ON THE ANNIHILATION OF WIMPS

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1409.4415 & 1411.xxxx

Fermilab Theory Seminar
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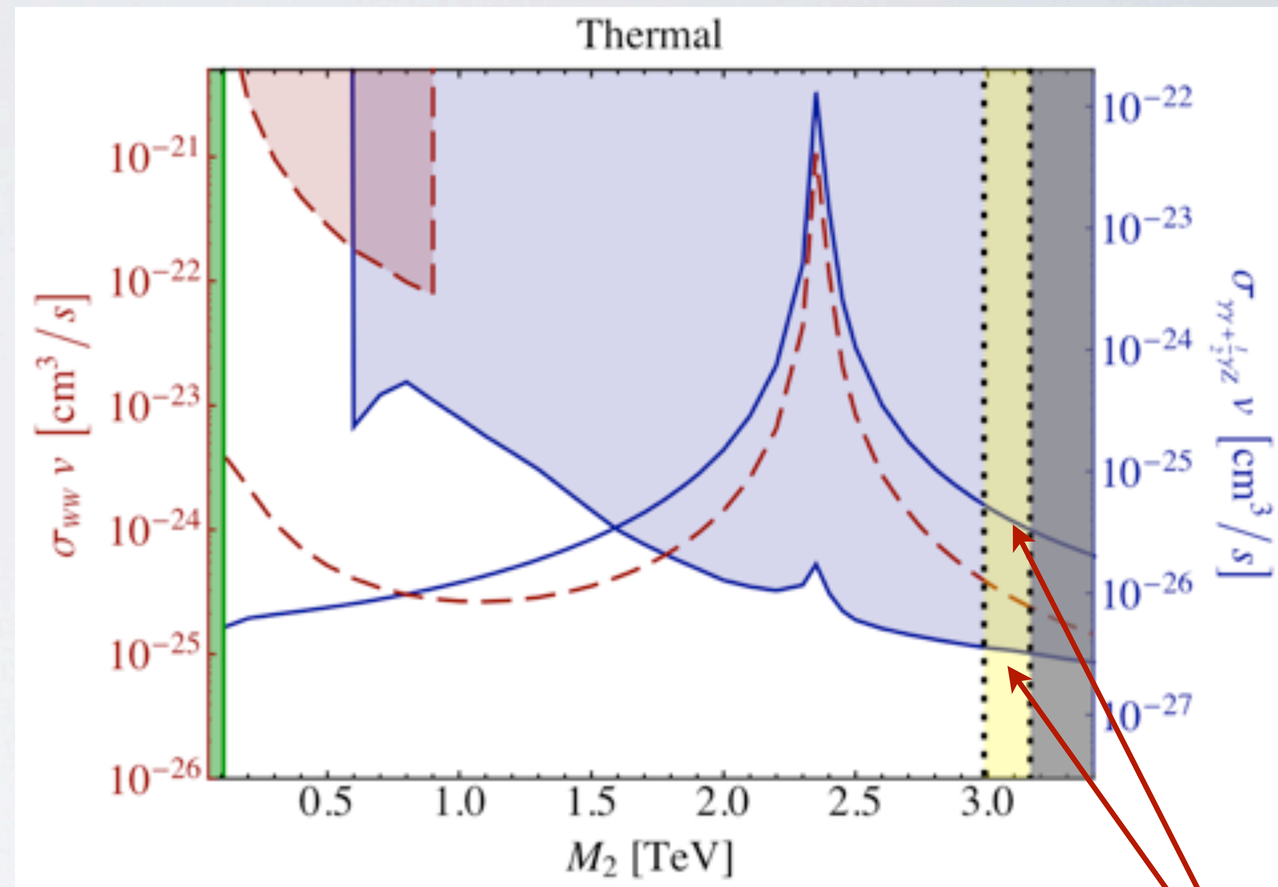
"WINO DARK MATTER UNDER SIEGE"*

- WIMP Miracle:
  - TeV-scale particle
  - Electroweak-strength coupling
  - Right relic abundance of dark matter from thermal freezeout
- SUSY gives such a WIMP



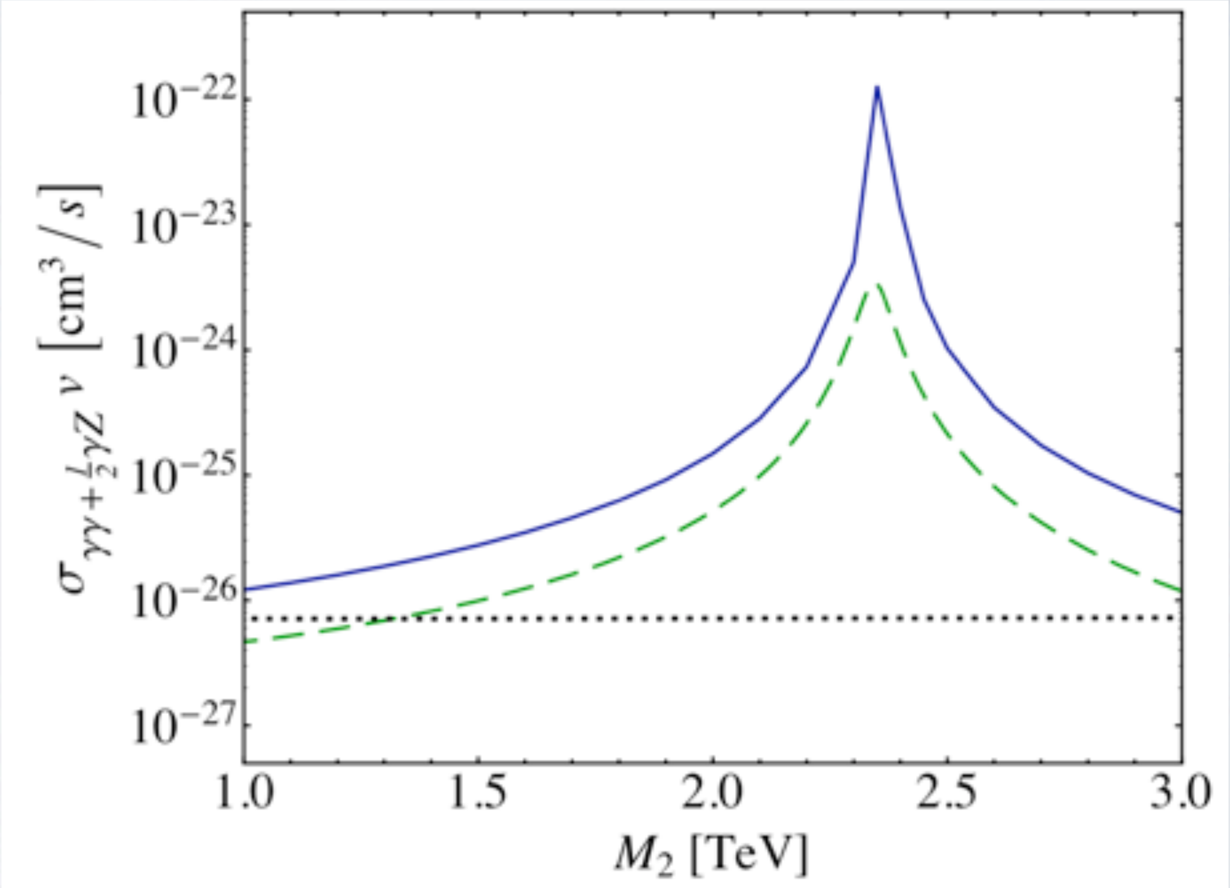
From 1307.4082:
Blue line: Wino annihilation rate
Blue shade: Exclusion from HESS
Yellow shade: All DM is thermal wino

Ruled out by 15x

*1307.4082: Cohen, Lisanti, Pierce, and Slatyer; see also "In Wino Veritas", 1307.4400: Fan & Reece

FORTRESS WINO?

