Global Constraints on an Invisible Heavy Neutrino

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What **do** we know about neutrinos?

Not much.

- The SM containing 3 fields (ν_e, ν_μ, ν_τ) with $SU(2)_L \times U(1)_Y$ charges {1/2, -1/2} fits the LEP data at the Z-pole extremely well.
- This means there ought to be *at least* 3 neutrinos (ν_1, ν_2, ν_3).
- Neutrino oscillations data can be fit with 3 neutrinos, at least 2 of them massive, where $\sum m_i \gtrsim 0.05$ eV or 0.1 eV.
- The CMB and ^{3}H decay suggest that the sum of light neutrino masses is $\sum m_i \lesssim \mathcal{O}(0.5 \text{ eV})$.
- Analysis of the CMB data is consistent with 3 relativistic neutrinos at the time of recombination.
- Oscillation data also allows measurement of the misalignment between interaction and mass bases.

That's about it...

What **don't** we know about neutrinos?

The list is formally infinite...

- Are neutrinos Majorana or Dirac fermions?
- What are the neutrino masses?
- What model best describes how neutrinos acquire mass?
- **How many mass eigenstates are there?**
- How are neutrino masses generated?
- How much do CP violation in the lepton sector?
- etc...

How many neutrinos?

The fields ν_e, ν_μ, and ν_τ can be expressed as linear combinations of mass eigenstates

$$\nu_\alpha = \sum_{i=1}^{3+k} U_{\alpha i} \nu_i, \qquad \alpha = e, \mu, \tau.$$

If $k > 0$, there are additional mass eigenstates, with unknown masses, that make up ν_e, ν_μ, and ν_τ.

Strong evidence from LEP that if there are more than 3 neutrinos ($k > 0$), then additional linear combinations are electroweak singlets:

$$\nu_s = \sum_{i=1}^{3+k} U_{si} \nu_i,$$

called “sterile neutrinos" (but you can get very far never using this term).

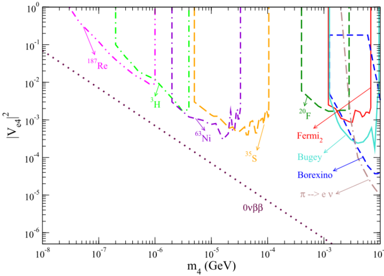
Additionally mass eigenstates affects, in principle, all phenomenology that involves neutrinos.

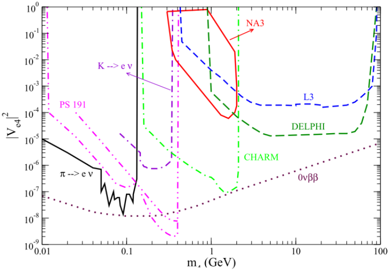
Let's consider a concrete example

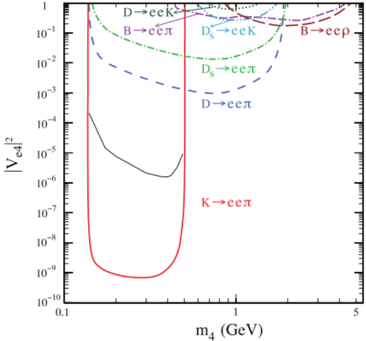
- One additional neutrino ν_4
- We look at the phenomenology that can take place between $10\text{ eV} \lesssim m_4 \lesssim 1\text{ TeV}$
- Observables will depend on $|U_{e4}|^2$, $|U_{\mu 4}|^2$, and $|U_{\tau 4}|^2$
- No sensitivity for CP invariance violation in this "heavy" mass range

Previous results for limits on $|U_{e4}|^2$

Atre, Han, Pascoli, Zhang, JHEP 0905 030 (2009)







Why revisit this?

There are some details that have been glossed over...

- Some experimental results ignore ν_4 decay, some do not.
- What is the difference in results between assuming Majorana or Dirac neutrinos?
- Implicit assumption that no other physics contributes to ν_4 decay
- Many inputs missing, like $\mu - e$ conversion, $\mu \to 3e$, $\tau \to \ell_1 \ell_2 \ell_3$, etc.
- Different CL's
- Nothing is marginalized
- What if we did a global combination of all the constraints?

