Super QCD on the Lattice in Two Dimensions

(JHEP 1510 (2015) 013)

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January 21, 2016

Outline

- Motivation
- Upshot
- Exact SUSY on the lattice
  - Challenges on the lattice
  - Nilpotent Supercharges
  - Topological Twisting
  - Kähler-Dirac formalism
  - SYM Model
- Quick Recap
- 2D SQCD Model on the Lattice
  - The Quiver Approach
  - SUSY breaking
- Summary

Motivation

- Provides a non-perturbative tool to study SUSY breaking
- Insight into dualities such as Seiberg duality, AdS/CFT etc
- Relevant for string theorists: possibility of the first numerical investigation Calabi-Yau models, probe into mirror symmetries (JHEP 0705:079,2007)

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Upshot

- Goal: Construct a lattice discretization of SQCD model which retains *exact* an supersymmetry
- Super QCD lattice theory must contain fermions transforming in the fundamental representation... fundamental fermions. Previous work was focused on models with just adjoints.
- Study the vacuum structure model, search for SUSY breaking

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Immediate challenges

- SUSY- a symmetry that extends Poincaré Symmetry
- Lattice constructions break Poincaré Symmetry
- Asymmetric gauge transformations for fermions and bosons
- Fermion "Doubling" problem
- Difficult to implement exact SUSY on the lattice

Nilpotent Supercharges

(Resolution of explicit broken Poincare Symmetry)

- Full SUSY algebra:

$$\left\{\mathcal{Q}^i_\alpha, \mathcal{Q}^j_\beta\right\} = 0, \left\{\overline{\mathcal{Q}}^i_{\dot\alpha}, \overline{\mathcal{Q}}^j_{\dot\beta}\right\} = 0, \left\{\mathcal{Q}^i_\alpha, \overline{\mathcal{Q}}^j_{\dot\beta}\right\} = 2P_m\sigma^m_{\alpha\dot\beta}\delta_{ij},$$

  where $i, j = 1, 2, .., \mathcal{N}$
- Can we preserve a <u>subalgebra</u> of the full SUSY algebra at <u>non-zero</u> lattice spacing?
- Supercharges in Euclidean space are independent. That is

$$\mathcal{Q}^i, \overline{\mathcal{Q}}^i \longrightarrow q^i$$

- Subalgebra $\left\{q^i, q^j\right\} = 0$ can be preserved on the lattice

Twisting

(Resolution of Fermion/Boson Asymmetric Transformation)

- $\{q^i, q^j\} = 0$ solves the Poincaré problem but $\in$ incomplete spinor representation of the Lorentz group
- Supercharges q^i must transform as scalars .. problem
- Consider the global symmetry group for SUSY:

$$SO(d)_E \; x \; SO(d)_R \subset SO(d)_E \; x \; G_R$$

- Twisting $\Longrightarrow$ choosing the diagonal subgroup

$$SO(d)' = \text{diag}\,[SO(d)_E \; x \; SO(d)_R]$$

- Notice: Fermions transform as integer spin under $SO(d)'_E$!

Kähler-Dirac formalism

Under *twisted* $SO(2)'$,

- Gamma basis expansion (in 2D): Spinors $\longrightarrow$ matrices

$$\Psi = \frac{\eta}{2} I + \psi_\mu \gamma_\mu + \chi_{\mu\nu} \gamma_\mu \gamma_\nu$$

- Twisting requires $\mathcal{N} = 2$ supersymmetry.
- Supercharge expansion:

$$q = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu} \gamma_\mu \gamma_\nu$$

- Scalar SUSY algebra:

$$\boxed{\{\mathcal{Q}, \mathcal{Q}\} = 0}$$

and

$$\{\mathcal{Q}, \mathcal{Q}_\mu\} = p_\mu$$

SYM Model

- Fermions: Twisted fields $\eta, \psi_\mu, \chi_{\mu\nu}$
- Bosons: Complexified gauge fields: $\mathcal{A}_\mu$, where,

$$\mathcal{A}_\mu = A_\mu + iB_\mu$$

- Notice: Scalars transform as integer spin under $SO(d)'_E$!
- Q-exact Action:

$$\boxed{S = \mathcal{Q}\Lambda \quad \Longrightarrow \quad \mathcal{Q}S = 0}$$

Twisted formalism $\longrightarrow$ Nilpotent SUSY algebra & Q-exact Action

SYM Model

- Fermions & Bosons transform as adjoints
- Q-exact action:

$$S = \frac{1}{g^2}\mathcal{Q}\int \mathrm{Tr}\left[\chi_{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x) + \eta(x)\left[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu\right] - \frac{1}{2}\eta(x)d\right],$$

