Precision predictions for single-top production at the LHC

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work in progress with K. Melnikov and M. Brucherseifer

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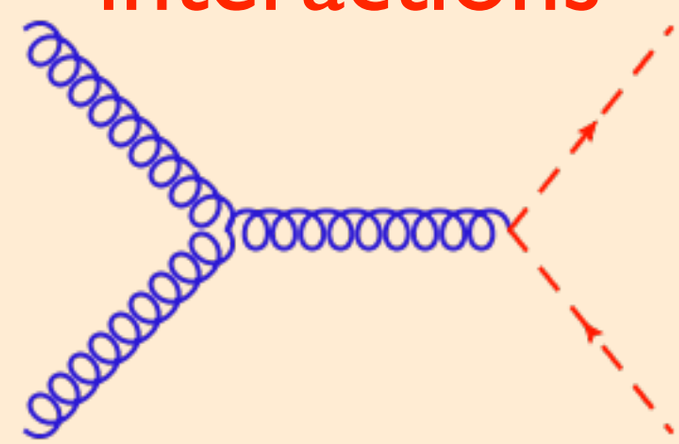
Top: a very special quark

The only 'natural' quark
$y_t \sim 1$

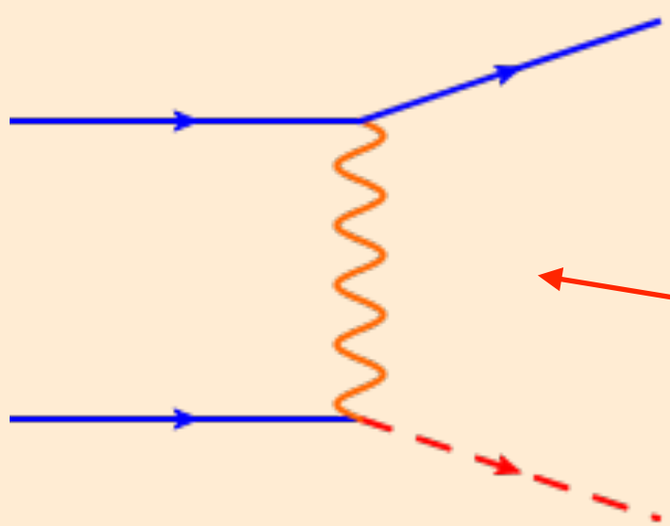
Strongly coupled to the
Higgs sector

Decay before
hadronization

Probe strong
interactions



Probe weak interactions

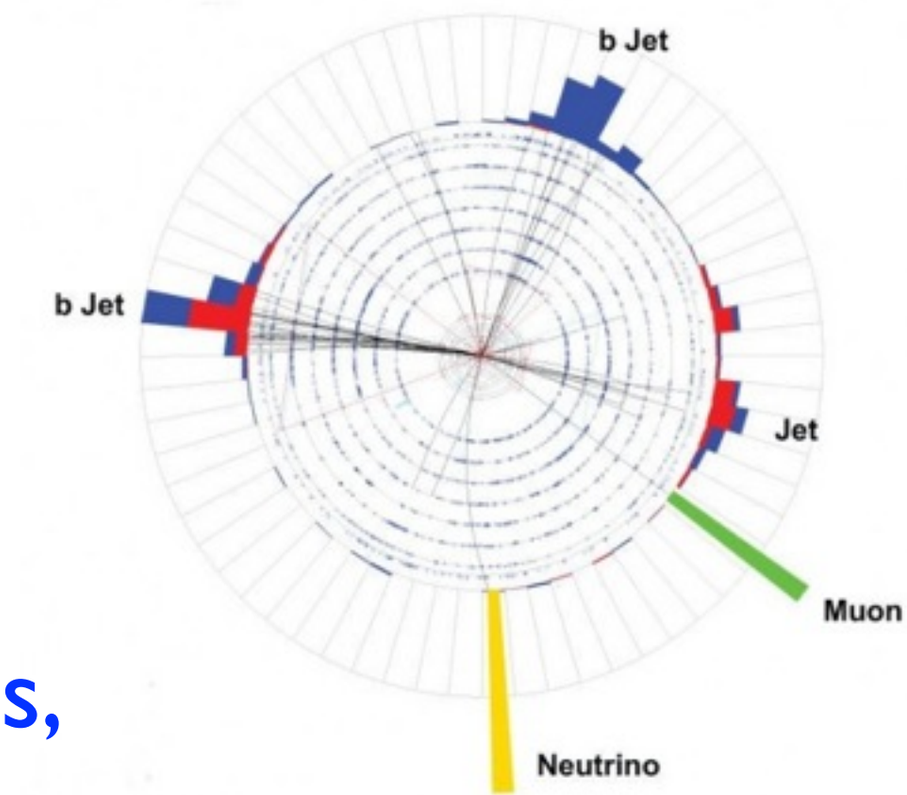


FOCUS OF THIS TALK

The top-quark: a Tevatron success

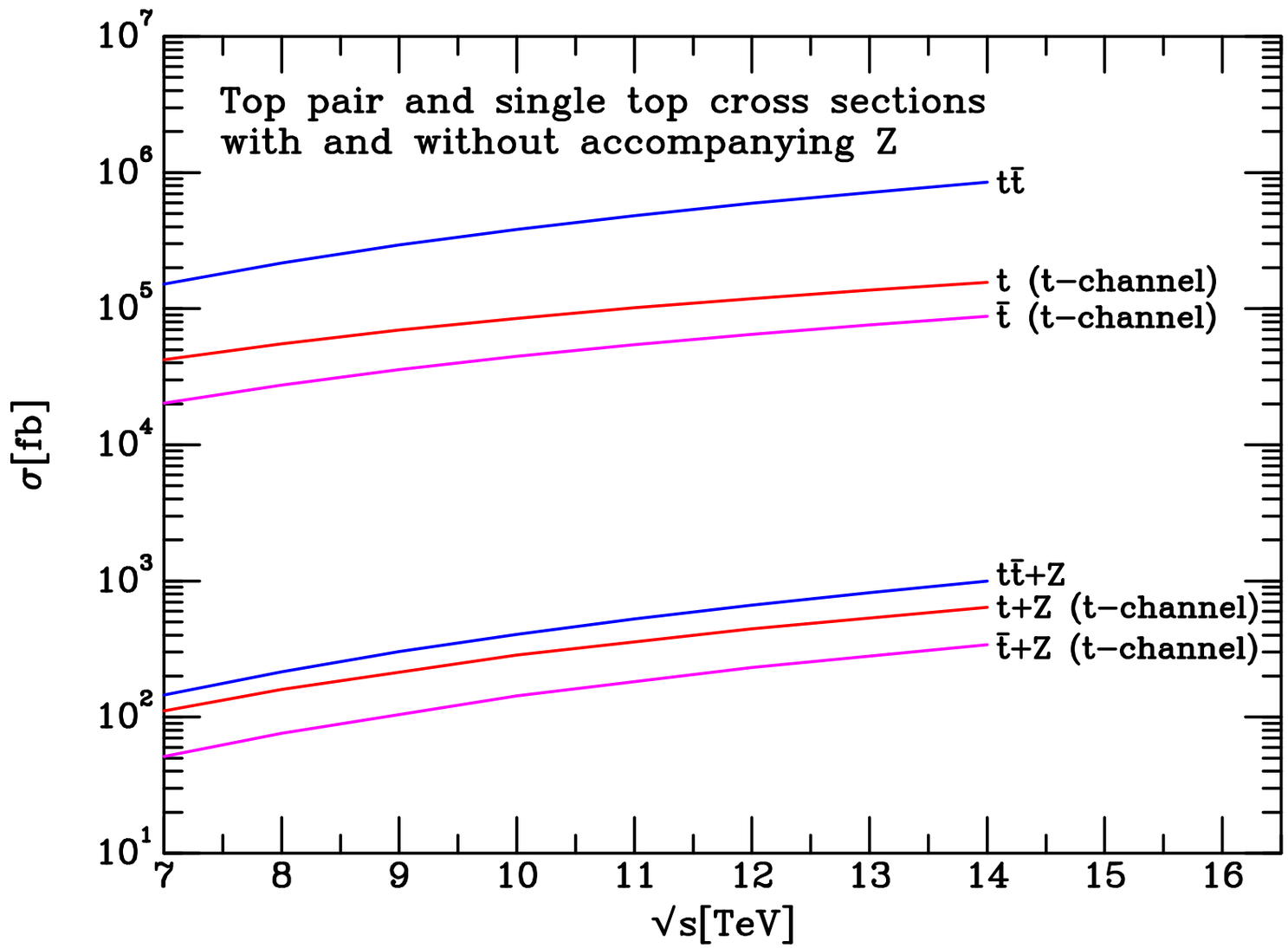
1995: milestone discovery

Single top production established as well



Mass, $\sigma_{t\bar{t}}$, Γ_t, W hel fractions, $|V_{tb}|$, spin correlations, s-t channel single top, A_{FB} …

LHC as a top factory



[Campbell, Ellis, Rontsch (2013)]

From discovery to PRECISION PHYSICS

$t\bar{t}$ theory: pretty good shape

QCD corrections: an old story [Nason et al (1988)]

More (~) recent developments:

- NLO with decay [Melnikov et al (2009), Campbell et al (MCFM)]
- Spin correlation studies [Mahlon and Parke (2010)]
- Resummations [Cacciari et al (2012), Beneke et al (2010), Czakon et al (2009), Bonciani et al (2008)]
- Top pair plus jets [Dittmaier et al (2008-2009), Melnikov et al (2010), Bredstein et al, Bevilacqua et al (2010)]
- Full NNLO for stable tops [Czakon et al (2013)]
- Electroweak corrections [Kuhn et al (2006)]
- Parton shower, automation…

FEW PERCENT ACCURACY IS POSSIBLE

Single-top: smaller rates, but sizable yield

A lot is known:

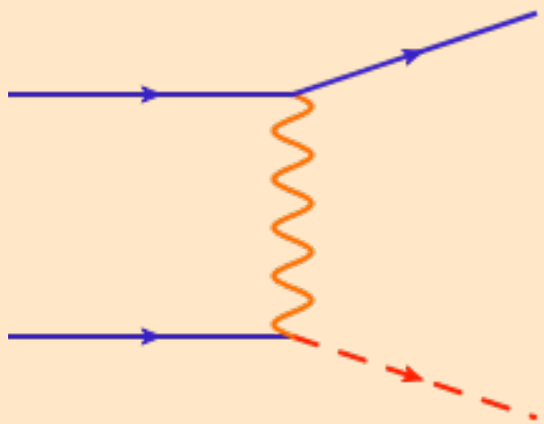
- NLO in the 4/5FS [Bordes et al (1995), Stelzer et al (1997), Harris et al (2002), Beccaria et al (2008), Heim et al (2010)]
- Corrections with decaying top [Campbell et al (2004-2009), Cao et al (2005), Schwienhorst et al (2011), Pittau(1996), Falgari et al (2010,2011), Papanastasiou et al (2013)]
- Soft gluon approximations [Kidonakis; Zhu et al (2010,2011)]
- NLO + PS, automation [Frederix et al (2012), Alioli et al (2009)]

Is it Enough to Claim ~ Percent Theoretical Accuracy?

Single-top: a closer look

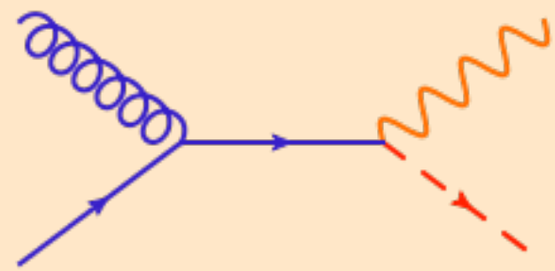
Three distinct production mechanisms

T-CHANNEL



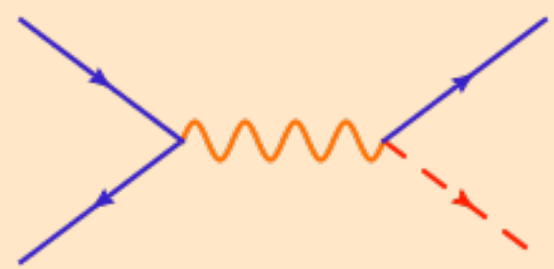
LHC8: ~ 82%
TEV: ~ 65%

ASSOCIATED PRODUCTION



LHC8: ~ 15%
TEV: ~ 0

S-CHANNEL



LHC8: ~ 5%
TEV: ~ 33%

requirement: ~ percent accuracy in the T-CHANNEL

t-channel single top: do we need NNLO?

