Towards event generation at NNLO

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Rudolf Peierls Centre for Theoretical Physics, University of Oxford

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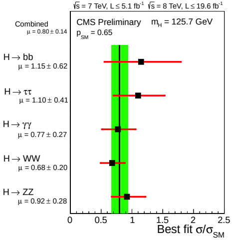
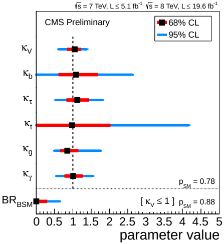
Status after LHC “run I”

- Scalar at 125 GeV found, study of properties begun





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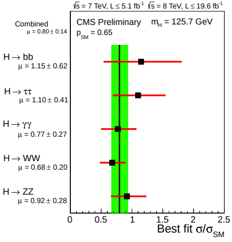
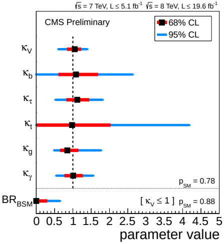
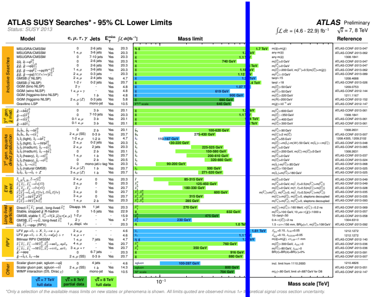
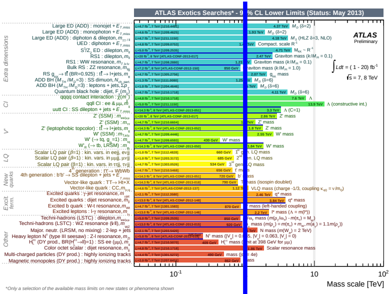




- In general no smoking-gun signal of new-physics





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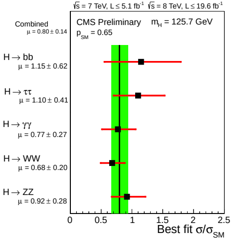

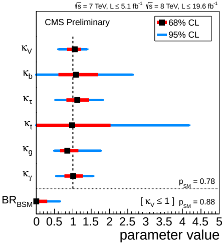

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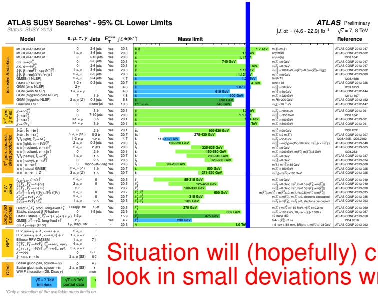
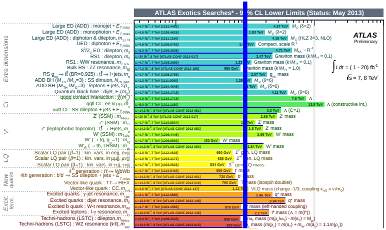
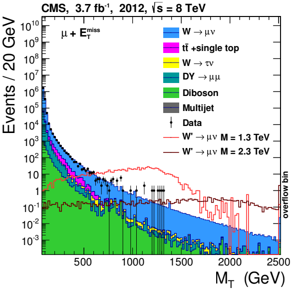
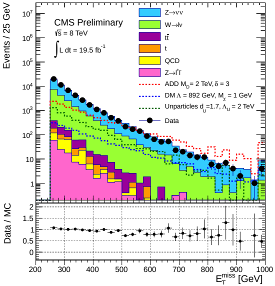


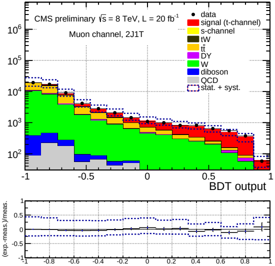


Situation will (hopefully) change at 14 TeV. If not, then we have to look in small deviations wrt SM: “precision physics”.

- examples of strategies to find new-physics / isolate SM processes:



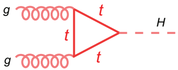




- Higgs discovery belongs to 1, but Higgs characterization requires theory inputs (rates,shapes,binned x-sections,...)
- For 2 and 3, we need to control as much as possible QCD effects (i.e. rates and shapes, and also uncertainties!)
- Some analysis techniques (e.g. 3) heavily relies on using MC event generators to separate signal and backgrounds
- In general, MC enter almost everywhere in LHC searches

Event generators: what they are?

ideal world: high-energy collision and detection of elementary particles



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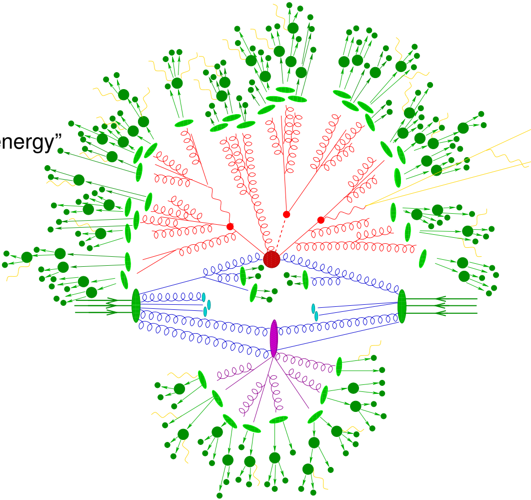
real world:

- collide non-elementary particles
- we detect e, μ, γ,hadrons, "missing energy"



[sherpa's artistic view]

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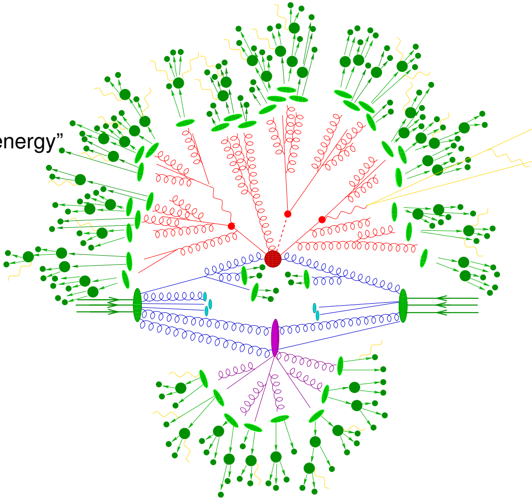
  - realistically
  - precisely
  - from first principles



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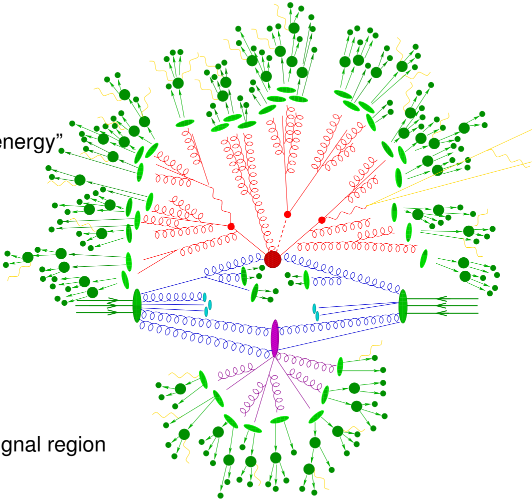
- full event simulation needed to:
  - compare TH and data
  - estimate how backgrounds affect signal region
  - test analysis strategies



[sherpa's artistic view]

Event generators: what's the output?