Maybe could we do a better job?

Our assumptions

We want to estimate conservative model-independent lower bounds on $|U_{e4}|^2$, $|U_{\mu 4}|^2$, and $|U_{\tau 4}|^2$ for 10 eV $\lesssim m_4 \lesssim$ 1 TeV

- ν_4 decays to other neutrinos or other light invisible states 100% of the time. If not, then the constraints, in general, will be stronger.
- The decay is prompt, e.g., $\lesssim 0.1$ s, which removes practically all cosmological constraints
- The values of $|U_{\alpha 4}|^2$ are constant as a function of m_4.

These are phenomenological choices, not necessarily based on a particular model.

Where can you look for additional neutrinos?

Constraints independent of how ν_4 decay:

- deviations from lepton universality
- lepton flavor violating processes
- neutrinoless double beta decay
- invisible Z width ($m_4 > m_Z$)
- dipole moments (current measurements are not precise enough)

Constraints that depend somewhat on how ν_4 decays:

- kinematics from β decay, muon decay, tau decay, meson decay
- invisible Z width
- neutrino oscillations

An example of a test for lepton universality

Consider the following ratio of branching fractions (not assuming lepton universality)

$$R \equiv \frac{\Gamma(\tau^- \to \mu^- \nu_\tau \overline{\nu}_\mu)}{\Gamma(\tau^- \to e^- \nu_\tau \overline{\nu}_e)} \simeq \left|\frac{g_\mu}{g_e}\right|^2 \frac{f\left(m_\mu^2/m_\tau^2\right)}{f\left(m_e^2/m_\tau^2\right)} = \left|\frac{g_\mu}{g_e}\right|^2 (0.9726)$$

Experiment measures this to be 0.9764 ± 0.0030.

If there is a heavy neutrino and $m_4 > m_\tau$, then

$$\left|\frac{g_\mu}{g_e}\right|^2 = \frac{1 - |U_{\tau 4}|^2 - |U_{\mu 4}|^2}{1 - |U_{\tau 4}|^2 - |U_{e4}|^2}$$

Thus, one could say experiment measures $|g_\mu/g_e|^2 = 1.0070 \pm 0.0031$, while the SM predicts $|g_\mu/g_e|^2 = 1$.

Tests for lepton universality, cont'

Observable	SM	Observed	$\lvert g_\ell/g_{\ell'}\rvert^2$
$\Gamma(\tau\to\mu\nu\bar\nu)/\Gamma(\tau\to e\nu\bar\nu)$	0.9726	0.9764 ± 0.0030	$\lvert g_\mu/g_e\rvert^2=1.0070\pm0.0031$
$\Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$	1.235×10^{-4} [15]	$(1.230\pm0.004)\times10^{-4}$	$\lvert g_e/g_\mu\rvert^2=0.9958\pm0.0032$
$\Gamma(K\to e\nu)/\Gamma(K\to\mu\nu)$	2.477×10^{-5} [15]	$(2.488\pm0.010)\times10^{-5}$	$\lvert g_e/g_\mu\rvert^2=1.0044\pm0.0040$
$\Gamma(K\to\pi\mu\nu)/\Gamma(K\to\pi e\nu)$	0.6591 ± 0.0031 [16]	0.6608 ± 0.0030	$\lvert g_\mu/g_e\rvert^2=1.0026\pm0.0065$
$\Gamma(K_L\to\pi\mu\nu)/\Gamma(K_L\to\pi e\nu)$	0.6657 ± 0.0031 [16]	0.6669 ± 0.0027	$\lvert g_\mu/g_e\rvert^2=1.0018\pm0.0062$
$\Gamma(W\to\mu\nu)/\Gamma(W\to e\nu)$	1.000 [17]	0.993 ± 0.019	$\lvert g_\mu/g_e\rvert^2=0.993\pm0.020$
$\Gamma(\tau\to e\nu\bar\nu)/\Gamma(\mu\to e\nu\bar\nu)$	1.345×10^{6}	$(1.348\pm0.004)\times10^{6}$	$\lvert g_\tau/g_\mu\rvert^2=1.002\pm0.003$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to\mu\nu)$	9771 ± 14 [18]	9704 ± 56	$\lvert g_\tau/g_\mu\rvert^2=0.993\pm0.006$
$\Gamma(\tau\to K\nu)/\Gamma(K\to\mu\nu)$	480 ± 1 [18]	469 ± 7	$\lvert g_\tau/g_\mu\rvert^2=0.977\pm0.014$
$\Gamma(D_s\to\tau\nu)/\Gamma(D_s\to\mu\nu)$	9.76 [14]	10.0 ± 0.6	$\lvert g_\tau/g_\mu\rvert^2=1.02\pm0.06$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to e\nu)$	$(7.91\pm0.01)\times10^{7}$ [15, 18]	$(7.89\pm0.05)\times10^{7}$	$\lvert g_\tau/g_e\rvert^2=1.000\pm0.007$
$\Gamma(\tau\to K\nu)/\Gamma(K\to e\nu)$	$(1.94\pm0.04)\times10^{7}$ [15, 18]	$(1.89\pm0.03)\times10^{7}$	$\lvert g_\tau/g_e\rvert^2=0.972\pm0.015$
$\Gamma(W\to\tau\nu)/\Gamma(W\to e\nu)$	0.999 [17]	1.063 ± 0.027	$\lvert g_\tau/g_e\rvert^2=1.063\pm0.027$

