# Coleman-Weinberg inflation

Iso, KK, Shimada (2014)

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# Introduction of Inflation

- Inflation paradigm is attractive

  - Solving horizon problem
  - Solving flatness problem

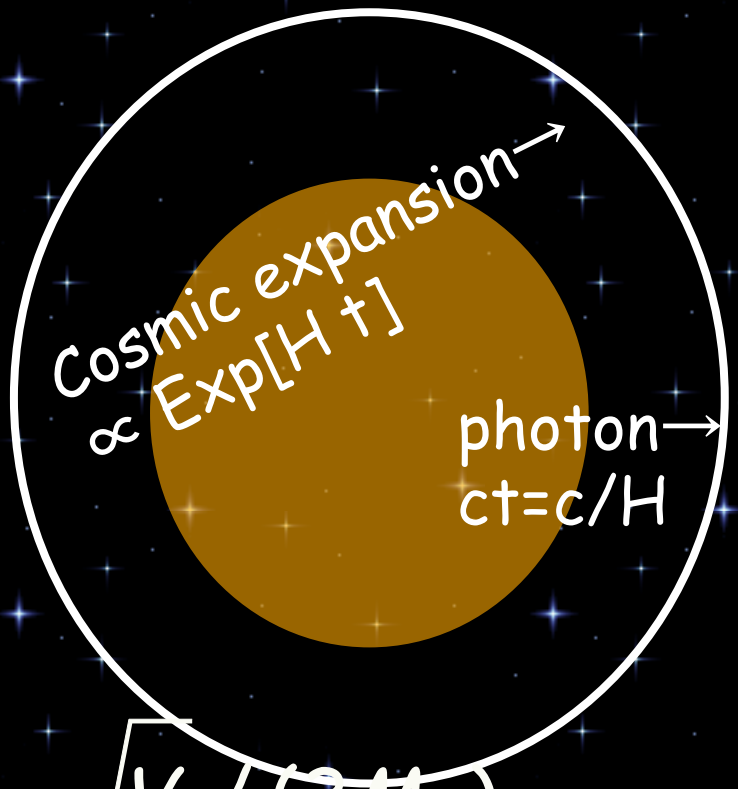
We need an e-folding number to be

$$N = \ln \frac{a(t_{end})}{a(t_*)} = \int_{t_*}^{t_{end}} \frac{da}{a} \sim 50 - 60$$

$a(t)$: scale factor

  - Solving GUT monopole problem
  - Producing temperature or density (curvature ) fluctuation with $\Delta T/T \sim 10^{-5}$

# To solve the horizon problem, we need Inflation




$$H = \dot{a}(t) / a(t) = \sqrt{V / (3M_G)}$$

# To solve the flatness problem, we need Inflation

$$\Omega_k(t) \equiv \frac{k}{a(t)^2 H^2} = \frac{k}{\dot{a}(t)^2} \to 0$$

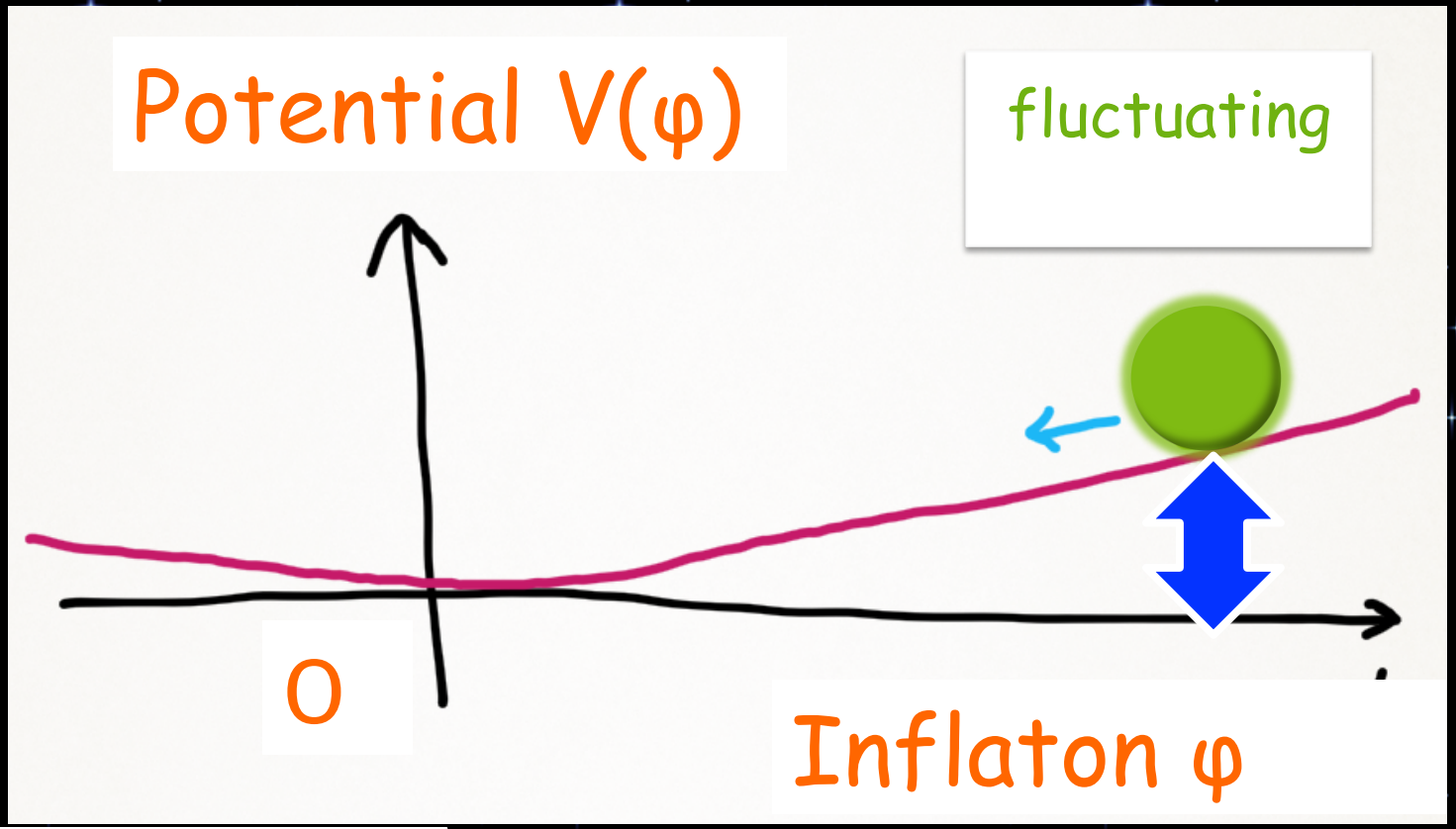
Very flat!!!