- Exclusion computed for LO +Sommerfeld Enhancement (SE)
- Maybe the wino saves itself at higher orders?
- If DM halo cored out to 1 (10) kpc, can reduce annihilation rate by 4x (15x)
- Exclusion assumed cusped, NFW profile

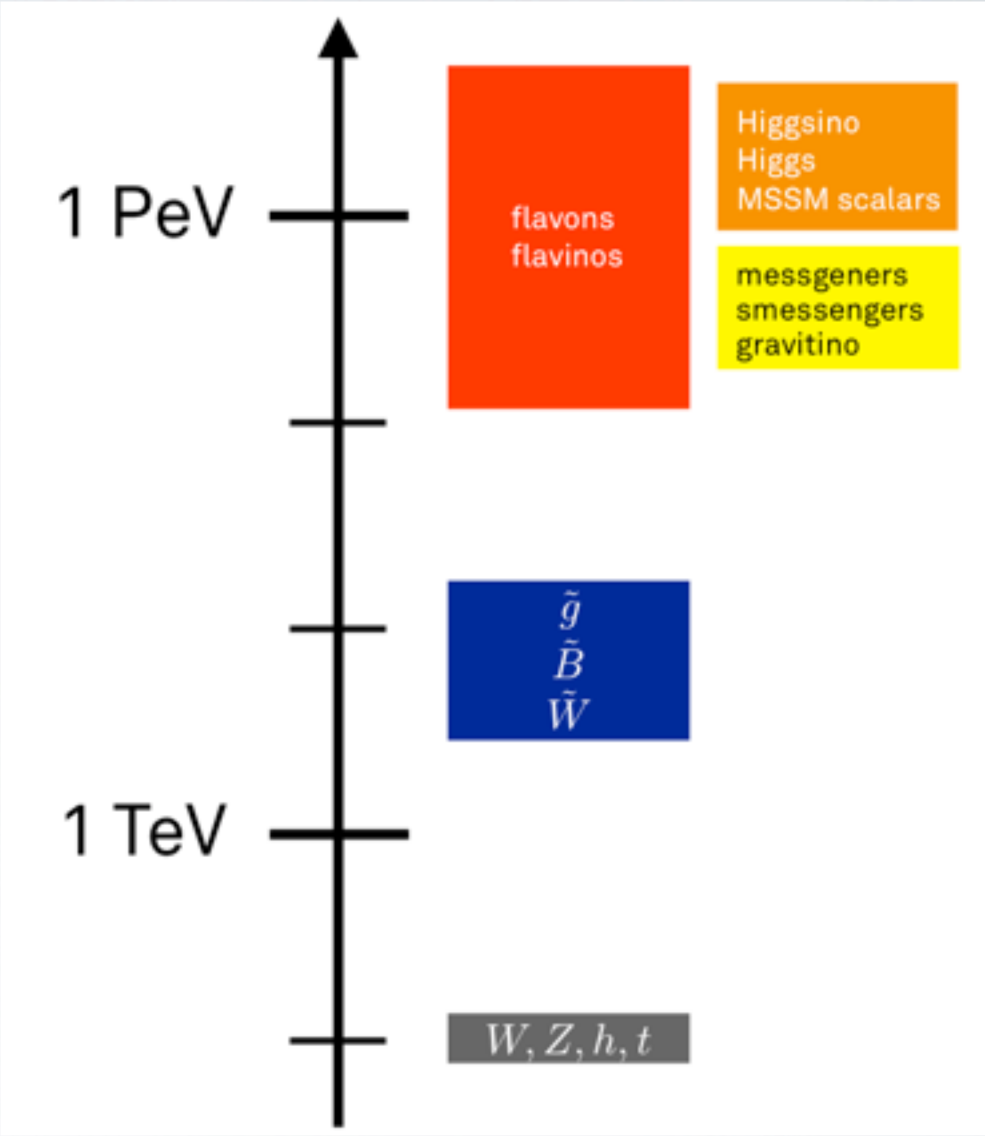


From 1307.4082
Blue: LO+SE
Green: NLO+SE (reduction by 4x?)
Dots: NLO only

Do we believe such a large, subleading effect?

WHY WINO DARK MATTER?

- Higgs + nothing else means?
  - SUSY to eliminate most fine-tuning? 10^{-8} instead of 10^{-32}
  - Higgs mass and flavor physics point to sfermions at 100-1000 TeV
  - Simpler SUSY model building (Gravity +Anomaly mediation)
  - Gauginos (w/ wino LSP) at right scale for WIMP Miracle
- Binos overclose universe, Higgsinos poorly constrained byt possible



From 1403.6118: MB, Stolarski, and Zorawski
Mini-split SUSY allows a radiative generation of flavor hierarchies
and allows for thermal WIMP-DM

Mini-split SUSY: 1112.2462: Ibe & Yanagida, 1210.0555: Arvanitaki et al., 1210.2395: Hall, Nomura, & Shirai, 1212.6971: Arkani-Hamed et al.

Also see Altmannshofer et al. 1409.2522 for radiative flavor

WINOS & BEYOND

- We want to understand higher order effects in wino annihilation
- Electroweak physics contains large, double-logarithms
- Resum them using Soft-Collinear Effective Theory (SCET)
- Treat slow (v~10-3), heavy DM nonrelativistically and compute Sommerfeld Enhancement with numerical Schrodinger Equation
- Results will be generic for any electroweak DM with M~1-10 TeV

SUDAKOV LOGARITHMS

- Emission of soft or collinear radiation can lead to infrared divergences

$$d\sigma(p \to p' + \gamma) = d\sigma(p \to p') \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} e^2 \left| \epsilon \cdot \left(\frac{p'}{p' \cdot k} - \frac{p}{p \cdot k} \right) \right|^2$$

$$= d\sigma(p \to p') \frac{\alpha}{\pi} \int_0^Q \frac{dk}{k}\, d\cos\theta \left(\frac{1}{1 - v\cos\theta} + \frac{1}{1 - v'\cos\theta} \right) f(v, v')$$

- Performing integrals gives us Sudakov double logarithm

$$d\sigma(p \to p' + \gamma) = d\sigma(p \to p') \frac{\alpha}{\pi} \log\left(Q^2/\mu^2\right) \log\left(Q^2/m^2\right)$$

- For wino annihilation at the thermal mass (3 TeV),

$$\frac{\alpha_W}{\pi} \log\left(M_{\rm wino}^2/m_W^2\right)^2 \approx 0.2$$

BLOCH-NORDSIECK THEOREM VIOLATION*

- Electroweak physics has infrared divergences, even in fully inclusive observables

$$S \equiv \mathcal{U}^F_{\alpha_F \beta_F} S^H_{\beta_F \beta_I} \mathcal{U}^I_{\alpha_I \beta_I}$$

