Large–N twisted volume reduction of QCD on the lattice

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QCD and related theories at large–N

We consider SU(N) gauge theory with n_f light Dirac fermions in the adjoint representation, in the large–N limit.

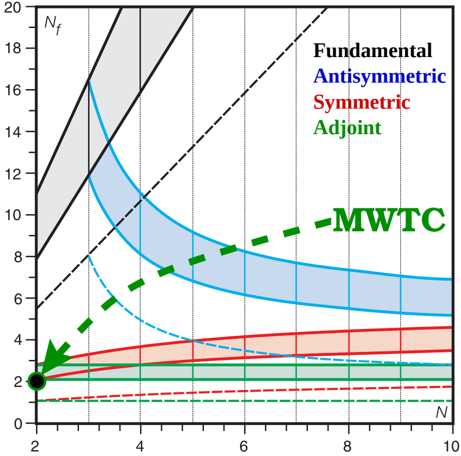
- $n_f = 0$: this is the large–N limit of QCD.
- $n_f = 1/2$: $\mathcal{N}=1$ supersymmetric Yang–Mills (SYM).
- $n_f = 1$: thought to be confining in the infrared.
- $n_f = 2$: thought to have an IRFP (InfraRed Fixed Point)

Using large–N volume independence (Eguchi–Kawai reduction), want to simulate these theories on a single site lattice.

Talk Outline

- Large–N twisted volume reduction
- $n_f = 0$: compare to QCD, test of reduction
- $n_f = 2$: very different to QCD

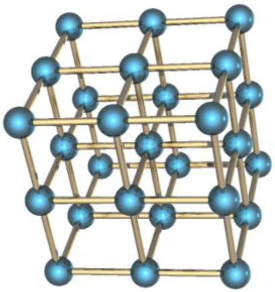
Why large N?



[arXiv:hep-ph/0611341]

- $n_f = 0$: Close relative of SU(3) QCD
- $n_f = 2$: Existence of fixed point in 2–loop perturbation theory is independent of N
- $n_f = 2$: γ_* in 2–loop perturbation theory is independent of N

Lattice Field Theory



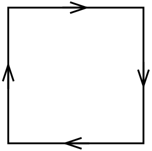
Formulate field theory on a discrete set of space–time points:

- $\hat{L}^4$ points, lattice spacing a
- Physical volume $L^4 = (\hat{L}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/L$

Lattice Field Theory



The simplest lattice discretisation of the Yang–Mills action is

$$S_{YM} = N_c b \sum_x \sum_{\mu<\nu} Tr \left(U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x) + h.c. \right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of U(N_c) Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} \equiv S_{EK} = N_c b \sum_{\mu<\nu} Tr \left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Conditions

...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Twisted Reduction

Gonzalez–Arroyo Okawa '83

Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

$$S_{TEK} = N_c b \sum_{\mu<\nu} Tr \left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

$$z_{\mu\nu} = exp\{2\pi i n_{\mu\nu}/N\} = z_{\nu\mu}^*$$

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

Twisted Reduction

Original TEK: $k = 1$, center-symmetry breaks for $N \gtrsim 100$

Choice of flux k

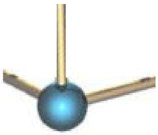
$$n_{\mu\nu} = k\sqrt{N}, \quad k\bar{k} = 1 \mod \sqrt{N}, \quad \tilde{\theta} = 2\pi\bar{k}/\sqrt{N}$$

To take $1/N \to 0$ limit, choose k such that

- $k/\sqrt{N} > 1/9$
- $\tilde{\theta} =$ constant

Garcia–Perez Gonzalez–Arroyo Okawa [arXiv:1307.5254]