# We need e-folding number to be N=50-60

$$N = \ln \frac{a(t_{end})}{a(t_*)} = \int_{t_*}^{t_{end}} \frac{da}{a} = \int_{t_*}^{t_{end}} H dt$$

$$N > 61 + \frac{2}{3} \ln \left( \frac{\rho_I^{1/4}}{10^{16} GeV} \right) + \frac{1}{3} \ln \left( \frac{T_R}{10^{16} GeV} \right)$$

# Inflaton field produces density perturbation



# Observations

$$\epsilon \equiv \frac{1}{2}\left(M_G \frac{V'}{V}\right)^2 = 2\left(\frac{M_G}{\phi}\right)^2$$

$$\eta \equiv M_G^2 \frac{V''}{V} = 2\left(\frac{M_G}{\phi}\right)^2$$

$$\xi^{(2)} \equiv V'''V' m_G^4 / V^2$$

- Planck2015 satellite reported;
  - Power spectrum of density fluctuation

$$M_G = M_p / \sqrt{8\pi}$$

$$\sqrt{P_\zeta} = \frac{V}{24\pi^2 m_G \varepsilon} = (3.091 \pm 0.025) \times 10^{-5} (\sim \Delta T / T)$$

  - Spectral index

$$n = \frac{d \ln P_\zeta}{d \ln k} + 1 == 1 + 2\eta - 6\varepsilon = 0.9639 \pm 0.0047$$

n-1 ~ - 0.04

  - Running of n(k)

$$\frac{dn}{d \ln k} = 24\varepsilon^2 - 16\varepsilon\eta - 2\xi^{(2)} = 0.009 \pm 0.010 \ (1\sigma)$$

# Abstract (1/3)

- Coleman-Weinberg conformal model is attractive to induce the EW phase transition. We need a B-L scalar to overcome the top Yukawa's negative contribution to $\beta_{\lambda H}$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

$$\beta(\phi) = \frac{\partial \lambda(\phi)}{\partial \ln \phi} > 0$$

$$V_{\rm cl} = \lambda_H |H|^4 + \lambda_\Phi |\Phi|^4 + \lambda_{mix} |H|^2 |\Phi|^2$$

$\lambda_{mix} \sim -10^{-14}$

$\lambda$ mix is negative at $\langle \phi \rangle = M$

We assume $\lambda_H \gg \lambda_\Phi$ and $|\lambda_{mix}|$

$\lambda_H(M_{UV}) = \lambda_{mix}(M_{UV}) = 0$ at the UV scale, e.g. $M_{UV} = M_{\rm Pl}$

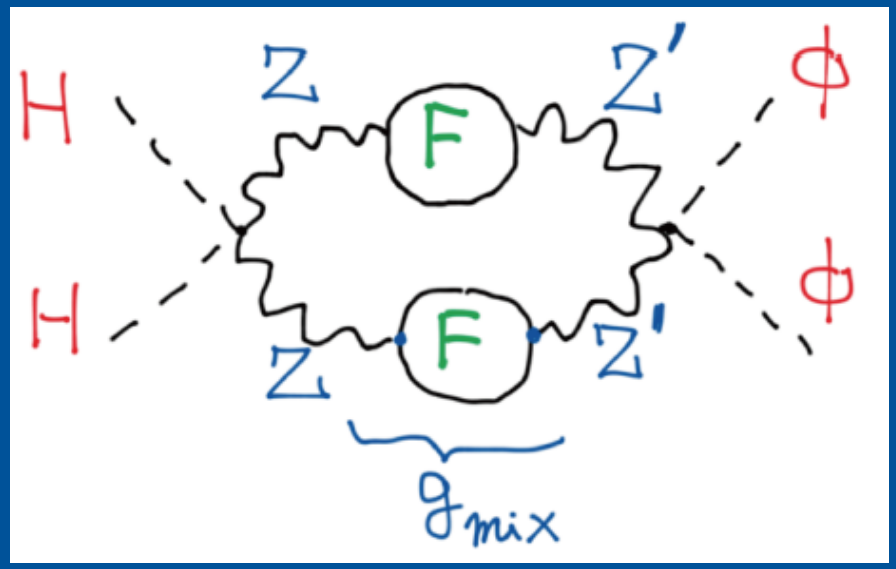
# Abstract (2/3)

- Mixing term (and even $\lambda_\Phi$ in flatland) is derived only by radiative corrections

$$\lambda_{mix} \approx -6 \cdot 10^3 \times \alpha_{B-L}^2 \alpha_Y^2$$

$\alpha_{B-L} \sim 10^{-7}$

$\lambda_{mix} \sim -10^{-14}$

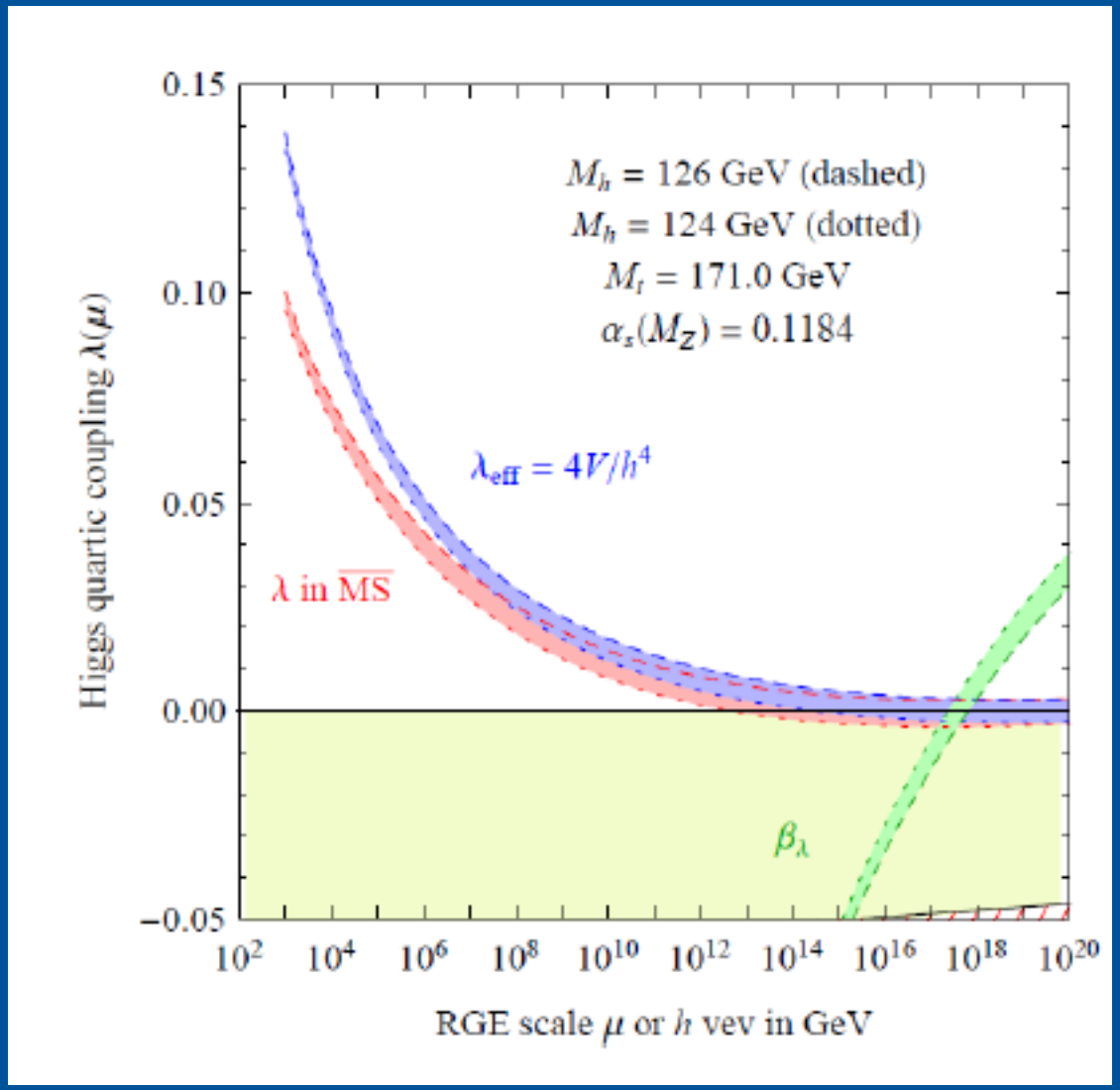



A figure by Iso-san

# Abstract (3/3)

- No other scales than Planck mass (no hierarchy problem)
- We have flat potentials for Higgs and B–L scalar $\phi$ up to the Planck scale,
- or zero at Planck scale (flatland scenarios)

