A Nonperturbative Regulator for Chiral Gauge Theories and Fluffy Mirror Fermions

DMG and David B. Kaplan
arXiv:1511.03649

Motivation: Lattice Regulate Chiral Gauge Theory

Big Question 1: Do chiral gauge theories (χGT) make sense beyond perturbation theory?

- Only known χGT is the Standard Model Electroweak sector
- What are the requirements to have a well-defined χGT

Big Question 2: What are the properties of strongly interacting chiral gauge theories?

- High energy extensions of the Standard Model

To answer these, must first find a nonperturbative regulator.

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Is this a technical issue or indicative of new physics or a problem with the Standard Model?

Technical Question: Define Fermion Measure

Observables are calculated by integrating over gauge fields with some measure

$$\langle F(A)\rangle = \frac{\int [DA] e^{-S(A)} \Delta(A) F(A)}{\int [DA] e^{-S(A)} \Delta(A)}$$

- F(A) is the observable
- S(A) is gauge action (Maxwell or Yang Mills)
- Δ(A) is due to fermions

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- Δ(A) is due to fermions
  - Δ(A) for Dirac fermion is well-known

$$\Delta_{DF}(A) = \det \not{D}(A)$$

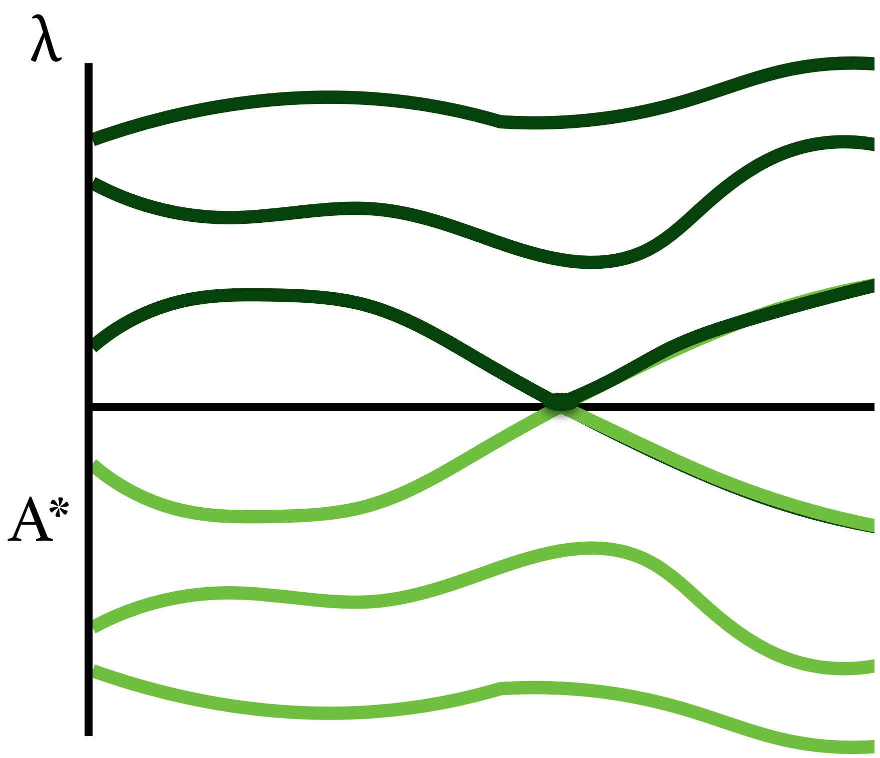
  - But it is not well know how to define Δ(A) for chiral fermion

$$\Delta_{\chi F} \Delta^*_{\chi F} = \Delta_{DF}$$

Technical Question: Define Fermion Measure

What is the fermion measure for Chiral Fermions

- Need definition so that effective action is local and analytic



$$\Delta_{\chi F}(A) = \prod_{\lambda_j > 0} \lambda_i$$

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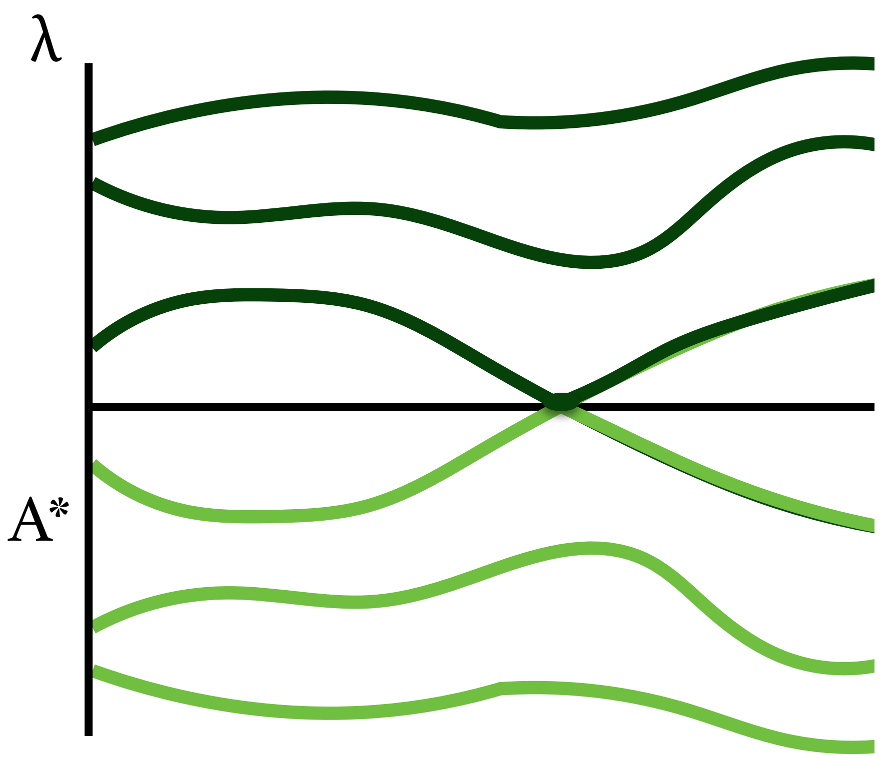

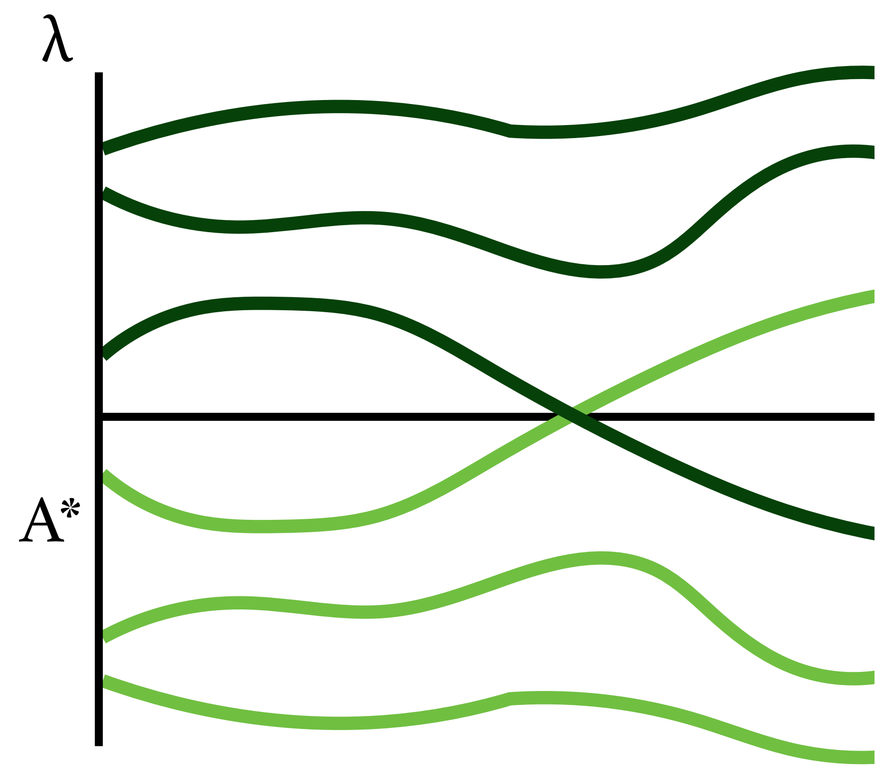

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Continuum Field Theory

- Theories with chiral symmetries can have anomalies
- Standard Model contains global anomalies
- Chiral gauge theories only well-behaved if no gauge anomalies

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- Lattice must explicitly break global chiral symmetry to reproduce anomaly
- Lattice must somehow distinguish anomalous and anomaly-free gauge theories

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How does one construct a lattice theory that has the correct continuum behavior?