Twisted Reduction



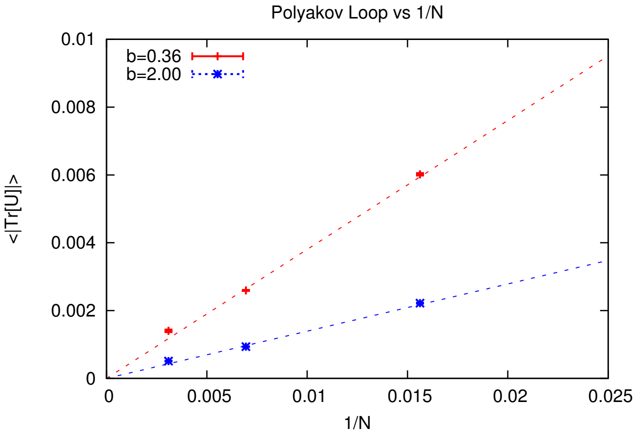
Twisted reduction: $\hat{L} \to \sqrt{N}$

- Single site lattice, lattice spacing a
- Physical volume $L^4 = (\sqrt{N}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/\sqrt{N}a$

Polyakov Loop vs 1/N



Wilson Flow

The Wilson flow evolves the gauge field according to

Flow Equation

$$\frac{\partial B_\mu}{\partial t} = D_\nu G_{\nu\mu}, \qquad B_\mu|_{t=0} = A_\mu$$

where A_μ is the gauge field, and t is the flow time.
This integrates out UV fluctuations above a scale $\mu = 1/\sqrt{8t}$ (i.e. smears observables over a radius $\sqrt{8t}$)

Lüscher [arXiv:0907.5491]

Wilson Flow of $\frac{1}{N}t^2\langle E\rangle$

The action density $E = G_{\mu\nu}G_{\mu\nu}$ as a function of flow time can be used to define a scale t_0

Definition of scale t_0

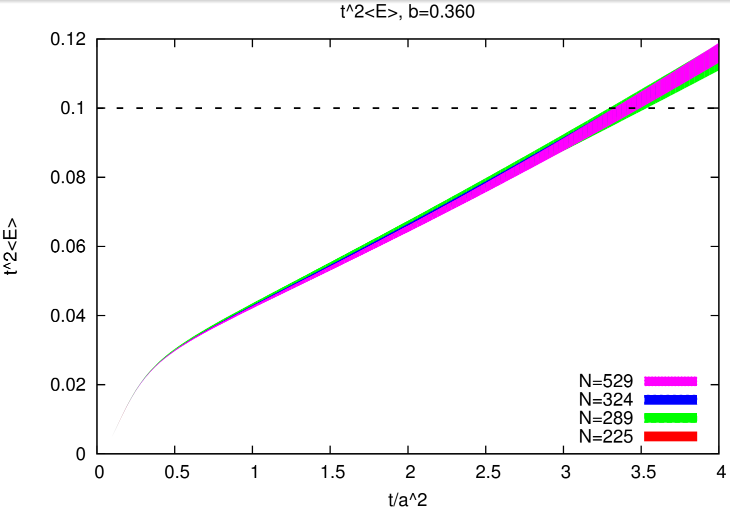
$$\frac{1}{N}t_0^2\langle E(t_0)\rangle = 0.1$$

Perturbative expansion of E at small flow time t

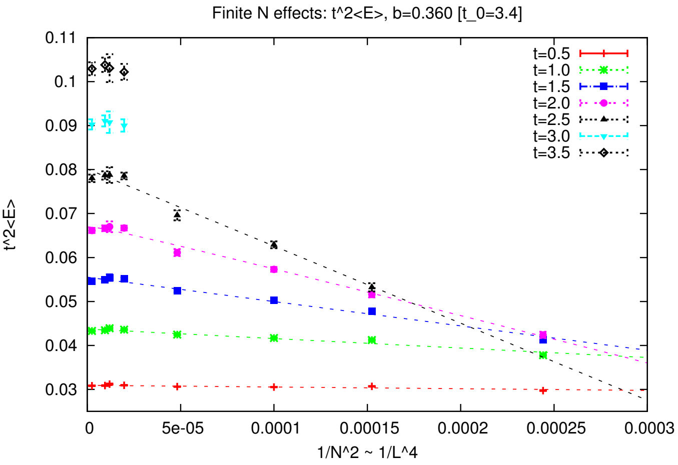
$$\frac{1}{N}t^2 E(t) = \frac{3\lambda}{128\pi^2}\left[1 + \frac{\lambda}{16\pi^2}(11\gamma_E/3 + 52/9 - 3\ln 3)\right]$$

