RADIATIVE CORRECTIONS TO MUON DECAY IN ORBIT SPECTRUM

ROBERT SZAFRON

UNIVERSITY OF ALBERTA

Fermilab Theory Seminar

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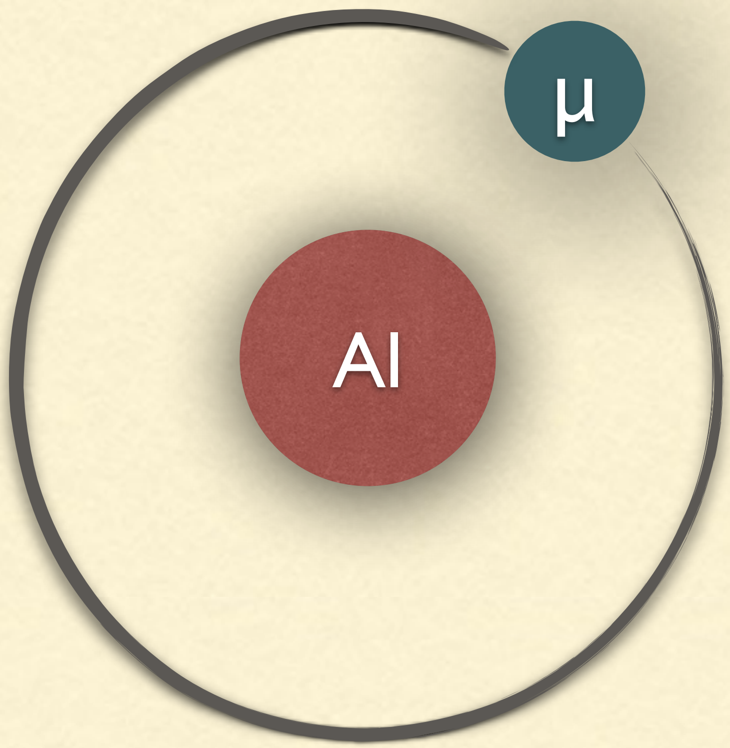
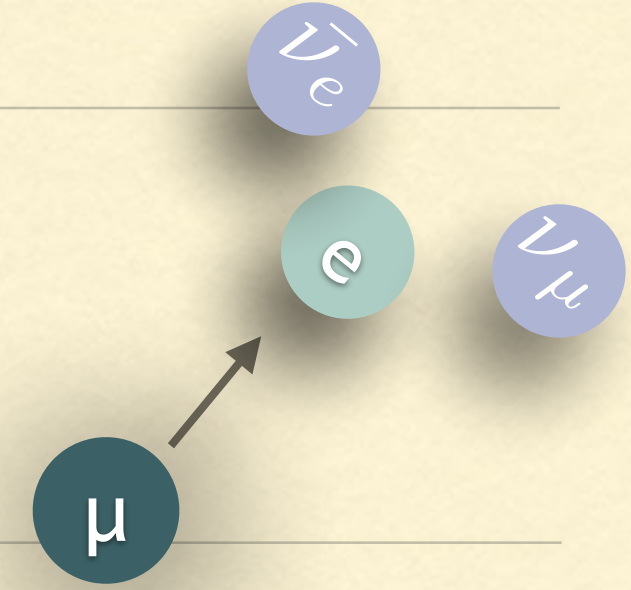
OUTLINE



- Muons
- Bound muons
- Decay in orbit spectrum:
  - central region
  - endpoint region
- Summary

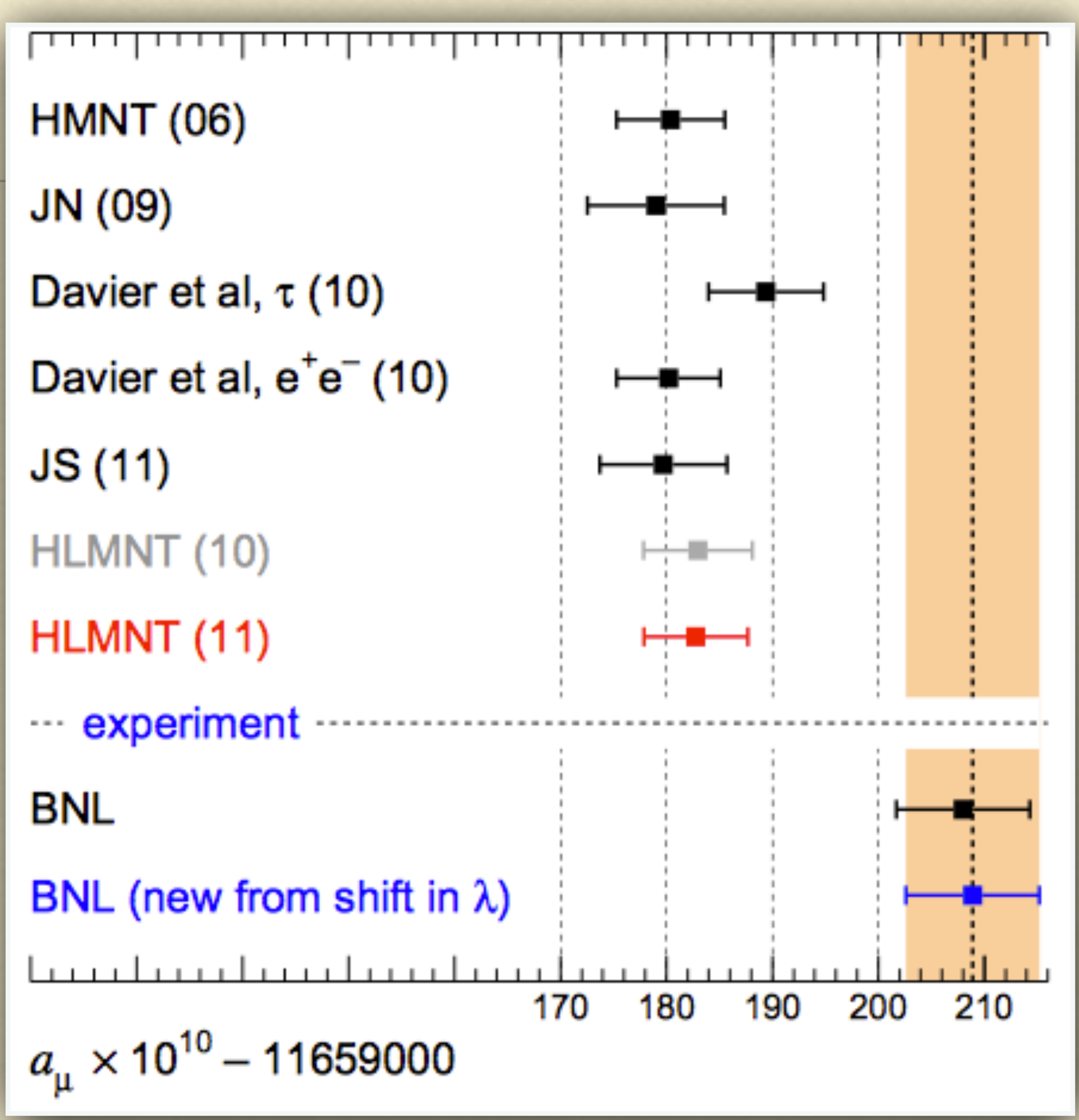
FREE MUON DECAY



	LO	NLO	NNLO
Decay Width	1947 E. Fermi, E. Teller, V. Weisskopf	1956 R. E. Behrends, R. J. Finkelstein, and A. Sirlin	1999 Timo van Ritbergen, Robin G. Stuart
Assymetry	1956 T.D. Lee, C.N. Yang	1959 T. Kinoshita and A. Sirlin	2014 F. Caola , A. Czarnecki, Y. Liang, K. Melnikov, R. S.
Spectrum	1950 Michel, L.	1958 S.M. Berman	2007 C. Anastasiou, K. Melnikov and F. Petriello

ANOMALUS MAGNETIC MOMENT

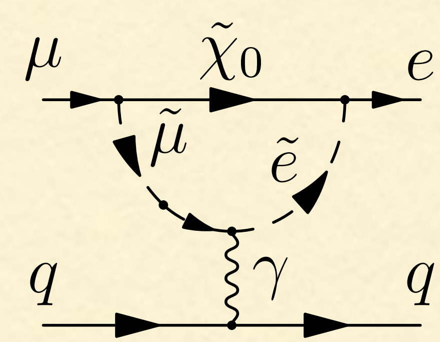
- Persistent ~3.5 sigma discrepancy
- Very high precision is required on both theoretical and experimental side
- Sensitive to hadronic corrections (VP and LBL)
- May indicate some New Physics contribution

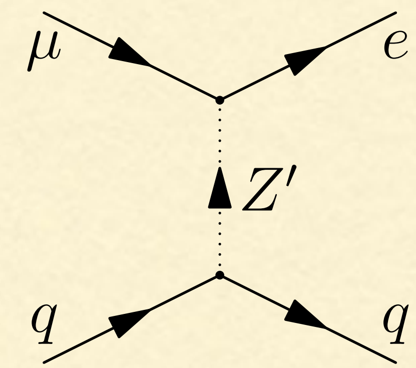


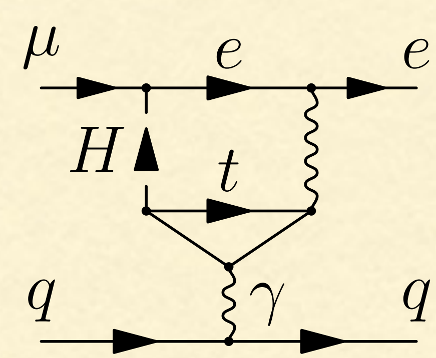
from T. Teubner et al. Nucl.Phys.Proc.Suppl. 225-227 (2012) 282-287

OFF—DIAGONAL DIPOLE MOMENTS

- Similar type of operators may contribute to g-2 and Charged Lepton Flavour Violation (CLFV)
- CLFV is suppressed in SM
- Three interesting CLFV processes
  - $\mu \to e\gamma$
  - muon electron conversion
  - $\mu \to eee$