- $\mathcal{D}_\mu$ and $\overline{\mathcal{D}}_\mu$ are covariant derivates.
  - $\mathcal{D}_\mu = \partial_\mu + \mathcal{A}_\mu, \quad \overline{\mathcal{D}}_\mu = \partial_\mu + \overline{\mathcal{A}}_\mu$
  - $\mathcal{F}_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu], \quad \overline{\mathcal{F}}_{\mu\nu} = \left[\overline{\mathcal{D}}_\mu, \overline{\mathcal{D}}_\nu\right]$

SYM Model

- The complexified gauge fields, *on the lattice* are defined as: $\mathcal{U}_\mu$, where,
$$\mathcal{U}_\mu = \mathbb{1} + \mathcal{A}_\mu$$
- Gauge covariant difference operators: $\mathcal{D}_\mu^{(\pm)}, \overline{\mathcal{D}}_\mu^{(\pm)}$
  - $+ \equiv$ Forward difference operator
    $- \equiv$ Backward difference operator
  - $\left[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu\right] = \overline{\mathcal{D}}_\mu^{(-)}\mathcal{U}_\mu, \quad \mathcal{F}_{\mu\nu} = \mathcal{D}_\mu^{(+)}\mathcal{U}_\nu, \quad \overline{\mathcal{F}}_{\mu\nu} = \overline{\mathcal{D}_\mu^{(+)}\mathcal{U}_\nu}$
- Q-exact action:
$$S = \frac{1}{g^2}\mathcal{Q}\sum_{x,\mu,\nu}\mathrm{Tr}\left[\chi_{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x) + \eta(x)\overline{\mathcal{D}}_\mu^{(-)}\mathcal{U}_\mu(x) - \frac{1}{2}\eta(x)d\right]$$

SYM Model

- Q-exact action:

$$S = \frac{1}{g^2}\mathcal{Q}\sum_{x,\mu,\nu}\mathrm{Tr}\left[\chi_{\mu\nu}(x)\mathcal{F}_{\mu\nu}(x) + \eta(x)\overline{\mathcal{D}}_{\mu}^{(-)}\mathcal{U}_{\mu}(x) - \frac{1}{2}\eta(x)d\right]$$

- Q-transformations:

$$\begin{aligned} \mathcal{Q}\mathcal{A}_{\mu} &= \psi_{\mu} \quad | \quad \mathcal{Q}\psi_{\mu} = 0 \\ \mathcal{Q}\eta &= d \quad | \quad \mathcal{Q}d = 0 \\ \mathcal{Q}\chi_{\mu\nu} &= -\overline{\mathcal{F}}_{\mu\nu} \quad | \quad \mathcal{Q}\overline{\mathcal{A}} = 0 \end{aligned}$$

$$\boxed{S = \frac{1}{g^2}\int \mathrm{Tr}\left[-\overline{\mathcal{F}}_{\mu\nu}\mathcal{F}_{\mu\nu} + \frac{1}{2}\left[\overline{\mathcal{D}}_{\mu},\mathcal{D}_{\mu}\right]^2 - \eta\overline{\mathcal{D}}_{\mu}\psi_{\mu} - \chi_{\mu\nu}\mathcal{D}_{[\mu}\psi_{\nu]}\right]}$$

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Recap

Twisting allows ...

- SUSY subalgebra to be preserved exactly on the lattice
- Spinors and scalars to transform as integer spins.
- Kähler-Dirac fermions can be discretized without the introducing fermion doubling problem.

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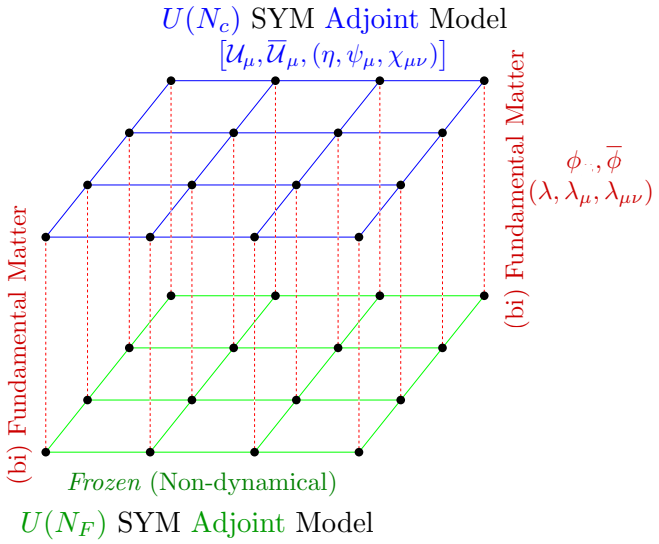
Quiver Approach: JHEP 1510 (2015) 013