# Flat at Planck scale?




$$\beta(h) = \frac{\partial\lambda(h)}{\partial \ln h}$$

In standard model (not included U(1)B-L scalar)

# Coleman-Weinberg potential

$$\beta(\phi) = \frac{\partial \lambda(\phi)}{\partial \ln \phi}$$

- One-loop potential

$$V(\phi) = \frac{A}{4}\phi^4 \left( \ln \frac{\phi^2}{M^2} - \frac{1}{2} \right) + V_0, \quad V_0 = \frac{AM^4}{8}$$

Minimum: $\phi = M$

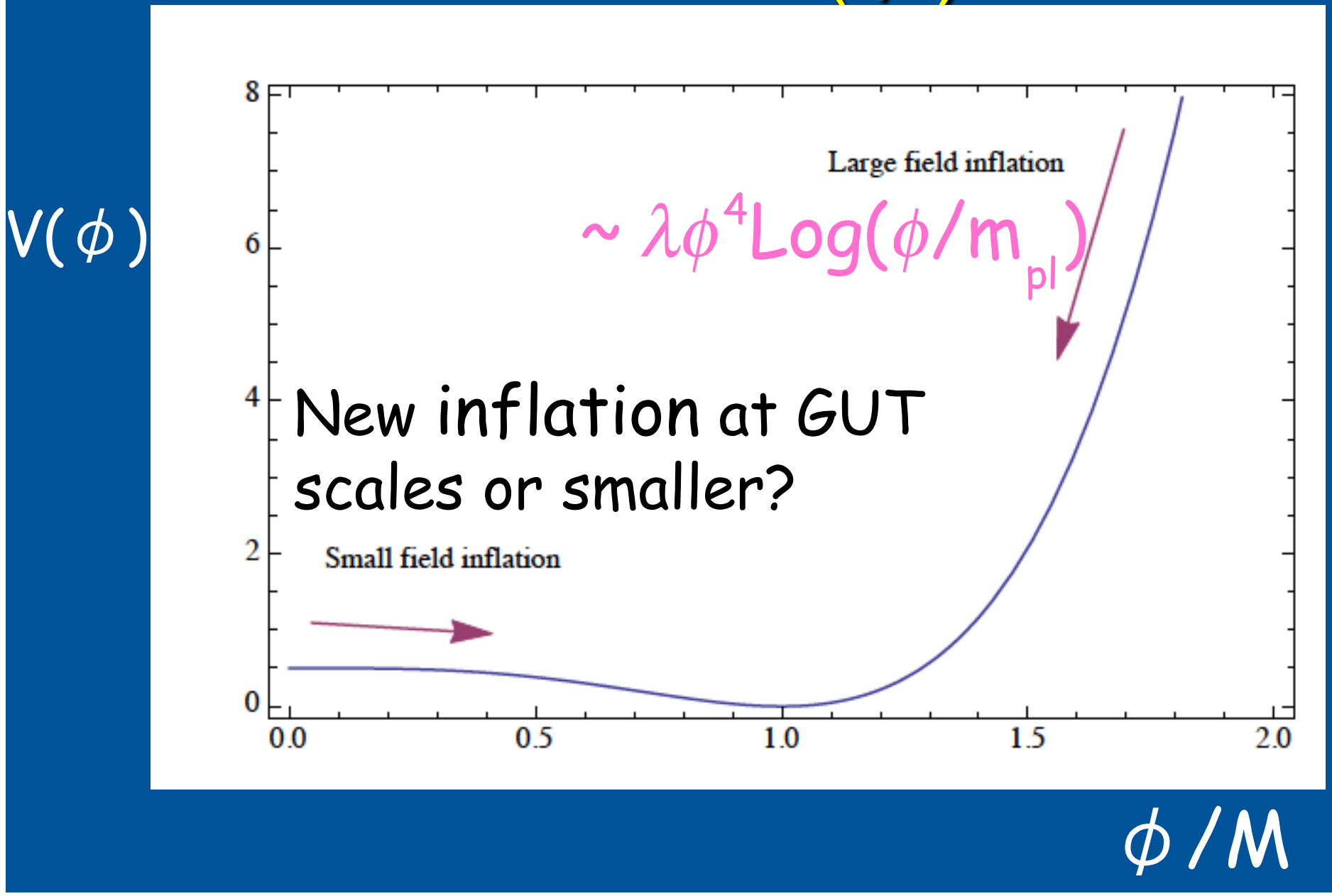
$$6\lambda = V^{(4)}(M) = 22A \text{ and } \beta_\phi = 2A$$

- Derivatives

$$V' = A\phi^3 \ln \frac{\phi^2}{M^2}$$

$$V'' = A\phi^2 \left( 2 + 3 \ln \frac{\phi^2}{M^2} \right)$$

# Potential $V(\phi)$



# Small-field Coleman-Weinberg Conformal models

Iso, KK, Shimada (2014)

# Problems in Coleman-Weinberg inflation

- Perturbation ($\propto$ V/ε) with a potential V($\phi$) at the GUT scale is too large for fixed e-folding number N ~ 50 -60, due to the smallness of ε

$$\Delta_R^2 \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^4 \epsilon}$$

- Even if we reduces the energy scale of V($\phi$), e.g. down to TeV scale, the amplitude of perturbation is well fitted, but the spectral index is too small, ns << 0.94 (A new severe problem)

# Slowroll parameters

- 1st slowroll parameter

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \approx 32\left(\frac{M_{\rm Pl}}{M}\right)^2\left(\frac{\phi}{M}\right)^6\left(\ln\frac{\phi^2}{M^2}\right)^2$$

- 2nd slowroll parameter

$$\eta = M_{\rm Pl}^2\left(\frac{V''}{V}\right) \approx 24\left(\frac{M_{\rm Pl}}{M}\right)^2\left(\frac{\phi}{M}\right)^2\ln\frac{\phi^2}{M^2}$$

$$\phi \ll M \sim 10^7\,\mathrm{GeV} - 10^{11}\,\mathrm{GeV}$$

$$\rightarrow \varepsilon \ll |\eta|$$

# Field values

$$\phi^2/M^2 \approx (|\eta|/24\ln(24M_{Pl}^2/|\eta|M^2))(M/M_{Pl})^2$$
$$\approx 10^{-3}|\eta|(M/M_{Pl})^2 \ll 1.$$

For $\phi \ll M$, and $\eta \sim -0.02$

$$\epsilon = \frac{|\eta|^3}{432\ \ln(24M_{Pl}^2/|\eta|M^2)}\left(\frac{M}{M_{Pl}}\right)^4 \ll 1$$