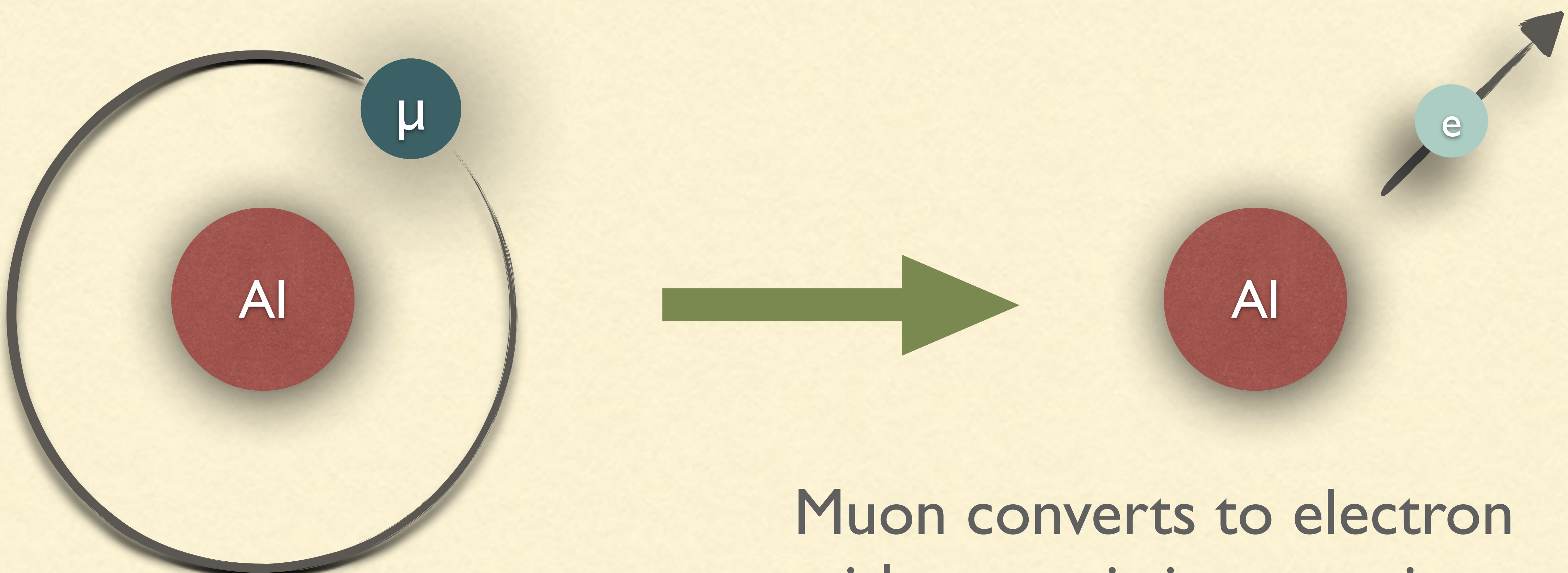




THREE PROCESSES WITH BOUND MUONS

PROCES		SM RATE	WHY IMPORTANT?
Conversion	$(\mu^- N) \to N + e^-$	Negligible	Observation indicates New Physics
Decay in Orbit DIO	$(\mu^- N) \to$ $N + e^- + \overline{\nu_e} + \nu_\mu$	Approximately equal to free muon decay rate	Background to conversion
Capture	$(\mu^- N) \to N' + \nu_\mu$	Depends on Z	Normalization factor for conversion

MUON ELECTRON CONVERSION



Muon converts to electron
without emitting neutrinos
Lepton family number not conserved

MUON ELECTRON CONVERSION

- Clean experimental signature — mono-energetic electron

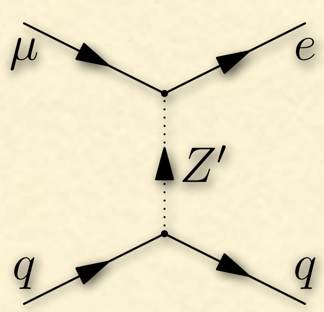
- Current limit on the ratio R of the conversion to the capture

$$R < 7 \times 10^{-13}$$

SINDRUM II
2003

- Planned experiments expect to improve R by ~4 orders of magnitude, this is equivalent to probing New Physics scale up to 10 000 TeV!

- Conversion can probe larger class of operators than $\mu \to e\gamma$

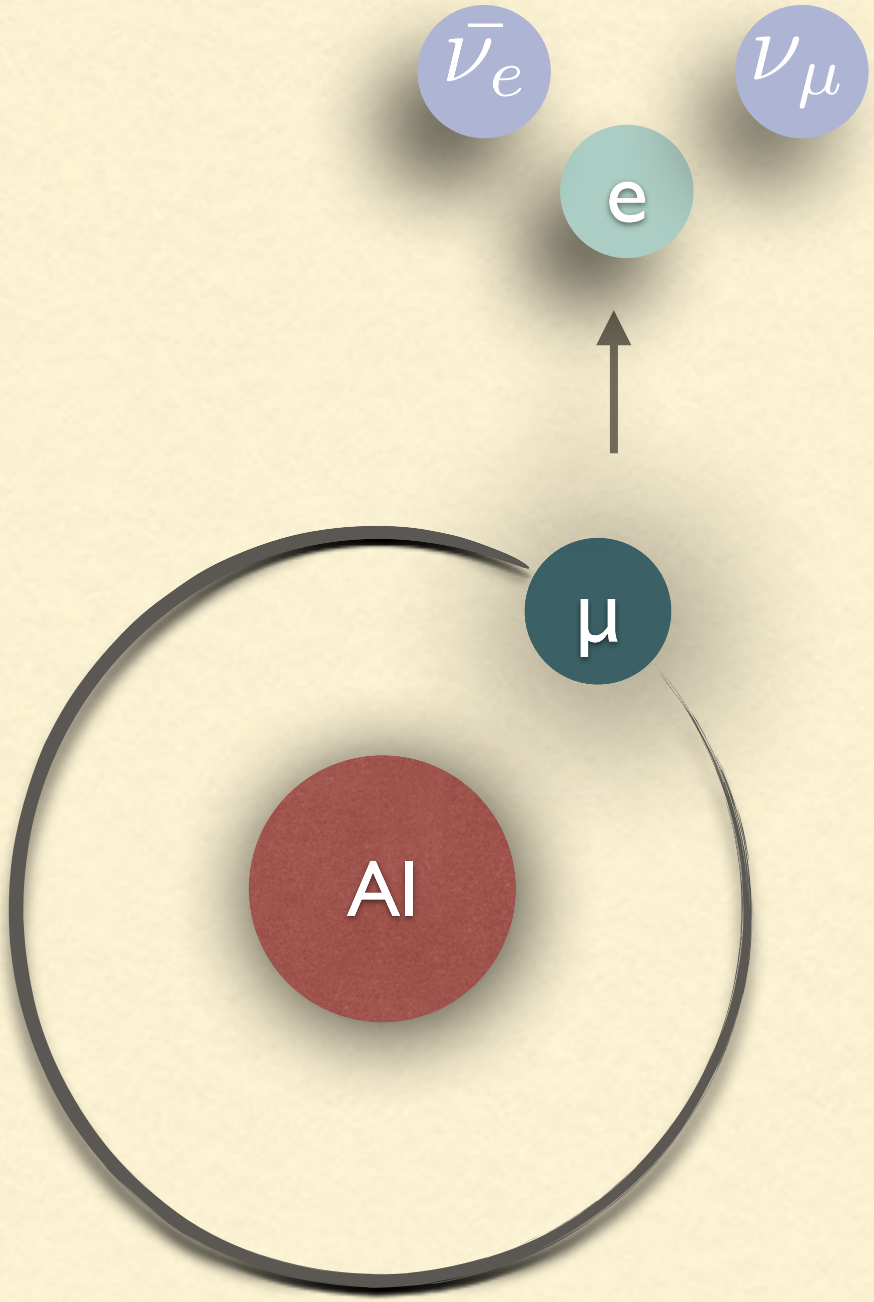


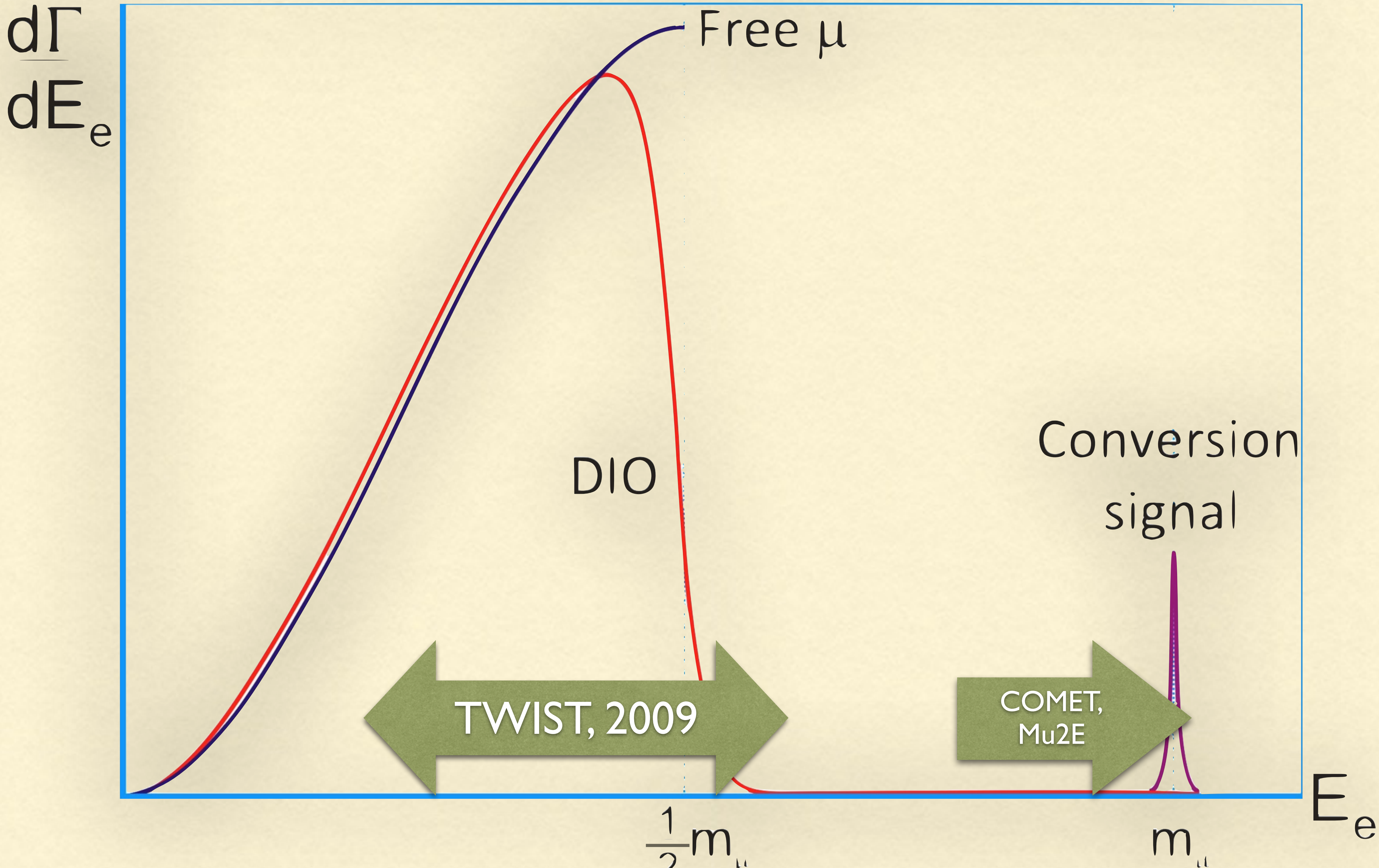
BOUND MUON DECAY

- Muon DIO: standard muon decay into an electron and two neutrinos, with the muon and a nucleus forming bound state

- For a free muon energy and momentum conservation restricts electron spectrum to $E_e < \frac{m_\mu}{2}$

- For DIO momentum can be exchanged between the nucleus and both the muon and the electron