The overall goodness of fit to the SM is $p\simeq0.06$ (χ^2/dof $\simeq 21.9/13$).

Tests for lepton universality, cont'

Observable	$\lvert g_\ell/g_{\ell'}\rvert^2$
$\Gamma(\tau\to\mu\nu\bar\nu)/\Gamma(\tau\to e\nu\bar\nu)$	$\lvert g_\mu/g_e\rvert^2=(1-\lvert U_{\tau4}\rvert^2-\lvert U_{\mu4}\rvert^2)/(1-\lvert U_{\tau4}\rvert^2-\lvert U_{e4}\rvert^2)$
$\Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$	$\lvert g_e/g_\mu\rvert^2=(1-\lvert U_{e4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2)$
$\Gamma(K\to e\nu)/\Gamma(K\to\mu\nu)$	$\lvert g_e/g_\mu\rvert^2=(1-\lvert U_{e4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2)$
$\Gamma(K\to\pi\mu\nu)/\Gamma(K\to\pi e\nu)$	$\lvert g_\mu/g_e\rvert^2=(1-\lvert U_{\mu4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$
$\Gamma(K_L\to\pi\mu\nu)/\Gamma(K_L\to\pi e\nu)$	$\lvert g_\mu/g_e\rvert^2=(1-\lvert U_{\mu4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$
$\Gamma(W\to\mu\nu)/\Gamma(W\to e\nu)$	$\lvert g_\mu/g_e\rvert^2=(1-\lvert U_{\mu4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$
$\Gamma(\tau\to e\nu\bar\nu)/\Gamma(\mu\to e\nu\bar\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2-\lvert U_{e4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2-\lvert U_{e4}\rvert^2)$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to\mu\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2)$
$\Gamma(\tau\to K\nu)/\Gamma(K\to\mu\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2)$
$\Gamma(D_s\to\tau\nu)/\Gamma(D_s\to\mu\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{\mu4}\rvert^2)$
$\Gamma(\tau\to\pi\nu)/\Gamma(\pi\to e\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$
$\Gamma(\tau\to K\nu)/\Gamma(K\to e\nu)$	$\lvert g_\tau/g_\mu\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$
$\Gamma(W\to\tau\nu)/\Gamma(W\to e\nu)$	$\lvert g_\tau/g_e\rvert^2=(1-\lvert U_{\tau4}\rvert^2)/(1-\lvert U_{e4}\rvert^2)$

When $m_4 > m_Z$, the overall goodness of fit is $p = 0.12$ ($\chi^2_{\rm min}/\mathrm{dof} \simeq 15.52/10$).