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Choice A: Explicit Gauge Violation

- Lattice theory is not gauge invariant
- Gauge invariance must be restored in continuum limit
- Sensible continuum limit only exists for anomaly-free theories

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We will go with Choice B

For our construction, only anomaly-free theories are local

Steps to Define Fermion Measure for χGT

Basic building block is Dirac fermion, in order to have well-defined eigenvalue problem

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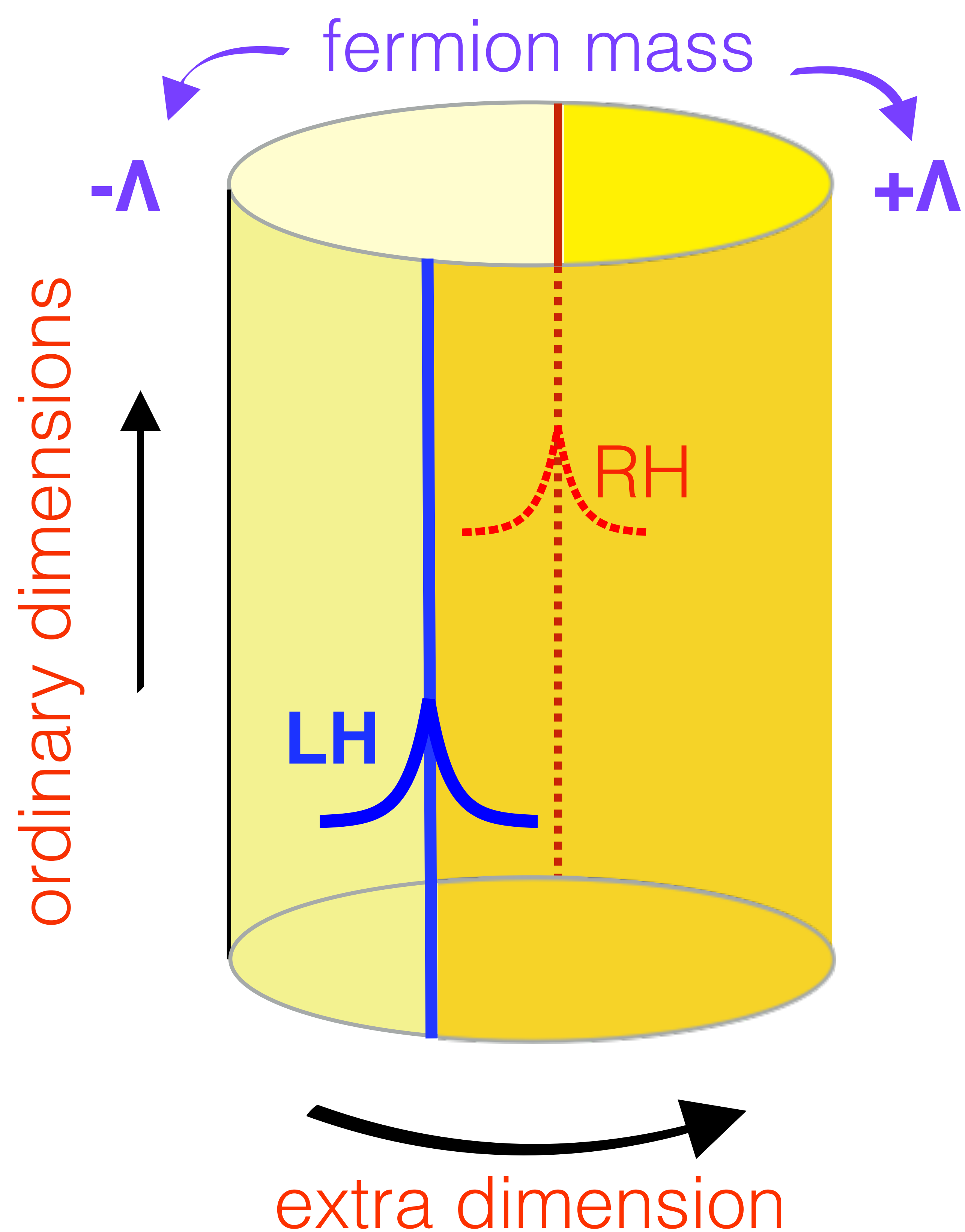
Global Chiral Symmetries

Domain Wall Fermions (DWF)

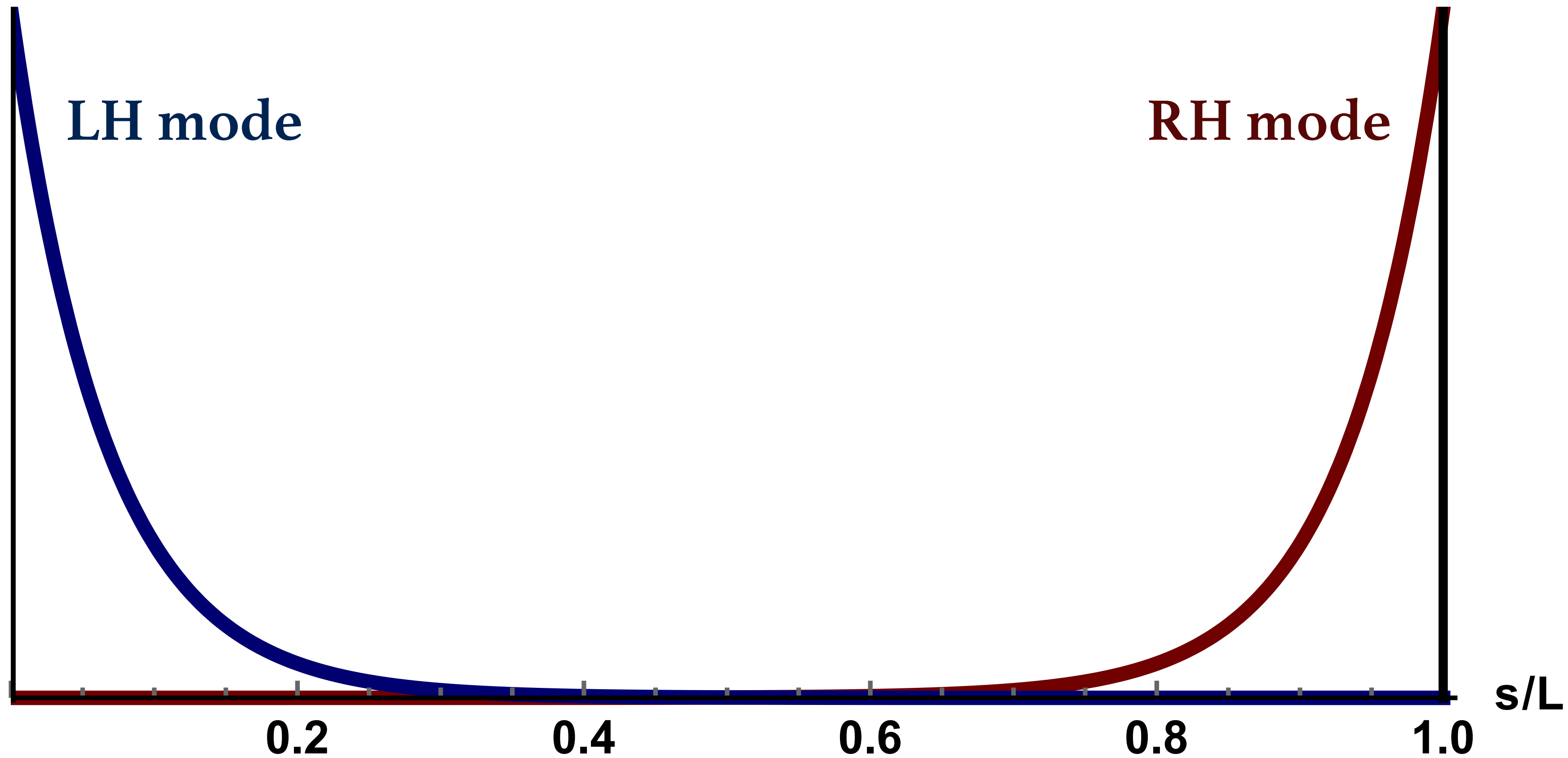
(DB Kaplan, 1992)

- Introduce extra (compact) dimension, x_5
- Fermion mass depends on x_5
- Massless modes localized on mass defects
- Gauge fields independent of x_5
- Anomaly due to bulk fermions carrying charge between mass defects
- Condensed matter physicists would call this a topological insulator



Global Chiral Symmetries

- In the mass basis, the 4d action is $S_F = \int d^4x \sum_{\tilde{n}=0}^{\infty} \overline{\psi}_{\tilde{n}} \left(\not{D}_4 + \mu_{\tilde{n}} \right) \psi_{\tilde{n}}$



