Calculating the long-distance contributions to neutral kaon mixing

Fermilab

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RBC and UKQCD Collaborations

Outline

- Review $K^0 - \overline{K^0}$ mixing
- Current lattice capability
- The $K_L - K_S$ mass difference
  - Euclidean space approach
  - Finite volume corrections
  - Numerical results
- Long-distance corrections to ε_K
  - Treating logarithmic divergences
- Outlook

RBC Collaboration

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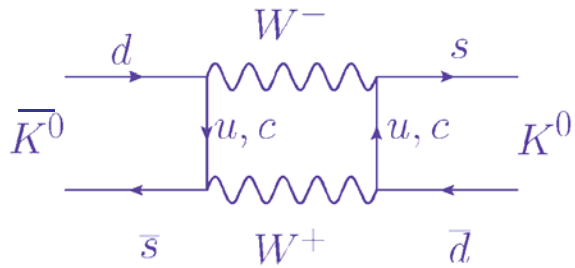
  - Marina Marinkovic (CERN)
  - Antonin Portelli
  - Chris Sachrajda
  - Matthew Spraggs
  - Tobi Tsang

K^0–$\overline{K^0}$ mixing

$K^0 - \overline{K}^0$ mixing

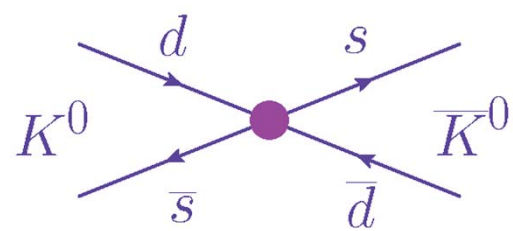
- ΔS=1 weak interactions allow $\overline{K}^0$ and K^0 to mix.



- New eigenstates are approximately:

$$K_+ = \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \qquad K_- = \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0)$$

- Small mass difference: ΔM_K = 3.483(6) x 10^{-12} MeV
- Effective $\frac{\alpha_{EM}}{\Lambda^2}(\bar{s}d)(\bar{s}d)$ operator would give ΔM_K = 3 x 10^{-12} MeV for Λ = 1000 TeV



$K^0 - \bar{K}^0$ mixing: Indirect CP Violation

- CP violation leads to K_L and K_S states which are not CP eigenstates:

$$K_S = \frac{K_+ + \bar{\epsilon} K_-}{\sqrt{1 + |\bar{\epsilon}|^2}} \qquad K_L = \frac{K_- + \bar{\epsilon} K_+}{\sqrt{1 + |\bar{\epsilon}|^2}}$$

- Here $\bar{\varepsilon}$ is closely related to $\epsilon_K = \bar{\epsilon} + i \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$

- Where $|\varepsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$

$K^0 - \overline{K}^0$ Mixing

- Time evolution of $K^0 - \overline{K}^0$ system given by familiar Wigner-Weisskopf formula:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where:

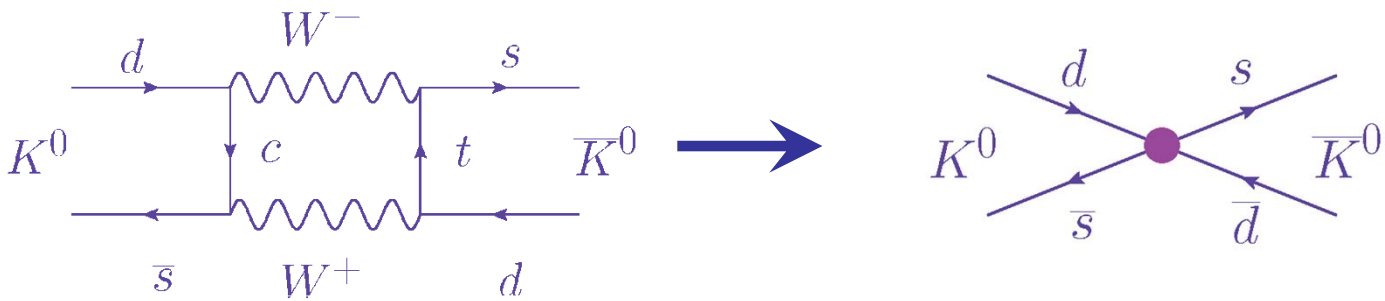
$$\Gamma_{ij} = 2\pi \sum_{\alpha} \int_{2m_\pi}^{\infty} dE \langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle \delta(E - m_K)$$

$$M_{ij} = \sum_{\alpha} \mathcal{P} \int_{m_\pi}^{\infty} dE \frac{\langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle}{m_K - E}$$

$K^0 - \overline{K}^0$ Mixing

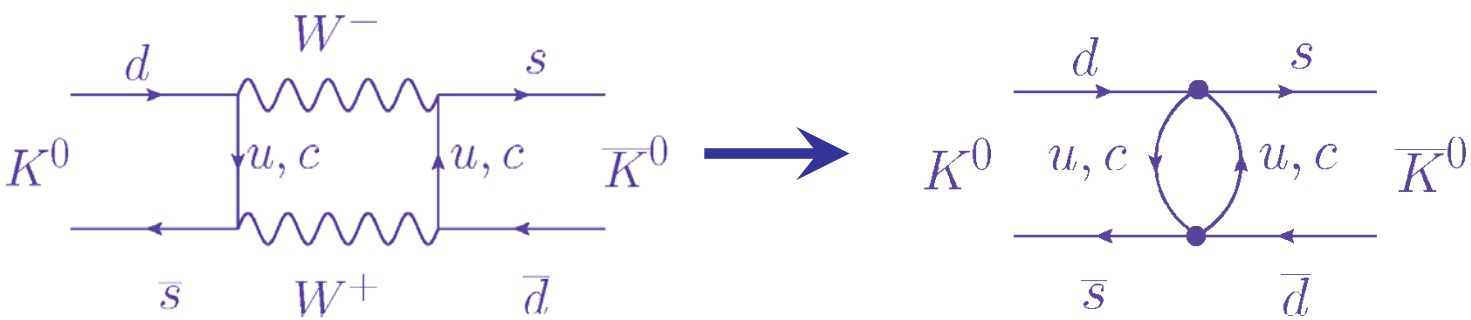
- CP violating: $p \sim m_t$

$$\overline{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\overline{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\overline{0}}}{\mathrm{Re}\,M_{0\overline{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\overline{0}}}\right\}$$



- CP conserving: $p \leq m_c$

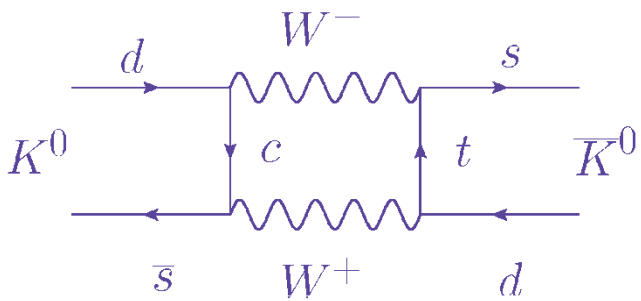
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\overline{0}}\}$$



$K^0 - \overline{K}^0$ Mixing

- CP violating: $p \sim m_t$

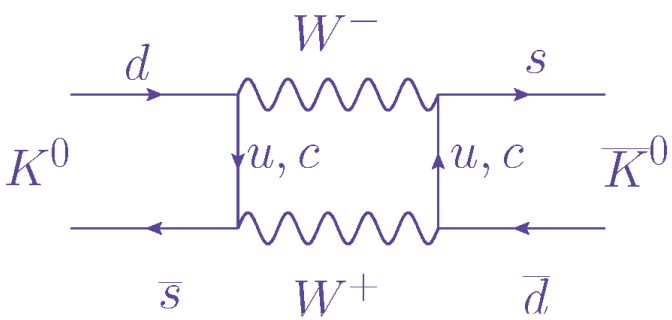
$$\overline{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\overline{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\overline{0}}}{\mathrm{Re}\,M_{0\overline{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\overline{0}}}\right\}$$



Long distance part is a small but important contribution

- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\overline{0}}\}$$

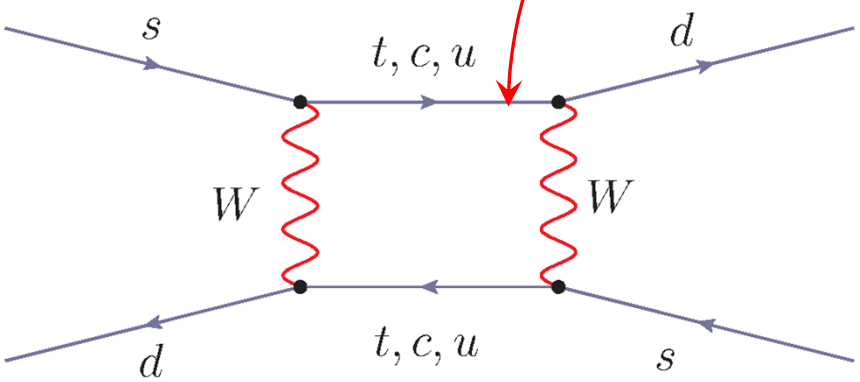


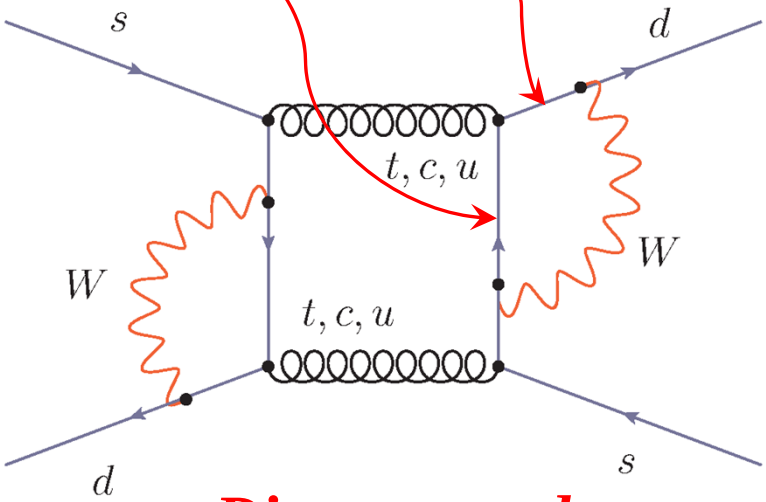
Long distance part is large. QCD perturbation theory fails at the 30% level.