- fully exclusive simulation: momenta of all outgoing leptons and hadrons:

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY
31	NU_E	12	1	29	22	0	0	60.53	37.24	-1185.0	1187.1
32	E+	-11	1	30	22	0	0	-22.80	2.59	-232.4	233.6
148	K+	321	1	109	9	0	0	-1.66	1.26	1.3	2.5
151	PI0	111	1	111	9	0	0	-0.01	0.05	11.4	11.4
152	PI+	211	1	111	9	0	0	-0.19	-0.13	2.0	2.0
153	PI-	-211	1	112	9	0	0	0.84	-1.07	1626.0	1626.0
154	K+	321	1	112	9	0	0	0.48	-0.63	945.7	945.7
155	PI0	111	1	113	9	0	0	-0.37	-1.16	64.8	64.8
156	PI-	-211	1	113	9	0	0	-0.20	-0.02	3.1	3.1
158	PI0	111	1	114	9	0	0	-0.17	-0.11	0.2	0.3
159	PI0	111	1	115	18	0	0	0.18	-0.74	-267.8	267.8
160	PI-	-211	1	115	18	0	0	-0.21	-0.13	-259.4	259.4
161	N	2112	1	116	23	0	0	-8.45	-27.55	-394.6	395.7
162	NBAR	-2112	1	116	23	0	0	-2.49	-11.05	-154.0	154.4
163	PI0	111	1	117	23	0	0	-0.45	-2.04	-26.6	26.6
164	PI0	111	1	117	23	0	0	0.00	-3.70	-56.0	56.1
167	K+	321	1	119	23	0	0	-0.40	-0.19	-8.1	8.1
186	PBAR	-2212	1	130	9	0	0	0.10	0.17	-0.3	1.0

- At some level, this enters in almost all experimental analyses.
  $\hookrightarrow$ The more precise we are, the smaller the impact of uncertainties on measured quantities

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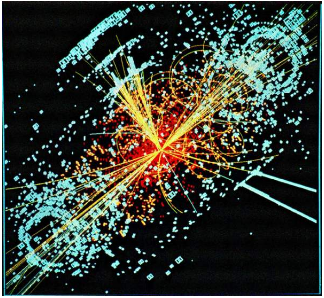
- hard scattering: QCD, EW, BSM (fixed order) $\mu \approx Q \gg \Lambda_{\text{QCD}}$
- multiple soft and collinear emissions $\Lambda_{\text{QCD}} < \mu < Q$
  $\hookrightarrow$ pQCD (parton shower approximation)
- large distance: hadronization $\mu \approx \Lambda_{\text{QCD}}$
  $\hookrightarrow$ non-perturbative QCD $\rightarrow$ phenomenological models, tuned on data.

Plan of the talk

- basics:
  - parton showers (LOPS)
  - fixed-order (NLO)
- matching NLO and PS (NLOPS)
  - POWHEG
- NLOPS merging
  - Multiscale improved NLO (MiNLO)
- matching NNLO with PS (NNLOPS)
  - Higgs production at NNLOPS



parton showers and fixed order

Parton showers I

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- need to simulate production of many quarks and gluons

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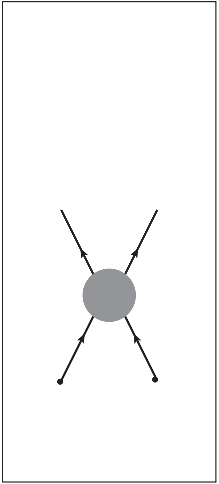

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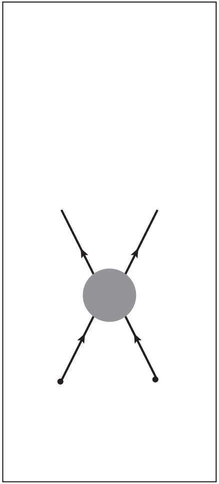

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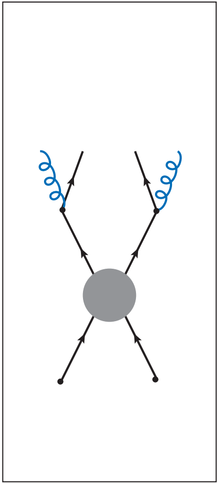

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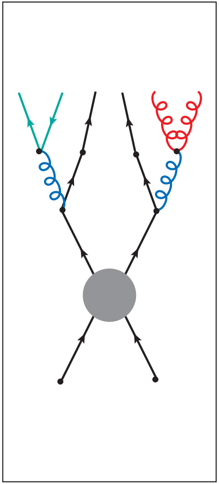

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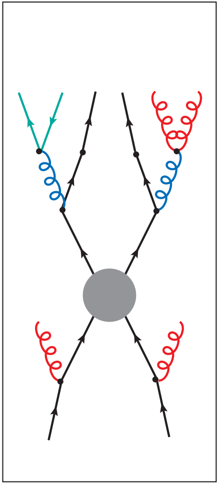

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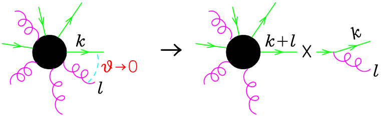
3. soft-collinear emissions are ennhanced:

$$\frac{1}{(p_1 + p_2)^2} = \frac{1}{2E_1 E_2 (1 - \cos\theta)}$$

4. in soft-collinear limit, factorization properties of QCD amplitudes

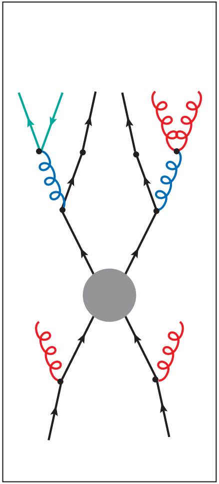


$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \; \boxed{\frac{\alpha_S}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi}}$$