Look at the NLO prediction

The total cross section at the 8 TeV LHC:

$$\sigma_{\rm LO} = 53.77 + 3.03 - 4.33 \text{ pb}$$
$$\sigma_{\rm NLO} = 55.13 + 1.63 - 0.90 \text{ pb}$$

Naively:

"Small ~ 2% corrections, no need to go further"

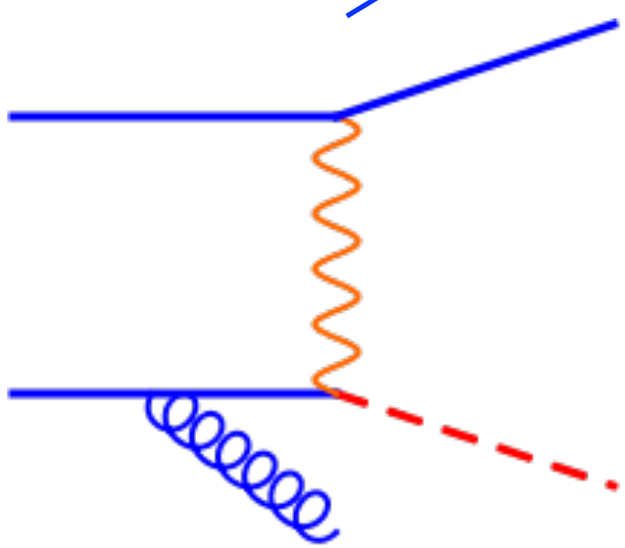
However…

T-channel single top: do we need NNLO?

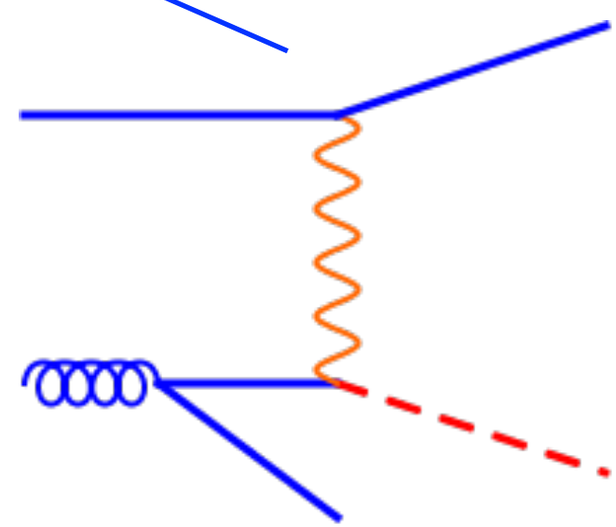
The total cross section at the 8 TeV LHC: A CLOSER LOOK

$$\sigma_{\rm LO} = 53.77 + 3.03 - 4.33 \text{ pb}$$

$$\sigma_{\rm NLO} = 55.13 + 1.63 - 0.90 \text{ pb}$$



+12%



-14%

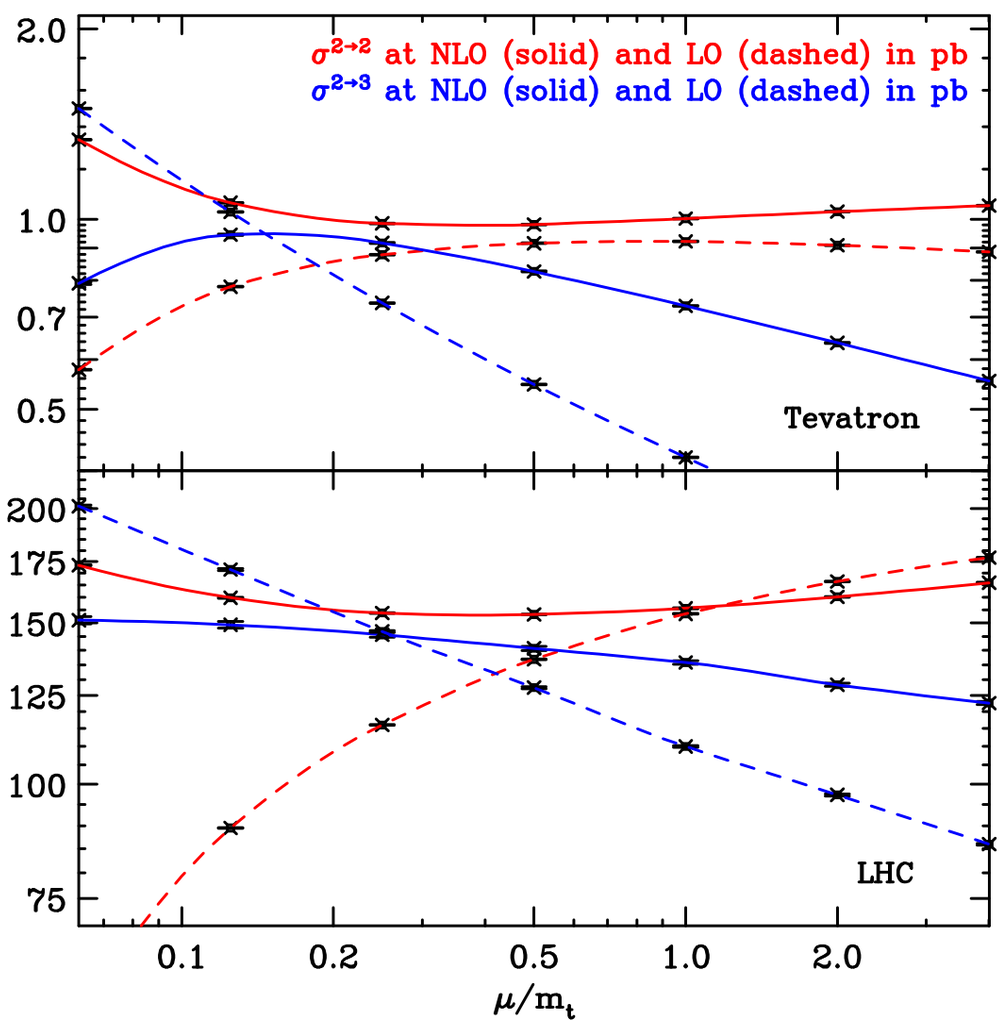
Large cancellations among channels

T-channel single top: do we need NNLO?

The total cross section at the 8 TeV LHC: A CLOSER LOOK

$$\sigma_{\rm LO} = 53.77 + 3.03 - 4.33 \text{ pb}$$

$$\sigma_{\rm NLO} = 55.13 + 1.63 - 0.90 \text{ pb}$$



[Campbell et al (2009)]

- Scale variation similar to corrections
- ~ percent difference between 4FNS/5FNS calculations

T-channel single top: do we need NNLO?

The total cross section at the 8 TeV LHC: A CLOSER LOOK

$$\sigma_{\rm LO} = 53.77 + 3.03 - 4.33 \text{ pb}$$

$$\sigma_{\rm NLO} = 55.13 + 1.63 - 0.90 \text{ pb}$$

- Large (accidental?) cancellations between channels
- Scale variation (~ NNLO!) as large as corrections
- Larger corrections for more exclusive observables

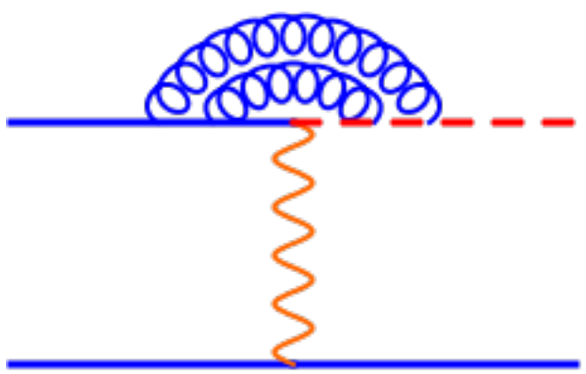
To control single-top production at the percent level:
NNLO CORRECTION TO T-CHANNEL PRODUCTION

Single-top t-channel prediction @ NNLO

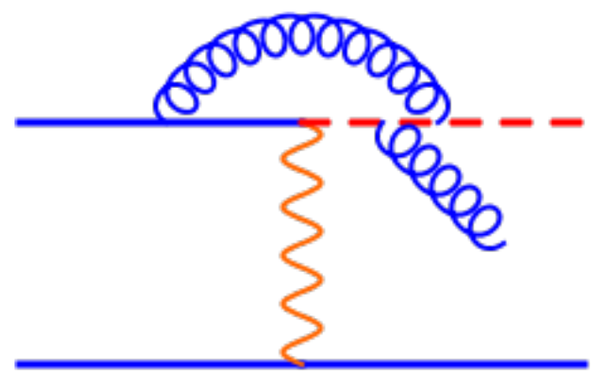
Anatomy of a NNLO computation

The ingredients:

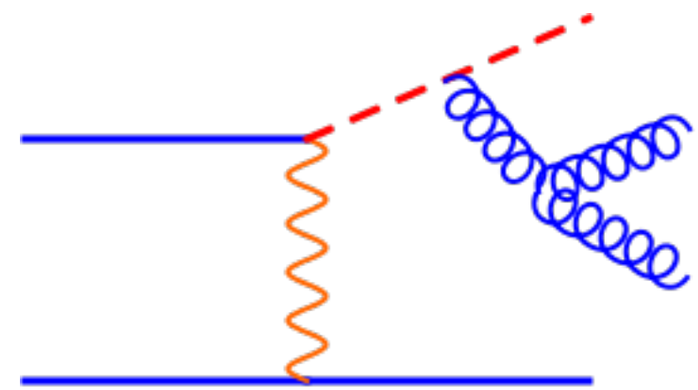
VV



RV



RR



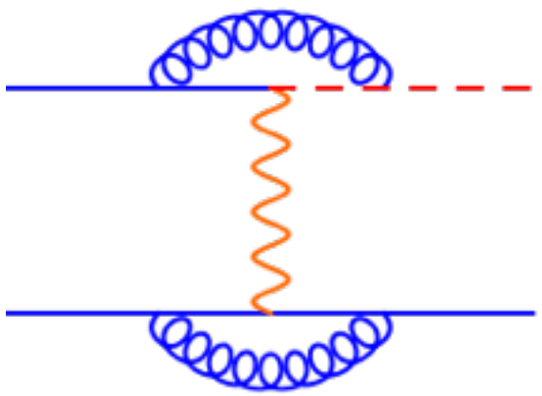
THE GOAL

To be useful, our computation must be:

- Fully differential in the top quark
- Top-decay friendly -> keep track of spin correlations

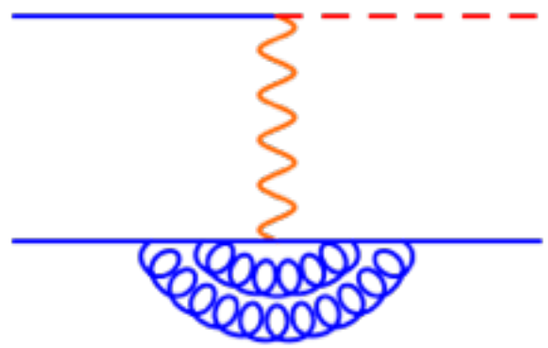
Single-top in the 'factorized' approximation

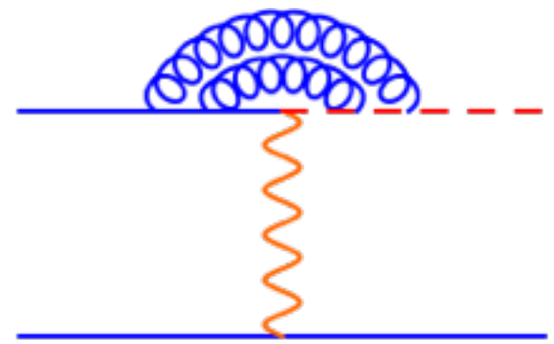
Two-loop amplitudes:



Trivial (~NLO^2)