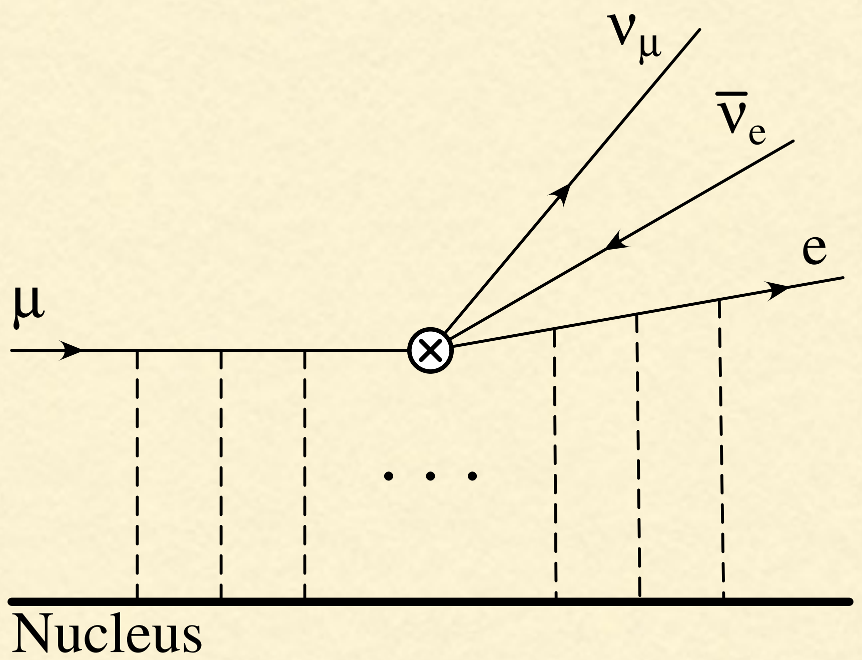




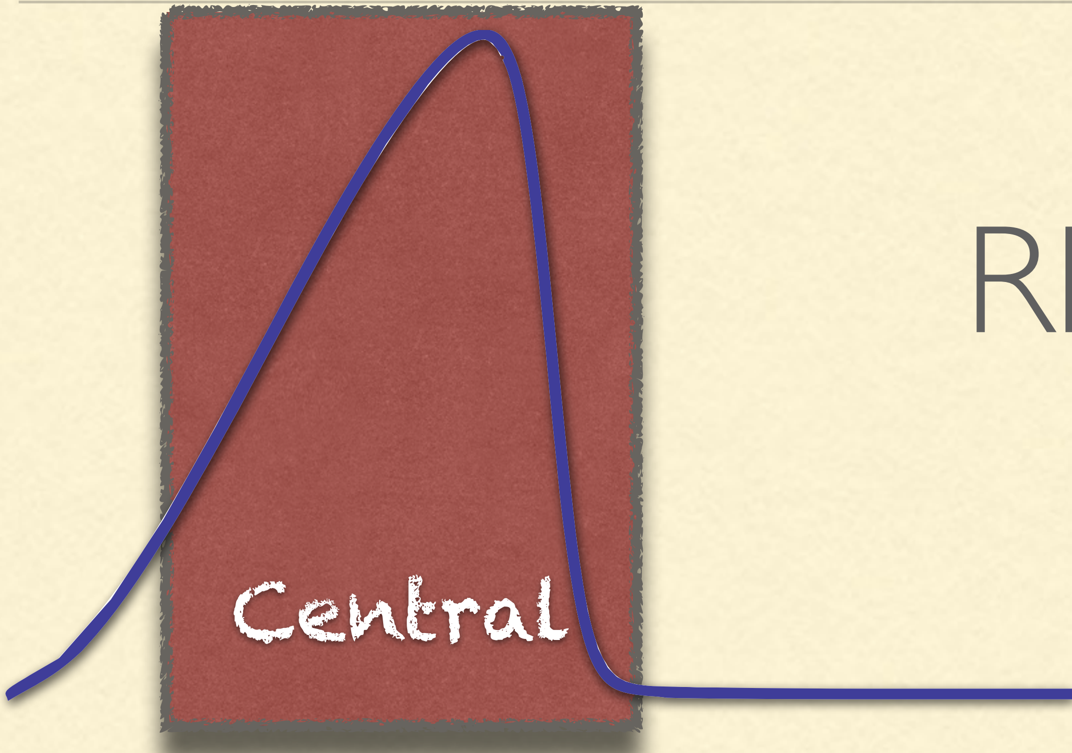
DIO SPECTRUM

HOW TO CALCULATE DIO SPECTRUM?

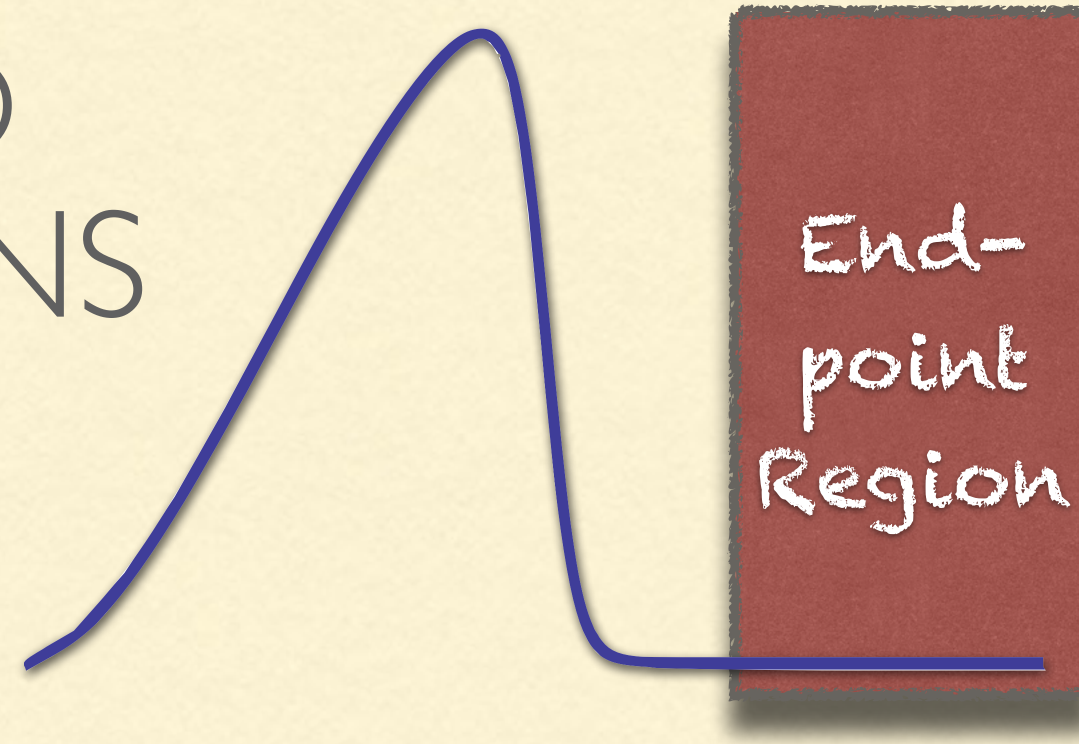
- Spectrum has to be calculated including effects of external (classical) electromagnetic field to all orders



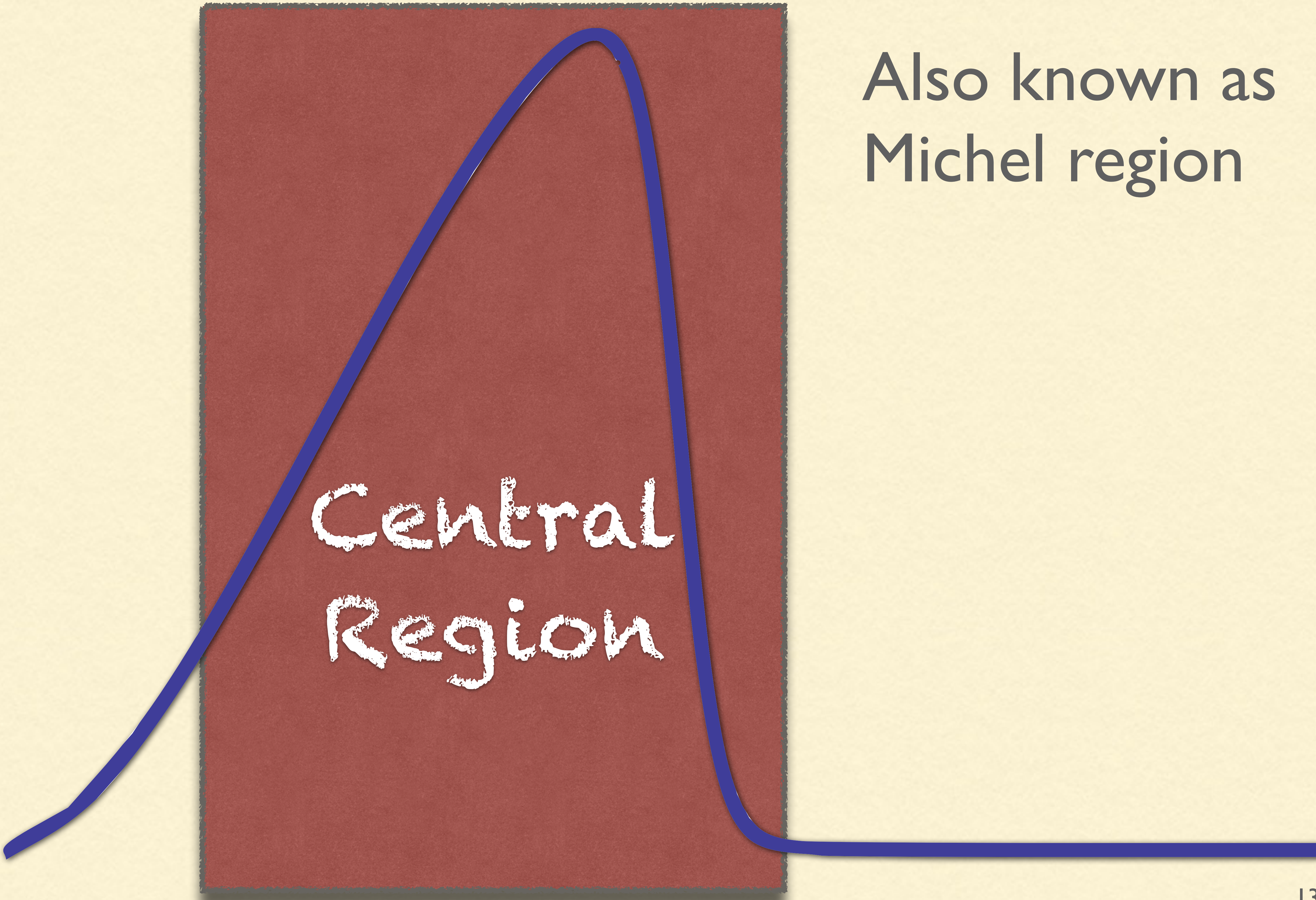
- LO is well known, calculated numerically, including finite size of nucleus
- We need also RC for muon DIO spectrum
- Some simplification can be obtained in different regions of the spectrum



TWO REGIONS



MOST IMPORTANT EFFECT:	
muon motion in an atom	exchange of a hard photon
IMPORTANT CORRECTIONS:	
final state interaction	finite size of the nucleus
	recoil effects
Radiative corrections!	