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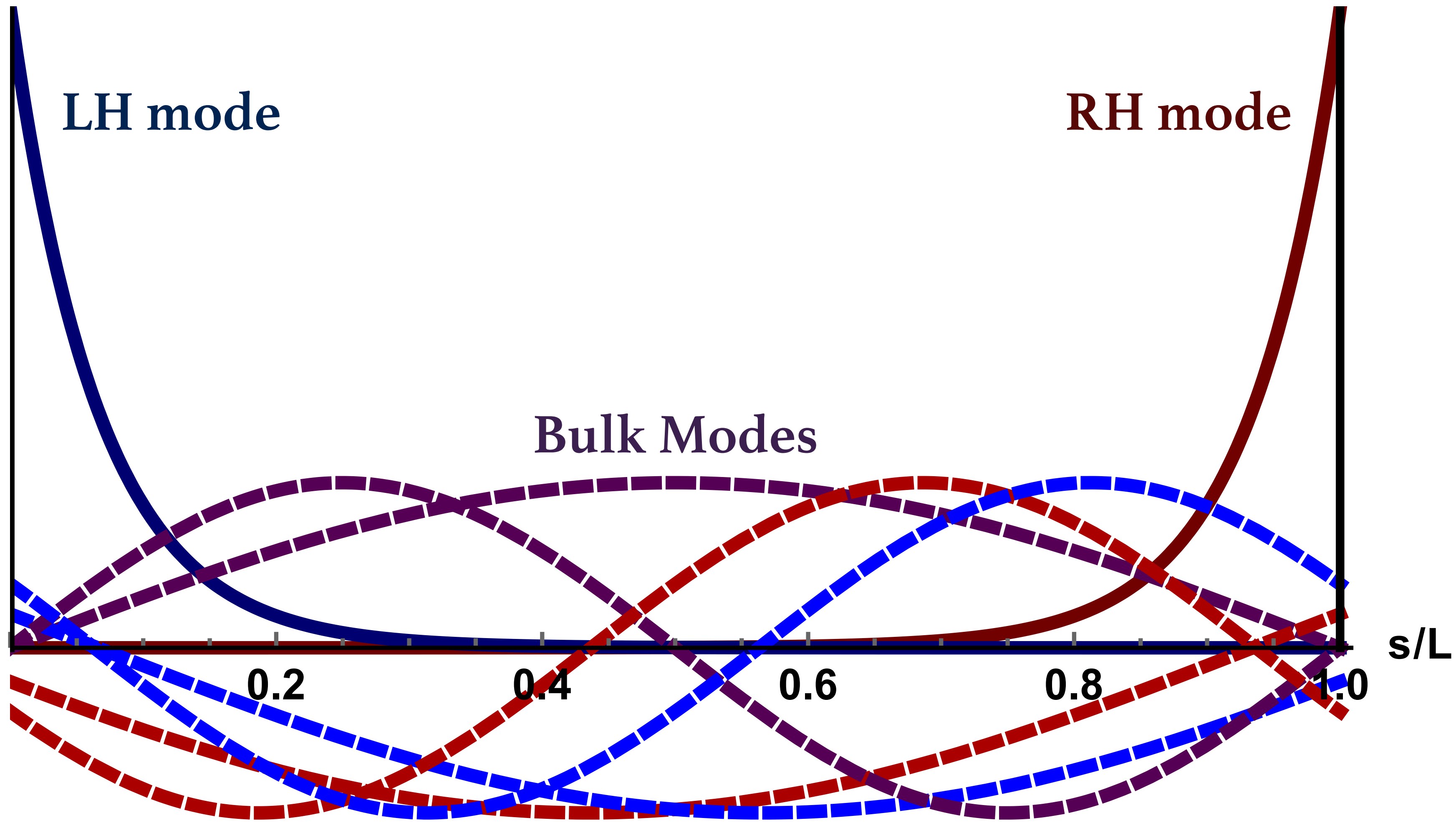


The theory contains a mass gap:

$$\mu_0 = 0 \qquad \mu_{\tilde{n}} = \sqrt{\left(\frac{\tilde{n}\pi}{L}\right)^2 + \Lambda^2} \qquad \tilde{n} = 1, 2, 3, \ldots$$

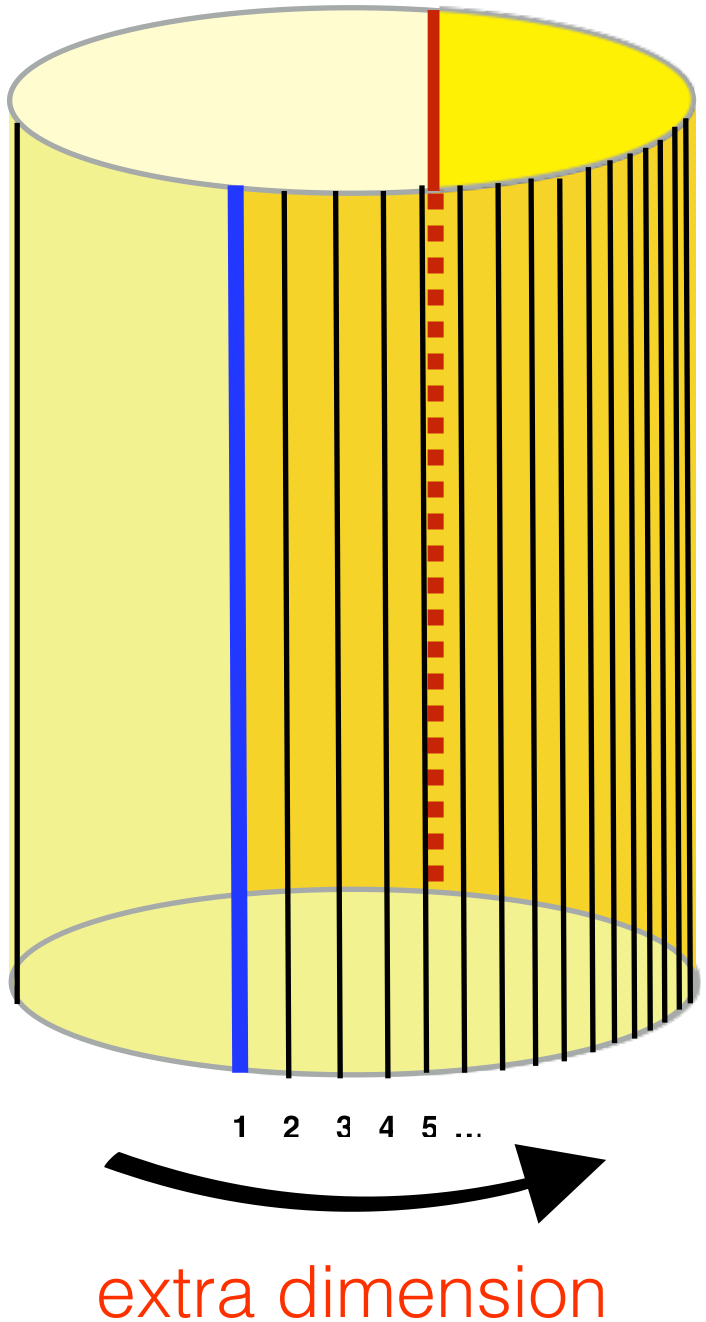
Global Chiral Symmetries

DWF always give rise to a vector gauge theory

- DWF 5d action is equivalent to action for an infinite number of 4d fermions
- If discretize extra dimension, x_5 is a flavor quantum number

$$\bar{\psi}\gamma_5\partial_5\psi \rightarrow \bar{\psi}_n\gamma_5\left(\psi_{n+1} - \psi_n\right)$$

Every flavor must be in same gauge group representation



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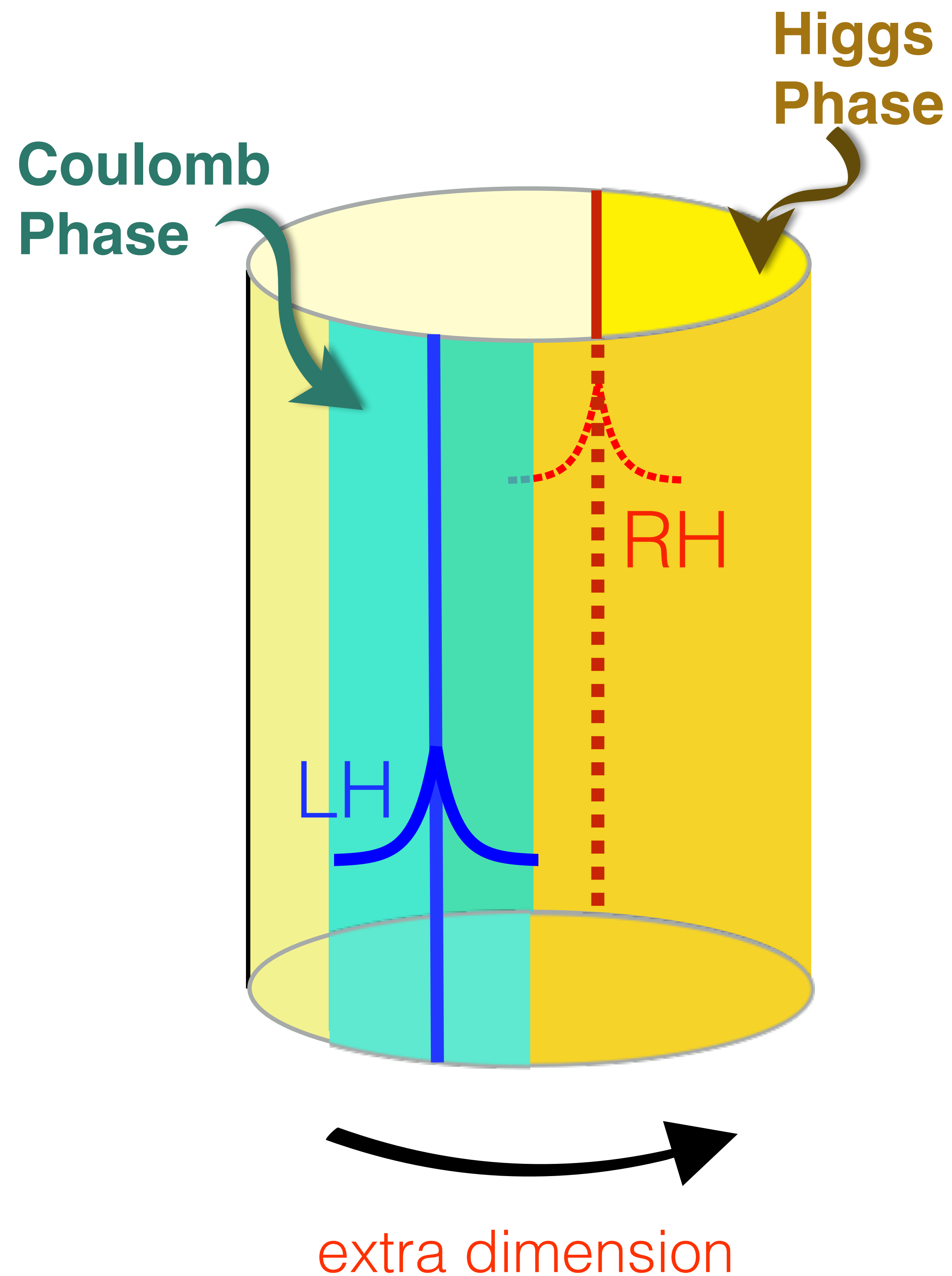
Gauged Chiral Symmetries (Previous Attempt)