Lepton flavor violation at 1 loop

Observable	Exp. Limit (90% CL)
$Br(\mu^- \to e^-\gamma)$	$< 1.0 \times 10^{-12}$
$Br(\tau^- \to e^-\gamma)$	$< 3.3 \times 10^{-8}$
$Br(\tau^- \to \mu^-\gamma)$	$< 4.4 \times 10^{-8}$
$Br(\mu^- \to e^-e^+e^-)$	$< 5.7 \times 10^{-13}$
$Br(\tau^- \to e^-e^+e^-)$	$< 2.7 \times 10^{-8}$
$Br(\tau^- \to \mu^-\mu^+\mu^-)$	$< 2.1 \times 10^{-8}$
$Br(\tau^- \to e^-\mu^+\mu^-)$	$< 2.7 \times 10^{-8}$
$Br(\tau^- \to \mu^-e^+e^-)$	$< 1.8 \times 10^{-8}$
$Br(\tau^- \to e^+\mu^-\mu^-)$	$< 1.7 \times 10^{-8}$
$Br(\tau^- \to \mu^+e^-e^-)$	$< 1.5 \times 10^{-8}$
$R^{\mathrm{Ti}}_{\mu\to e}$	$< 4.3 \times 10^{-12}$
$R^{\mathrm{Au}}_{\mu\to e}$	$< 7 \times 10^{-13}$
$R^{^{32}\mathrm{S}}_{\mu\to e}$	$< 7 \times 10^{-11}$
$R^{\mathrm{Pb}}_{\mu\to e}$	$< 4.6 \times 10^{-11}$

Heavy neutrinos can contribute to all of these processes at 1 loop.

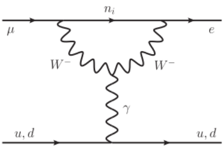
Radiative charged lepton decays

The lepton flavor violating process $\ell \to \ell' \gamma$ can occur more often if there is a heavy neutrino, via penguin diagrams:

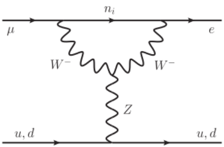
$$Br(\ell \to \ell' \gamma) \simeq \frac{\alpha_W^3 s_W^2}{256\pi^3} \frac{m_\ell^5}{M_W^4 \Gamma_\ell} \left| U^*_{\ell' 4} U_{\ell 4} G_\gamma \left(\frac{m_4^2}{M_W^2} \right) \right|^2$$

$\mu - e$ conversion

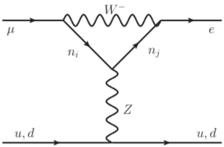
We don't ignore diagram (c)



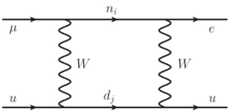
(a) Photon Penguin Diagram



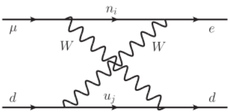
(b) Z Penguin Diagram



(c) Z Penguin Diagram



(d) Box Diagram



(e) Box Diagram

(stolen with implicit permission from Alonso, et al., JHEP 01 118, 2013)

Limits on $\mu - e$ conversion on Ti give strongest limits.

Other LFV processes

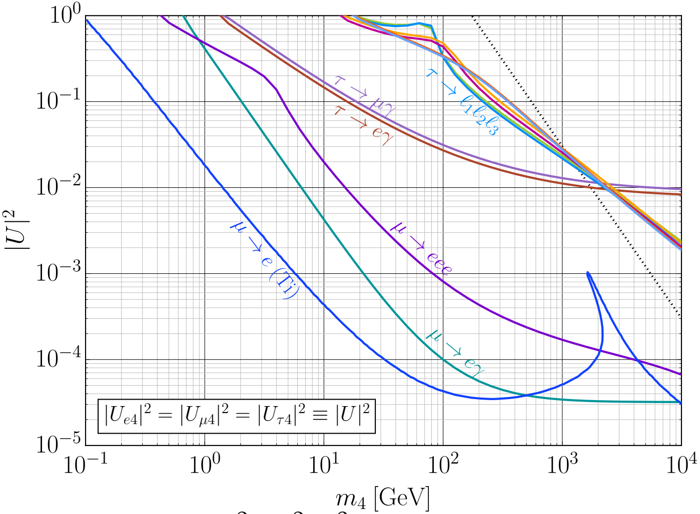
Diagrams for $\mu - e$ conversion are almost the same to those that contribute to $\mu \to 3e$ and the six $\tau \to \ell_1 \ell_2 \ell_3$ processes.