Lüscher [arXiv:1006.4518]

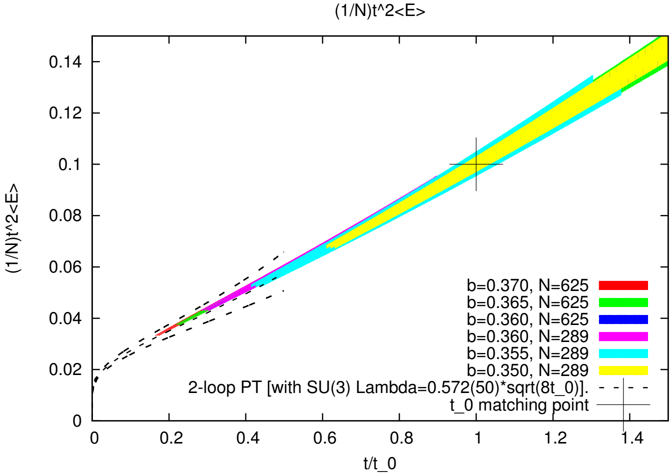
Setting the scale with t_0



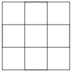
Finite Volume Effects of $\frac{1}{N}t^2\langle E\rangle$



Comparison to SU(3) Perturbation Theory



Running of the coupling: Step Scaling



- Step scaling - change in coupling from $\hat{L}$ to $s\hat{L}$
- $u = \overline{g}^2(b, s\hat{L})$
- $\sigma(u, s) = u' = \overline{g}^2(b, s\hat{L})$
- Now tune bare parameters until $\overline{g}^2(b', \hat{L}) = u'$
- Repeat

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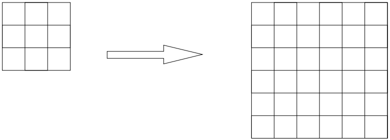

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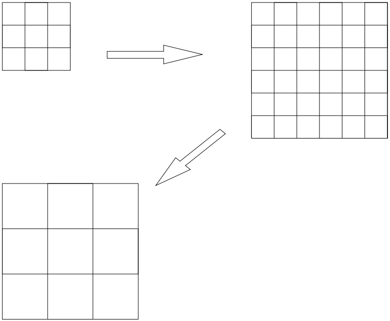

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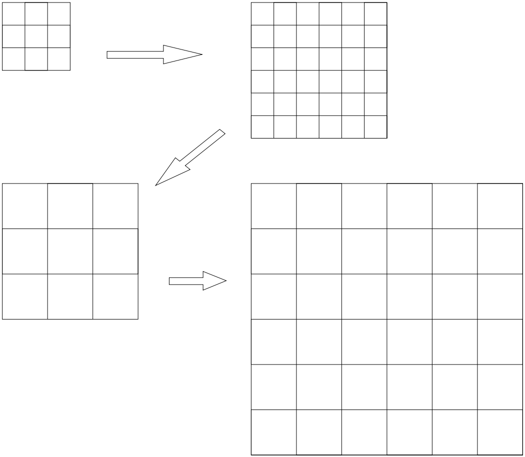

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Twisted Gradient Flow Scheme

Define a renormalised coupling in terms of E at positive flow time:

Definition of renormalised coupling λ_{TGF}

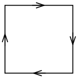
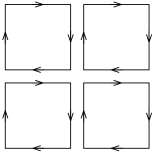
$$\lambda_{TGF}(L) = \mathcal{N}_T^{-1}(c)t^2\langle E\rangle\big|_{t=c^2N/8} = \lambda_{\overline{\mathrm{MS}}} + \mathcal{O}(\lambda^2_{\overline{\mathrm{MS}}})$$

- Smearing radius is a fraction c of the lattice size $\sqrt{8t} = cL = c\sqrt{N}a$
- Renormalisation scale is the inverse of the box size $\mu = 1/L$.
- c is a free parameter, defines renormalisation scheme.