Examine in more detail

- Two types of diagram (most gluons not shown):

up type quarks





Connected

(two quark lines are connected by W's)

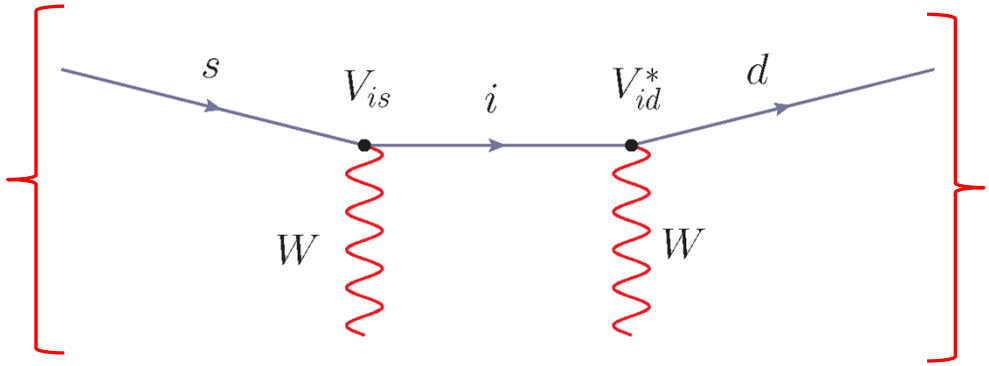
Disconnected

(each quark line is connected to itself by W's)

Standard Model Review

- Three up-type propagators:

$$\sum_{i=u,c,t}$$



- GIM subtraction:

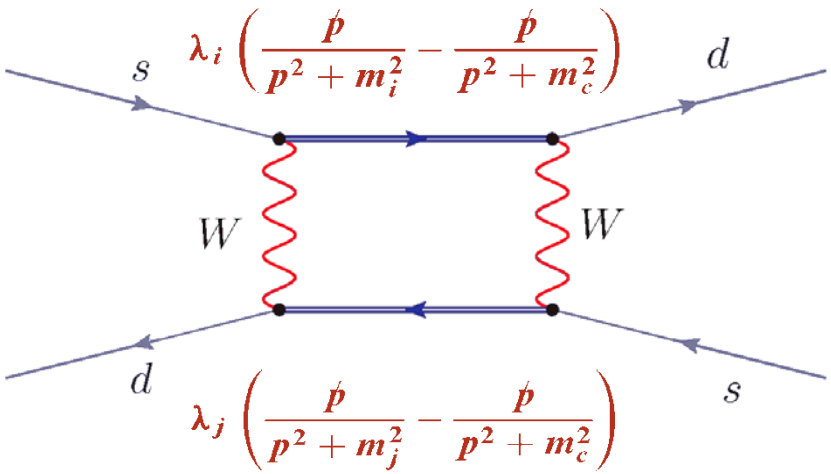
$$\sum_{i=u,c,t} \left\{ V_{i,d}^* \frac{\not{p}}{p^2+m_i^2} V_{i,s} - V_{i,d}^* \frac{\not{p}}{p^2+m_c^2} V_{i,s} \right\}$$

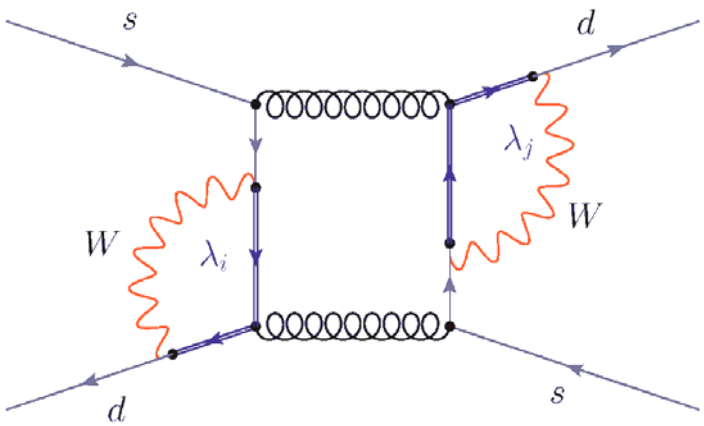
$$= \lambda_t \left\{ \frac{\not{p}}{p^2+m_t^2} - \frac{\not{p}}{p^2+m_c^2} \right\} + \lambda_u \left\{ \frac{\not{p}}{p^2+m_u^2} - \frac{\not{p}}{p^2+m_c^2} \right\}$$

$$\lambda_i = V_{i,d}^* \, V_{i,s}$$

Six contributions to ΔM_K and ε_K

- Six types of diagram:





$\lambda_i \lambda_j = \lambda_t \lambda_t$, $\lambda_u \lambda_u$ and $\lambda_t \lambda_u$

- Masses & CKM coefficients: $(m_t / m_c)^2 = 2.1 \times 10^4$

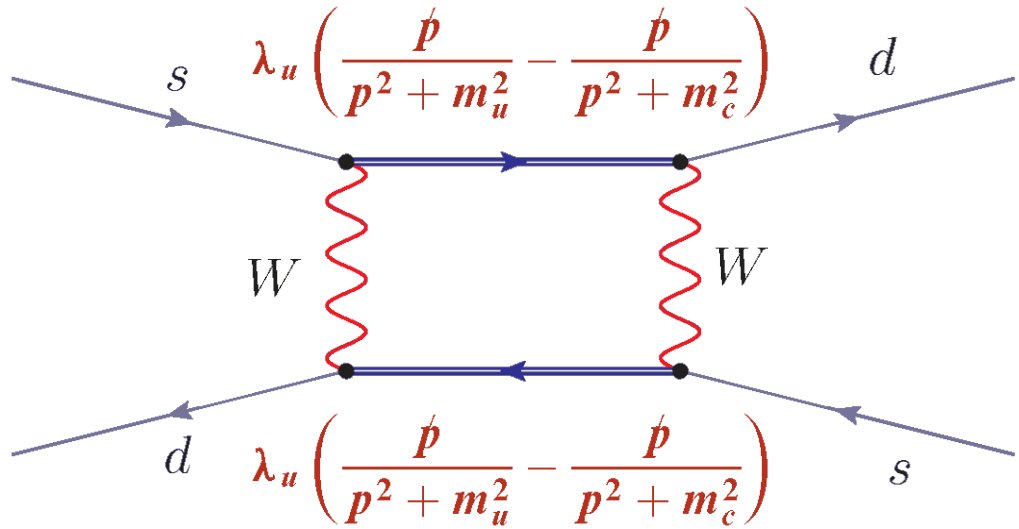
$\lambda_u = 0.22 \qquad \lambda_c = -0.22 + 1.34 \times 10^{-4}\, i$

$\lambda_t = 3.2 \times 10^{-4} - 1.34 \times 10^{-4}\, i$

uu box diagram

$\sim \lambda_u \, \lambda_u$

$\sim (0.22 + i\,0)^2$



$$\lambda_u^2 G_F^2 \int d^4k \left[\frac{\not k \, (m_c^2 - m_u^2)}{(k^2 + m_u^2)(k^2 + m_c^2)} \right]^2 \sim G_F^2 \, m_c^2 \, [0.22 + i0]^2$$

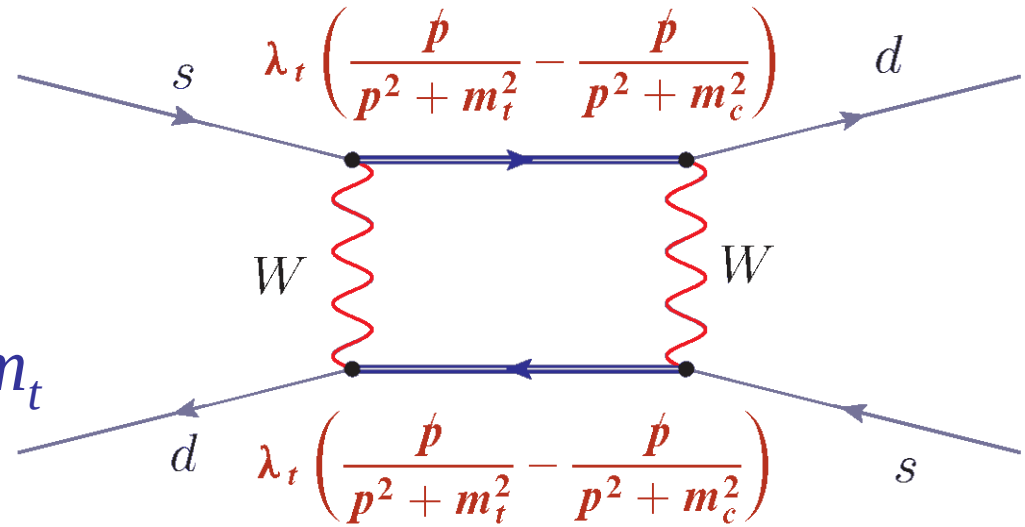
- Large contribution to ΔM_K
- No imaginary part!

