Event Shapes for Exclusive Jet Processes: From the LHC to the EIC

Sonny Mantry
Northwestern University and Argonne National Lab

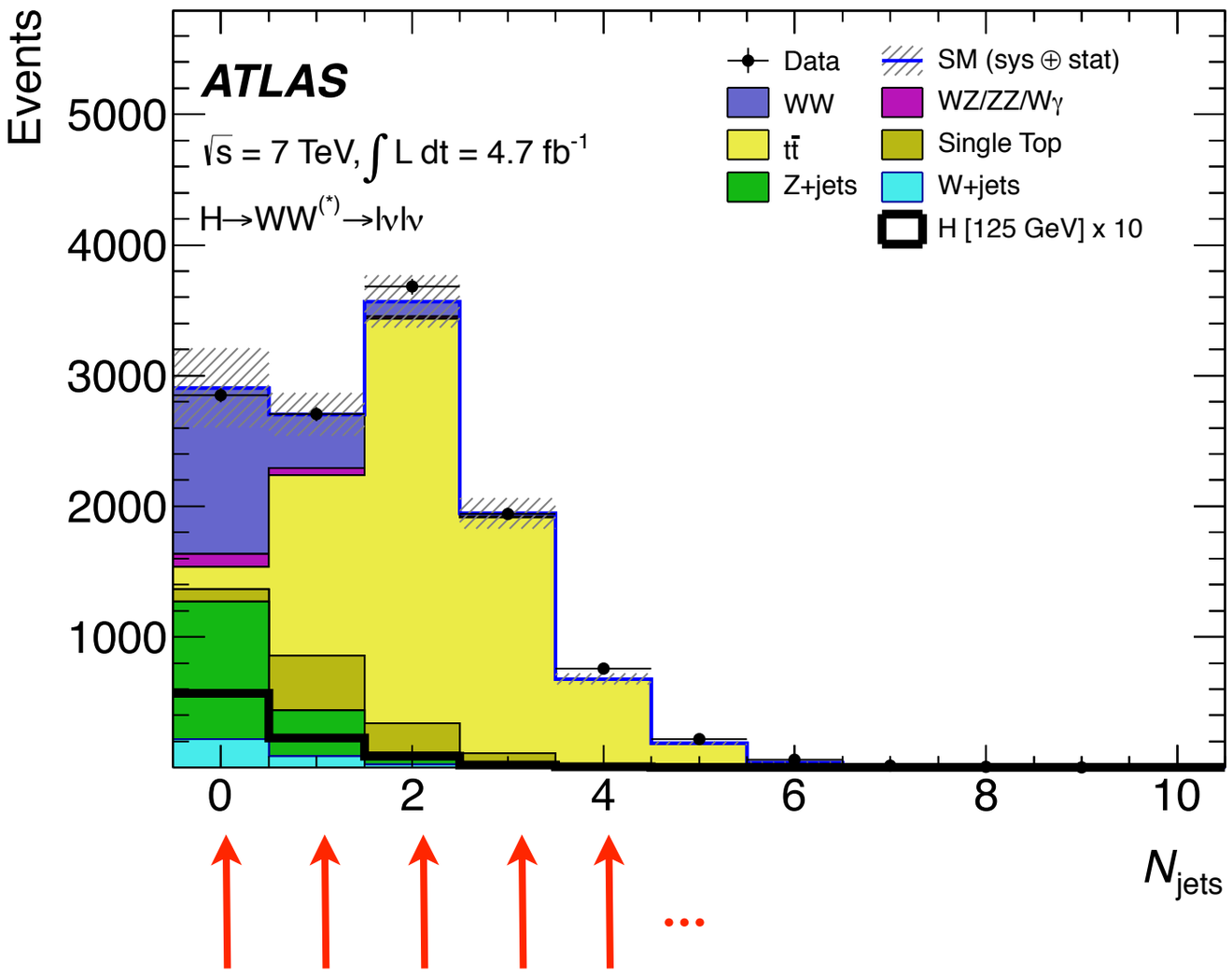
Xiaohui Liu, SM, Frank Petriello, Phys.Rev. D86 (2012) 074004

Zhongbo Kang, SM, Jianwei Qiu, Phys.Rev. D86 (2012) 114011

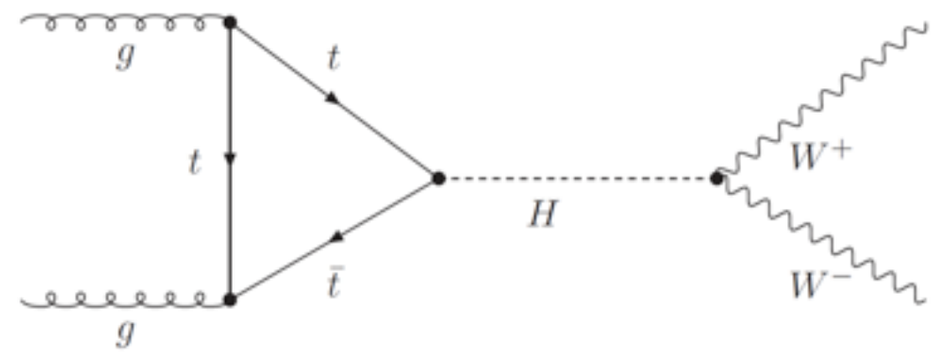
Zhongbo Kang, Xiaohui Liu, SM, Jianwei Qiu, arXiv:1303.3063

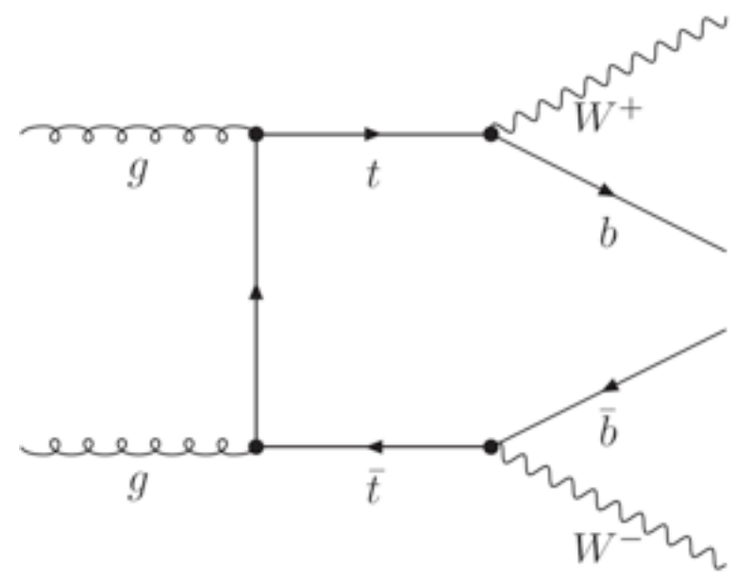
Fermilab
Theory Seminar
May 9th, 2013

LHC data grouped by jet bins



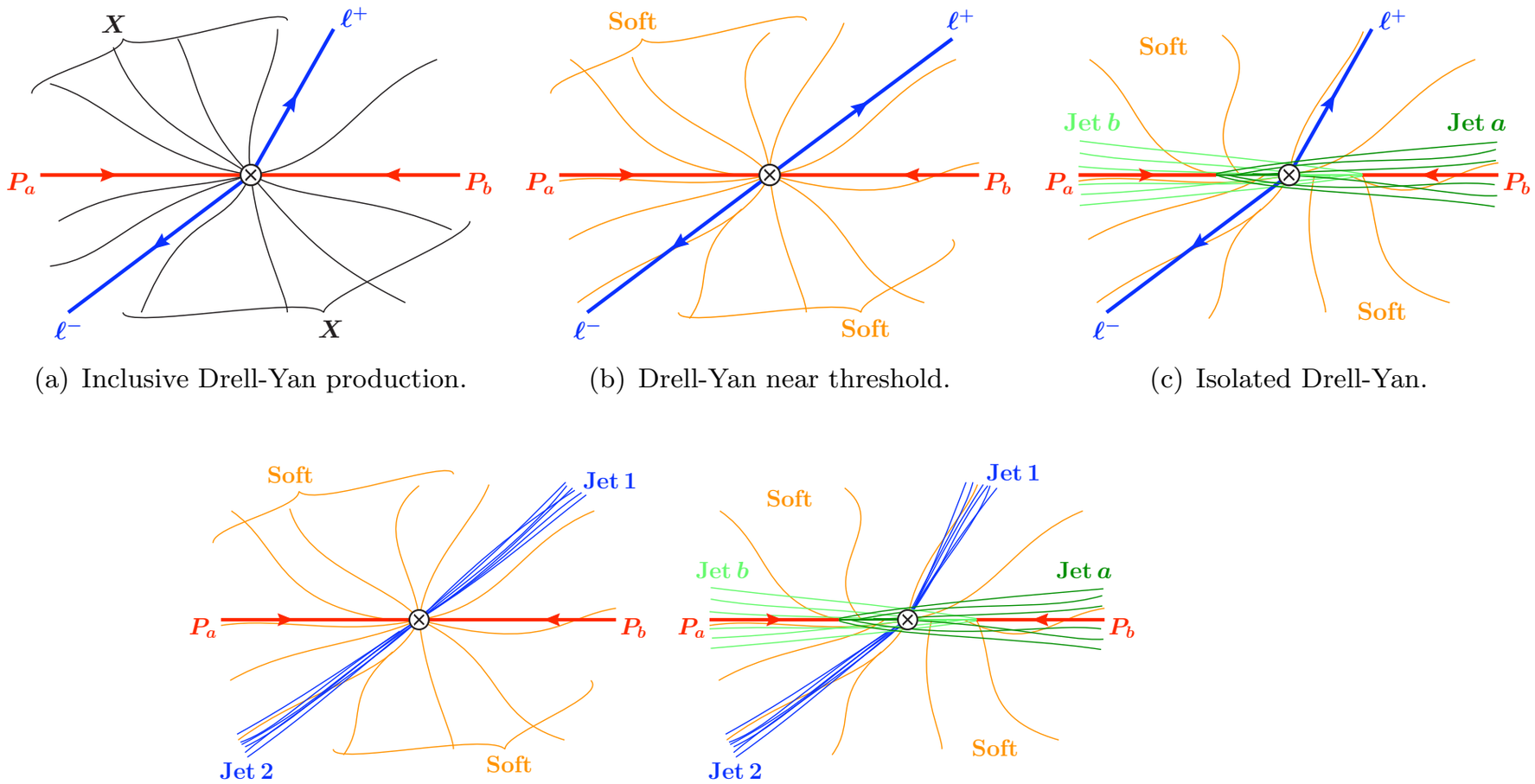
Higgs + 0j 1j 2j 3j 4j ...





- Data is categorized into jet bins.
- The background composition depends on the jet bin.
- Jet binning can enhance signal over background.

Jet veto logarithms



(a) Inclusive Drell-Yan production. (b) Drell-Yan near threshold. (c) Isolated Drell-Yan.

(d) Dijet production near threshold. (e) Isolated dijet production.

- Veto on additional jets introduces sensitivity to soft and collinear emissions.
- Jet veto Sudakov logarithms arise and can affect perturbation theory.

Higgs + 0j : $$\sigma_0(p_T^{\rm cut}) = \sigma_B\left(1 - \frac{3\alpha_s}{\pi}\, 2\ln^2\frac{p_T^{\rm cut}}{m_H} + \cdots\right)$$

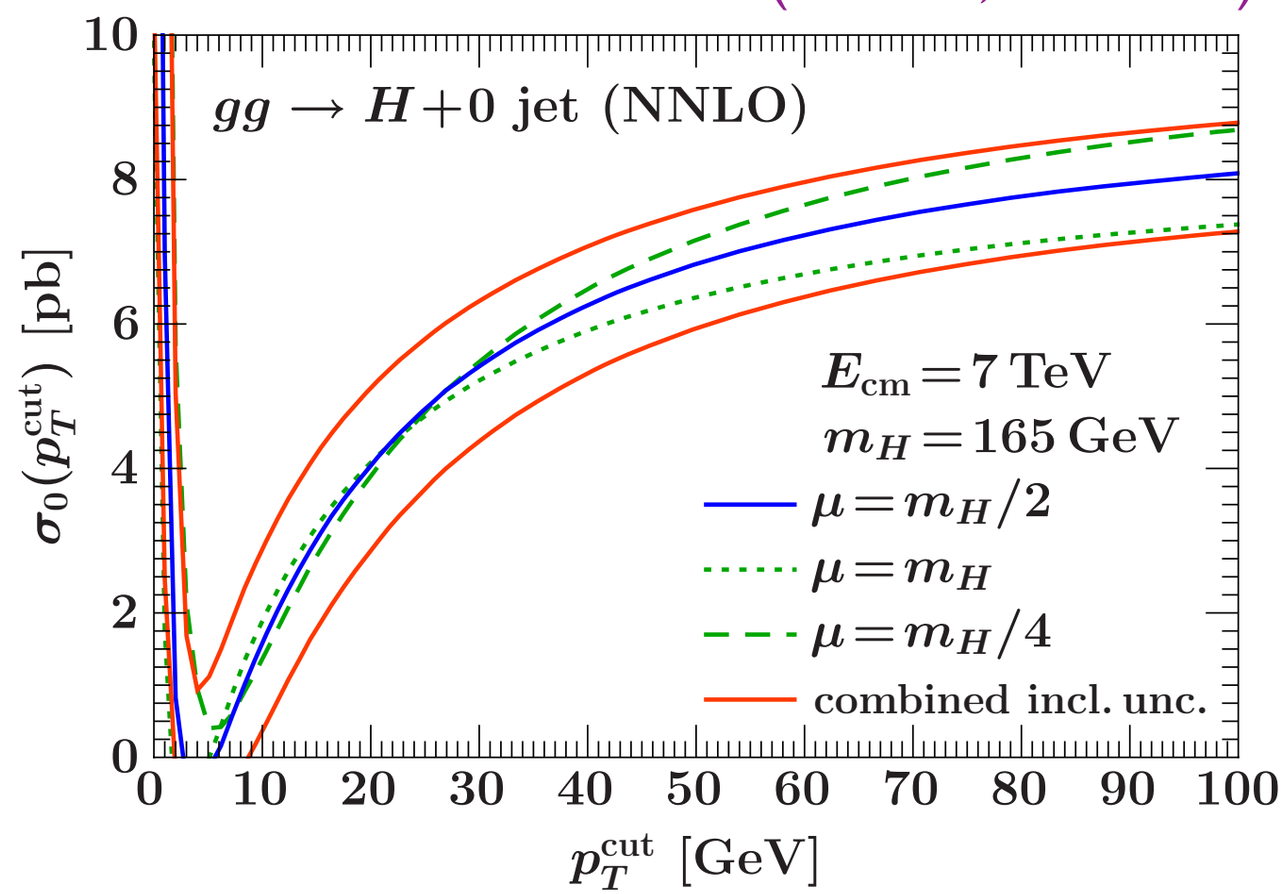
- Resummation of jet veto logarithms is needed.

Jet vetoes affect theoretical uncertainties

(Stewart, Tackmann)

- Exclusive 0j cross-section:

$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}$$



- Accidental Cancellations between large K factors and jet veto logarithms:

$$\sigma_{\text{total}} \simeq \sigma_B \left[1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)\right]$$

$$\sigma_{\geq 1}(p^{\text{cut}}) \simeq \sigma_B \left[\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)\right]$$

- Uncertainties in total and 1-jet inclusive cross-sections are uncorrelated:

$$\Delta_0^2 = \Delta_{\text{total}}^2 + \Delta_{\geq 1}^2$$

Jet algorithm dependence

- Jet algorithm dependence begins at NNLO for two emissions:

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n)\,\frac{\sqrt{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}}{R}$$

$$d_{iB} = p_{Ti}^n\,,$$

$\longrightarrow$

$n = 1$	k_T
$n = 0$	Cambridge-Aachen
$n = -1$	anti-k_T

- Jet algorithm dependence begins at NNLO for two emissions:

$$\alpha_s^2\, f_{\rm alg}^{(2)}(R)\,\ln\lambda$$

$$\lambda = p_T^{\rm cut}/m_H$$

$\longrightarrow$ $\alpha_s^2 R^2 \ln\lambda$

$\longrightarrow$ $\alpha_s^2 \ln R \ln\lambda$

(Banfi, Salam,Zanderighi; Becher,Neubert; Tackmann, Walsh,Zuberi; Liu,Petriello; ...)

- Jet algorithm dependence can make analytic control difficult. Going to higher orders in perturbation theory or resummation becomes more challenging. Often numerical methods must be employed.

Event Shape Analysis

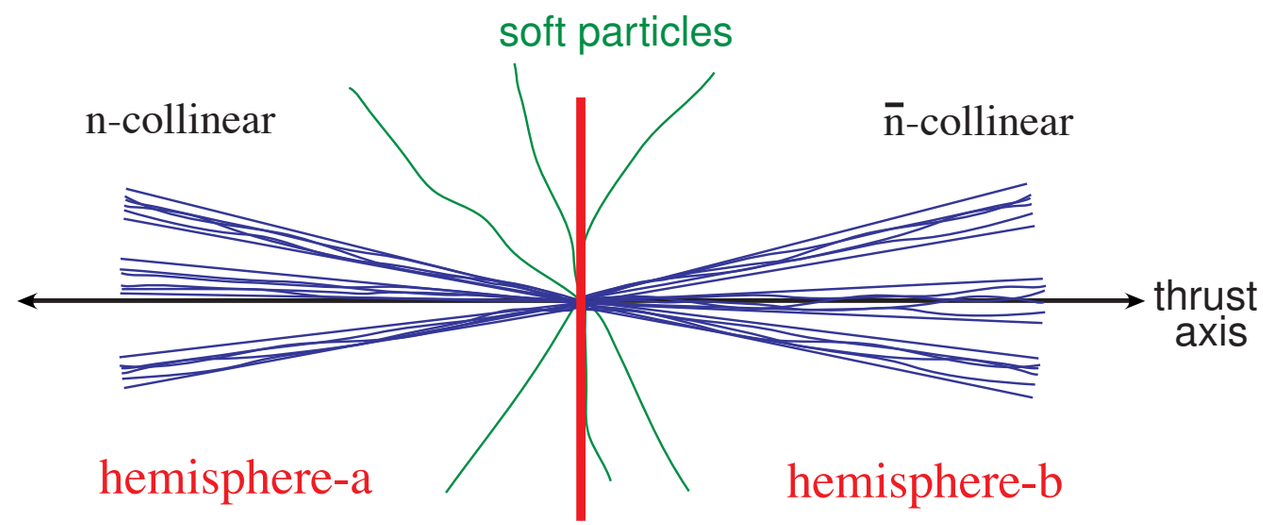
Sonny Mantry, NU & ANL

Event Shapes

- Event shapes are theoretically cleaner and allow for better analytic control.

- Thrust:

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



- Two jet configurations obtained in the limit:

$$T \to 1$$

- Thrust acts as a jet veto.

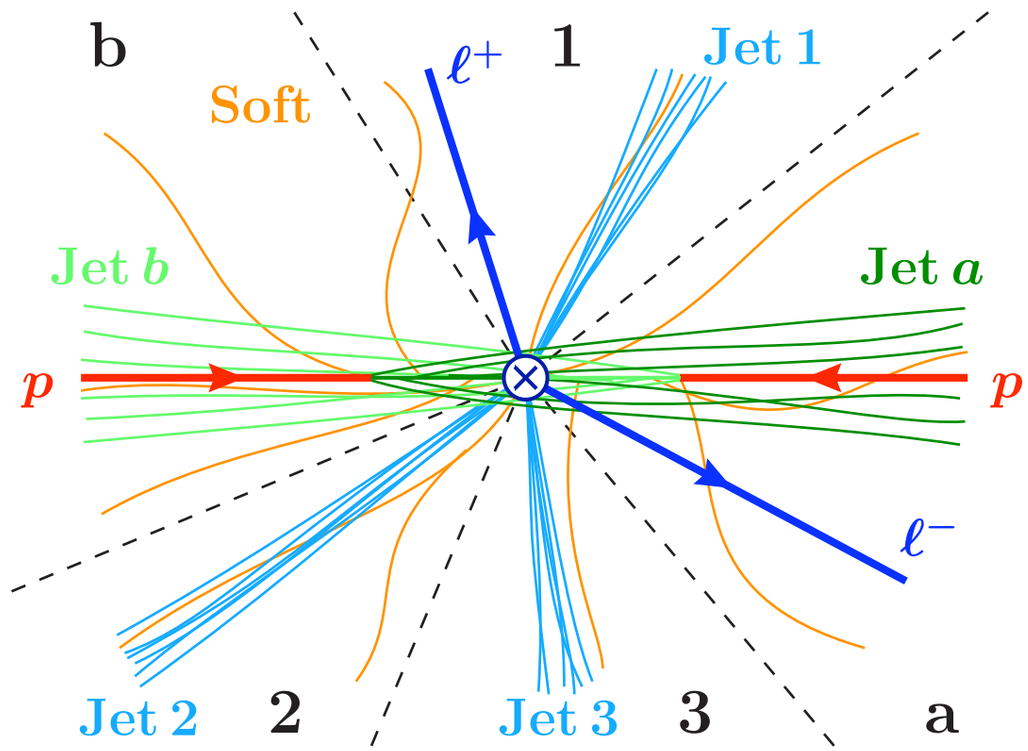
- Thrust resummation corresponds to resumming Sudakov jet veto logs.

N-Jettiness

Sonny Mantry, NU & ANL

N-Jettiness

(Stewart, Tackmann, Waalewijin)



(c) $pp \to$ leptons plus jets.

- N-jettiness allows for an event-shape based analysis of multi-jet events (generalization of Thrust)

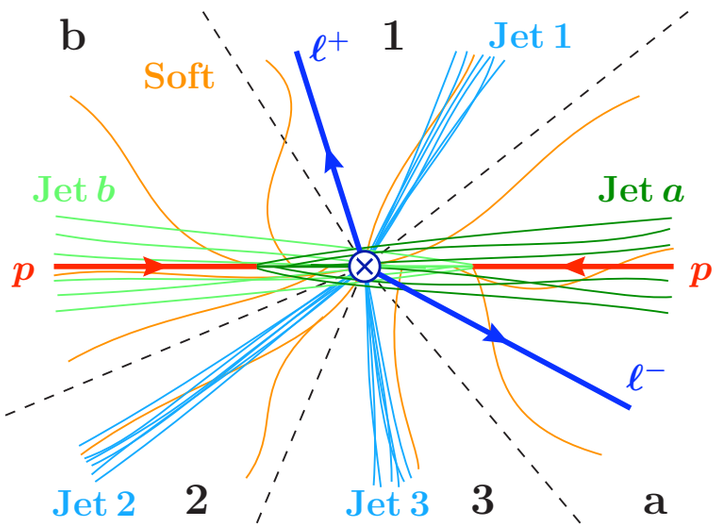
$$\tau_N = \sum_k \min_i \left\{ \frac{2 q_i \cdot p_k}{Q_i} \right\}$$

- Limit of N infinitely narrow jets is given by (Jet Veto)

$$\tau_N \to 0$$

N-Jettiness

(Stewart, Tackmann, Waalewijin)



(c) $pp \to$ leptons plus jets.

- Jet reference vectors can be obtained by minimizing:

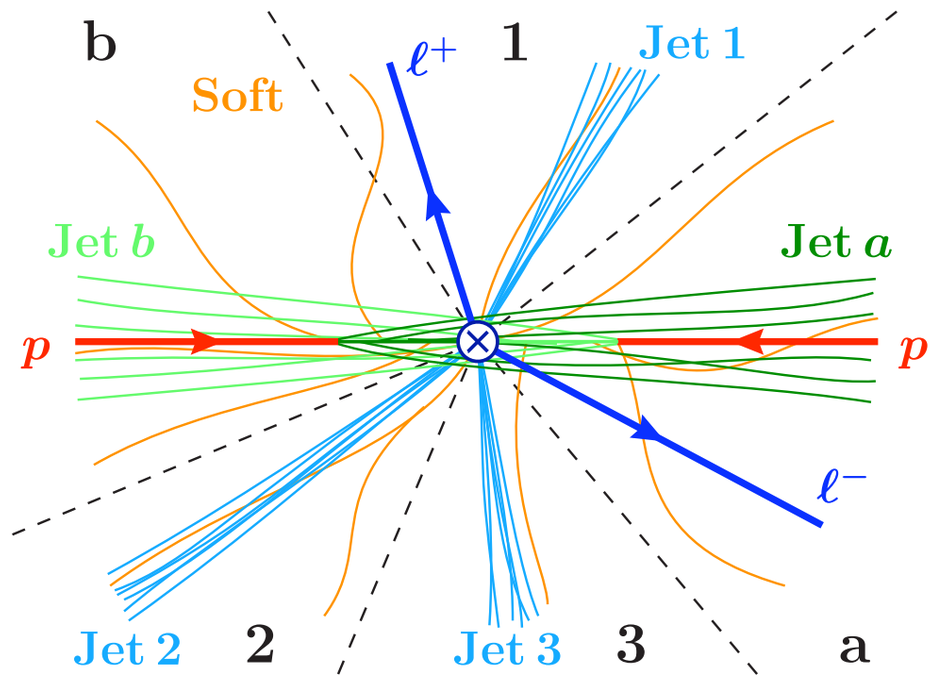
$$\tau_N = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

- No jet algorithm dependence.

- Implementation with a jet algorithm:
  - Use a standard jet algorithm to find N-jets.
  - The momenta of the N-jets and the beam directions give reference vectors.
  - The reference vectors are the only information used from the jet algorithm.
  - Calculate value for the N-jettiness global event shape.
  - Select events with N narrow well-separated jets and impose veto on additional jets: $\tau_N \to 0$
- Jet algorithm dependence is power suppressed.