- Ilakovac, Pilaftsis, Nucl. Phys. B 437 491 (1995).

Semileptonic tau decays $\tau^- \to \nu h^-$ (where h stands for the hadronic system), follow from the same diagrams. But these are technically difficult to calculate. They likely do not contribute much to our analysis.

Comparing LFV processes

All limits at 99% CL.



Black dashed line is $|U|^2 = v^2/m_4^2$.

Neutrinoless double beta decay

The most conservative constraint from $0\nu 2\beta$ decay comes when $m_{\text{light}} \equiv m_1 \approx m_2 \approx m_3$, and the “light" and “heavy" contributions destructively interfere:

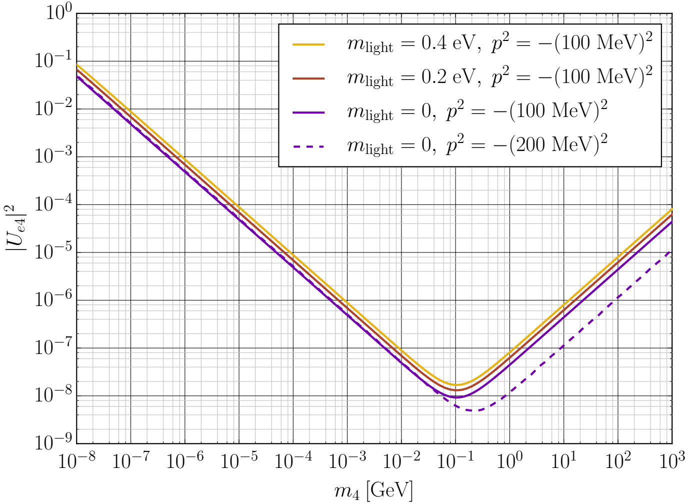
$$|m_{\beta\beta}| > \left| m_{\text{light}}(1 - |U_{e4}|^2) - \left(\frac{m_4}{1 - m_4^2/p^2} \right) |U_{e4}|^2 \right|$$

A combination analysis of experimental limits (Guzowski et al., PRD 92 012002, 2015), conservatively estimates

$$m_{\beta\beta} < 310 \text{ meV, at } 90\% \text{ CL.}$$

Only holds if neutrinos are Majorana fermions.

$0\nu2\beta$ constraints on $|U_{e4}|^2$



Invisible Z width

One can "measure" the number of electroweak neutrino fields:

$$N_\nu \equiv \left.\frac{\Gamma(Z \to \mathrm{inv})}{\Gamma(Z \to \ell\ell)}\right|_{\mathrm{exp}} \times \left.\frac{\Gamma(Z \to \ell\ell)}{\Gamma(Z \to \nu\nu)}\right|_{\mathrm{SM}}$$

$\Gamma(Z \to \ell\ell)/\Gamma(Z \to \nu\nu)$ is predicted in the SM to be 1.9913 ± 0.0008, and $N_\nu = 3$.
But LEP measures $N_\nu = 2.9840 \pm 0.0082$. About a 2σ discrepancy.

If there's a heavy neutrino and $m_4 > m_Z$:

$$N_\nu = 3\left(1 - |U_{e4}|^2 - |U_{\mu 4}|^2 - |U_{\tau 4}|^2\right) + \mathcal{O}\left(|U_{\alpha 4}|^4\right)$$

Which means $|U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2 < 9.5 \times 10^{-3}$ at 90% CL.