$z = k^0/(k^0 + l^0)$ quark energy fraction

$t = \left\{(k+l)^2, l_T^2, E^2\theta^2\right\}$ splitting hardness

$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$ AP splitting function



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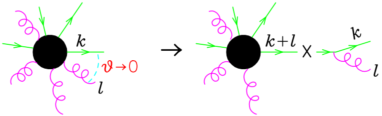

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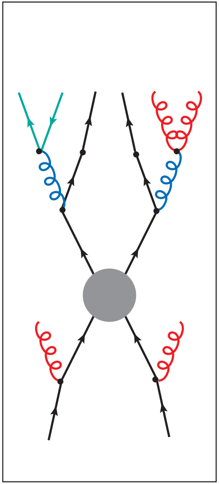
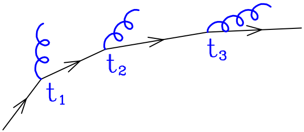


probabilistic interpretation!

5. dominant contributions for multiparticle production due to strongly ordered emissions

$$t_1 > t_2 > t_3 ...$$



6. at any given order, we also have virtual corrections: for consistency we should include them with the same approximation

- LL virtual contributions included by assigning to each internal line a Sudakov form factor:

$$\Delta_a(t_i, t_{i+1}) = \exp\left[-\sum_{(bc)} \int_{t_{i+1}}^{t_i} \frac{dt'}{t'} \int \frac{\alpha_s(t')}{2\pi} P_{a,bc}(z)\, dz\right]$$

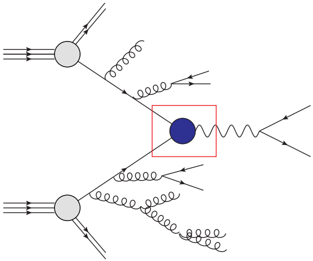
- Δ_a corresponds to the probability of having no emission between t_i and t_{i+1} off a line of flavour a

☞ resummation of collinear logarithms

7. At scales $\mu \approx \Lambda_{\text{QCD}}$, hadrons form: non-perturbative effect, simulated with models fitted to data.

- parton shower: algorithm to resum (some classes of) collinear/soft logs in a "fully-exclusive" way.
- based on description of multiple real and virtual radiative corrections using a probabilistic language

$$d\sigma_{\mathrm{SMC}} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \qquad \Big\}$$



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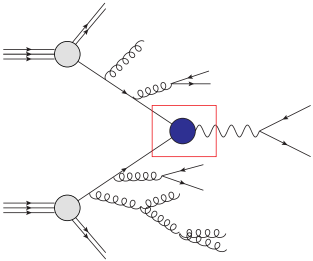
$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \Delta(t_{\rm max}, t_0) \Big\}$$



$$\Delta(t_{\rm max}, t) = \exp\left\{ - \int_t^{t_{\rm max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

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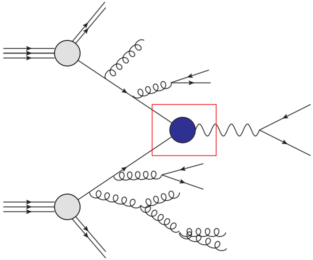

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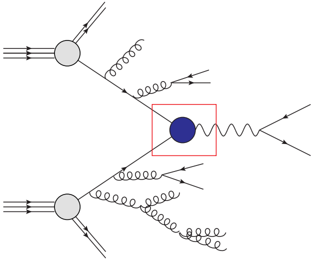

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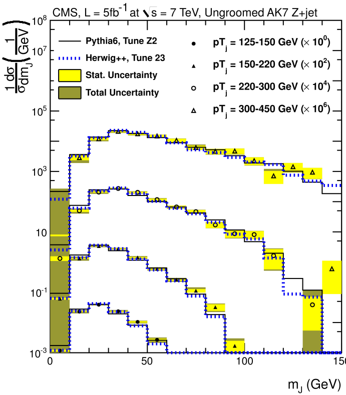
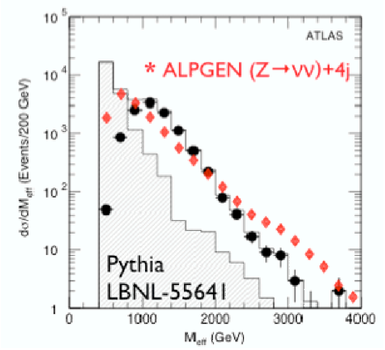
This is "LOPS"

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)

Do they work?





[Gianotti,Mangano 0504221]

✓ ok when observables dominated by soft-collinear radiation

✗ Not surprisingly, they fail when looking for hard multijet kinematics

✗ they are only LO+LL accurate (whereas we can compute up to (N)NLO QCD corrections)

⇒ Not enough if interested in precision (10% or less), or in multijet regions

Next-to-Leading Order I

$\alpha_S \sim 0.1 \Rightarrow$ to improve the accuracy, use exact perturbative expansion

$$\boxed{d\sigma = d\sigma_{\text{LO}} + \left(\frac{\alpha_S}{2\pi}\right) d\sigma_{\text{NLO}} + \left(\frac{\alpha_S}{2\pi}\right)^2 d\sigma_{\text{NNLO}} + \ldots}$$

LO: *Leading Order*
NLO: *Next-to-Leading Order*
...

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LO: *Leading Order*
NLO: *Next-to-Leading Order*
...

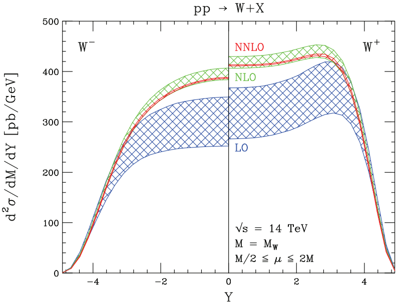
Why NLO is important?

- first order where rates are reliable
- shapes are, in general, better described
- possible to attach sensible theoretical uncertainties

- over the years, the success of MCFM clearly demonstrates the importance of NLO accuracy

---

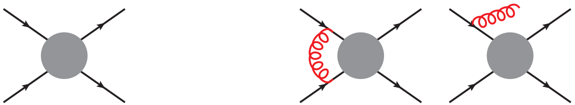
☞ when NLO corrections large, NNLO is desirable (as in Higgs production!)