- SQCD- a theory with fundamental fermions
- Start with a $(2+1)$ D twisted SUSY model

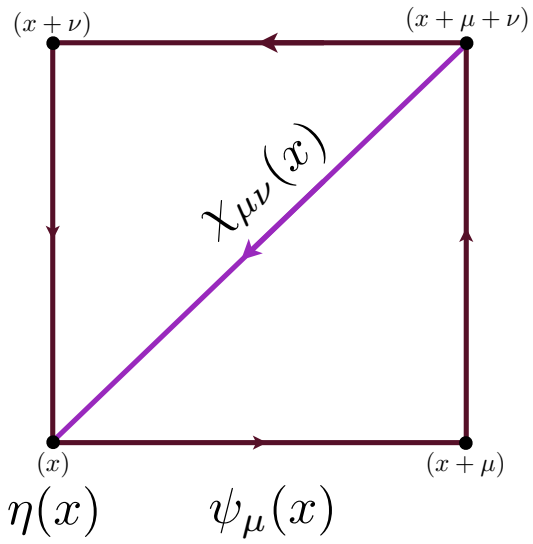


Quiver Approach

- $U(N_c)$ Adjoint Matter:

$$\eta(x) \to G(x)\eta(x)G^\dagger(x)$$
$$\psi_\mu(x) \to G(x)\psi_\mu(x)G^\dagger(x+\mu)$$
$$\mathcal{U}_\mu(x) \to G(x)\mathcal{U}_\mu(x)G^\dagger(x+\mu)$$
$$\chi_{\mu\nu}(x) \to G(x+\mu+\nu)\chi_{\mu\nu}(x)G^\dagger(x)$$

$$\mathrm{G(x)} \in U(N_c)$$

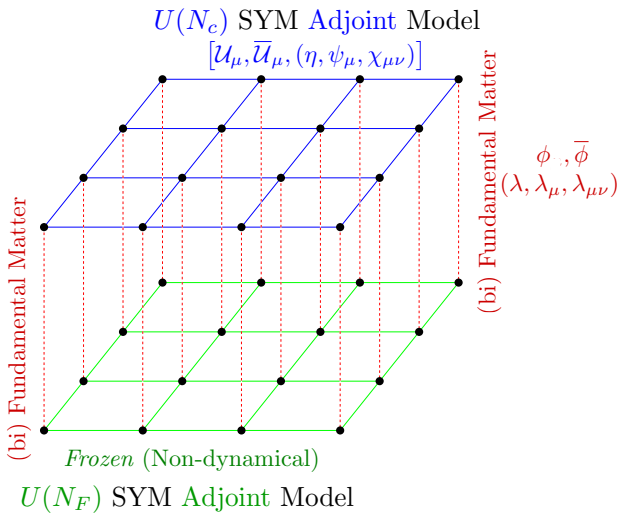


- Orientation of the fields ensures G.I

$$\boxed{\eta \longrightarrow \text{site}; \quad \psi_\mu \longrightarrow \text{link}; \quad \chi_{\mu\nu} \longrightarrow \text{diagonal link}}$$

Quiver Approach

(bi)Fundamental Matter:

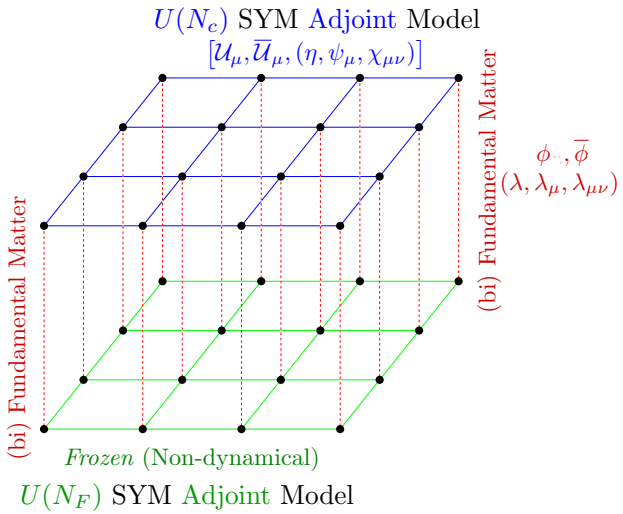


$$\phi(x) \to G(x)\ \phi(x)\ H^\dagger(x)$$
$$\lambda(x) \to G(x)\ \lambda(x)\ H^\dagger(x)$$
$$\lambda_\mu(x) \to H(x+\mu)\ \lambda_\mu(x)\ G^\dagger(x)$$
$$\lambda_{\mu\nu}(x) \to G(x)\ \lambda_{\mu\nu}(x)\ H^\dagger(x+\mu+\nu)$$