N-Jettiness

(Stewart, Tackmann, Waalewijin)



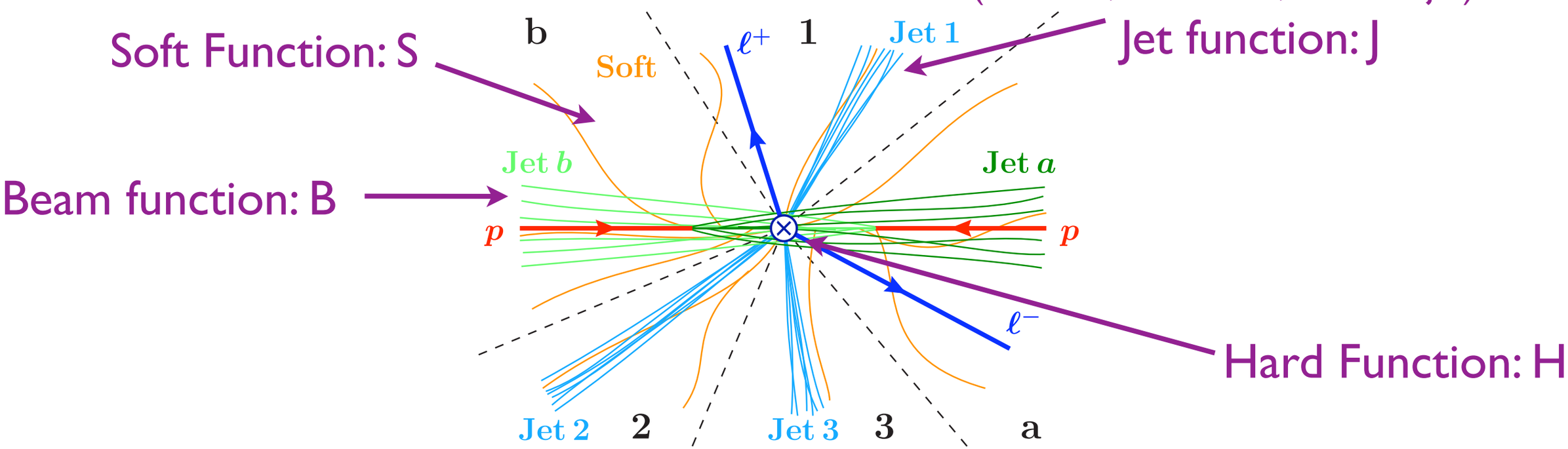
(c) $pp \to$ leptons plus jets.

- "Jet" momenta given by sum of momenta in jet regions

$$P_i^\mu = \sum_k p_k^\mu \prod_{j \neq i} \theta\big(\hat{q}_j \cdot p_k - \hat{q}_i \cdot p_k\big)$$

N-Jettiness

(Stewart, Tackmann, Waalewijin)



(c) $pp \to$ leptons plus jets.

- Physics dominated by the soft-collinear limit of QCD.
- Naturally described by the Soft-Collinear Effective Theory (SCET)

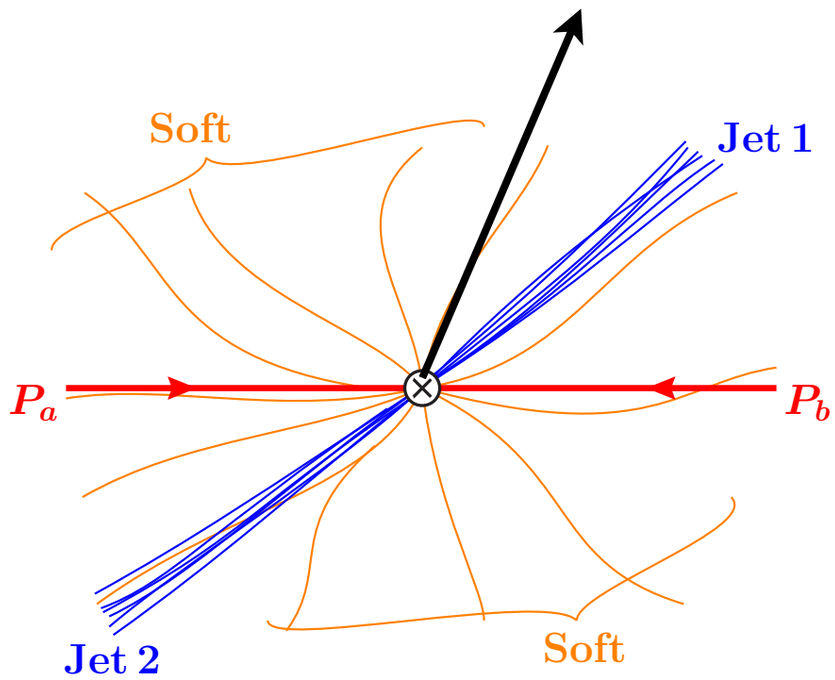
(Bauer, Fleming, Luke, Pirjol, Stewart,...)

- Factorization of soft and collinear regions

(Jouttenus, Stewart, Tackmann, Waalewijin)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N^a\,\mathrm{d}\mathcal{T}_N^b\cdots\mathrm{d}\mathcal{T}_N^N} = \int \mathrm{d}x_a\mathrm{d}x_b \int \mathrm{d}^4q\,\mathrm{d}\Phi_L(q) \int \mathrm{d}\Phi_N(\{q_J\})\, M_N(\Phi_N,\Phi_L)\,(2\pi)^4\delta^4\big(q_a+q_b-q_1-\cdots-q_N-q\big)$$
$$\times \sum_\kappa \int \mathrm{d}t_a\, B_{\kappa_a}(t_a,x_a,\mu) \int \mathrm{d}t_b\, B_{\kappa_b}(t_b,x_b,\mu) \prod_{J=1}^N \int \mathrm{d}s_J\, J_{\kappa_J}(s_J,\mu) \qquad (14)$$
$$\times \vec{C}_N^{\kappa\dagger}(\Phi_N,\Phi_L,\mu)\,\widehat{S}_N^\kappa\Big(\mathcal{T}_N^a-\frac{t_a}{Q_a},\mathcal{T}_N^b-\frac{t_b}{Q_b},\mathcal{T}_N^1-\frac{s_1}{Q_1},\ldots,\mathcal{T}_N^N-\frac{s_N}{Q_N},\{\hat{q}_i\},\mu\Big)\,\vec{C}_N^\kappa(\Phi_N,\Phi_L,\mu)\,.$$

Two Applications

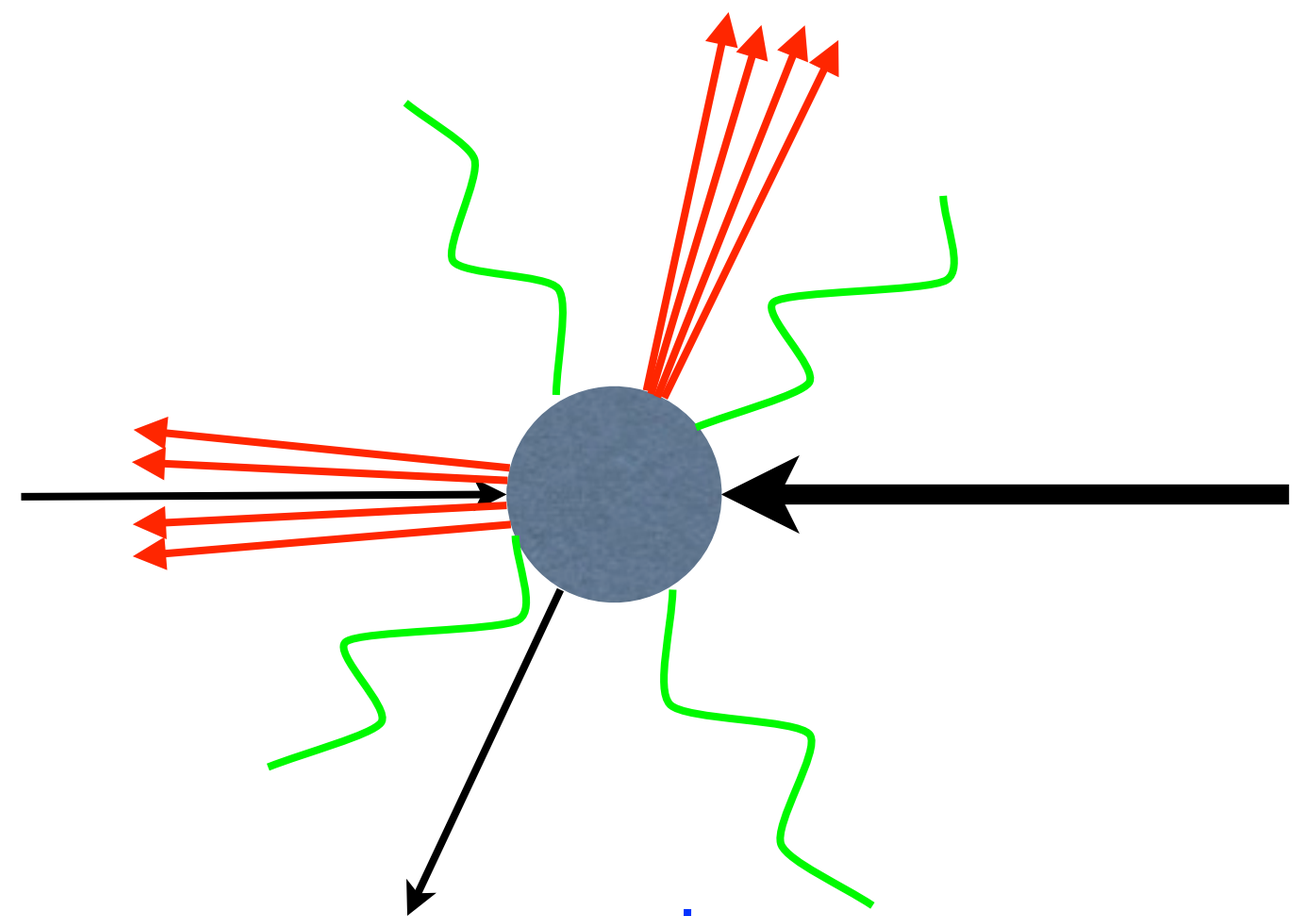


Gauge Boson Threshold Production with two jets

↓

Background to new physics searches

DIS with one jet



↓

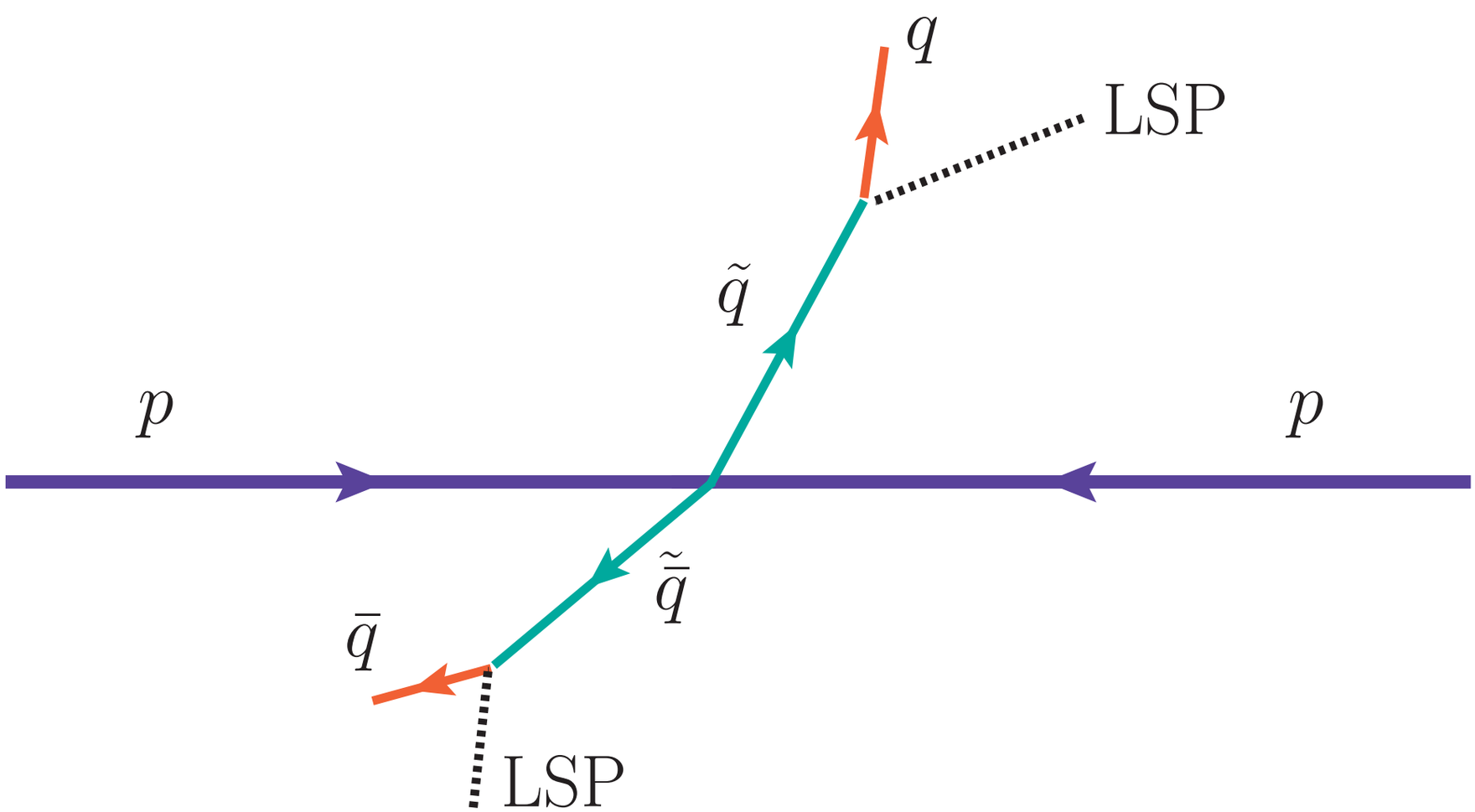
Probe of nuclear physics

Gauge Boson Production with multiple jets near threshold

Sonny Mantry, NU & ANL

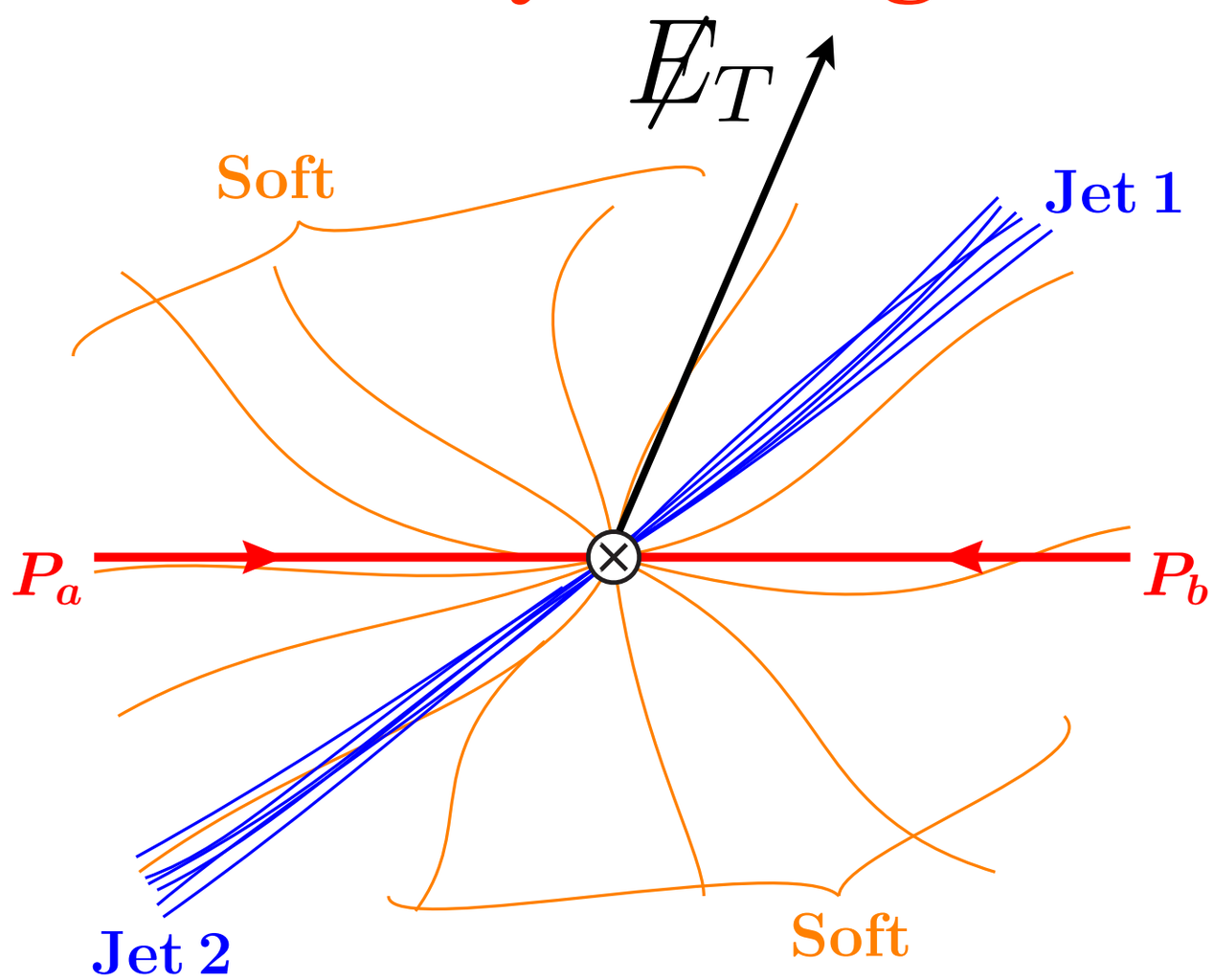
New Physics Processes

- Gluino pair production in SUSY



- Signatures involve jets and missing energy.
- Reconstruction of invariant mass peak not possible.

New Physics Signal



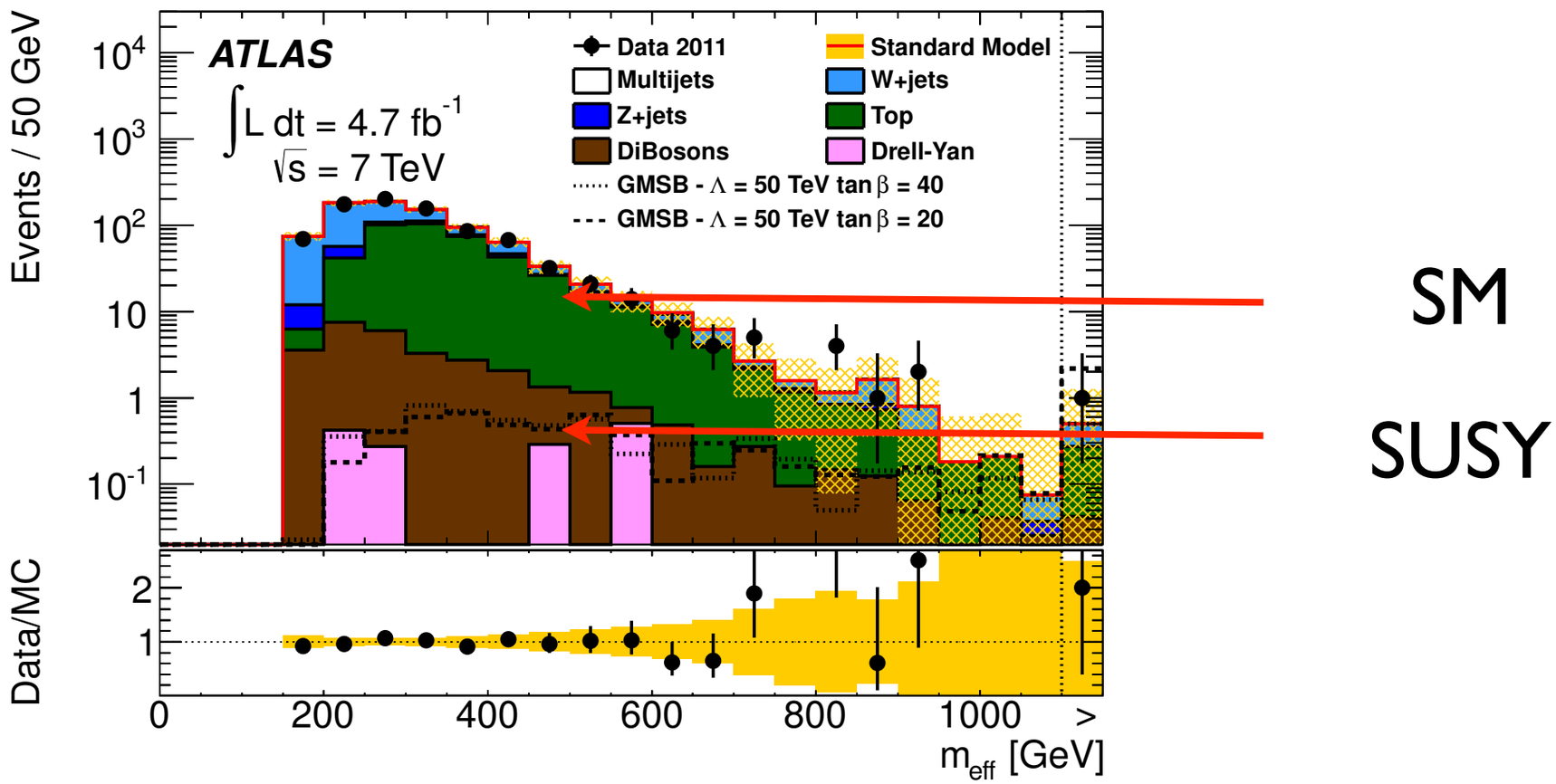
$$M_{\rm eff} = \not{E}_T + \sum_{\rm jets\ j} p_{Tj}$$

- Signatures involve jets and missing energy.
- Such signatures also appear through SM background processes.

Meff Distributions

- New physics signals can appear as excesses in Meff distributions:

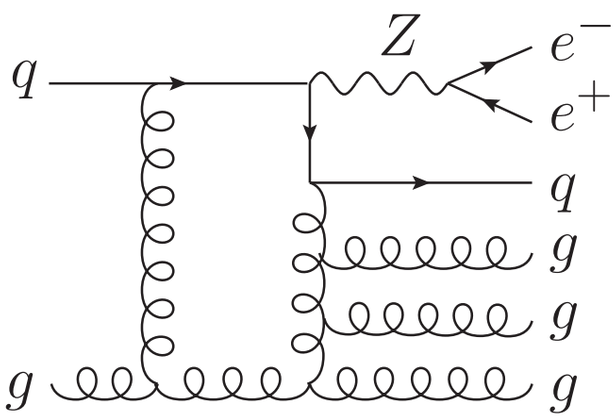
$$M_{\text{eff}} = \not{E}_T + \sum_{\text{jets } j} p_{Tj}$$

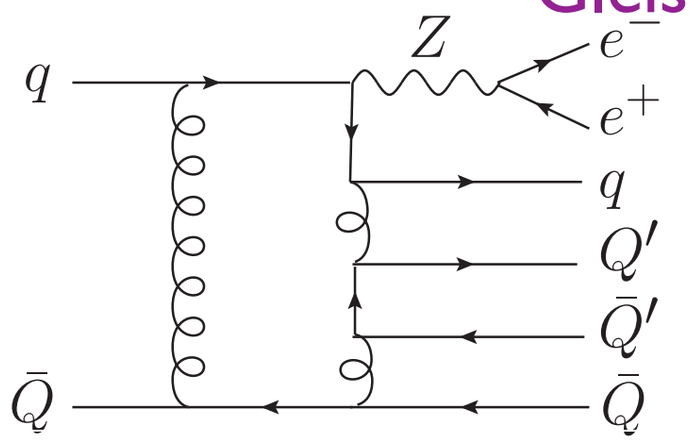


- Precise SM predictions for Meff distribution required for interpreting new physics.

Fixed Order Calculations

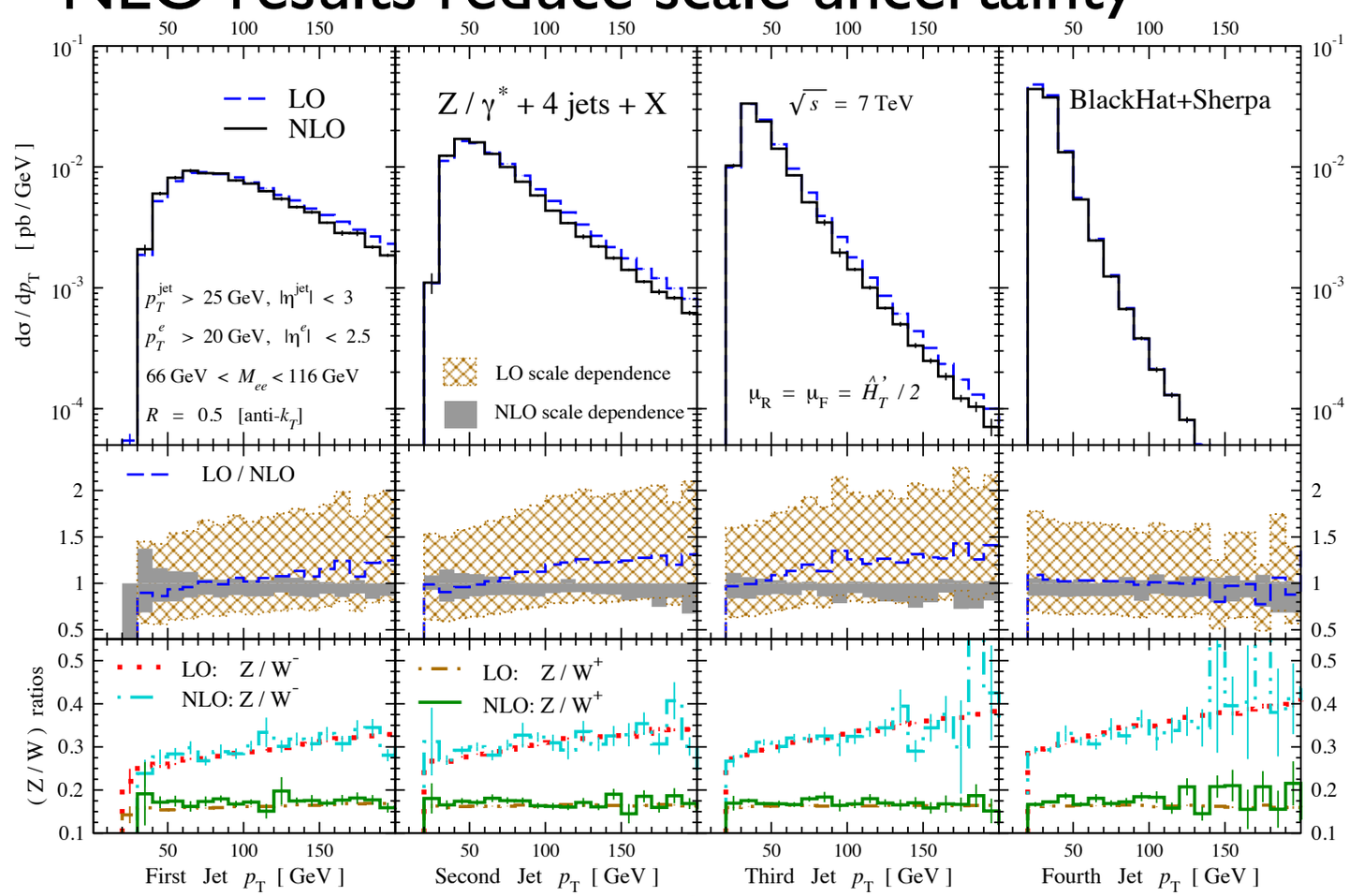
- W,Z +2,3,4 jets known at NLO (Ellis, Campbell; Ellis, Melnikov,Zanderighi,Dixon; Berger, Bern, Dixon, Febres Cordero,Forde, Gleisberg,Ita, Kosower, Maitre,...)



