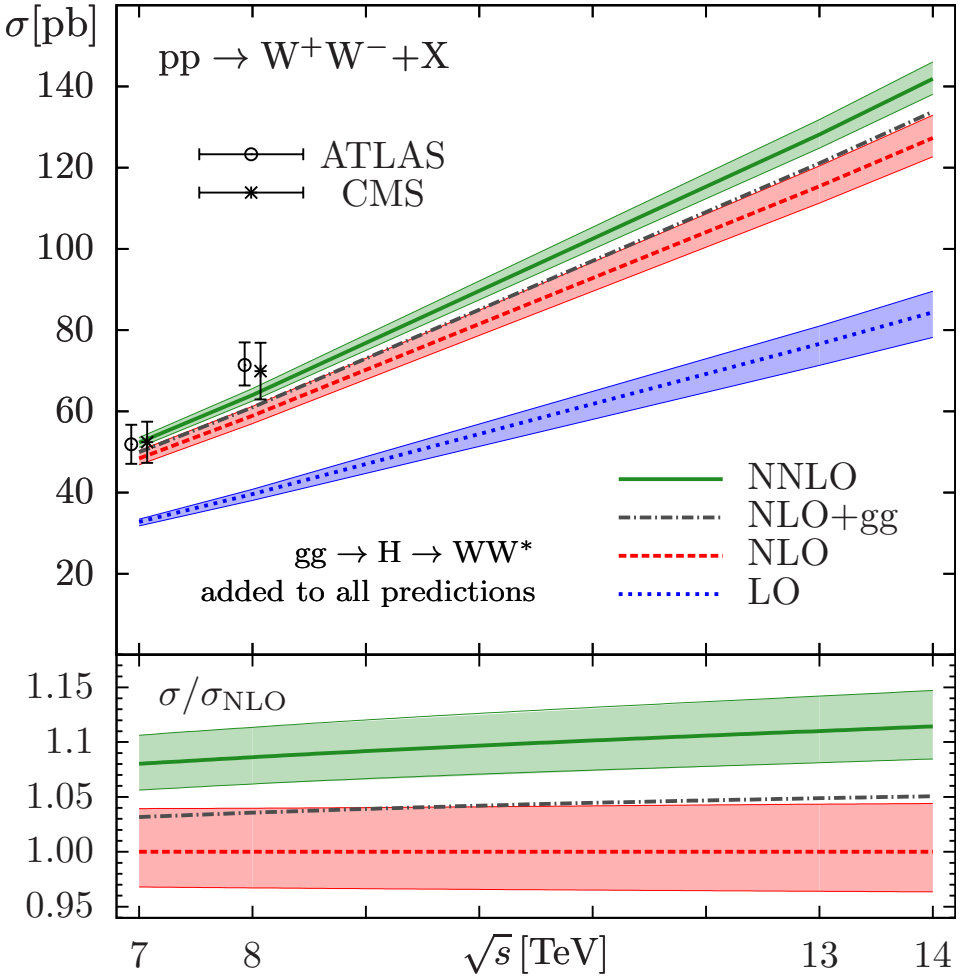


new physics ?

make sure to understand SM prediction !

Result: W^+W^- production at NNLO

first NNLO prediction: significantly reduces tension with data

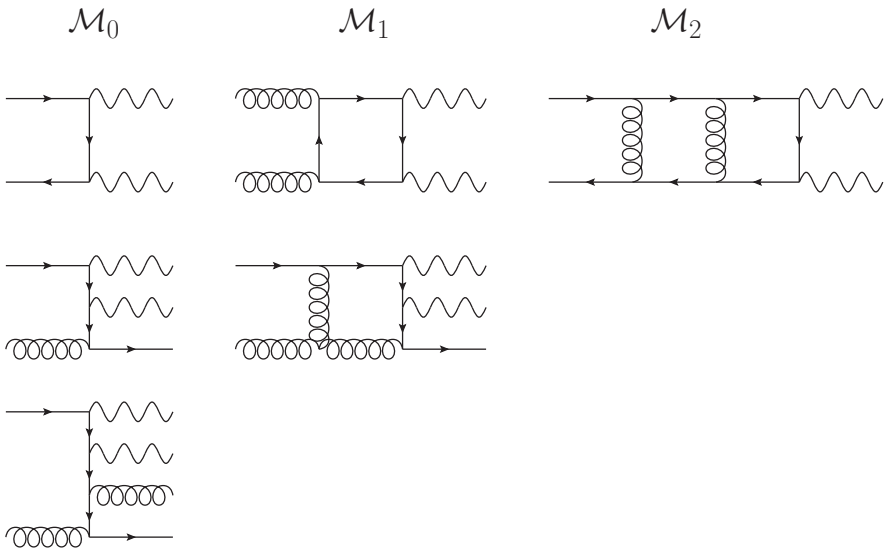


[Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

NNLO corrections: 9-12% (scale variation: 3%, *gg* fraction: 35%)

ingredients for $VV' + X$ production at NNLO QCD:

	LO	NLO	NNLO
$2 \to 2$	$\mathcal{M}_0^* \mathcal{M}_0$ qq	$\mathcal{M}_0^* \mathcal{M}_1$ qq	$\mathcal{M}_0^* \mathcal{M}_2$, $\mathcal{M}_1^* \mathcal{M}_1$ qq, gg
$2 \to 3$	-	$\mathcal{M}_0^* \mathcal{M}_0$ qq, qg	$\mathcal{M}_0^* \mathcal{M}_1$ qq, qg
$2 \to 4$	-	-	$\mathcal{M}_0^* \mathcal{M}_0$ qq, qg, gg



note: some channels contribute only at higher orders:

- qg starting at NLO
- gg starting at NNLO $\to$ control error by computing N^{3}LO contributions from this channel

subtraction terms: up to 2 unresolved partons needed

- q_T subtraction: [Catani, Grazzini '07; Catani, Cieri, de Florian, Ferrera, Grazzini '13]
- N-jettiness subtraction: [Boughezal, Foecke, Liu, Petriello '15; Boughezal, Foecke, Giele, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15]
- antenna subtraction: [Gehrmann-De Ridder, Gehrmann, Glover '05]
- sector-improved subtraction: [Czakon '10]

QCD full NNLO (equal masses):