Idea: Localize Gauge Fields at one defect

- Waveguide Model
- Gauge fields **depend** on x_5
- Need spontaneous symmetry breaking to preserve gauge invariance
- New RH mode appears at location of SSB
- Spectrum has **Dirac** fermions

Golterman, Jansen, Vink 1993



Gauged Chiral Symmetries (New Attempt)

New Idea: Localize gauge fields around one defect via gradient flow (DMG and DB Kaplan, arXiv:1511.03649)

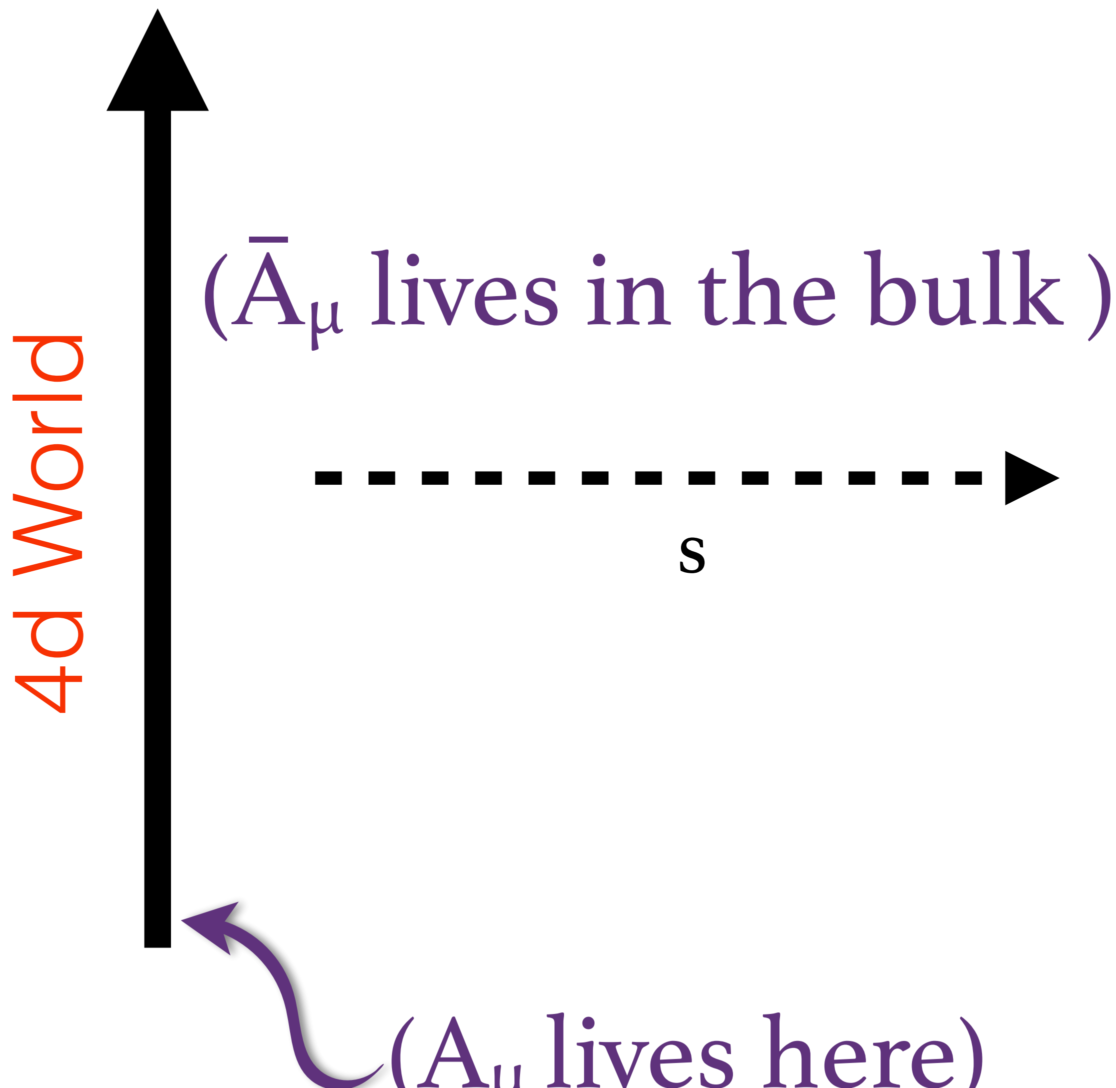
Gradient Flow

- Utilizes extra dimension
- Start with any gauge field, A_μ
- Extend gauge field into the bulk

Flow Eq: $\partial_s \bar{A}_\mu = D_\nu \bar{F}_{\nu\mu}$ BC: $\bar{A}_\mu(x,0) = A_\mu(x)$

- Behaves like heat equation
- **Damps out high momentum modes**

Quantum Gradient Flow (Lüscher, 2010)



- Gauge Covariant Flow Eq.

$$\partial_s \bar{A}_\mu = D_\nu \bar{F}_{\nu\mu}$$

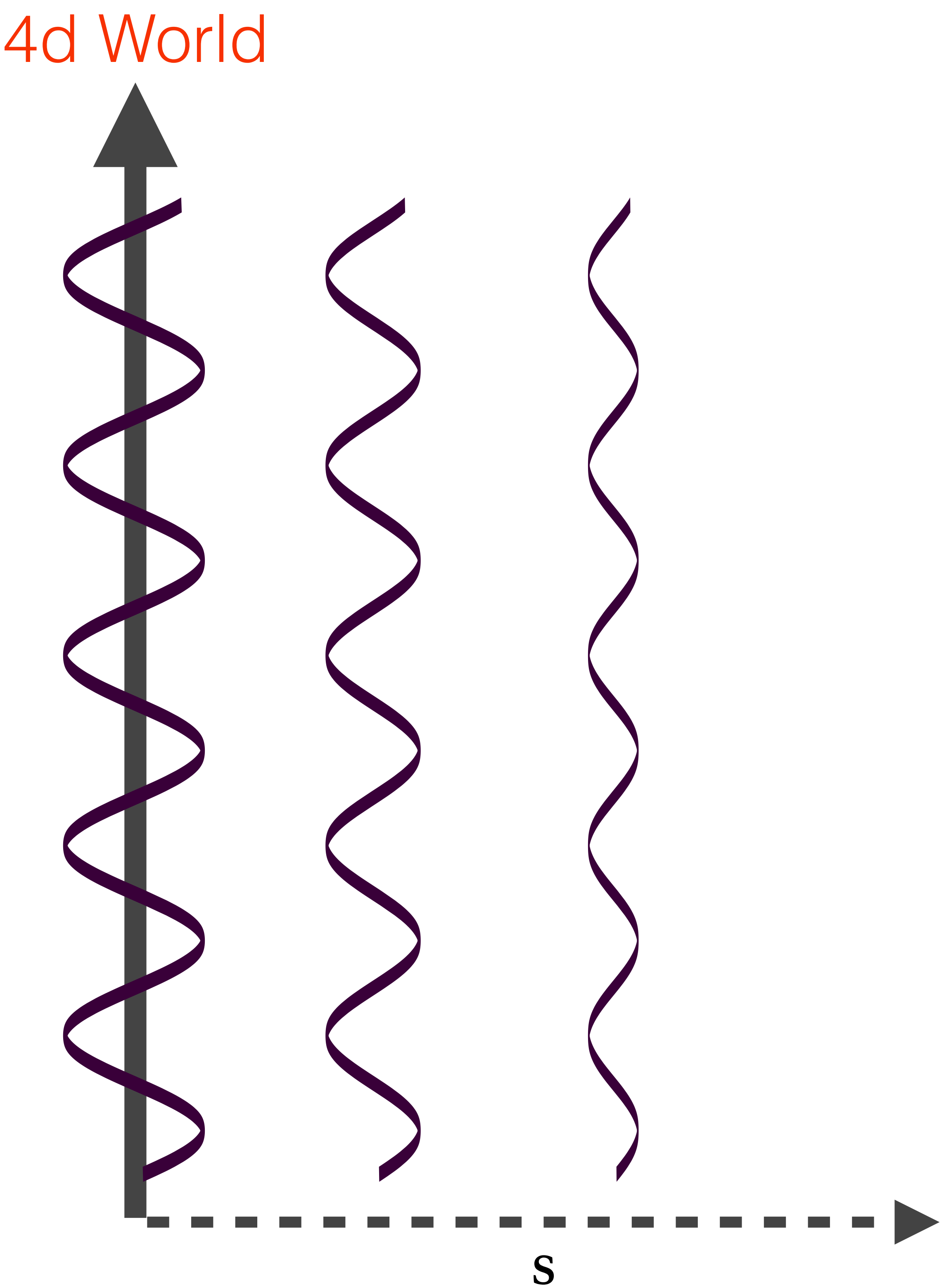
- Boundary Condition

$$\bar{A}_\mu(x,0) = A_\mu(x)$$

- Gauge action only depends on $\mathbf{A}_\mu$

$$S_G = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu}$$

Quantum Gradient Flow: 2d/3d QED Example



Write A_μ in terms of gauge and physical degree of freedom

$$\bar{A}_\mu = \partial_\mu \bar{\omega} + \epsilon_{\mu\nu}\partial_\nu \bar{\lambda}$$

Flow Eqs.