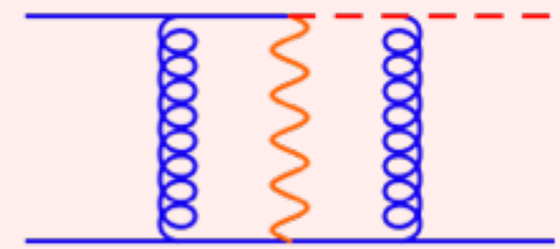
Simple





~OK

(very) hard



Must be interfered with tree-level -> COLOR SINGLET

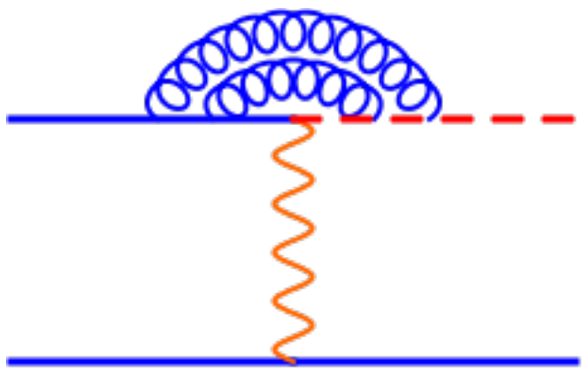
The 'hard' amplitude contribution is suppressed by $1/N_c^2$

In the following, I will CONSISTENTLY NEGLECT IT

['factorized/structure function/DIS' approximation]

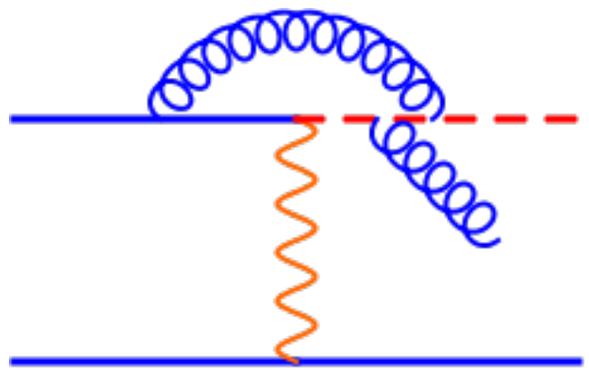
Anatomy of a NNLO computation

VV



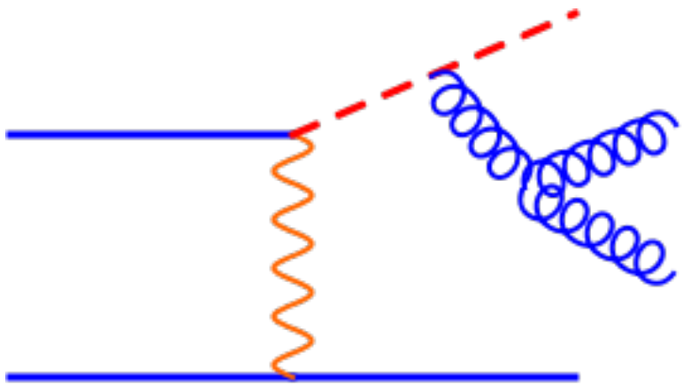
[Bonciani et al (2008), Beneke et al (2009)]

RV



[Campbell et al (2005)]

RR



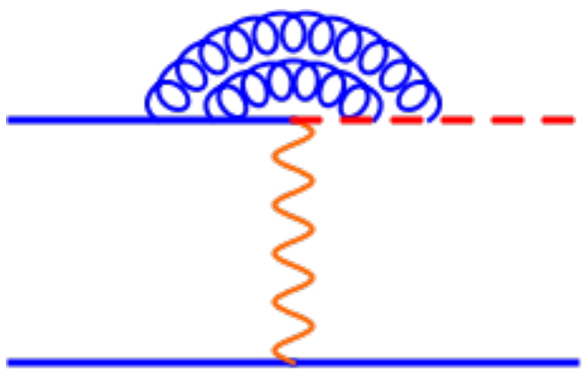
Individual ingredients known for a while.

What prevented from doing the computation?

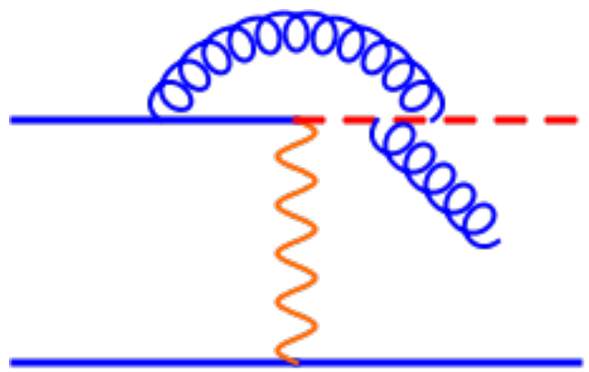
A (generic) procedure to extract IR poles from RV and RR was unknown until very recently

The big problem: extracting IR divergences

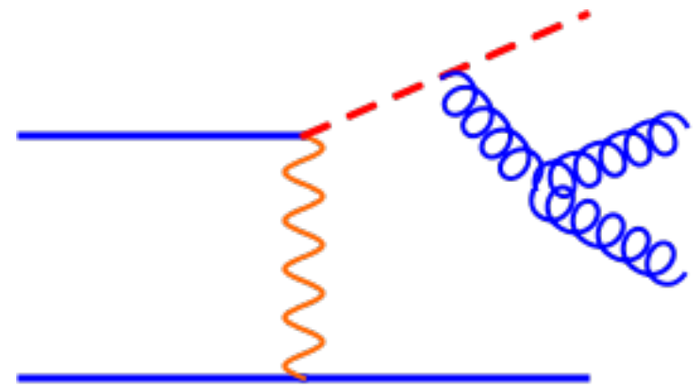
VV



RV



RR



$$\int \left[\frac{\mathrm{vv}_4}{\epsilon^4} + \frac{\mathrm{vv}_3}{\epsilon^3} + \frac{\mathrm{vv}_2}{\epsilon^2} + \frac{\mathrm{vv}_1}{\epsilon} + \mathrm{vv}_0\right] d\phi_2$$

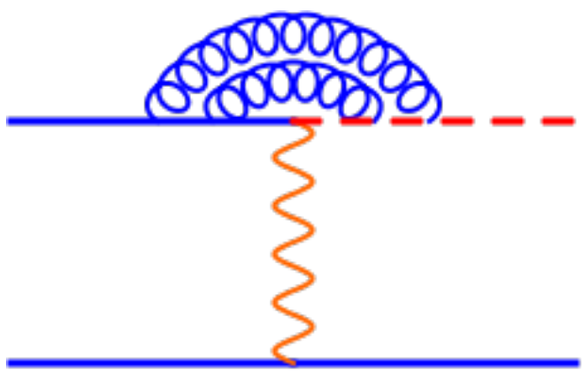
$$\int \left[\mathrm{rr}_0\right] d\phi_4$$

$$\int \left[\frac{\mathrm{rv}_2}{\epsilon^2} + \frac{\mathrm{rv}_1}{\epsilon} + \mathrm{rv}_0\right] d\phi_3$$

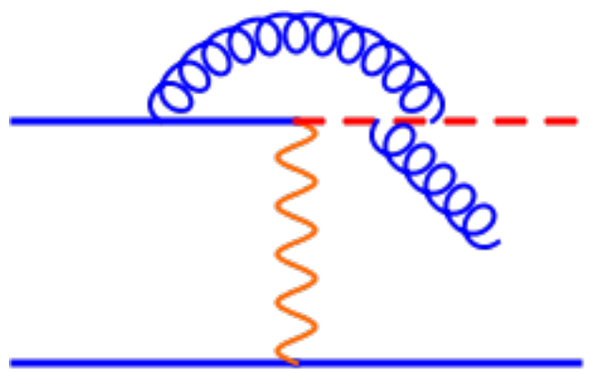
Divergences are "hidden" in the PS, only become manifest after PS integration

The big problem: extracting IR divergences

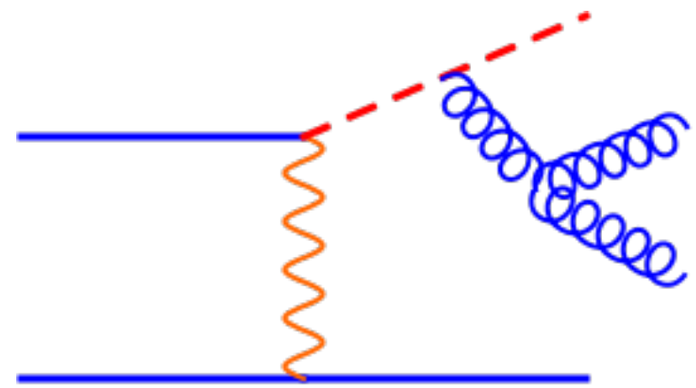
VV



RV



RR



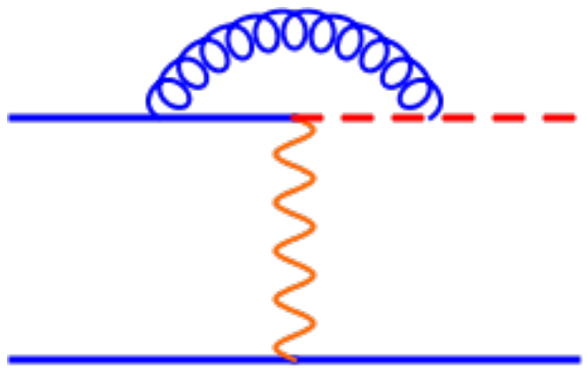
Divergences only manifest after PS integration

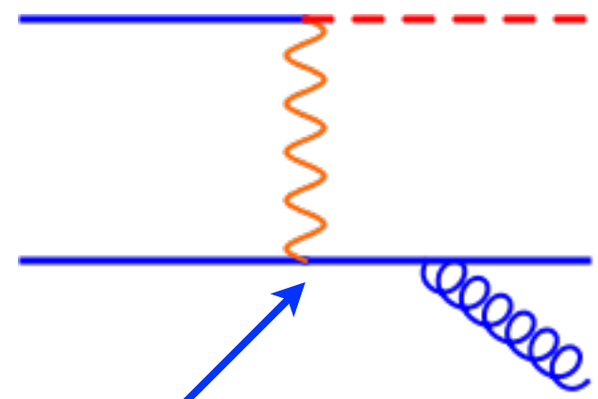
What we want: arbitrary cuts // arbitrary distributions -> fully differential, no PS integration

We need a way to extract IR singularities without performing the full PS integration

One step back: the standard NLO solution

Add and subtract local counterterms





$$\sigma = \sigma_v + \sigma_r \to \int \mathcal{V} d\phi_2 + \int [\mathcal{R} - \mathcal{S}]\, d\phi_3 + \int \mathcal{S} d\phi_3$$

Requirements for the Counterterms:

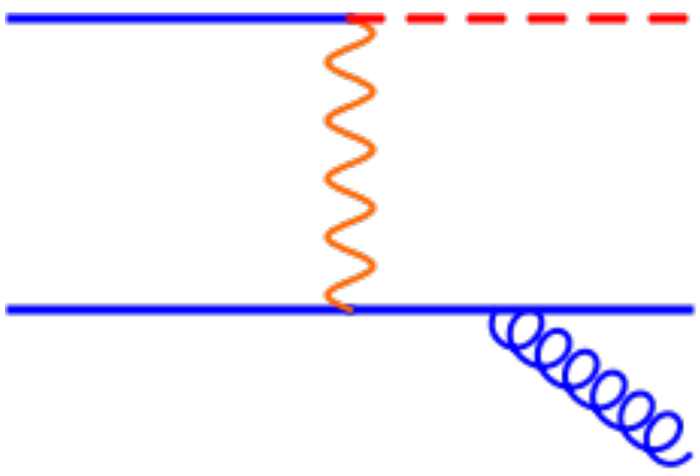
- $\mathcal{S}$ approximates $\mathcal{R}$ in any unresolved configuration -> $\mathcal{R} - \mathcal{S}$ is integrable without further regularization
- $\mathcal{S}$ should be integrable analytically for a generic configuration of the H+jet system -> $\int \mathcal{S} d\phi_3 \to \int [\mathcal{S} d\phi_1] \otimes d\phi_2$ and $[\mathcal{S} d\phi_1]$ exactly cancels the poles in $\mathcal{V}$ (KLN theorem)

The subtraction paradigm: NLO vs NNLO

Nice and universal solution. The problem:

- find appropriate counterterms (~ easy, use universal soft/collinear behavior of QCD)
- INTEGRATE THEM OVER UNRESOLVED PHASE SPACE