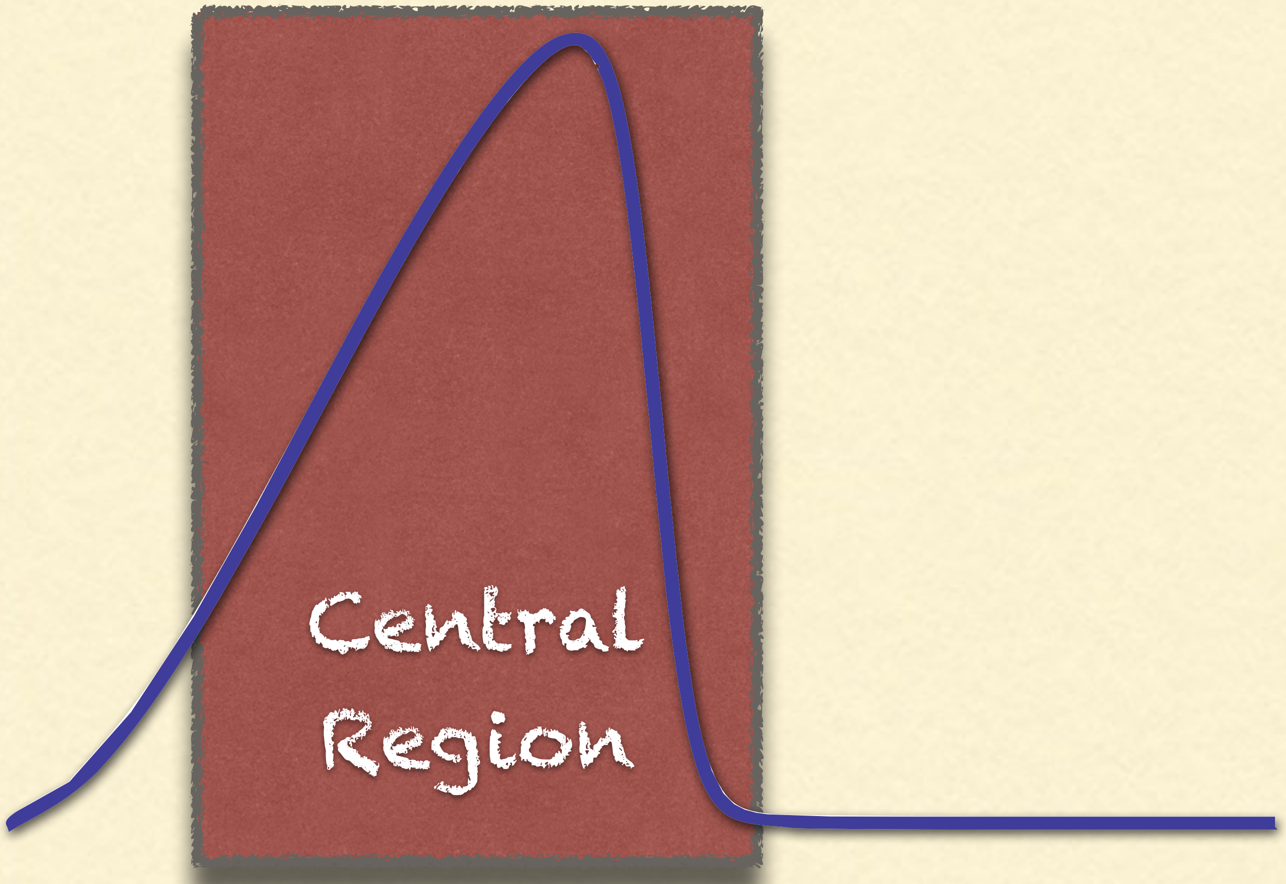


Also known as Michel region

CENTRAL REGION

$$m_\mu Z\alpha \ll E_e \lesssim \frac{m_\mu}{2}$$

- Typical momentum transfer between nucleus and muon is of the order of $m_\mu Z\alpha$
- External field effects have to be resumed
- Dominant effect — muon motion in the initial state



HQEFT & MUONIC ATOM

- Similar problems appear for semileptonic $B \to Xl\nu$ decays of mesons containing Heavy Quarks

M. Neubert 94
I. Bigi et al. 94

- Heavy Quarks in QCD: scale separation $M \gg \Lambda_{QCD}$
- Muonic atom: $m_\mu \gg m_\mu Z\alpha$
- HQEFT solution: shape function, a non-perturbative object
- QED solution: we can define an analog of the shape function and, what is more important, we can calculate it!

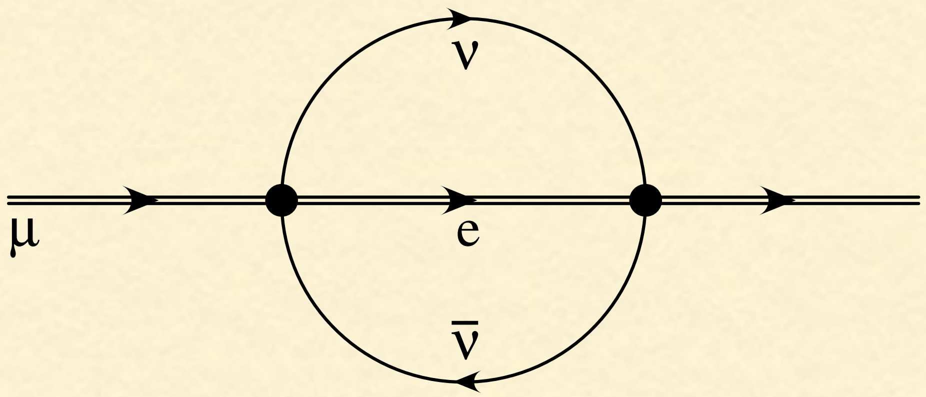
QED SHAPE FUNCTION

Electron is almost on-shell

- Electron propagator in the external field

$$\frac{1}{(p_e+\pi)^2} \approx \frac{1}{p_e^2 + 2p_e\cdot\pi} \to \delta(p_e^2 + 2p_e\cdot\pi)$$

$$\pi_\mu = i\partial_\mu - eA_\mu$$



- We are interested only in the leading corrections

KINEMATICS

- Electron is off-shell $p_e^2 \sim m_\mu^2 Z\alpha$
- so p_e can be written in terms of a light-like $n^2 = 0$ vector

$$\boxed{p_e^\mu = E_e n^\mu + \delta p_e^\mu} \qquad \delta p_e \sim m_\mu Z\alpha$$

- This justifies resummation $p_e^2 \sim p \cdot \pi$
- $\delta\left(p_e^2 + 2p_e \cdot \pi\right) = \delta\left(p_e^2 + 2E_e n \cdot \pi\right)$

A USEFUL TRICK

- We can rearrange the delta function:

$$\delta\left(p_e^2 + 2E_e n\cdot\pi\right) = \int \mathrm{d}\lambda \delta\left(p_e^2 + 2E_e\lambda\right)\delta\left(\lambda - n\cdot\pi\right)$$

Only kinematical, commuting variables

Non perturbative, function of λ

QED SHAPE FUNCTION

- Shape function is defined as an expectation value:

Momentum distribution

$$S(\lambda) = \int \mathrm{d}^3x \psi^\star(x)\delta(\lambda - n\cdot\pi)\psi(x)$$

Final state interaction

- We chose lightcone gauge $n \cdot A = 0$

$$S(\lambda) = \int \frac{d^3k}{(2\pi)^3} \left|\psi_{l.c.}(k)\right|^2 \delta(\lambda + \vec{n}\cdot\vec{k})$$

- Normalization: $\int_{-\infty}^{\infty} d\lambda S(\lambda) = 1$

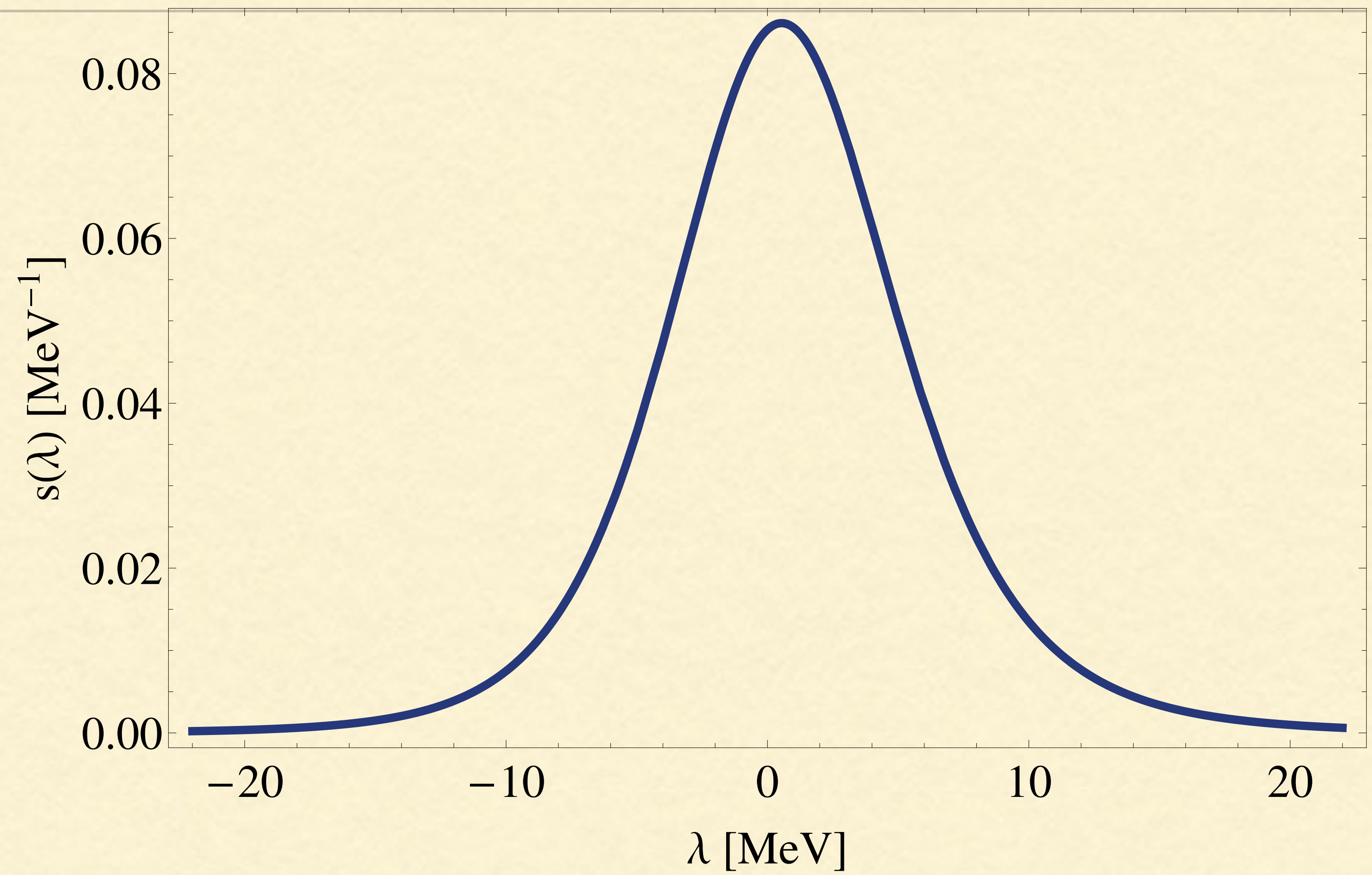
POWER COUNTING

- $\lambda \sim \frac{p_e^2}{2E_e} \sim m_\mu Z\alpha$ (muon momentum in an atom)
- Shape function behaves as $S(\lambda) \sim \frac{1}{Z\alpha}$
- First moment is zero in the leading order $\int d\lambda \lambda S(\lambda) \sim (Z\alpha)^2$
- Second moment $\int d\lambda \lambda^2 S(\lambda) = \frac{1}{3}(m_\mu Z\alpha)^2$