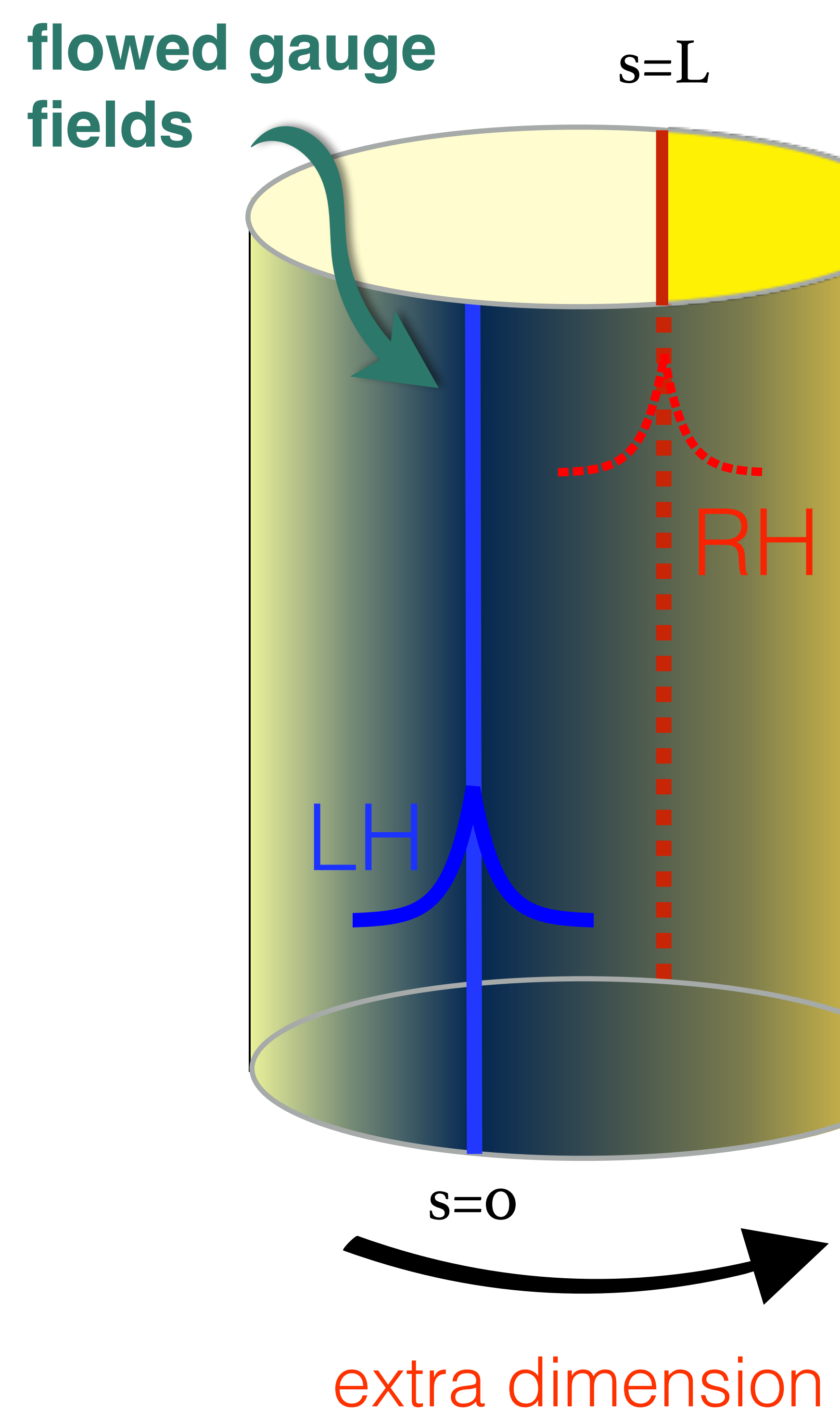
$$\partial_s \bar{\lambda} = \Box \bar{\lambda} \qquad \partial_s \bar{\omega} = 0$$

Flow in extra dimension damps out high momenta modes

Combine Domain Wall Fermions and Gradient Flow

Idea: Localize Gauge Fields at one defect via gradient flow

- Gauge field at s= 0 is quantum gauge field $A_\mu(x)$
- Bulk gauge field $\bar{A}_\mu(x,s)$ obeys flow equation
- Flow is symmetric around s=0
- RH modes have soft form factor coupling to physical degrees of freedom
- LH and RH modes couple equally to gauge degrees of freedom



Fluffy Mirror Fermions

How does theory look in flavor picture?

- All fermions couple with same strength to gauge degree of freedom
- Bulk and boundary fermions have different form factors for dynamical degree of freedom
- RH mode has form factor

$$e^{-\xi p^2 L/\Lambda}$$

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Flow parameter Wall seperation

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Photon Momentum   Bulk fermion mass

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- Idea: Decouple mirror fermions on far boundary with soft form factors

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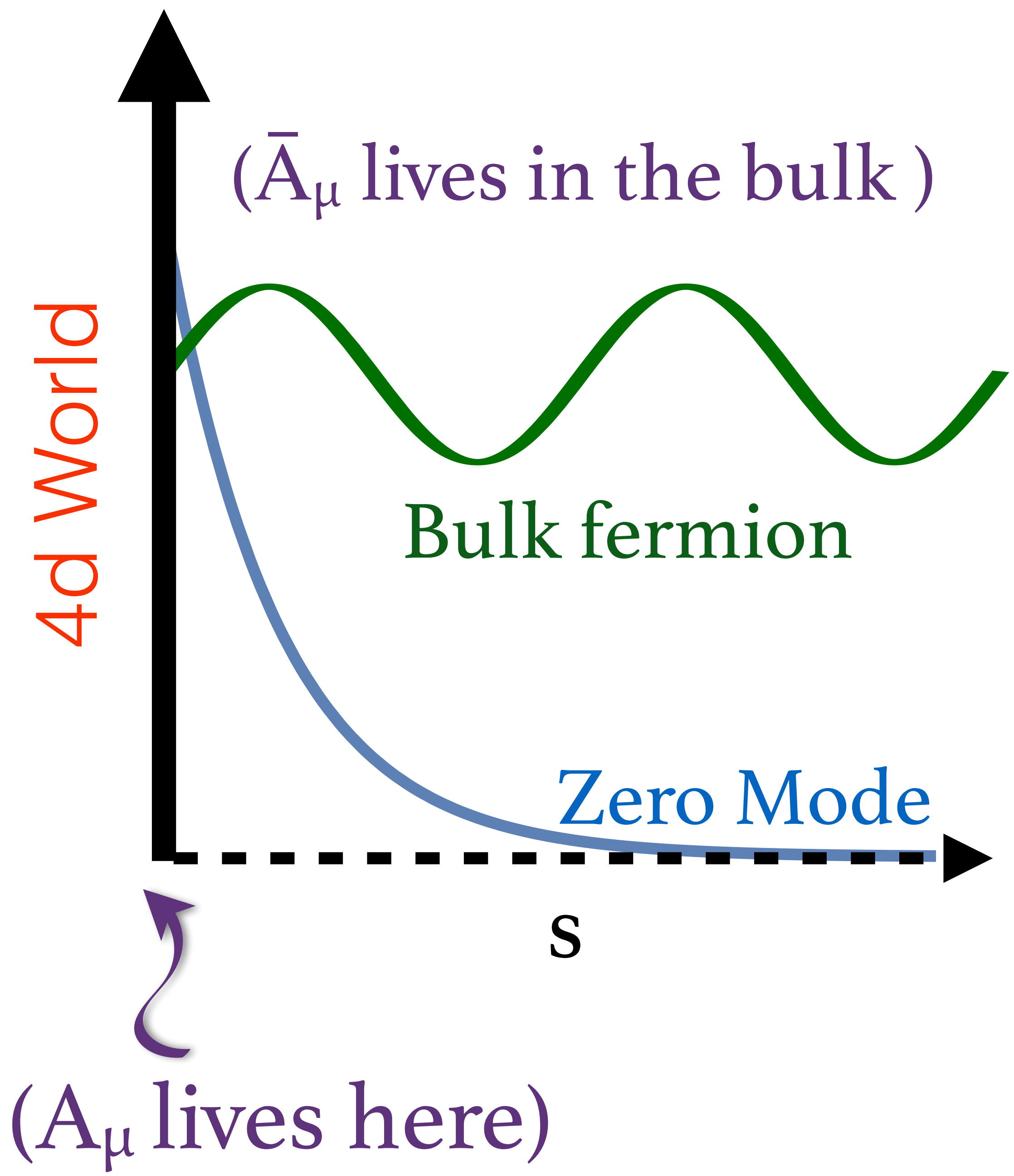
Anomalies and Callan-Harvey Mechanism (Callan and Harvey, 1984)