NLO



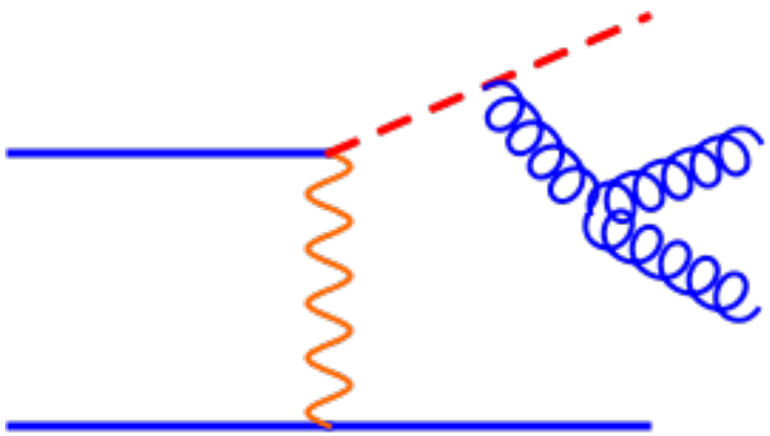
one extra gluon, can be soft and/or collinear

Relatively easy to find $\mathcal{S}$ and compute $[\mathcal{S} d\phi_1]$

[Catani, Seymour (1996)]

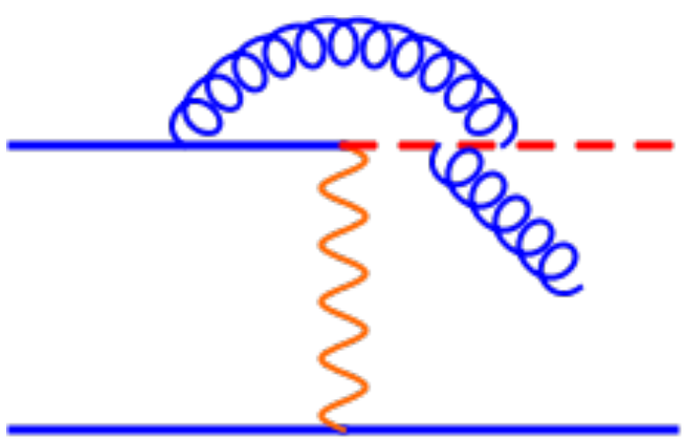
The subtraction paradigm: NLO vs NNLO

NNLO



2 unresolved gluons, many different unresolved configurations

$[\mathcal{S} d\phi_2]$ extremely challenging task



Same configurations as NLO, but complicated analytical structure of the amplitude (branch cuts in soft/collinear variables)

MUCH MORE COMPLICATED THAN NLO

The subtraction paradigm @ NNLO: a long journey

- proof of concept: IR cancellations for e^+e^- -> dijet

  [Gehrmann-de Ridder, Gehrmann, Glover (2004)]

- dijet and trijet in e^+e^- collisions

  [Gehrmann-de Ridder, Gehrmann, Glover, Heinrich (2007)]

- extension to hadron collider:
  DIJET PRODUCTION AT THE LHC (gg only)

  [Gehrmann-de Ridder, Gehrmann, Glover, Pires (2013)]

- processes involving massive particles: work in progress

The subtraction paradigm @ NNLO: nice shortcuts for simpler processes

For specific processes, the subtraction terms can be obtained much more easily from resummation formulas

- qT resummation: Higgs, W/ZH, Drell-Yan, γγ, Zγ
  [Catani, Cieri, De Florian, Ferrera, Grazzini, Kallweit, Torre (2007-2013)]
- SCET + phase space slicing: top decay
  [Gao, Li, Zhu (2012)]

Powerful techniques, but BOUND TO THE EXISTENCE OF A RESUMMATION FORMULA

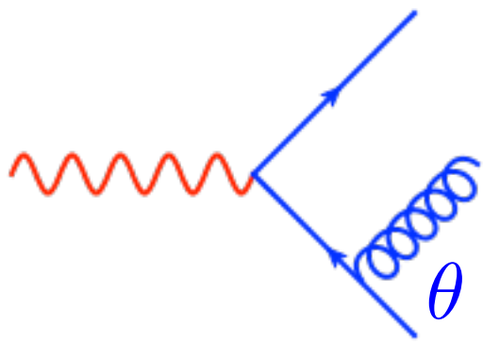
(common lore at the LHC: production of colorless states (?))

The subtraction paradigm @ NNLO:

Can we find a fully generic approach to NNLO computation which does not involve analytic integration over complicated counterterms?

A different approach:
LET THE PHASE SPACE INTEGRATE FOR YOU

Again a simpler NLO example:



$$\frac{d^{d-1}g}{(2\pi)^{d-1}2E_g} \sim \left(1-\cos^2\theta\right)^{-\epsilon} d\cos\theta$$

$$|M|^2 \sim \frac{1}{1-\cos\theta}$$

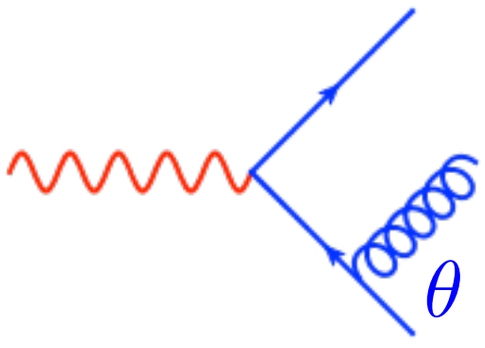
$$\int |M|^2 d\phi_3 \sim \int \frac{d\cos\theta}{(1-\cos\theta)^{1+\epsilon}} f(\cos\theta, \{y\})\{dy\}_2 \xrightarrow{\cos\theta \to 1-2x}$$

$$\int \frac{dx}{x^{1+\epsilon}} f(x,\{y\}) = -\frac{f(0,\{y\})}{\epsilon} + \int_0^1 dx \frac{f(x,\{y\}) - f(0,\{y\})}{x}$$

With this parametrization, (integrated) subtractions arise naturally from the PS, no need for analytic integration

A different approach:
Let the Phase Space Integrate for You

Again a simpler NLO example:



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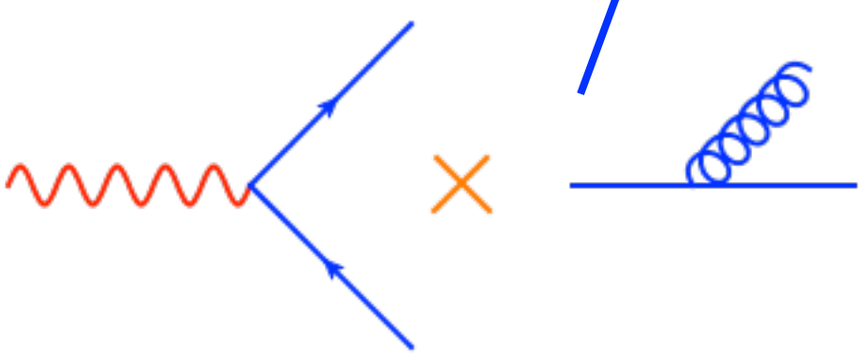
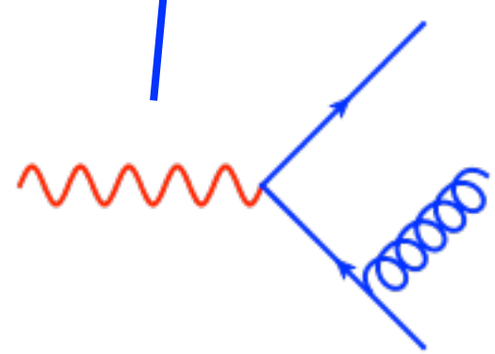




A different approach:
LET THE PHASE SPACE INTEGRATE FOR YOU

Basic idea: find a clever parametrization of the PS which makes

IR SINGULARITIES MANIFEST:

$$\int |M|^2 d\Phi \rightarrow$$

$$\int [|M|^2 x]\{dy\} \frac{dx}{x^{1+\epsilon}} = -\frac{1}{\epsilon} F(0) + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

$$F(x) = \int [|M|^2 x]\{dy\}$$

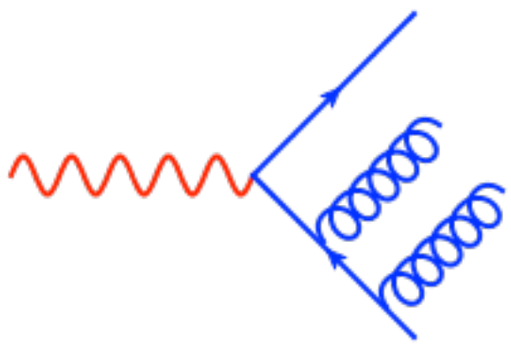
Remap singular denominators on the hypercube

- Singularities are extracted before integration
- Subtraction terms given in terms of reduced (=lower multiplicity) ME and universal soft/collinear factors

NNLO: same spirit, new problems to solve

Overlapping divergences $\longrightarrow$ SECTOR DECOMPOSITION

[Binoth, Heinrich; Anastasiou, Melnikov, Petriello (2004)]



$$|M|^2 \sim \frac{1}{s_{ijk}} = \frac{1}{s_{ij} + s_{ik} + s_{jk}}$$

$$\int |M|^2 d\Phi \sim \int \frac{dx_1 dx_2}{x_1^{1+\epsilon} x_2^{1+\epsilon} (x_1 + x_2)^\epsilon} F(\vec{x}; \{y\})\{dy\}$$

- Sector I: $x_1 > x_2 \rightarrow x_2 = z x_1$

$$\int |M|^2 d\Phi \sim \int \frac{dx_1 dz}{x_1^{1+3\epsilon} z^{1+\epsilon} (1+z)^\epsilon} F(\vec{x}; \{y\})\{dy\}$$

- Sector II: $x_1 < x_2 \rightarrow x_1 = t x_2$

$$\int |M|^2 d\Phi \sim \int \frac{dt dx_2}{t^{1+\epsilon} x_2^{1+3\epsilon} (1+t)^\epsilon} F(\vec{x}; \{y\})\{dy\}$$

SECTOR DECOMPOSITION: pro et contra

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

Subtraction and integrated subtraction terms are for free

(no need for analytic PS integrations)

Powerful tool for fully differential NNLO computations:

- dijet production at LEP [Anastasiou, Melnikov, Petriello (2004)]
- Higgs production at hadron colliders [Anastasiou, Melnikov, Petriello (2005)]
- DY production at hadron colliders [Melnikov, Petriello (2006)]
- bb->H [Bühler, Herzog et al (2012)]

BUT

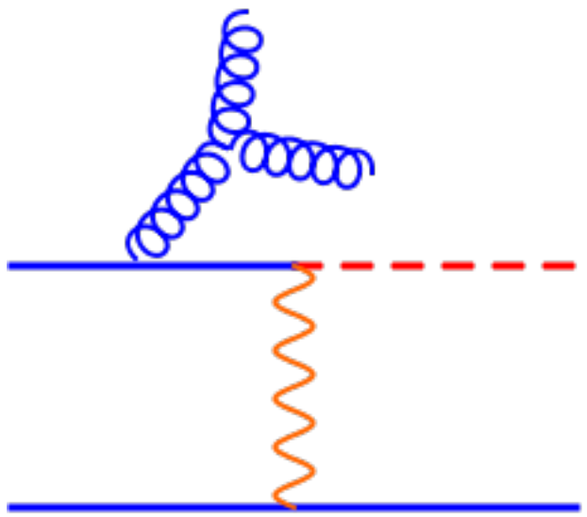
Parametrization become challenging for more complicated processes

Parametrization known only for ONE COLLINEAR DIRECTION

As it is, highly process-dependent framework

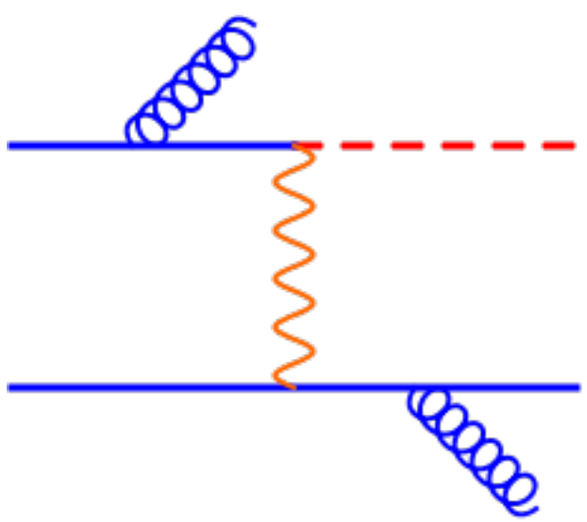
Higgs plus jet: singularity structure

Much more complicated singularity structure. Collinear:

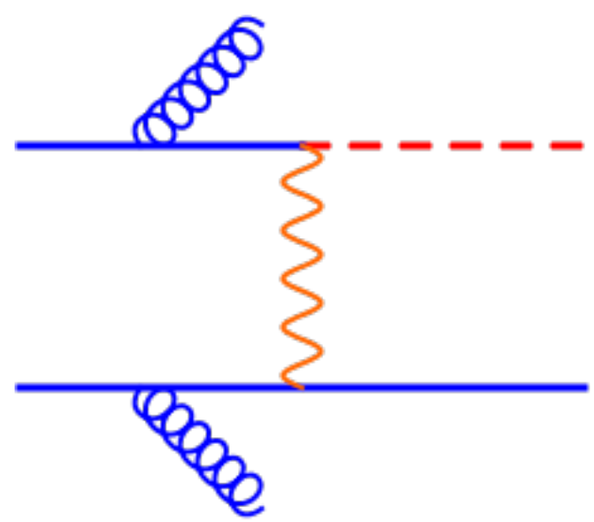


$$\sim \frac{P_{qgg} \cdot |M|^2}{s_{igg}}, \quad \frac{P_{gg} \otimes |M_j|^2}{s_{gg}}$$

x3



x2,



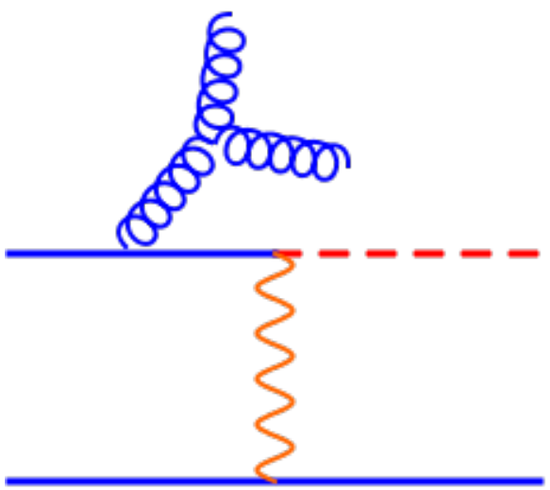
$$\sim \frac{P_{qq} P_{qq} \cdot |M|^2}{s_{ig} s_{jg}}$$

Potential troubles: $s_{1g}, s_{2g}, s_{3g}, s_{gg}, s_{1gg}, s_{2gg}, s_{3gg}$ and combinations

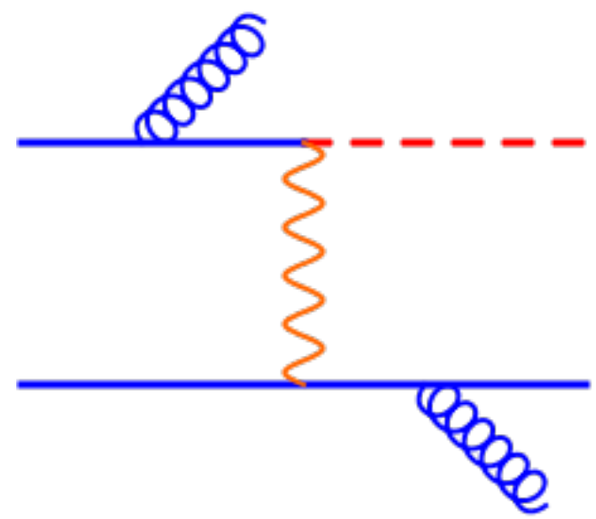
Finding a 'good' global parametrization is (very) hard

Sector-improved subtraction scheme

HOWEVER: collinear sing. cannot occur all together [Czakon (2010)]



Troubles:
s_{igg}, s_{gg} only



Troubles:
s_{ig}, s_{jg} only

Can we make use of it, i.e.
can we single out different collinear directions?

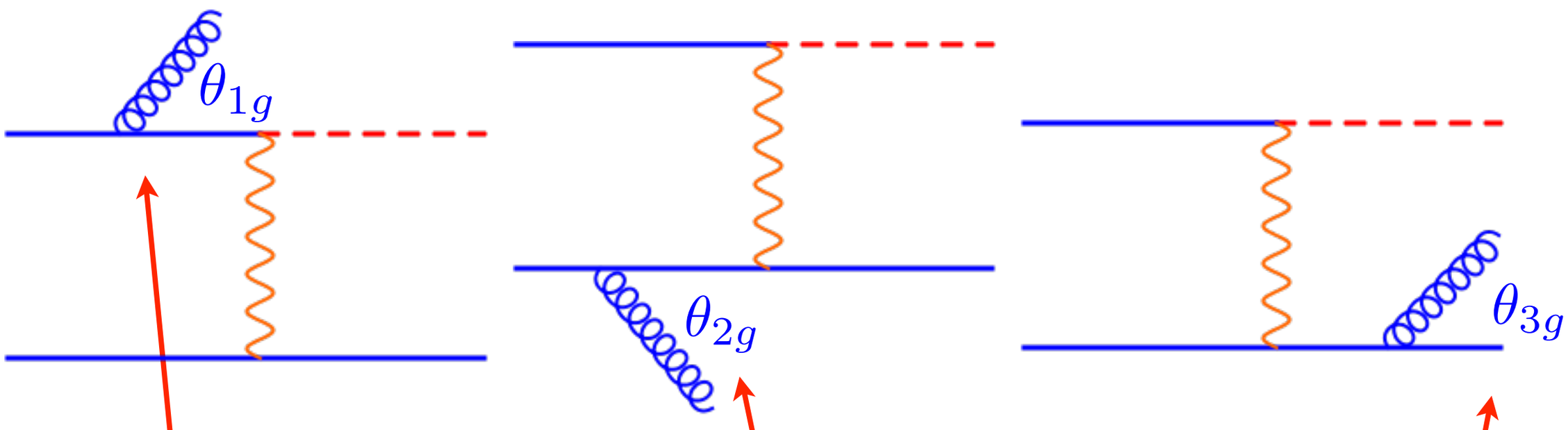
YES, just use the Frixione-Kunszt-Signer (FKS) partitioning
[Czakon (2010)]

$$1 = \sum \Delta^{g_1||i, g_2||j}$$

$$\Delta_s^{g_1||i, g_2||j} \to 0 \text{ when } g_1||p_l,\ g_2||p_m,\ l \neq i,\ m \neq j$$

FKS redux

Again the NLO case [Frixione, Kunszt, Signer (1995)]



$$\rho_i = 1 - \cos\theta_{ig}$$

$$1 = \frac{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2}$$

$$\int |M|^2 \cdot 1 \; d\phi_3 =$$

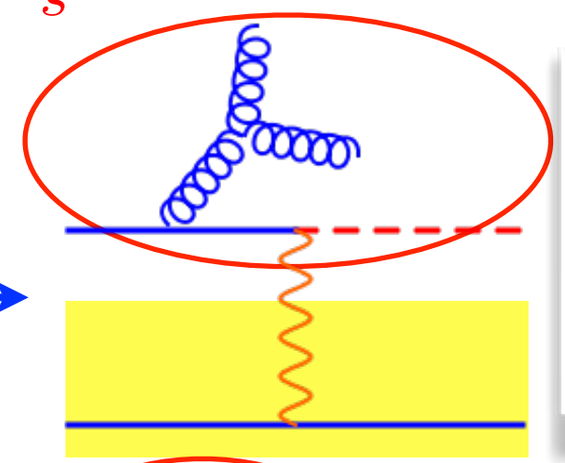
$$\int \frac{|M|^2 \rho_2\rho_3 \; d\phi_3}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2} + \int \frac{|M|^2 \rho_1\rho_3 \; d\phi_3}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2} + \int \frac{|M|^2 \rho_1\rho_2 \; d\phi_3}{\rho_2\rho_3 + \rho_1\rho_3 + \rho_1\rho_2}$$

Sector-improved subtraction scheme

Sector decomposition + FKS

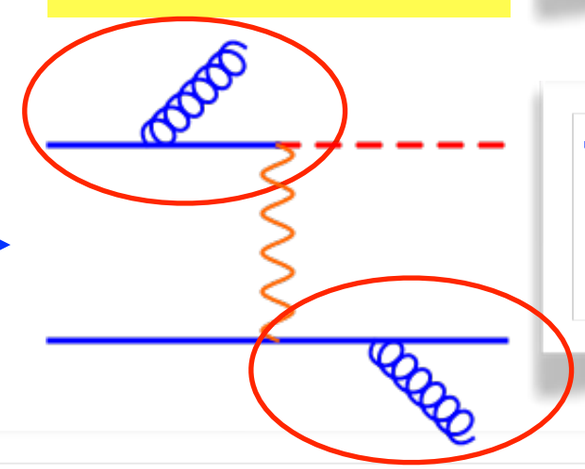
$$\int |M|^2 d\phi = \sum_s \int |M|^2 d\phi \Delta_s^{g_1||i,g_2||j}$$

$$\int |M|^2 d\phi \Delta^{g_1||1,g_2||1} \longrightarrow$$



Single collinear direction
~ parametrization of
ggH, DY, $e^+e^- \rightarrow$ dijets

$$\int |M|^2 d\phi \Delta^{g_1||1,g_2||3} \longrightarrow$$



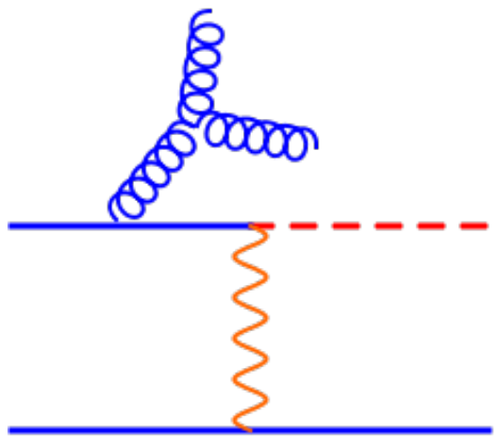
Two (~uncorrelated) dir.
~ NLO^2

No matter how complicated the process is,
it can be reduced to the sum of individual contributions. For each of them, we know a sector decomposition-friendly PS parametrization
[Czakon (2010)]

Sector-improved subtractio: single-top

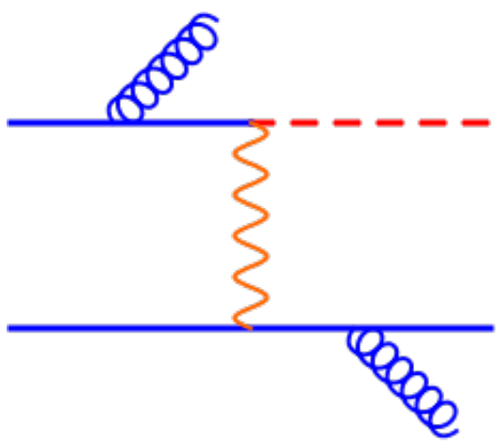
Worked-out details for RR: [Czakon (2010)]

(Although we use a slightly different parametrization and sector definition)



Three triple-collinear partitions

Each: 5 sectors



Six double-collinear (energy ordering)

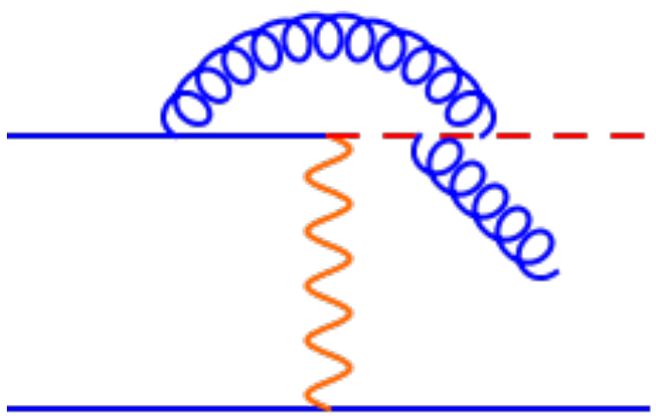
No sector decomposition required

$$\mathrm{RR}_i = \int F_i(x_1, x_2, x_3, x_4, \{y\}) \prod \frac{dx_i}{x_i^{1+a_i\epsilon}} \{dy\} =$$

$$\int \{dy\} \left\{ \frac{F_i(\vec{0}, \{y\})}{a\epsilon^4} + \frac{1}{\epsilon^3} \left[\left(\frac{F_i(x_1, 0, 0, 0, \{y\}) - F_i(\vec{0}, \{y\})}{b x_1} \right) dx_1 + \ldots \right] + \ldots \right\}$$