QED SHAPE FUNCTION

- Finally a factorization formula can be derived

$$\boxed{\frac{d\Gamma_{DIO}}{dE_e} = \frac{d\Gamma_{Free}}{dE_e} * S}$$



Momentum distribution of muon in an atom

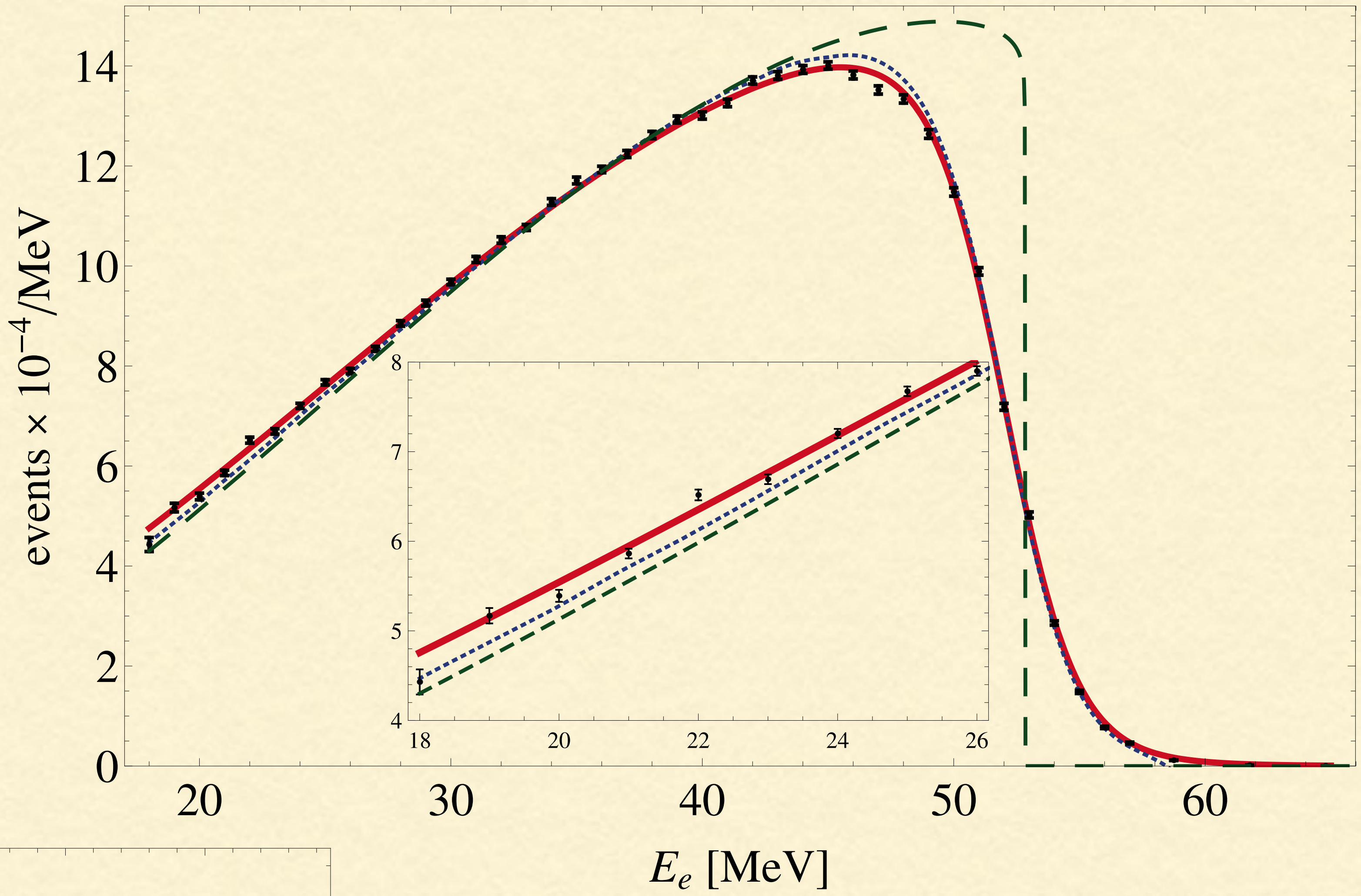
$$S(\lambda) = \int \mathrm{d}^3 x \psi^\star(x) \delta(\lambda - n \cdot \pi) \psi(x)$$

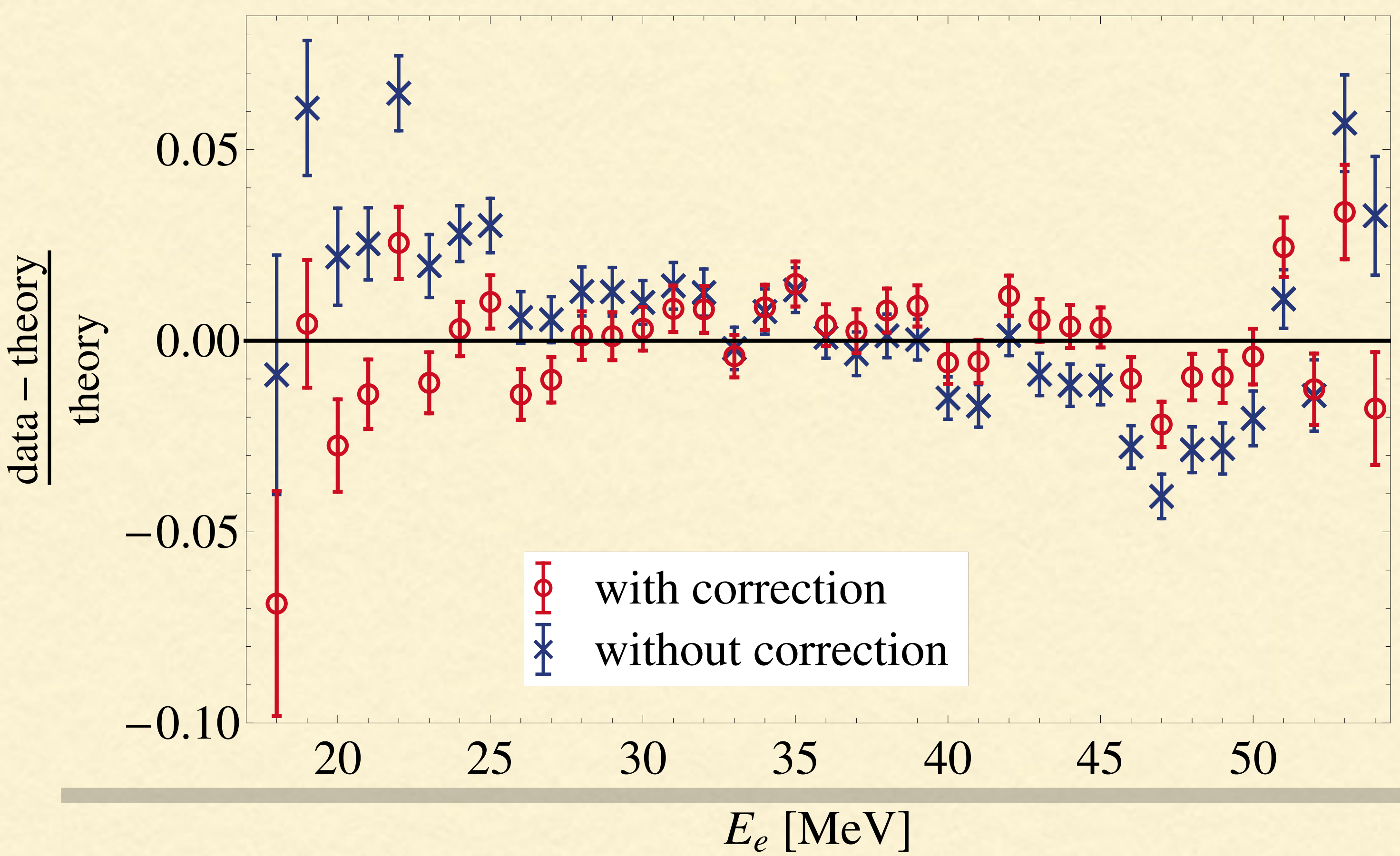
n - a lightlike vector,
plot for Aluminium, Z=13