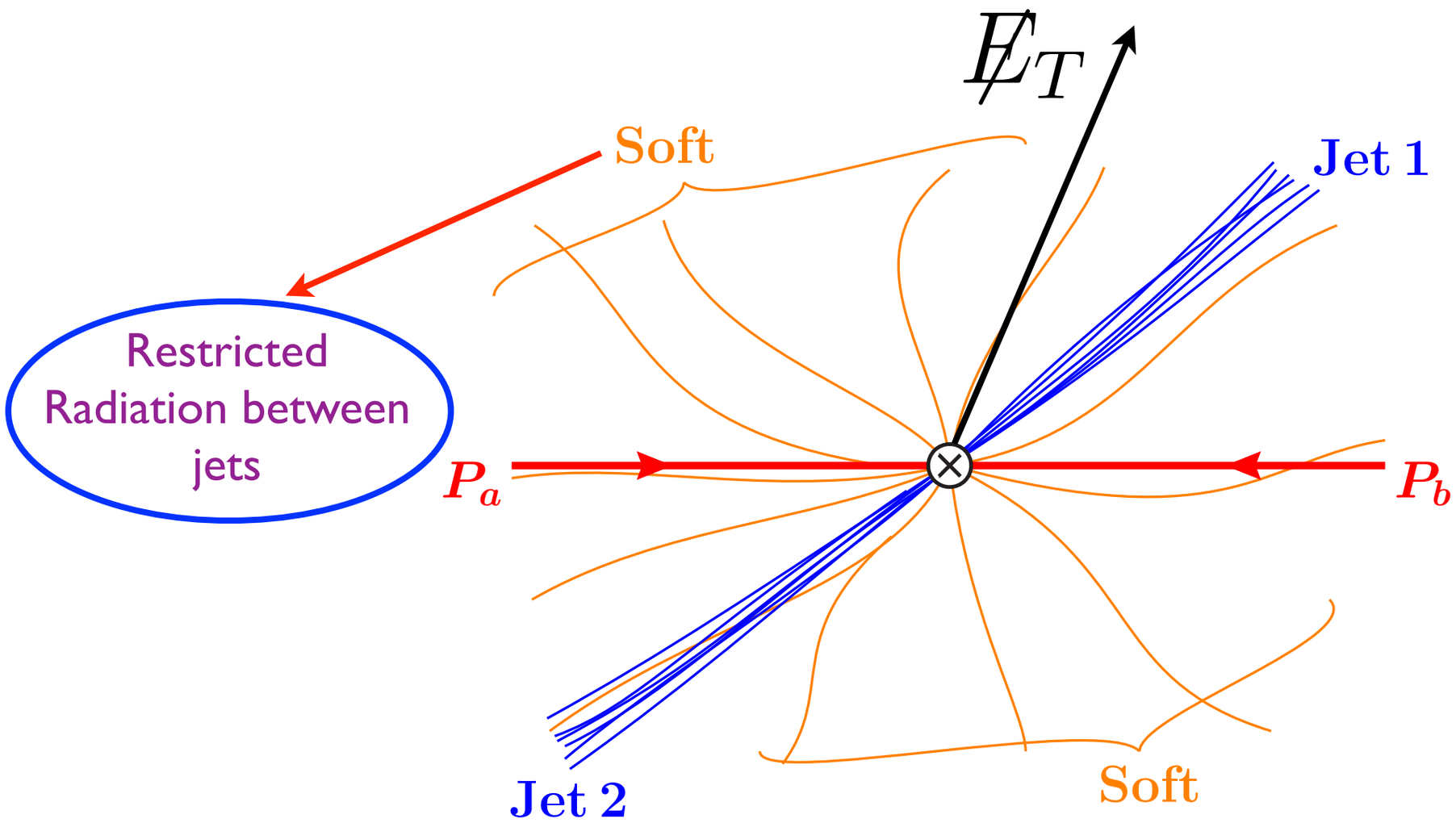


- Diboson+1-jet at NLO (Campbell,Hartanto,Williams)
- ...

- NLO results reduce scale uncertainty



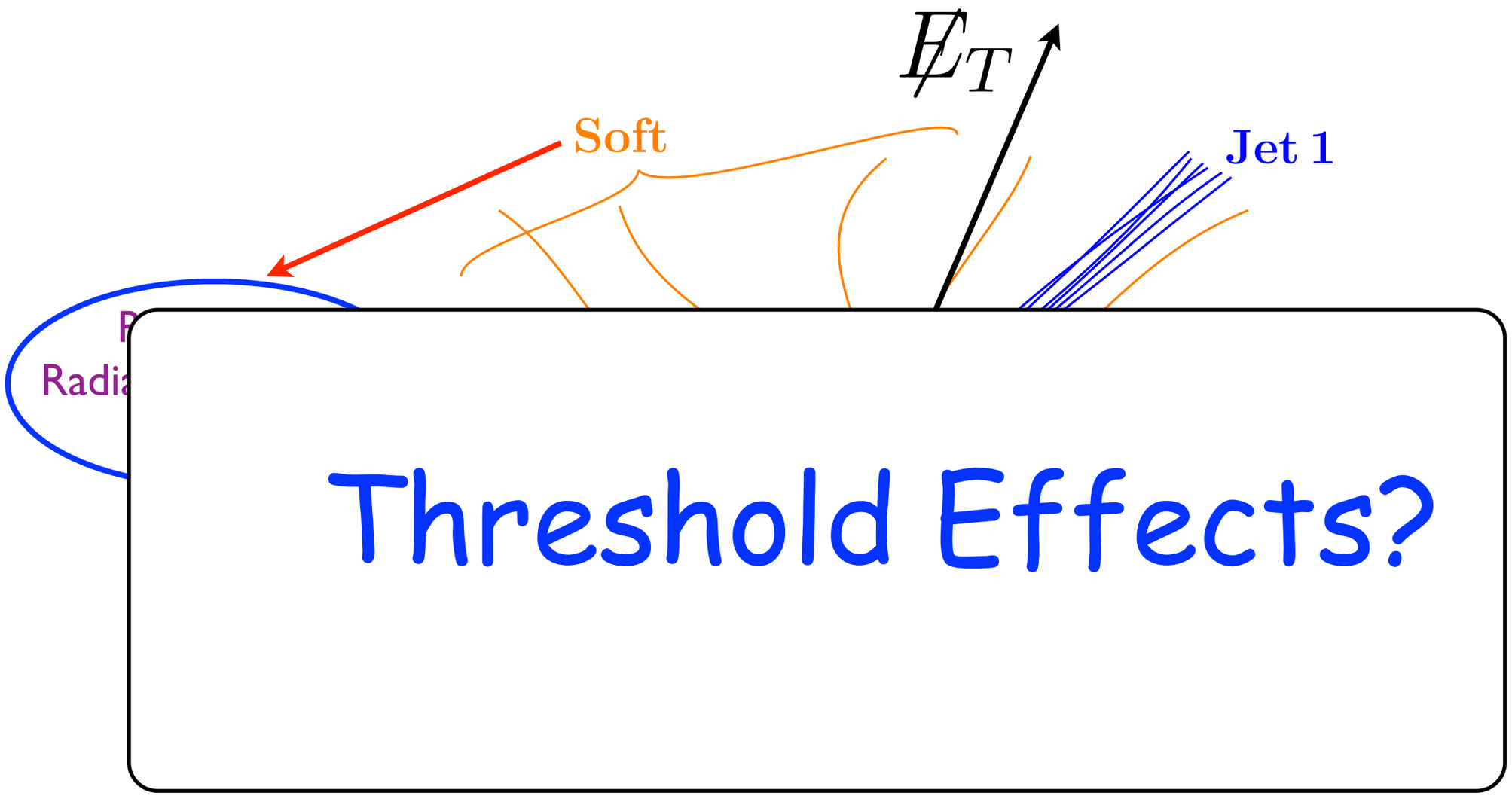
Jet Production Near Threshold



$$M_{\text{eff}} = \not{E}_T + \sum_{\text{jets } j} p_{Tj}$$

- Soft radiation between jets is restricted for large Meff.

Jet Production Near Threshold



$$M_{\text{eff}} = \not{E}_T + \sum_{\text{jets } j} p_{Tj}$$

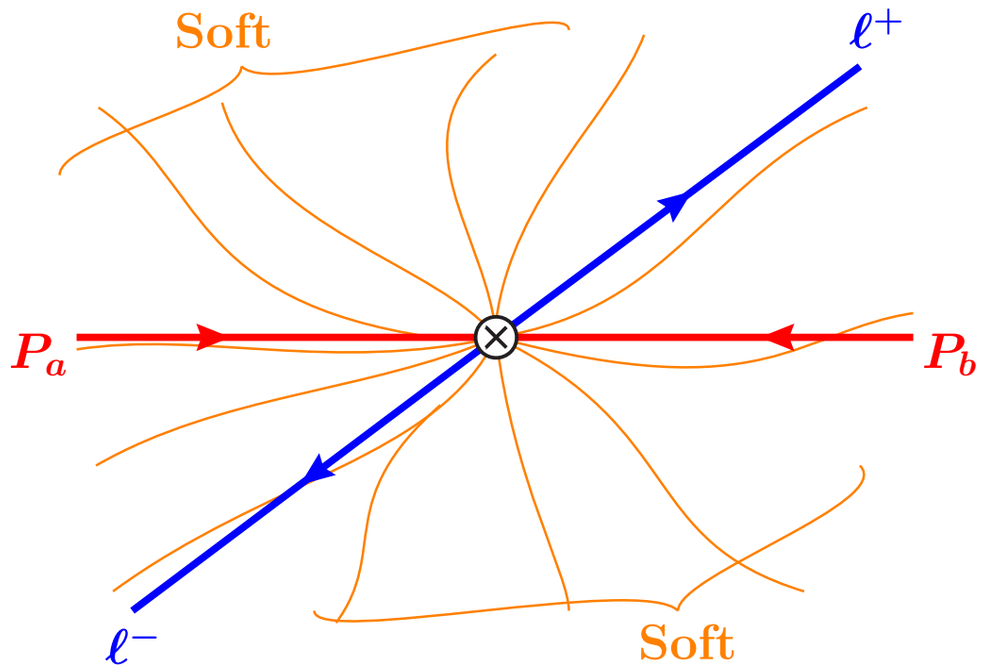
- Soft radiation between jets is restricted for large Meff.

Drell-Yan at Threshold

(A more familiar example)

Sonny Mantry, NU & ANL

Drell-Yan at Threshold



(Sterman, Catani, Trentadue,...)

$$\tau = \frac{M^2}{s} \quad , \quad z = \frac{M^2}{\hat{s}} = \frac{\tau}{x_1 x_2} \quad , \quad \tau \leq z \leq 1$$

- Hadronic threshold

$$\tau \longrightarrow 1$$

- Partonic threshold

$$z \longrightarrow 1$$

Drell-Yan at Threshold

- Factorized cross-section near threshold:

$$\frac{d^2\sigma}{dM^2 dY} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_{i,j} \int dx_1\, dx_2\, \widetilde{C}_{ij}(x_1, x_2, s, M, \mu_f)\, f_{i/N_1}(x_1, \mu_f)\, f_{j/N_2}(x_2, \mu_f)$$

- NLO expression for the partonic cross-section

$$\begin{aligned}
\frac{C_{q\bar{q}}}{e_q^2} &= \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right] \\
&+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ + 1-z\right]\right. \\
&\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right\}
\end{aligned}$$

(Altarelli; Anastasiou, Dixon, Melnikov, Petriello)

Drell-Yan at Threshold

- Factorized cross-section near threshold:

$$\frac{d^2\sigma}{dM^2 dY} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_{i,j} \int dx_1\, dx_2\, \widetilde{C}_{ij}(x_1, x_2, s, M, \mu_f)\, f_{i/N_1}(x_1, \mu_f)\, f_{j/N_2}(x_2, \mu_f)$$

- NLO expression for the partonic cross-section

$$\begin{aligned}\frac{C_{q\bar{q}}}{e_q^2} &= \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right] \\ &+ \frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ + 1 - z\right]\right. \\ &\left. + \frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right) - 2(1-z)\right]\right\}\end{aligned}$$

(Altarelli; Anastasiou, Dixon, Melnikov, Petriello)

- Two disparate scales appear in the partonic cross-section

Drell-Yan at Threshold

- Factorized cross-section near threshold:

$$\frac{d^2\sigma}{dM^2 dY} = \frac{4\pi\alpha^2}{3N_c M^2 s} \sum_{i,j} \int dx_1\, dx_2\, \widetilde{C}_{ij}(x_1, x_2, s, M, \mu_f)\, f_{i/N_1}(x_1, \mu_f)\, f_{j/N_2}(x_2, \mu_f)$$

- NLO expression for the partonic cross-section

$$\frac{C_{q\bar{q}}}{e_q^2} =$$

Threshold Logarithms

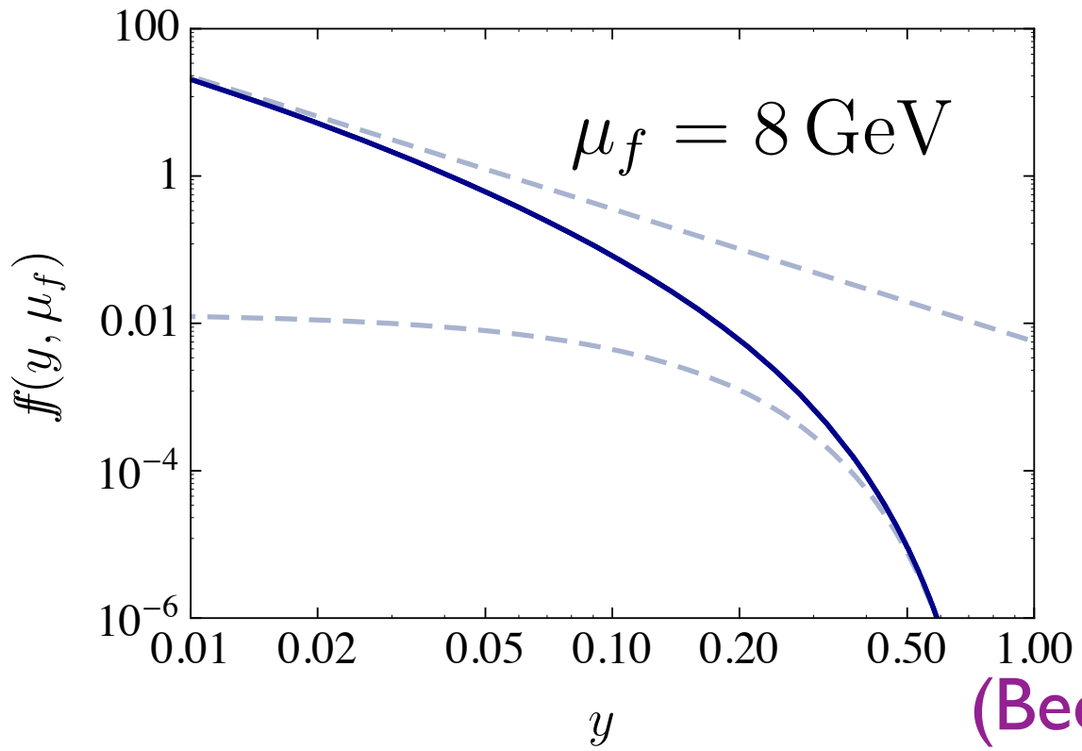
$$\alpha_s^n \left(\frac{\log^m(1-z)}{1-z} \right)_+$$

(Altarelli; Anastasiou, Dixon, Melnikov, Petriello)

- Two disparate scales appear in the partonic cross-section

Dynamical Enhancement of Threshold Effects

(Appell,Sterman,Mackenzie; Catani,Mangano,Nason)



(Becher,Neubert)

$$\tau = \frac{M^2}{s} \quad , \quad z = \frac{M^2}{\hat{s}} = \frac{\tau}{x_1 x_2} \quad , \quad \tau \leq z \leq 1$$

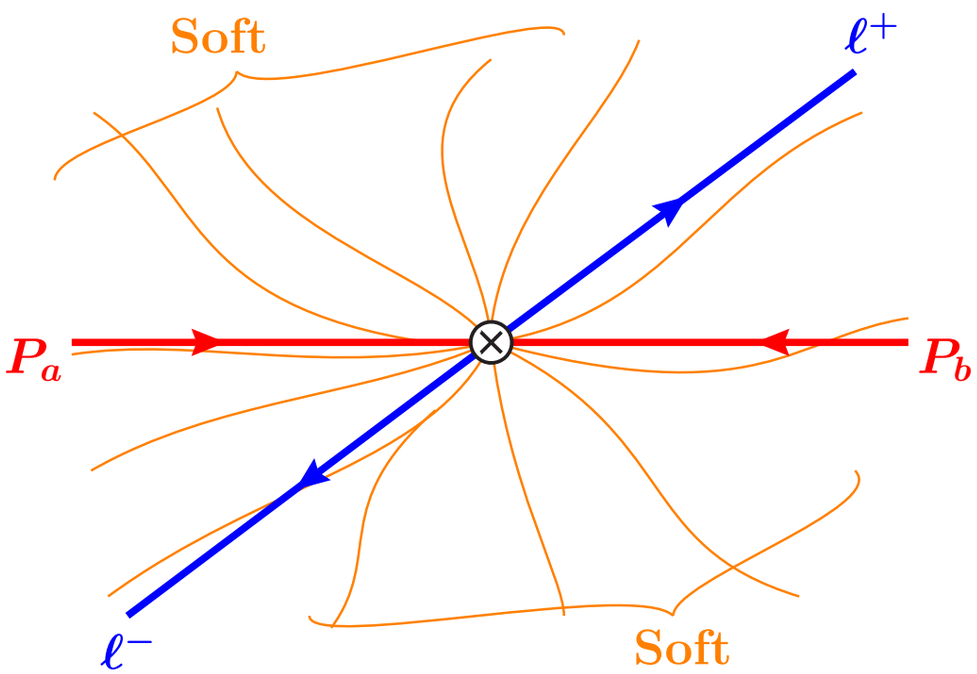
- Threshold effects can be important even away from hadronic threshold

$$\frac{d\sigma^{\mathrm{thresh}}}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^2 s} \int_\tau^1 \frac{dz}{z} C(z, M, \mu_f)\, \mathbb{f}(\tau/z, \mu_f)$$

- Parton luminosity function favors large z region enhancing partonic threshold effects.

Factorization for Drell-Yan at Threshold

(Sterman, Catani, Trentadue)



$$\mu_h \sim M \longrightarrow \text{Hard Scale}$$

$$\mu_s \sim \sqrt{\hat{s}}\,(1-z) \longrightarrow \text{Soft scale}$$

$$\tau = \frac{M^2}{s} \quad , \quad z = \frac{M^2}{\hat{s}} = \frac{\tau}{x_1 x_2} \quad , \quad \tau \leq z \leq 1$$

- Cross-section

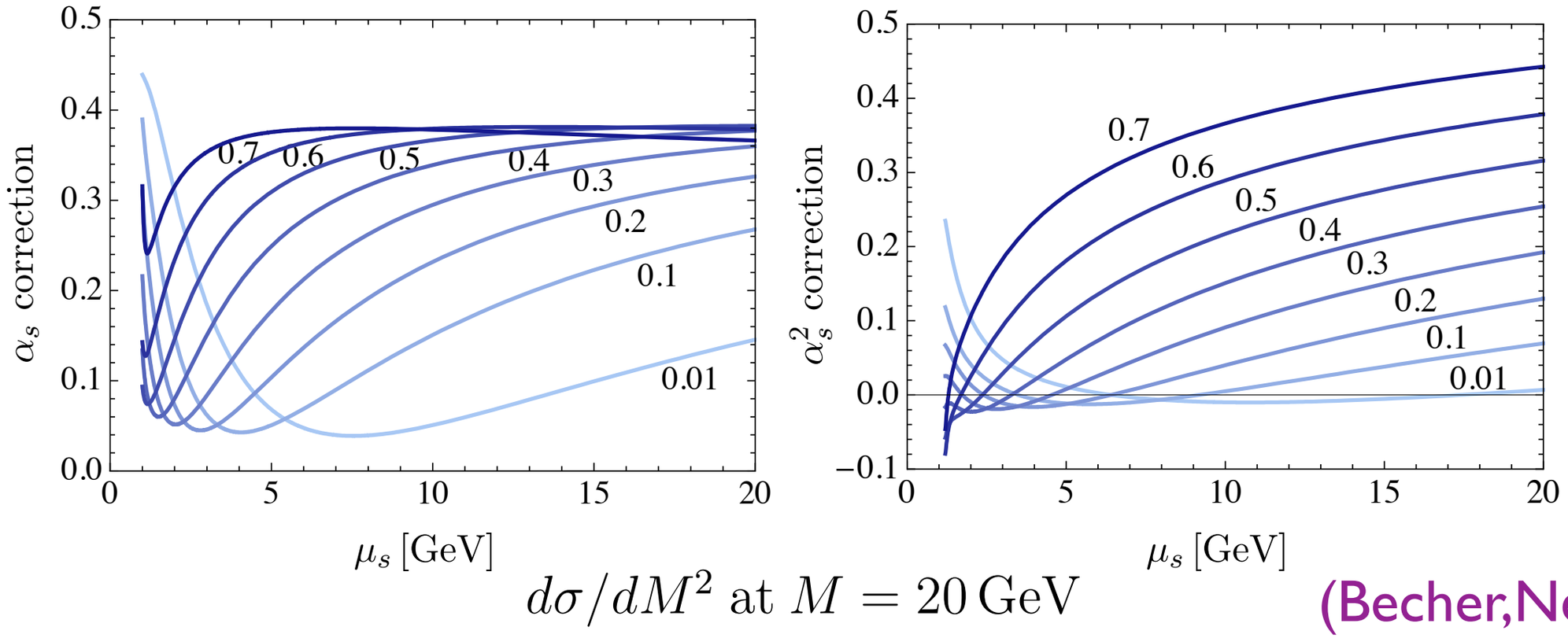
$$\frac{d\sigma^{\text{thresh}}}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^2 s} \int_\tau^1 \frac{dz}{z}\, C(z, M, \mu_f)\, \mathbb{f}(\tau/z, \mu_f)$$