- We sum over degenerate final states, but not initial

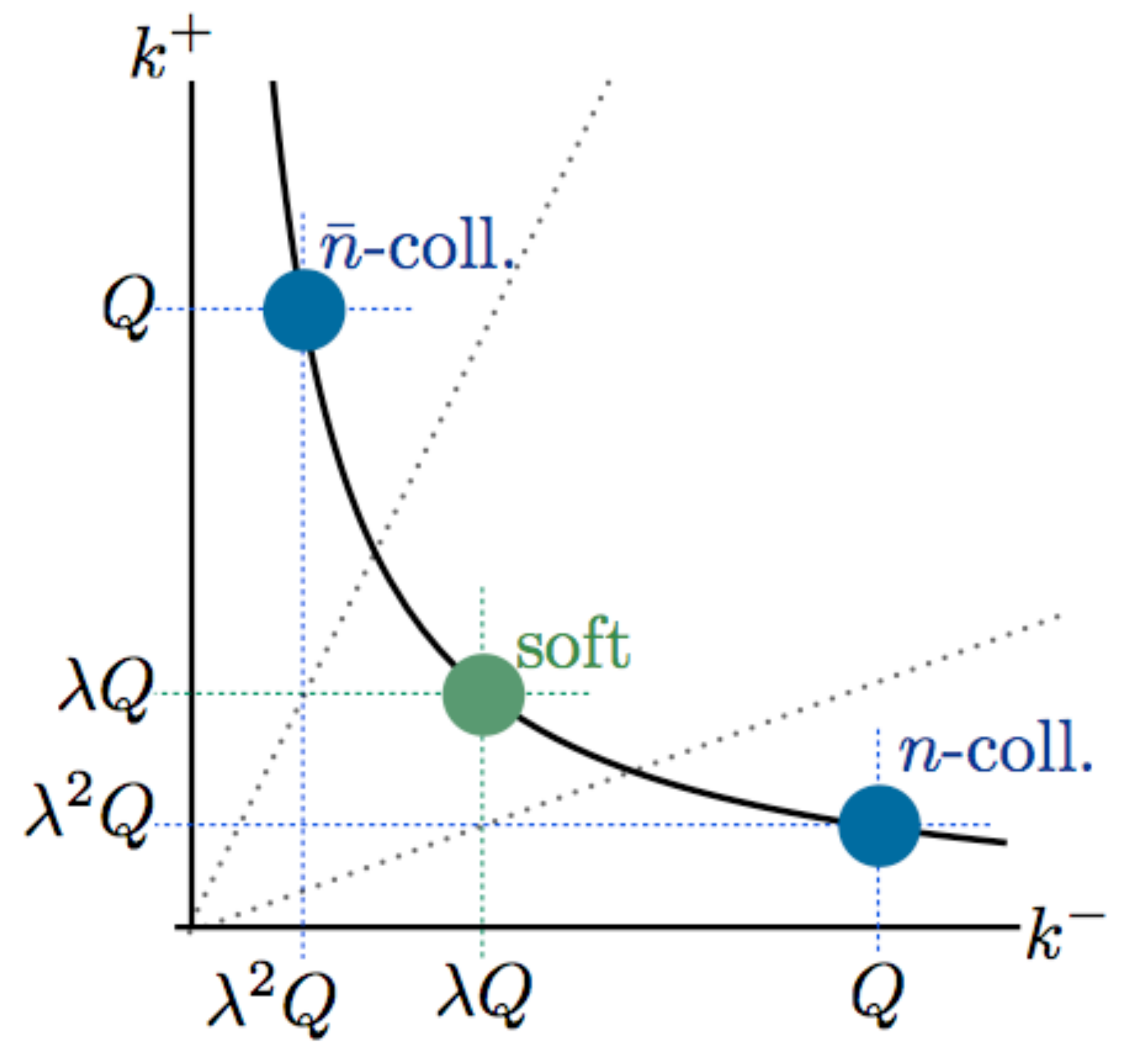
$$\sum_{f \in \Delta(p_F)} |\langle f|S|i\rangle|^2 = {}_S\langle 0|\mathcal{U}^{I\dagger}_{\alpha_I \beta_I'} (S^{H\dagger} S)_{\beta_I' \beta_I} \mathcal{U}^I_{\beta_I \alpha_I} |0\rangle_S = (S^{H\dagger} S)_{\alpha_I \alpha_I} + \Delta\sigma_{\alpha_I}$$

- We avoid pathology because
  - QED: Abelian lets us commute U^I and cancel with final sum
  - QCD: Singlets let us average over initial colors
  - Electroweak: Gauge boson masses cut off divergence, but allow for $\log(Q^2/m_W^2)^2$

*hep-ph/0001142: Ciafaloni, Ciafaloni, & Comelli

SOFT-COLLINEAR EFFECTIVE THEORY

- SCET converts kinematic logarithms into anomalous dimensions
- We can use RG to resum them
- Originally developed for B-decays and jet physics, but extension to electroweak is trivial



For process of interest, split fields into relevant modes and expand around appropriate kinematic limit

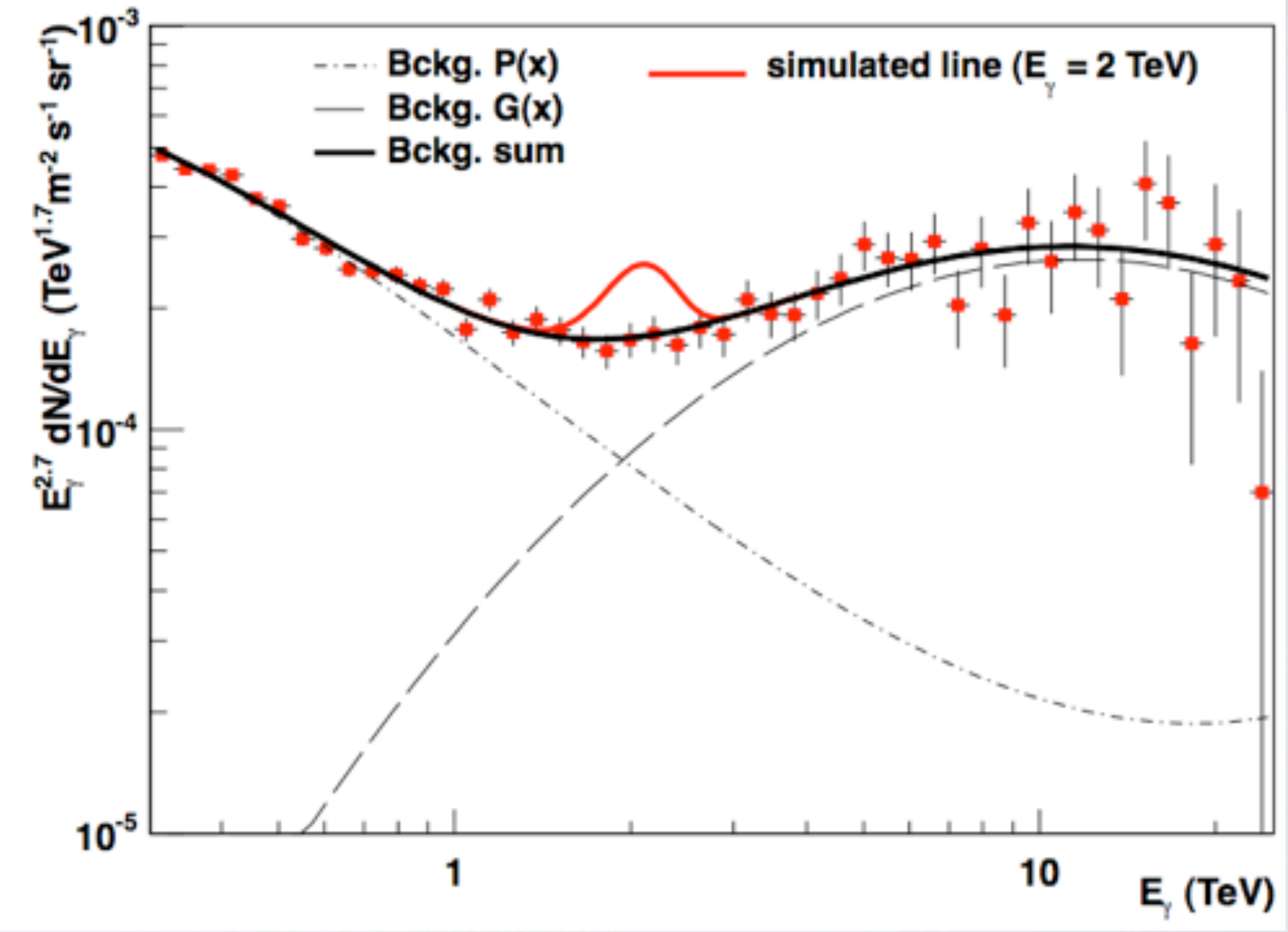
MODES & FACTORIZATION

- Large logs → disparate scales ($\lambda \sim m_W/M_{wimp}$) → factorization
- Factorization lets us separate 2 nontrivial behaviors (Sommerfeld and Sudakov)
- Setting up EFT requires list of relevant modes