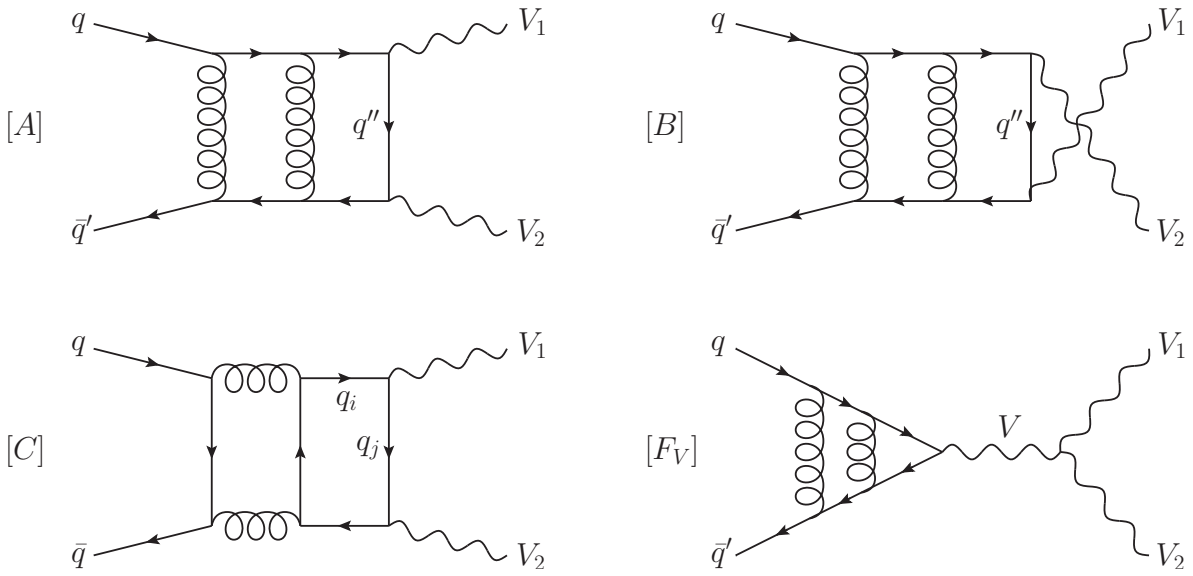
- master integrals: [Gehrmann, Tancredi, Weihs '13; Gehrmann, AvM, Tancredi, Weihs '14]
- amplitudes: [Gehrmann, AvM, Tancredi (unpublished)]
- ZZ@NNLO [Cascioli,Gehrmann,Grazzini,Kallweit,Maierhöfer,AvM,Pozzorini,Rathlev,Tancredi,Weihs'14]
- WW@NNLO [Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

QCD full NNLO (different masses):

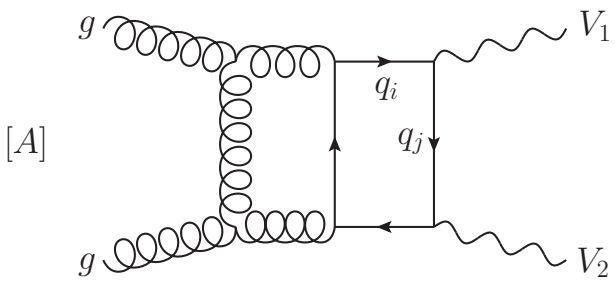
- master integrals: [Henn, Melnikov, Smirnov '14; Caola, Henn, Melnikov, Smirnov '14]; [Papadopoulos, Tammasini, Wever '14]; [Gehrmann, AvM, Tancredi '15]
- amplitudes $q\bar{q}' \to VV'$: [Caola, Henn, Melnikov, Smirnov '14]; [Gehrmann, AvM, Tancredi '15]
- amplitudes $gg \to VV'$: [Caola, Henn, Melnikov, Smirnov '15]; [AvM, Tancredi '15]
- $\gamma^*\gamma^*$@NNLO (partial): [Anastasiou, Cancino, Chavez, Duhr, Lazopoulos, Mistlberger, Müller '14]
- fully differential off-shell ZZ production [Grazzini, Rathlev, Kallweit '15]
- gg channel for W^+W^-, ZZ @NLO [Caola, Melnikov, Roentsch, Tancredi '15, '15]
- p_T spectra for W^+W^-, ZZ @NNLL+NNLO [Grazzini, Rathlev, Kallweit, Wiesemann '15]

Feynman diagrams

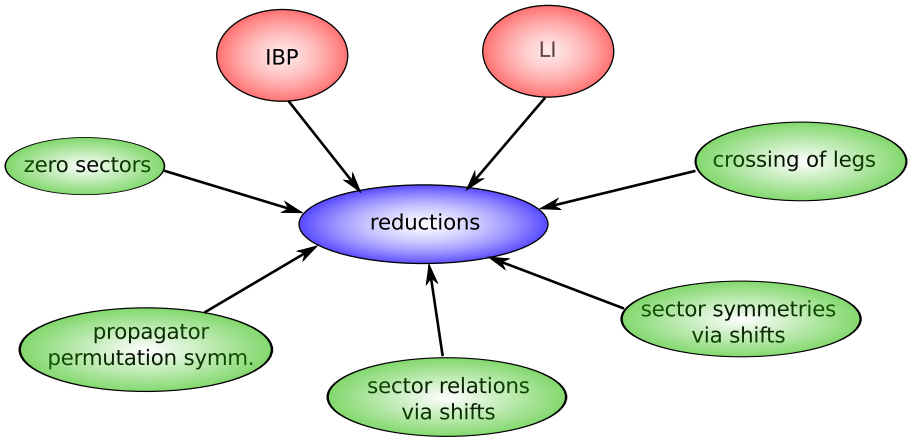
$q\bar{q}'$ channel:

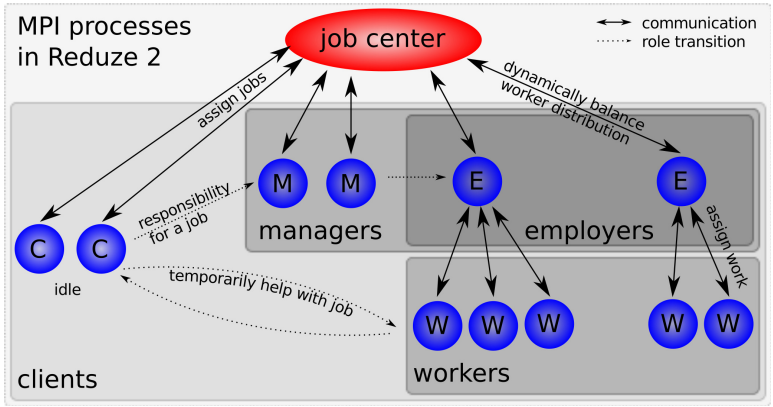


gg channel:



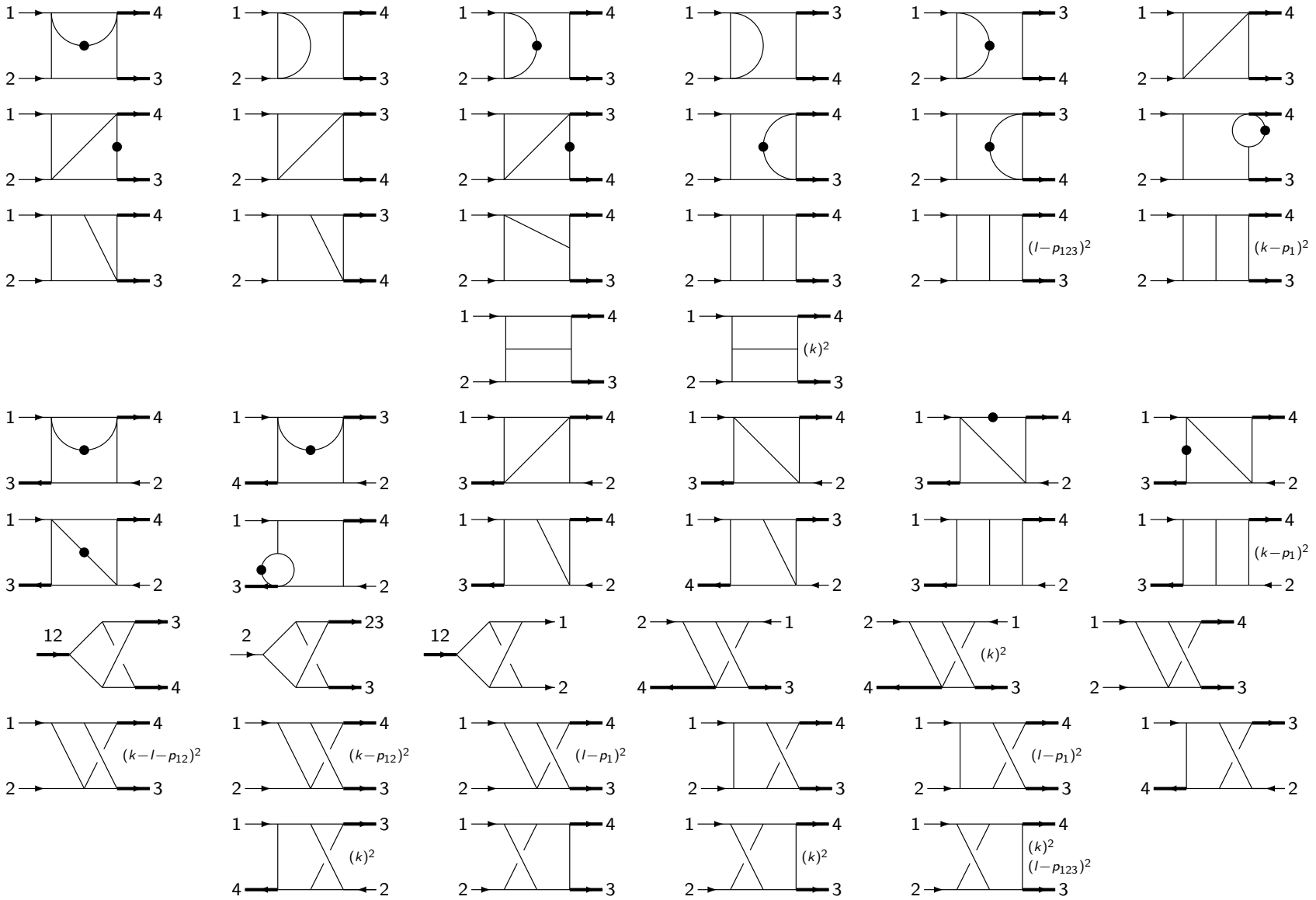
Computer algebra: integral reduction





- integration-by-parts identities: systematic reductions [Chetyrkin, Tkachov 81; Laporta '01]
- C++ program Reduze [Studerus '10], [AvM, Studerus '12]
- solved systems with e.g. 10^7 integrals and algebraic coefficients
- new algorithms based on graph matroids
- phase space integrals, dimension shifts, mapping to finite integrals, ...
- open source: powered cutting-edge calculations also of various third-party research groups

Master integrals for $q\bar{q}' \to VV'$ and $gg \to VV'$



An improved basis for differential equations

- method by [Kotikov '91]; [Gehrmann, Remiddi '99], relies on IBP reduction
- system of diff. eqns for basis integrals wrt external invariants ($\epsilon = (4-d)/2$):

$$\frac{\partial}{\partial s_i}\vec{M}(\epsilon, s) = \bar{\mathbf{A}}^{(s_i)}(\epsilon, s)\vec{M}(\epsilon, s)$$

- in certain cases proper choice of basis achieves [Kotikov '10]; [Henn '13]:

$$\bar{\mathbf{A}}^{(s_i)}(\epsilon, s) = \epsilon\, \mathbf{A}^{(s_i)}(s)$$

such that

$$\boxed{\mathrm{d}\vec{M}(\epsilon, s) = \epsilon \sum_n \mathbf{A}^{(n)}\, \mathrm{dln}\, l_n(s)\ \vec{M}(\epsilon, s)}$$

- every term of ϵ expansion: multiple polylogs of uniform weight

Structure of result

vector of 111 master integrals in canonical basis with alphabet:

$$\begin{aligned}\{\bar{l}_1,\ldots,\bar{l}_{20}\} = \{&2,\bar{x},1+\bar{x},1-\bar{y},\bar{y},1+\bar{y},1-\bar{x}\bar{y},1+\bar{x}\bar{y},1-\bar{z},\bar{z},\\ &1+\bar{y}-2\bar{y}\bar{z},1-\bar{y}+2\bar{y}\bar{z},1+\bar{x}\bar{y}-2\bar{x}\bar{y}\bar{z},1-\bar{x}\bar{y}+2\bar{x}\bar{y}\bar{z},\\ &1+\bar{y}+\bar{x}\bar{y}+\bar{x}\bar{y}^2-2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z},1+\bar{y}-\bar{x}\bar{y}-\bar{x}\bar{y}^2-2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z},\\ &1-\bar{y}-\bar{x}\bar{y}+\bar{x}\bar{y}^2+2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z},1-\bar{y}+\bar{x}\bar{y}-\bar{x}\bar{y}^2+2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z},\\ &1-2\bar{y}-\bar{x}\bar{y}+\bar{y}^2+2\bar{x}\bar{y}^2-\bar{x}\bar{y}^3+4\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z}+2\bar{x}\bar{y}^3\bar{z},\\ &1-\bar{y}-2\bar{x}\bar{y}+2\bar{x}\bar{y}^2+\bar{x}^2\bar{y}^2-\bar{x}^2\bar{y}^3+2\bar{y}\bar{z}+4\bar{x}\bar{y}\bar{z}+2\bar{x}^2\bar{y}^3\bar{z}\}\end{aligned}$$

in parametrisation which rationalizes root of Källén function $\sqrt{s^2+p_3^4+p_4^4-2(s\,p_3^2+p_3^2\,p_4^2+p_4^2\,s)}$:

$$s=\bar{m}^2(1+\bar{x})^2,\quad t=-\bar{m}^2\bar{x}((1+\bar{y})(1+\bar{x}\bar{y})-2\bar{z}\bar{y}(1+\bar{x})),\quad p_3^2=\bar{m}^2\bar{x}^2(1-\bar{y}^2),\quad p_4^2=\bar{m}^2(1-\bar{x}^2\bar{y}^2)$$

- integrate in terms of multiple polylogarithms
- all difficult boundary values fixed by regularity conditions

Computer algebra: special functions

Multiple polylogarithms [Remiddi, Gehrmann]; [Goncharov]

$$G(w_1, \ldots, w_n; x) = \int_0^x \frac{\mathrm{d}t}{t - w_1} G(w_2, \ldots, w_n; t)$$

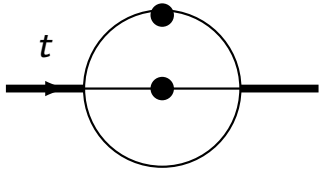
- generalisation of log, Li_n, ...
- new methods: symbol calculus for functional identities

  e.g.: $\mathrm{Li}_2(1/x) = -\mathrm{Li}_2(x) - \ln^2(x)/2 + i\pi \ln x + \pi^2/3$

  [Goncharov, Spradlin, Vergu, Volovich '10], [Brown '11], [Duhr '12], [Duhr, Gangl, Rhodes '11]
- computer algebra package [AvM]

in traditional functional basis:

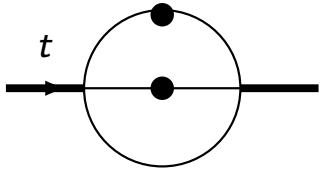
$$- \epsilon^2 \bar{m}^{2\epsilon} t$$



$$
\begin{aligned}
&= 1 \\
&\quad + \epsilon\big[- 2G(-1,\bar{y}) - 2G(0,\bar{x}) - 2G(-1/\bar{y},\bar{x}) - 2G(((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),\bar{z})\big] \\
&\quad + \epsilon^2\big[4G(0,\bar{x})G(((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),\bar{z}) + 4G(-1/\bar{y},\bar{x})G(((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),\bar{z}) \\
&\qquad + G(-1,\bar{y})(4G(0,\bar{x}) + 4G(-1/\bar{y},\bar{x}) + 4G(((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),\bar{z})) + 4G(-1,-1,\bar{y}) \\
&\qquad + 4G(0,0,\bar{x}) + 4G(0,-1/\bar{y},\bar{x}) + 4G(-1/\bar{y},0,\bar{x}) + 4G(-1/\bar{y},-1/\bar{y},\bar{x}) \\
&\qquad + 4G(((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),((1+\bar{y})(1+\bar{x}\bar{y}))/(2(1+\bar{x})\bar{y}),\bar{z})\big] \\
&\quad + O(\epsilon^3)
\end{aligned}
$$

in optimised functional basis for numerical evaluation:

$$- \epsilon^2 m^{2\epsilon} t$$

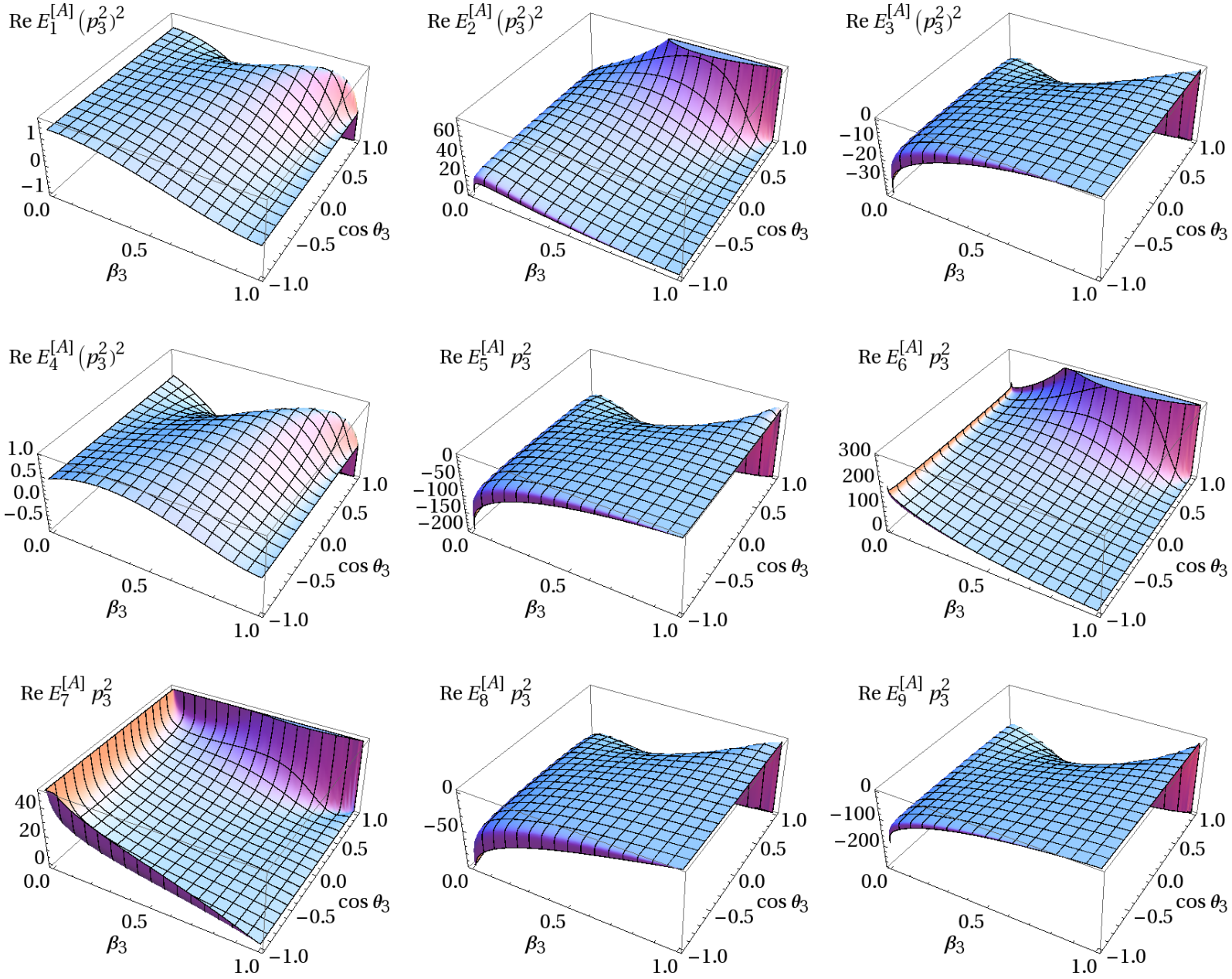


$$= 1 + \epsilon\big[- 2\ln(l_1) - 2\ln(l_5)\big] + \epsilon^2\big[2\ln^2(l_1) + 4\ln(l_1)\ln(l_5) + 2\ln^2(l_5)\big] + O(\epsilon^3)$$

systematic rewriting:

- $O(150ms)$ full off-shell helicity amplitudes for $q\bar{q}' \to VV' \to 4$ leptons
- orders of magnitude faster and more stable than traditional representation

helicity amplitudes for $q\bar{q}' \to VV'$ @ 2-loops [Gehrmann, AvM, Tancredi '15]
helicity amplitudes for $gg \to VV'$ @ 2-loops [AvM, Tancredi '15]



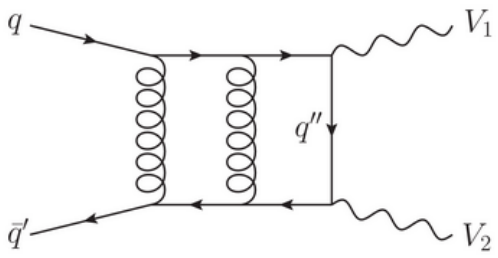
HTTPS://VVAMP.HEPFORGE.ORG

VVamp project

This is the web page of the VVamp project. We provide the two-loop helicity amplitudes for electroweak vector boson pair production and their decay into 4 leptons in quark-antiquark annihilation and in gluon-gluon fusion.

You can download our analytical results for the master integrals and the amplitudes. Moreover, we provide C++ implementations for the fast and reliable numerical evaluation of the amplitudes.

Quark channel



Reference

- Thomas Gehrmann, Andreas von Manteuffel, Lorenzo Tancredi: "*The two-loop helicity amplitudes for qq' -> V1V2 -> 4 leptons*", arXiv:1503.04812

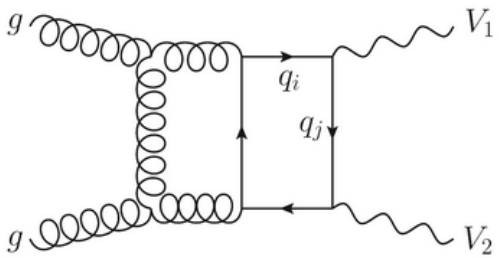
Downloads: amplitudes

- bare form factors exact in d: class A, class B, class C (Form format)
- finite form factors in qt-scheme: class A, class B, class C (Form format)
- relations for projectors: Aj of taui, taui of Aj (Form format)
- numerical implementation of form factors: qqvvamp package (C++, requires GiNaC)

Downloads: master integrals

- master integral definitions: Mathematica, Form format
- master integral traditional solutions: Mathematica, Form format
- master integral optimised solutions: Mathematica, Form format
- master integral crossing relations: Mathematica, Form format
- integral families, kinematics (in Reduze 2 format)

Gluon channel



Reference

- Andreas von Manteuffel, Lorenzo Tancredi: "*The two-loop helicity amplitudes for gg -> V1V2 -> 4 leptons*", arXiv:1503.08835