# Field values

- Normalization of the perturbation at pivot

$$\Delta_R^2 \approx \frac{V_0}{24\pi^2 M_{\rm Pl}^4 \epsilon} = \frac{9A\ln(24M_{Pl}^2/|\eta|M^2)}{4\pi^2|\eta|^3}$$

$$= 2.215 \times 10^{-9}$$

$$k_0 = k_{\rm CMB} = 0.05\ {\rm Mpc}^{-1}$$

- Parameters

$$A \sim 10^{-15}$$

$$V_0^{1/4} \sim 10^{-4} M$$

100 TeV for $M=10^9$GeV

# E-folding number

- E-folding number

$$N = \frac{1}{M_{\rm Pl}^2}\int_{\phi_{end}}^{\phi}\frac{V}{V'}d\phi \approx \frac{3}{2}\left(\frac{1}{|\eta|}-\frac{1}{|\eta_{end}|}\right)$$

$$N = 3/(1-n_s) - 3/2 = 73.5$$

For $1-n_s=0.04$

- Required value

$$N > N_{CMB} = 61 + \frac{2}{3}\ln\left(\frac{V_0^{1/4}}{10^{16}\,GeV}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{16}\,GeV}\right)$$

$$\sim 30 + \frac{2}{3}\ln\left(\frac{V_0^{1/4}}{10^{3}\,GeV}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{3}\,GeV}\right)$$

# Fermion condensate

Iso, KK, Shimada (2014)

with $H_{inf} << 100\text{MeV}$

- We need a negative linear term

$$V_{linear} \sim -C\phi \sim -C_0 h$$

- Quark condensates and mixing between Higgs and $\phi$

$$\lambda_{mix}\varphi^2 h^2$$

$$V_{linear} \sim -y < \bar{q}q > h$$

$$C_0 \sim y\langle\bar{q}q\rangle$$

y: Yukawa coupling

$$C = \sqrt{|\lambda_{mix}|/2\lambda_h}C_0 = (246/M[\text{GeV}])C_0$$

# E-folding number

$$N = \frac{1}{M_{Pl}^2} \int_{\phi}^{\phi_{end}} \frac{V_0 d\phi}{A\phi^3 \ln(M^2/\phi^2) + C}$$

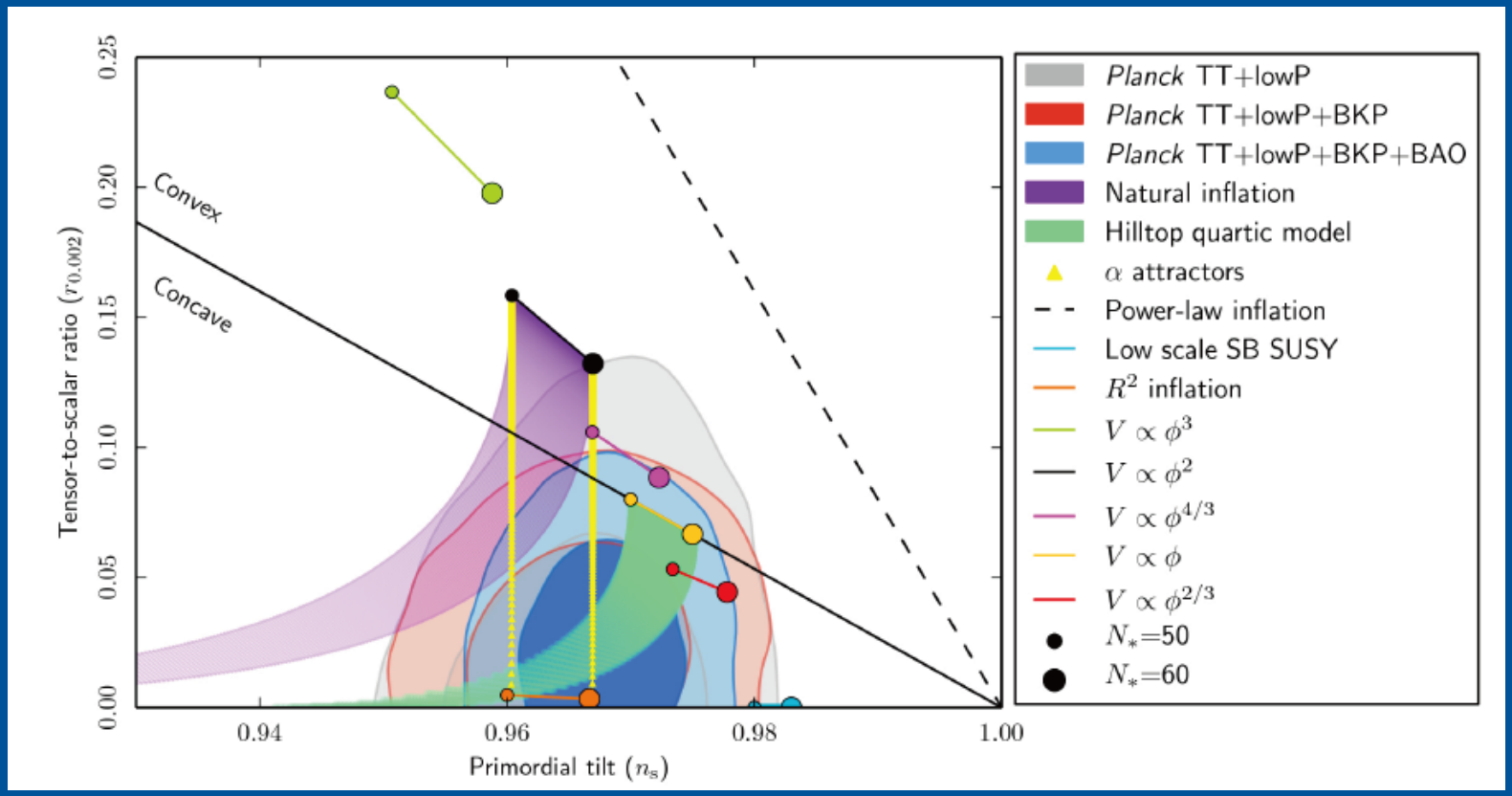
$$C \equiv A\tilde{C}M^3(M/M_{\rm Pl})^3$$

the denominator of $N$ balance for $C \sim 10^{-6}$ and $\phi_{\rm CMB}$, or for $\tilde{C} \sim 10^{-3}$ and $\phi_{end}$.

$$y \sim 10^{-6}, \quad \langle \bar{q}q \rangle \sim (100\text{MeV})^3, \quad \tilde{C} \sim 10^{-5} \leftrightarrow M = 10^8\,\text{GeV}$$

# Tensor to scalar ratio by Planck and BICEP/Keck + BAO

Planck 2015 results. XX. Constraints on inflation




Strong point: monomial $1/2m^2\varphi^2$ was excluded at two sigma

# Running of Spectral Index

$$\alpha \equiv \frac{dn_s}{d\ln k} = -24\varepsilon^2 + 16\varepsilon\eta - 2\xi^{(2)}$$