Ramos [arXiv:1308.4558]

Lattice Discretisation Effects

Need to choose a discretisation for E:

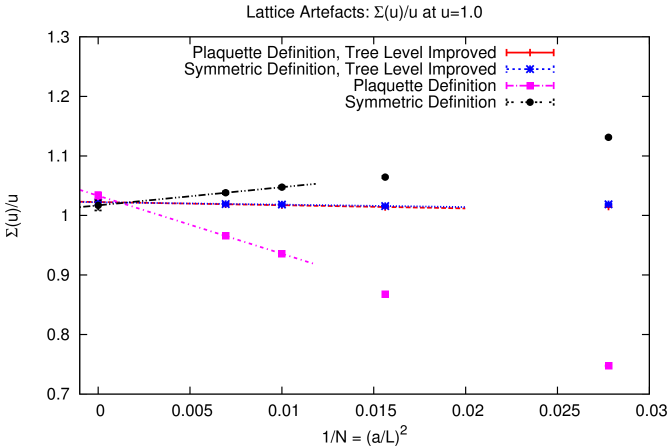
- Plaquette  or Symmetric 

Also have a choice for $\mathcal{N}_T$:

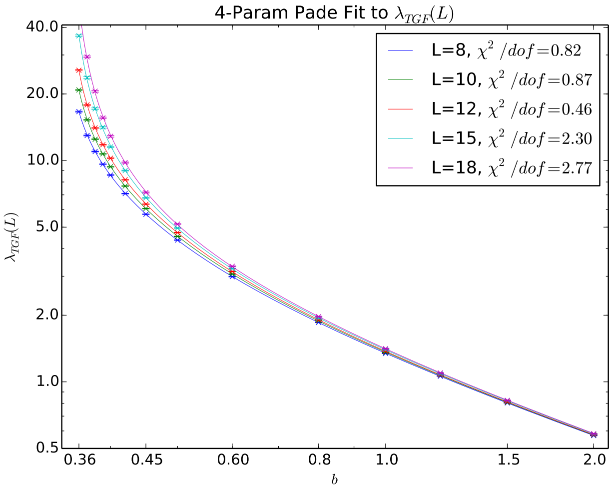
- Continuum definition
- Tree level lattice definition

All equivalent up to $\mathcal{O}(a/L)^2$ lattice artefacts.

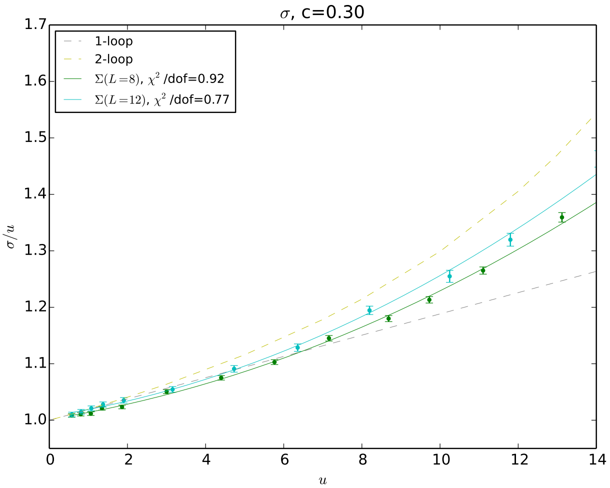
Lattice Artefacts, $u = 1, c = 0.30$



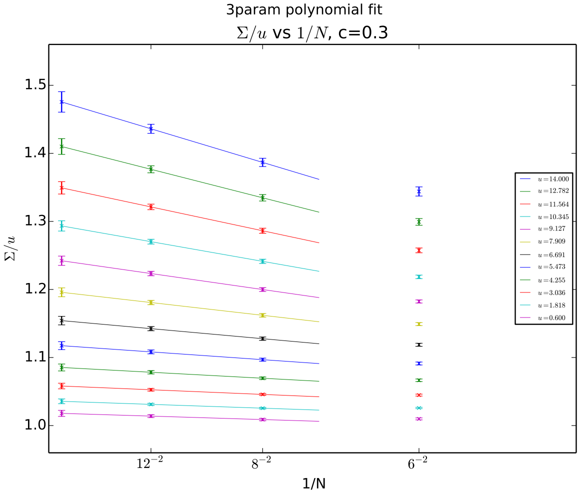
Twisted Gradient Flow Coupling for $c = 0.30$