Downloads: amplitudes

- bare form factors exact in d: class A (Form format)
- finite form factors in qt-scheme: class A (Form format)
- relations for projectors: Pj of Tjdag (Form format)
- numerical implementation of form factors: ggvvamp package (C++, requires GiNaC)

available NNLO predictions of different research groups employ these amplitudes

Fiducial cross sections for ZZ production

- consider 4 lepton final states
- take off-shell effects and spin correlations into account
- careful with extrapolations to Sudakov region [Monni, Zanderighi '14]
- fiducial cross sections for ATLAS cuts
  - 8 TeV data

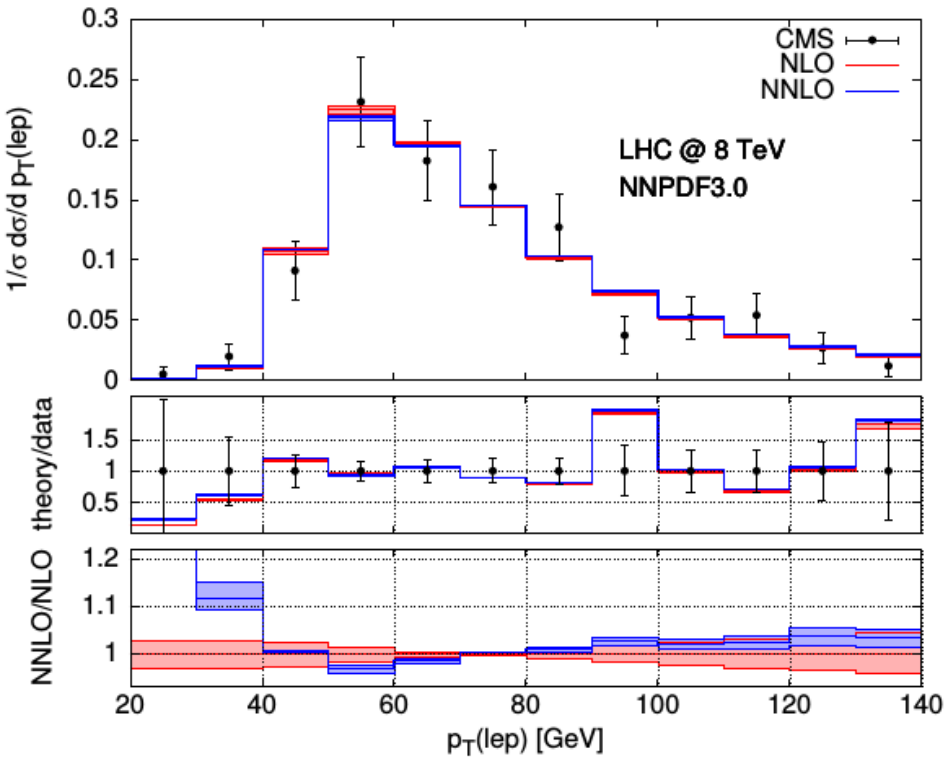
Channel	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma_{\rm exp}$ (fb)
$e^+e^-e^+e^-$	$3.547(1)^{+2.9\%}_{-3.9\%}$	$5.047(1)^{+2.8\%}_{-2.3\%}$	$5.79(2)^{+3.4\%}_{-2.6\%}$	$4.6^{+0.8}_{-0.7}(\text{stat})^{+0.4}_{-0.4}(\text{syst.})^{+0.1}_{-0.1}(\text{lumi.})$
$\mu^+\mu^-\mu^+\mu^-$				$5.0^{+0.6}_{-0.5}(\text{stat})^{+0.2}_{-0.2}(\text{syst.})^{+0.2}_{-0.2}(\text{lumi.})$
$e^+e^-\mu^+\mu^-$	$6.950(1)^{+2.9\%}_{-3.9\%}$	$9.864(2)^{+2.8\%}_{-2.3\%}$	$11.31(2)^{+3.2\%}_{-2.5\%}$	$11.1^{+1.0}_{-0.9}(\text{stat})^{+0.5}_{-0.5}(\text{syst.})^{+0.3}_{-0.3}(\text{lumi.})$

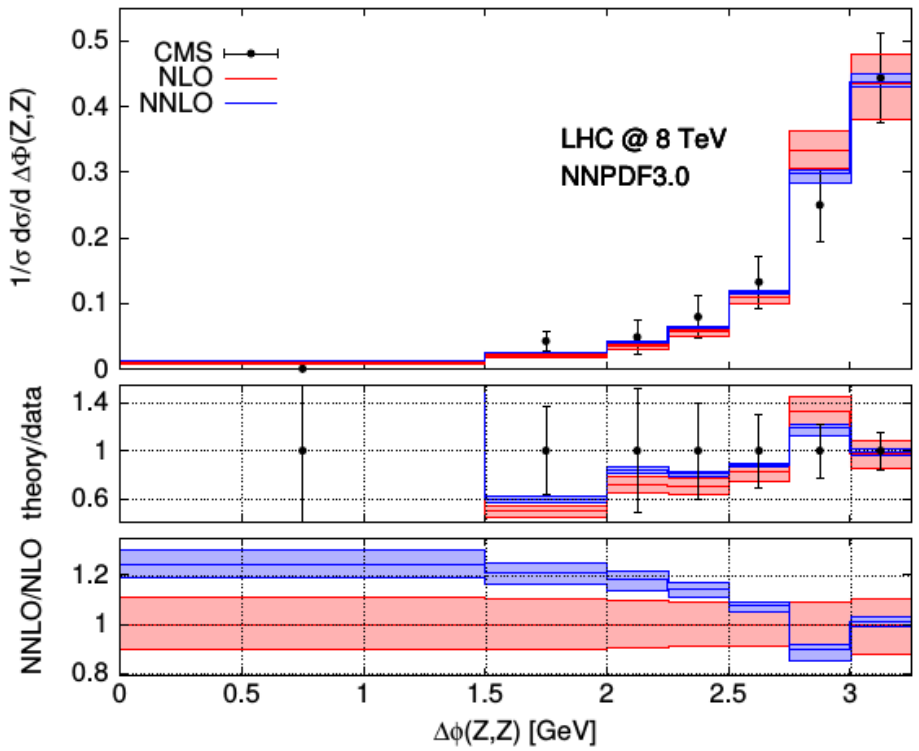
  - 13 TeV data

Channel	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma_{\rm exp}$ (fb)
$e^+e^-e^+e^-$	$5.007(1)^{+4\%}_{-5\%}$	$6.157(1)^{+2\%}_{-2\%}$	$7.14(2)^{+2\%}_{-2\%}$	$8.4^{+2.4}_{-2.0}(\text{stat})^{+0.4}_{-0.2}(\text{syst.})^{+0.5}_{-0.3}(\text{lumi.})$
$\mu^+\mu^-\mu^+\mu^-$				$6.8^{+1.8}_{-1.5}(\text{stat})^{+0.3}_{-0.3}(\text{syst.})^{+0.4}_{-0.3}(\text{lumi.})$
$e^+e^-\mu^+\mu^-$	$9.906(1)^{+4\%}_{-5\%}$	$12.171(2)^{+2\%}_{-2\%}$	$14.19(2)^{+2\%}_{-2\%}$	$14.7^{+2.9}_{-2.5}(\text{stat})^{+0.6}_{-0.4}(\text{syst.})^{+0.9}_{-0.6}(\text{lumi.})$

  [Grazzini, Kallweit, Rathlev '15] (with two-loop amplitudes [Gehrmann, AvM, Tancredi '15])
- note: NNLO effects up to 20% (gg fraction: 60%)