$$\sim -O(10) \times O(10^{-2})^2$$

$$\sim -O(10^{-3})$$

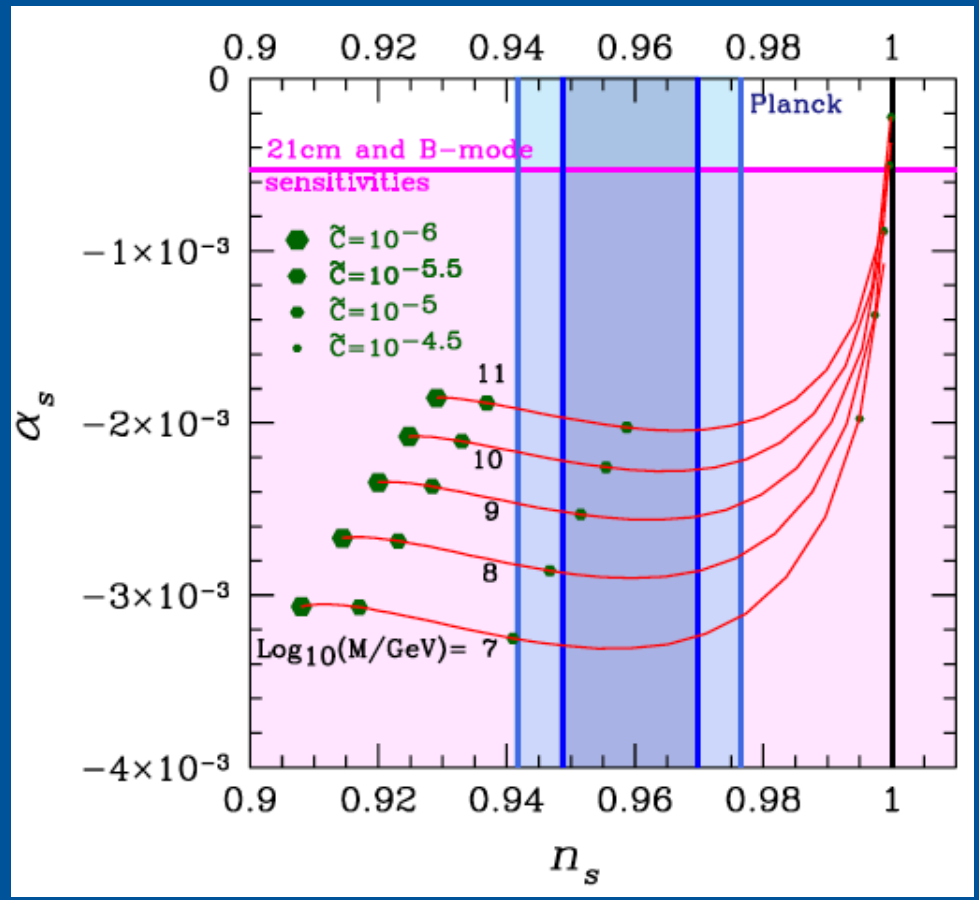
$$\xi^{(2)} \equiv \frac{V'''V'}{V^2} M_{\mathrm{P}}^4$$

# Running spectral index

$$n_s = 0.9569 \pm 0.0077 \quad (0.9586 \pm 0.0056)\,,$$
$$dn_s/d\ln k = 0.011^{+0.014}_{-0.013} \quad (0.009 \pm 0.010)\,,$$
$$d^2n_s/d\ln k^2 = 0.029^{+0.015}_{-0.016} \quad (0.025 \pm 0.013)\,,$$

$$\alpha_s \approx -2\xi^{(2)} \text{ with } \xi^{(2)} \equiv V'V'''M_{\rm Pl}^4/V^2$$

$$C \equiv A\tilde{C}M^3(M/M_{\rm Pl})^3$$




Iso, KK, Shimada (2014)

# What about smaller scales?

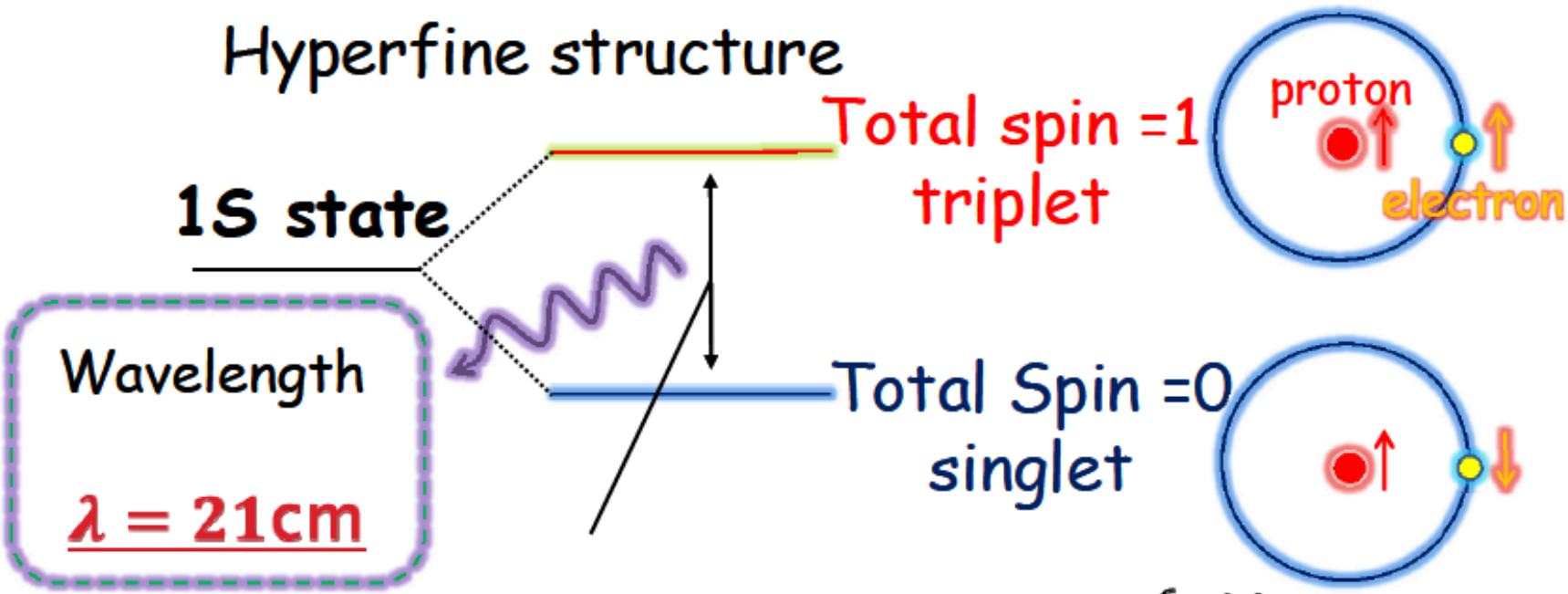
故宮博物院

# Cosmological 21cm line observations

KK, Oyama, T.Takahashi, T.Sekiguchi, 2013

# ◇ 21cm line

◆proton-electron's spin-spin interaction




$$\Delta E = 2\pi\nu_{21} = 5.8 \times 10^{-6}\text{eV}$$

# 21 cm brightness temperature

$$\delta T_b^{obs}\left(\frac{\nu_{21}}{1+z}, \boldsymbol{r}, z\right) \approx A\frac{x_{HI}n_H}{(1+z)H(z)}\left[1-\frac{T_\gamma}{T_S}\right]\left[1-\frac{1+z}{H(z)}\frac{d\mathrm{v}_{p||}}{dr}\right]$$

**Peculiar velocity**

Optical depth is small

$1-e^{-\tau_{\nu_{21}}} \approx \tau_{\nu_{21}}$

$$A \equiv \frac{3c^3\hbar A_{21}}{16\nu_{21}^2 k_B}$$

$\chi_{HI}$:neutral fraction

## Spin temperature

$$\frac{n_1}{n_0} \equiv \frac{g_1}{g_0}\exp\left(-\frac{h\nu_{21}}{k_B T_S}\right)$$

# ◇ 21cm brightness temperature fluctuation $\delta_{21}$

$$\delta_{21} \equiv \frac{\delta T_b^{obs} - \delta \bar{T}_b^{obs}}{\delta \bar{T}_b^{obs}}$$