Integrating out bulk fermions generates a Chern-Simons term



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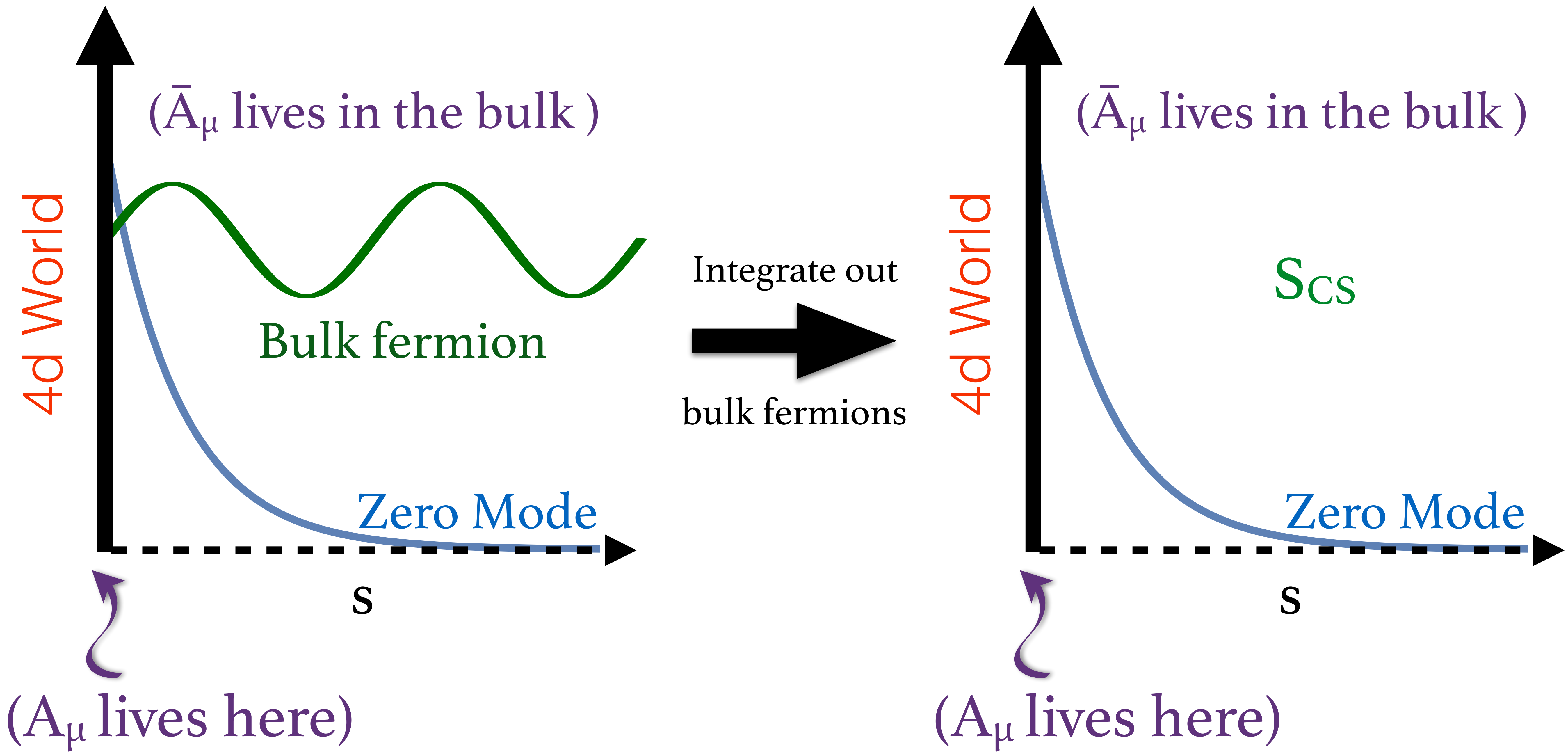

Anomalies and Callan-Harvey Mechanism (Callan and Harvey, 1984)

- Integrating out bulk fermions generates a Chern-Simons term
- In 3 dimensions, the Chern Simons action is

$$S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1)\, \mathrm{Tr}\left(\bar{F}\bar{A} - \tfrac{1}{3}\bar{A}^3\right)$$

- This approximation is only valid far away from domain wall

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CS only depends on sign of domain wall mass

Fermion Contribution

Pauli Villars Contribution

- This approximation is only valid far away from domain wall

Anomalies and Callan-Harvey Mechanism

- Consider 3 dimensional QED with flowed gauge fields

$$S_3^{\text{bulk}} = 2e^2 c_3 \frac{\Lambda}{|\Lambda|} \int dx^2 dy^2 \left(\frac{\partial_\mu \partial_\alpha}{\Box} A_\alpha(x) \right) \Gamma(x-y) \left(\frac{\partial_\mu \partial_\beta}{\Box} \epsilon_{\beta\gamma} A_\gamma(y) \right)$$

  - No gauge field in 3rd dimension

- Effective two point function is nonlocal

$$\Gamma(r) = \left(\delta^2(r) - \frac{\mu^2}{4\pi} e^{-\mu^2 r^2/4} \right) \qquad \mu \equiv \sqrt{\frac{\Lambda}{\xi L}}$$

  - When flow is turned off ($\mu \to \infty$), Γ vanishes

Serves as an IR cutoff

Anomaly Cancellation and Nonlocality

- DWF with flowed gauge fields gives rise to a nonlocal 2d theory

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Fermion Chirality

- The theory is local if this prefactor vanishes

This is exactly equivalent to the requirement that the chiral fermions be in an anomaly free representation

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Basic building block is Dirac fermion, in order to have well-defined eigenvalue problem

1. Global chiral symmetry (massless Dirac fermions)
   - Domain Wall Fermions
2. Decouple mirror fermions
   - Gradient Flow
3. Mechanism for distinguishing anomalous versus anomaly free fermion representation
   - Theory is local if fermions are in an anomaly free representation

Proposal for chiral fermion measure

Recall that the goal is to be able to define a chiral fermion measure

$$\langle F(A)\rangle = \frac{\int [DA] e^{-S(A)} \Delta(A) F(A)}{\int [DA] e^{-S(A)} \Delta(A)}$$

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One factor for each species of fermion

5d Dirac operator with flowed gauge field

Pauli-Villars

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- Effective action is what one would expect for chiral fermion (did not show here)

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Fluffy Mirror Fermions

Question: What if this construction is taken to be physical?

- Standard Model fermions have mirror partners that couple extremely weakly
- Detectable via ultra-soft probes: "low-energy" frontier
  - Mirror fermions couple like Standard fermions at low momenta

$$\lim_{p \to 0} e^{-p^2/\mu^2} \to 1$$

- Many possible phenomenological applications
- Can also do flow for gravity - Ricci Flow

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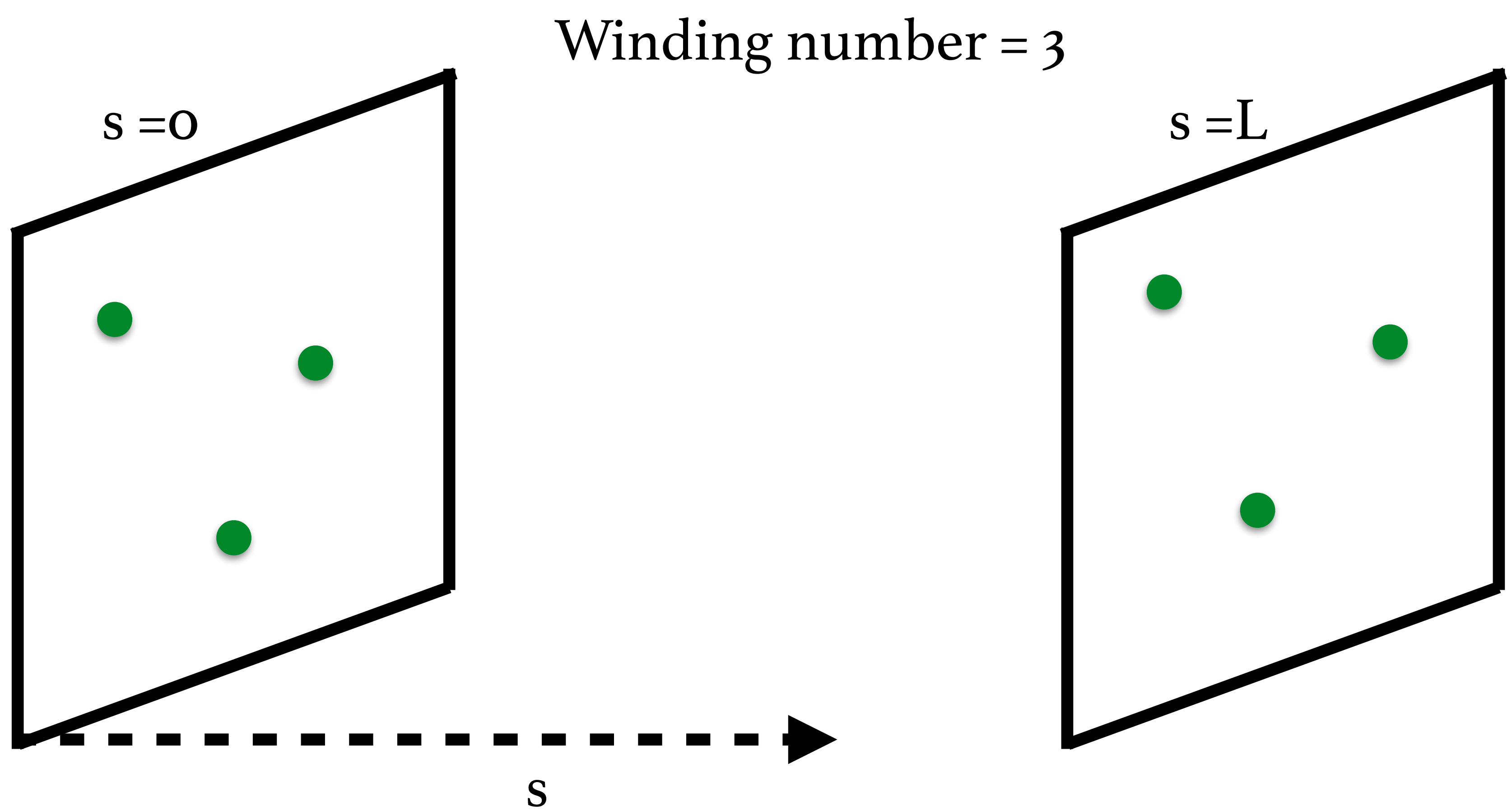
Topological Gauge Configurations