tt box diagram

$\sim\lambda_t\,\lambda_t$

$\sim(3.2 - i\,1.34)^2 \times 10^{-8}$

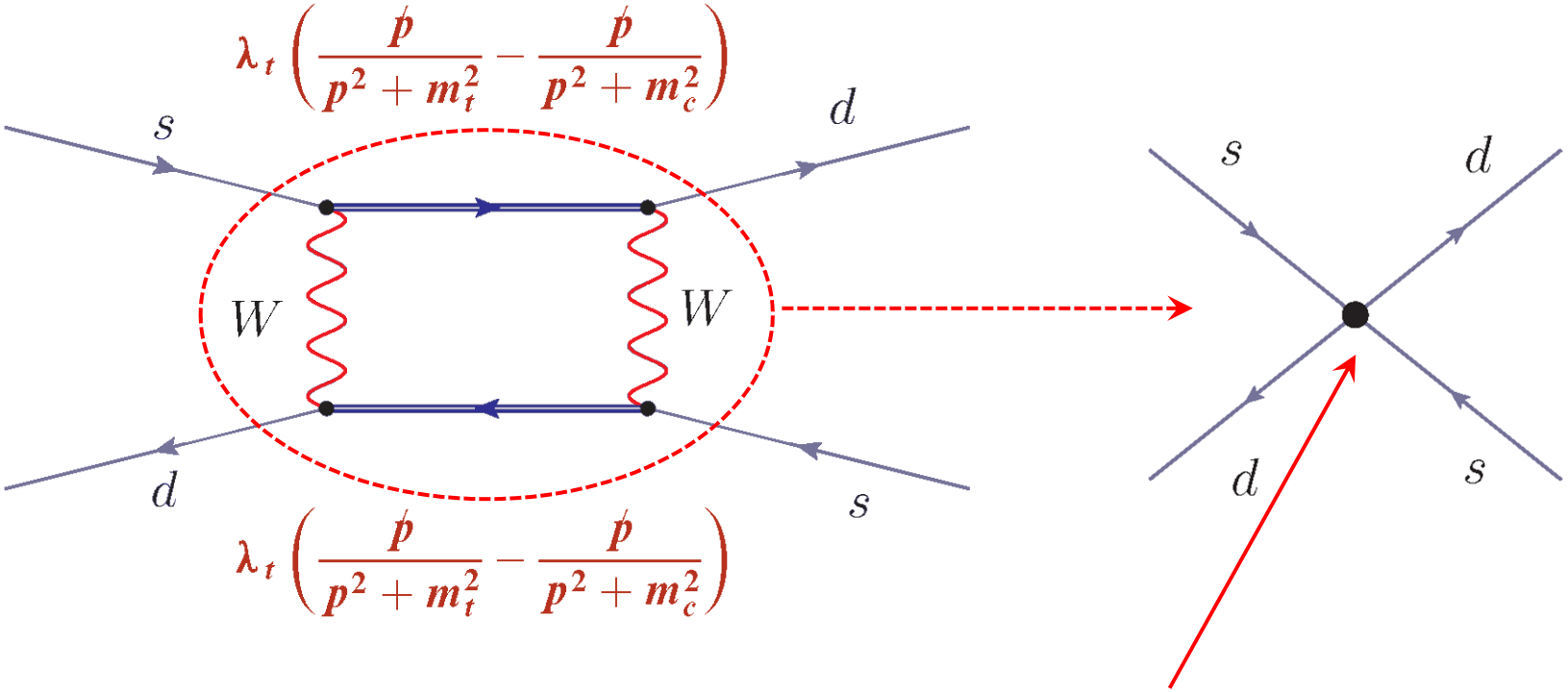
- Dominated by p ~ m_t



$$\lambda_t^2 G_F^2 \int d^4k \left[\frac{\not k\,(m_c^2-m_t^2)}{(k^2+m_t^2)(k^2+m_c^2)}\right]^2 \sim G_F^2 m_t^2 \left[(3.2-1.34i)\times 10^{-4}\right]^2$$

- Largest contribution to ε_K
- Contributes ~4% to ΔM_K, familiar pert. th. x B_K

tt box diagram



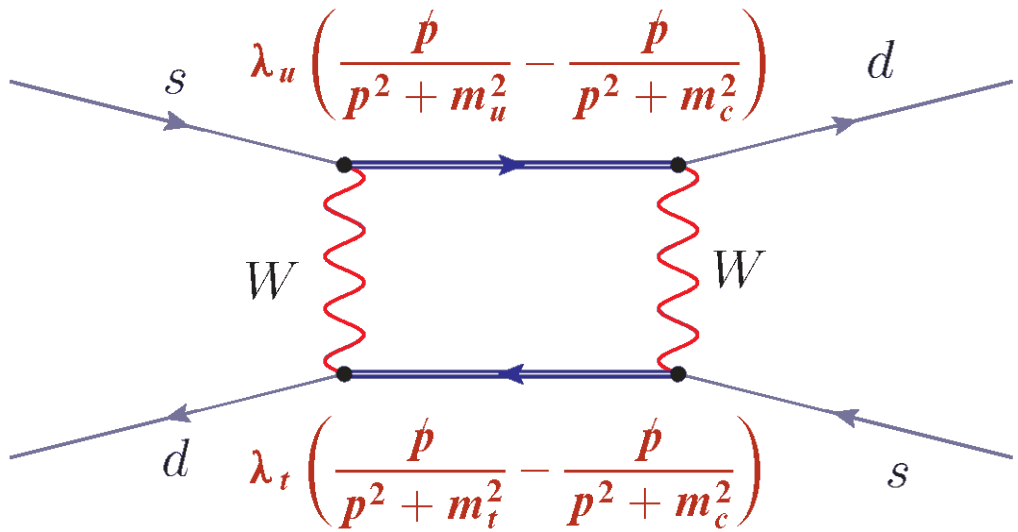
$$O_{LL} = \bar{s}_a \gamma^\mu (1 - \gamma^5) d_a \, \bar{s}_b \gamma^\mu (1 - \gamma^5) d_b$$

- Requires only the usual lattice calculation of B_K

ut box diagram

$\sim\lambda_u\,\lambda_t$

$\sim 0.22\text{x}(3.2 - i\,1.34)\text{x}10^{-4}$



- Treat top and charm pieces separately

- Charm $\lambda_u\lambda_t G_F^2\int d^4k\frac{\not k\,(m_c^2-m_u^2)}{(k^2+m_u^2)(k^2+m_c^2)}\,\frac{\not k}{(k^2+m_c^2)}$

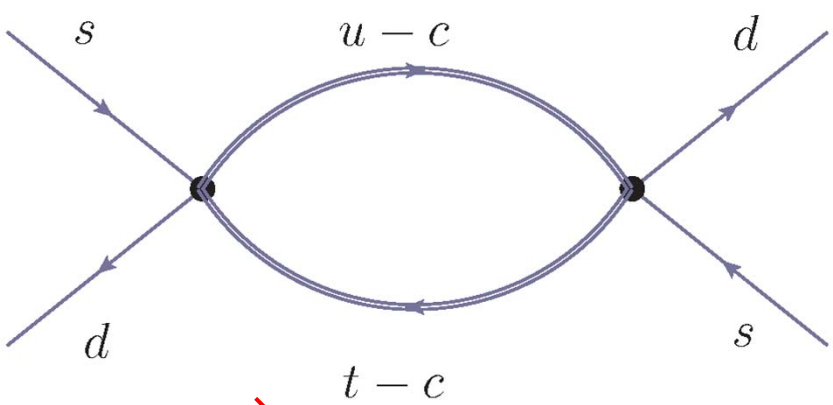
$$\sim G_F^2\,m_c^2\,\log(m_c a)\left[0.22\times(3.2-1.34i)\times10^{-4}\right]$$

- Top $\lambda_u\lambda_t G_F^2\int d^4k\frac{\not k\,(m_c^2-m_u^2)}{(k^2+m_u^2)(k^2+m_c^2)}\,\frac{\not k}{m_t^2}\sim G_F^2\,m_c^2\left[0.22\times(3.2-1.34i)\times10^{-4}\right]$

- Contributes ~ 2% to ε_K, ~ 0.2% to ΔM_K

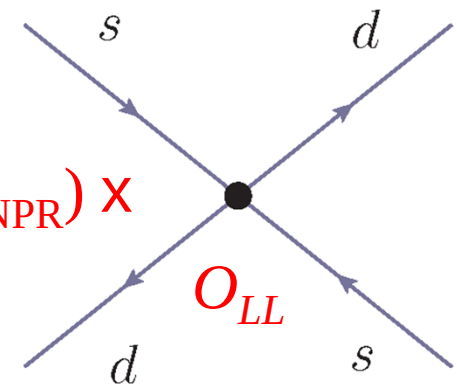
ut box diagram

$$\lambda_u \left(\frac{\not{p}}{p^2 + m_u^2} - \frac{\not{p}}{p^2 + m_c^2} \right)$$



$$\lambda_t \left(\cancel{\frac{\not{p}}{p^2 + m_t^2}} - \frac{\not{p}}{p^2 + m_c^2} \right)$$