$$G(x) \in U(N_c)$$
$$H(x) \in U(N_F)$$

Quiver Approach

(bi)Fundamental Matter:



$$\phi(x) \to G(x)\ \phi(x)\ H^\dagger(x)$$
$$\lambda(x) \to G(x)\ \lambda(x)\ H^\dagger(x)$$
$$\lambda_\mu(x) \to H(x+\mu)\ \lambda_\mu(x)\ G^\dagger(x)$$
$$\lambda_{\mu\nu}(x) \to G(x)\ \lambda_{\mu\nu}(x)\ H^\dagger(x+\mu+\nu)$$

The N_F *freeze* $\Longrightarrow$ $H(x) \approx \mathbb{1}$

Matter fields $\phi, \lambda, \lambda_\mu, \lambda_{\mu\nu}$ transform as fundamentals under $U(N_c)$

Action

- Dimensionally reduce from a **pure 3D** action:

$$S_{3d} = \frac{1}{g^2}\int \quad \mathrm{Tr} \quad \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right]^2\right]$$
$$- \quad \mathrm{Tr} \quad \left[\eta\overline{\mathcal{D}}_a\psi_a - \chi_{ab}\mathcal{D}_{[a}\psi_{b]} - \theta_{abc}\overline{\mathcal{D}}_a\chi_{bc}\right]$$

- Adjoint Action: $a = \mu = 1, 2$ and $b = \nu = 1, 2$

$$S_{adj} = \frac{1}{g^2}\int \mathrm{Tr}\left[-\overline{\mathcal{F}}_{\mu\nu}\mathcal{F}_{\mu\nu} + \frac{1}{2}\left[\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu\right]^2 - \eta\overline{\mathcal{D}}_\mu\psi_\mu - \chi_{\mu\nu}\mathcal{D}_{[\mu}\psi_{\nu]}\right]$$

- Fundamental Action: $a, b = 3$

$$S_{fund} = \frac{1}{g^2}\int \quad \mathrm{Tr} \quad \left[-\overline{\mathcal{F}}_{a3}\mathcal{F}_{a3} + \frac{1}{2}\left[\overline{\mathcal{D}}_3, \mathcal{D}_3\right]^2\right]$$
$$- \quad \mathrm{Tr} \quad \left[\eta\overline{\mathcal{D}}_3\psi_3 - \chi_{a3}\mathcal{D}_{[a}\psi_{3]} - \theta_{ab3}\overline{\mathcal{D}}_a\chi_{b3}\right]$$

Action

- Fundamental fields:

Fields in the 3rd direction	Fundamental fields
$\mathcal{U}_3(x)$	$\phi(x)$
$\psi_3(x)$	$\lambda(x)$
$\chi_{\mu 3}(x)$	$\lambda_\mu(x)$
$\theta_{\mu\nu 3}(x)$	$\lambda_{\mu\nu}(x)$

- Fundamental fields have inherited the 2D Kähler-Dirac structure

Scalars

- Scalars arise from the commutator term:

$$S_{3d} = \frac{1}{g^2}\int \quad \mathrm{Tr} \quad \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left[\overline{\mathcal{D}}_a, \mathcal{D}_a\right]^2\right]$$
$$- \quad \mathrm{Tr} \quad \left[\eta\overline{\mathcal{D}}_a\psi_a - \chi_{ab}\mathcal{D}_{[a}\psi_{b]} - \theta_{abc}\overline{\mathcal{D}}_a\chi_{bc}\right]$$

- The commutator term results from $\mathcal{Q}\mathrm{Tr}\ \left(\eta\overline{\mathcal{D}}_a\mathcal{U}_a\right)$
- Deformation:

$$\mathcal{Q}\ \mathrm{Tr}\ \left[\eta\left\{\overline{\mathcal{D}}_\mu^{(-)}\mathcal{U}_\mu + \left(\phi\overline{\phi} - r\mathbb{1}_{Nc}\right)\right\}\right]$$