$$\begin{aligned}
\mathrm{WIMPs}\,(\chi): &\ (E \sim \lambda^2,\ p \sim \lambda) \\
\mathrm{Potential}: &\ (E \sim \lambda^2,\ p \sim \lambda) \\
\mathrm{Collinear}\,(B^{\mu\perp}): &\ (k_+ \sim 1,\ k_- \sim \lambda^2,\ k_\perp \sim \lambda) \\
\mathrm{Soft}\,(S_{ab}): &\ (k_+ \sim \lambda,\ k_- \sim \lambda,\ k_\perp \sim \lambda)
\end{aligned}$$

WHAT TO COMPUTE?

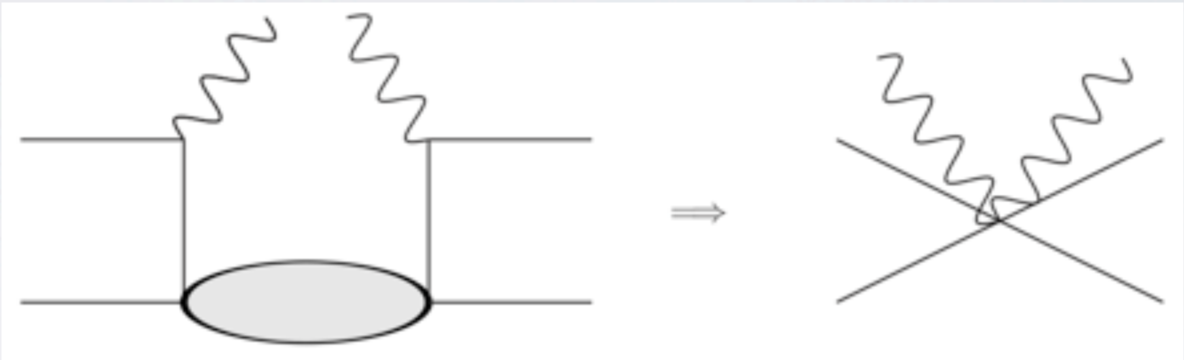
- Our interest is in setting limits from indirect detection
- HESS is an air Cherenkov telescope that observes photons colliding with the atmosphere
- Therefore, we compute $\chi\chi \to \gamma + X$



From HESS collaboration 1301.1173
at 3 TeV, energy resolution is 400 GeV

OPERATOR BASIS

- Since our interest is semi-inclusive processes, it is useful to work with the OPE



- At tree level with one-loop running, we generate:

$$O_1 = (\bar{\chi}\gamma^5\chi)\,(\bar{\chi}\gamma^5\chi)\,B^{\mu A\perp}B^{A\perp}_{\mu}$$

$$O_2 = \frac{1}{2}\Big\{\,(\bar{\chi}\gamma^5\chi)\,(\bar{\chi}_A\gamma^5\chi_B) + (\bar{\chi}_A\gamma^5\chi_B)\,(\bar{\chi}\gamma^5\chi)\,\Big\}B^{\perp A}_{\mu}B^{\mu B\perp}$$

$$O_3 = (\bar{\chi}_C\gamma^5\chi_D)\,(\bar{\chi}_D\gamma^5\chi_C)\,B^{\mu A\perp}B^{A\perp}_{\mu}$$

$$O_4 = (\bar{\chi}_A\gamma^5\chi_C)\,(\bar{\chi}_C\gamma^5\chi_B)\,B^{\perp A}_{\mu}B^{\mu B\perp}$$

We implicitly work with SU(2) adjoint, so basis reduced by having Majorana fermion

WILSON LINES, GAUGE INVARIANCE, SOFTS

- SCET has rich gauge structure (separate for soft and collinear sectors) and collinear fields contain implicit collinear Wilson lines

$$B_\mu^{a\perp} \equiv f^{abc} W_n^T (D_\mu^\perp)^{bc} W_n$$

- We have soft Wilson lines for both WIMP and collinear fields

$$S_{(v,n)} = P[e^{ig\int_{-\infty}^0 (v,n)\cdot A((v,n)\lambda)d\lambda}]$$

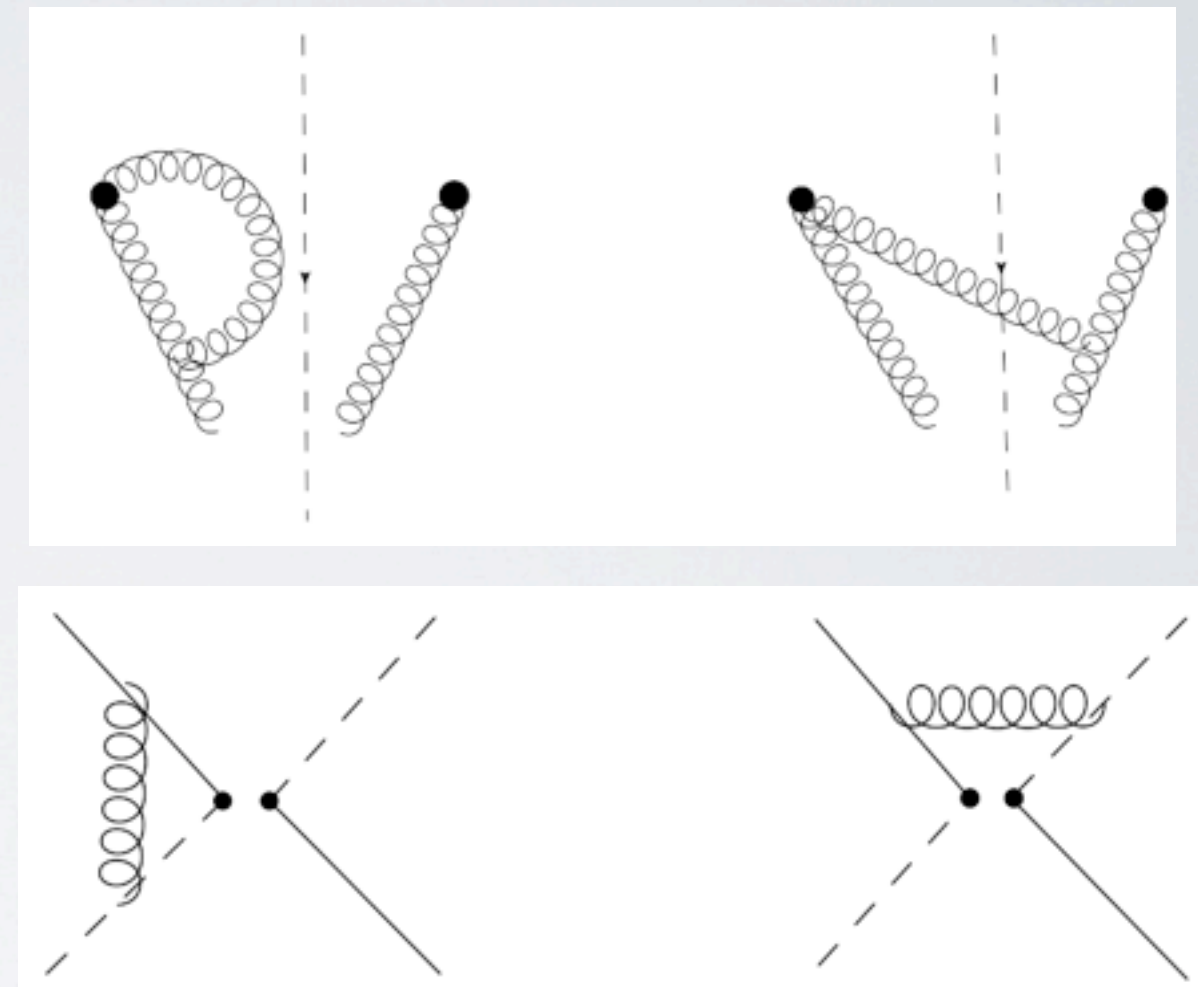
- Soft&Collinear-gauge-boson structure is therefore