+ $(E_{\mathrm{PT}} - E_{\mathrm{NPR}})$ x

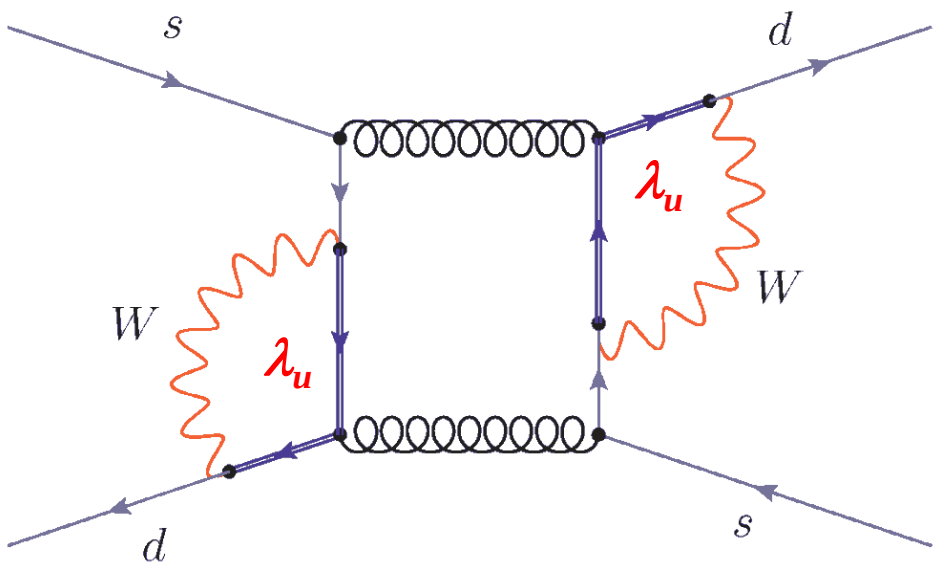


- O_{LL} subtraction replaces lattice short distance piece with perturbative short distance piece.

uu disconnected diagram

$\sim\lambda_u\,\lambda_u$

$\sim(0.22 + i\,0)^2$

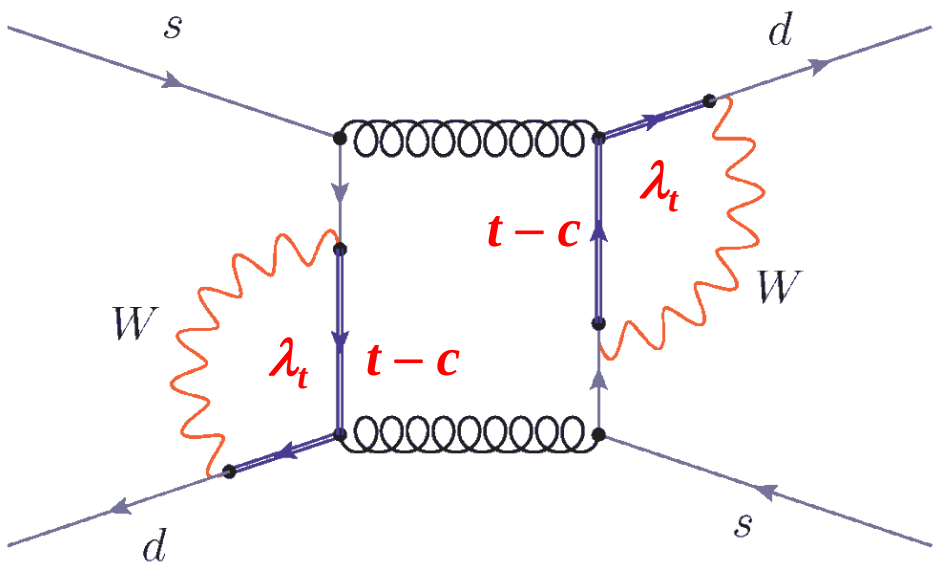


- Convergent $\sim \alpha_s^2\, G_F^2\, m_c^2\, (0.22 + i\,0)^2$
- Large contribution to ΔM_K
- No imaginary part!

tt disconnected diagram

$\sim\lambda_t\,\lambda_t$

$\sim(3.2 - i\,1.34)^2 \times 10^{-8}$

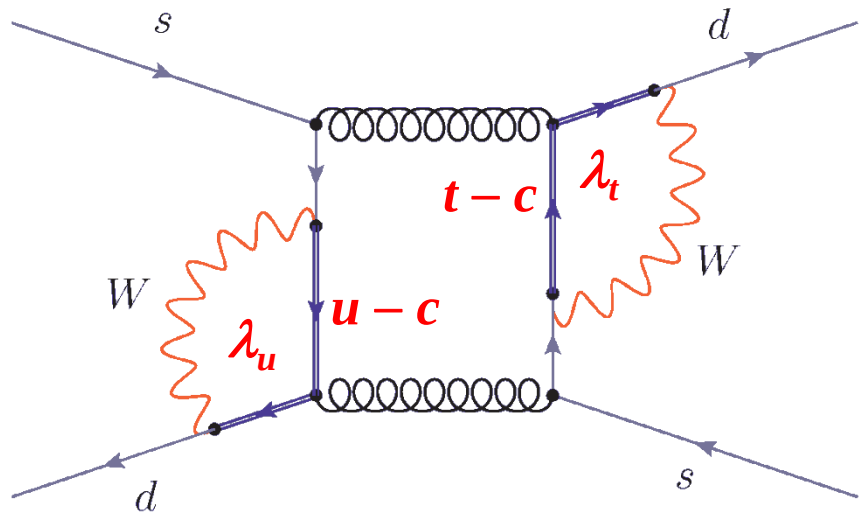


- Given by O_{LL} matrix element, accurate to 10^{-4}
- ~4% correction to ΔM_K
- Standard NNLO contribution to ε_K

ut disconnected diagram

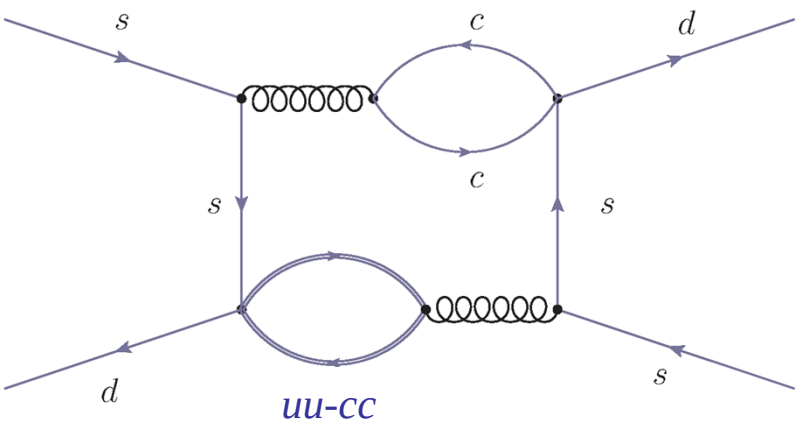
$\sim \lambda_u \lambda_t$

$\sim 0.22 \times (3.2 - i\, 1.34) \times 10^{-4}$



- Up factor: convergent ($\sim m_c^2/k^2$)
- Top factor:
  - charm vertex: requires gluonic penguin subtraction
  - top vertex: represented by gluonic penguin operator
- Final overall log(a) will require O_{LL} subtraction.

ut disconnected diagram

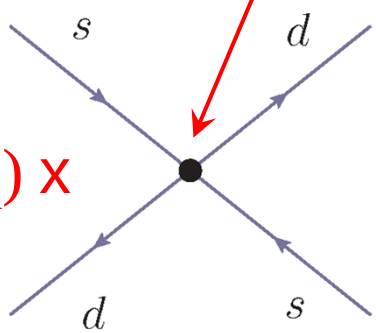


uu-cc

convergent

+ $(E_{\mathrm{PT}} - E_{\mathrm{NPR}})$ x

O_{LL} subtraction



O_{LL}

Approximate Summary

quarks	$M_{\bar{0}0}$	Re($M_{\bar{0}0}$)	Im($M_{\bar{0}0}$)
$(u\text{-}c)(u\text{-}c)$	$\lambda_u^2\,(m_c/m_w)^2$	1.1×10^{-5}	0
$(t\text{-}c)(t\text{-}c)$	$\lambda_t^2\,(m_t/m_w)^2$	4.0×10^{-7}	4.1×10^{-7}
$(u\text{-}c)(t\text{-}c)$	$\lambda_u\lambda_t\,(m_c/m_w)^2$	1.6×10^{-8}	6.6×10^{-9}

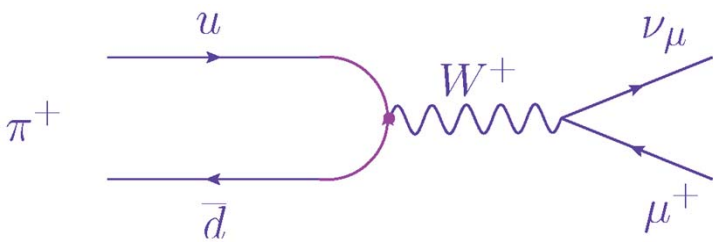
- All six types of diagrams can be computed using lattice methods.
- Examine
  - ΔM_K
  - Long distance parts of ε_K

Lattice QCD in 2015

Current state-of-the-art

- Use chiral fermions: Mobius DWF
- Physical m_π=135 MeV and L = 4 – 6 fm.
- Generate 48^3 x 96 and 64^3 x 128.
- Many complex ingredients:
  - Highly optimized code (64 threads, SPI comms., wide-vector FP)
  - Sophisticated algorithms (deflation, FG $(\Delta t)^3$ integrator, multigrid)
  - Complex measurement strategies (NPR, G-parity BC, 5-pt functions, all-mode-averaging)
- Complete set of measurements took 5.3 hours on a 32-rack BG/Q machine (sustains 1 Pflops)

Simple state-of-the-art example: f_π



$$\langle 0|\bar{d}\gamma^5\gamma^\mu u|\pi^+(\vec{p})\rangle = f_\pi \frac{p^\mu}{\sqrt{4E_\pi(\vec{p})}}$$