Lattice Discrete Beta Function [preliminary]



Continuum Extrapolation [preliminary]



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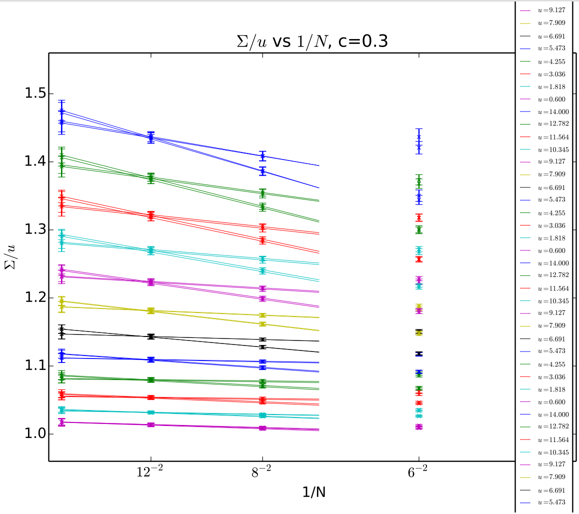

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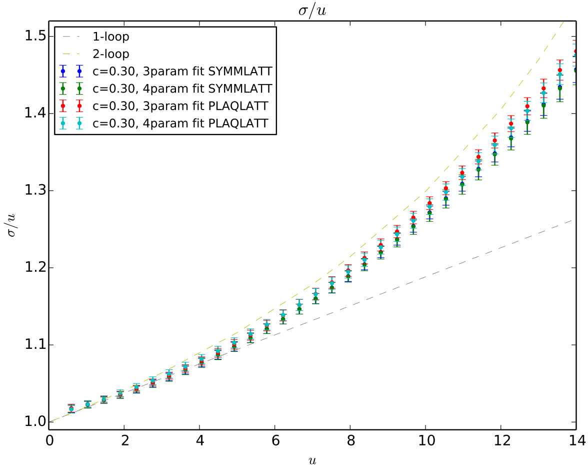
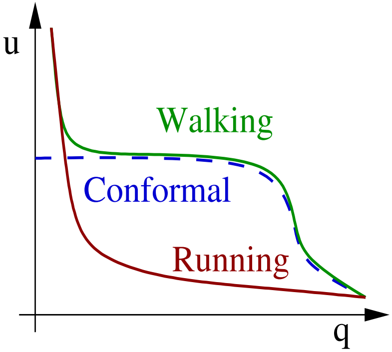
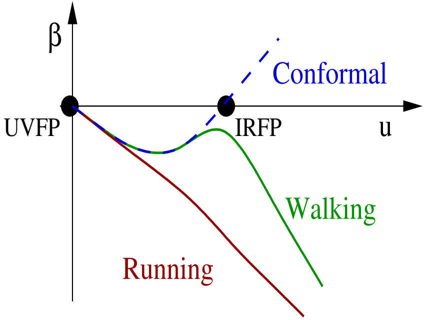


Confining vs Conformal Cartoon

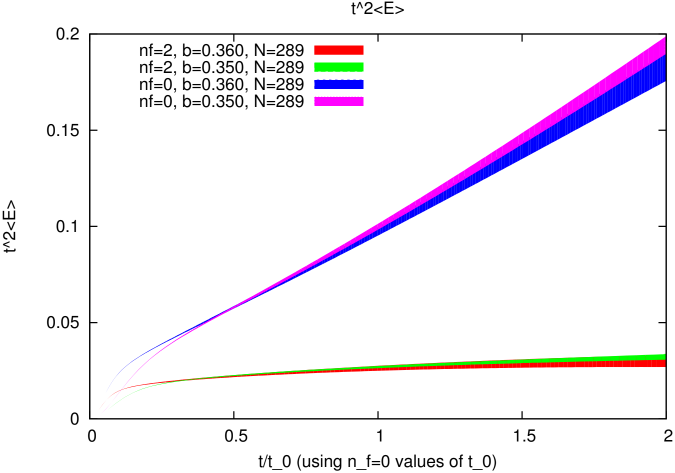




Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
  - **Existence** of fixed point
  - **Mass anomalous dimension** at the fixed point