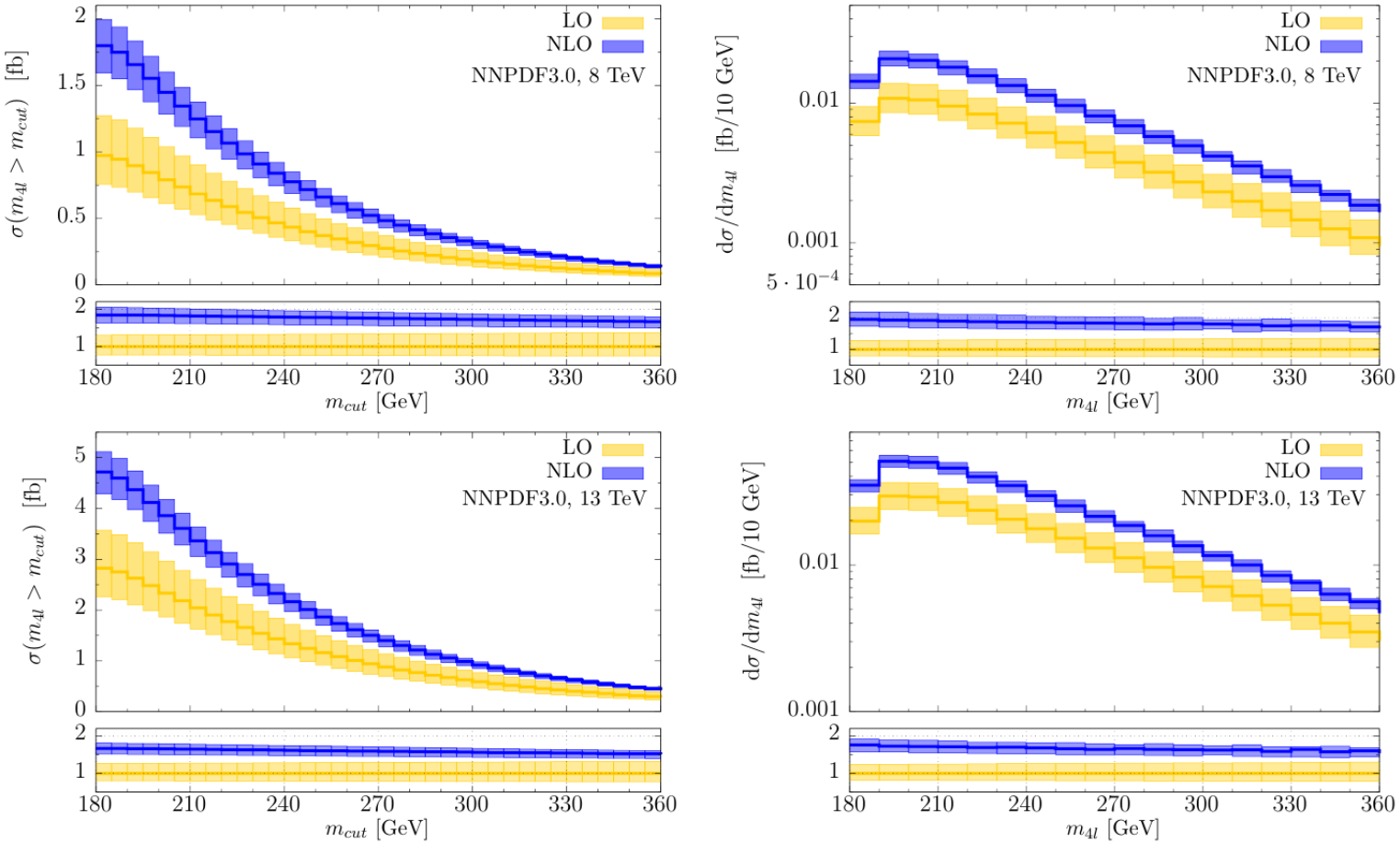
Distributions for ZZ production





[Grazzini, Kallweit, Rathlev '15]