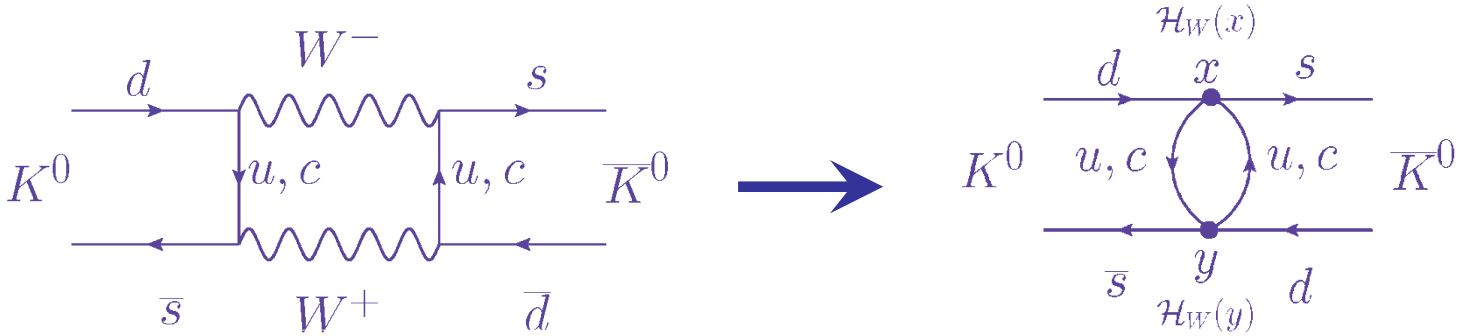
$$f_\pi = N \sum_{\vec{r}} \frac{\langle A^0(\vec{r},t)\, O_\pi(t=0)\rangle}{\langle O_\pi^\dagger(t)\, O_\pi(t=0)\rangle^{\frac{1}{2}}} e^{m_\pi t/2}$$

- 2012 (elaborate chiral fit): $f_\pi = 127(3)_{\text{stat}}(3)_{\text{sys}}$ MeV
- 2013 (m_π=135 MeV): $f_\pi = 130.0(0.3)_{\text{stat}}$ MeV (40 configs.)
- Experiment: $f_\pi = 130.4(0.04)(0.2)$ MeV

ΔM_K from Euclidean space

ΔM_K from Euclidean space

- Replace $W^{\pm}$ exchange by four-Fermi operator:



- Use 4-flavor theory incorporating GIM cancellation
- Amplitude is regular when the two weak operators collide: $x \rightarrow y$
- This long distance calculation is self-contained with no subtraction or perturbative correction needed!

ΔM_K from Euclidean space

- Start with $H = H_{\mathrm{QCD}} + H_{\mathrm{W}}$
- Calculate:

$$\langle K_\pm | e^{-\left(H_{\mathrm{QCD}}+H_{\mathrm{W}}\right)t} | K_\pm \rangle = e^{-\left(M_K+\Delta M_{K_\pm}\right)t} + \ldots$$

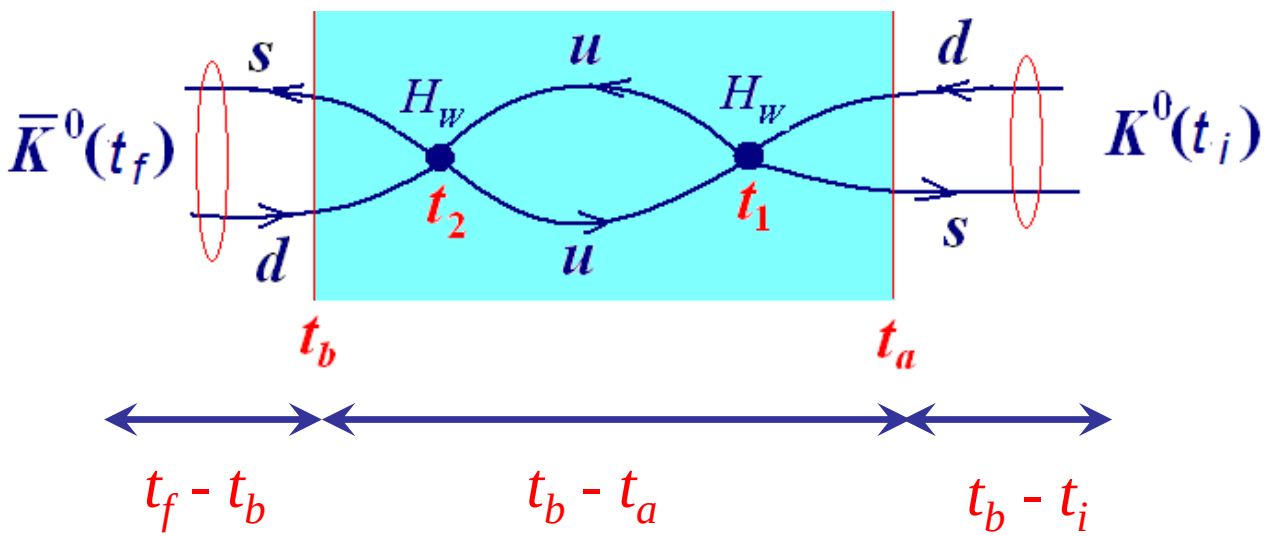
$$= e^{-M_K t}\left(1 + \Delta M_{K_\pm} t\right) + \ldots$$

- The … terms include |0>, $|\pi>$ and $|\pi\,\pi>$ states with energy below M_K
- These fall with increasing t exponentially less rapidly that the term of interest!

ΔM_K from Euclidean space

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\dagger}(t_i)\right)|0\rangle$$



Interpret Lattice Result

1. 2.

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(-(t_b - t_a) - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)(t_b - t_a)}}{M_K - E_n} \right)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential:

   $E_n < m_K$ must be removed!

3.

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} - 2 \frac{d(\phi + \delta_0)}{dk}\bigg|_{m_K} |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0)\big|_{M_K}$$

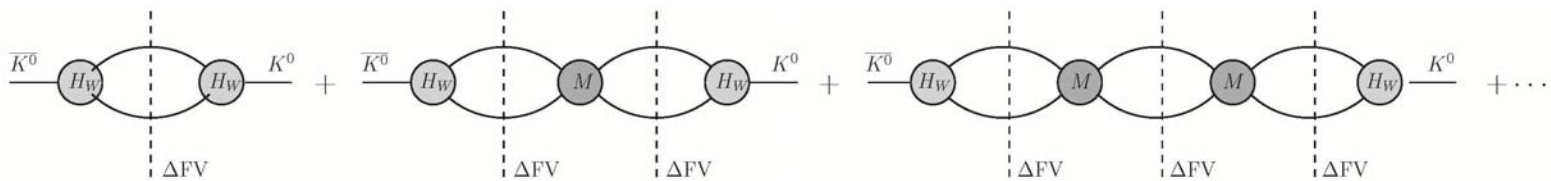
N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda

Finite volume correction

- Exploit Kim, Sachrajda and Sharpe (arXiv:hep-lat/0507006)

$$\Delta_{\rm FV}\left\{\left\langle T\left(K^0(0)\int_{-\infty}^{\infty}H_W(t_1)dt_1\int_{-\infty}^{\infty}H_W(t_2)dt_2\int d^4x\,\overline{K}^0(x)e^{iqx}\right)\right\rangle\right\}\left(\frac{(q^2+M_K^2)}{i}\right)^2$$

$=$



$$= \langle\overline{K}^0|H_W|(\pi\pi)^{in}\rangle F\frac{1}{1+\frac{i}{2}MF}\langle(\pi\pi)^{out}|H_W|K^0\rangle$$

$$= \frac{qM_K\pi}{2}\left|\langle\overline{K}^0|H_W|(\pi\pi)^{in}\rangle\right|\left(\cot(\phi+\delta)+i\right)\left|\langle(\pi\pi)^{out}|H_W|K^0\rangle\right|$$

$$M_{K_L}-M_{K_S}=2\sum_n\frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n}-2\frac{d(\phi+\delta_0)}{dk}\bigg|_{m_K}|\langle n_0|H_W|K^0\rangle|^2\cot(\phi+\delta_0)\big|_{M_K}$$

N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda

ΔM_K Numerical Experiments

Choice of weak operators

- Use four-Fermi operators in the four-flavor theory:

$$Q_1^{qq'} = (\overline{q}_i d_i)_{V-A} (\overline{q}'_j s_j)_{V-A} \qquad Q_2^{qq'} = (\overline{q}_i d_j)_{V-A} (\overline{q}'_j s_i)_{V-A}$$

$$\mathcal{H}_W = \frac{G_F}{2} \sum_{q,q'=u,c} V_{qd} V_{q's}^* \left(C_1 Q_1^{qq'} + C_2 Q_2^{qq'} \right)$$

- Use Rome-Southampton NPR and 4-flavor RI/SMOM / $\overline{\text{MS}}$-NDR matching from Lehner and Sturm
- Assume Cabibbo unitarity:

$0 = \lambda_u + \lambda_c + \lambda_t \approx \lambda_u + \lambda_c$ where $\lambda_q = V_{qd} V_{qs}^*$

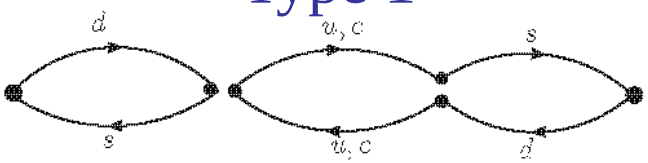
Lattice setup

- Must include charm quark (GIM *u–c* cancellation)
- Three calculations performed:

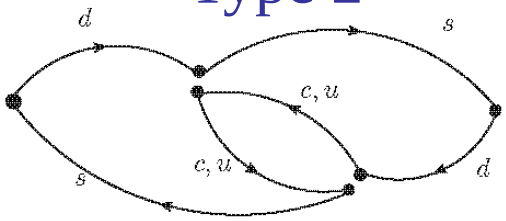
Jianglei Yu
- – $16^3 \times 32$, m_p = 420 MeV, types 1 & 2 (arXiv:1212.5931)
- – $24^3 \times 64$, m_p = 330 MeV, all graphs (arXiv:1406.0916)