Sector-improved subtraction: single-top

Worked-out details for RV: [Boughezal, Melnikov, Petriello (2011)]



Three collinear partitions
(same of NLO)

Phase-space is simple (same of NLO), but amplitudes have non trivial branch-cuts

$$\mathrm{RV}_i = \int \{dy\} \frac{dx_1}{x_1^{1+2\epsilon}} \frac{dx_2}{x_2^{1+\epsilon}} \left(F_{i,1} + (x_1^2 x_2)^{-\epsilon} F_{i,2} + \cancel{x_1^{-2\epsilon} F_{i,3}} \right) =$$

$$= \int \{dy\} \left[\frac{A}{\epsilon^4} + \frac{B}{\epsilon^3} + \frac{C}{\epsilon^2} + \frac{D}{\epsilon} + E \right]$$

Sector-improved subtraction: building blocks

Recall the general structure: $F(x) = \int [|M|^2 x]\{dy\}$

$$\int |M|^2 d\phi = \frac{F(0)}{\epsilon} + \int dx \frac{F(x) - F(0)}{x} + \ldots$$

We need to provide

- $F(\vec{x}; \{y\})$: fully-resolved matrix element (RR and RV)
- $\lim_{x_i \to 0} F(\vec{x}; \{y\})$: matrix element in a singular configuration

$\downarrow$

$\lim_{x_i \to 0} F(\vec{x}; \{y\})$: reduced (=lower multiplicity) matrix element times universal eikonals / splitting functions

[Catani, Grazzini (1998, 2000); Kosower, Uwer (1999)]

At the end: ~ 100 different configurations to consider

Single-top: building blocks

Because of gluon spin correlations, we chose to work in full CDR

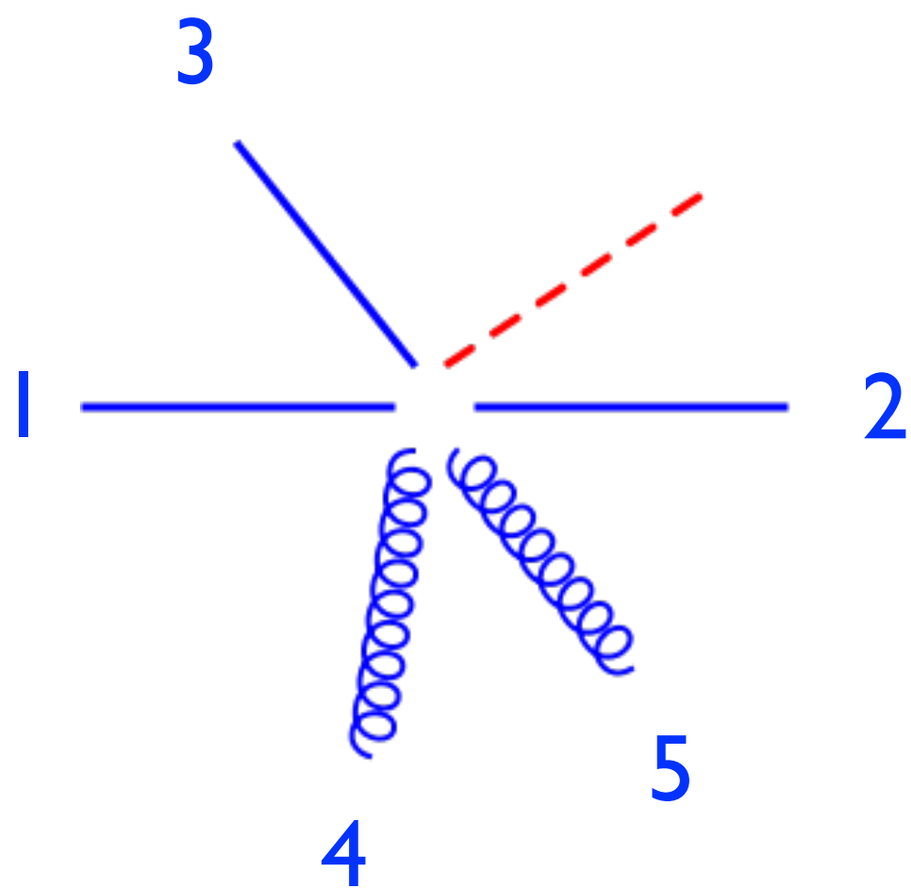
Apart from eikonals/splitting functions, we require

- tree-level t+3j
- tree-level t+2j up to $\mathcal{O}(\epsilon)$ (although $\mathcal{O}(\epsilon)$ cancels out)
- tree-level t+j
- one-loop t+2j [Campbell et al (2009)]
- one-loop t+1j up to $\mathcal{O}(\epsilon^2)$ (although see [Weinzierl (2011)])
- two-loop t+1j [Bonciani, Ferroglia (2008); Beneke et al (2009)]
- renormalization, collinear subtractions

Amplitudes are evaluated near to singular configurations:
have to be very stable (and possibly fast) →
ANALYTIC RESULTS, SPINOR-HELICITY FORMALISM

EXTREMELY GRATEFUL TO MCFM FOR PROVIDING EXCELLENT AMPLITUDES ALREADY AS A FORTRAN CODE!

A glimpse into technical details: D-dim angles



$p_{12} = (x,0,0,x)$
$p_3 = (x,x,0,x)$
$p_4 = (x,x,x,x)$
$p_5 = (x,x,x,x;\, x)$

Fundamental for proper spin correlations
in the collinear limit (the price of locality...)

$$|M|^2 \to P^{\mu\nu}\widetilde{|M|}^2_{\mu\nu} \qquad \langle P^{\mu\nu}\widetilde{|M|}^2_{\mu\nu}\rangle_{d=4} \neq P_{AP}\widetilde{|M|}^2$$

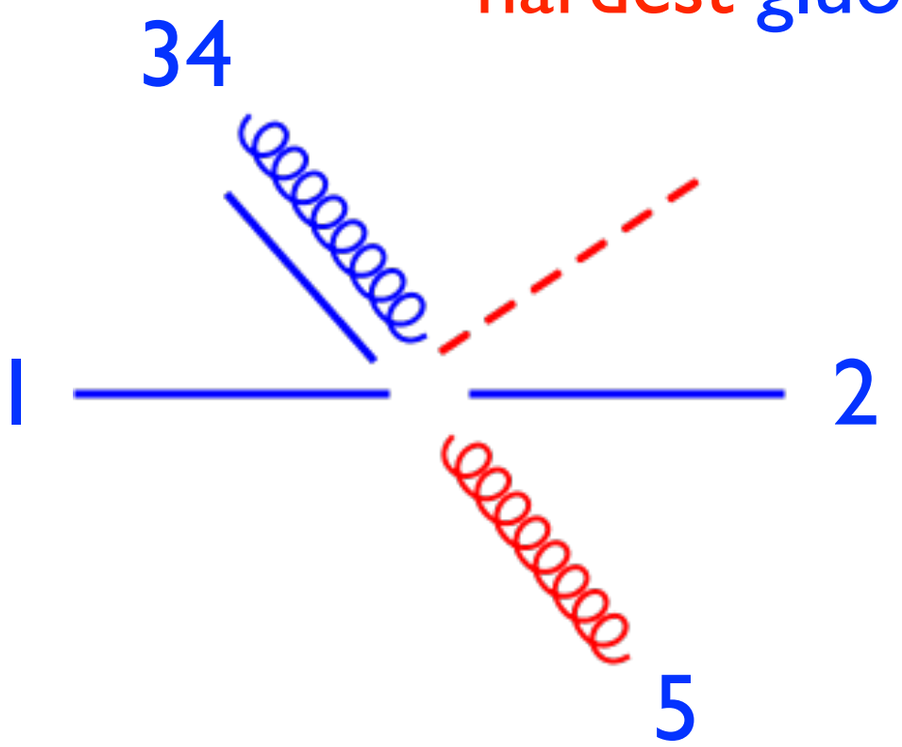
NLO: force correct behavior by hand. NNLO: more involved correlations; soft/collinear FKS dampings do not collapse

Should we define D-dim observables and compute amplitudes with D-dim momenta?

A trick: rotate such that the softest gluon has 5d

Then, only potential problem:

hardest gluon collinear, softest gluon generic



One direction is lost ->

ROTATE AGAIN

$p_{12} = (x,0,0,x)$
$p_3 = (x,x,0,x)$
$p_4 = (x,x,0,x)$
$p_5 = (x,x,x,x;\, x)$

$\xrightarrow{\mathcal{R}_{y5}}$

$p_{12} = (x,0,0,x)$
$p_3 = (x,x,0,x)$
$p_4 = (x,x,0,x)$
$p_5 = (x,x,x,x;\, 0)$

D-dim momenta always decouple from reduced ME and observables, at any stage of the computation

Do NOT decouple from eikonals/splitting functions

D-dim angles: spinor-helicity in higher dimension

Because of gluon spin correlations, we are forced to work in full CDR

We need $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2)$ for tree and 1-loop amplitudes

Naively: just square a bunch of diagrams.

But we want to KEEP TRACK OF SPIN CORRELATIONS

Dimensional reconstruction: $\mathcal{O}(\epsilon)$ from spinor-helicity in higher D

Scalar-like gluons with polarization vectors pointing in the D=5,6 subspaces

Similar to what is done for 1-loop in D-dimensional unitarity

- although slightly more tricky if quarks are around [$\bar{u}\gamma^\mu\hat{p}_1\ldots\hat{p}_n\gamma^\mu v$ (1-loop) vs $\bar{u}\gamma^\mu\hat{p}_1\ldots\hat{p}_k v$ (here)]
- and analytic-friendly