$$O_s^a = S^T_{vA'A} S_{vBB'} S^T_{n\tilde{A}A} S_{nB\tilde{B}} \qquad O_s^b = \mathbf{1}\,\delta_{\tilde{A}\tilde{B}}\delta_{A'B'}$$

$$O_c^a = B^\perp_{\tilde{A}} B^\perp_{\tilde{B}} \qquad O_c^b = B^\perp \cdot B^\perp \delta_{\tilde{A}\tilde{B}}$$

ANOMALOUS DIMENSIONS

- Compute soft & collinear anomalous dimensions separately
- OPE converted amplitude-squared to operator whose expectation value gives rate
- Thus, real & virtual corrections to $\chi\chi \rightarrow \gamma + X$ will appear as loops



Above: Collinear contributions (real & virtual)
Below: Interactions between
soft Wilson lines (solid - timelike; dashed - lightlike)

RAPIDITY RG

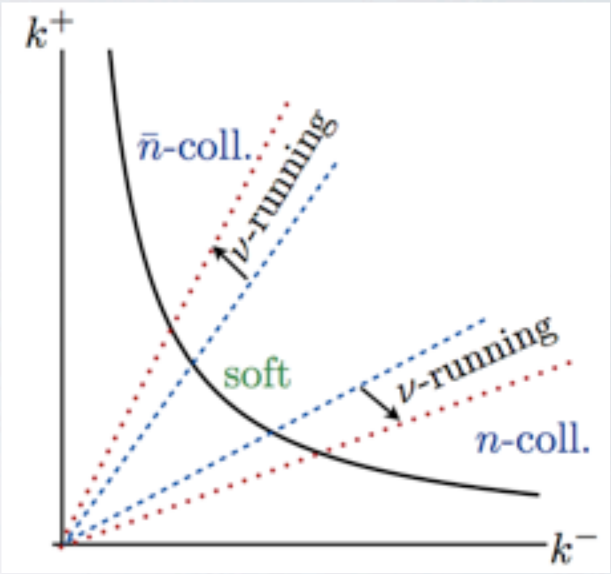
- SCET is a "modal" theory

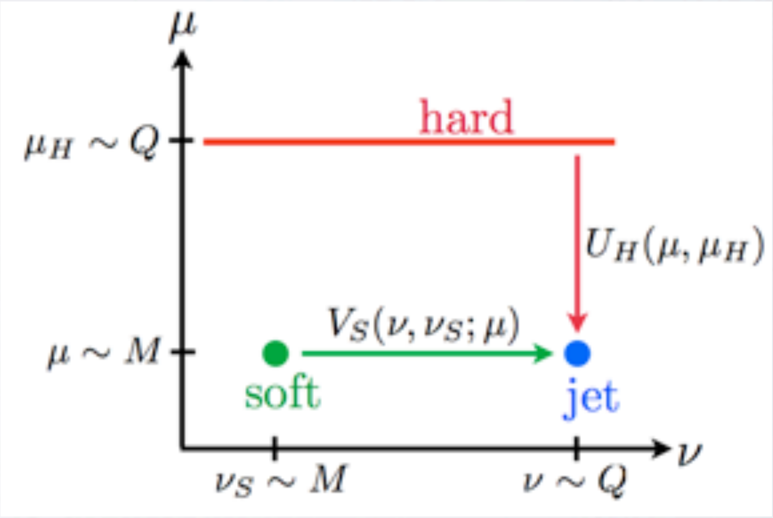
$$A_\mu = A_\mu^{c,n} + A_\mu^{c,\bar{n}} + A_\mu^{soft} + \dots$$

- We thus get divergences when integrals invade other sectors

- Regulating sets up RG for resumming these rapidity logs

$$W_n = \sum_{\text{perms}} \exp\left[-\frac{g}{\bar{n}\cdot P}\frac{\nu^\eta}{|\bar{n}\cdot P|^\eta}\bar{n}\cdot A_n\right]$$





From Chiu et al. 1202.0814: In SCETII, soft and collinear modes have same virtuality
ν-running lets us minimize log between soft & collinear scales

ANOMALOUS DIMENSION RESULTS

- For the collinear and soft sectors of our operators, (a - nonsinglet and b - singlet)

$$\gamma^c_{aa} = \frac{3g^2}{4\pi^2}\log(\frac{\nu^2}{4M_\chi^2}), \quad \gamma^s_{aa} = \frac{-3g^2}{4\pi^2}\log(\frac{\nu^2}{\mu^2}),$$

$$\gamma^c_{ba} = \frac{-g^2}{4\pi^2}\log(\frac{\nu^2}{4M_\chi^2}), \quad \gamma^s_{ba} = \frac{g^2}{4\pi^2}\log(\frac{\nu^2}{\mu^2})$$

Note **ν** dependence drops out in soft-collinear sum

- We get the following RG equation (2,4 - nonsinglet and 1,3 - singlet)

$$\mu\frac{d}{d\mu}C_{2,4}(\mu) = -(\gamma^c_{aa} + \gamma^s_{aa})C_{2,4}$$

$$\mu\frac{d}{d\mu}C_{1,3}(\mu) = -(\gamma^c_{ba} + \gamma^s_{ba})C_{2,4}.$$

Logs minimized for (**μ**,**ν**)
soft: (m_W, m_W)
collinear: (M_{wino}, m_W)

WILSON COEFFICIENTS

- We have four operators, but controlled by one tree-level matching coefficient

  - Just two dimension-five operators in "square root" of OPE, $\chi\chi B_n B_{\underline{n}}$ and $\chi^C \chi^D B_n^C B_{\underline{n}}^D$ and

  - Their coefficients equal and opposite to cancel, e.g. $\chi^3\chi^3 \rightarrow W^3 W^3$