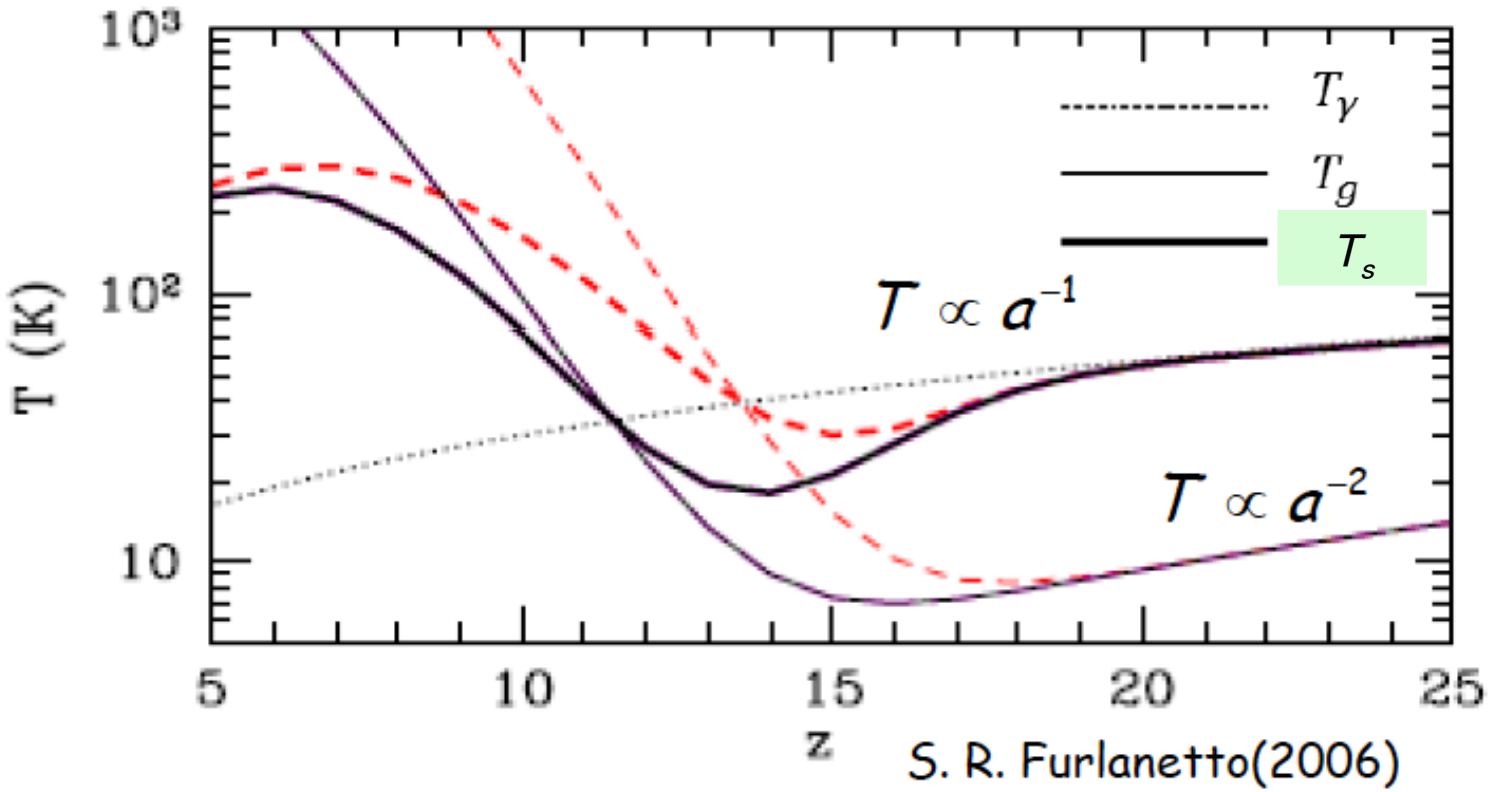
$$\delta_{\partial v} \equiv \frac{1+z}{H(z)} \frac{dv_{p||}}{dr}$$

$$\delta_{21} = \left(\frac{1}{1+\delta_{T_s}}\right)\left[1 + \delta_H + \delta_{x_{HI}} + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_s} - \frac{\bar{T}_\gamma}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_\gamma} - \delta_{\partial v}\right]$$

$$+\left[\delta_{x_{HI}}\delta_H + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{x_{HI}}\delta_{T_s} - \frac{\bar{T}_\gamma}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_\gamma}\delta_{x_{HI}} - \delta_{x_{HI}}\delta_{\partial v} + \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{x_H}\delta_{T_s} - \frac{\bar{T}_S}{\bar{T}_S - \bar{T}_\gamma}\delta_{T_s}\delta_{\partial v}\right] - 1$$

$$\delta_{x_{HI}}, \delta_{T_s} \approx \mathcal{O}(1)$$

# Evolution of spin temperature after star formation




S. R. Furlanetto(2006)

Gas was heated by X-ray emission

$$T_g > T_\gamma$$

**z ≈ 10** by Ly-α heating $T_S \rightarrow T_g \gg T_\gamma$

# 21cm line power spectrum $P_{21}(k,\mu)$

$$\left\langle \tilde{\delta}_{21}(\boldsymbol{k})\tilde{\delta}_{21}^{*}(\boldsymbol{k}')\right\rangle = (2\pi)^3\delta^D(\boldsymbol{k}-\boldsymbol{k}')\,P_{21}(k,\mu)$$

$M_\nu$ < 0.1 eV $\delta_{x_{HI}} \ll 1$

$$P_{21}(k,\mu) = (1+\mu^2)^2 P_{\delta_H\delta_H}(k)$$

$P_{\delta_H\delta_H}(k)$ : matter power spectrum

**Detail of ionization history gives us power spectrum**

# SKA ( Square kilometer Array )

## Location : Australia and South Africa

Antenna number
5000

Effective total
Antenna area
$6 \times 10^5 \ \mathrm{m}^2$

http://www.skatelescope.org/

**Construction Phase (2016 – )**

# Omniscope

Max Tegmark, Matias Zaldarriaga arXiv:0805.4414v2 (2008)
Max Tegmark, Matias Zaldarriaga Phys. Rev. D 82, 103501 (2010)

Lower cost than usual interferometers

- J. R. Pritchard, E. Pierpaoli, Phys Rev D 78, 065009

| Antenna number | Effective total anntena area |
|---|---|
| $10^6$ | $10^6$ m |

# 21cm Fisher matrix

M.McQuinn, O.Zahn, M.Zaldarriaga, L.Hernquist, S.R. Furlanetto (2006) Astrophys.J.653:815-830,2006

$$F_{ij} = \frac{1}{2}\sum_{i}^{N} \frac{1}{P_{T_b}^{tot}(k,\mu)^2} \frac{\partial P_{T_b}^{tot}(k,\mu)}{\partial \theta_i} \frac{\partial P_{T_b}^{tot}(k,\mu)}{\partial \theta_j}$$

$$P_{T_b}^{tot} \equiv \left(\delta \bar{T}_b^{obs}\right)^2 P_{21} + P_{Noise}$$

$$P_{Noise} \equiv \left(\frac{\lambda^2 T_{sys}}{A_e}\right)^2 \frac{1}{n_b t_0}$$ Detector Noise

# CMB B-mode polarization

By Planck (ESA), PolarBEAR (USA,Japan), CMPpol (USA)

- Fisher matrices

$$\mathbf{F}_{ij}^{\mathrm{CMB}} = \sum_{l} \frac{(2\ell+1)}{2} f_{\mathrm{sky}} \times \mathrm{Trace}\left[\mathbf{C}_{\ell}^{-1}\frac{\partial \mathbf{C}_{\ell}}{\partial p_i}\mathbf{C}_{\ell}^{-1}\frac{\partial \mathbf{C}_{\ell}}{\partial p_j}\right]$$

$$\mathbf{C}_{\ell} = \begin{pmatrix} C_{\ell}^{\mathrm{TT}} + N_{\ell}^{\mathrm{TT}} & C_{\ell}^{\mathrm{TE}} & C_{\ell}^{\mathrm{Td}} \\ C_{\ell}^{\mathrm{TE}} & C_{\ell}^{\mathrm{EE}} + N_{\ell}^{\mathrm{EE}} & 0 \\ C_{\ell}^{\mathrm{Td}} & 0 & C_{\ell}^{\mathrm{dd}} + N_{\ell}^{\mathrm{dd}} \end{pmatrix}$$