[Anastasiou et al., '03]

Next-to-Leading Order II

NLO how-to



$$d\sigma = d\Phi_n \Big\{ \underbrace{B(\Phi_n)}_{\text{LO}} + \underbrace{\frac{\alpha_s}{2\pi}\Big[V(\Phi_n) + R(\Phi_{n+1})\, d\Phi_r\Big]}_{\text{NLO}} \Big\}$$

- Inputs: tree-level n-partons (B), 1-loop n-partons (V), tree-level $n+1$ partons (R)
- V and $\int R$ separately divergent, but finite when computing IR-safe observables
- truncated series $\Rightarrow$ result depends on "unphysical" scales

---

Limitations:

- Results are at the parton level only ($5-6$ final-state partons is the frontier)
- In regions where collinear emissions are important, they fail (no resummation)
- Choice of scale is an issue when multijets in the final states

matching NLO and PS

NLO

- ✓ precision
- ✓ nowadays this is the standard
- ✗ limited multiplicity
- ✗ (fail when resummation needed)

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll's
- ✗ limited precision (LO)
- ✗ (fail when multiple hard jets)

☞ can merge them and build an NLOPS generator?

Problem:

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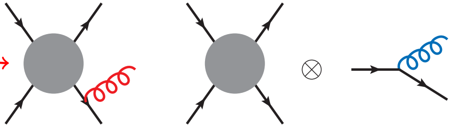
Problem: overlapping regions!

NLO:



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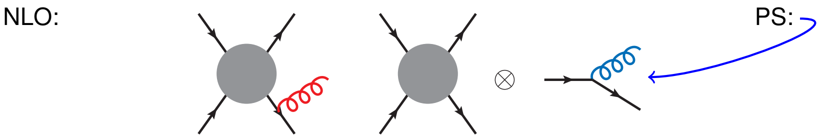

PS vs NLO

NLO

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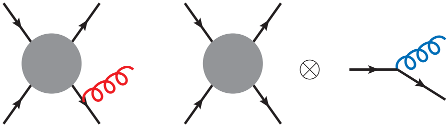
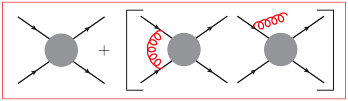


✓ 2 methods available to solve this problem:
MC@NLO and POWHEG [Frixione-Webber '03, Nason '04]

$$d\sigma_{\mathrm{POW}} = d\Phi_n \quad \bar{B}(\Phi_n) \quad \left\{\Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r\right\}$$

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$



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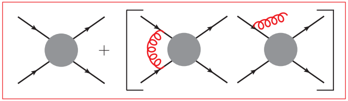

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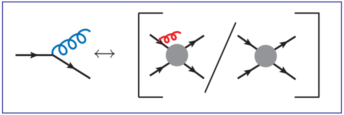

$$\Delta(t_{\mathrm{m}}, t) \Rightarrow \Delta(\Phi_n; k_{\mathrm{T}}) = \exp\left\{-\frac{\alpha_s}{2\pi}\int \frac{R(\Phi_n,\Phi_r')}{B(\Phi_n)}\theta(k_{\mathrm{T}}' - k_{\mathrm{T}})\, d\Phi_r'\right\}$$

$$d\sigma_{\rm POW} = d\Phi_n\ \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\ d\Phi_r \right\}$$

+ $p_{\rm T}$-*vetoing subsequent emissions*, to avoid double-counting.

- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs
- extra jets in the shower approximation

This is "NLOPS"

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POWHEG BOX

[Alioli,Nason,Oleari,Re ’10]

- large library of SM processes, (largely) automated
- widely used by LHC collaborations: in general, it works well

NLOPS: POWHEG II

$$d\sigma_{\mathrm{POW}} = d\Phi_n\ \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\min}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)}\ d\Phi_r \right\}$$

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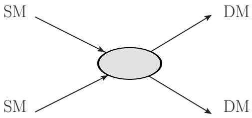
☞ Notice: when doing $X+$ jet(s) @ NLO, $\bar{B}(\Phi_n)$ is not finite !
$\hookrightarrow$ need of a generation cut on Φ_n (or variants thereof)
$\hookrightarrow$ POWHEG for $H+1$ jet cannot be used for inclusive Higgs production

Following work of Fox & Williams, we studied DM production at the LHC, including PS effects

[Haisch,Kahlhoefer,Re '13]



✗ nothing to detect ⇒ not visible !

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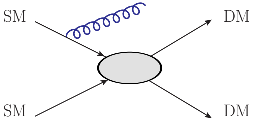
[Haisch,Kahlhoefer,Re '13]



✓ can emit extra SM particle

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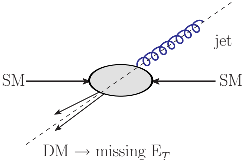

monojet signal !

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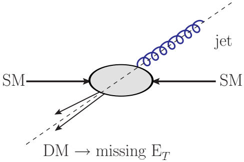

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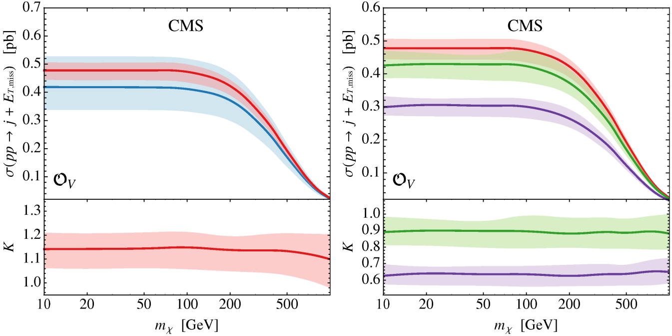

NLOPS merging

MiNLO: intro

- for processes with widely different scales (*e.g.* $X+$ jets close to Sudakov regions) choice of scales is not straightforward
- scale often chosen a posteriori, requiring typically
  - NLO corrections to be small
  - sensitivity upon scale choice to be minimal ($\rightarrow$ plateau in $\sigma(\mu)$ vs. μ)