LEADING CORRECTIONS

and their relation to the experimental data

TWIST 2009, PRD80,052012

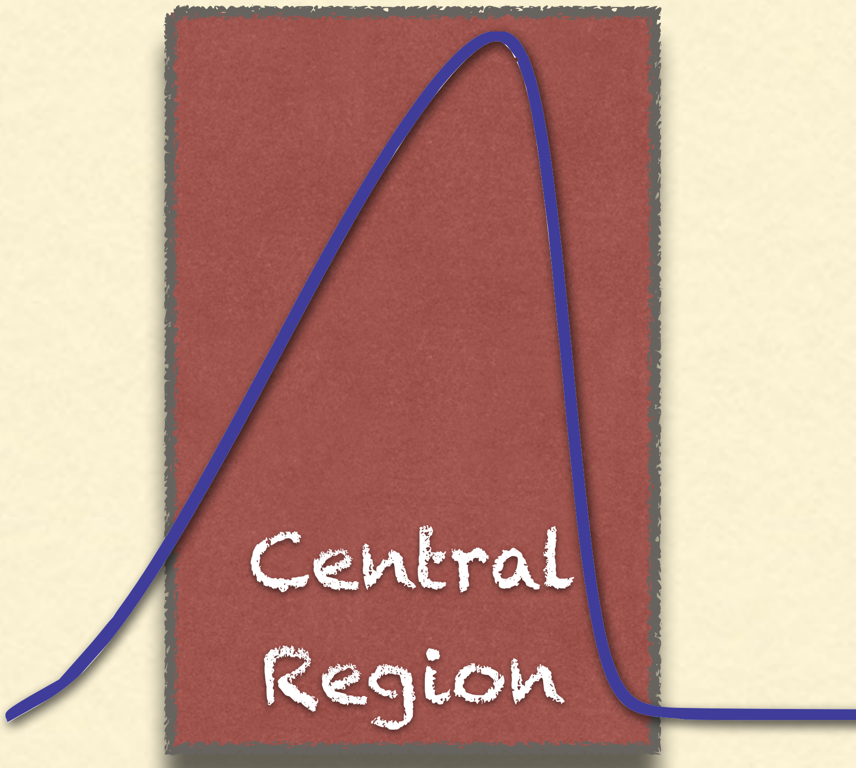


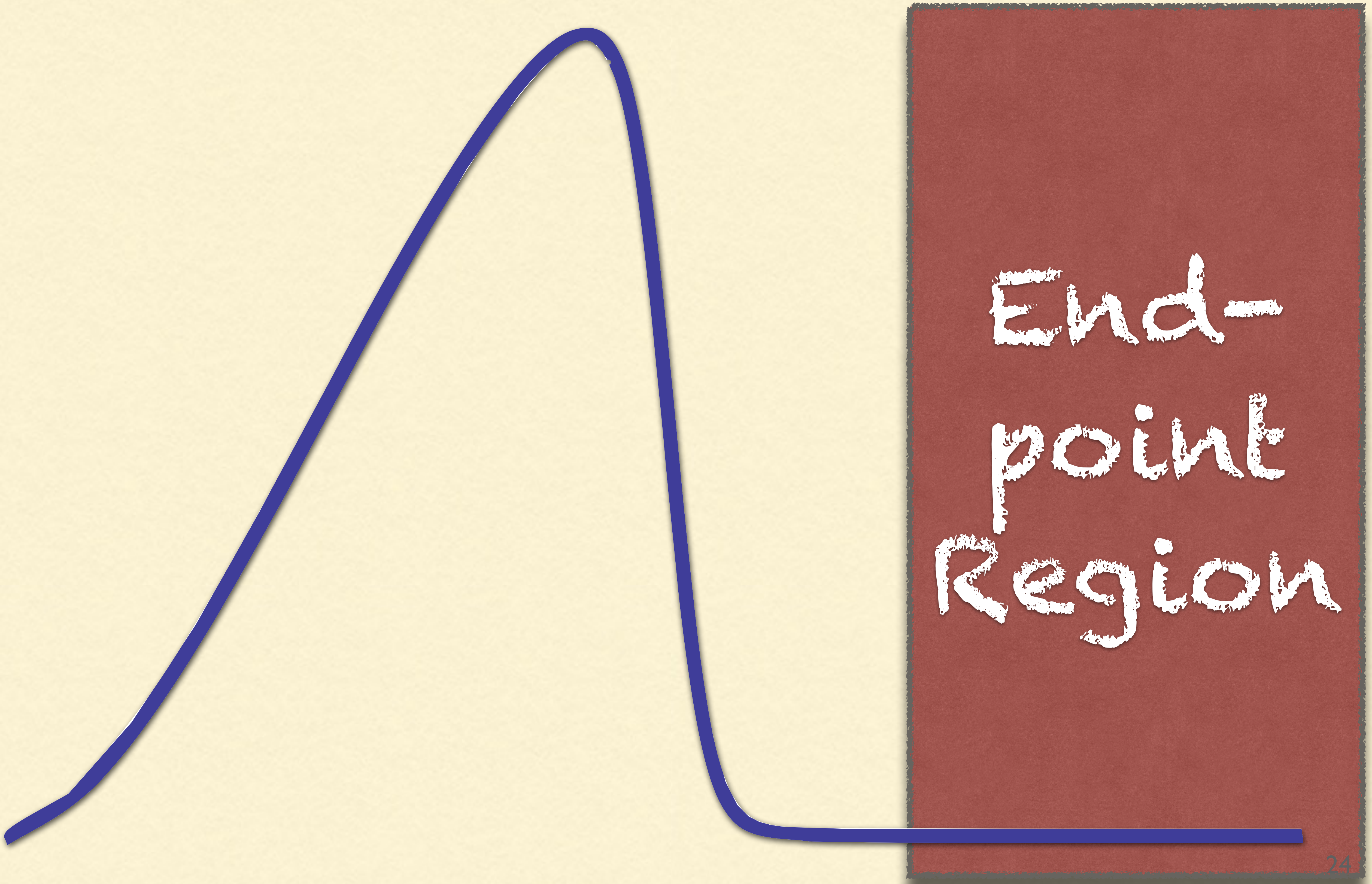


A. Czarnecki, M. Dowling, X. Garcia i Tormo , W. Marciano, RS
Phys. Rev. D90 (2014), 093002

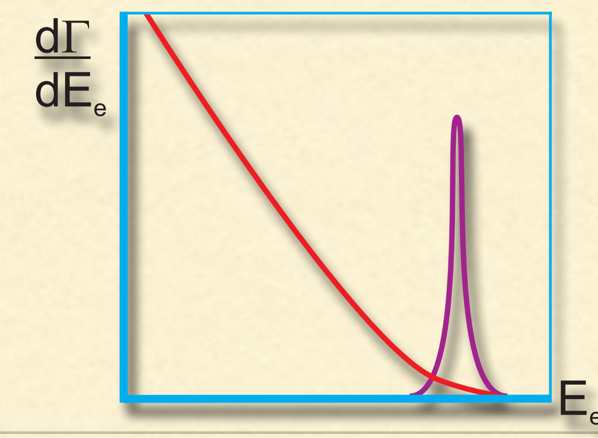
CENTRAL REGION SUMMARY

- Radiative corrections are more important than the higher order binding effects
- Heavy Quark factorization approximation works also for muons
- Gauge invariance requires that the final state interaction is also included
- Momentum transfer to nucleus is small




End-point Region

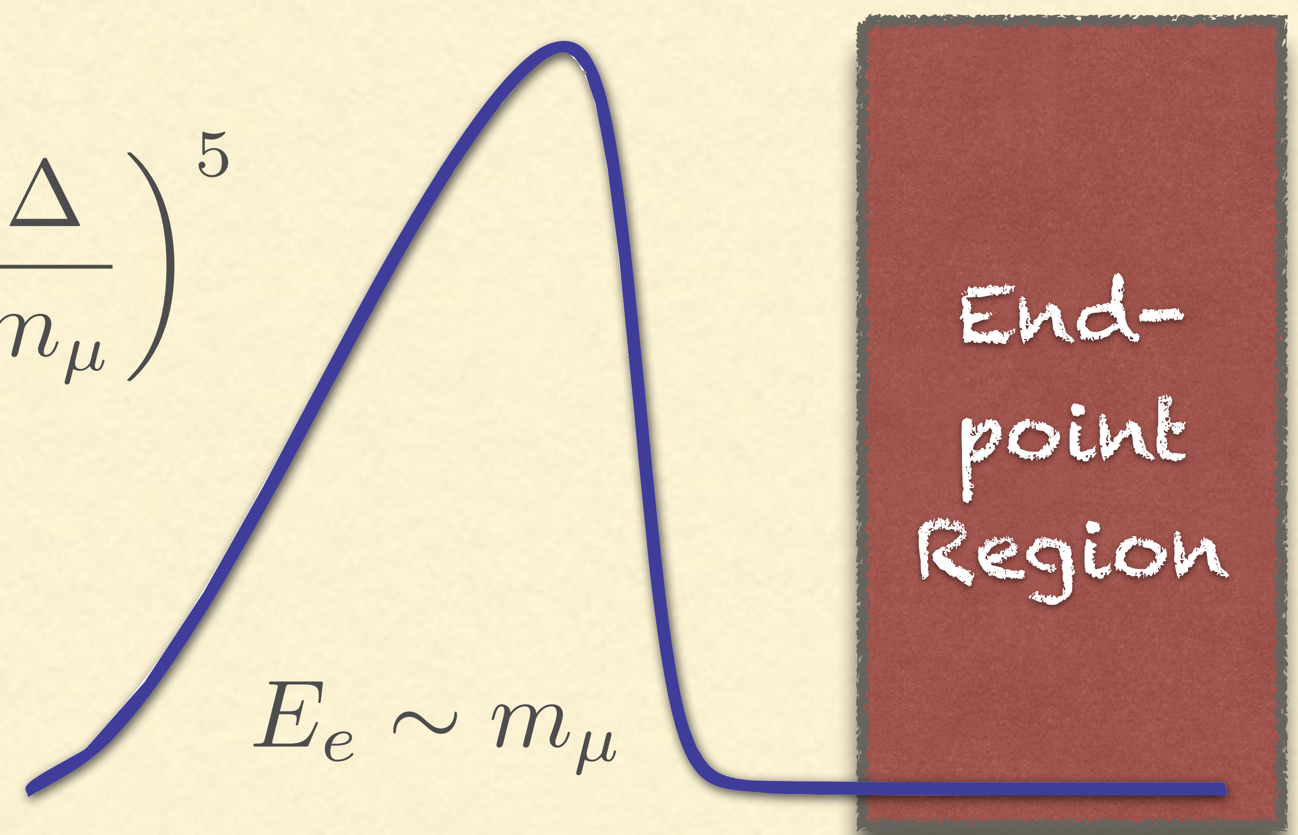
ENDPOINT REGION



- Typical momentum transfer between the nucleus and the muon is of the order of the muon mass
- Both wave functions and propagators can be expanded in powers of $Z\alpha$

$$\frac{m_\mu}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi}(Z\alpha)^5\left(\frac{\Delta}{m_\mu}\right)^5$$

$$\Delta = E_{max} - E_e$$



ENDPOINT ENERGY

$$E_{max} = m_\mu + E_b + E_{rec}$$

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$

Binding energy

$$E_{rec} \approx -\frac{m_\mu^2}{2m_N}$$

Recoil energy

(kinetic energy of the nucleus)

Both corrections decrease the endpoint energy

HOW TO OBTAIN THE LEADING TERM

- Electron WF: Plain wave + leading corrections

$$\overline{\psi}_p(\vec{q}) = \bar{u}(p)\left[\delta^3(\vec{p}-\vec{q}) + \not{V}\left((\vec{p}-\vec{q})^2\right)\frac{1}{\not{q}-m_e}\right]$$

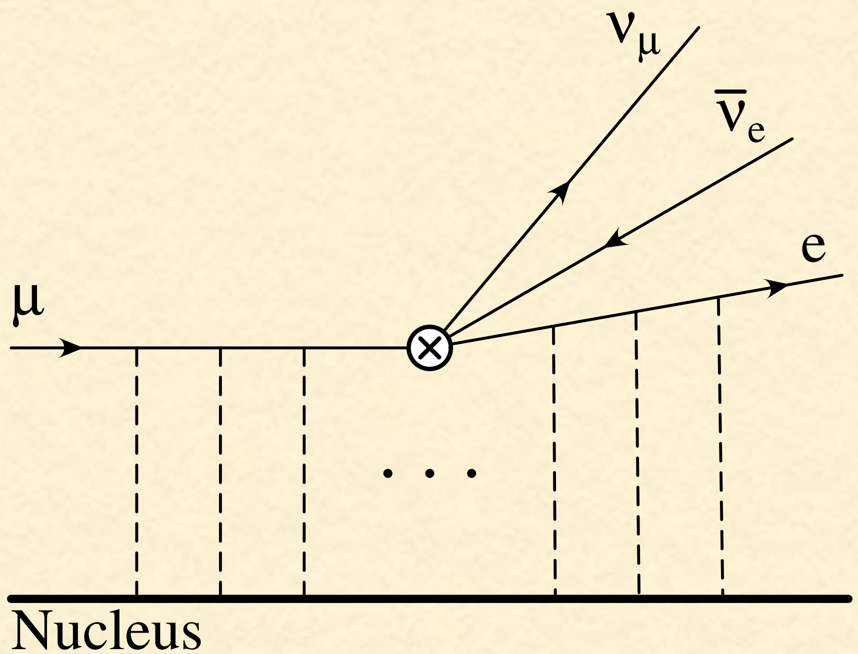
- Muon WF: Non relativistic correction + first correction

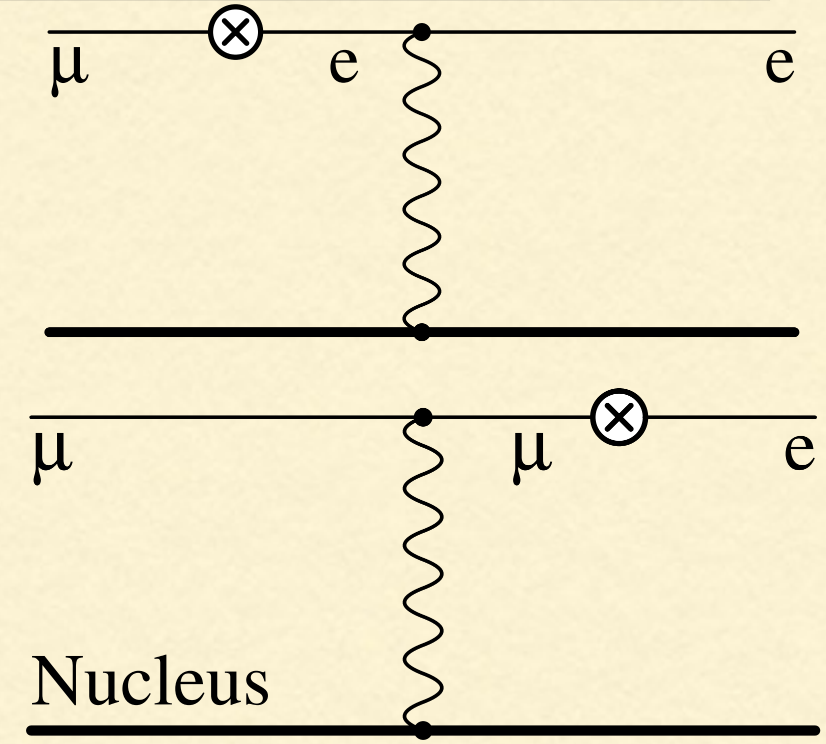
$$\psi(\vec{q}) = \psi_{NR}(\vec{q})\left(1 + \frac{\vec{q}\cdot\vec{\gamma}}{2m_\mu}\right)u(P)$$