Ziyuan Bai
- – $32^3 \times 64$, m_p = 170 MeV, all graphs

Type 1



Type 2



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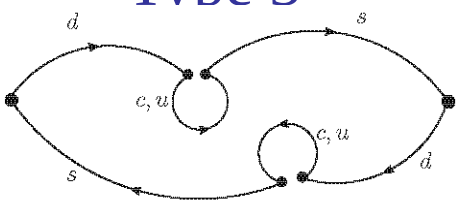

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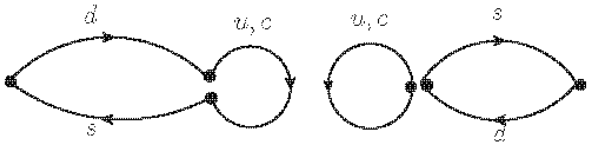
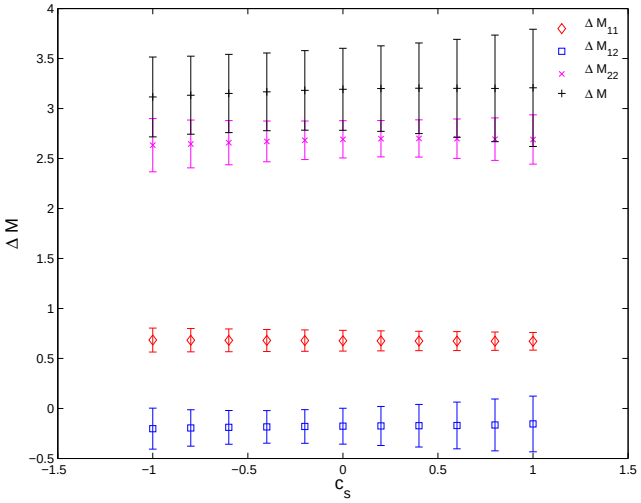
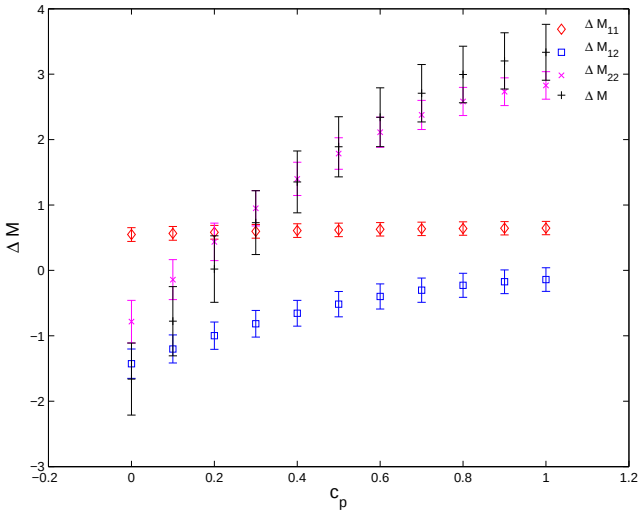


Exponentially growing terms

- The vacuum, π^0 and η require special treatment:
  - Calculate $\langle$ X | H_W | K^0 $\rangle$ directly and subtract, X = $|0\rangle$, π^0, η
  - Fit the exponential time dependence in the 4-point function
  - Adjust $c_s\, \bar{s}\, d$ or $c_p\, \bar{s}\, \gamma^5 d$ terms to completely remove two unwanted states.



vary c_s



vary c_p

Remove extra η contribution

- Calculate $\langle\ \eta\ |\ H_W\ |\ K^0\ \rangle$ directly and remove
- Has an ~10% effect on the result

PRL **105,** 241601 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

η and η' Mesons from Lattice QCD

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The large mass of the ninth pseudoscalar meson, the η', is believed to arise from the combined effects of the axial anomaly and the gauge field topology present in QCD. We report a realistic, 2 + 1-flavor, lattice QCD calculation of the η and η' masses and mixing which confirms this picture. The physical eigenstates show small octet-singlet mixing with a mixing angle of $\theta = -14.1(2.8)^\circ$. Extrapolation to the physical light quark mass gives, with statistical errors only, $m_\eta = 573(6)$ MeV and $m_{\eta'} = 947(142)$ MeV, consistent with the experimental values of 548 and 958 MeV.

DOI: 10.1103/PhysRevLett.105.241601 PACS numbers: 12.38.Gc, 11.15.Ha, 11.30.Rd, 14.40.Be

The relatively large mass of the ninth pseudoscalar meson, the η', provides a significant challenge for quantum chromodynamics (QCD), the component of the standard model which describes the interactions of quarks and gluons. On a naive classical level, there are nine conserved axial-vector currents. Given the vacuum breaking of the symmetries which these currents generate, this should diagrams to decrease exponentially with increasing time separation. For mesons this falloff roughly matches the exponential time dependence of the massive, Euclidean-space meson propagator, and good numerical signals can be seen over a large range of times. For terms in which the source and sink of the meson propagator are not joined by quark propagators, the needed exponential decrease comes

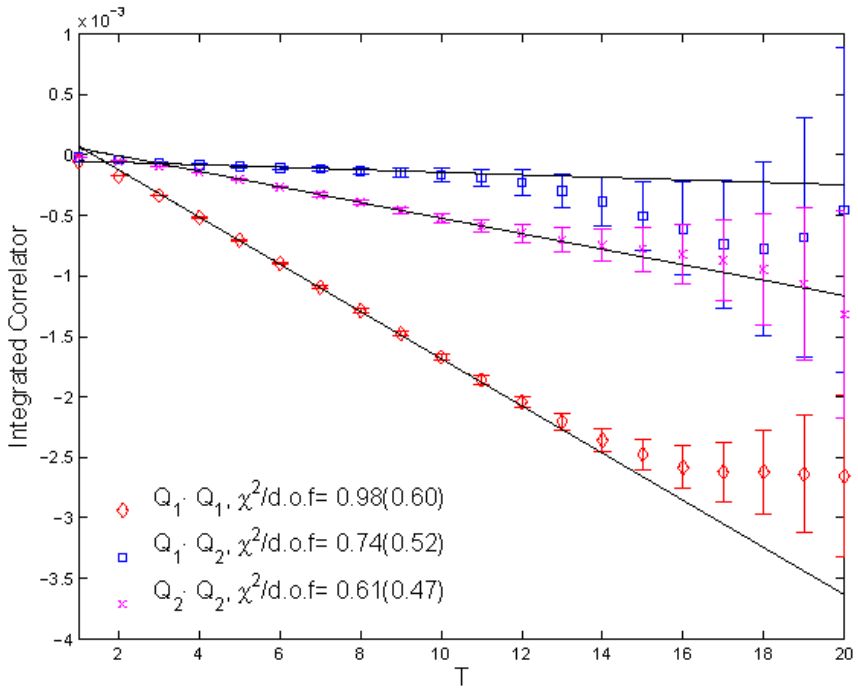
$$\eta' = \frac{1}{\sqrt{3}}\left(\bar{u}u + \bar{d}d + \bar{s}s\right)$$

$$\eta = \frac{1}{\sqrt{6}}\left(\bar{u}u + \bar{d}d - 2\bar{s}s\right)$$

$$\frac{\left|\langle\eta|H_W|K^0\rangle\right|^2}{M_K - M_\eta}\left(-(t_b - t_a) - \frac{1}{M_K - M_\eta} + + \frac{e^{(M_K - M_\eta)(t_b - t_a)}}{M_K - M_\eta}\right)$$

Lattice results
(Jianglei Yu)

- N_f=2+1, 24^3 x 64, m_π = 330 MeV, $m_c^{\overline{MS}}$(2 GeV) = 949 MeV
- Incorporate GIM cancellation



- Large statistics (800 configurations, 64 measurements each).

Results

Δ_K	T_{min}	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
7	6	0.754(42)	-0.16(15)	2.70(18)	3.30(34)
	7	0.755(42)	-0.18(15)	2.66(18)	3.23(34)
	8	0.751(42)	-0.18(15)	2.62(19)	3.18(35)

x 10^{-12} MeV

Diagrams	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
Type 1,2	1.485(80)	1.567(38)	3.678(56)	6.730(96)
All	0.754(42)	-0.16(15)	2.70(18)	3.30(34)

x 10^{-12} MeV

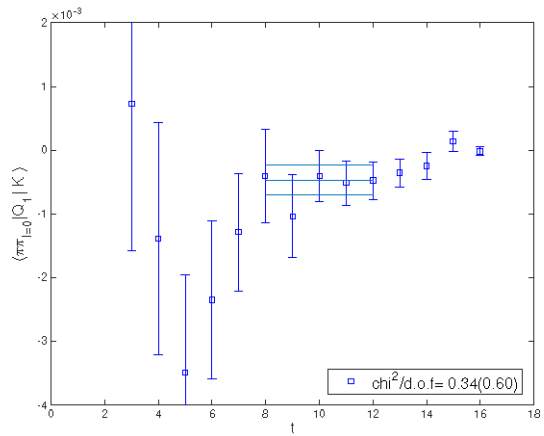
- Unphysical, m_π = 330 MeV
- Active charm but $m_c\, a = 0.55$
- 80^2 x 96 x 192, 1/a=3.0 GeV calculations planned.

- Result:
  ΔM_K = 3.30(34) 10^{-12} MeV
- ΔM_K^{expt} = 3.483(6) 10^{-12} MeV
- Agreement fortuitous!