  - Squaring in OPE leads trivial color contraction, giving

$$C_1 \equiv C \quad C_2 = -2C$$
$$C_3 = 0 \quad C_4 = C$$

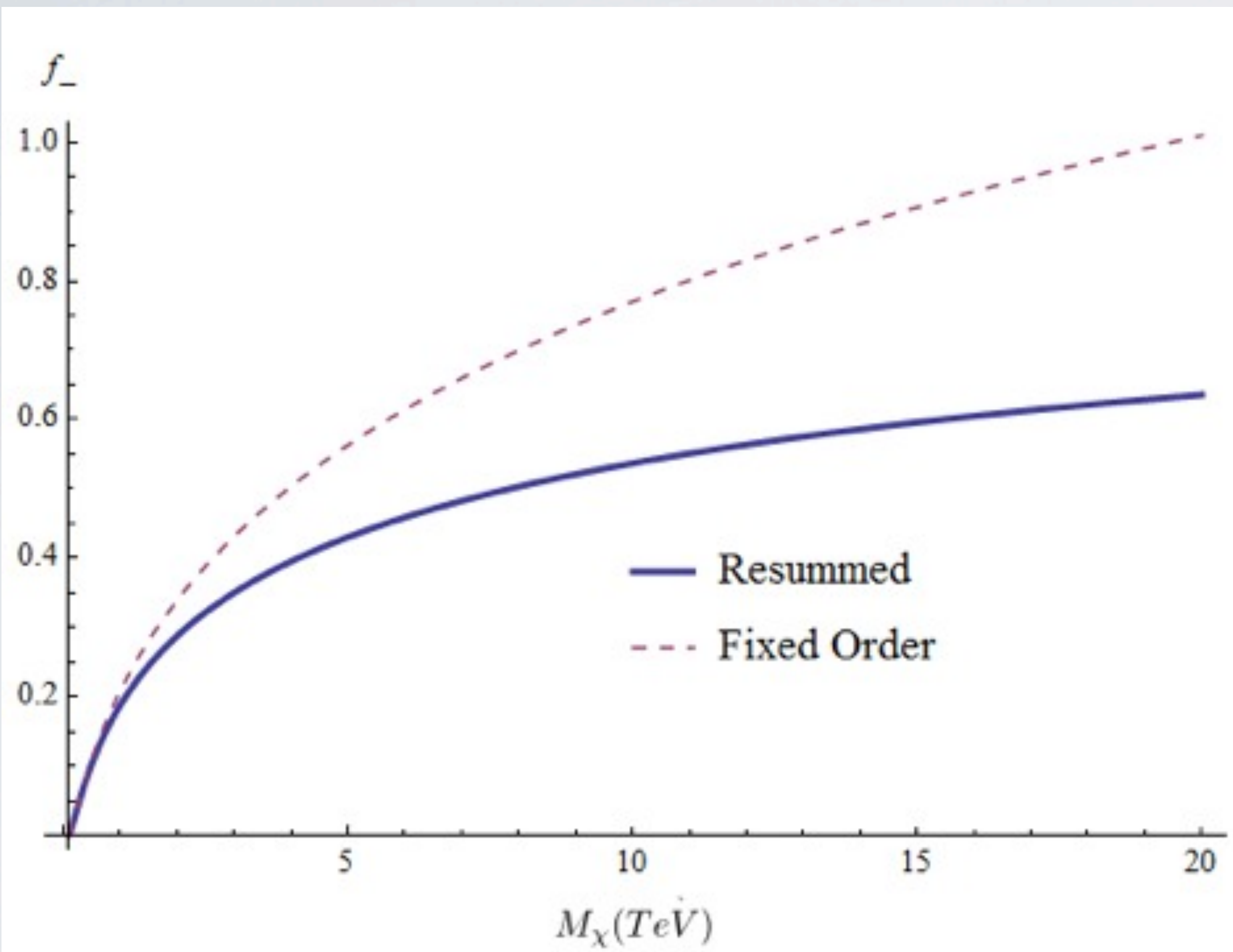
TOTAL RATE

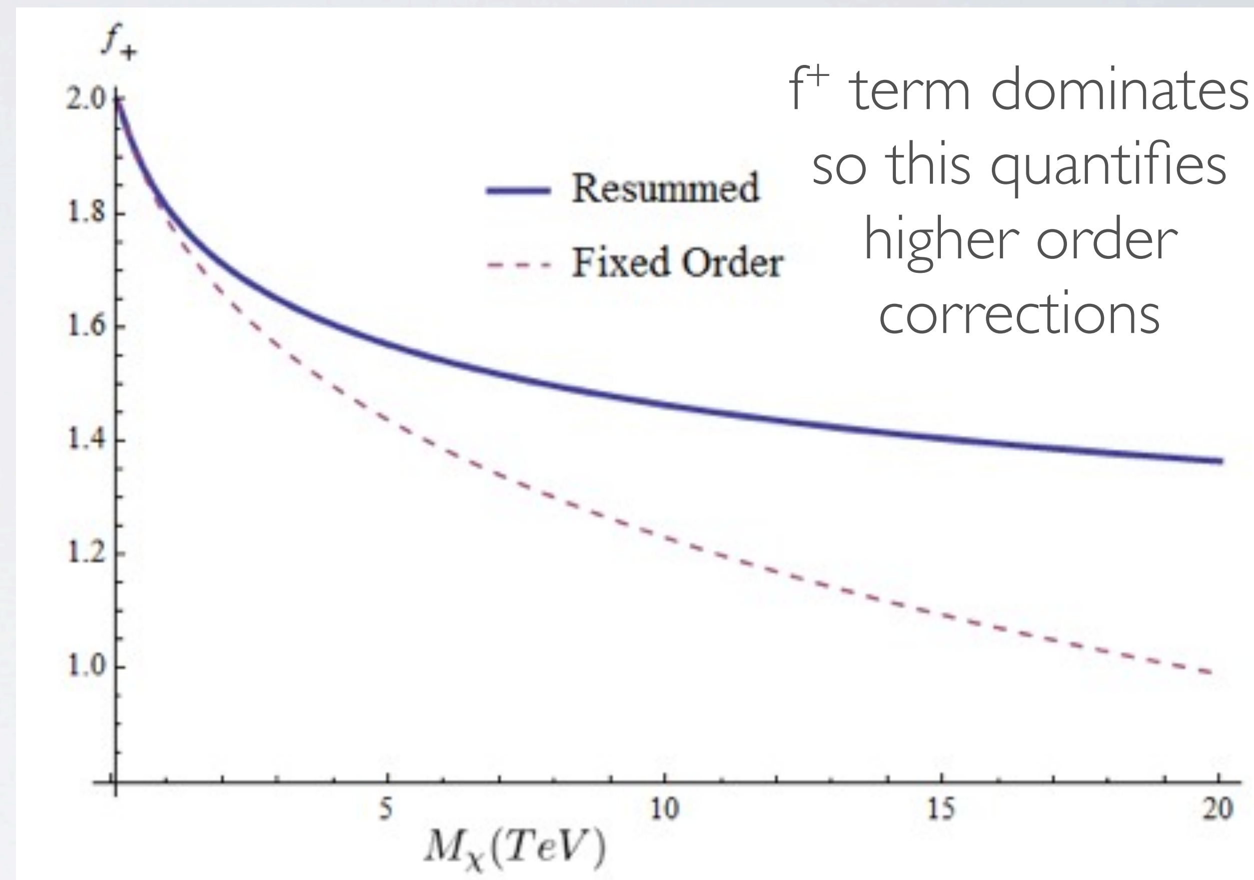
$$\frac{1}{E_\gamma}\frac{d\sigma}{dE_\gamma} = C_1(\mu = E_\gamma)\delta(E_\gamma - M_\chi)\left(\frac{2}{3}f_-\,|\,s^{00}_{00}\,\psi_{00}(0)\,|^2\right.$$
$$\left. +2f_+\,|\,s^{00}_{+-}\psi_{+-}(0)\,|^2 + \frac{2}{3}f_-(s^{00}_{00}\,s^{00}_{+-}\psi_{00}\psi_{+-} + h.c.)\right)$$