Wilson Flow: $n_f = 0$ vs $n_f = 2$



Mode Number Method

In the infinite volume, chiral limit, and for small eigenvalues,

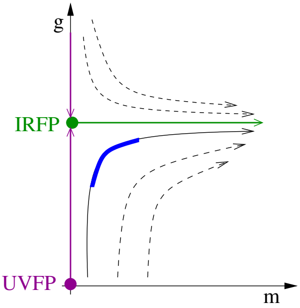
Spectral density of the Dirac Operator

$$\lim_{m\to 0}\lim_{V\to\infty}\rho(\omega)\propto\omega^{\frac{3-\gamma_*}{1+\gamma_*}}+\dots$$

- Integral of this is the mode number, which is just counting the number of eigenvalues of the Dirac Operator on the lattice.
- Fitting this to the above form can give a precise value for γ, as done recently for MWT by Agostino Patella.

DeGrand [arXiv:0906.4543], Del Debbio et. al. [arXiv:1005.2371], Patella [arXiv:1204.4432], Hasenfratz et. al. [arXiv:1303.7129]

Mode Number Fit Range

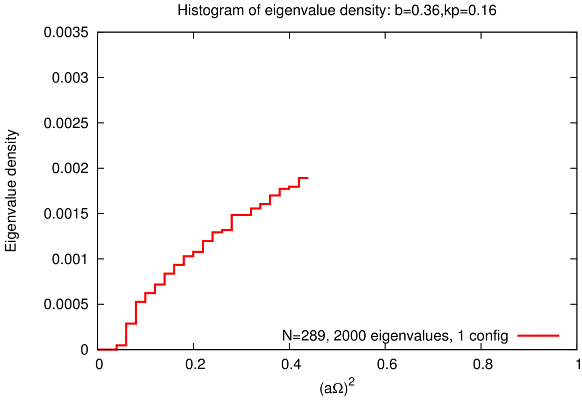


RG flows in mass–deformed CFT:

- Flow from UV (high eigenvalues) to IR (low eigenvalues)
- Finite mass drives us away from FP in the IR
- Interested in intermediate blue region
- $\frac{1}{\sqrt{N}} \ll m \ll \Omega_{IR} < \Omega < \Omega_{UV} \ll \frac{1}{a}$

Eigenvalue density histogram

Histogram shows change between the two regimes as the volume is increased.



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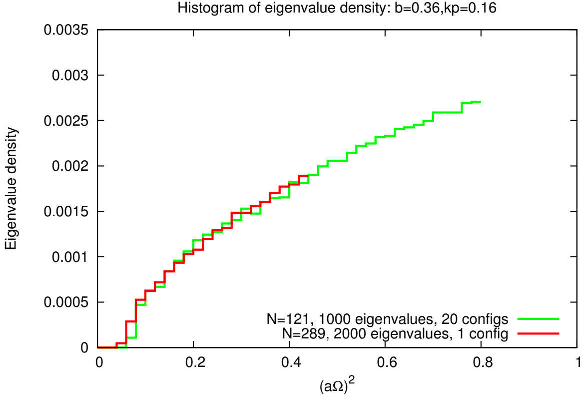

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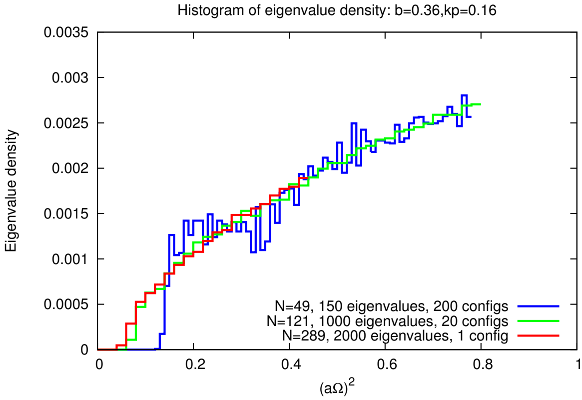