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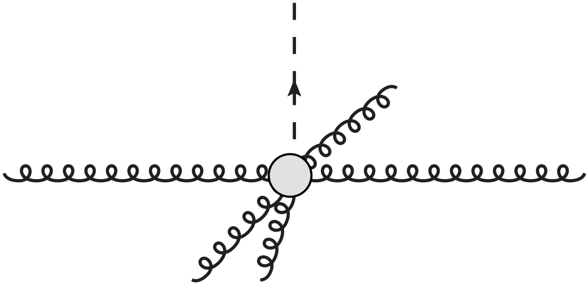
MiNLO: Multiscale Improved NLO [Hamilton,Nason,Zanderighi, 1206.3572]

- aim: method to a-priori choose scales in NLO computation

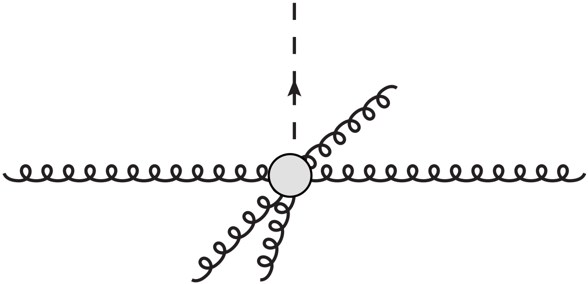
- idea: at LO, the CKKW procedure allows to take these effects into account: modify the LO weight $B(\mathbf{\Phi}_n)$ in order to include (N)LL effects.

$\Rightarrow$ "Use CKKW" on top of NLO computation that potentially involves many scales

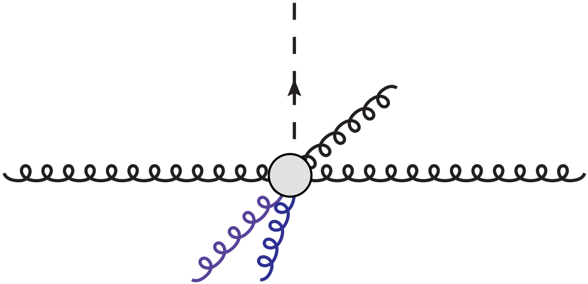


- start from ME weight $B(\mathbf{\Phi}_n)$

CKKW in a nutshell I



- find “most-likely” shower history (via k_T-algo)



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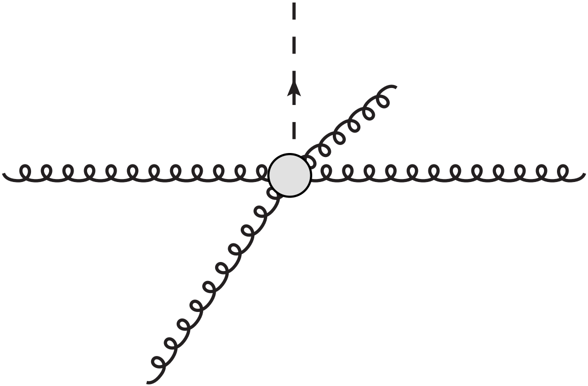

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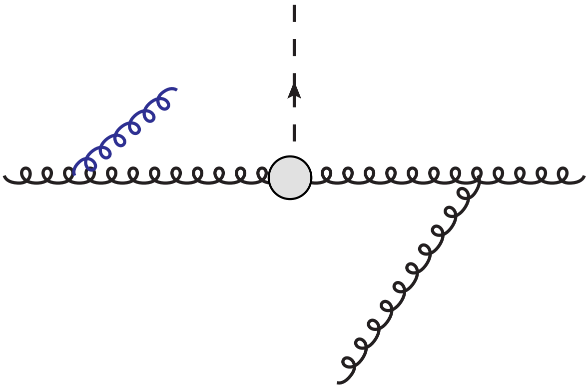
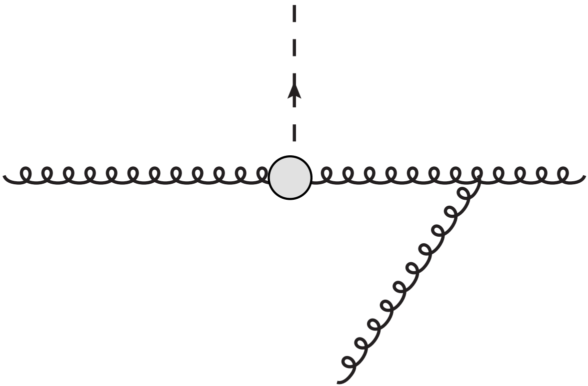


- find “most-likely” shower history (via k_T-algo)
- clustering scale $q_2 = k_T$



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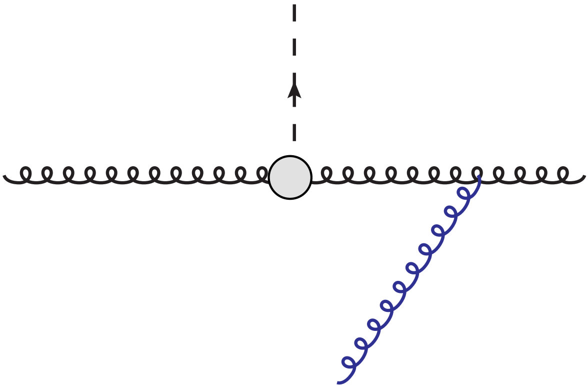
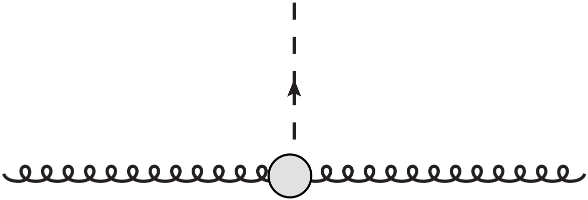
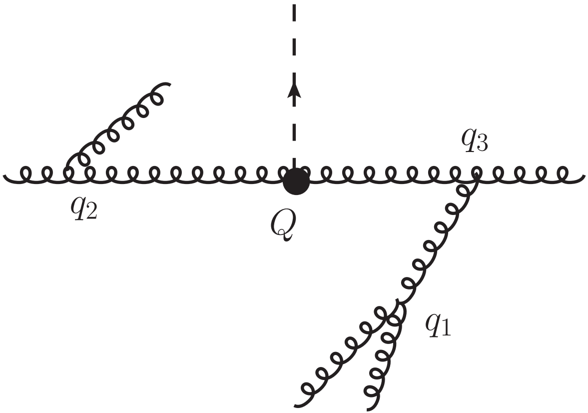


- find "most-likely" shower history (via k_T-algo)
- clustering scale $q_3 = k_T$