- Factorization of hard and soft scales (Belitsky; Becher, Neubert)

$$C(z, M, \mu_f) = H(M, \mu_f)\, S(\sqrt{\hat{s}}\,(1-z), \mu_f)$$

Dynamic Soft Scale

Relative contribution from NLO and NNLO soft function

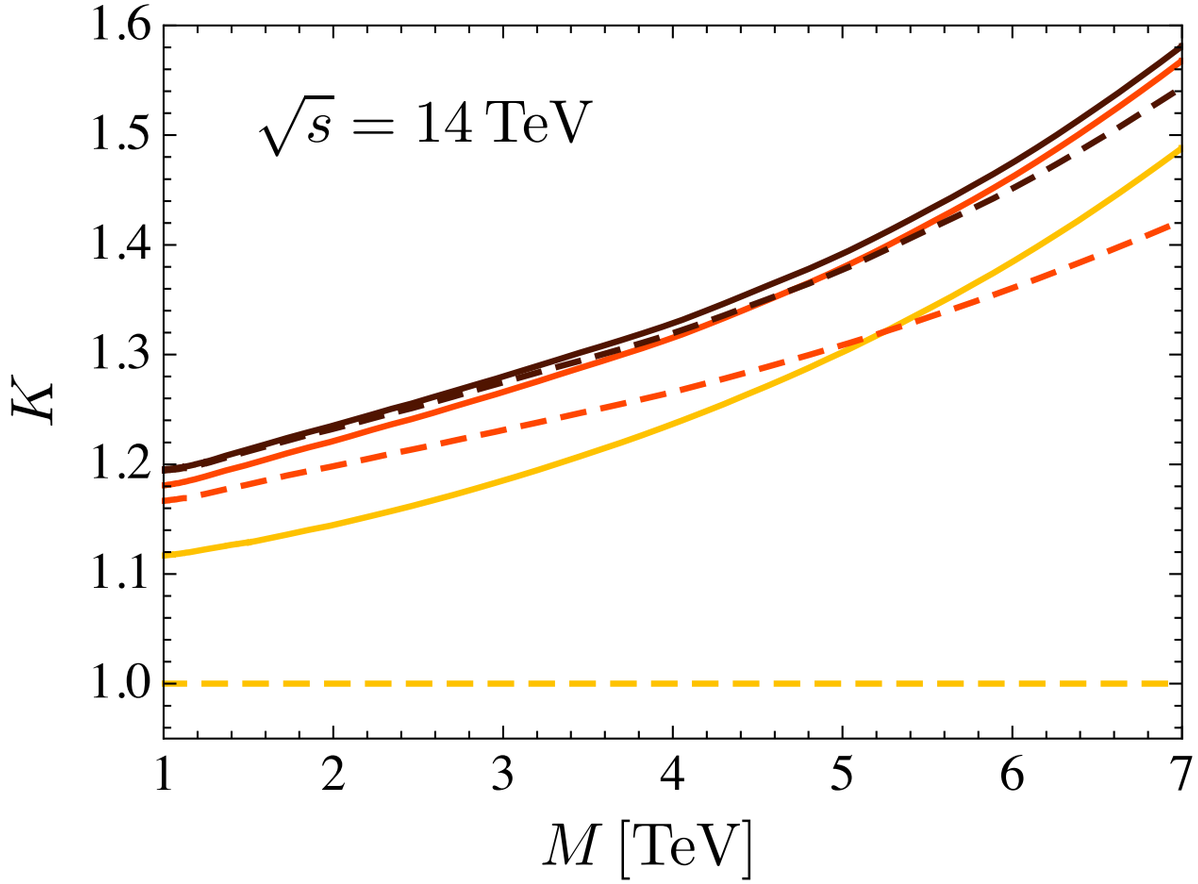


(Becher,Neubert)

$$\mu_s \approx M(1-\tau)$$ → Naive Choice

$$\mu_s \approx \frac{M(1-\tau)}{(2+b_d+b_{\bar{d}})} \approx \frac{M(1-\tau)}{13}$$ → Dynamical Choice

Threshold Resummation for Drell Yan



$$\frac{d\sigma}{dM^2} = K(M^2, \tau)\, \frac{d\sigma}{dM^2}\bigg|_{\mathrm{LO}}$$

(Becher,Neubert)

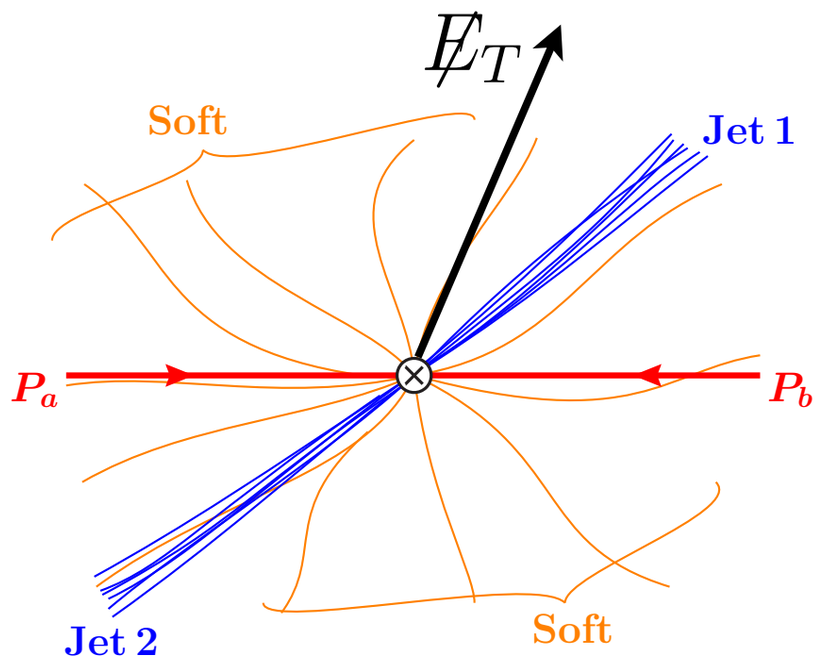
- Resummation known to NNNLL accuracy.

(Mukherjee, Vogelsang; Bolzoni; Ravindran,Smith,van Neervan;Becher,Neubert)

Multi-jet Production near Threshold

Sonny Mantry, NU & ANL

Jet Production Near Threshold



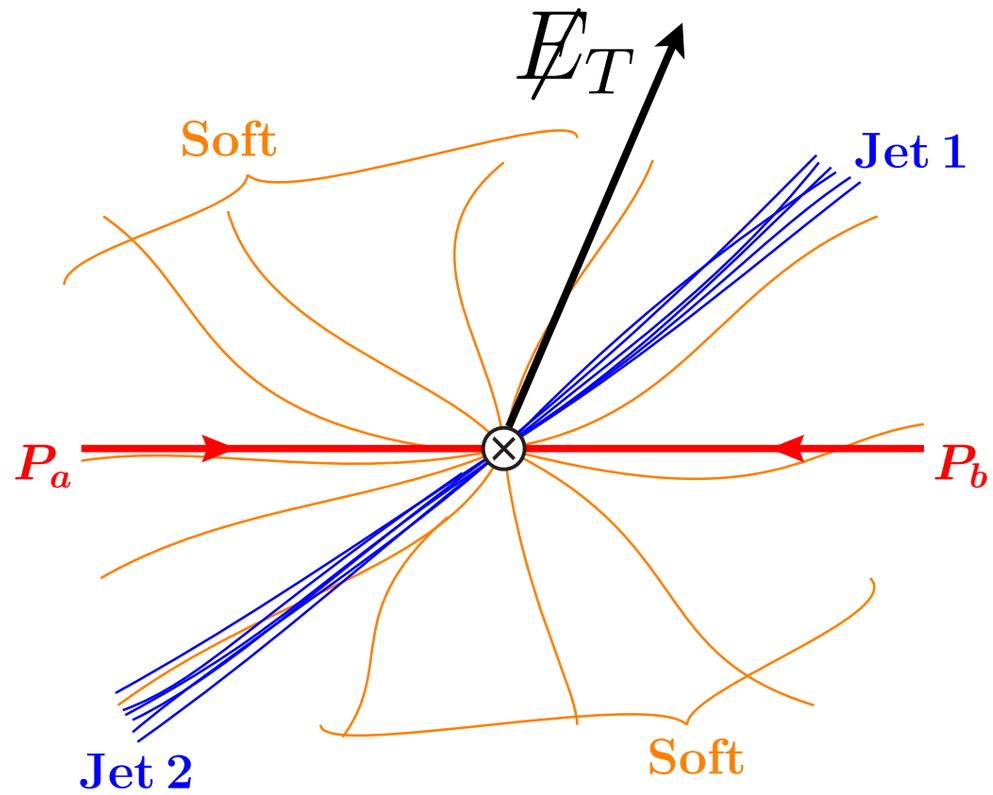
- Factorization and NLL resummation has been studied for dijet production. (Kidonakis,Oderda,Sterman)

- The treatment of multiple jets usually requires jet algorithms to be implemented.

- Dijet production at machine threshold was recently studied using event-shapes. (Kelley,Schwartz)

- The structure of factorization with jet algorithms was studied in SCET. (Bauer,Dunn,Hornig)

Jet Production Near Threshold



$$\hat{s}_{min} = (q + \sum_{i=1}^{N} p_J^i)^2, \qquad p_J \equiv (p_J^T \cosh \eta_J, \mathbf{p}_J^T, p_J^T \sinh \eta_J)$$

$$z = \frac{\hat{s}_{min}}{\hat{s}}, \qquad \tau = \frac{\hat{s}_{min}}{s}, \qquad \tau \leq z \leq 1$$

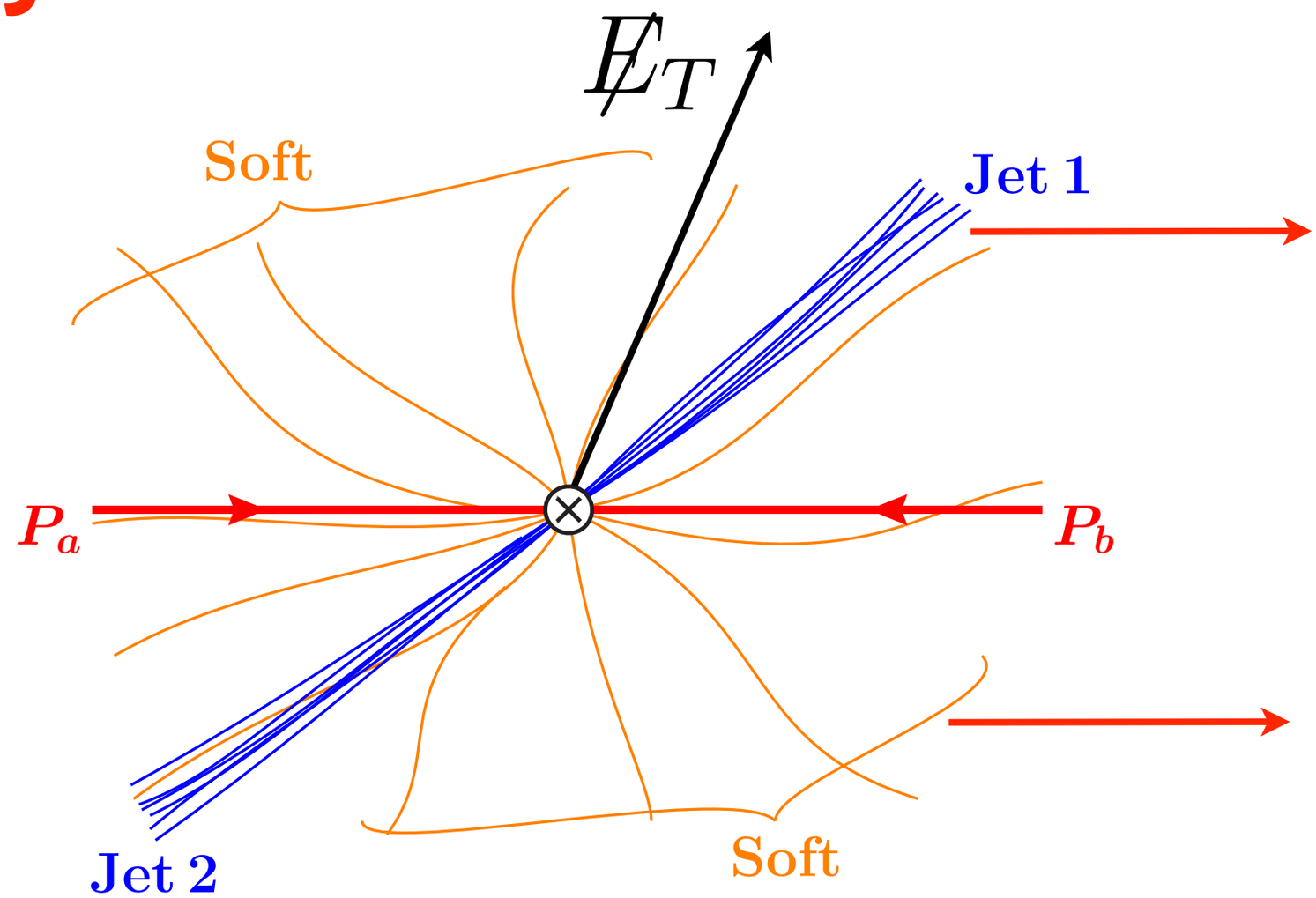
- Hadronic threshold

$$\tau \longrightarrow 1$$

- Partonic threshold

$$z \longrightarrow 1$$

Jet Production Near Threshold



Collinear modes

$$p_c \sim \sqrt{\hat{s}}(\lambda^2, 1, \lambda)$$

$$\lambda \sim \sqrt{1-z}.$$

Soft modes

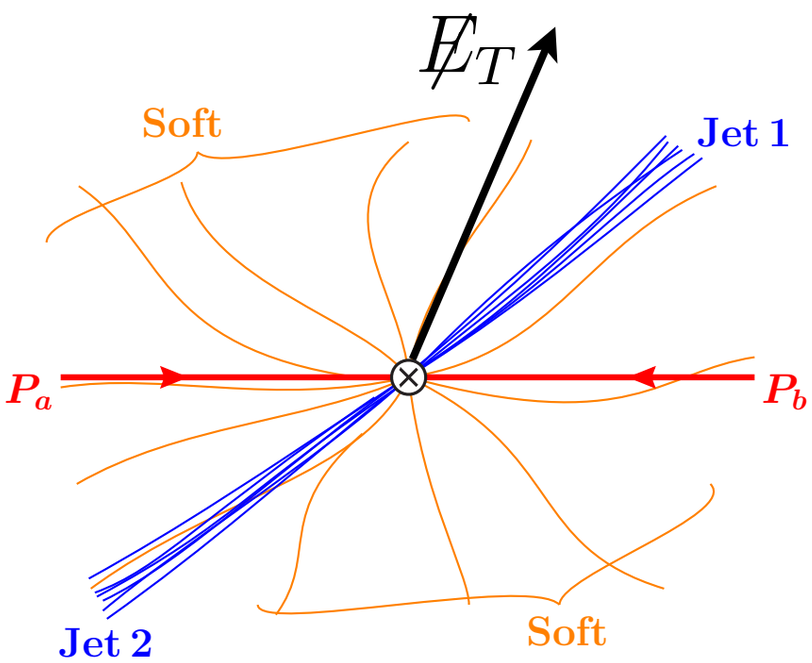
$$k_s \sim \sqrt{\hat{s}}(\lambda^2, \lambda^2, \lambda^2)$$

- Physics dominated by soft radiation and collinear radiation of jets.

$$1 - z = \frac{2}{\hat{s}}\left[p_I \cdot k_{out} + p_I^0 \sum_i^N (p_i^{c+} + k_i^+)\right]$$

Meff Distributions

$$M_{\rm eff} = \not{E}_T + \sum_{\text{jets } j} p_{Tj}$$



Multiple Scales

$$m_{eff} \gg m_{eff}\sqrt{1-z} \gg m_{eff}(1-z) \gg \Lambda_{QCD},$$

Hard | Jet | Soft | Non-Perturbative

- Schematic factorization formula

$$d\sigma = H \otimes J_1 \otimes J_2 \cdots \otimes J_N \otimes S \otimes f \otimes f$$

Hard function

Jet functions; jets defined by jettiness event shape

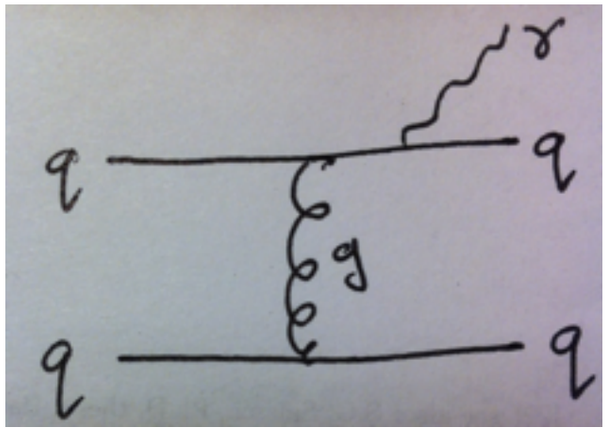
Threshold Soft function

PDFs

Photon + 2-Jets

Sonny Mantry, NU & ANL

Photon + 2-Jets



- Partonic channels

$$qq' \to qq'\gamma, \qquad \bar{q}\bar{q}' \to \bar{q}\bar{q}'\gamma, \qquad q\bar{q}' \to q\bar{q}'\gamma,$$

$$qg \to qg\gamma, \qquad \bar{q}g \to \bar{q}g\gamma, \qquad q\bar{q} \to gg\gamma, \qquad gg \to q\bar{q}\gamma.$$

- Color structure of SCET operators

$$\begin{aligned}
\theta_1^{\alpha\beta\gamma\delta} &= (\bar{\chi}_2^\alpha\, t^a\, \chi_1^\beta)\,(\bar{\chi}_4^\gamma\, t^a\, \chi_3^\delta), \\
\theta_2^{\alpha\beta\gamma\delta} &= (\bar{\chi}_2^\alpha\, \mathbf{1}\, \chi_1^\beta)\,(\bar{\chi}_4^\gamma\, \mathbf{1}\, \chi_3^\delta).
\end{aligned}$$

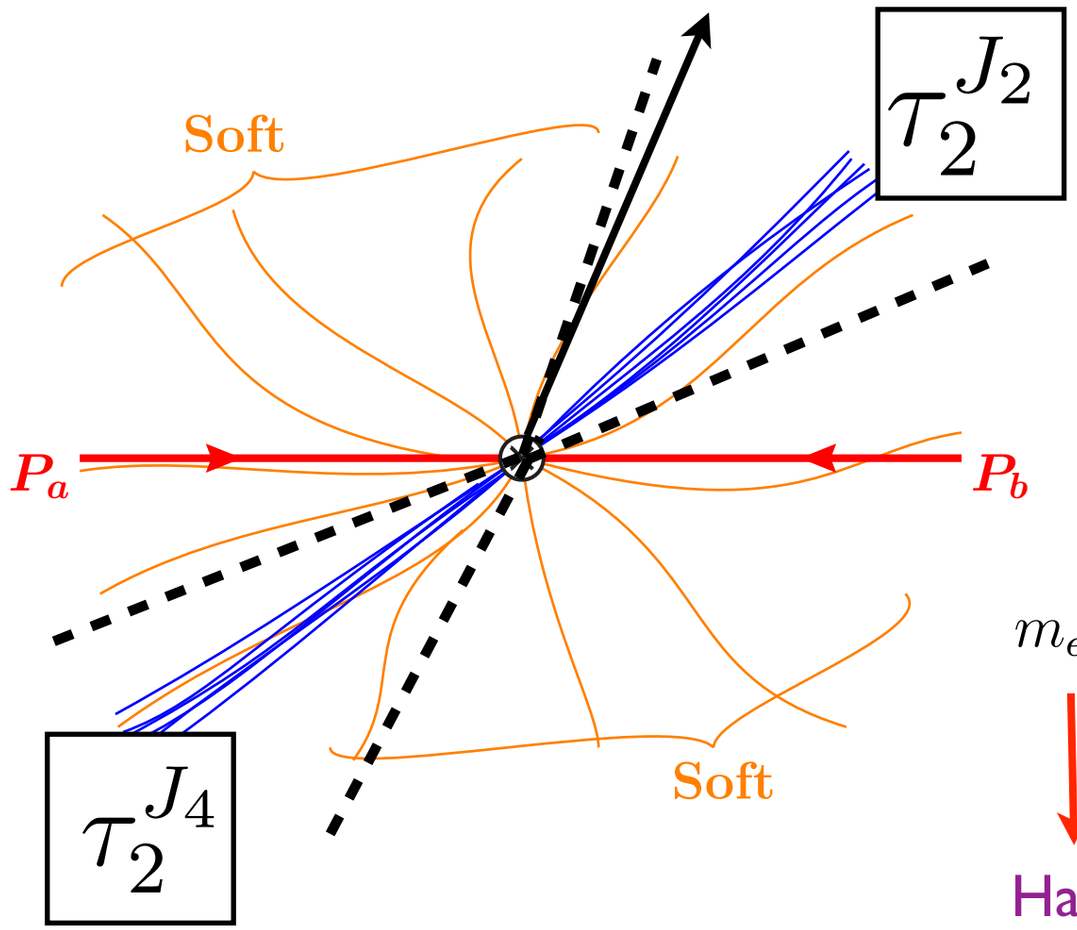
$$\begin{aligned}
\Theta_{1,\mu\nu}^{\alpha\beta} &= (\bar{\chi}_2^\alpha\, t^{a_1} t^{a_3}\, \chi_4^\beta)\, A_\mu^{a_1} A_\nu^{a_3} \\
\Theta_{2,\mu\nu}^{\alpha\beta} &= (\bar{\chi}_2^\alpha\, t^{a_3} t^{a_1}\, \chi_4^\beta)\, A_\mu^{a_1} A_\nu^{a_3} \\
\Theta_{3,\mu\nu}^{\alpha\beta} &= (\bar{\chi}_2^\alpha\, \delta^{a_1 a_3}\, \chi_4^\beta)\, A_\mu^{a_1} A_\nu^{a_3}.
\end{aligned}$$

- RG evolution of these SCET operators was studied at NNLL

(Kelley,Schwartz)

Photon + 2-Jets

$$\hat{\tau}_2 \equiv \tau_2^{J_2} + \tau_2^{J_4}$$



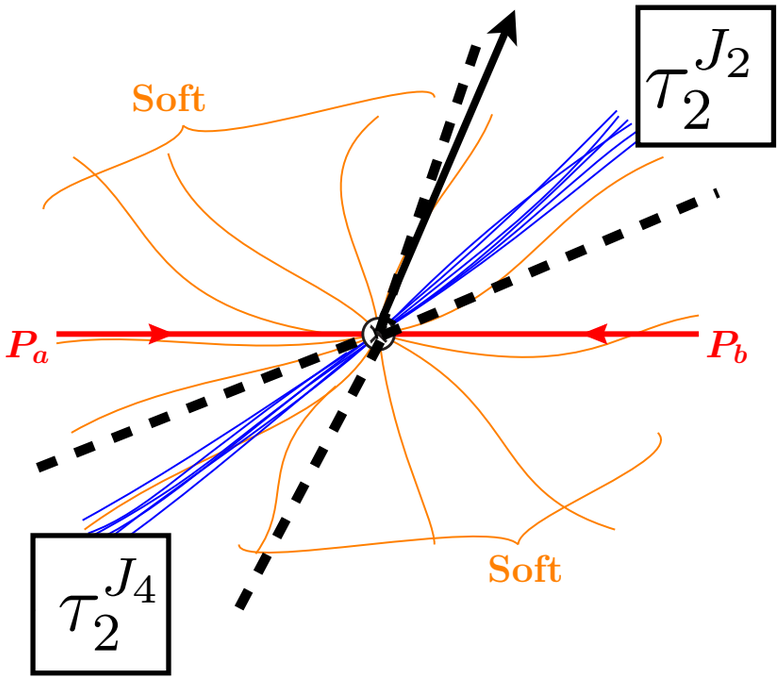
$$m_{eff} \gg m_{eff}\sqrt{1-z} \gg m_{eff}(1-z) \gg \Lambda_{QCD}$$

Hard | Jet | Soft | Non-Perturbative

- Factorization formula differential in phase space and jettiness

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{q_{J_2}}\,\mathrm{d}\Phi_{q_{J_4}}\,\mathrm{d}\Phi_q \mathrm{d}\hat{\tau}_2}$$

Photon + 2-Jets



$$\hat{\tau}_2 \equiv \tau_2^{J_2} + \tau_2^{J_4}$$

- Factorization formula differential in phase space and jettiness

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{q_{J_2}}\,\mathrm{d}\Phi_{q_{J_4}}\,\mathrm{d}\Phi_q \mathrm{d}\hat{\tau}_2} \;=\; & \frac{1}{N_s}\frac{(2\pi)^4}{Q^4\tau}\int_\tau^1 \mathrm{d}z \int_{-\frac{1}{2}\ln\frac{z}{\tau}}^{\frac{1}{2}\ln\frac{z}{\tau}} \mathrm{d}y \int \mathrm{d}s_{J_2}\,\mathrm{d}s_{J_4} \int \mathrm{d}k^0_{out} \int \mathrm{d}\tau^2_{2,s}\,\mathrm{d}\tau^4_{2,s} \\
& \times\delta^2(q^T_{J_2}+q^T_{J_4}+q^T)\,\delta\left(y-\tanh^{-1}\left(\frac{q^z_{J_2}+q^z_{J_4}+q^z}{q^0_{J_2}+q^0_{J_4}+q^0}\right)\right) \\
& \times\delta\left(1-z-\frac{2\cosh y}{\sqrt{\hat{s}}}\,\hat{\tau}_2-\frac{2}{\sqrt{\hat{s}}}k^0_{out}\right)\,\delta\left(\hat{\tau}_2-\frac{s_{J_2}}{\bar{q}_{J_2}}-\frac{s_{J_4}}{\bar{q}_{J_4}}-\tau^{J_2}_{2,s}-\tau^{J_4}_{2,s}\right) \\
& \times H_{IJ}(\bar{q}_i,n_i,\mu_S;\mu_H)\,S_{JI}(k^0_{out},\tau^{J_2}_s,\tau^{J_4}_s,\mu_S) \\
& \times f(x_a,\mu_S)\,f(x_b,\mu_S)\,J_{J_2}(s_{J_2},\mu_S;\mu_J)\,J_{J_4}(s_{J_4},\mu_S;\mu_J)\;,
\end{aligned}$$