Can Fluff decouple from topological gauge configurations ?

- Flow was multiple attractive fixed points
  - Ex: Instanton Solution
- Is there a 't Hooft vertex that links Standard Model and Fluff?
- If Fluff cannot decouple, is this proposal doomed?
  - Need to look at how construction behaves at both strong and weak coupling

Topological Gauge Configurations - Weak Coupling



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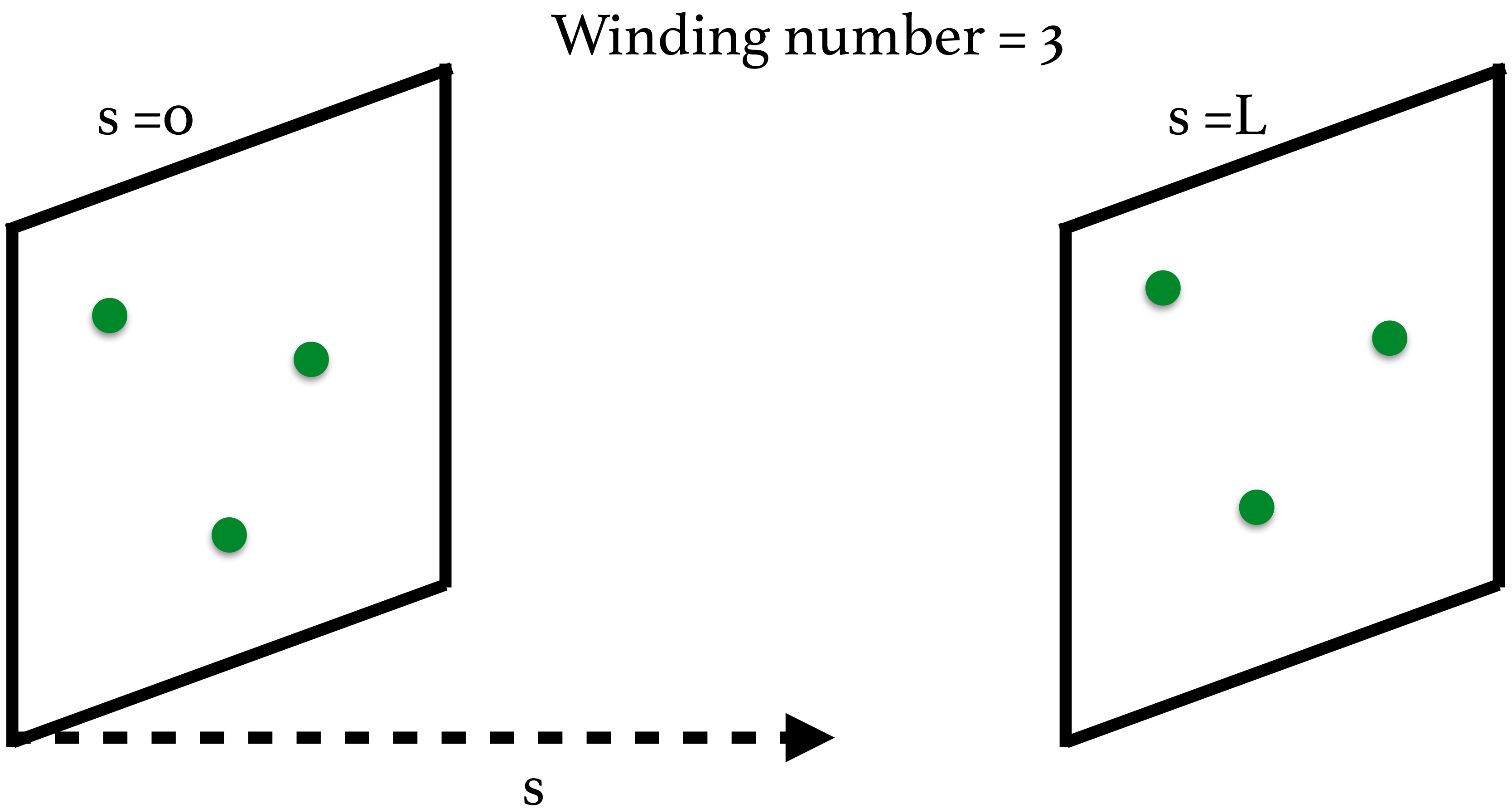
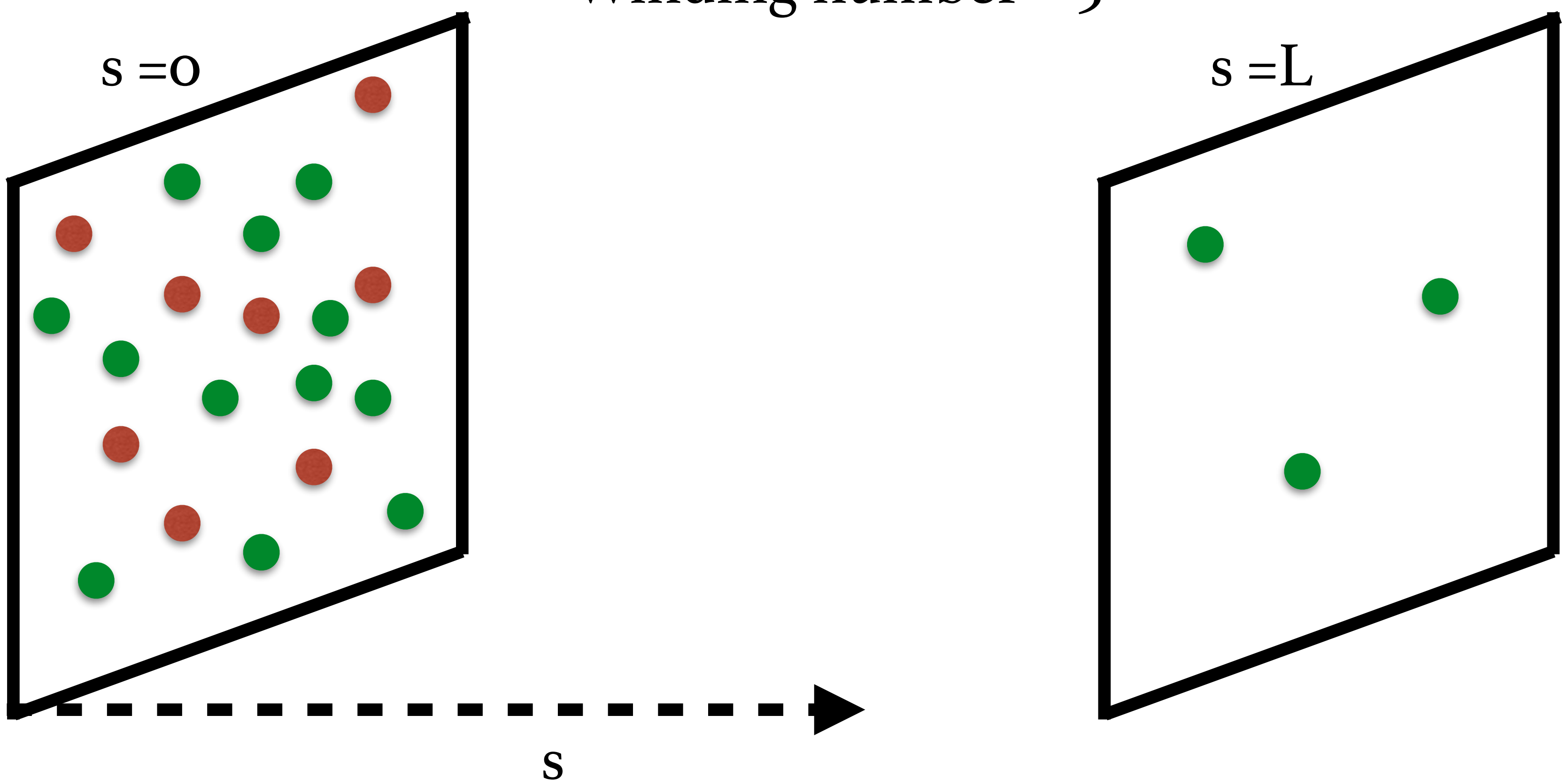