New m_π = 170 MeV calculation
(Ziyuan Bai)

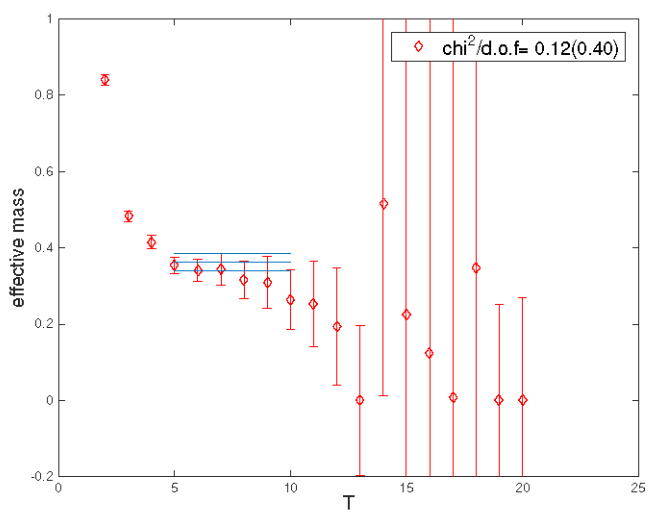
- N_f=2+1, 32^3 x 64, 1/a=1.37 GeV
- Charm: m_c = 592 and 750 MeV

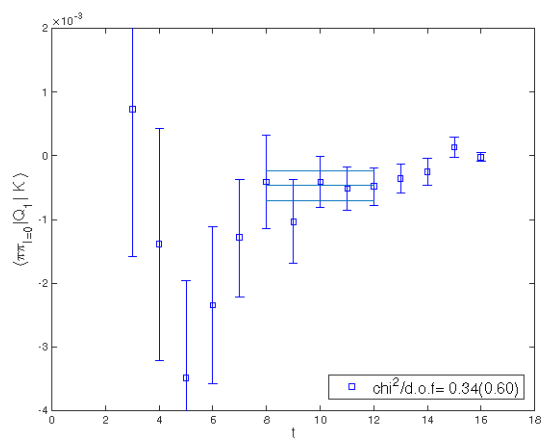
M_K	M_π	M_η	$E_{\pi\pi,I=0}$	$E_{\pi\pi,I=2}$
492.8(2)	171.9(2)	496(32)	335.8(13)	346.8(7)



Q1

m_η

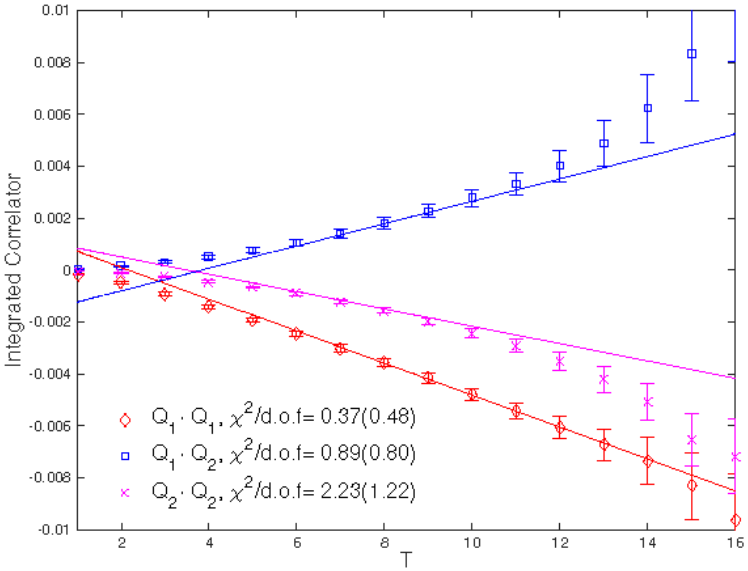




Q2

m_π = 170 MeV results

(Ziyuan Bai)



	ΔM_K x 10^{+12}
Types 1-4	5.76(73)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, I=0	-0.097(49)
$\pi\pi$, I=2	-6.56(6) x 10^{-4}
Δ_{FV}	0.029(19)

- Use m_c = 750 MeV, fit for t ≥ 8
- Disconnected contribution small
- $\pi\pi$ contribution ~2% and FV correction ~0.5%

Long distance part of ε_K

ΔS = 1, Four flavor operators
(Ziyuan Bai)

- Choose appropriate N_f=4 effective Hamiltonian:

$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V_{ts}^* V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

$$\left.\begin{aligned} Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\ Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \end{aligned}\right\} \text{current x current}$$

$$\left.\begin{aligned} Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\ Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\ Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \end{aligned}\right\} \text{QCD penguin}$$

Focus on $\lambda_t\lambda_u$ contribution to ε_K
(Ziyuan Bai)

- Construct $\boldsymbol{H_W}$(x) x $\boldsymbol{H_W}$(y) and extract the $\boldsymbol{\lambda_t\lambda_u}$ term

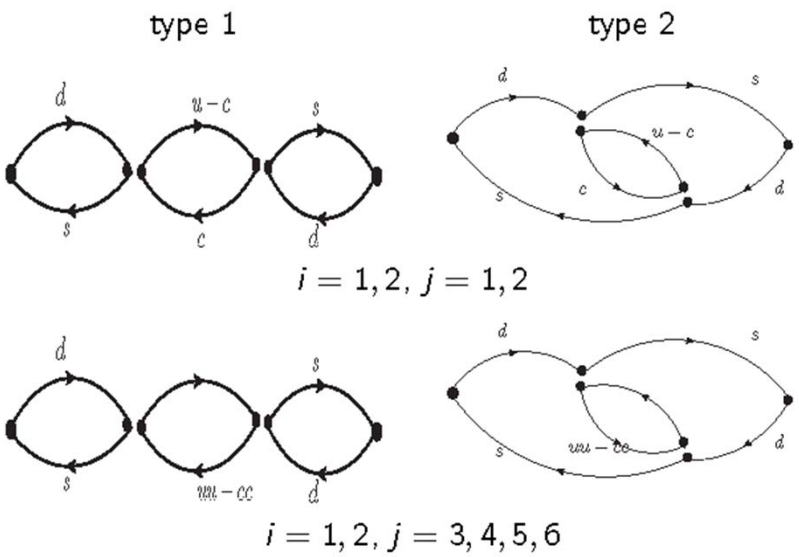
$$H_W(x)H_W(y) = \frac{G_F^2}{2}\lambda_t\lambda_u \sum_{i=1}^{2}\sum_{j=1}^{6} C_i C_j Q_{ij}$$

$$Q_{ij} = \begin{cases} 2Q_i^{cc}(x)Q_j^{cc}(y) - Q_i^{uu}(x)Q_j^{cc}(y) - Q_i^{cc}(x)Q_j^{uu}(y) & \text{if } j = 1,2 \\ \quad -Q_i^{uc}(x)Q_j^{cu}(y) - Q_i^{cu}(x)Q_j^{uc}(y) & \\ \left(Q_i^{cc}(x) - Q_i^{uu}(x)\right)Q_j(y) & \text{if } j = 3,\ldots,6 \\ \quad +Q_j(y)\left(Q_i^{cc}(y) - Q_i^{uu}(y)\right) & \end{cases}$$

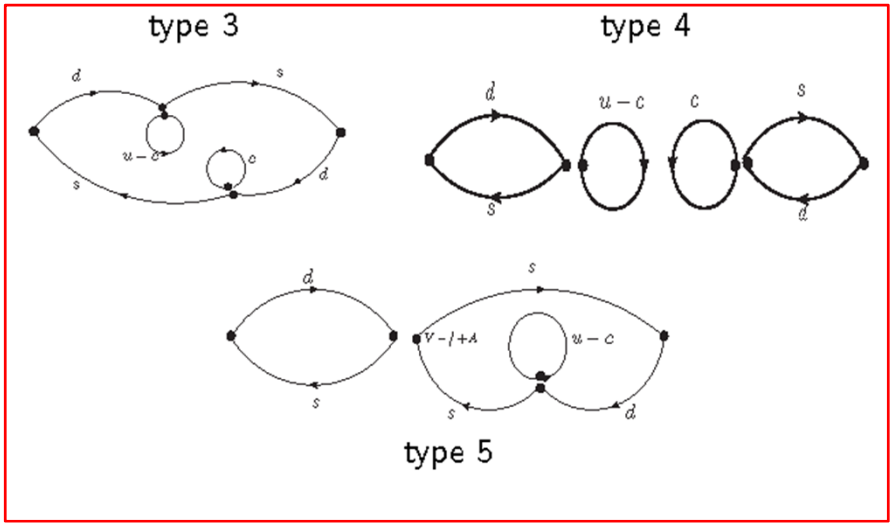
- Identify five types of diagrams

Diagrams for $\lambda_t \lambda_u$ contribution to ε_K
(Ziyuan Bai)

- Identify five types of diagrams

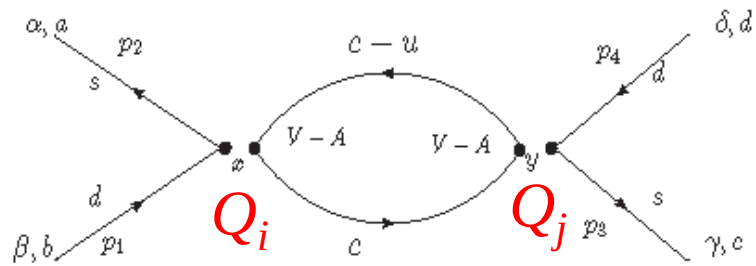


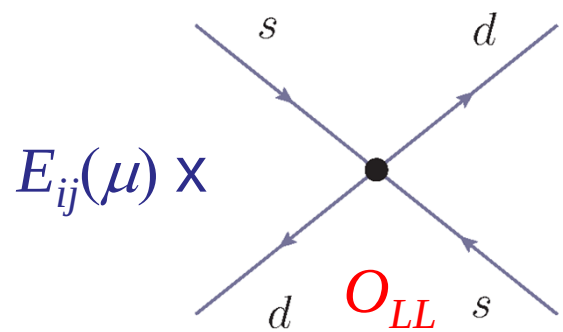
Omit from 1st study



Removing lattice short distance part
(Ziyuan Bai)