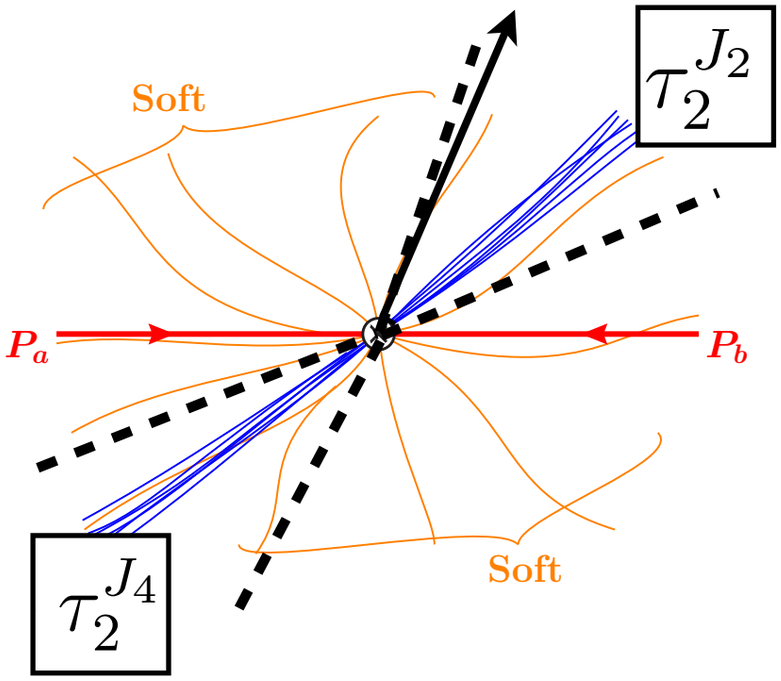
Hard function

Soft function

PDFs

Jet functions

Photon + 2-Jets



$$\hat{\tau}_2 \equiv \tau_2^{J_2} + \tau_2^{J_4}$$

- Factorization formula differential in phase space and jettiness

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{q_{J_2}}\,\mathrm{d}\Phi_{q_{J_4}}\,\mathrm{d}\Phi_q \mathrm{d}\hat{\tau}_2} \;=\; & \frac{1}{N_s}\frac{(2\pi)^4}{Q^4\tau}\int_\tau^1 \mathrm{d}z \int_{-\frac{1}{2}\ln\frac{z}{\tau}}^{\frac{1}{2}\ln\frac{z}{\tau}} \mathrm{d}y \int \mathrm{d}s_{J_2}\,\mathrm{d}s_{J_4} \int \mathrm{d}k^0_{out} \int \mathrm{d}\tau^2_{2,s}\,\mathrm{d}\tau^4_{2,s} \\
& \times\delta^2(q^T_{J_2}+q^T_{J_4}+q^T)\,\delta\left(y-\tanh^{-1}\left(\frac{q^z_{J_2}+q^z_{J_4}+q^z}{q^0_{J_2}+q^0_{J_4}+q^0}\right)\right) \\
& \times\delta\left(1-z-\frac{2\cosh y}{\sqrt{\hat{s}}}\,\hat{\tau}_2-\frac{2}{\sqrt{\hat{s}}}k^0_{out}\right)\,\delta\left(\hat{\tau}_2-\frac{s_{J_2}}{\bar{q}_{J_2}}-\frac{s_{J_4}}{\bar{q}_{J_4}}-\tau^{J_2}_{2,s}-\tau^{J_4}_{2,s}\right) \\
& \times H_{IJ}(\bar{q}_i,n_i,\mu_S;\mu_H)\,S_{JI}(k^0_{out},\tau^{J_2}_s,\tau^{J_4}_s,\mu_S) \\
& \times f(x_a,\mu_S)\,f(x_b,\mu_S)\,J_{J_2}(s_{J_2},\mu_S;\mu_J)\,J_{J_4}(s_{J_4},\mu_S;\mu_J)\;,
\end{aligned}$$

Hard function

Soft function

PDFs

Jet functions

Scale Choices

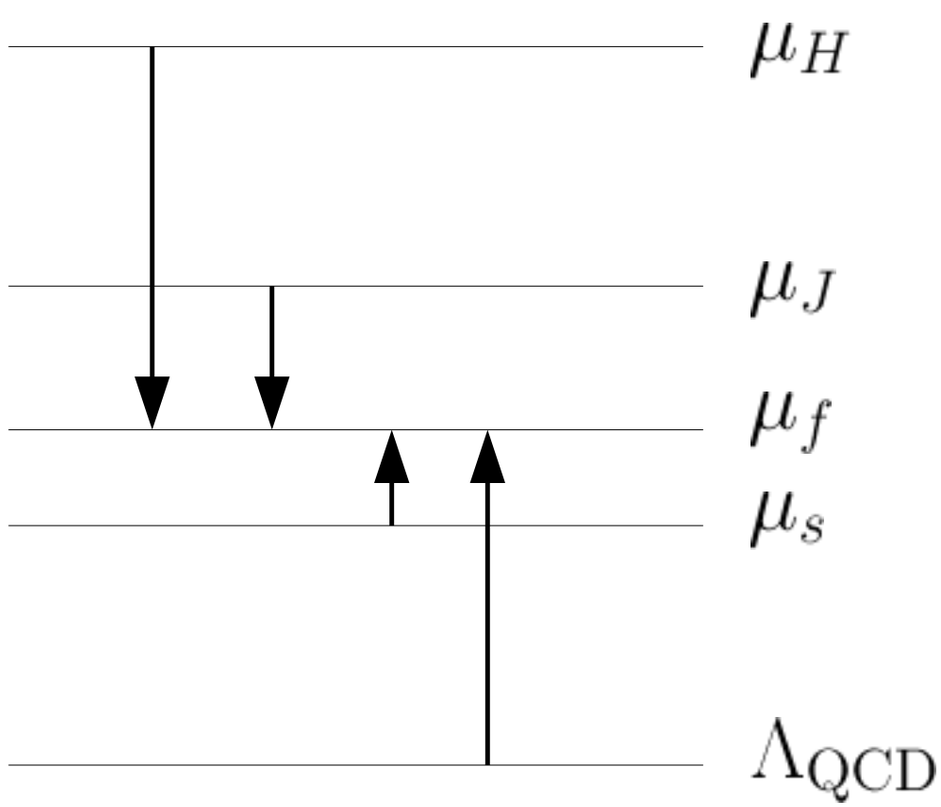
$$d\sigma = H \otimes J_1 \otimes J_2 \cdots \otimes J_N \otimes S \otimes f \otimes f$$

Hard

Jet functions

Soft function

PDFs



- Optimal scale choices can be determined by minimizing the NLO contributions of the hard function, jet function, and soft function.

Scale Choices

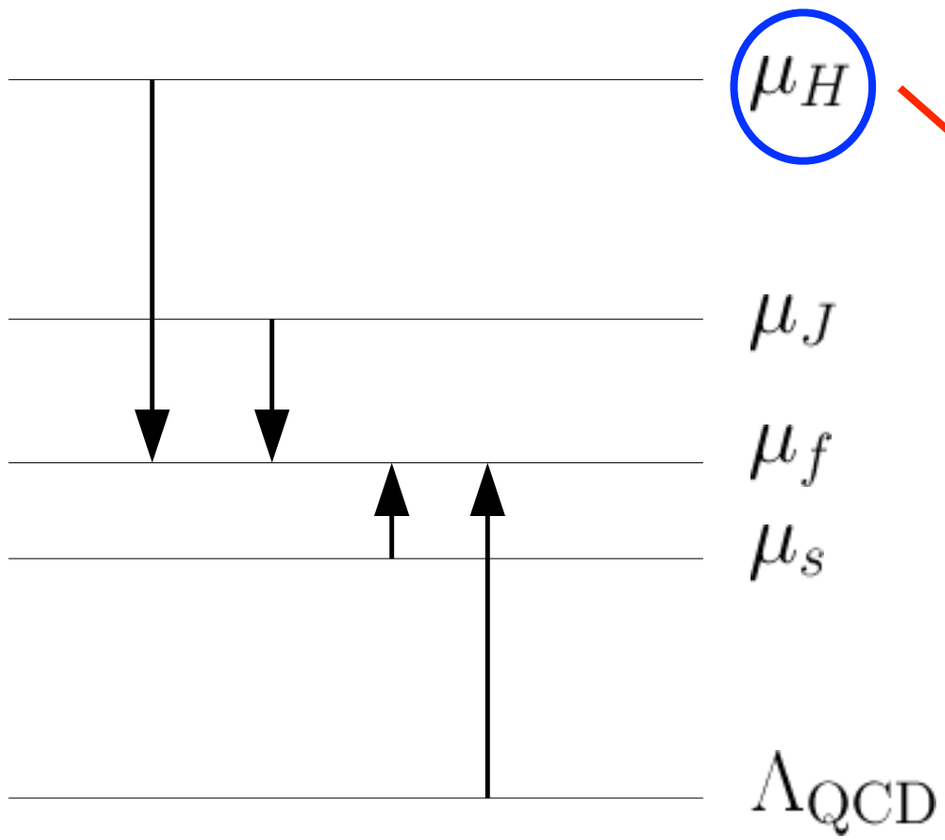
$$d\sigma = H \otimes J_1 \otimes J_2 \cdots \otimes J_N \otimes S \otimes f \otimes f$$

Hard

Jet functions

Soft function

PDFs



- We made the scale choice

$$\mu_H \sim \sqrt{P_{J1}^T P_{J2}^T}$$

Scale Choices

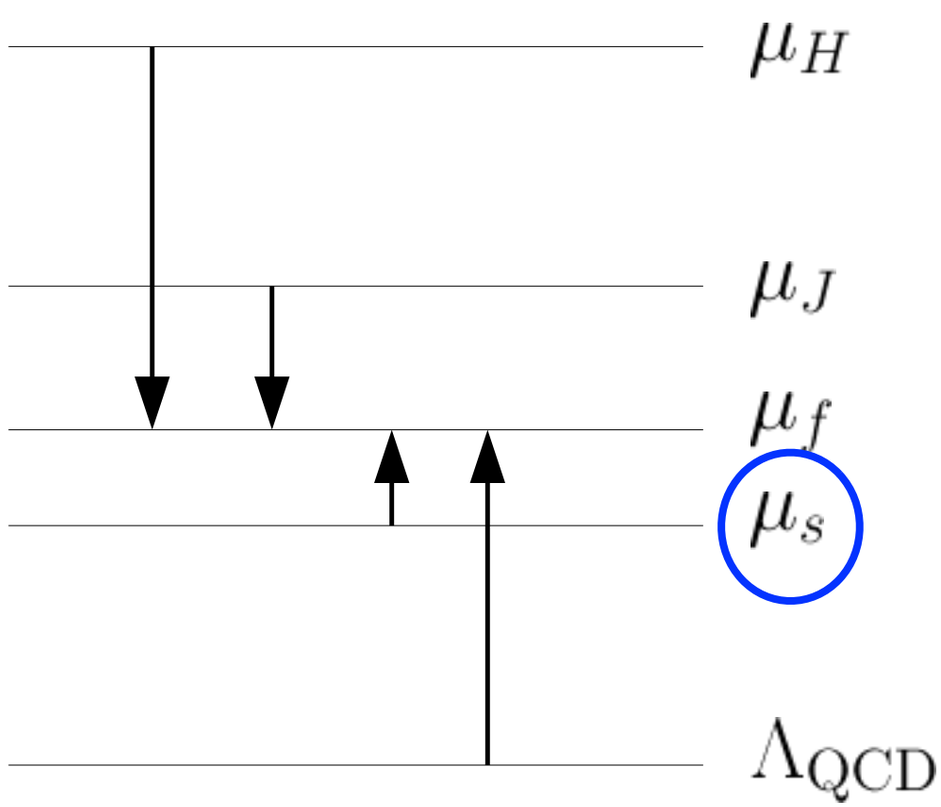
$$d\sigma = H \otimes J_1 \otimes J_2 \cdots \otimes J_N \otimes S \otimes f \otimes f$$

Hard

Jet functions

Soft function

PDFs



- Soft scale determined by see-saw relation:

$$\mu_S \sim \frac{\mu_J^2}{\mu_H}$$

Scale Choices

$$d\sigma = H \otimes J_1 \otimes J_2 \cdots \otimes J_N \otimes S \otimes f \otimes f$$

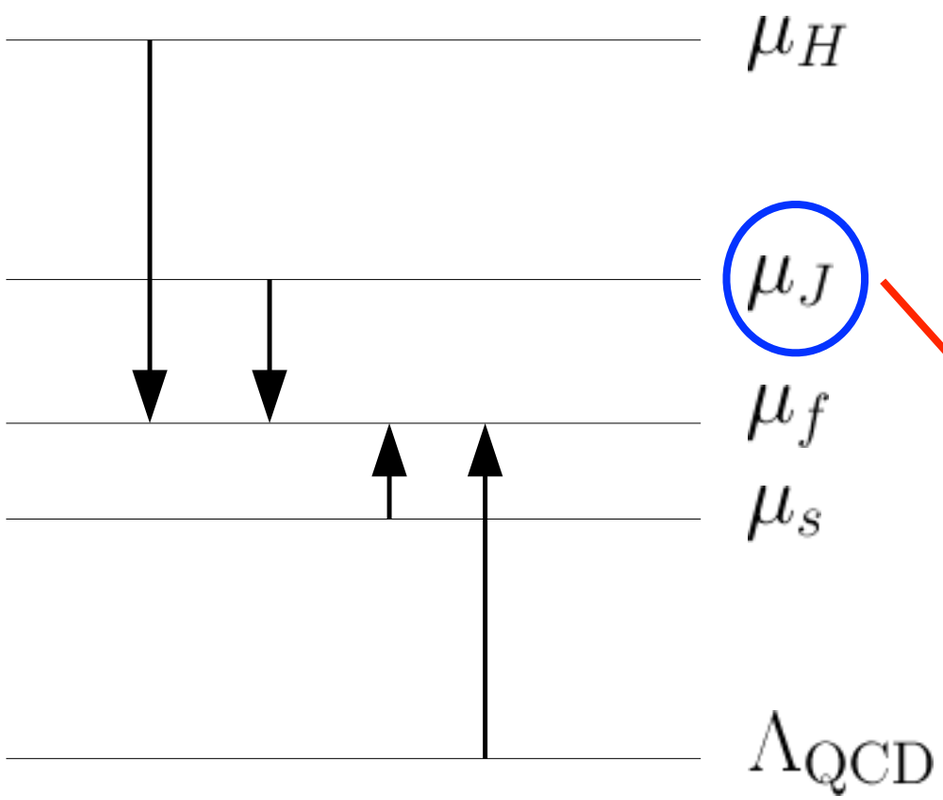
Hard

Jet functions

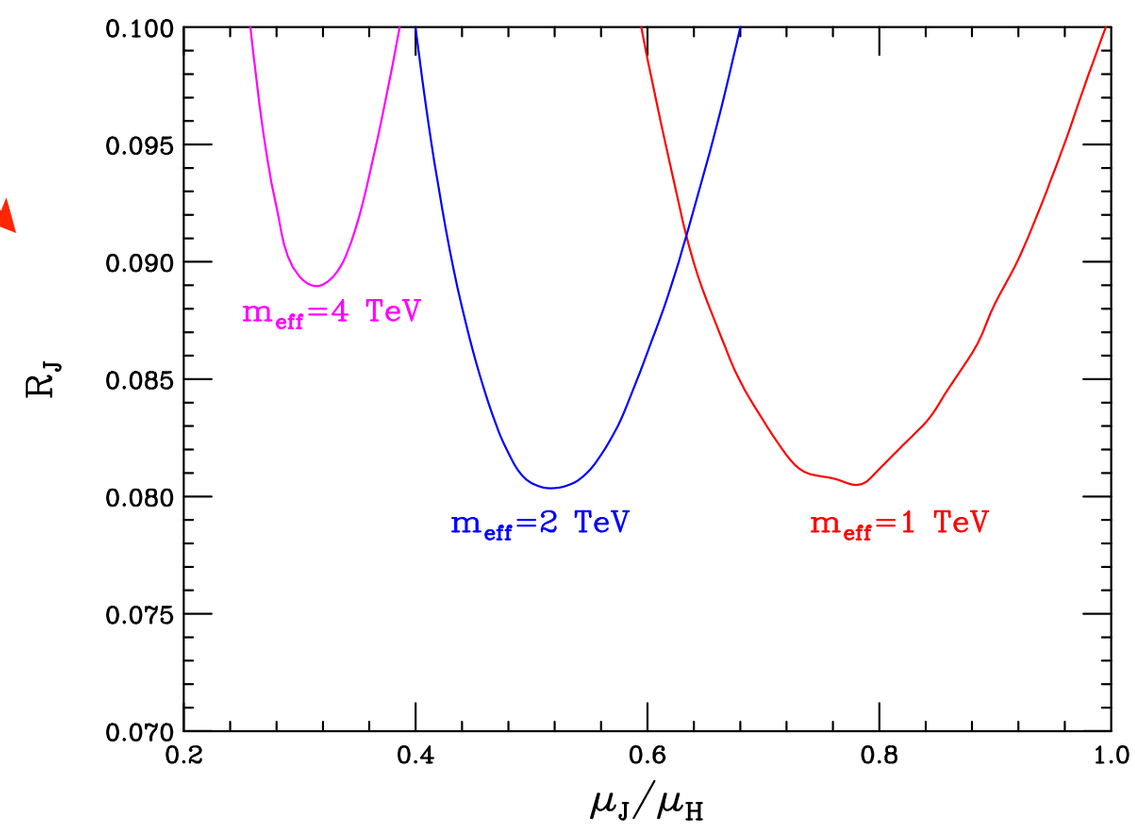
Soft function

PDFs

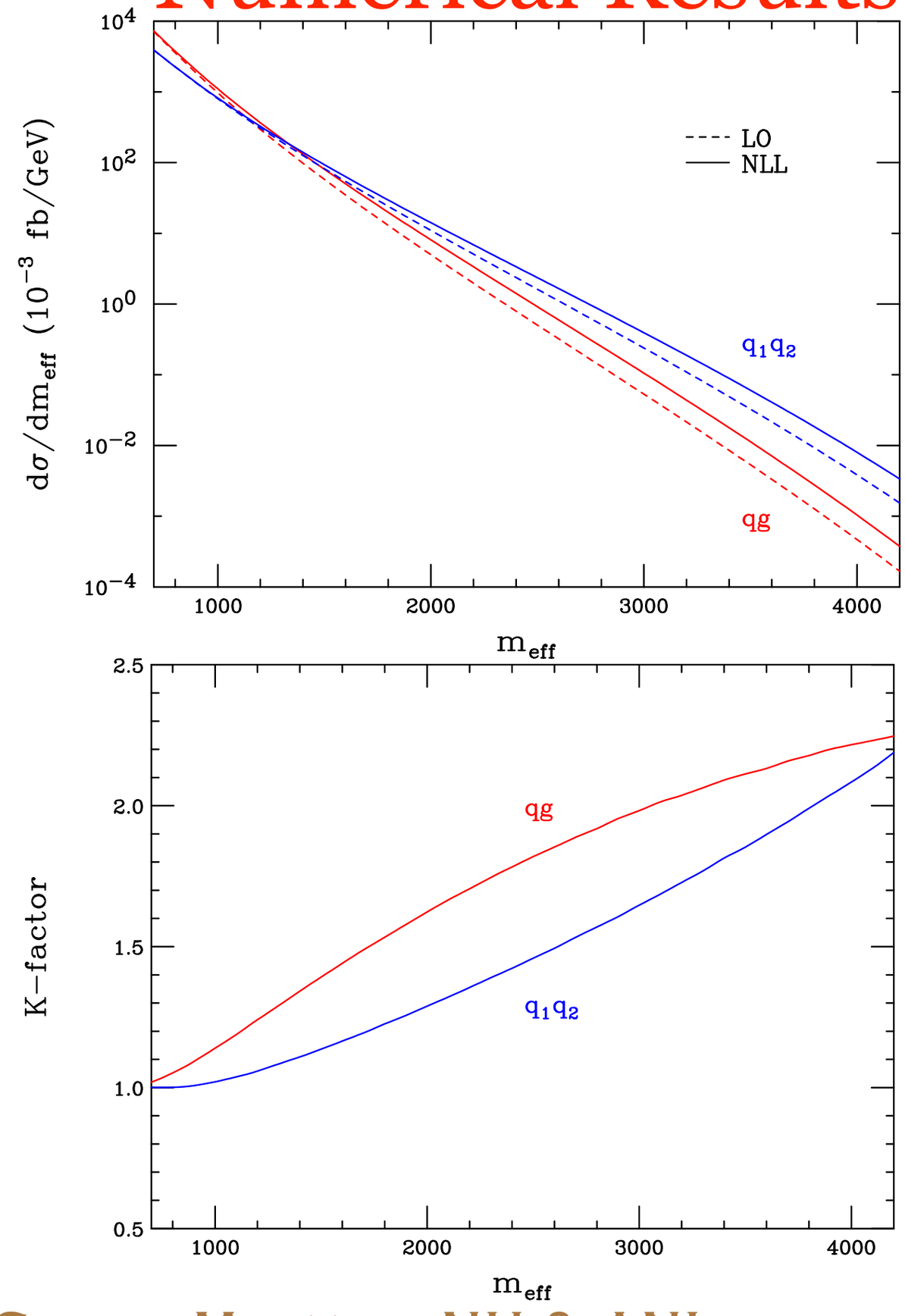
- Minimize NLO jet function contribution.



$$R_J = \frac{d\sigma^{NLO_{jet-logs}}(\mu_H = \mu_S = \mu_J)}{d\sigma^{LO}(\mu_H = \mu_J = \mu_S = \sqrt{q^T_{J_2} q^T_{J_4}})}$$



Numerical Results: NLL Resummation



Meff Distribution

$$pp \to \gamma + 2 \text{ jets}$$

$$\sqrt{s} = 7 \text{ TeV}$$

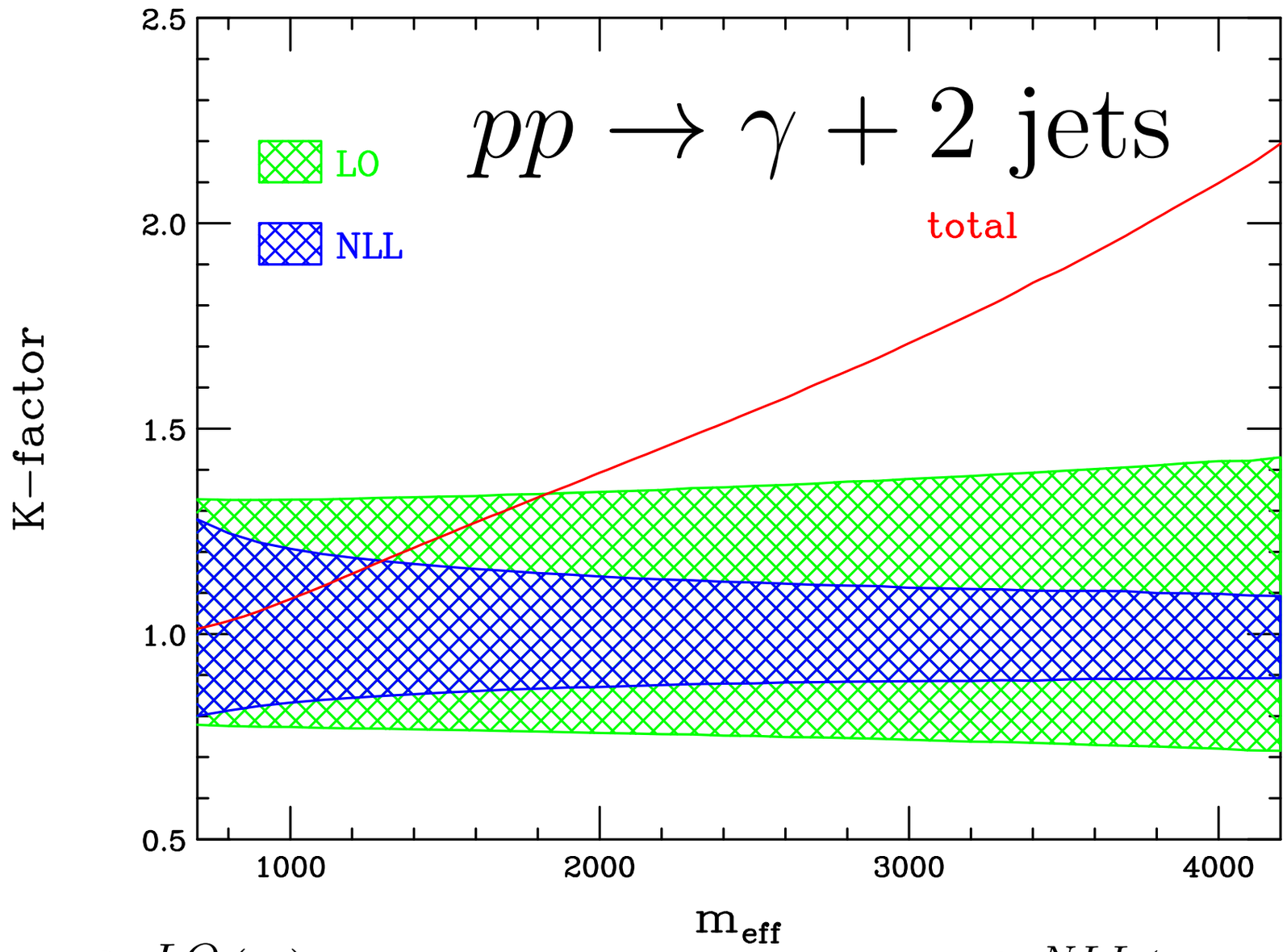
$$\Delta R_{ij} > 0.4$$

$$p_T > 100 \text{ GeV}$$

CTEQ6 LO PDFs

Improved Scale uncertainties

Meff Distribution



$$K_{LO} = \frac{d\sigma^{LO}(\mu)}{d\sigma^{LO}(\mu^c = \sqrt{q_{J_2}^T q_{J_4}^T})}, \quad K_{NLL} = \frac{d\sigma^{NLL}(\mu_H, \mu_J, \mu_S)}{d\sigma^{NLL}(\mu_H^c = \sqrt{q_{J_2}^T q_{J_4}^T}, \mu_J^c, \mu_S^c)}$$

Jet Veto Dependence

- Formula at NLL :

$$\frac{\mathrm{d}\sigma^{\mathrm{NLL}}(pp \to \gamma + 2 \text{ jets})}{\mathrm{d}\Phi_{q_{J_2}}\,\mathrm{d}\Phi_{q_{J_4}}\,\mathrm{d}\Phi_q} = \frac{(2\pi)^4}{Q^4\tau}\int_{z_{min}}^{1}\mathrm{d}z\,\delta^2(q^T_{J_2}+q^T_{J_4}+q^T)\,\mathrm{Tr}\left[\tilde{H}^{\mathrm{NLL}}(\mu_S;\mu_H)\tilde{S}^{(0)}\right]$$
$$\times\exp\left(-4(C_{J_2}+C_{J_4})S(\mu_J,\mu_S) - A_{J_2}(\mu_J,\mu_S) - A_{J_4}(\mu_J,\mu_S)\right)$$
$$\times\left(\frac{\mu_H}{\bar{q}_{J_2}}\right)^{-2C_{J_2}A_\gamma(\mu_J,\mu_S)}\left(\frac{\mu_H}{\bar{q}_{J_4}}\right)^{-2C_{J_4}A_\gamma(\mu_J,\mu_S)}$$
$$\times\left(\frac{2\mu_J^2\cosh y}{\mu_H\sqrt{\hat{s}}}\right)^{\omega}\frac{(e^{\gamma_E})^{\omega}}{\Gamma(-\omega)}\left(\frac{1}{1-z}\right)^{1+\omega} f_a(x_a)f_b(x_b)\,, \qquad (28)$$