- The ψ-factors quantify short-distance annihilation
- The s-factors quantify overlap between annihilating and asymptotic states (Sommerfeld), and we will calculate them by numerical Schrodinger solutions
- The f-factors arise from running Wilson coefficients and resum Sudakov double-logs

$$f_\pm \equiv 1 \pm \exp(-\frac{3\alpha}{\pi}\log^2(\frac{M_W}{E_\gamma}))$$

RESUMMATION





Sudakov factor vs. Dark Matter mass

Resummation is modest ~5% affect for thermal Wino (3 TeV)

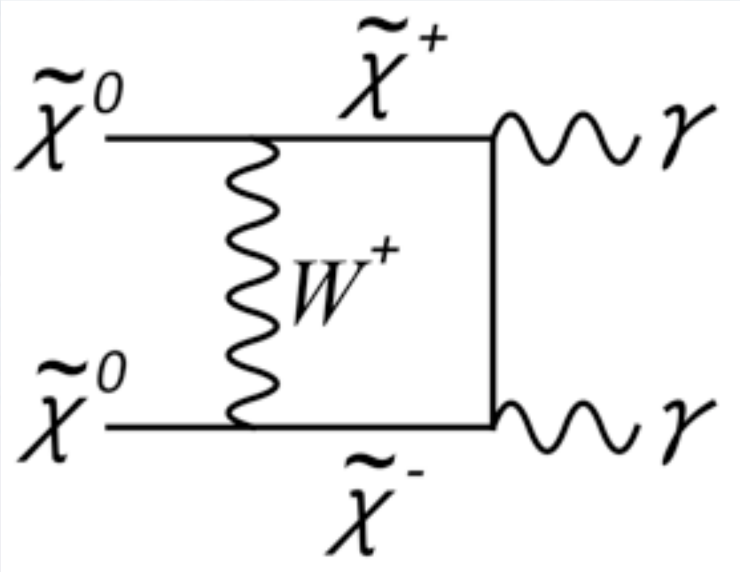
NONPERTURBATIVE EFFECTS

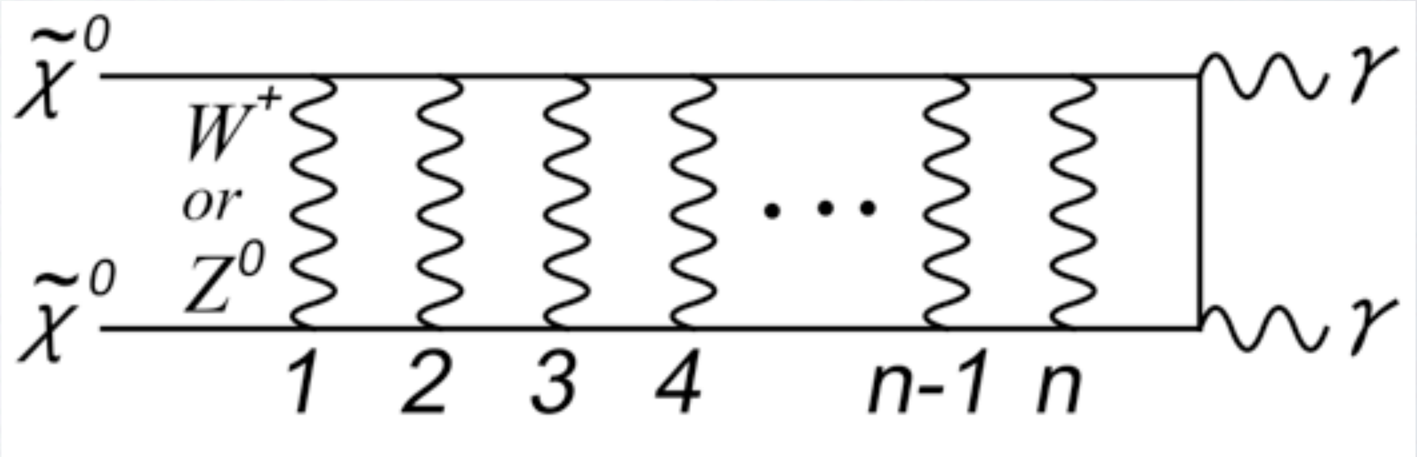
- So far we have focused on loops involving final-state bosons
- W-exchanges flip between $\chi^0\chi^0$ and $\chi^+\chi^-$ states

$$\sigma_{\mathrm{one-loop}} v = \frac{4\pi\alpha^2\alpha_W^2}{m_W^2}$$

- Subsequent ladder exchanges unsuppressed

$$A_n \simeq \alpha \left(\frac{\alpha_2 M}{m_W}\right)^n$$





From Hisano et al. hep-ph/0412403

SOMMERFELD ENHANCEMENT*

- Slowly-moving objects in a potential can have much larger cross section than perturbative treatment suggests

- In nonrelativistic regime, summing infinite ladder exchange equivalent to solving Schrodinger equation for appropriate potential

- Annihilation rate becomes

$$\sigma v = |s|^2 \Gamma$$

- s given by $\psi(\infty) / \psi(0)$ for boundary conditions $\psi(0) = 1$, $\psi'(\infty) = i k \psi(\infty)$

*Hisano et al.: hep-ph/0412403, Arkani-Hamed et al.: 0810.0713, Slatyer: 0910.5713

ELECTROWEAK POTENTIAL*

- Potential accounts for Yukawa exchange of Ws, Zs, Coulomb exchange of γs, and χ^+ - χ^0 mass splitting = (0.17 GeV)

$$\begin{pmatrix} 2\delta M - \frac{\alpha}{r} - \alpha_W c_W^2 \frac{e^{-m_Z r}}{r} & -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} & 0 \end{pmatrix}$$