Gluon induced N^{3}LO contributions to ZZ production

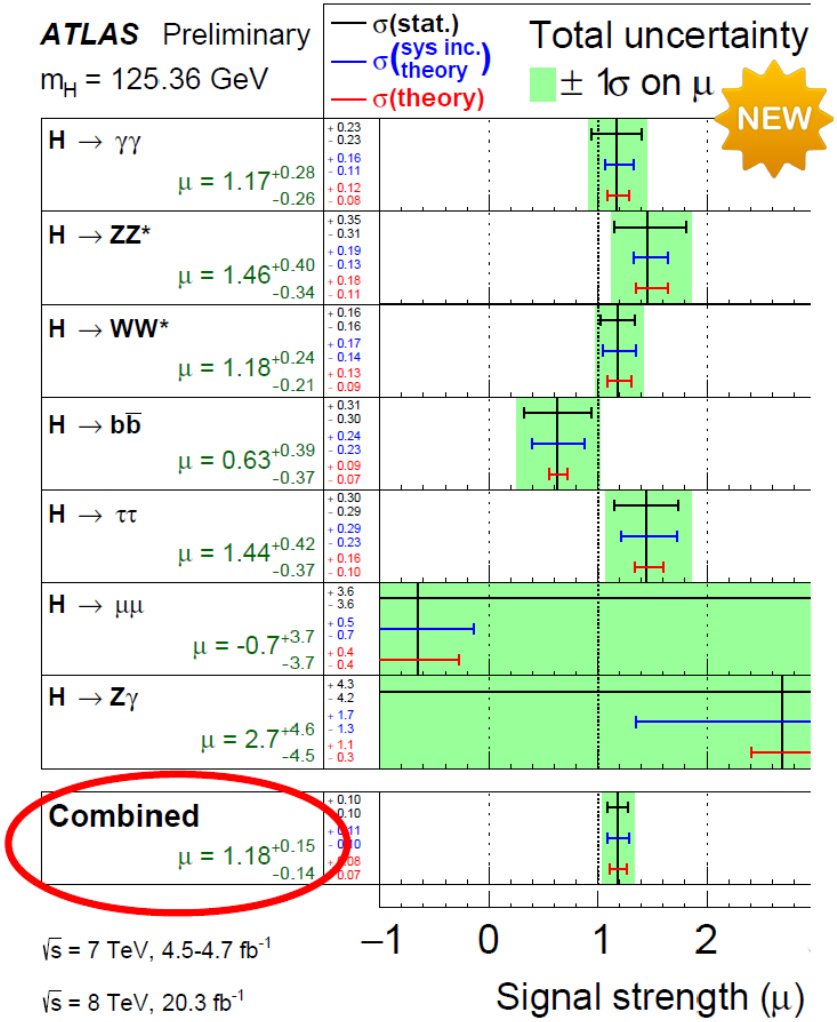


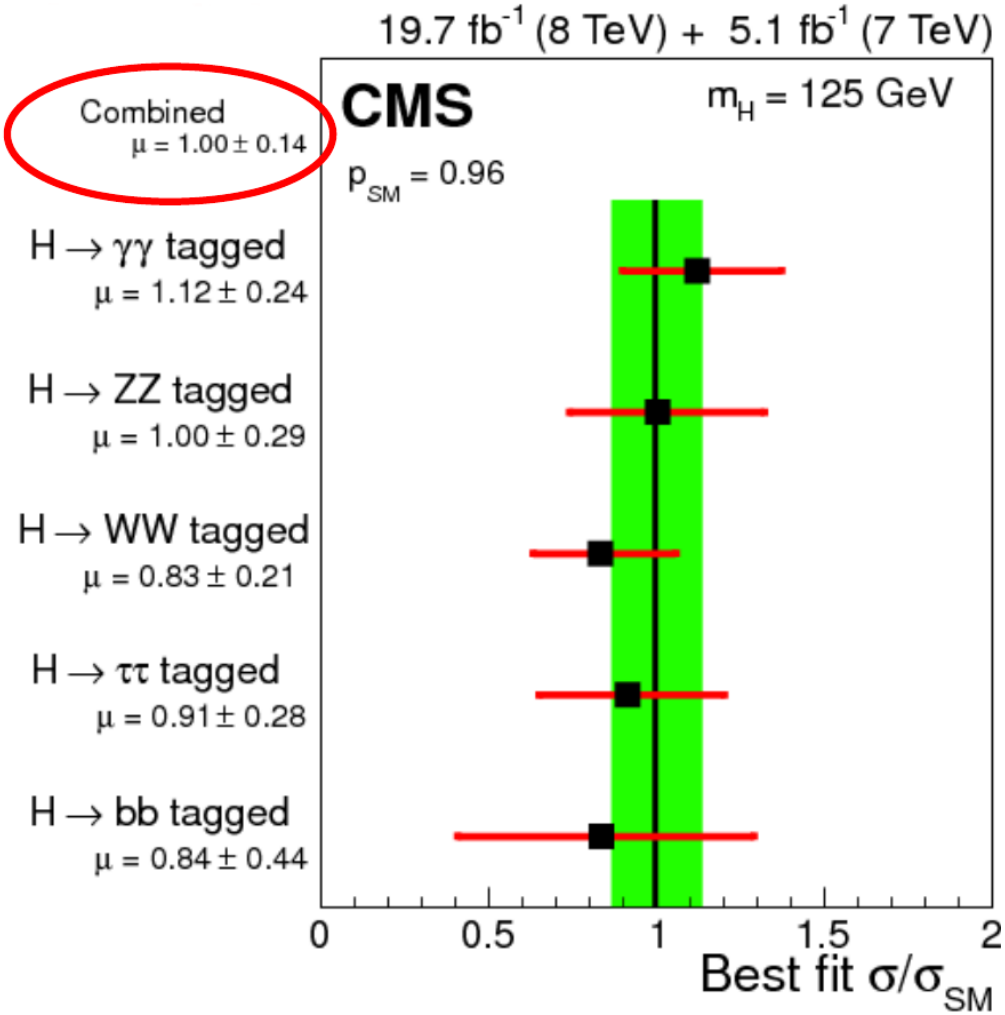
[Caola, Melnikov, Röntsch, Tancredi '15] (with two-loop amplitudes [AvM, Tancredi '15])

- invariant 4 lepton mass important for Higgs width measurement
- increase NNLO prediction by 6-8%
- remaining uncertainty: 2%

Status of Higgs cross section

[Duehrssen Moriond '15]

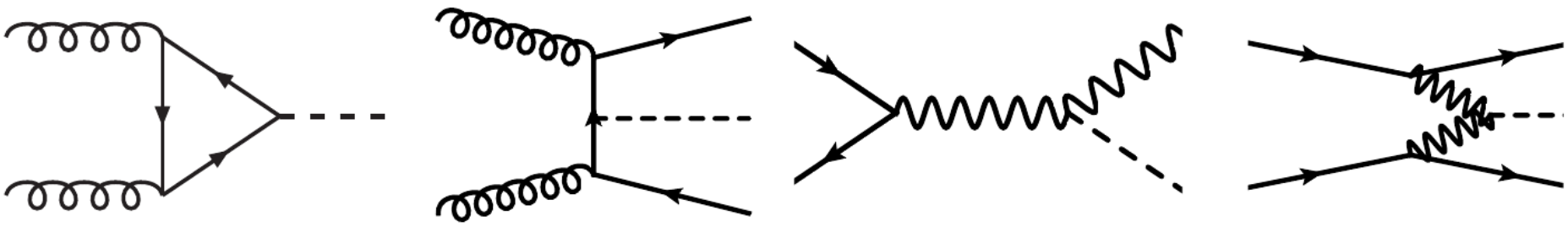




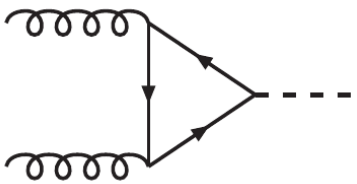
- good agreement SM and data
- severe limits on NP contributions

Higgs production at the LHC

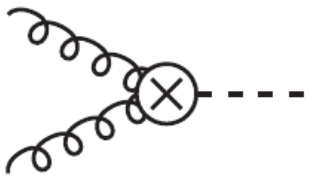
production channels:



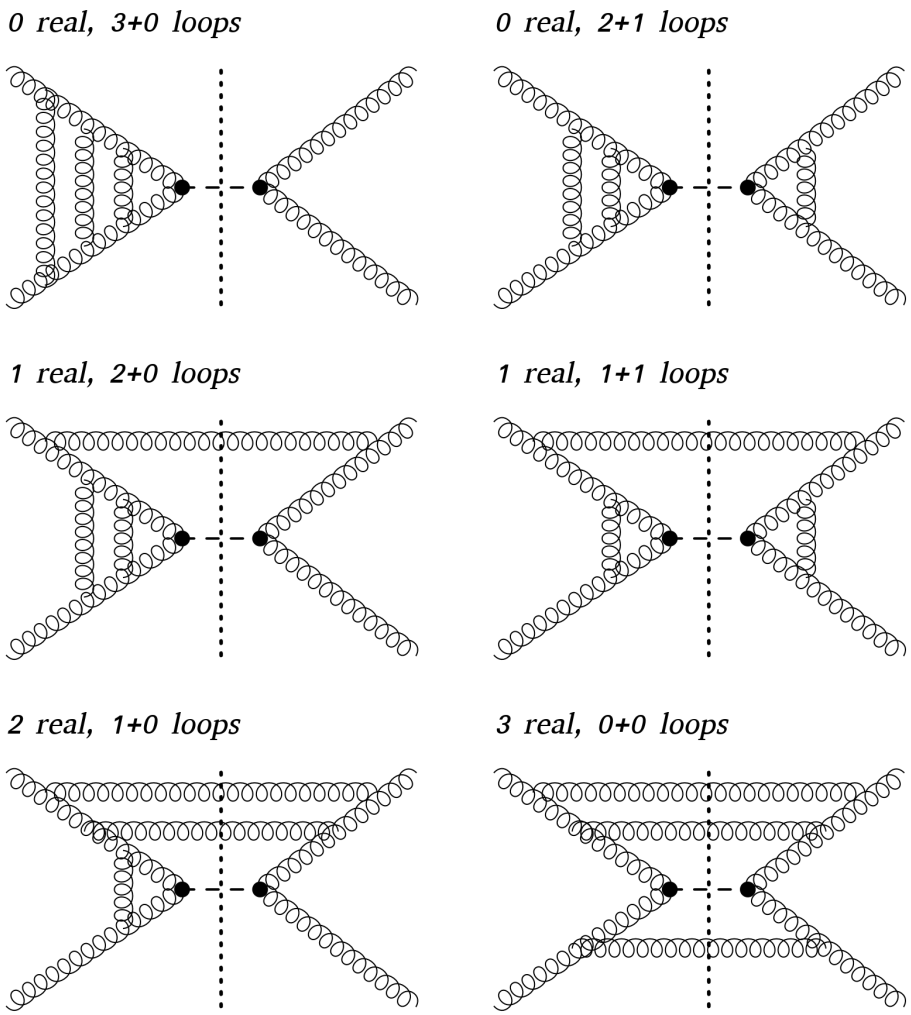
gluon fusion most important



problem: LO inprecise and inaccurate
solution: higher orders in effective theory (inf. top mass)