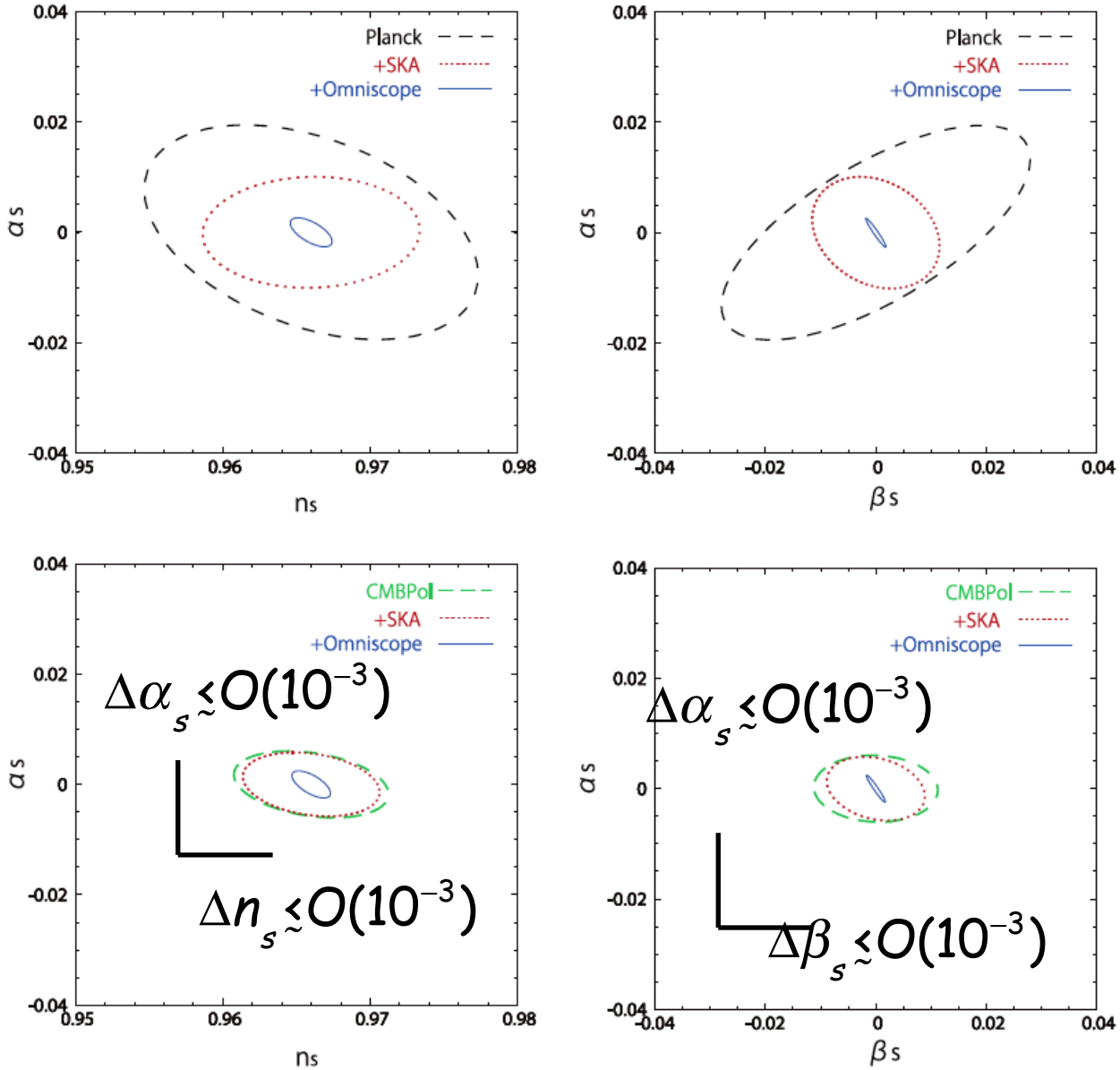
$$\mathbf{F}^{\mathrm{21cm+CMB}} \simeq \mathbf{F}^{\mathrm{CMB}} + \mathbf{F}^{\mathrm{21cm}}$$

$$\mathbf{F}_{ij}^{\mathrm{21cm}} = \sum_{\mathrm{pixels}} \frac{1}{[\delta P_{21}(\mathbf{u})]^2}\left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_i}\right)\left(\frac{\partial P_{21}(\mathbf{u})}{\partial p_j}\right)$$

# Running and Running of

Kohri, Oyama, Sekiguchi, T.Takahashi (2013)

$$\alpha_s \equiv \frac{d\ln P_\zeta}{d\ln k}$$




$$\beta_s \equiv \frac{d\alpha_s}{d\ln k}$$

# Sensitivities

| $k_{\rm ref}$ [Mpc$^{-1}$] | 0.002 | 0.01 | 0.05 | 0.1 | 0.2 | 0.5 |
|---|---|---|---|---|---|---|
| $\delta n_s$ | $3.81 \times 10^{-3}$ | $2.62 \times 10^{-3}$ | $5.53 \times 10^{-4}$ | $4.01 \times 10^{-4}$ | $4.68 \times 10^{-4}$ | $3.33 \times 10^{-4}$ |
| $\delta \alpha_s$ | $1.47 \times 10^{-3}$ | $1.87 \times 10^{-3}$ | $1.00 \times 10^{-3}$ | $5.57 \times 10^{-4}$ | $2.64 \times 10^{-4}$ | $6.65 \times 10^{-4}$ |
| $\delta \beta_s$ | $2.43 \times 10^{-4}$ | $5.94 \times 10^{-4}$ | $6.86 \times 10^{-4}$ | $6.88 \times 10^{-4}$ | $6.87 \times 10^{-4}$ | $6.79 \times 10^{-4}$ |

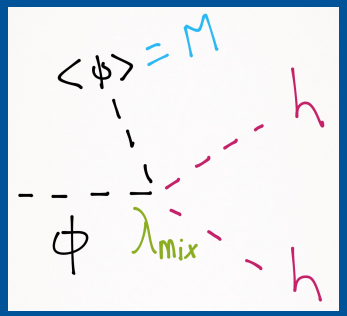
Table 5: Expected $1\sigma$ uncertainties of $n_s$, $\alpha_s$ and $\beta_s$ from CMBpol+Omniscope for several values of $k_{\rm ref}$.