- After Q-transformation and integrating out the auxiliary field:

$$\frac{1}{2}\mathrm{Tr}\ \left[\overline{\mathcal{D}}_\mu^{(-)}\mathcal{U}_\mu + \left(\phi\overline{\phi} - r\mathbb{1}_{Nc}\right)\right]^2$$

SUSY breaking

- D-term :

$$\boxed{S_D = \sum_{i=1}^{N_F} \mathrm{Tr}\ \left[\ \phi_i \overline{\phi}_i\ -\ r\mathbb{1}_{Nc}\ \right]^2}$$

- $S_D = 0 \Longrightarrow$ supersymmetry is preserved
- N_c/N_F Dependence:
  - $N_c > N_F$
    - Rank of $\left(\phi(x)\overline{\phi}(x)\right)$ is at most N_F
    - **cannot** cancel $\mathbb{1}$
    - $S_D \neq 0 \longrightarrow$ *SUSYbreaking*
  - $N_c \leq N_F$
    - Rank of $\left(\phi(x)\overline{\phi}(x)\right)$ is at most N_F
    - **can** cancel $\mathbb{1}$
    - $S_D = 0 \longrightarrow$ *SUSYpreserving*

Soft SUSY breaking

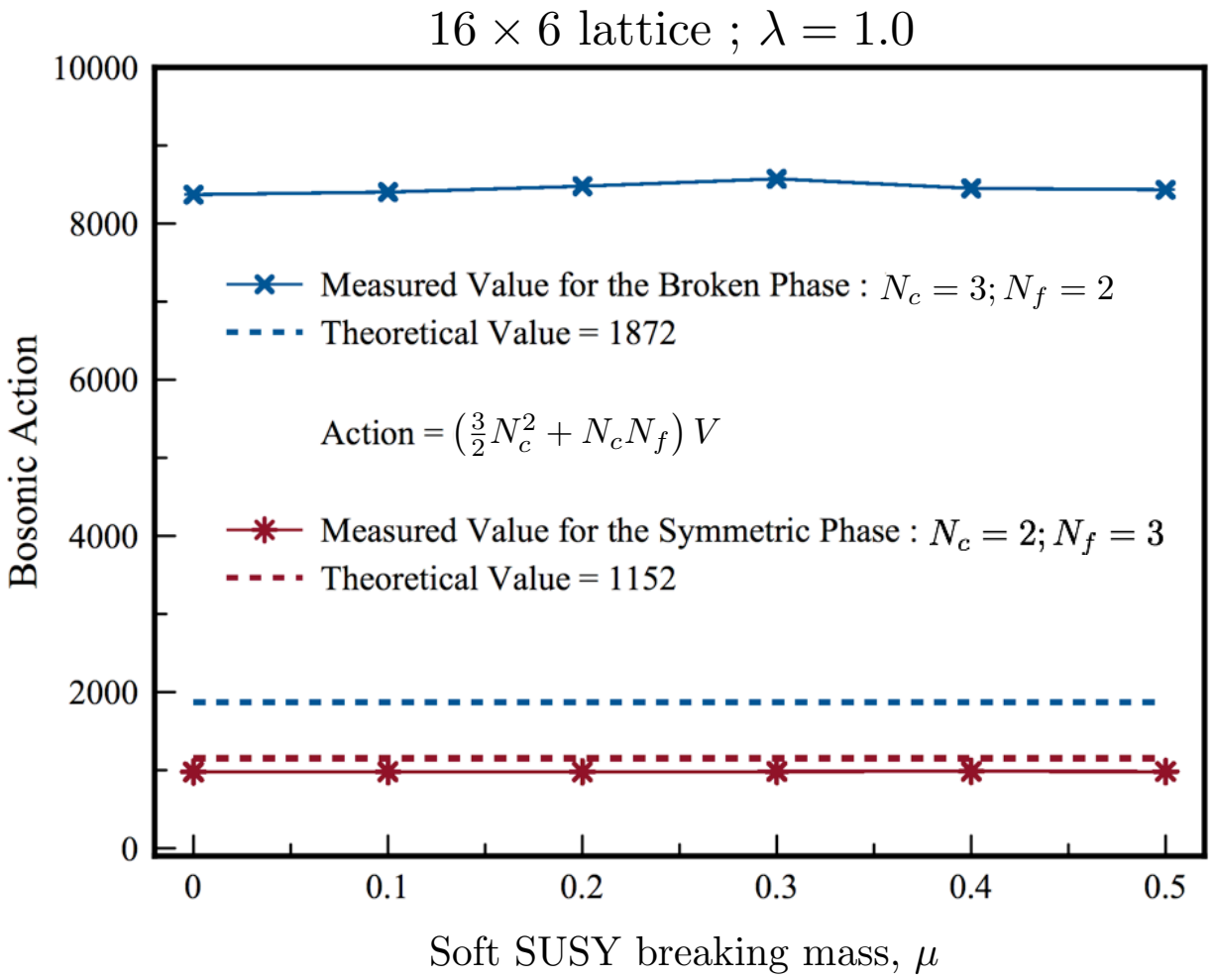
- Complexified gauge field is expanded in the *algebra* as