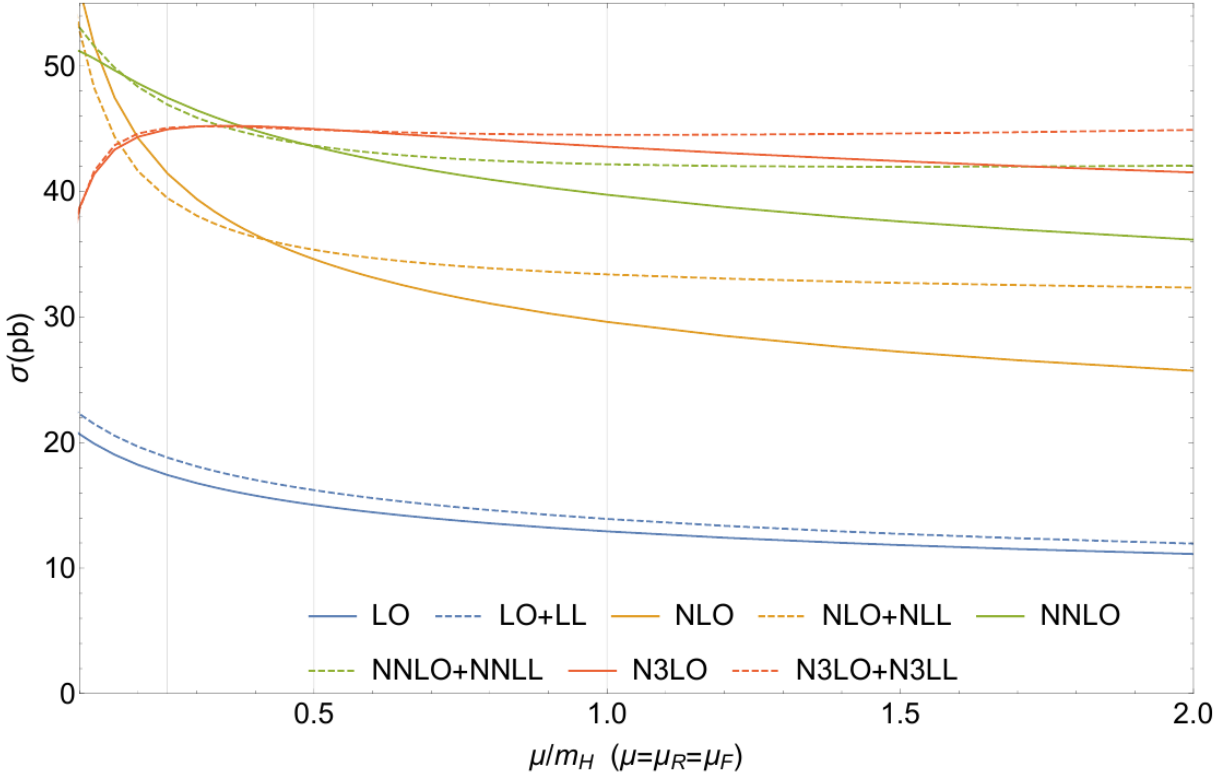
Building blocks for N^{3}LO prediction



first result at N^{3}LO in 2014: soft-virtual approximation

- [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger '14]
- [Li, AvM, Schabinger, Zhu '14]
  - SCET based calculation, also for Drell-Yan and $\mathcal{N} = 4$ SYM

Approximate N^{3}LO and resummations



[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, Lazopoulos '16]

- important: subleading terms in threshold expansion
- missing: exact z-dependence
- resummation improves convergence of perturbative expansion
- missing for N^{3}LL: cusp anomalous dimension @ 4 loops !
- fundamental question about IR behaviour of QCD:
  Casimir scaling of quark and gluon cusp anomalous dimension $\Gamma^q_{\text{cusp}} = \frac{C_F}{C_A}\Gamma^g_{\text{cusp}}$?

gluon and quark form factors: virtual corrections to Higgs and Drell-Yan production

- cusp anomalous dimension can be extracted from poles in ϵ
- previous milestone calculation: the 3-loop form factors
  - [Baikov, Chetyrkin, A. Smirnov, V. Smirnov, Steinhauser '09]
  - [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10, '10]
- 4-loop form factors challenging

new idea for master integrals:

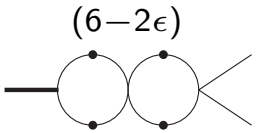
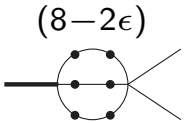
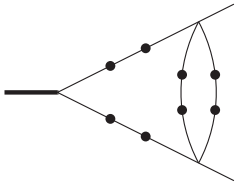
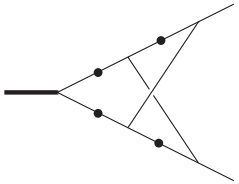
- always possible to use finite instead of infinite loop integrals !
  [Panzer '14], [AvM, Panzer, Schabinger '14]
  application: direct integrations (analytical or numerical)

new idea for reductions:

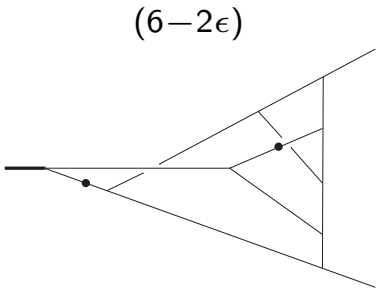
- reconstruct reductions from finite field samples
  [AvM, Schabinger '14]

setting the stage: [AvM, Panzer, Schabinger '15]

- rederived all literature results with our method of finite integrals, e.g.:

$$\mathcal{F}_2^q(\epsilon) = \frac{1}{\epsilon^4}\Bigg[c_1$$  $$+ c_2$$  $$\Bigg] + \frac{1}{\epsilon^3}\Bigg[c_3$$  $$\Bigg] + \frac{1}{\epsilon}\Bigg[c_4$$  $$\Bigg]$$

- first 4-loop results:



$$= \frac{18}{5}\zeta_2^2\zeta_3 - 5\zeta_2\zeta_5 + \Big(24\zeta_2\zeta_3 + 20\zeta_5 - \frac{188}{105}\zeta_2^3 - 17\zeta_3^2 + 9\zeta_2^2\zeta_3 - 47\zeta_2\zeta_5 - 21\zeta_7 + \frac{6883}{2100}\zeta_2^4 + \frac{49}{2}\zeta_2\zeta_3^2 + \frac{1}{2}\zeta_3\zeta_5 - 9\zeta_{5,3}\Big)\epsilon + \mathcal{O}\left(\epsilon^2\right)$$

- work in progress (N_f^3 for quarks just completed)

will unambiguously determine fundamental objects of QCD subject to many speculations:

- gluon and quark cusp anomalous dimensions at 4-loops

Conclusions

- high-energy collider physics crucial to understand Nature at small distances
- LHC run II with lots of precision data ahead
- theory predictions
  - match experimental precision
  - requires multiloop calculations for key processes
  - crucial: modern methods & tools for loop amplitudes & integrals
- this talk:
  - diboson production: NNLO fully differential
  - Higgs and Drell-Yan production: N^3LO corrections
  - work in progress: four-loop form factor