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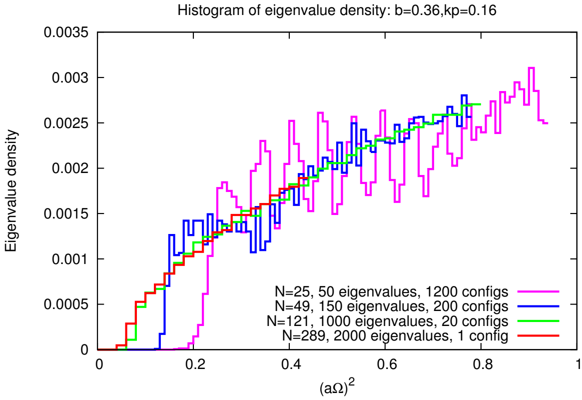
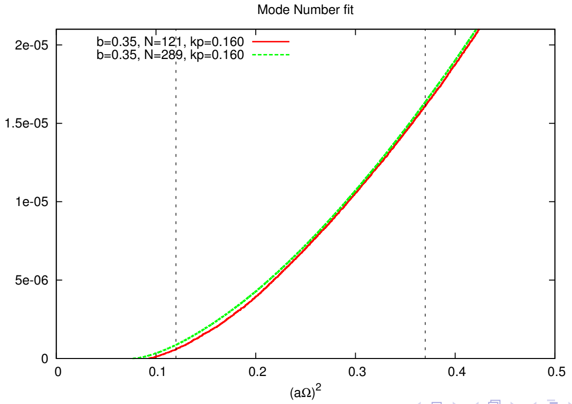


Mode Number Example Fit $b = 0.35, \kappa = 0.16$

$$N = 289: \ A = 1.16 \times 10^{-4}, (am)^2 = 0.068, \gamma = 0.258$$
$$N = 121: \ A = 1.04 \times 10^{-4}, (am)^2 = 0.108, \gamma = 0.417$$



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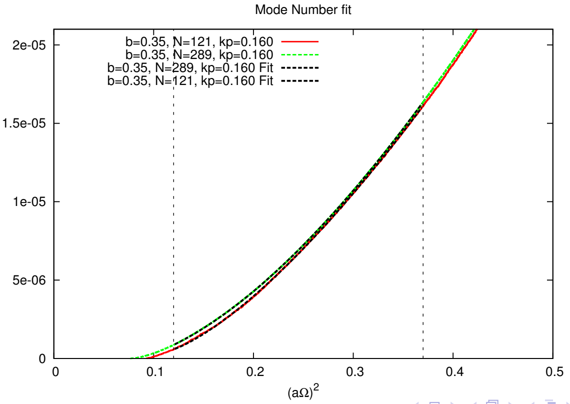
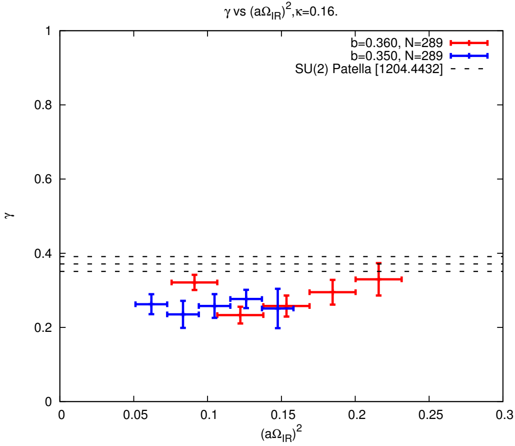


Mass anomalous dimension results [preliminary]



Conclusion and Future Work

- Promising initial results.
  - Twisted volume reduction seems to work
  - $n_f = 0$ at large N in very good agreement with N=3

Future Work / In Progress:

- $n_f = 2$: Running coupling study, add lighter masses, different bare couplings.
- Comparison with $n_f = 1, n_f = 1/2$