Kohri, Oyama, T.Takahashi, T.Sekiguchi, 2013

# Reheating

- Decay

$$\Gamma_{\phi\to hh} \sim \frac{\lambda_{mix}^2}{16\pi}\frac{\langle\phi\rangle^2}{m_\phi} \sim 10^{-2}\frac{(\lambda_{mix}M)^2}{m_\phi} \sim 10^{-2}\frac{m_h^4}{m_\phi M^2}$$




$$\sim 10^{-14}\,\mathrm{GeV}\left(\frac{10^9\,\mathrm{GeV}}{M}\right)^2$$

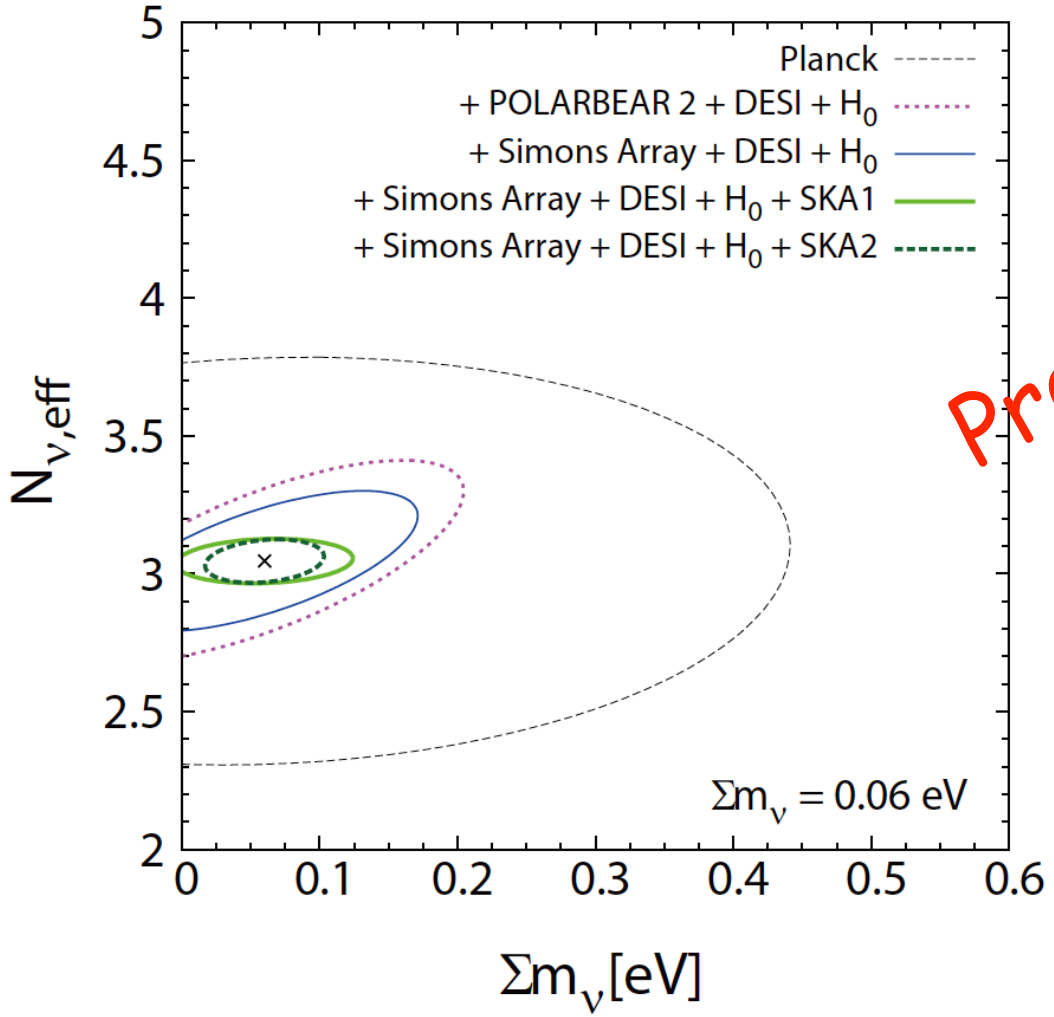
$$m_h = \sqrt{|\lambda_{\rm mix}|}M$$

- Reheating

$$\Gamma_{\phi\to hh} \equiv 3H(T_R)$$

$$T_R \sim 100\,\mathrm{GeV}\left(\frac{10^9\,\mathrm{GeV}}{M}\right)$$

# Future constraints on neutrino mass by 21cm, CMB, and BAO




Preliminary

Oyama, Kohri, Hazumi (2015) in preparation

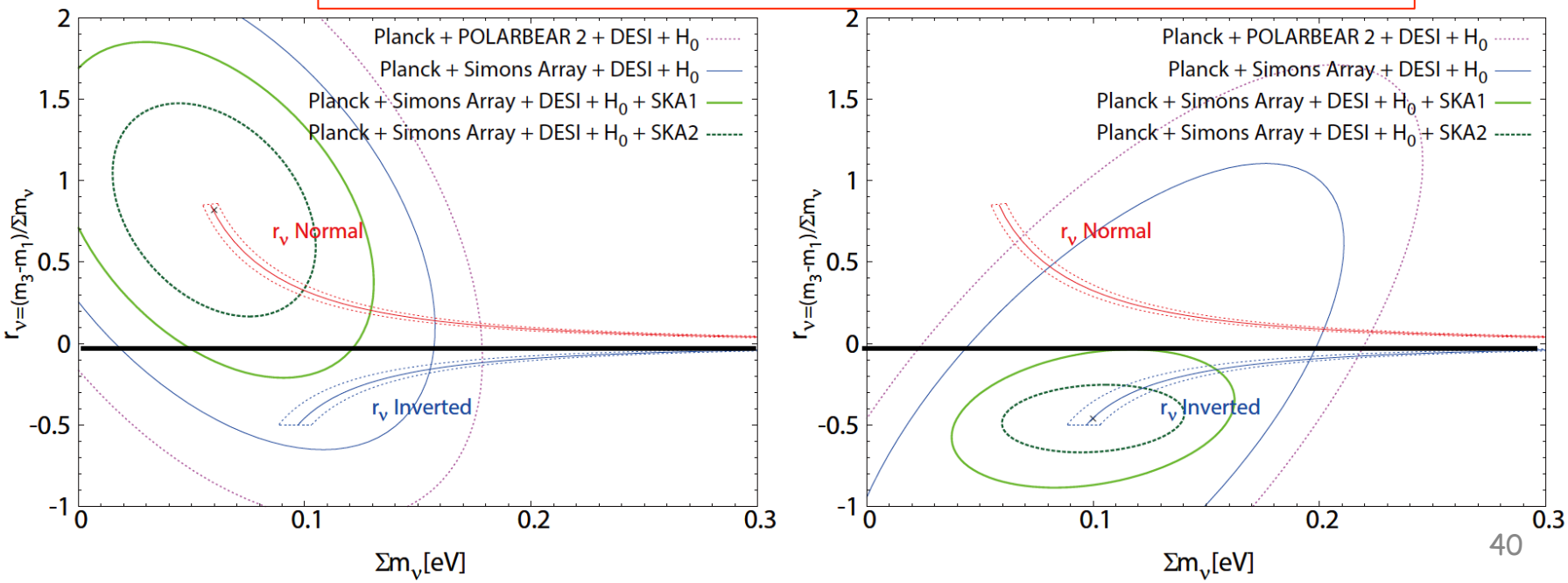
# Future constraints on neutrino hierarchy by 21cm, CMB and BAO

Oyama, Kohri, Hazumi (2015) in preparation

Preliminary

- Hierarchy parameter

$$r_\nu \equiv \frac{m_3 - m_1}{\Sigma m_i} = \begin{cases} > 0 & \text{normal hierarchy} \\ < 0 & \text{inverted hierarchy} \end{cases}$$



# Conclusion

- Conformal infltion models are attractive in terms of both large and small field inflations. After Planck's 2015 data release, we found that only the small-field models fit the observations

- In near future we can discriminate this model from others by using new 21cm and CMB B-mode polarization observations

# Outstanding issues

- The trajectory is not so trivial.
- Tensor to scalar ratio is small ($r \ll 10^{-20}$), which must not be observed forever
- A possible large-field preceding inflation could occur before the small-field CW conformal inflation