- Cut dependence related to threshold condition:

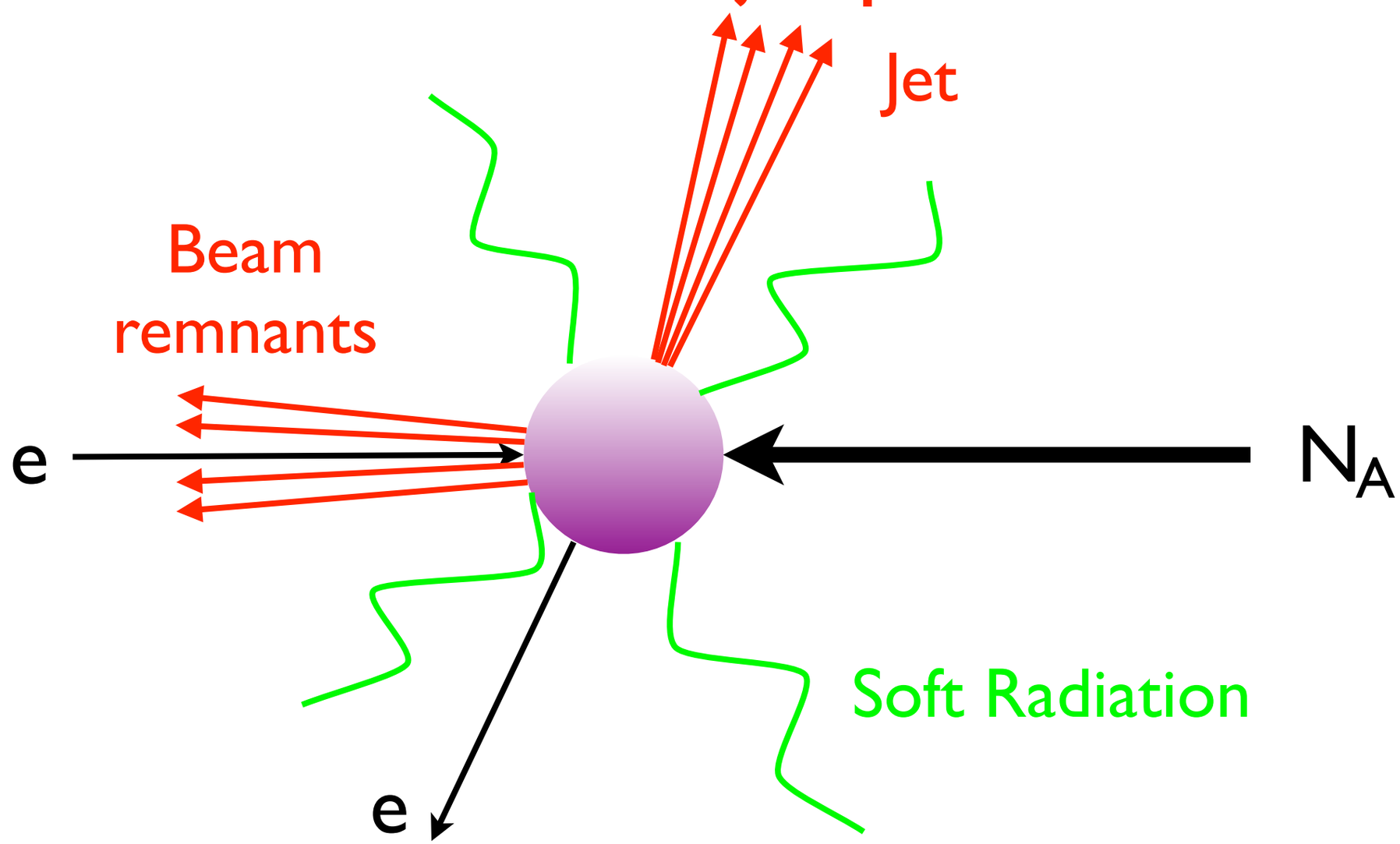
$$z_{min} = \max\left[1-\frac{1}{2}A(\sqrt{A^2+4}-A),\tau\exp(2|y|)\right]$$

$$A = \frac{2\cosh y\,\hat{\tau}_2^{\mathrm{cut}}}{\sqrt{\tau s}}$$

Probing Nuclear Dynamics in Single Jet Production in DIS

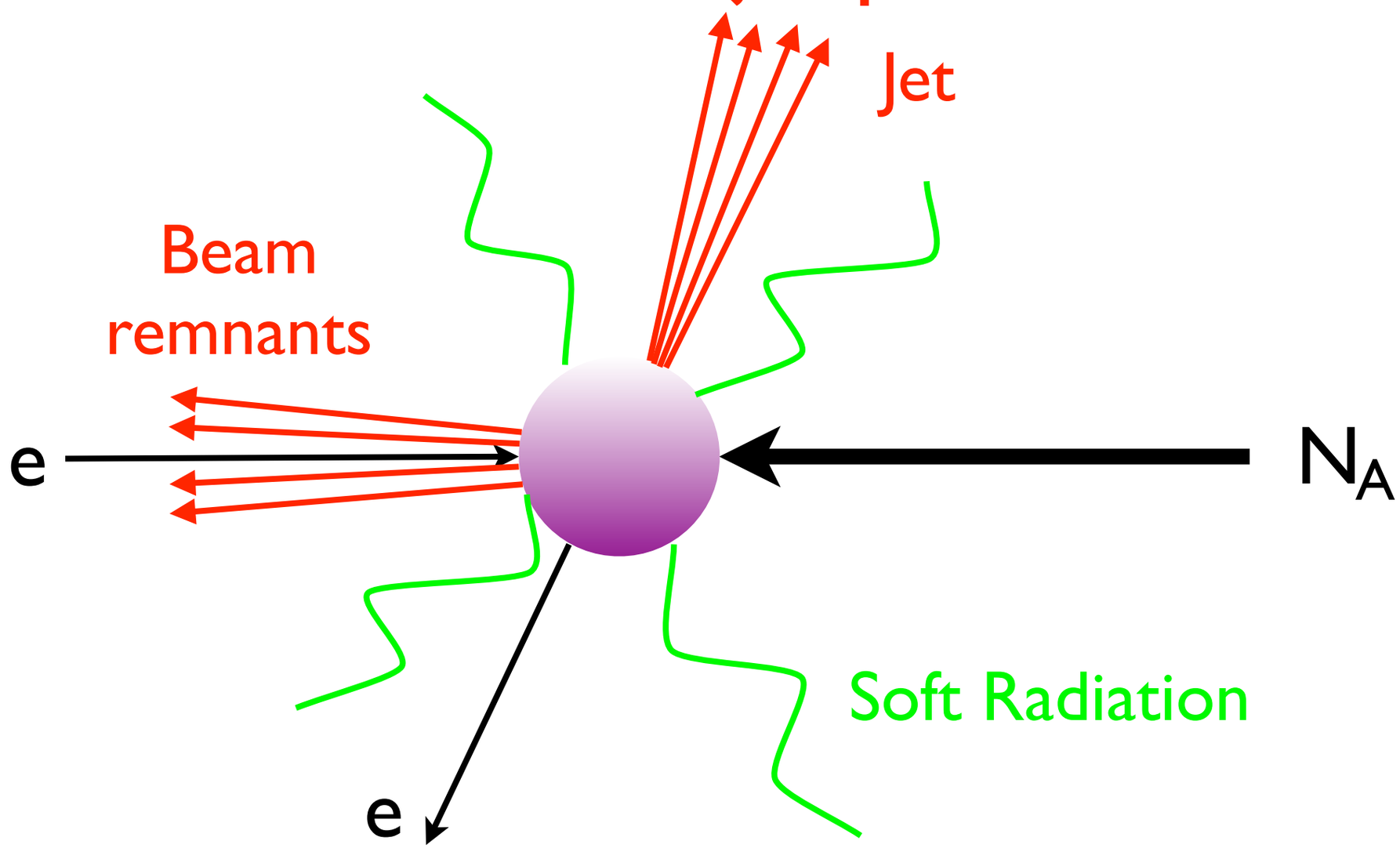
Sonny Mantry, NU & ANL

Exclusive jet production in DIS



$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

Exclusive jet production in DIS

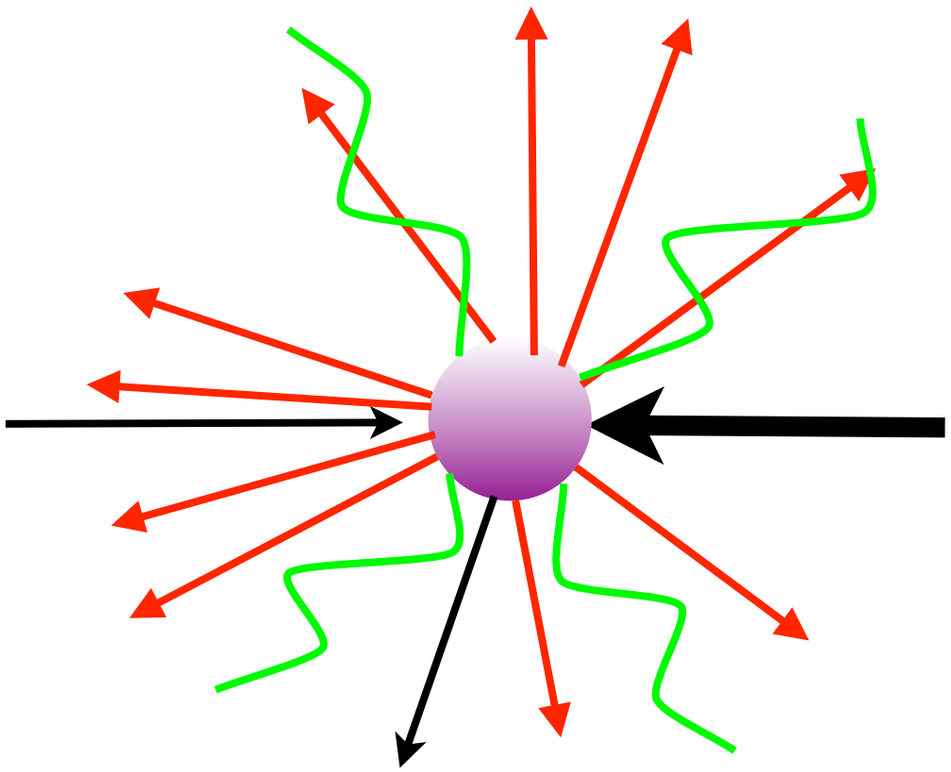


$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

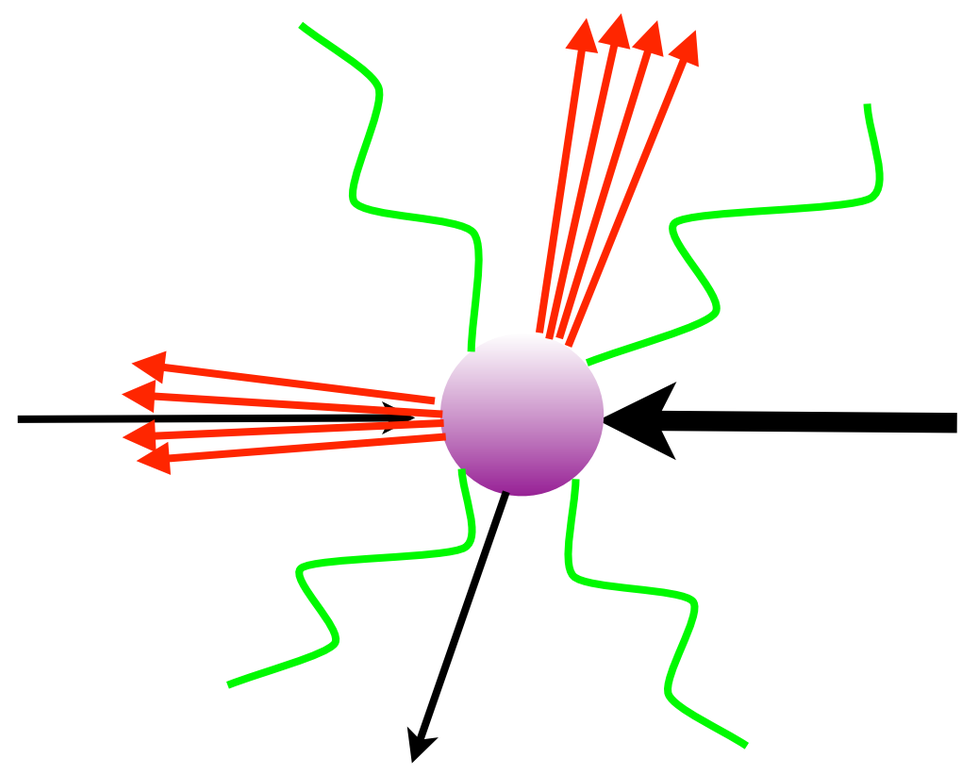
1-jettiness: global event shape

1-Jettiness Event Shape

- Configurations of large and small 1-jettiness:



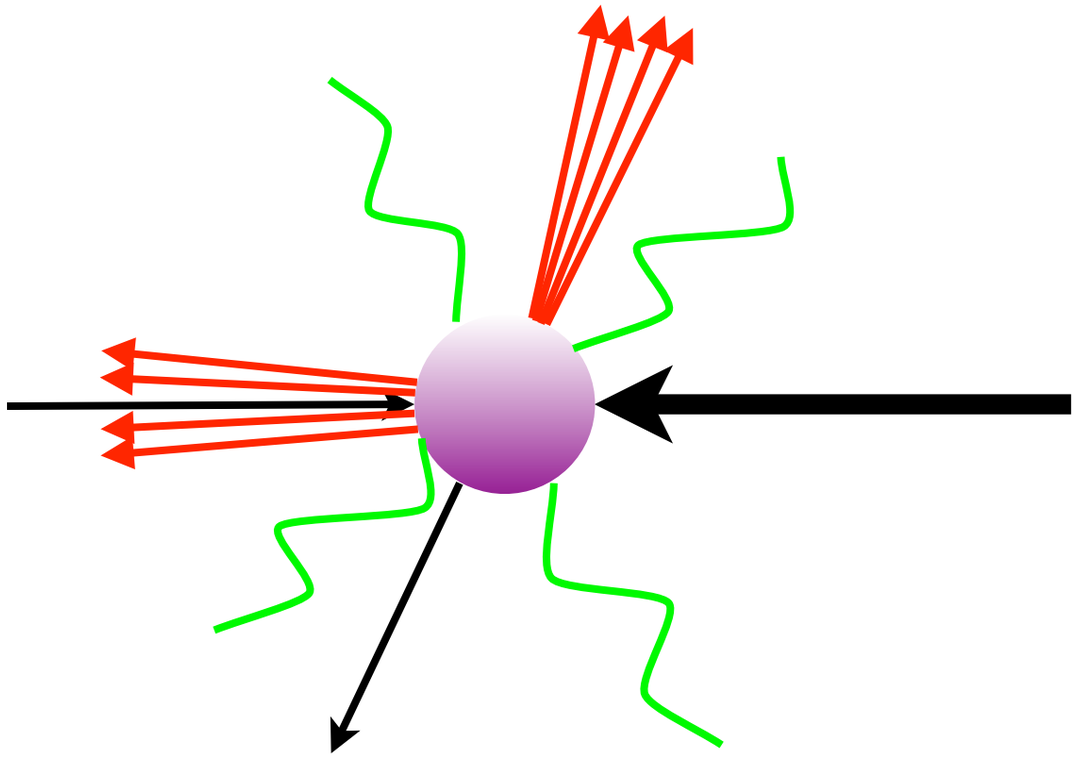
$$\tau_1 \sim P_{J_T}$$

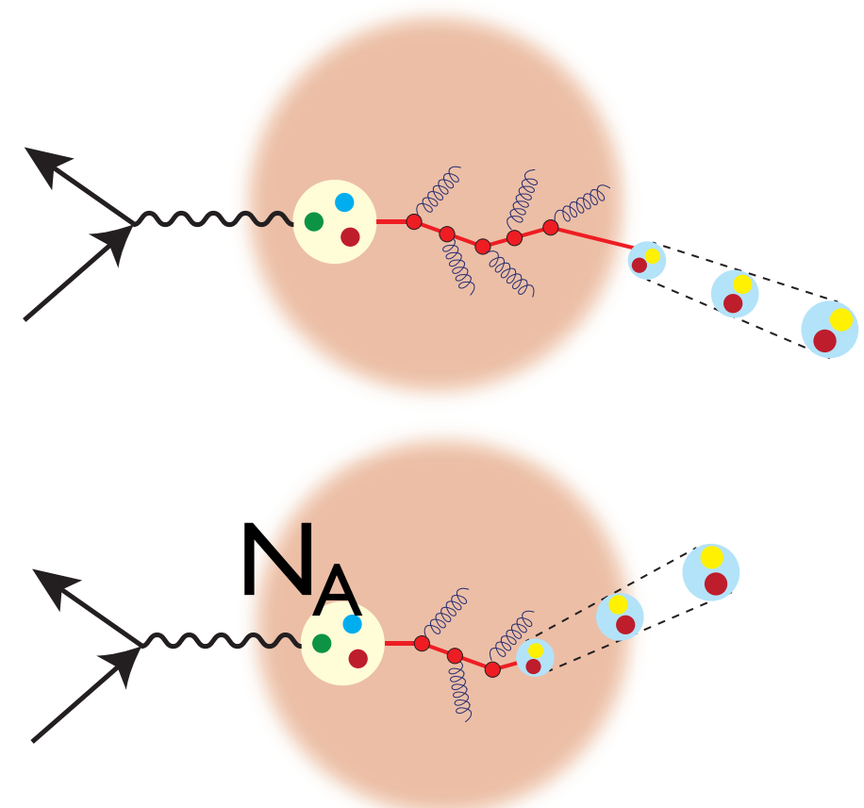


$$\tau_1 \ll P_{J_T}$$

- 1-jettiness distributions can be a probe of nuclear structure and dynamics.

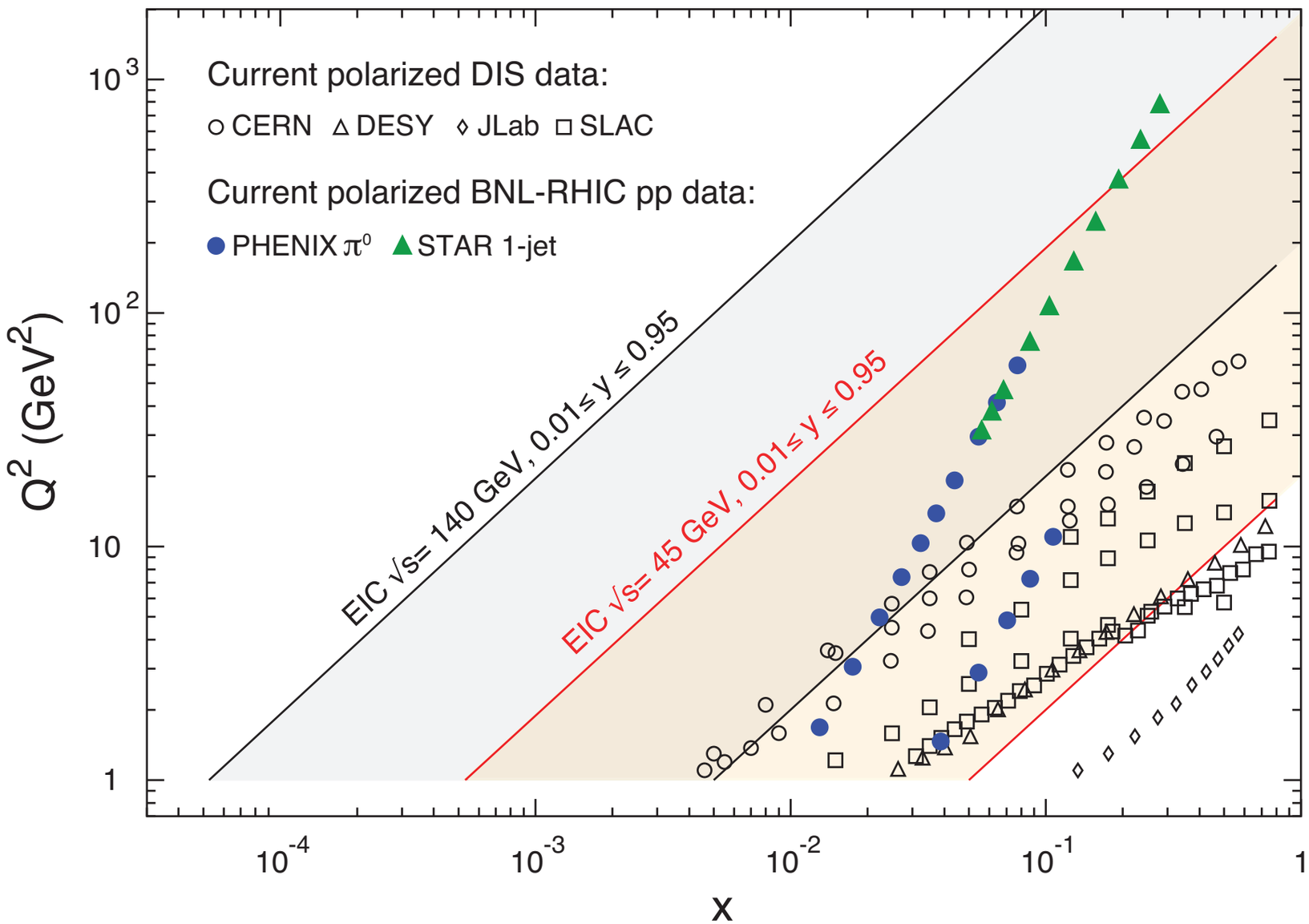
Jet Production as a probe of Nuclear Physics





- Study jet distributions in e-A collisions.
- Probe of nuclear PDFs at leading twist.
- Higher twist correlations.
- Parton propagation through cold nuclear matter.
- Energy loss mechanisms.
- Nuclear medium effects.
- ...

Future Electron-Ion Collider



- Two US proposals
  - -eRHIC
  - -MEIC/ELIC
- European proposal
  - -LHeC

$$\left(\sqrt{s} \sim 800 GeV \right)$$

- High Luminosity.
- Wide kinematic coverage.
- Variety of nuclear targets.