$$\mathcal{U}_\mu = \mathbb{1} + \mathcal{A}_\mu \tag{1}$$

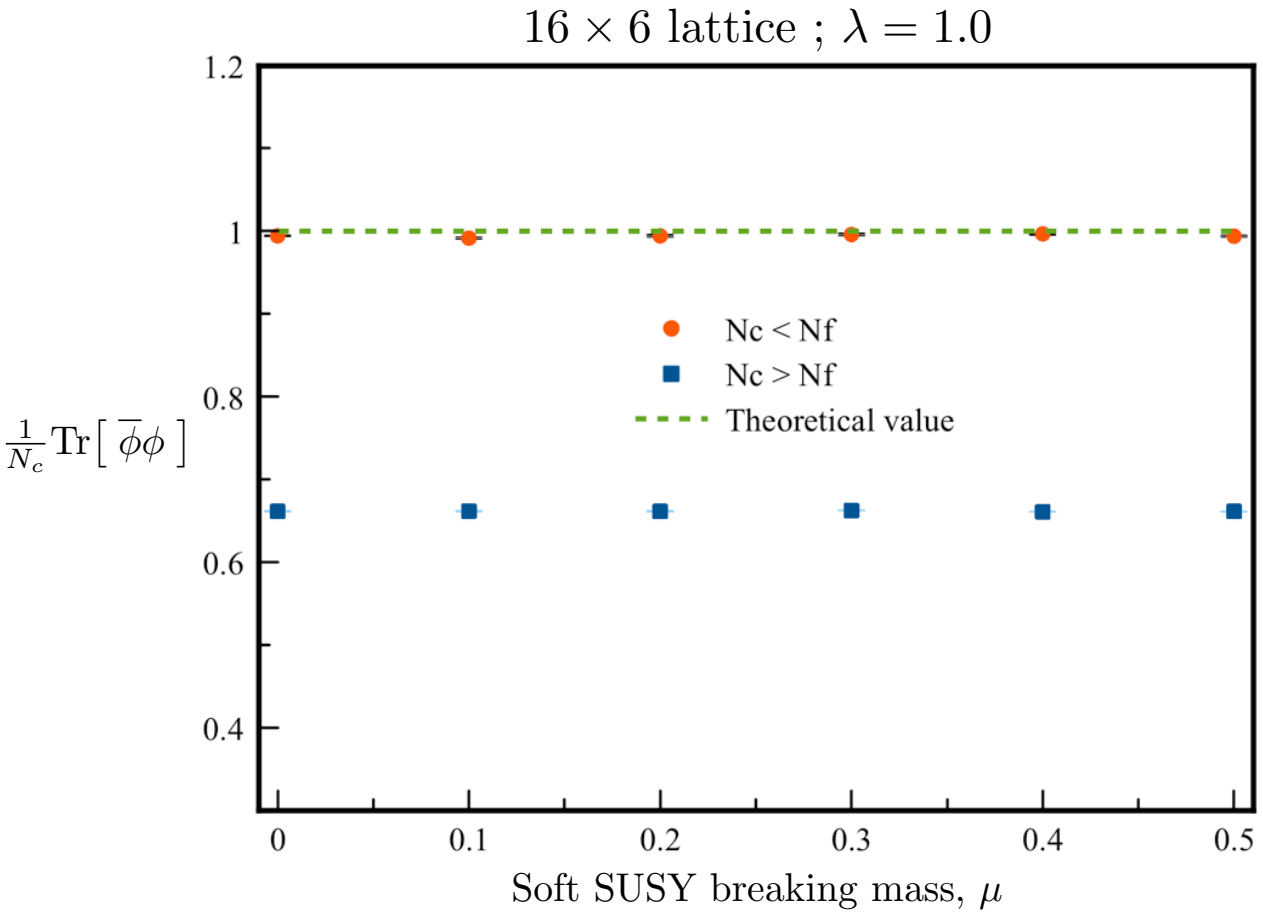
- $\mathbb{1}$ can be interpreted as arising from higgsing the trace mode of the adjoint scalars

$$S_{\text{soft}} = \mu^2 \left[\frac{1}{N} \text{Tr} \ \left(\overline{\mathcal{U}}_\mu \mathcal{U}_\mu \right) - 1 \right]^2 . \tag{2}$$