At weak coupling, instanton contribution is most important

- Flow does not affect location of instantons
- Correlation between location of instantons on the two boundaries allows for exchange of energy/momentum
- Highly suppressed process, so difficult to observe

Topological Gauge Configurations - Strong Coupling

Winding number = 3



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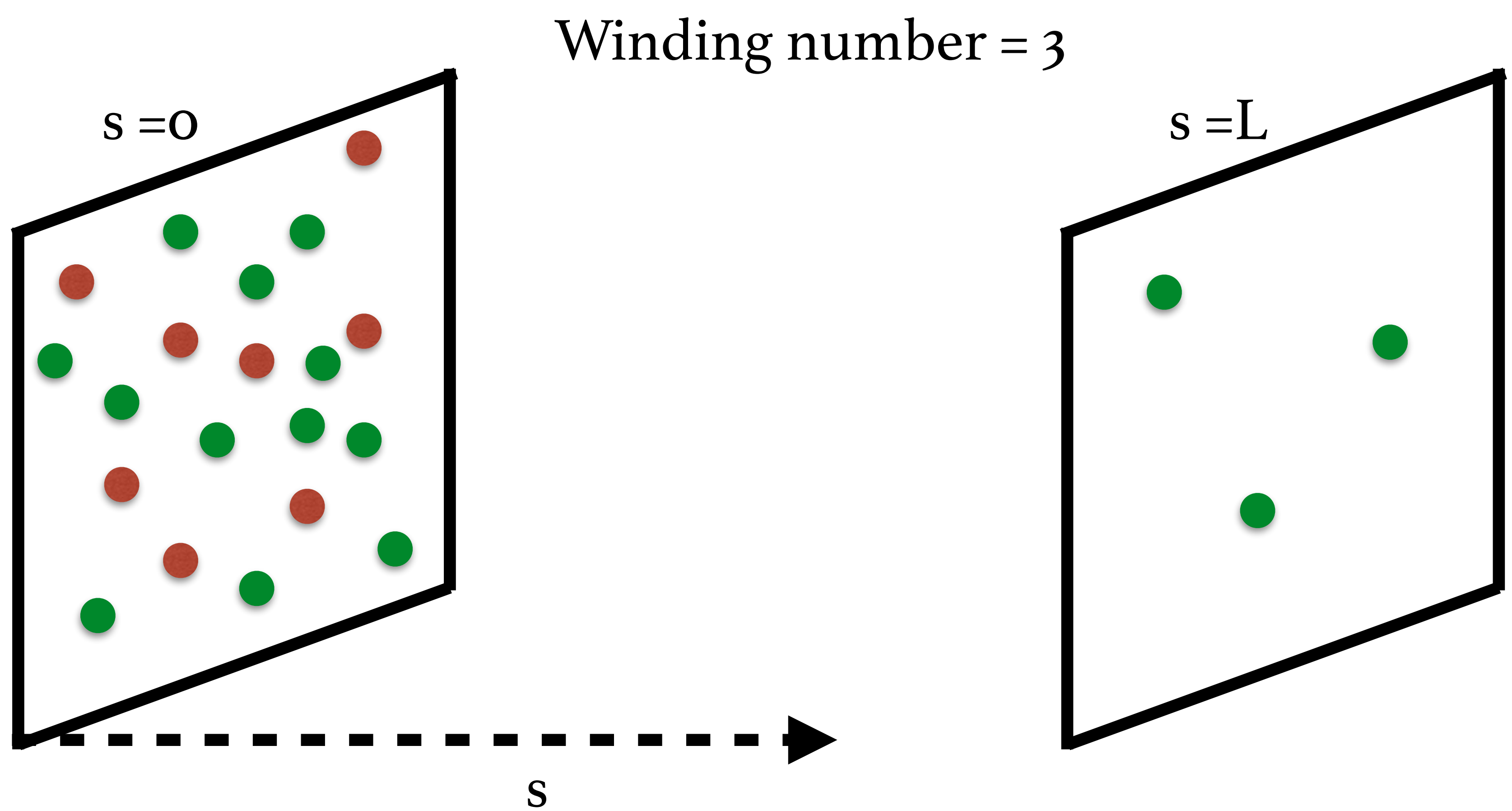
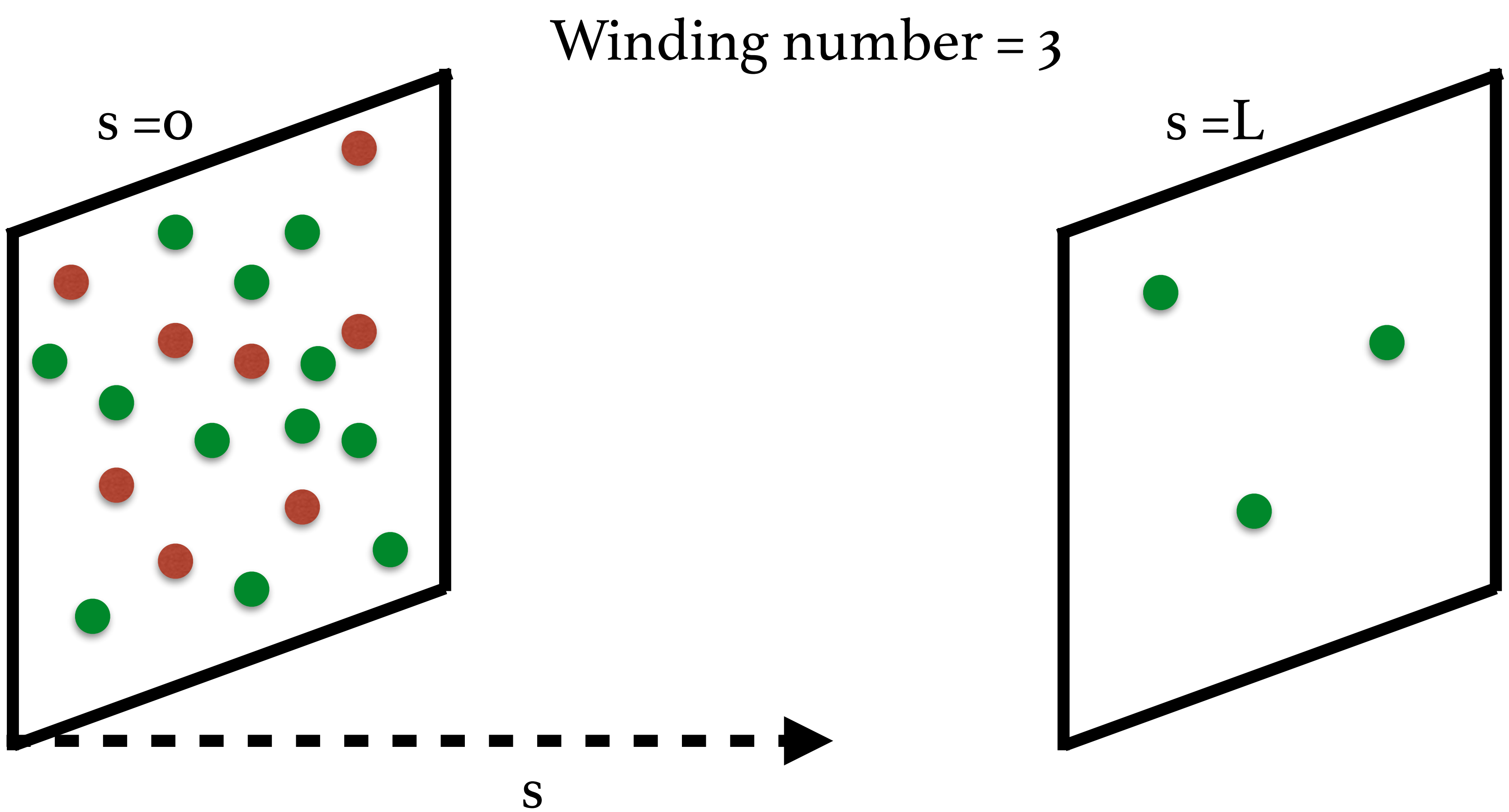


At strong coupling, need to include instanton-anti instanton pairs

- I-A pairs DO NOT satisfy equations of motion
- If flow for sufficiently long time, all pairs will annihilate
- If no correlation between location of instantons on the two boundary, standard fermions and Fluff do not exchange energy/momentum

Topological Gauge Configurations - Strong Coupling



At strong coupling, need to include instanton-anti instanton pairs

- Chaotic dynamics lead to uncorrelated operators on the two boundaries

$$\frac{1}{V_4}\int d^4x\,\mathcal{O}(x)\int d^4y\,\mathcal{O}'(y)$$

Summary

- Proposal for fermion measure for chiral gauge theory

$$\Delta(A) = \prod_i \frac{\det(\not{D} - \Lambda_i \epsilon(s))}{\det(\not{D} - \Lambda_i)}$$

  - Combines domain wall fermions and gradient flow
  - Local theory if chiral fermion representation is anomaly free
  - Mirror fermions decouple due to exponentially soft form factors to gauge fields (gradient flow) and possible gravity (Ricci flow)

- Is there Fluff hiding in the Standard Model?

- Is this the beginning of the low energy frontier?