- Hard process scale Q

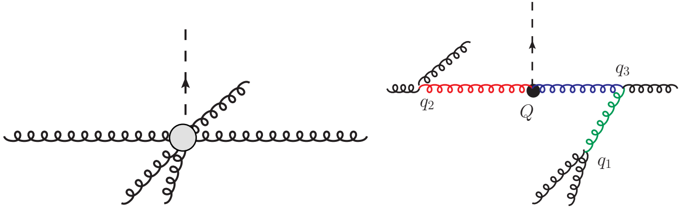
CKKW in a nutshell I



- most-likely shower history

CKKW in a nutshell II

- original weight $B(\mathbf{\Phi}_n) \Rightarrow$ “most-likely” shower history (via k_T-algo):
  $Q > q_3 > q_2 > q_1 \equiv Q_0$



- New weight:

$$\alpha_S^5(Q)B(\mathbf{\Phi}_3) \quad \rightarrow \quad \alpha_S^2(Q)B(\mathbf{\Phi}_3)\,\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_2)}\,\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_3)}\,\frac{\Delta_g(Q_0,q_3)}{\Delta_g(Q_0,q_1)}$$
$$\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_3)\Delta_g(Q_0,q_1)\Delta_g(Q_0,q_1)$$
$$\alpha_S(q_1)\alpha_S(q_2)\alpha_S(q_3)$$

  where typically

$$\log\Delta_{\mathrm{f}}(q_T,Q) = -\int_{q_T^2}^{Q^2}\frac{dq^2}{q^2}\,\frac{\alpha_S(q^2)}{2\pi}\left[A_{1,\mathrm{f}}\log\frac{Q^2}{q^2}+B_{1,\mathrm{f}}\right]$$

- Fill phase space below Q_0 with vetoed shower

From CKKW to MiNLO

Next-to-Leading Order accuracy needs to be preserved

1. Scale dependence shows up at NNLO ["scale compensation"]:

$$O(\mu') - O(\mu) = \mathcal{O}(\alpha_S^{n+2}) \quad \text{if} \quad O \sim \alpha_S^n \quad \text{at LO}$$

2. Away from soft-collinear regions, exact NLO recovered:

$$O_{\text{MiNLO}} = O_{\text{NLO}} + \mathcal{O}(\alpha_S^{n+2}) \qquad [\, i.e. \;\; \alpha_S^n \;\; \& \;\; \alpha_S^{n+1} \;\; \text{reproduce plain NLO} \,]$$

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- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\mathbf{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\mathbf{\Phi}_n)$$

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

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* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

- Sudakov FFs in internal and external lines of Born "skeleton"

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)...\}$$

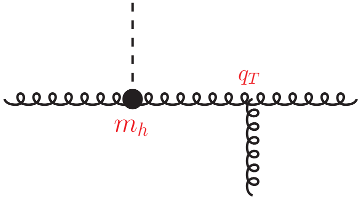
* Upon expansion, $\mathcal{O}(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n)\Big(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + ...\Big)$$

Example, in 1 line: $H+1$ jet

- Pure NLO:

$$d\sigma = \bar{B}\, d\mathbf{\Phi}_n = \alpha_{\mathrm{S}}^3(\mu_R)\Big[B + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} V(\mu_R) + \alpha_{\mathrm{S}}^{(\mathrm{NLO})} \int d\Phi_{\mathrm{rad}} R\Big]\, d\mathbf{\Phi}_n$$



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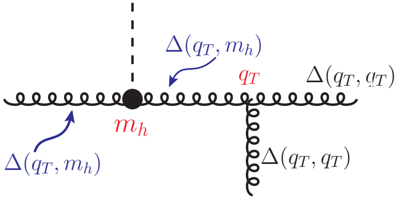
- MiNLO:

$$\bar{B} = \alpha_S^2(m_h)\alpha_S(q_T)\Delta_g^2(q_T, m_h)\Big[B\left(1 - 2\Delta_g^{(1)}(q_T, m_h)\right) + \alpha_S^{(\mathrm{NLO})} V(\bar{\mu}_R) + \alpha_S^{(\mathrm{NLO})}\int d\Phi_{\mathrm{rad}} R\Big]$$



MiNLO: example

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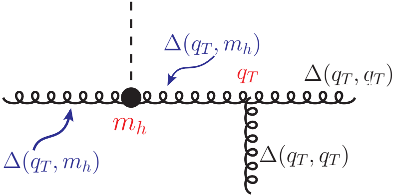

* $\bar{\mu}_R = (m_h^2 q_T)^{1/3}$
* $\log \Delta_{\mathrm{f}}(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi}\Big[A_f \log\frac{Q^2}{q^2} + B_f\Big]$
* $\Delta_{\mathrm{f}}^{(1)}(q_T, Q) = -\alpha_S^{(\mathrm{NLO})} \frac{1}{2\pi}\Big[\frac{1}{2} A_{1,\mathrm{f}} \log^2\frac{Q^2}{q_T^2} + B_{1,\mathrm{f}} \log\frac{Q^2}{q_T^2}\Big]$
* $\mu_F = Q_0 (= q_T)$

☞ Sudakov FF included on Born kinematics

MiNLO: example

Example, in 1 line: $H+1$ jet

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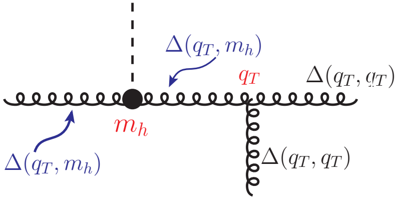
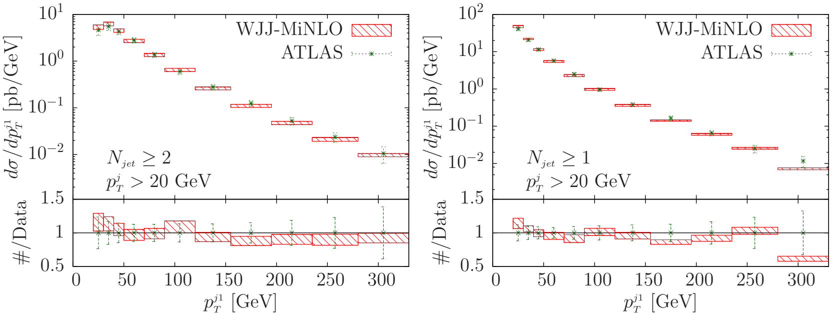


☞ $X+$ jets cross-section finite without generation cuts
$\hookrightarrow$ $\bar{B}$ with `MiNLO` prescription: ideal starting point for NLOPS (`POWHEG`) for $X+$ jets

MiNLO: results

VJJ−MiNLO [Campbell,Ellis,Nason,Zanderighi, 1303.5447]



☞ Start from $W+2$ jets @ NLO, good agreement with data also when requiring $N_{\text{jet}} \geq 1$!