Bosonic Action



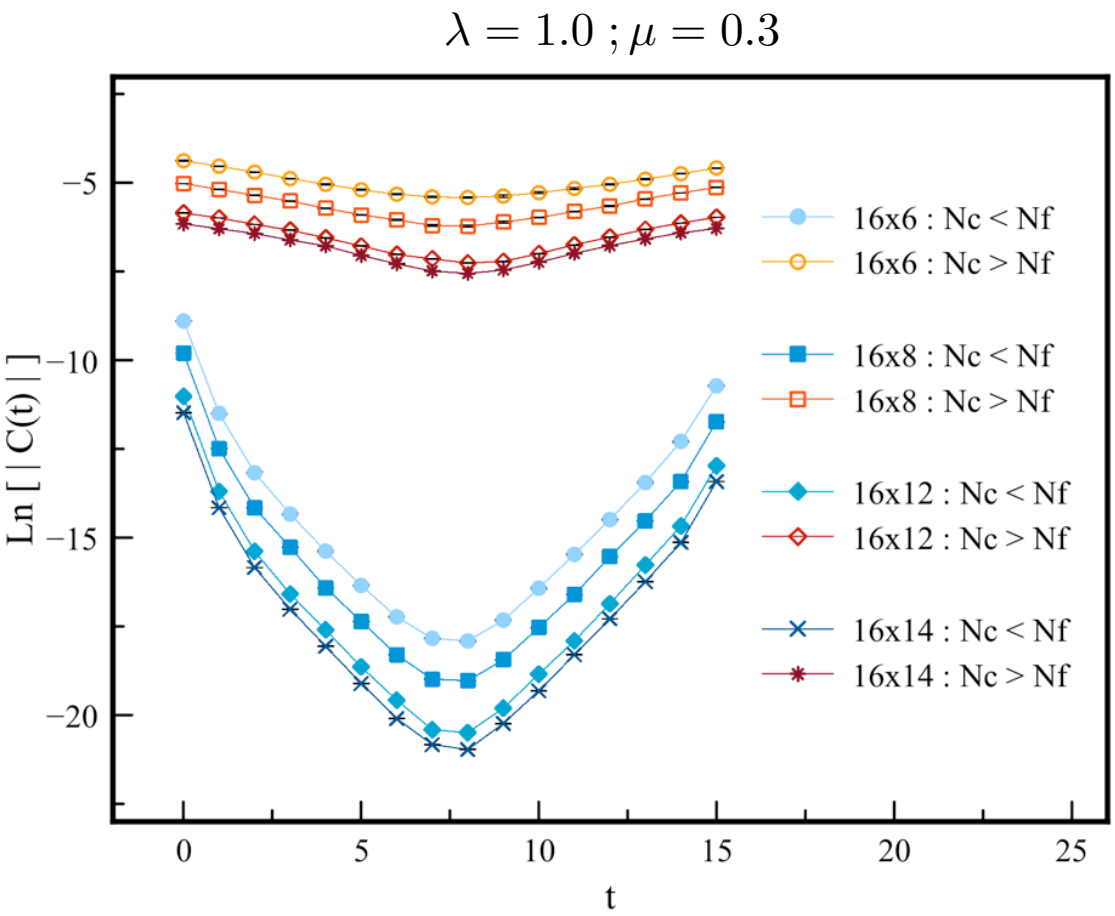
Scalars


16×6 lattice ; $\lambda = 1.0$
1.2
1
0.8
0.6
0.4
$\frac{1}{N_c}\mathrm{Tr}[\,\overline{\phi}\phi\,]$
Nc < Nf
Nc > Nf
Theoretical value
0
0.1
0.2
0.3
0.4
0.5
Soft SUSY breaking mass, μ