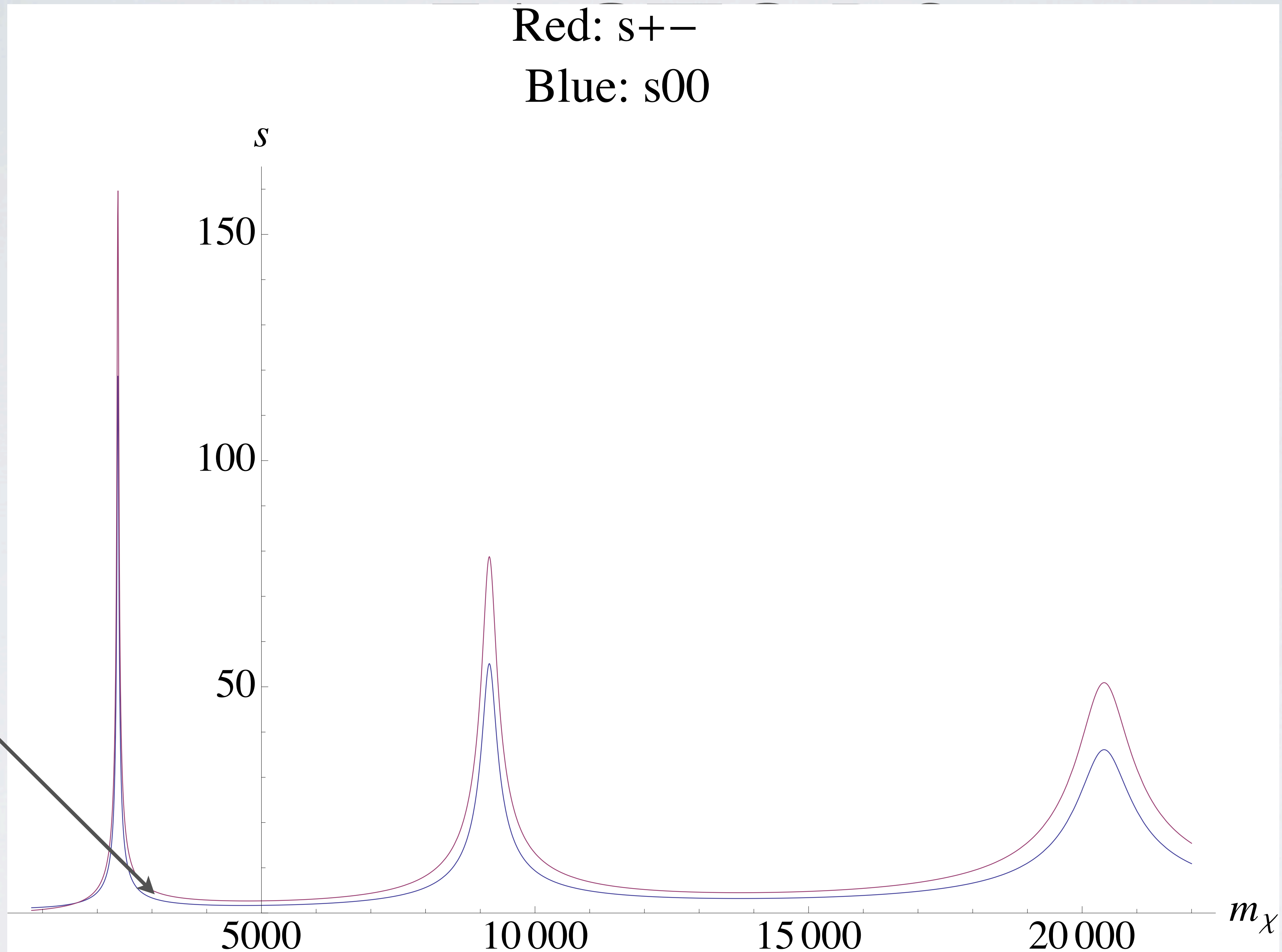
Mass splitting means +- state decays exponentially

- Between the two-body $^1S_0$ states

$$\frac{1}{\sqrt{2}}\chi^+(\vec{x}-\vec{r}/2)\chi^-(\vec{x}-\vec{r}/2) \qquad \frac{1}{2}\chi^0(\vec{x}-\vec{r}/2)\chi^0(\vec{x}-\vec{r}/2)$$

We follow Hisano et al.: hep-ph/0412403 and Cohen et al 1307.4082

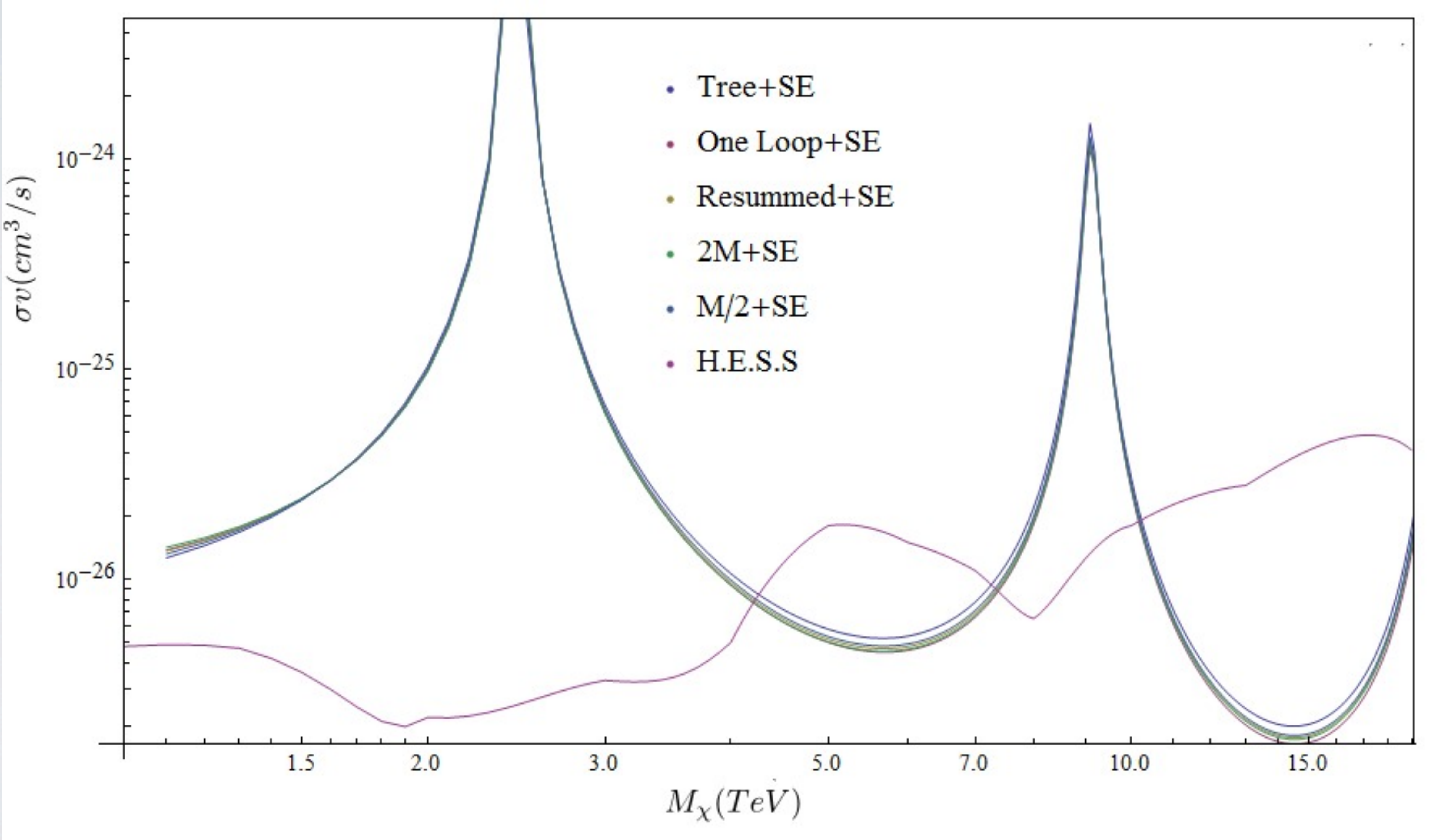
WINO SOMMERFELD



Thermal
3 TeV
value

Solving Schrodinger equation for v = 10^{-3} gives us resonance regions with $O(10^4)$ enhancement

TOTAL RATE & EXCLUSION



Exclusion curve taken from Ovanesyan, Slatyer, and Stewart (1409.8294); HESS data with NFW profile

COMMENTS & CONCLUSION

- Viability of thermal wino dark matter requires vastly different profile than that preferred by simulation community (core ~ 10 kpc)
- When computing observationally relevant semi-inclusive rate, NLO effects are small (~10%) compared to ~50% found in exclusive analyses: Ovanesyan, Slatyer, and Stewart (1409.8294); Bauer et al. (1409.7392)
- Accounting for real-emission leads to LL operator mixing and Wilson coefficients with structure 1+Exp[-Log^2]. Exclusive calculation has no mixing and therefore gets simply Exp[-Log^2]
- Resumming inclusive electroweak double logs at hadron colliders where resummation could be important for precision (LHC) or crucial (100 TeV)