This is not possible in a standard NLO...

“Improved” MiNLO & NLOPS merging I

[Hamilton et al., 1212.4504]

- accuracy of $\texttt{BJ+MiNLO}$ for inclusive observables carefully investigated
- $\texttt{BJ+MiNLO}$ describes inclusive boson observables at order α_S (relative to $B + 0j$ at LO)
- to reach genuine NLO when inclusive, higher terms must be of relative order α_S^2, *i.e.*

$$O_{\mathrm{BJ+MiNLO}} = O_{\mathrm{B@NLO}} + \mathcal{O}(\alpha_S^{b+2})$$

  if O is inclusive (B@LO $\sim \alpha_S^b$).
- “Original MiNLO” contains ambiguous $\mathcal{O}(\alpha_S^{b+3/2})$ terms.
- Possible to improve $\texttt{BJ+MiNLO}$ such that NLO $B + 0j$ (NLO$^{(0)}$) is recovered, without spoiling NLO accuracy for $B + 1j$ (NLO$^{(1)}$) .
  - proof based on careful comparisons of general resummation formula with $\texttt{MiNLO}$ ingredients
  - need to include B_2 in $\texttt{MiNLO}$-Sudakovs
  - need to evaluate $\alpha_S{}^{(\mathrm{NLO})}$ in $\texttt{BJ+MiNLO}$ at scale q_T, and $\mu_F = q_T$

Effectively it is like if we merged NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used).

Other NLOPS-merging approaches: [Hoeche,Krauss, et al.,1207.5030] [Frederix,Frixione,1209.6215] [Lonnblad,Prestel,1211.7278 - Platzer,1211.5467] [Alioli,Bauer, et al.,1211.7049]

"Improved" MiNLO & NLOPS merging II

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2}\Big\{[C_{ga}\otimes f_a](x_A,q_T)\times[C_{gb}\otimes f_b](x_B,q_T)\times \exp S(q_T,Q)\Big\} + R_f$$

- If $C_{ij}^{(1)}$ included and R_f is LO$^{(1)}$, then upon integration we get NLO$^{(0)}$
- Take derivative, then compare with MiNLO :

$$\sim \sigma_0 \frac{1}{q_T^2}[\alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L] \exp S(q_T,Q) + R_f \qquad L = \log(Q^2/q_T^2)$$

- highlighted terms are needed to reach NLO$^{(0)}$:

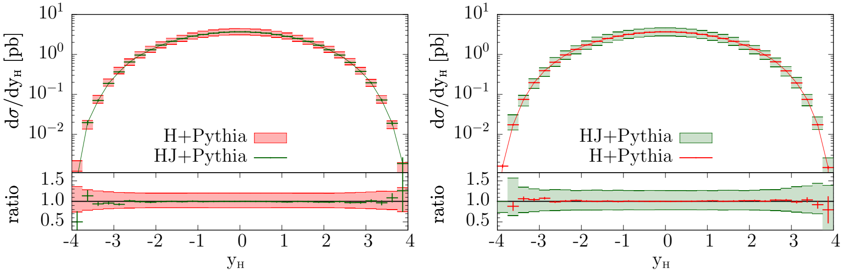
$$\int^{Q^2} \frac{dq_T^2}{q_T^2} L^m {\alpha_S}^n(q_T) \exp S \sim \left(\alpha_S(Q^2)\right)^{n-(m+1)/2}$$

- if I don't include B_2 in MiNLO Δ_g, I miss a term $(1/q_T^2)\alpha_S^2 B_2 \exp S$
- upon integration, violate NLO$^{(0)}$ by a term of relative $\mathcal{O}(\alpha_S^{3/2})$
- "wrong" scale in $\alpha_S^{(\mathrm{NLO})}$ in MiNLO produces again same error

Alternative proof also available in the paper.

MiNLO merging: results

[Hamilton et al., 1212.4504]



- "H+Pythia": standalone POWHEG ($gg \to H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- "HJ+Pythia": HJ−MiNLO* + PYTHIA (PS level) [7pts band, μ from MiNLO]

✓ very good agreement (both value and band)

☞ Notice: band is $\sim 20 - 30\%$

matching NNLO with PS

- Higgs production at NNLOPS

NNLO+PS I

- HJ−MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  * obvious for y_H, by construction
  * α_S^4 accuracy of HJ−MiNLO* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_S^2)$
  * if we had NLO$^{(0)}$ $+ \mathcal{O}(\alpha_S^{2+3/2})$, 1-jet region spoiled because

$$[\text{NLO}^{(1)}]_{\text{NNLOPS}} = \text{NLO}^{(1)} + \mathcal{O}(\alpha_S^{4.5})$$

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$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

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---

* Variants for W are possible:

$$W(y, p_T) = h(p_T)\frac{\int d\sigma_A^{\mathrm{NNLO}}\delta(y - y(\mathbf{\Phi}))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(y - y(\mathbf{\Phi}))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma\; h(p_T), \qquad d\sigma_B = d\sigma\;(1 - h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

* $h(p_T)$ controls where the NNLO/NLO K-factor is spread
* β cannot be too small, otherwise resummation spoiled

NNLO+PS II

In 1309.0017, we used

$$W(y,p_T) = h(p_T)\frac{\int d\sigma^{\rm NNLO}\delta(y-y(\mathbf{\Phi})) - \int d\sigma_B^{\rm MiNLO}\delta(y-y(\mathbf{\Phi}))}{\int d\sigma_A^{\rm MiNLO}\delta(y-y(\mathbf{\Phi}))} + (1-h(p_T))$$

$$d\sigma_A = d\sigma\, h(p_T), \qquad d\sigma_B = d\sigma\,(1-h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