Event Shapes for DIS

- Event shapes have been studied before in DIS:

- Breit Frame
- Thrust, NLL +NLO (Anyonelli, Dasgupta, Salam, 99)
- Broadening, NLL +NLO (Dasgupta, Salam, 01)
- Non-Global Event Shapes (Dasgupta, Salam, 01, 02)

- 1-jettiness global event shape for DIS was first introduced about a year ago:

(Z.Kang, SM, Qiu, 12)

- 1-jettiness factorization in SCET
- Proposed as probe of nuclear physics
- Proton target, NLL results

- More recently: (Z. Kang, Liu, SM, Qiu, 13)

- NNLL resummation
- Variety of nuclear targets: Proton, C, Ca, Fe, Au, Ur

- Most recently: (D. Kang, Lee, Stewart, 13)

- Also, considered 1-jettiness with NNLL resummation
- Introduced two new variations of 1-jettiness and their factorization
- Analysis restricted to proton target

Event Shapes for DIS

- Event shapes have been studied before in DIS:
  - Breit Frame
  - Thrust, NLL +NLO (Anyonelli, Dasgupta, Salam, 99)
  - Broadening, NLL +NLO (Dasgupta, Salam, 01)
  - Non-Global Event Shapes (Dasgupta, Salam, 01, 02)
- 1-jettiness global event shape for DIS was first introduced about a year ago:
  - 1-jettiness factorization in SCET (Z.Kang, SM, Qiu, 12)
  - Proposed as probe of nuclear physics
  - Proton target, NLL

Focus of this talk

- More recently: (Z. Kang, Liu, SM, Qiu, 13)
  - NNLL resummation
  - Variety of nuclear targets: Proton, C, Ca, Fe, Au, Ur
- Most recently: (D. Kang, Lee, Stewart, 13)
  - Also, considered 1-jettiness with NNLL resummation
  - Introduced two new variations of 1-jettiness and their factorization
  - Analysis restricted to proton target

1-Jettiness in DIS

Kinematics

- Electron momentum:

$$p_e^\mu = (p_e^0, \vec{p}_e)$$

- Nucleus momentum:

$$P_A^\mu = A(p_e^0, -\vec{p}_e)$$

- Electron energy:

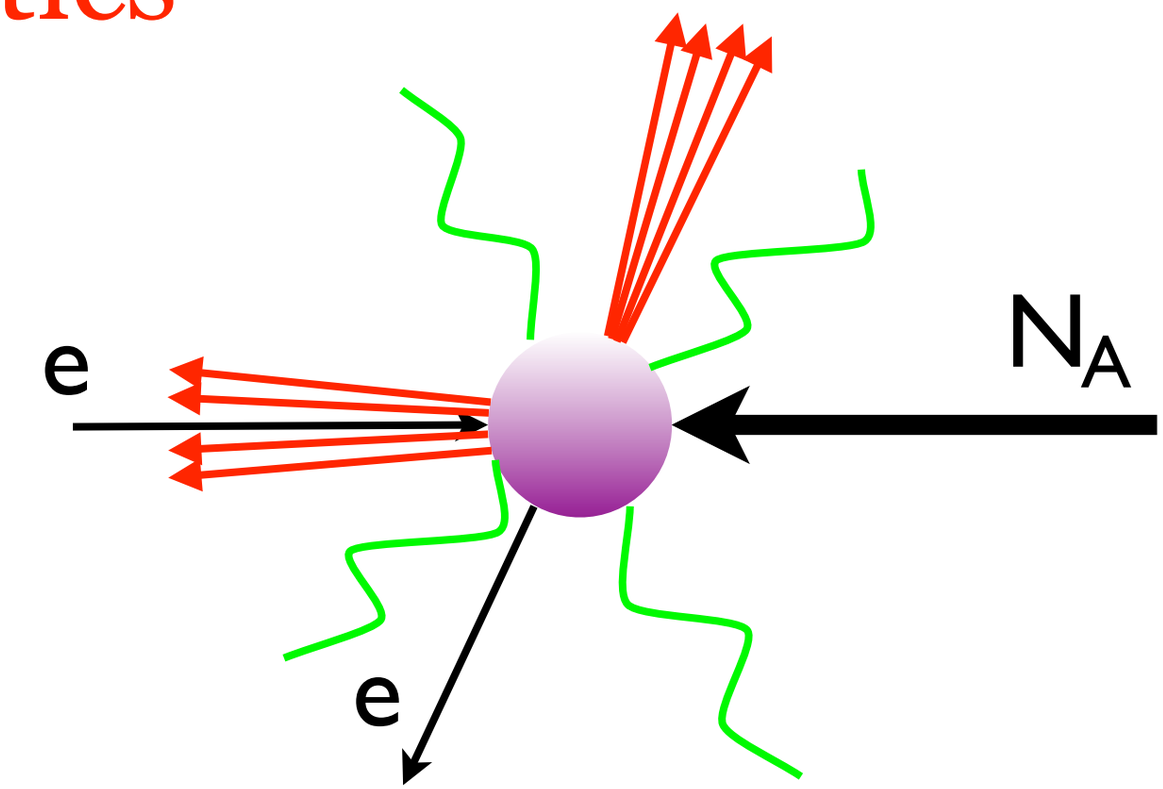
$$p_e^0 = |\vec{p}_e| = \frac{Q_e}{2}$$

- Center of mass energy squared:

$$s = (p_e + P_A)^2 = A\, Q_e^2$$

- Center of mass energy per nucleon:

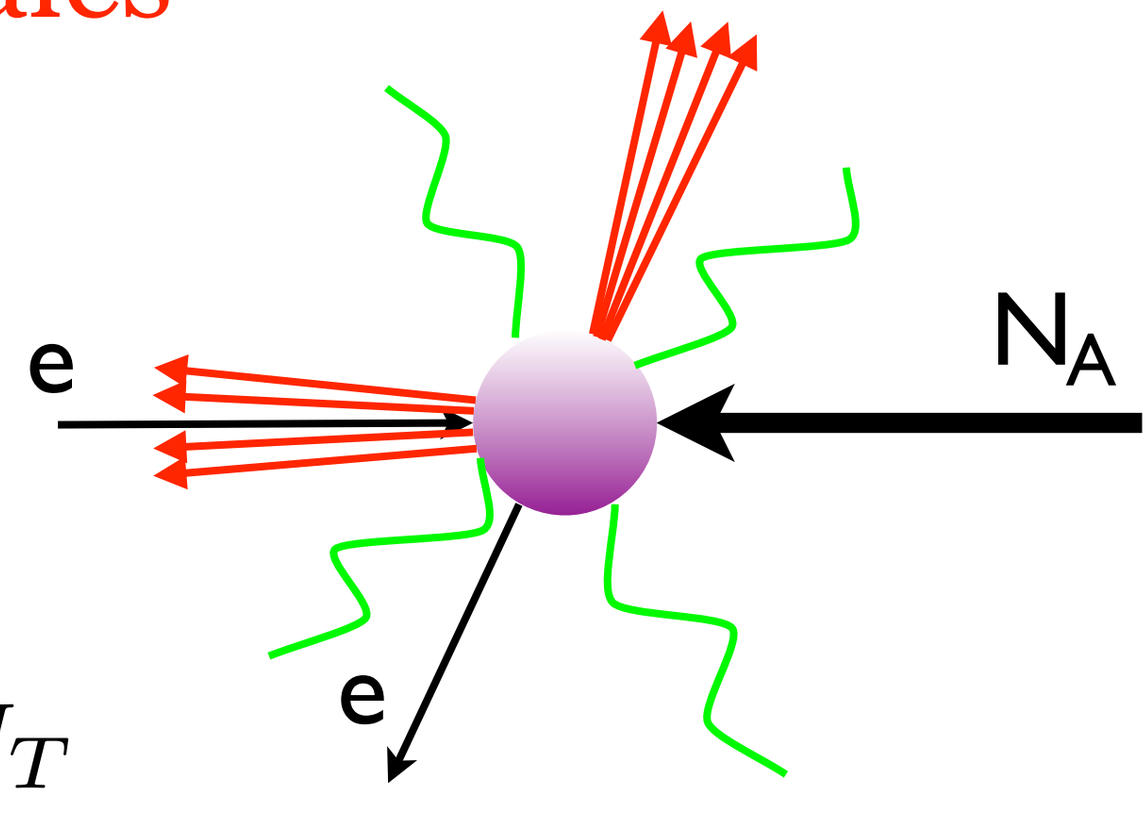
$$Q_e$$



$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

Energy Scales

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$



- Hierarchy of scales:

$$Q_s, \Lambda_{QCD} \ll \tau_1 \ll P_{J_T}$$

Nuclear scales

1-jettiness

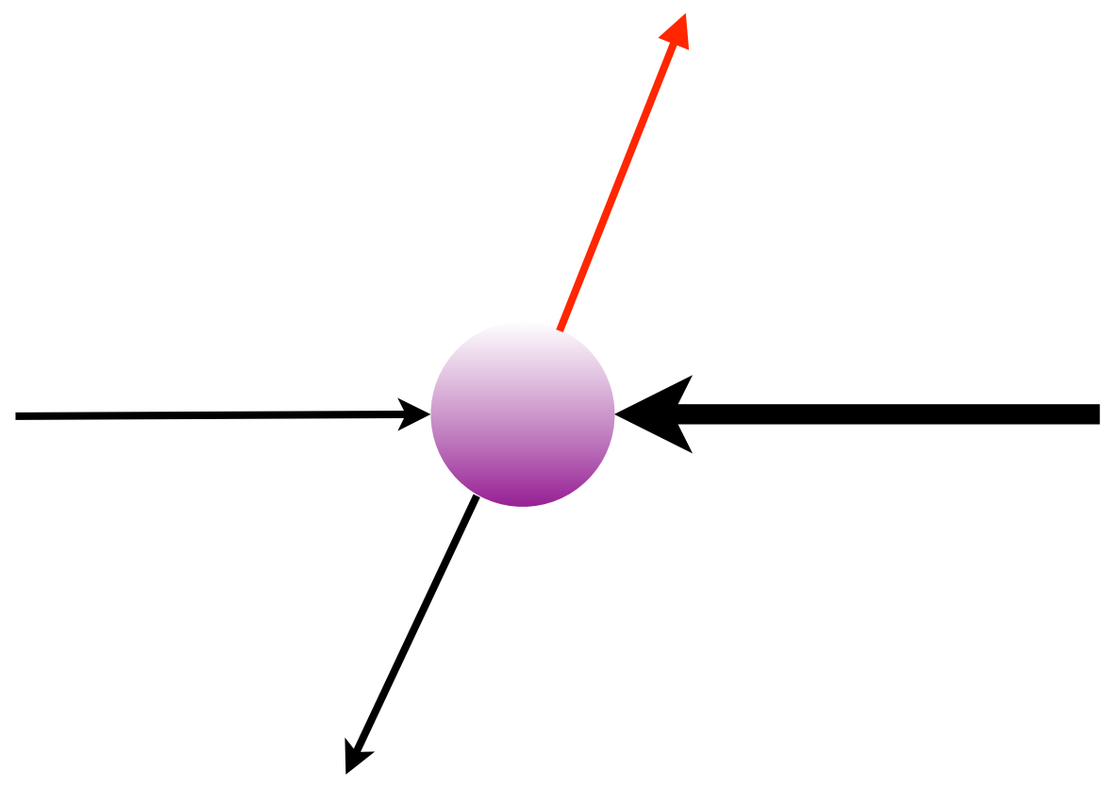
Jet transverse momentum

- Jet-veto Sudakov logarithms:

$$\sim \alpha_s^n \ln^{2n}(\tau_1/P_{J_T})$$

Tree Level 1-jettiness Distribution

$$d\sigma_A \equiv \frac{d^3\sigma(e^- + N_A \to J + X)}{dy\, dP_{J_T}\, d\tau_1}$$

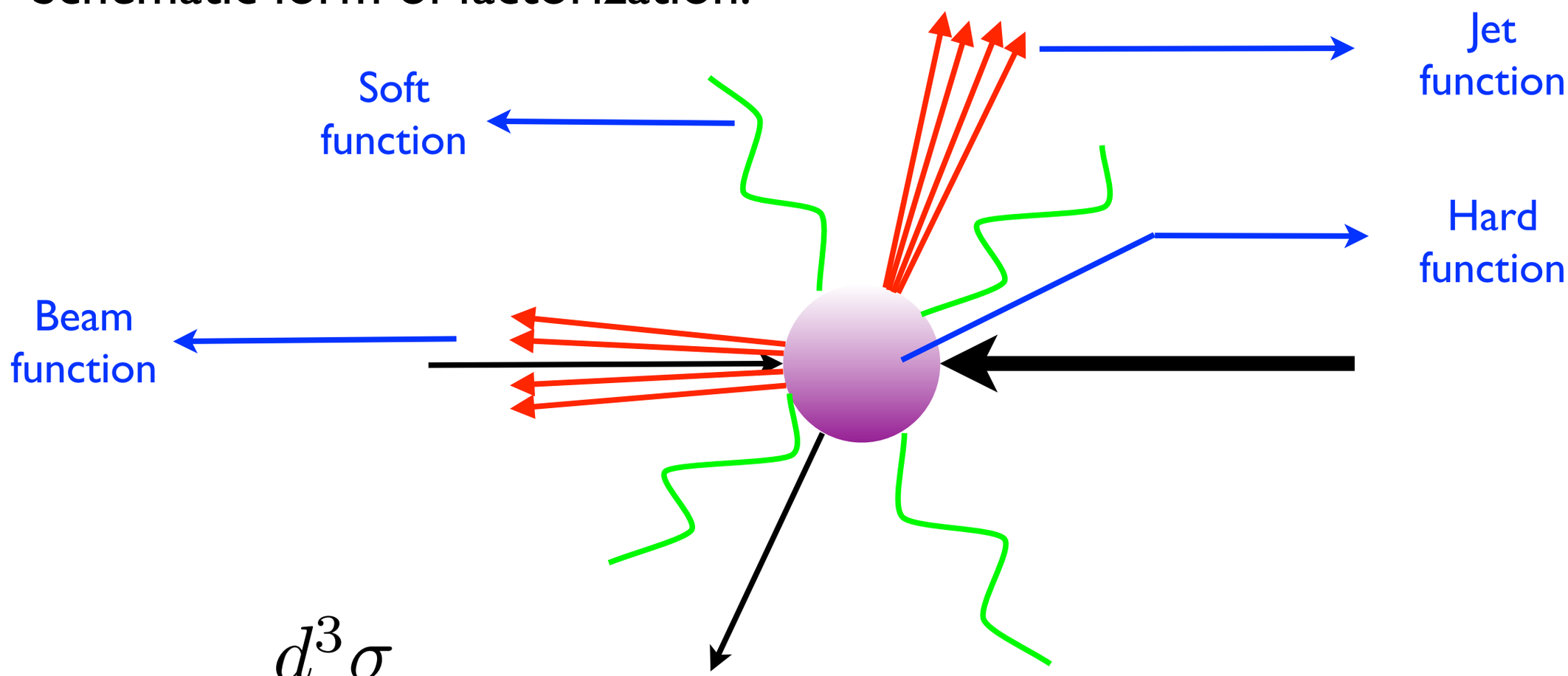


- Tree-level distribution in 1-jettiness:

$$\frac{d^3\sigma^{(0)}}{dy dP_{JT} d\tau_1} = \sigma_0 \delta(\tau_1) \sum_q e_q^2 \frac{1}{A} f_{q/A}(x_A, \mu)$$

Factorization

- Schematic form of factorization:



$$\frac{d^3\sigma}{dy\,dP_{JT}\,d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S},$$

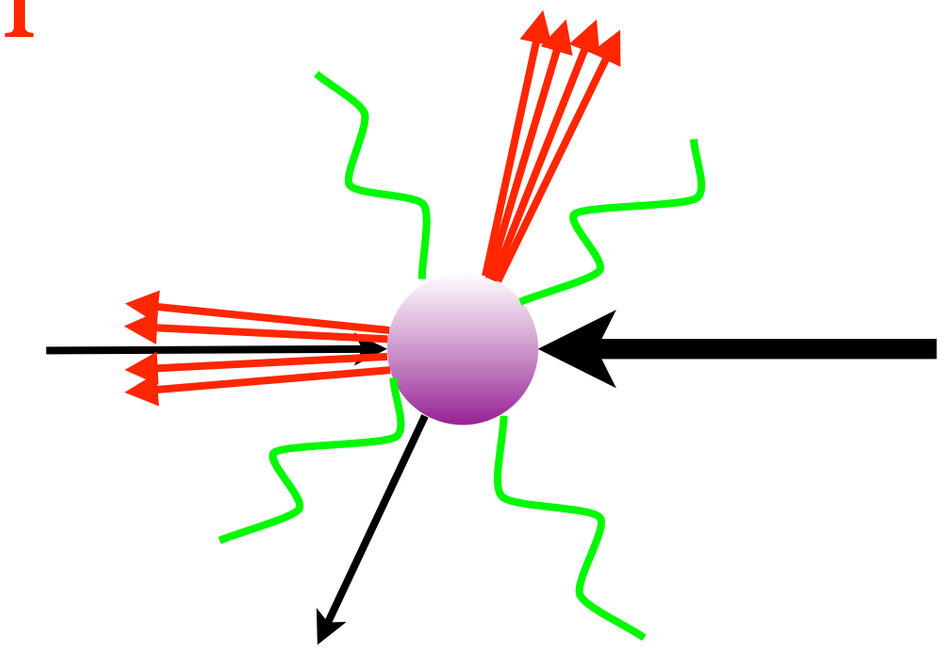
Hard function | Beam function | Jet function | Soft function

Factorization

- Schematic form of factorization:

$$\frac{d^3\sigma}{dy dP_{JT} d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S},$$

Hard function — Beam function — Jet function — Soft function

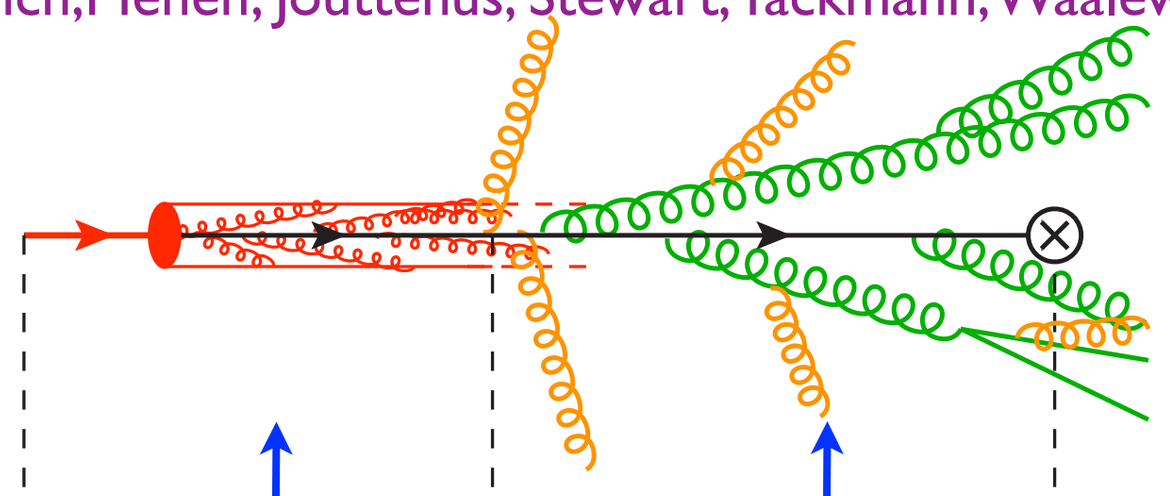


- Separate physics of PDF and perturbative initial state radiation:

(Fleming, Leibovich, Mehen; Jouttenus, Stewart, Tackmann, Waalewijin)

$$B_A \sim \mathcal{I} \otimes f_A$$

Wilson coefficient — Nuclear PDF



Nuclear PDF — Initial state radiation

Factorization

- Detailed form of factorization:

$$\frac{d^3\sigma}{dy dP_{JT} d\tau_1} = \frac{\sigma_0}{A}\sum_{q,i} e_q^2 \int_0^1 dx \int ds_J \int dt_a$$

Hard function → $\times H(xAQ_e P_{J_T} e^{-y}, \mu; \mu_H)\delta\left[x - \frac{e^y P_{J_T}}{A(Q_e - e^{-y}P_{J_T})}\right]$

Jet function → $\times J^q(s_J, \mu; \mu_J) B^q(x, t_a, \mu; \mu_B)$ ← Beam function

$\times \mathcal{S}\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right),$ ← Soft Function

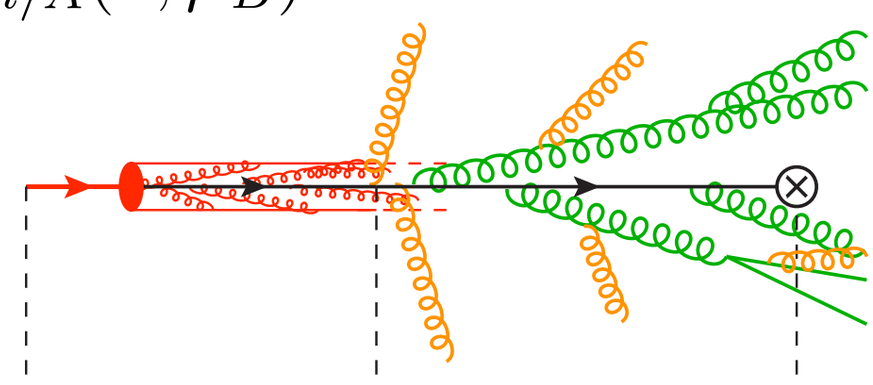
- Beam function matching onto the PDF:

(Fleming, Leibovich, Mehen; Jouttenus, Stewart, Tackmann, Waalewijin)

$$B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} \mathcal{I}^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B)$$

- Tree-level matching:

$$B^q(x, t_a, \mu_B) = \delta(t_a) f_{q/A}(x, \mu_B)$$



Power Corrections

- Many different sources of power corrections.

- Dominant nuclear-dependent power corrections come from the OPE of the beam function

$$B^q(x, t_a, \mu; \mu_B) = \int_x^1 \frac{dz}{z} \mathcal{I}^{qi}\left(\frac{x}{z}, t_a, \mu; \mu_B\right) f_{i/A}(z, \mu_B) + \mathcal{O}\left(\frac{Q_s^2(A)}{t_a}\right)$$

- Size of power corrections controlled by

$$\frac{Q_s^2(A)}{t_a} \sim \frac{A^\alpha \Lambda_{QCD}^2}{\tau_1 P_{J_T}}$$

-Higher twist correlations

-Nuclear medium effects: energy loss, multiple scattering,...

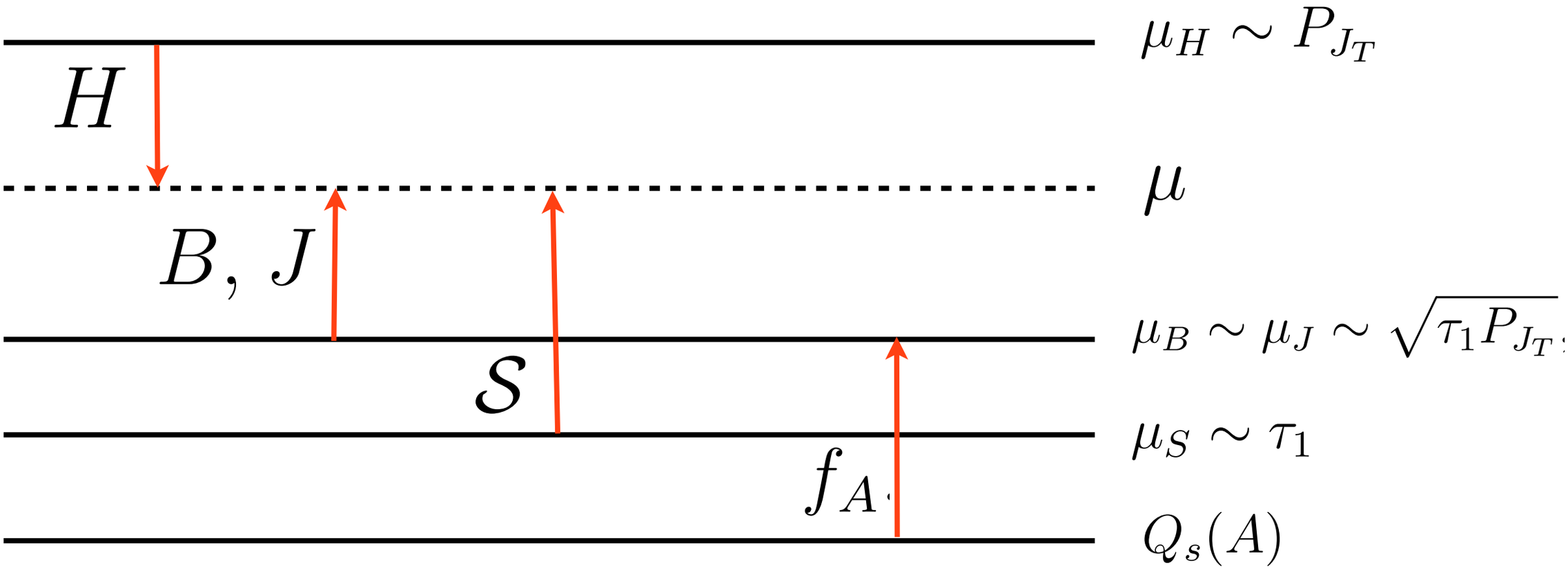
- Power corrections can be studied as a function of: $\{A, \tau_1, P_{J_T}\}$

Numerical Results: Proton

Sonny Mantry, NU & ANL

Resummation

- Resummation achieved through renormalization group equations:



- All objects in factorization formula have well defined evolution equations:

$$\mu\frac{d}{d\mu}H(Q^2,\mu) \;=\; \gamma_H\, H(Q^2,\mu),$$

$$\mu\frac{d}{d\mu}B_A^q(x,t,\mu) \;=\; \int dt'\,\gamma_B(t-t',\mu)\,B_A^q(x,t',\mu),$$

$$\mu\frac{d}{d\mu}J(s,\mu) \;=\; \int ds'\,\gamma_J(s-s',\mu)J(s',\mu),$$

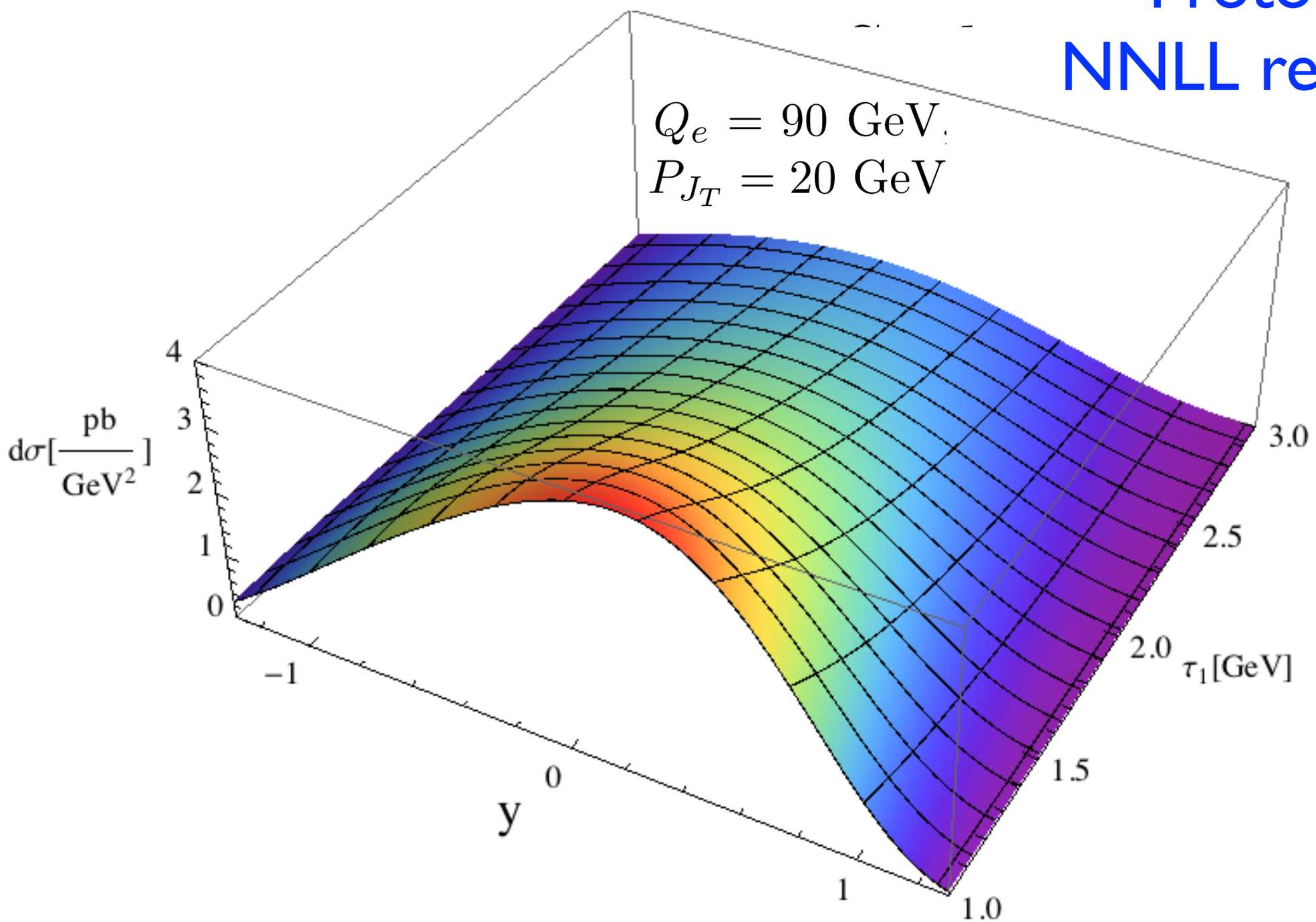
$$\mu\frac{d}{d\mu}\mathcal{S}(k_a,k_J,\mu) \;=\; \int dk_a' \int dk_J'\,\gamma_S(k_a-k_a',k_J-k_J',\mu)\mathcal{S}(k_a',k_J',\mu)$$