Michel electron spectrum from muon decay

We don't know of an analysis that fit the TWIST measurement of the energy spectrum of Michel electrons from muon decay with a heavy-neutrino model. But the expresses exist (Dixit, et al., PRD 27 2216, 1983):

$$\frac{d\Gamma(\mu \to e\bar{\nu}\nu)}{dx} = \frac{G_F^2 m_\mu^5}{192\pi^3} \mathscr{F}\left(x, \delta, \rho, |U_{e4}|^2, |U_{\mu 4}|^2\right) + \text{radiative corrections}$$

where $x \equiv 2E_e/m_\mu$, $\delta \equiv m_4/m_\mu$, and

$$\mathscr{F}\left(x, \delta, \rho, |U_{e4}|^2, |U_{\mu 4}|^2\right) \equiv \left(1 - |U_{e4}|^2 - |U_{\mu 4}|^2\right) f(x, 0, \rho) + \left(|U_{e4}|^2 + |U_{\mu 4}|^2\right) f(x, \delta, \rho)$$

$$f(x, \delta, \rho) \equiv \frac{x^2}{2} \left[6(1-x) + \frac{4}{3}\rho(4x-3) - 3\delta^2 - \frac{3\delta^4}{(1-x)^2} - \frac{(x-3)\delta^6}{(1-x)^3} \right] \Theta(1 - x - \delta^2)$$

The value of ρ is predicted to be $\rho_{\mathrm{SM}} = 3/4$ in the SM, and the TWIST experiment measures it to be $\rho_{\mathrm{exp}} = 0.74997 \pm 0.00026$.

Michel spectrum cont'

Non-zero values of $|U_{\mu 4}|^2$, and $|U_{e4}|^2$ could affect the fit to the data that determines the measurements of ρ.

We define a χ^2 function to compare the Michel electron energy spectrum two functions, one where $\rho = \rho_{\text{SM}}$ and $\delta = 0$, and another where $\rho = \rho_{\text{exp}}$ and δ is set to a constant value. We use 1 MeV bins, and the uncertainty on each bin is the uncertainty on ρ_{exp}.

We find that for 10 MeV $\lesssim m_4 \lesssim$ 70 MeV,

$$|U_{\mu 4}|^2 + |U_{e4}|^2 < \mathcal{O}(10^{-3}) \text{ at 99\% CL}$$

Neutrino oscillations

Because we focus on an fourth neutrino with mass $m_4 \gtrsim 10$ eV, the associated oscillations typically too rapid to resolve experimentally. Even so, the oscillation probability for only three light neutrinos has a distinctly different shape from the oscillation probability for three light neutrinos and one heavy neutrino:

$$P_{\nu_\alpha \to \nu_\beta} \simeq \left| \delta_{\alpha\beta} - U_{\alpha 4} U^*_{\beta 4} + U_{\alpha 2} U^*_{\beta 2} \left(e^{-i\Delta_{12}} - 1 \right) + U_{\alpha 3} U^*_{\beta 3} \left(e^{-i\Delta_{13}} - 1 \right) \right|^2 + |U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

This assumes that neutrinos do not decay along their flight path.

- KARMEN: $4|U_{e4}|^2|U_{\mu 4}|^2 < 1.3 \times 10^{-3}$ at 90% CL
- FNAL-E531: $4|U_{\mu 4}|^2|U_{\tau 4}|^2 < 4 \times 10^{-3}$ at 90% CL
- FNAL-E531: $4|U_{e4}|^2|U_{\tau 4}|^2 \lesssim 0.2$ at 90% CL

Collecting other kinematic constraints

β decay: ^{187}Re, ^{3}H, ^{63}Ni, ^{35}S, ^{45}Ca, ^{64}Cu, ^{20}F (along with the super-allowed Fermi decays).

- $|U_{e4}|^2 < \mathcal{O}(10^{-3})$ when 1 keV $\lesssim m_4 \lesssim$ 450 keV

$\pi \to e\nu$

- TRIUMF: $|U_{e4}|^2 < \mathcal{O}(10^{-8})$ at 90% CL for 10 MeV $\lesssim m_4 \lesssim$ 55 MeV

$\pi \to \mu\nu$

- Abela et al (PLB 105 263, 1981): $|U_{\mu 4}|^2 < \mathcal{O}(10^{-4})$ for 10 $\lesssim m_4 \lesssim$ 30 MeV

$K \to \mu\nu_4$

- E949: $|U_{\mu 4}|^2 < \mathcal{O}(10^{-8})$ at 90% CL for 175 MeV $\lesssim m_4 \lesssim$ 300 MeV
- KEK: $|U_{\mu 4}|^2 < \mathcal{O}(10^{-5})$ at 90% CL for 70 MeV $\lesssim m_4 \lesssim$ 300 MeV