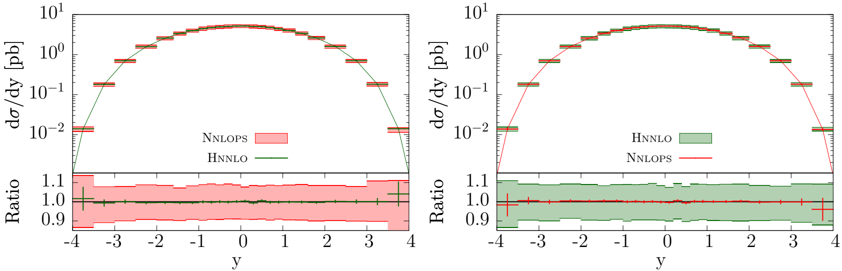
- one gets exactly $(d\sigma/dy)_{\rm NNLOPS} = (d\sigma/dy)_{\rm NNLO}$ (no $\alpha_{\rm S}^5$ terms)
- we used $h(p_T^{j_1})$

---

inputs for following plots:

- results are for 8 TeV LHC
- scale choices: NNLO input with $\mu = m_H/2$, HJ-MiNLO "core scale" m_H (other powers are at q_T)
- PDF: everywhere MSTW8NNLO
- NNLO always from HNNLO
- events reweighted at the LH level
- plots after $k_{\rm T}$-ordered PYTHIA 6 at the PS level (hadronization and MPI switched off)

- NNLO with $\mu = m_H/2$, `HJ-MiNLO` "core scale" m_H [NNLO from `HNNLO`, Catani,Grazzini]
- $(7_{\mathrm{Mi}} \times 3_{\mathrm{NN}})$ pts scale var. in `NNLOPS`, 7pts in NNLO

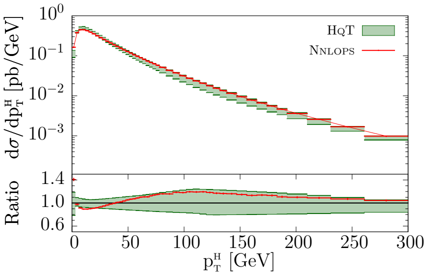


☞ Notice: band is 10%

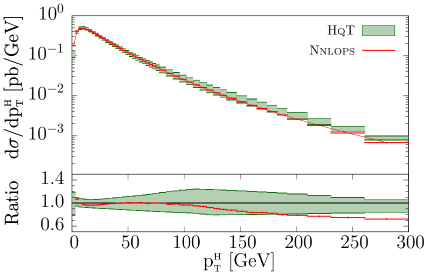
[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]

NNLO+PS (p_T^H)

$\beta = \infty$ (W indep. of p_T)



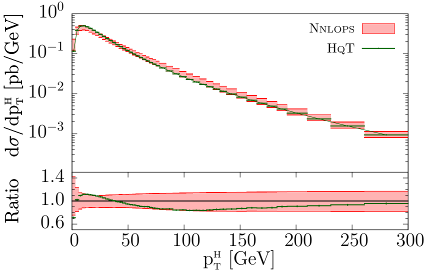
$\beta = 1/2$



- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\rm res} \equiv m_H/2$ [HqT, Bozzi et al.]
- ✓ $\beta = 1/2$ & ∞: uncertainty bands of HqT contain NNLOPS at low-/moderate p_T
- $\beta = 1/2$: HqT tail harder than NNLOPS tail ($\mu_{\rm HqT} <$ "$\mu_{\rm MiNLO}$")
- $\beta = 1/2$: very good agreement with HqT resummation ["$\sim$ expected", since $Q_{\rm res} \equiv m_H/2$]

NNLO+PS (p_T^H)

$\beta = \infty$ (W indep. of p_T)



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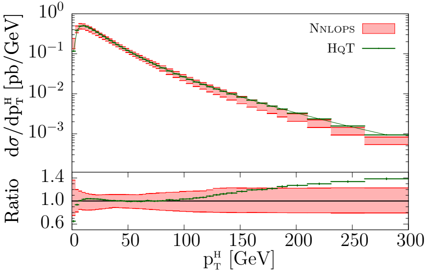
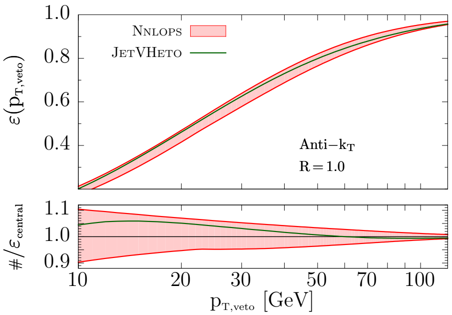
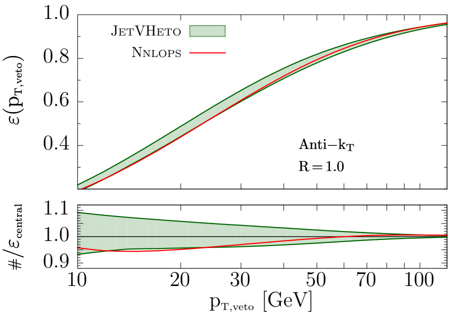


- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\text{res}} \equiv m_H/2$
- $\beta = 1/2$: NNLOPS tail $\rightarrow$ NLOPS tail [$W(y, p_T \gg m_H) \rightarrow 1$] larger band (affected just marginally by NNLO, so it's $\sim$ genuine NLO band)

NNLO+PS ($p_T^{j_1}$)





$$\varepsilon\left(p_{\mathrm{T,veto}}\right) = \frac{\Sigma(p_{\mathrm{T,veto}})}{\sigma^{\mathrm{tot}}} = \frac{1}{\sigma^{\mathrm{tot}}} \int d\sigma\, \theta\left(p_{\mathrm{T,veto}} - p_{\mathrm{T}}^{\mathrm{j}_1}\right)$$

- `JetVHeto`: NNLL resum, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\mathrm{res}} \equiv m_H/2$, (a)-scheme only
  [`JetVHeto`, Banfi et al.]
- nice agreement, differences never more than 5-6 %

☞ Separation of $H \to WW$ from $t\bar{t}$ bkg: x-sec binned in N_{jet}
0-jet bin ⇔ jet-veto accurate predictions needed !

Conclusions

- Especially in absence of very clear singals of new-physics, accurate tools are needed for LHC phenomenology
- In the last decade, impressive amount of progress: new ideas, and automated tools
  - "NLO revolution" $\rightarrow$ NLO "Les Houches" wishlist closed
- Shown results of merging NLOPS for different jet-multiplicities *without* merging scale
- Shown first working example of NNLOPS
- A lot of QCD theoretical work entering in these improvements

But of course there is still a lot to be done !

Conclusions

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Thank you for your attention!