Goldstino

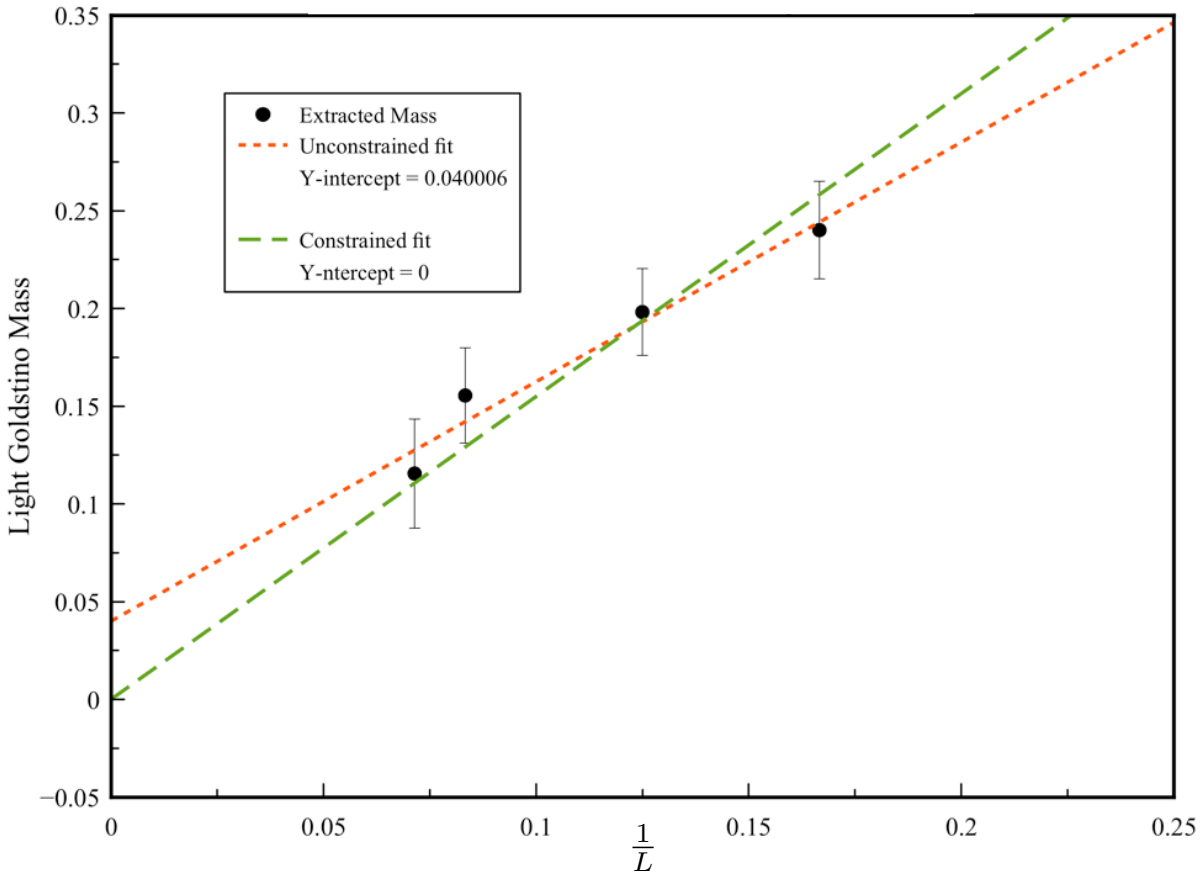
- Non-zero vacuum energy:
  $\langle 0|H|0\rangle \neq 0 \longrightarrow \langle 0|\{\mathcal{Q},\mathcal{Q}_0\}|0\rangle \neq 0$
- Gauge invariant super currents
- $\mathcal{O}(x) = \mathrm{Tr}\left[\sum_\mu \psi_\mu(x)\overline{\mathcal{U}}_\mu(x)\left(\phi(x)\overline{\phi}(x) - \mathrm{rI_{N_c}}\right)\right]$
  $\mathcal{O}'(y) = \mathrm{Tr}\left[\eta(y)\left(\phi(y)\overline{\phi}(y) - \mathrm{rI_{N_c}}\right)\right].$
- $C(t) = \langle \mathcal{O}'(y,t)\mathcal{O}(x,0)\rangle$

Goldstino


$\lambda = 1.0\ ; \mu = 0.3$
Ln [| C(t) |]
t
−5
−10
−15
−20
0
5
10
15
20
25
16x6 : Nc < Nf
16x6 : Nc > Nf
16x8 : Nc < Nf
16x8 : Nc > Nf
16x12 : Nc < Nf
16x12 : Nc > Nf
16x14 : Nc < Nf
16x14 : Nc > Nf

Goldstino mass

$$\lambda = 1.0\ ;\mu = 0.3$$



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Summary

- Constructed a 2 dimensional SQCD model on the lattice
- First numerical investigation of this model
- We see the first signs of SUSY breaking for the scenario $N_c > N_F$
- Future Work :
  - Include a super potential term on the lattice
  - Investigate the dualities inherent in this 2-dimensional model such as Witten's duality