More complicated processes:
$|M|^2$ vs higher dimensional spinor helicity

$$\sum_{pol} |M|^2$$

```
File Edit Options Buffers Tools F90 Help
      amp2_ep =  + ep * (  - 58.0_dp - 10.0_dp*s12**(-2)*s13**(-1)*s14*s23*&
      s24**(-1)*s34**2 - 15.0_dp*s12**(-2)*s13**(-1)*s14*s23*s34 + &
      5.0_dp*s12**(-2)*s13**(-1)*s14*s23*s34**2*s124**(-1) - 5.0_dp&
      *s12**(-2)*s13**(-1)*s14*s23*s24 + 5.0_dp*s12**(-2)*s13**(-1)&
      *s14*s23*s24*s34*s124**(-1) - 13.0_dp*s12**(-2)*s13**(-1)*s14&
      *s23**2*s24**(-1)*s34 + 7.0_dp*s12**(-2)*s13**(-1)*s14*s23**2&
      *s24**(-1)*s34**2*s123**(-1) - 10.0_dp*s12**(-2)*s13**(-1)&
      s14*s23**2 + 12.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s34*&
      s123**(-1) + 8.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s34&
      s124**(-1) - 8.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s34*&
      s134**(-1) - 2.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s34**2*&
      s123**(-1)*s124**(-1) + 5.0_dp*s12**(-2)*s13**(-1)*s14*s23**2&
      *s24*s123**(-1) + 5.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s24*&
      s124**(-1) - 5.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s24*&
      s134**(-1) - 2.0_dp*s12**(-2)*s13**(-1)*s14*s23**2*s24*s34*&
      s123**(-1)*s124**(-1) )
      amp2_ep = amp2_ep + ep * ( 3.0_dp*s12**(-2)*s13**(-1)*s14*&
      s23**2*s24*s34*s124**(-1)*s134**(-1) - 3.0_dp*s12**(-2)*&
      s13**(-1)*s14*s23**3*s24**(-1) + 7.0_dp*s12**(-2)*s13**(-1)*&
      s14*s23**3*s24**(-1)*s34*s123**(-1) + 7.0_dp*s12**(-2)*&
      s13**(-1)*s14*s23**3*s123**(-1) + 3.0_dp*s12**(-2)*s13**(-1)*&
      s14*s23**3*s124**(-1) - 3.0_dp*s12**(-2)*s13**(-1)*s14*s23**3&
      *s134**(-1) - 2.0_dp*s12**(-2)*s13**(-1)*s14*s23**3*s34*&
      s123**(-1)*s124**(-1) + 5.0_dp*s12**(-2)*s13**(-1)*s14*s23**3&
      *s34*s123**(-1)*s134**(-1) - 2.0_dp*s12**(-2)*s13**(-1)*s14*&
      s23**3*s24*s123**(-1)*s124**(-1) + 5.0_dp*s12**(-2)*s13**(-1)&
      *s14*s23**3*s24*s123**(-1)*s134**(-1) + 3.0_dp*s12**(-2)*&
      s13**(-1)*s14*s23**3*s24*s124**(-1)*s134**(-1) - 13.0_dp*&
      s12**(-2)*s13**(-1)*s14**2*s23*s24**(-1)*s34 + 7.0_dp*&
      s12**(-2)*s13**(-1)*s14**2*s23*s24**(-1)*s34**2*s124**(-1) - &
      8.0_dp*s12**(-2)*s13**(-1)*s14**2*s23 + 12.0_dp*s12**(-2)*&
      s13**(-1)*s14**2*s23*s34*s124**(-1) )
      amp2_ep = amp2_ep + ep * (  - 10.0_dp*s12**(-2)*s13**(-1)*&
      s14**2*s23**2*s24**(-1) + 10.0_dp*s12**(-2)*s13**(-1)*s14**2*&
      s23**2*s24**(-1)*s34*s123**(-1) + 10.0_dp*s12**(-2)*s13**(-1)&
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      s13**(-1)*s14**2*s23**2*s24**(-1)*s34**2*s123**(-1)*&
      s124**(-1) + 8.0_dp*s12**(-2)*s13**(-1)*s14**2*s23**2*&
      s123**(-1) + 12.0_dp*s12**(-2)*s13**(-1)*s14**2*s23**2*&
      s124**(-1) - 7.0_dp*s12**(-2)*s13**(-1)*s14**2*s23**2*&
      s134**(-1) - 6.0_dp*s12**(-2)*s13**(-1)*s14**2*s23**2*s34*&
      s123**(-1)*s124**(-1) + 3.0_dp*s12**(-2)*s13**(-1)*s14**2*&
      s23**2*s34*s124**(-1)*s134**(-1) - 2.0_dp*s12**(-2)*s13**(-1)&
      *s14**2*s23**2*s24*s34**(-1)*s134**(-1) + 7.0_dp*s12**(-2)*&
      s13**(-1)*s14**2*s23**3*s24**(-1)*s123**(-1) + 3.0_dp*&
      s12**(-2)*s13**(-1)*s14**2*s23**3*s24**(-1)*s124**(-1) - 4.0_dp&
-UU-:----F1  out_gggg_me2colsum.f90   Top L1     (F90)----------------------
```

...

```
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      s124**(-1)*s234**(-1) - 5.0_dp*s12**3*s23**(-1)*s34**(-1)&
      s134**(-1) - 4.0_dp*s12**3*s23**(-1)*s123**(-1)*s134**(-1) - &
      6.0_dp*s12**3*s23**(-1)*s134**(-1)*s234**(-1) + 4.0_dp*s12**3&
      *s23**(-1)*s24*s34**(-1)*s134**(-1)*s234**(-1) - 2.0_dp*&
      s12**3*s24**(-2)*s34*s124**(-1)*s234**(-1) - 5.0_dp*s12**3*&
      s24**(-1)*s34**(-1)*s134**(-1) )
      amp2_ep = amp2_ep + ep * (  - 4.0_dp*s12**3*s24**(-1)*&
      s124**(-1)*s134**(-1) - 6.0_dp*s12**3*s24**(-1)*s134**(-1)*&
      s234**(-1) + 16.0_dp*s12**3*s34**(-1)*s134**(-1)*s234**(-1)&
      + 4.0_dp*s12**3*s23*s24**(-1)*s34**(-1)*s134**(-1)*&
      s234**(-1) - 7.0_dp*s12**3*s13*s14**(-1)*s23**(-1)*s34**(-1)*&
      s134**(-1) - 4.0_dp*s12**3*s13*s14**(-1)*s23**(-1)*s134**(-1)&
      *s234**(-1) + 8.0_dp*s12**3*s13*s14**(-1)*s23**(-1)*s24*&
      s34**(-1)*s134**(-1)*s234**(-1) - 2.0_dp*s12**3*s13*s14**(-1)&
      *s24**(-1)*s124**(-1)*s134**(-1) - 2.0_dp*s12**3*s13*&
      s14**(-1)*s24**(-1)*s134**(-1)*s234**(-1) + 4.0_dp*s12**3*s13&
      *s14**(-1)*s34**(-1)*s134**(-1)*s234**(-1) - 2.0_dp*s12**4*&
      s13**(-1)*s14**(-1)*s23**(-1)*s24**(-1) - 2.0_dp*s12**4*&
      s13**(-1)*s23**(-1)*s134**(-1)*s234**(-1) - 4.0_dp*s12**4*&
      s13**(-1)*s24**(-1)*s134**(-1)*s234**(-1) - 4.0_dp*s12**4*&
      s14**(-1)*s23**(-1)*s134**(-1)*s234**(-1) - 2.0_dp*s12**4*&
      s14**(-1)*s24**(-1)*s134**(-1)*s234**(-1) )
-UU-:----F1  out_gggg_me2colsum.f90   39% L5890   (F90)----------------------
```

Lots of $\sim 1/s_{ij}^2$ spurious singularities

$\mathcal{O}(\epsilon)$: 5890 lines

Typical amplitude from spinor-helicity in higher dimension

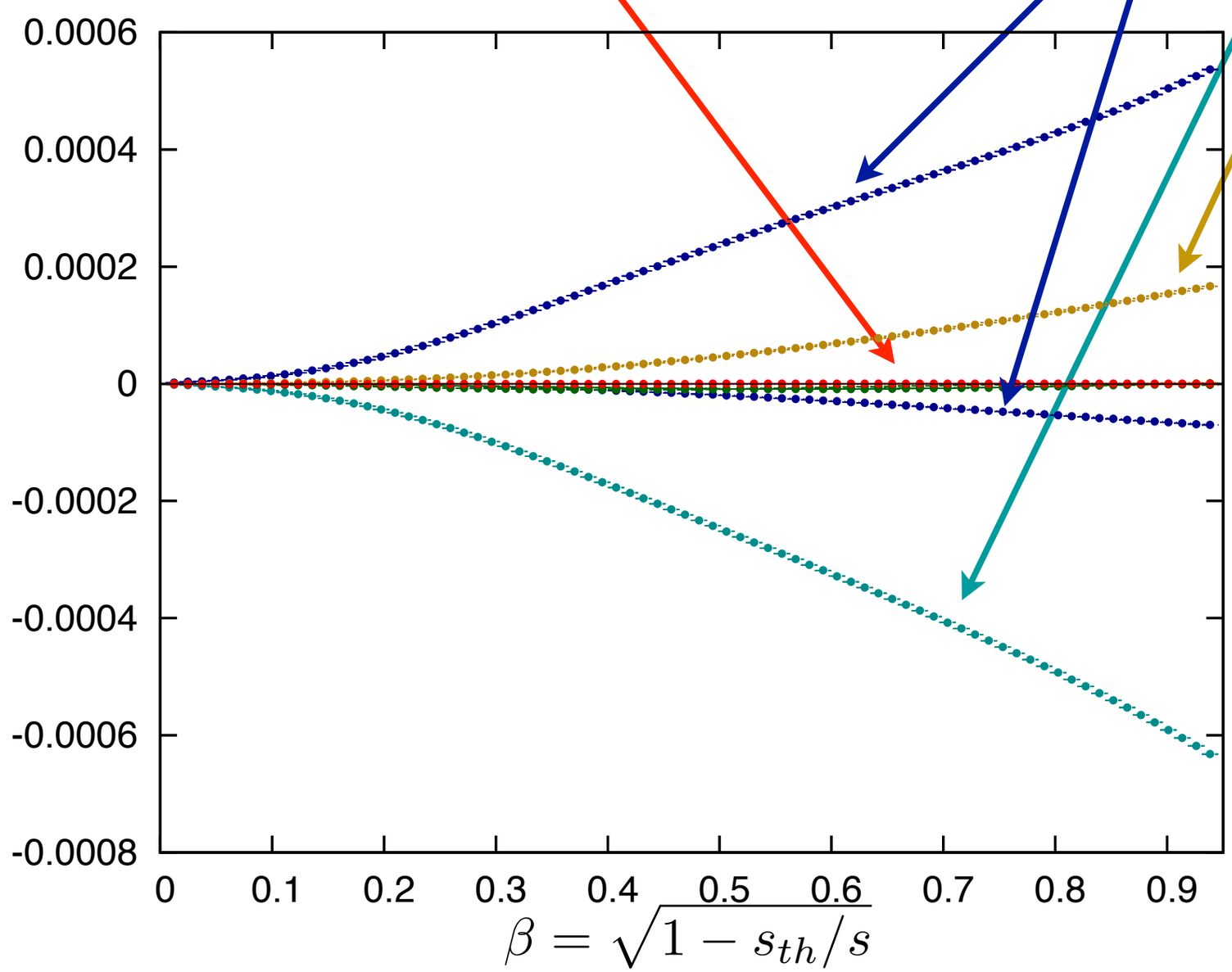
$$A(1^s, 2^+, 3^s, 4^+) = -\frac{\langle 1|p_h|4]\langle 3|p_h|4]}{s_{123}\langle 12\rangle\langle 23\rangle} + \frac{\langle 1|p_h|2]\langle 3|p_h|2]}{s_{134}\langle 14\rangle\langle 34\rangle} + \frac{m_h^2\langle 13\rangle^2}{\langle 12\rangle\langle 14\rangle\langle 23\rangle\langle 34\rangle},$$

$$A(1^-_{\bar{q}}, 2^+_q, 3^s, 4^{s'}) = \frac{\langle 23\rangle}{2\langle 13\rangle}\left(1 - \frac{m_h^2}{s_{123}}\right) - \frac{[41]}{2[42]}\left(1 - \frac{m_h^2}{s_{124}}\right)$$