- Evaluate off-shell Green's function at $\boldsymbol{p_i^2} = \mu^2$
- Forces internal momentum also to the scale μ or greater
- This is a definition of the short-distance part of diagram.
- Add $E_{ij}(\mu)$ $(\bar{s}\gamma^\nu(1-\gamma^5)d)$ $(\bar{s}\gamma^\nu(1-\gamma^5)d)$ with $E_{ij}(\mu)$ chosen to make SD part agree with perturbation theory.
- $p_i^2 = 2p_i \cdot p_j = \mu^2$





Simulation details
(Ziyuan Bai)

- Use 24^3 x 64, $1/a$ = 1.73 GeV, MeV ensemble
- m_π = 329 MeV, m_K = 575 MeV, m_c = 949 MeV (0.363/a)
- Average over 64 separate, time-translated measurements.
- Use low-mode deflation with 300 Lanczos eigenvectors
- Study 1.4 GeV $\leq \mu \leq$ 2.6 GeV

Wilson coefficients
(Ziyuan Bai)

- Begin with $\overline{\text{MS}}$ Wilson coefficients:

$$C^{\overline{MS}} = \begin{pmatrix} -0.2967 & 1.1385 & 0.0217 & -0.0518 & 0.0102 & -0.0671 \end{pmatrix}$$

- Use Rome-Southampton RI/SMOM method to determine equivalent coefficients for our 6 lattice operators:

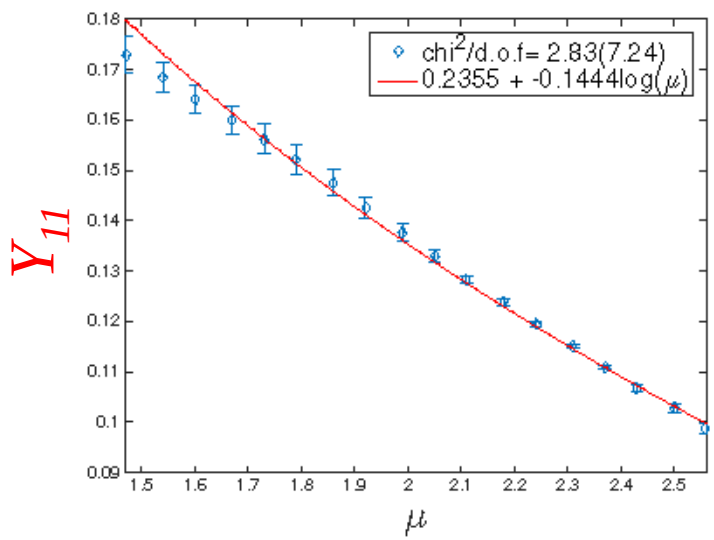
$$\sum_{i=1}^{6} C_i^{\overline{MS}} Q_i^{\overline{MS}} = \sum_{i=1}^{6} C_i^{lat} Q_i^{lat},$$

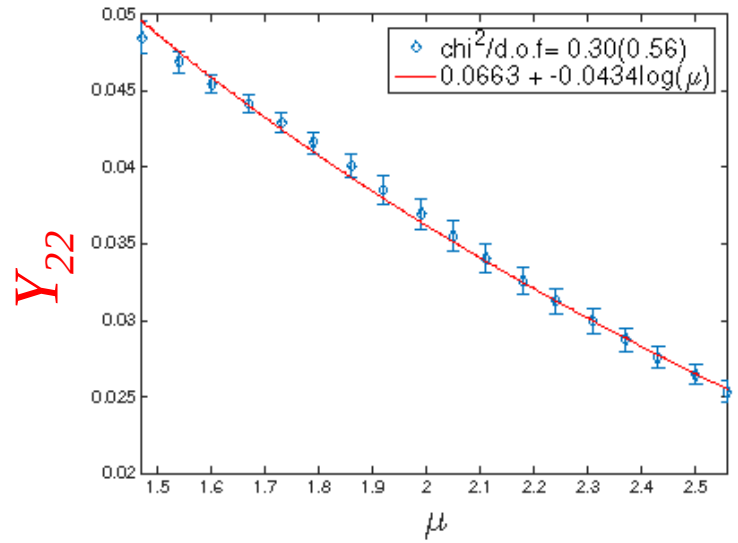
$$C^{\overline{lat}} = \begin{pmatrix} -0.2373(1) & 0.6885(1) & 0.0113(8) & -0.0213(10) & 0.0085(8) & -0.0256(8) \end{pmatrix}$$

- Calculate $\langle \overline{K^0} | \int_0^T dt_2 \int_0^T dt_1 T \left\{ Q_i(t_1) Q_j(t_2) \right\} | K^0 \rangle$ numerically.

Short-distance lattice correction
(Ziyuan Bai)

- Results for short-distance coefficient Y_{11} and Y_{22} of O_{LL} for the products Q_1Q_1 and Q_2Q_2:





- Effect of a cutoff radius $|x - y| < R$ at $\mu = 1.93$ GeV

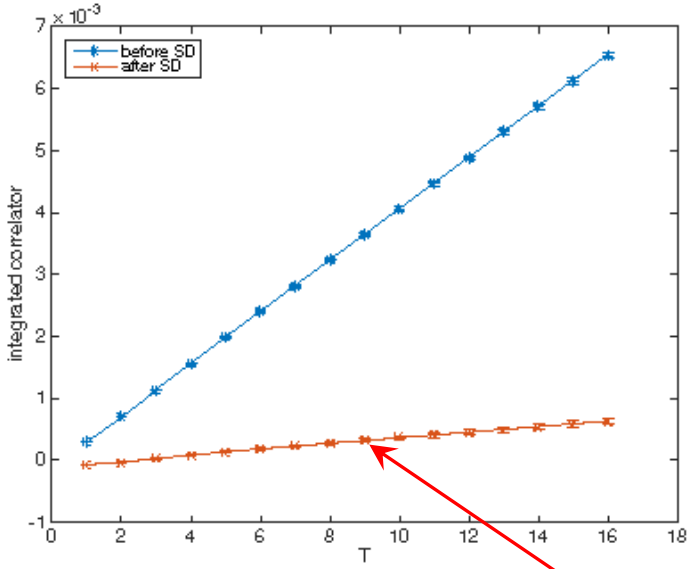
Cutoff	3	4	5	6	none
E_{11}^{lat}	0.1462	0.1501	0.1493	0.1489	0.1489
E_{11}^{lat}	0.0418	0427	0.0425	0.0425	0.0425

Integrated Correlators

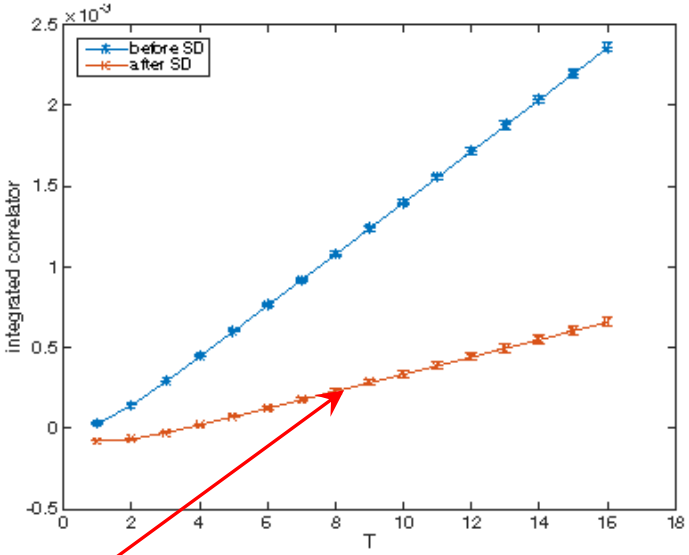
(Ziyuan Bai)

Preliminary

$$\langle \overline{K^0} | \int_0^T dt_2 \int_0^T dt_1 T \left\{ Q_i(t_1) Q_j(t_2) \right\} | K^0 \rangle$$



$Q_1 Q_1$



$Q_2 Q_2$

After short-distance subtraction

Progress toward long-distance part of ε_K

Preliminary

(Ziyuan Bai)

- Examine only type 1 and 2 diagrams
- Result for $\lambda_t\lambda_u$ contribution with lattice cutoff:

$$|\varepsilon| = 2.69(8) \times 10^{-4}$$

- Result for $\lambda_t\lambda_u$ contribution after SD subtraction:

$$|\varepsilon| = 0.152(84) \times 10^{-4}$$

- Full experimental value

$$|\varepsilon| = 22.28(11) \times 10^{-4}$$

- Now working with Christoph Lehner and ***PhySyHCAl*** to add back the correct perturbative short distance part at LO.

Outlook

- The $K_L - K_S$ mass difference in the standard model (and beyond) is a practical target for lattice QCD.
- Controlled 15-20% errors should be possible in ~2 years, when $1/a \geq 3$ GeV ensembles become available.
- Sub-percent accuracy possible in 5-10 years!
- Similar precision possible for LD contribution to ε_K:
  - Include all diagrams
  - Do fully NNLO perturbation theory for SD part.
- An exciting time for lattice QCD:
  - $K \to \pi\pi$, ΔI=3/2 and 1/2, ε'
  - $m_{K_L} - m_{K_S}$ and ε_K
  - $K^0 \to \pi^0 \bar{l} l$, $K^\pm \to \pi^\pm \bar{\nu}\nu$
  - Quark effects on g_μ - 2 at $O(\alpha^3)$