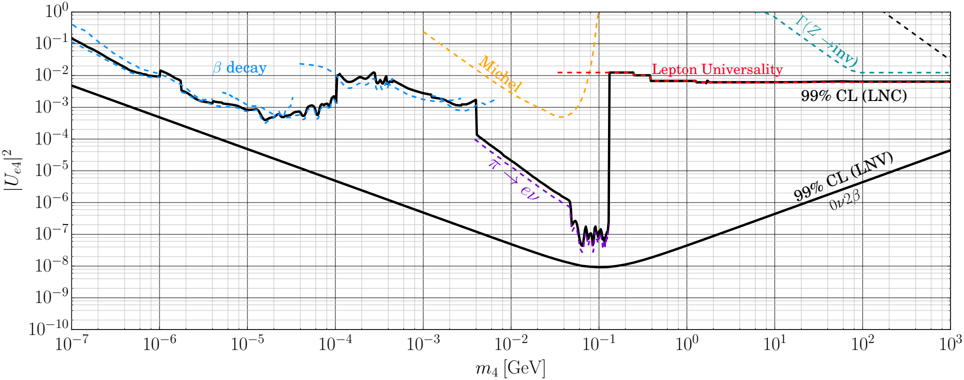
$\tau^- \to \nu\pi^-\pi^+\pi^-$

- The only candidate method for kinematic constraints on $|U_{\tau 4}|^2$ (AK, Dobbs, PRD 91 053006, 2015)

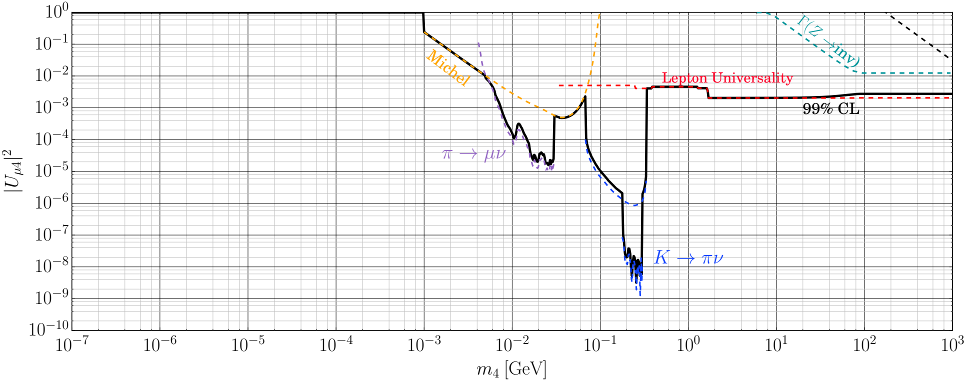
Combination assumptions

- We apply the constraints from lepton universality tests when m_4 is larger than the mass of the parent particle.
- The constraints on N_ν from invisible Z decays are applied for all relevant values of m_4.
- We choose $p = -(100\ \mathrm{MeV})^2$, $m_{\mathrm{light}} = 0.05$ eV, and $|m_{\beta\beta}| < 310$ meV (90% CL) when applying the constraints from neutrinoless double beta decay.
- If details are unavailable, the individual χ^2's for each constraint are assumed to be zero when $|U_{\alpha 4}|^2 = 0$.
- To utilize experimentally-calculated limits, we assume the χ^2 function scales like $|U_{\alpha 4}|^4$.
- Marginalize over unseen variables.