t-channel single-top at NNLO: (preliminary) results

Checks: poles cancellation

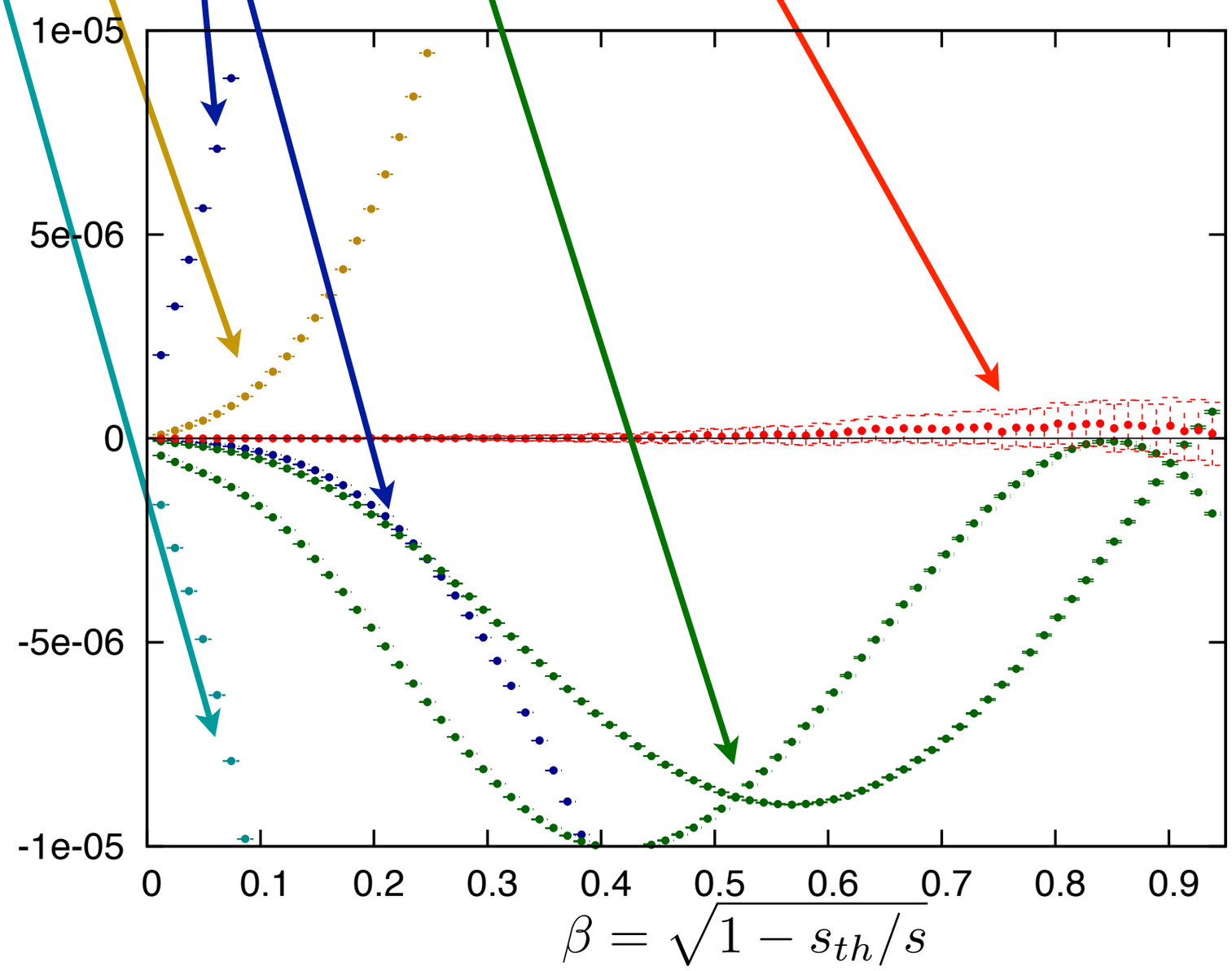
NUMERICAL CANCELLATION between
renormalization and coll. couterterms, RR, RV, VV



$1/\epsilon$ poles, summing individual contributions

Checks: poles cancellation

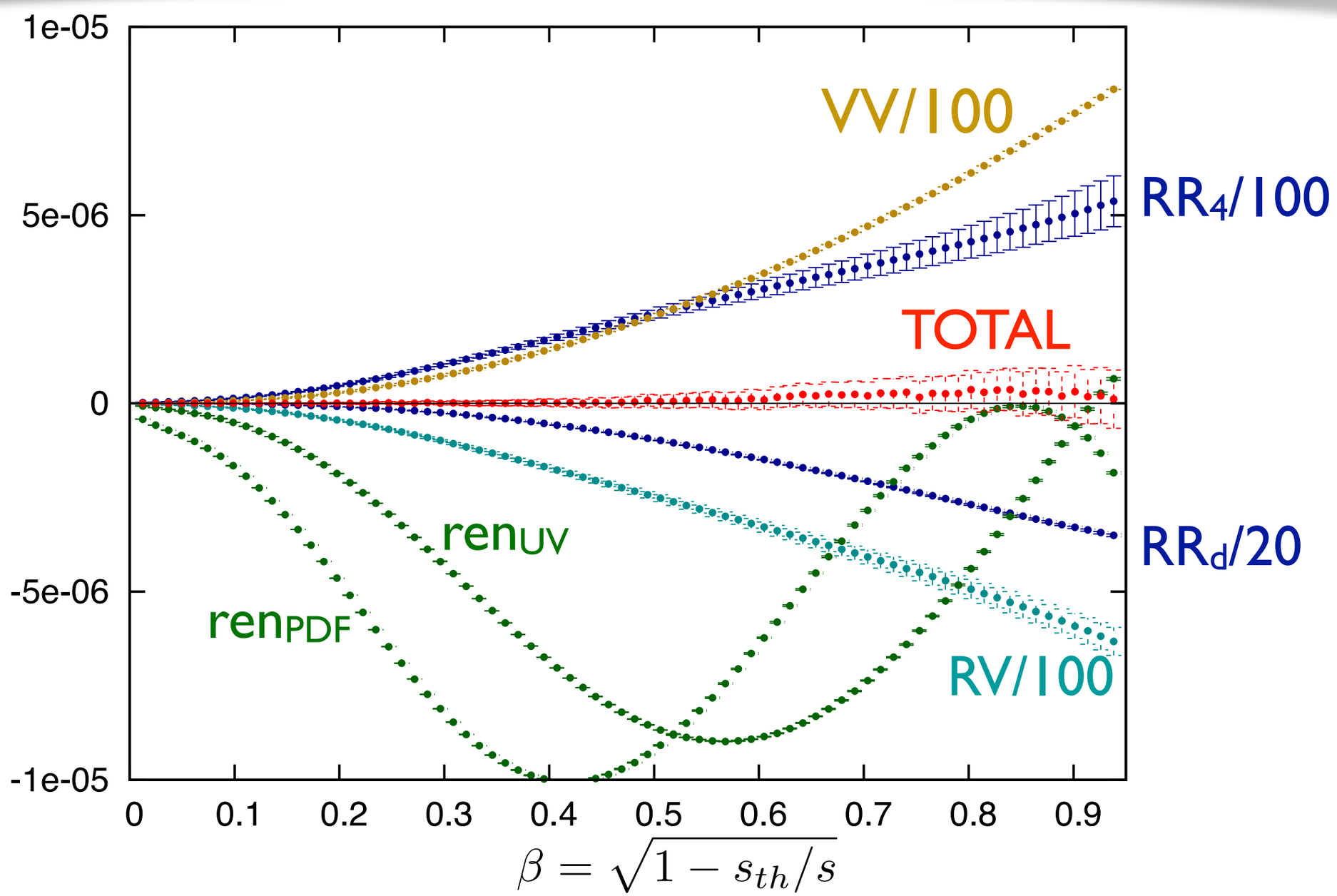
NUMERICAL CANCELLATION between
RV, VV, RR, renormalization and coll. couterterms



$1/\epsilon$ poles, summing individual contributions

Checks: poles cancellation

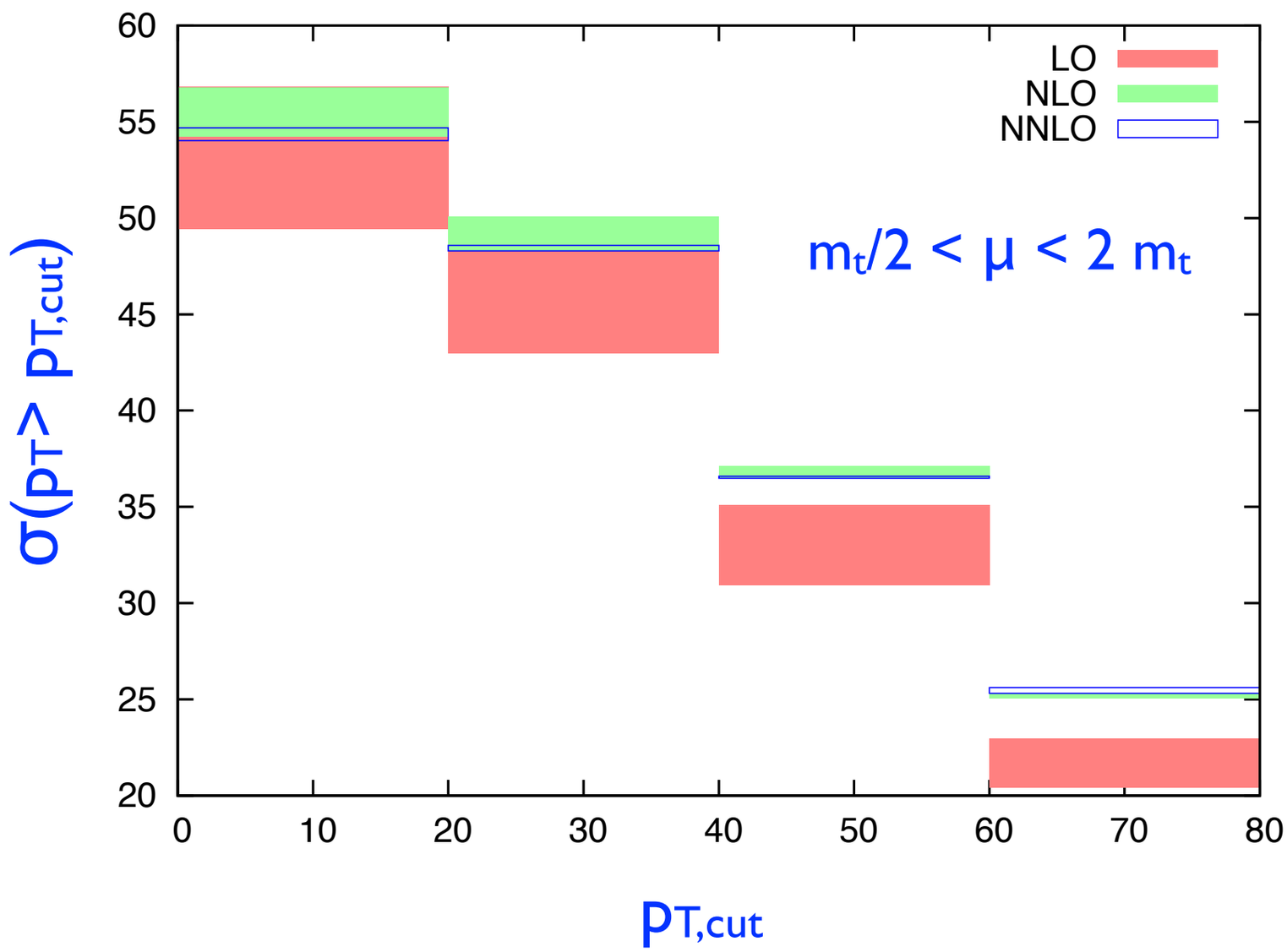
NUMERICAL CANCELLATION between
RV, VV, RR, renormalization and coll. couterterms



$1/\epsilon$ poles, summing individual contributions

Single-top @ NNLO

8 TeV LHC, m_t = 173.2 GeV



$$\sigma_{\rm LO} = 53.8^{+3.0}_{-4.3}\ \ {\rm pb}$$

$$\sigma_{\rm NLO} = 55.1^{+1.6}_{-0.9}\ \ {\rm pb}$$

$$\sigma_{\rm NNLO} = 54.2^{+0.5}_{-0.2}\ {\rm pb}$$

$$\sigma_{t,{\rm LO}}/\sigma_{\bar{t},{\rm LO}} = 1.85$$

$$\sigma_{t,{\rm NLO}}/\sigma_{\bar{t},{\rm NLO}} = 1.83$$

$$\sigma_{t,{\rm NNLO}}/\sigma_{\bar{t},{\rm NNLO}} = 1.83$$

Conclusions

- We presented results for (DIS-like) t-channel single-top
  - O(82%) of the total cross-section at the LHC
  - Neglected term color-suppressed -> subdominant
- Result needed for reliable ~ percent theoretical control
  - Already now, LHC can measure the total cross-section at the ~10 % level (Tevatron final combinations: ~ 20%)
- Corrections are small -> perturbative expansion behaving well
- NNLO dependence on μ very small -> use for PDFs?

Conclusions

NNLO QCD for generic fully exclusive observables is feasible

- In theory:
  - The antenna method [Gehrmann-de Ridder, Gehrmann, Glover (2004)]
  - FKS+sector decomposition [Czakon (2010), Boughezal, Melnikov, Petriello (2011)]
  - Semi-analytic subtraction (for e^+e^-) [Bolzoni, Del Duca, Somogyi, Trocsanyi (2010)
- In practice:
  - $t\bar{t}$ production [Czakon, Fiedler, Mitov (2013)]
  - dijet production [Currie, Gehrmann-De Ridder, Gehrmann, Glover, Pires (2013,2014)
  - Higgs plus 1 jet [Boughezal, FC, Melnikov, Petriello, Schulze (2013)]

Outlook

- For real phenomenology: include top-quark decay
  - spin information is always traced -> simple to interface
  - we know top-decay@NNLO [Brucherseifer, FC, Melnikov (2013)]
  - missing step: combining everything
- Compute interesting differential distributions
  - lepton/jet distributions, angular correlations…
- Results with ATLAS/CMS setup

INTERESTING LHC PHENOMENOLOGY

BEHIND THE CORNER!

Thank you for your attention!