- **Born approximation is not applicable**

Compare photoelectric effect, Sauter 1931

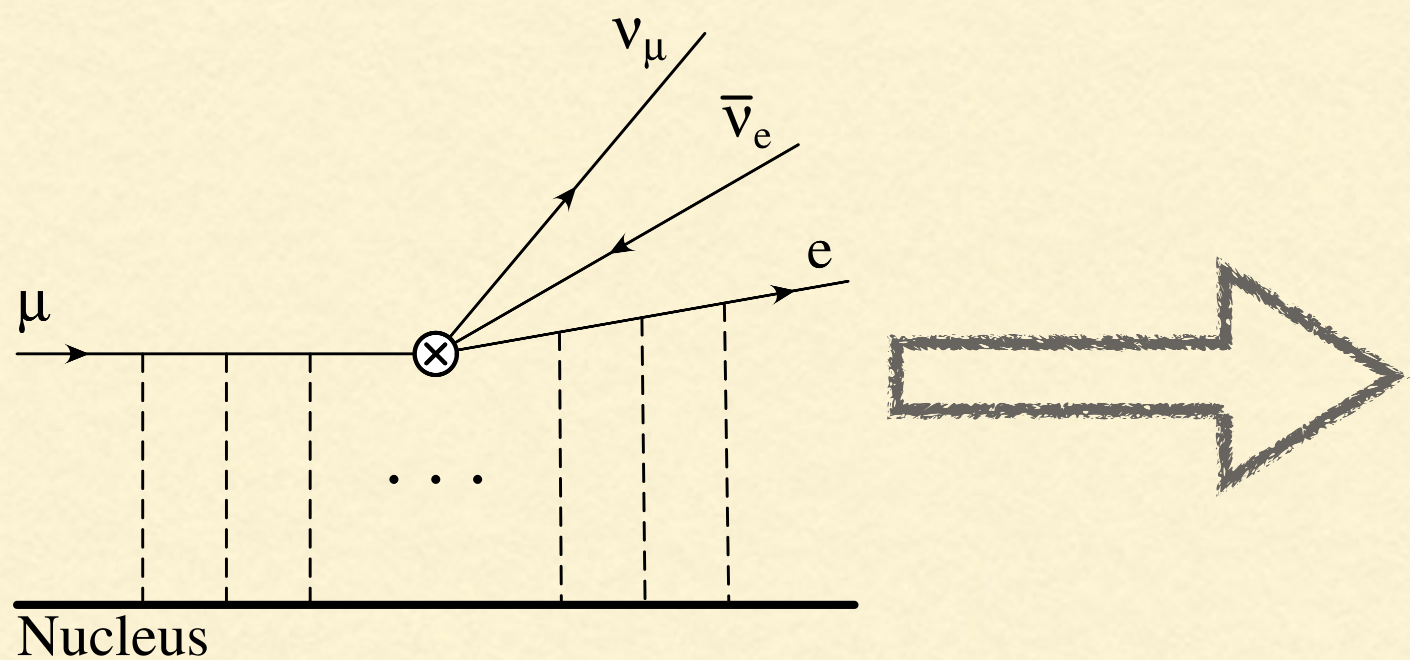
ENDPOINT EXPANSION

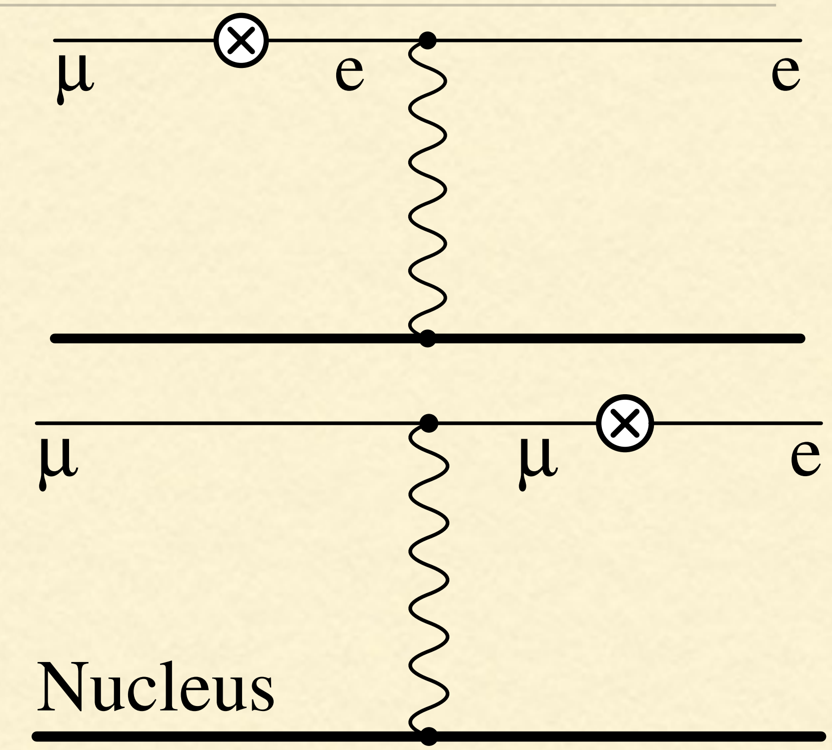




$$\int \frac{\mathrm{d}^3\nu}{\nu_0}\frac{\mathrm{d}^3\overline{\nu}_0}{\overline{\nu}_0}\delta\left(\Delta-\nu_0-\overline{\nu}_0\right)\ldots\psi\ldots\overline{\psi}\sim\Delta^5$$

ENDPOINT EXPANSION



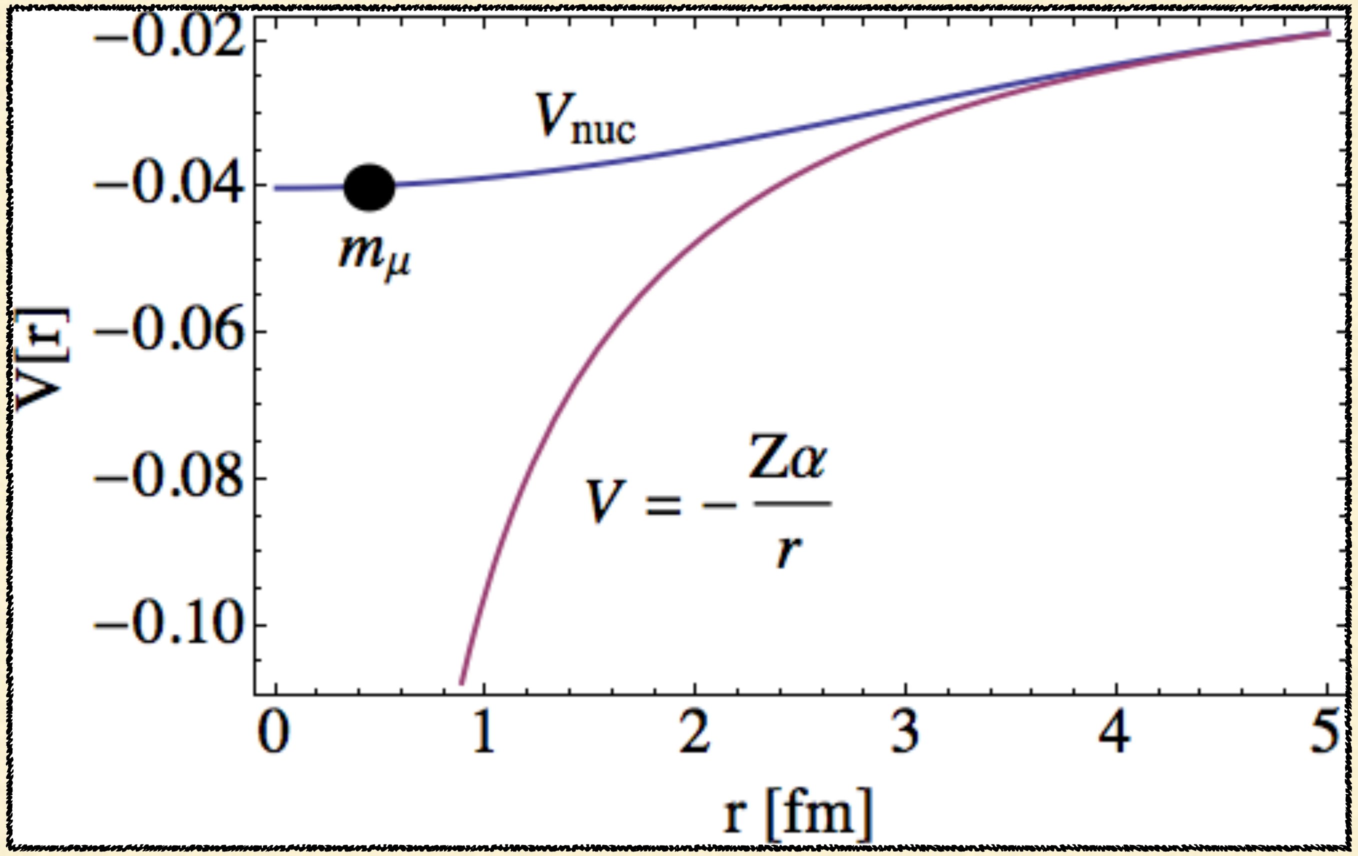


$$V\left(\vec{k}^2\right) = -\frac{Z\alpha}{2\pi^2\vec{k}^2}$$

$$\mathcal{A} \sim \psi(0) \times V(m_\mu^2) \sim (Z\alpha)^{\frac{3}{2}} \times Z\alpha = (Z\alpha)^{\frac{5}{2}}$$

FINITE NUCLEUS SIZE CORRECTION

- In the endpoint region momentum transfer is of the order of the muon mass, this is much smaller than the size of the Aluminium nucleus



- Two main effects:

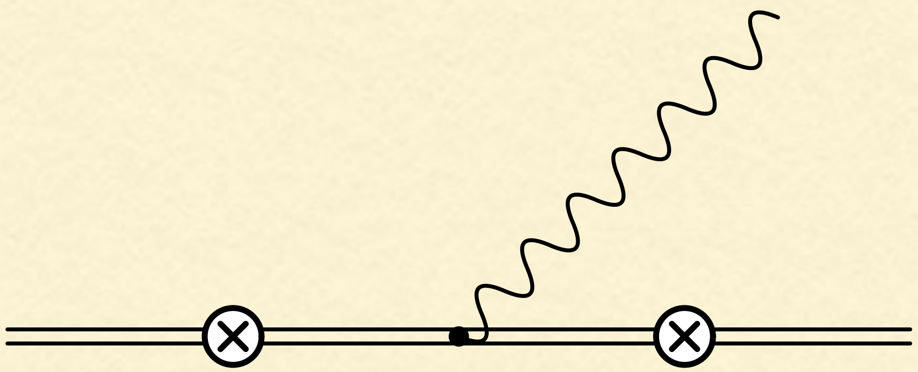
$$F_\rho(\vec{k}^2) = \frac{V_\rho\left(\vec{k}^2\right)}{V\left(\vec{k}^2\right)}$$

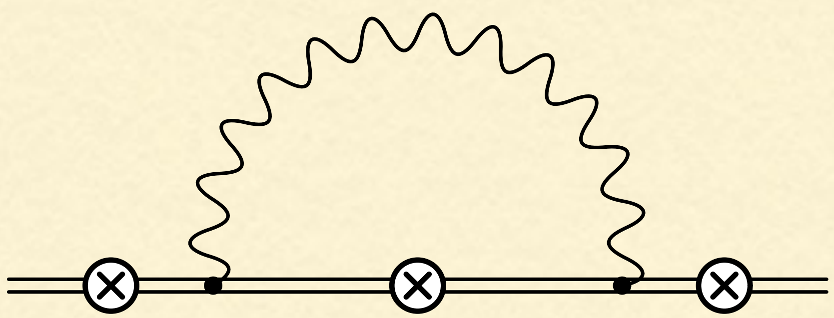
Nucleus formfactor

$$R = \frac{\chi(0)^2}{\Psi(0)^2} = 0.71.$$

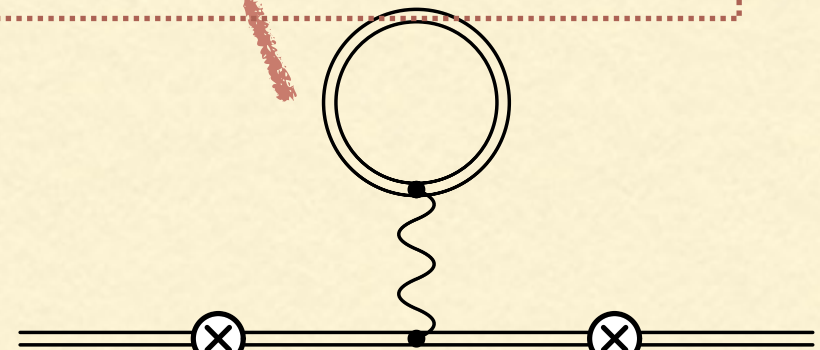
Wave function at zero

RADIATIVE CORRECTIONS





$$\frac{1}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} = \Delta^5 \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5 \left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S} \left(1 + \frac{\alpha}{\pi}\delta_{VP} + \frac{\alpha}{\pi}\delta_H\right)$$



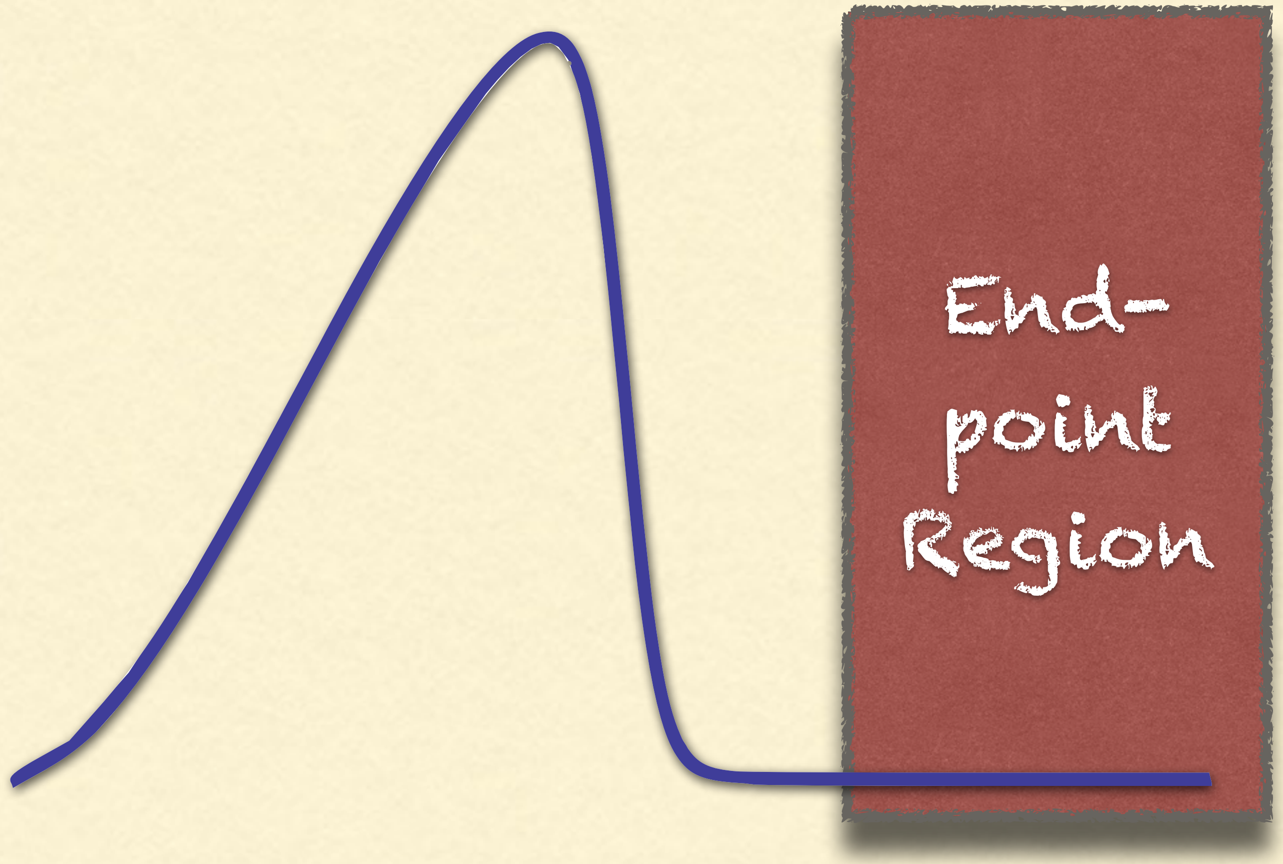
$$\delta_S \approx 10.0, \quad \delta_H \approx -15.6, \quad \delta_{VP} \approx 6.1$$

HIGHER ORDER TERMS

- Expansion parameter is $\pi Z\alpha$, this is again very similar to the calculations of photoelectric effect
- Higher order terms were calculated numerically, they give -21% correction for a point-like nucleus
- Finite-Size nucleus corrections suppress higher order terms
- Also higher orders in Δ may be required for precise determination of experimental background

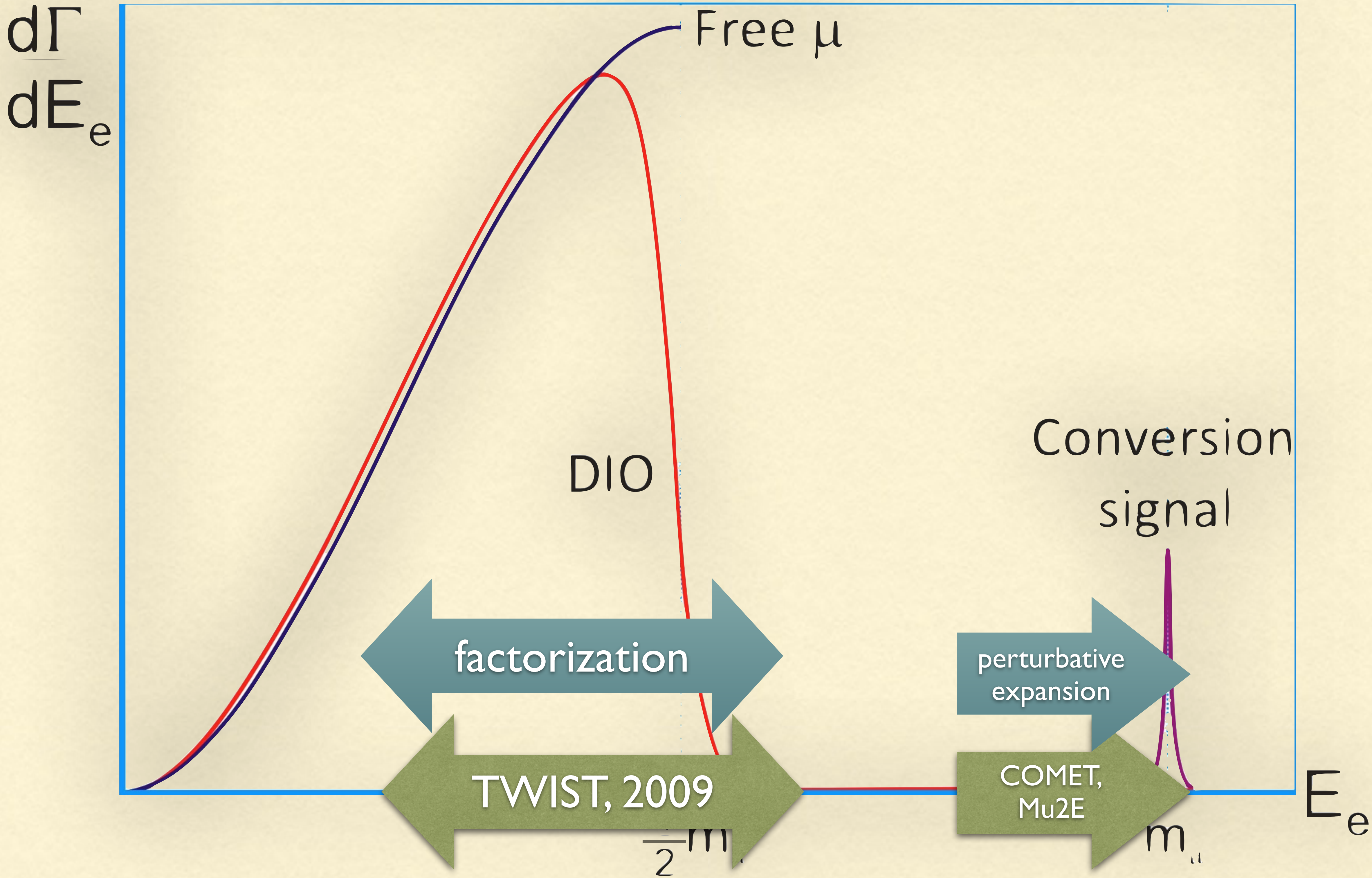
ENDPOINT REGION SUMMARY

- Large momentum transfer — we need to worry about finite nucleus size and recoil
- We can use Feynman diagrams
- Higher order terms are still a challenge for the future



SUMMARY

- Searches for rare decays require accurate predictions for the SM background
- TWIST measurement of the DIO spectrum is sensitive to RC
- Muon DIO spectrum:
  - We have expansion in different regions
  - Ultimate goal is a correction to spectrum in the whole energy range
- Numerical evaluation of the spectrum at the one loop level may be necessary



dΓ
dE_e
Free μ
DIO
Conversion
signal
factorization
TWIST, 2009
perturbative
expansion
COMET,
Mu2E
E_e

THANK YOU!