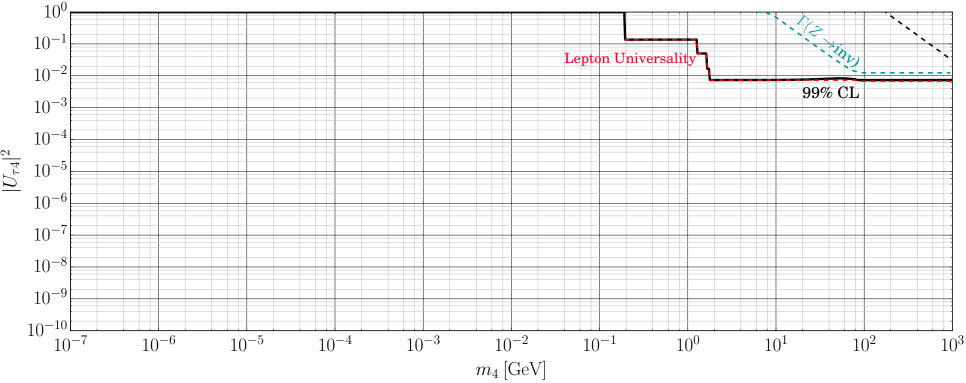
Results for $|U_{e4}|^2$



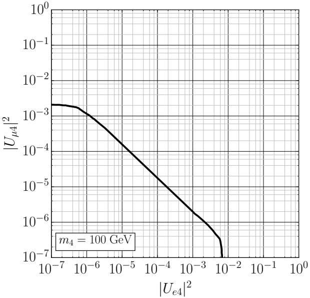
Results for $|U_{\mu 4}|^2$

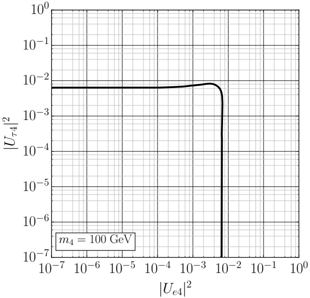


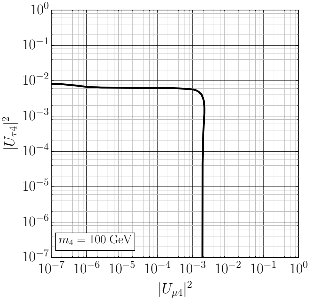
Results for $|U_{\tau 4}|^2$



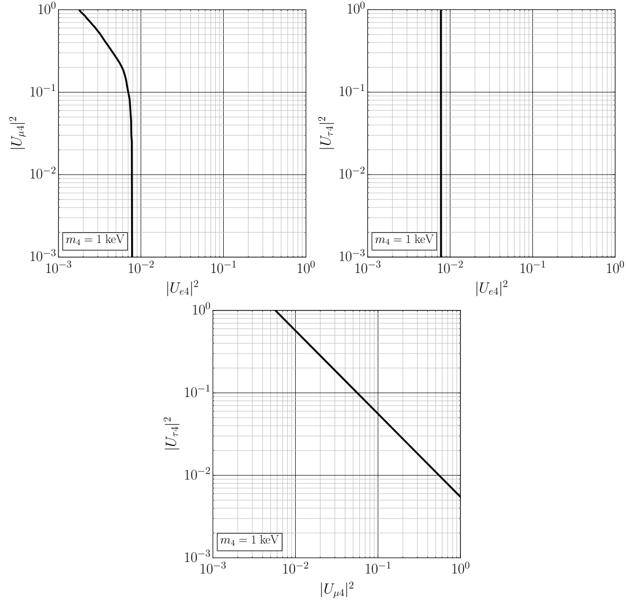
2D results when $m_4 = 100$ GeV, 99% CL







2D results when $m_4 = 1$ keV, 99% CL



Summary

- Focused on a fourth "invisible" neutrino, with mass 10 eV $\lesssim m_4 \lesssim$ 1 TeV, in order to estimate rough lower bounds on $|U_{e4}|^2$, $|U_{\mu 4}|^2$, and $|U_{\tau 4}|^2$.
- Global combination, including constraints from tests for lepton universality, lepton flavor violating processes, $0\nu 2\beta$, electroweak precision measurements, neutrino oscillations, kinematic searches from the decay of nuclei, pseudoscalar mesons, muons, and tau leptons.
- Neutrino oscillations and beta decay dominate constraints when $m_4 \lesssim 1$ MeV.
- Kinematic searches dominate for 1 MeV $\lesssim m_4 \lesssim$ 1 GeV.
- Tests for lepton universality and $\mu - e$ conversion dominate constraints when $m_4 \gtrsim 1$ GeV.
- The phase space is much less constrained if ν_4 does not decay via the weak interactions.
- Still a work in progress. But should be ready for public viewing in about a month.