1-jettiness and rapidity distribution

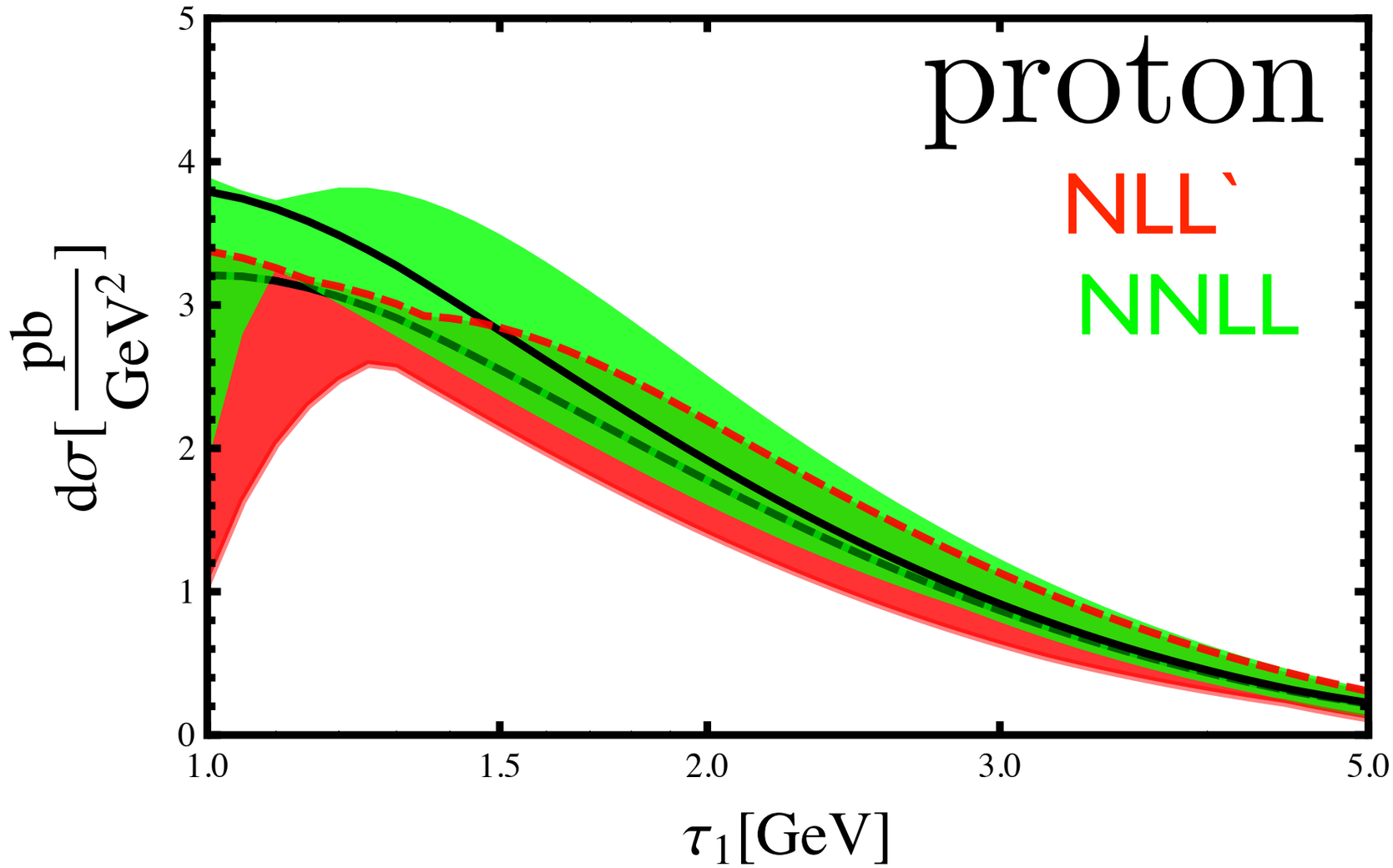
- One can study distributions in the space of :

$$\{A, Q_e, P_{J_T}, y, \tau_1\}$$

Proton target:
NNLL resummation



1-jettiness distribution for the Proton



$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$y = 0$$

Numerical Results: Nuclei

Sonny Mantry, NU & ANL

Nuclear PDFs

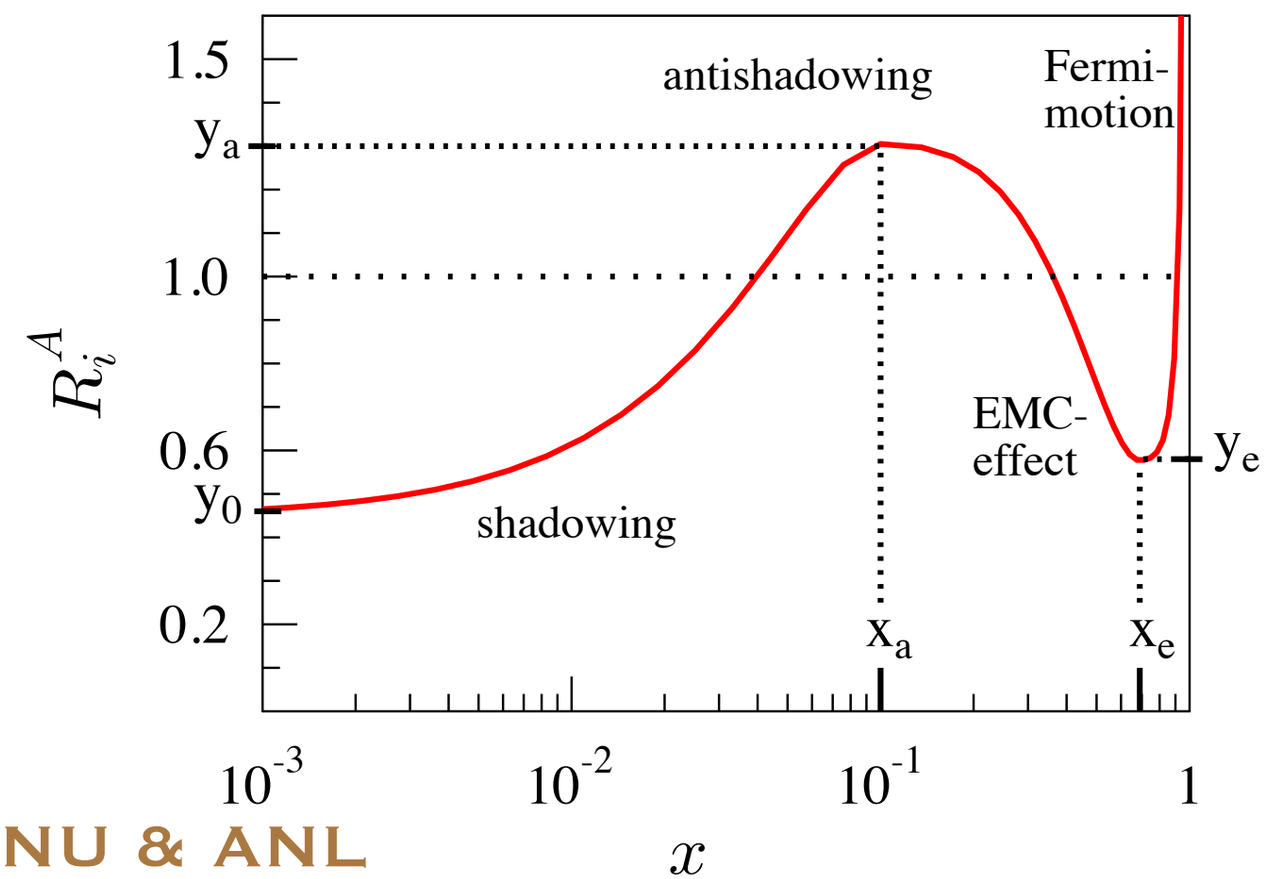
- At leading twist, we directly probe nuclear PDFs

(Eskola, Paukunnen, Salgado)

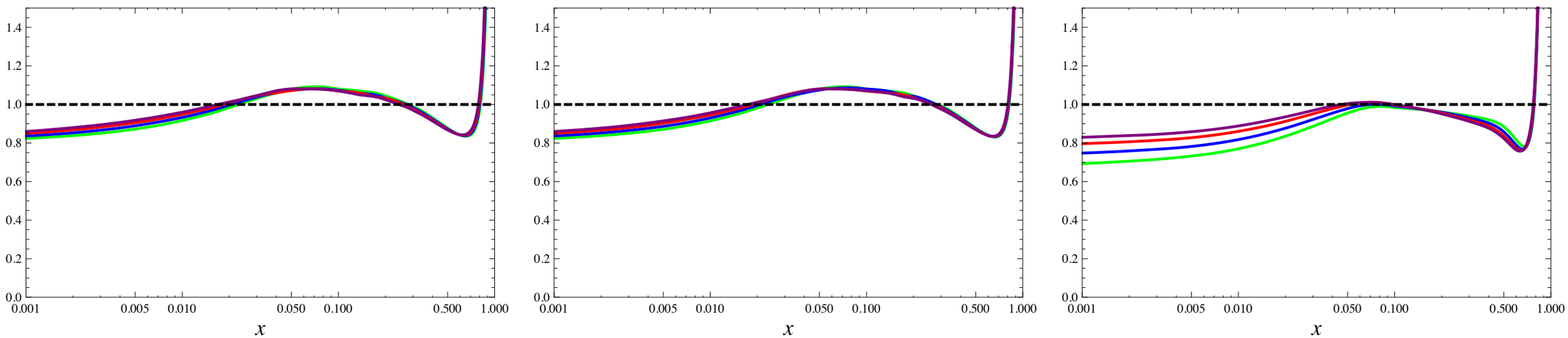
$$
\begin{aligned}
f_{u/A}^{EPS09}(x,\mu) &= \frac{Z}{A}\, R_u^A(x,\mu)\, f_{u/p}(x,\mu) + \frac{A-Z}{A}\, R_d^A(x,\mu)\, f_{d/p}(x,\mu), \\
f_{d/A}^{EPS09}(x,\mu) &= \frac{Z}{A}\, R_i^A(x,\mu)\, f_{d/p}(x,\mu) + \frac{A-Z}{A}\, R_u^A(x,\mu)\, f_{u/p}(x,\mu), \\
f_{s,c,b/A}^{EPS09}(x,\mu) &= R_{s,c,b}^A(x,\mu)\, f_{s,c,b/p}(x,\mu), \\
f_{g/A}^{EPS09}(x,\mu) &= R_g^A(x,\mu)\, f_{g/p}(x,\mu),
\end{aligned}
$$

EPS09 PDF set

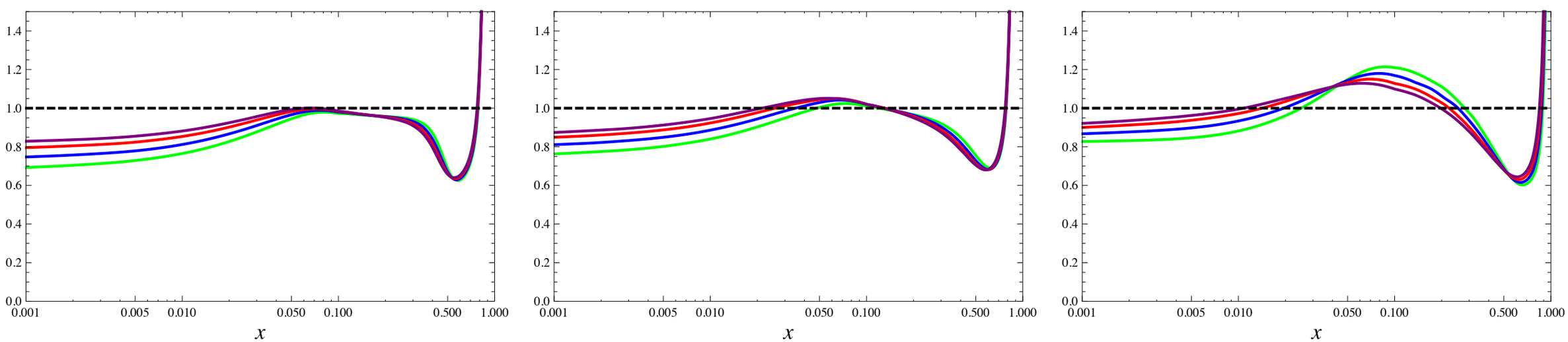
- Schematic behavior of nuclear modification factors



Nuclear Modification Factors for Uranium



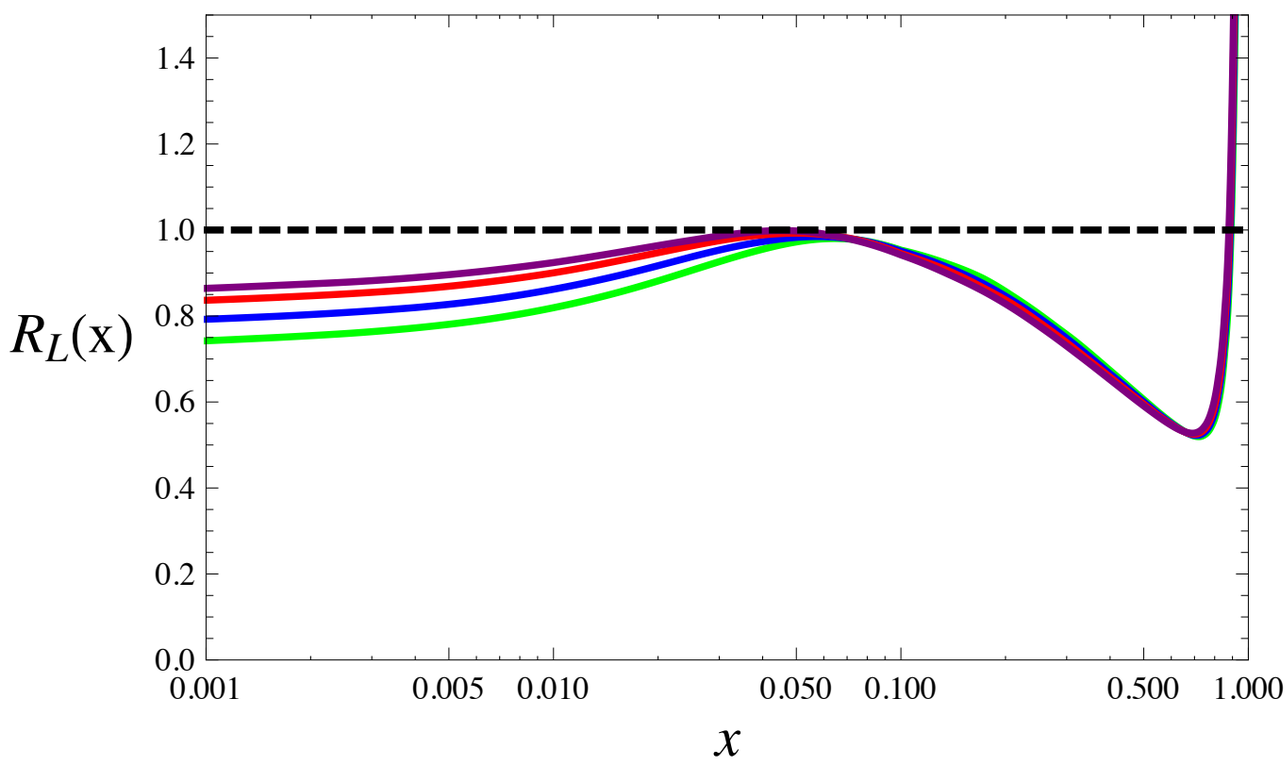
(a) $R_u^{\mathrm{Ur(V)}}$ (b) $R_d^{\mathrm{Ur(V)}}$ (c) $R_u^{\mathrm{Ur(S)}}$



(d) $R_d^{\mathrm{Ur(S)}}$ (e) R_s^{Ur} (f) R_g^{Ur}

$\mu = 3$ GeV (Green) $\mu = 5$ GeV (Blue) $\mu = 10$ GeV (Red) $\mu = 20$ GeV (Purple)

Ratio of Luminosity factors: Uranium to proton



$$R_L(x,\mu) = \frac{\sum_q e_q^2 f_{q/A}^{EPS09}(x,\mu)}{\sum_q e_q^2 f_{q/p}(x,\mu)}$$

- Factorization formula:

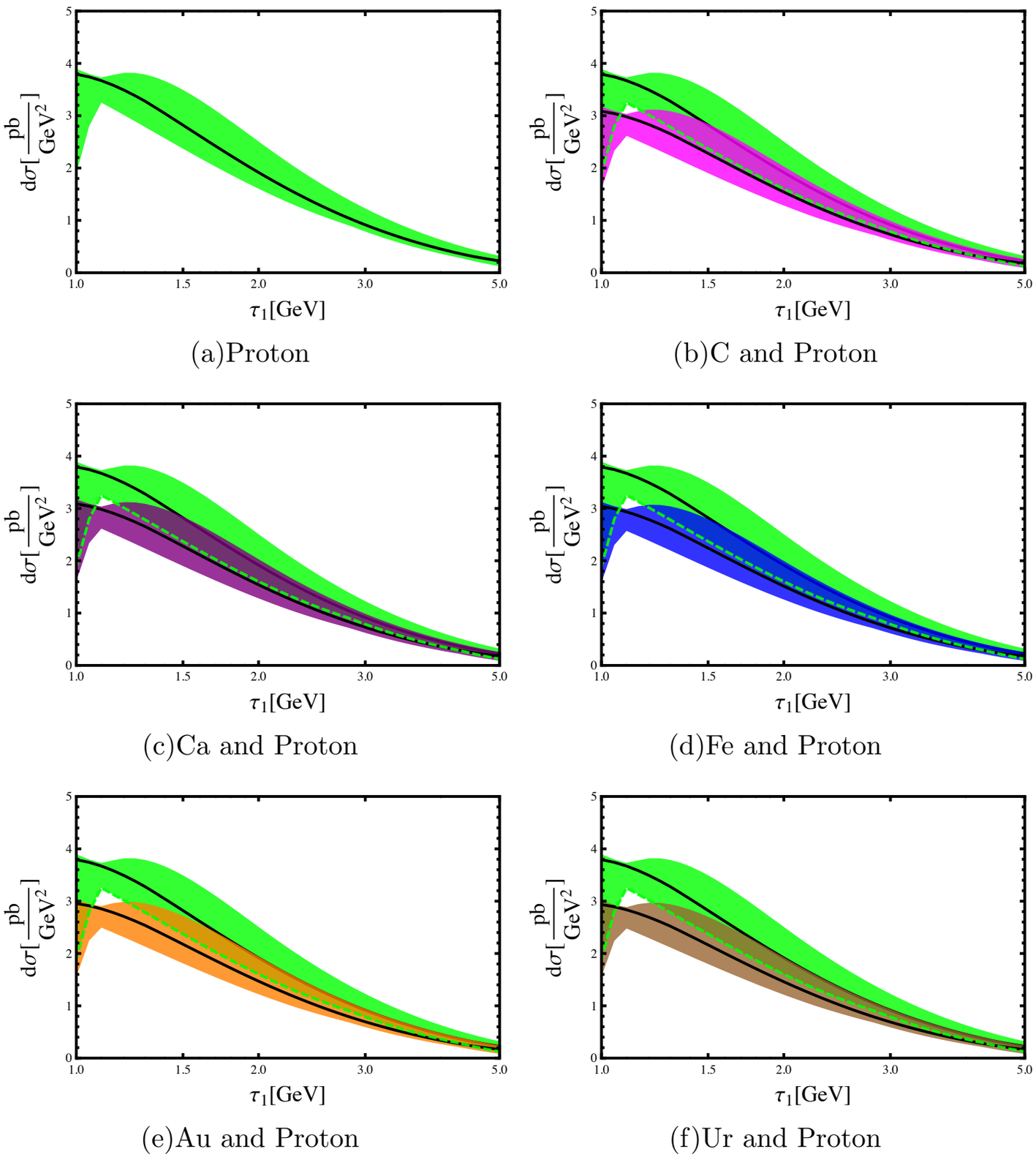
$$\left.\frac{d^3\sigma}{dy dP_{J_T} d\tau_1}\right|_{EPS09} = \sigma_0 \sum_{q,i} e_q^2 \int_{x_*}^1 \frac{dx}{x} \int ds_J \int dt_a$$
$$\times H(\xi^2,\mu;\mu_H) J^q(s_J,\mu;\mu_J) \mathcal{I}^{qi}\left(\frac{x_*}{x}, t_a, \mu; \mu_B\right)$$
$$\times \mathcal{S}\left(\tau_1 - \frac{t_a}{Q_a} - \frac{s_J}{Q_J}, \mu; \mu_S\right) f_{i/A}^{EPS09}(x,\mu_B),$$

- Lower limit of Bjorken-x integration:

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

Determines the Bjorken-x region

1-jettiness distribution for various nuclei



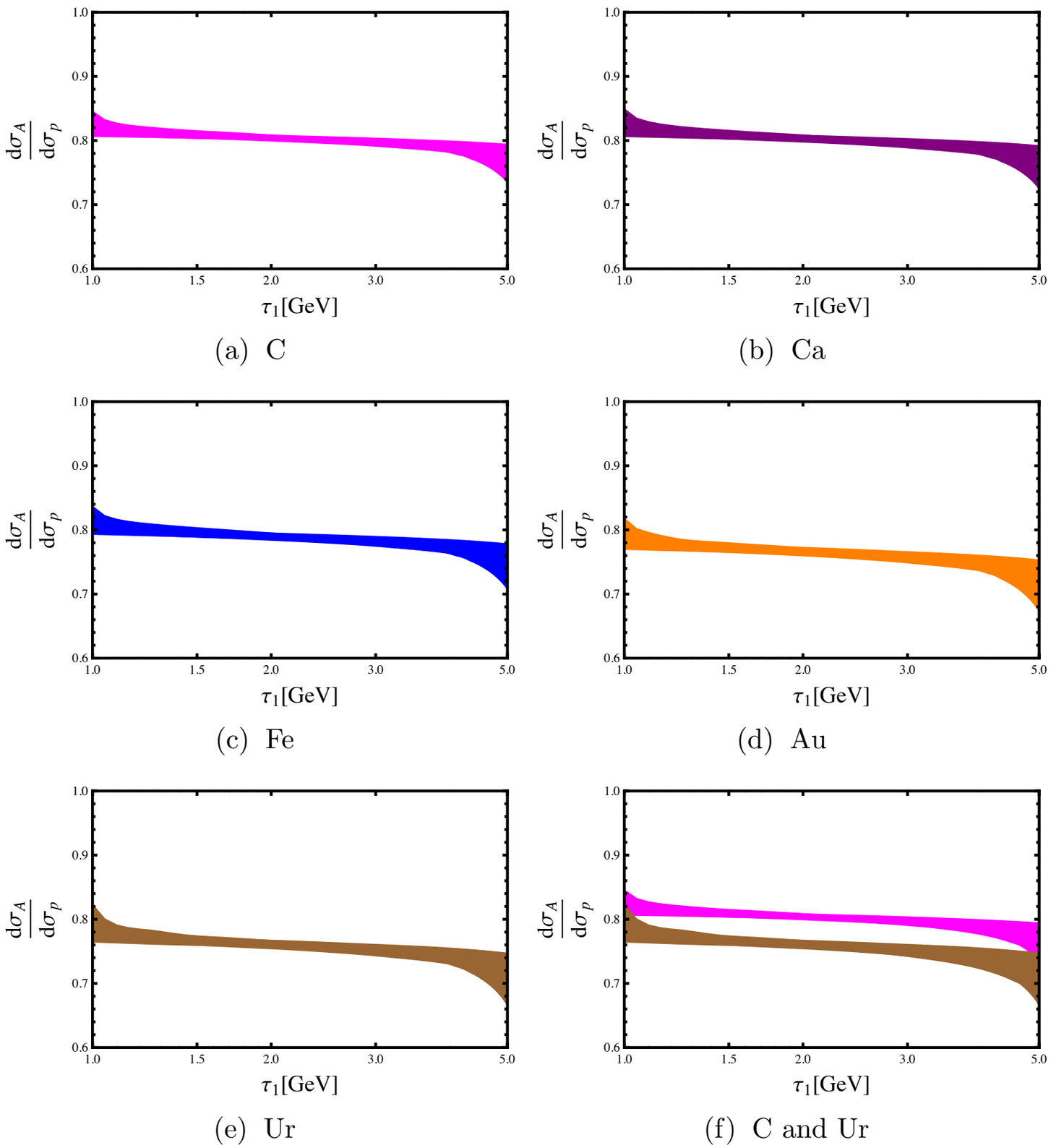
NNLL resummation

$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$y = 0$$

1-jettiness: Nucleus to proton ratios



$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

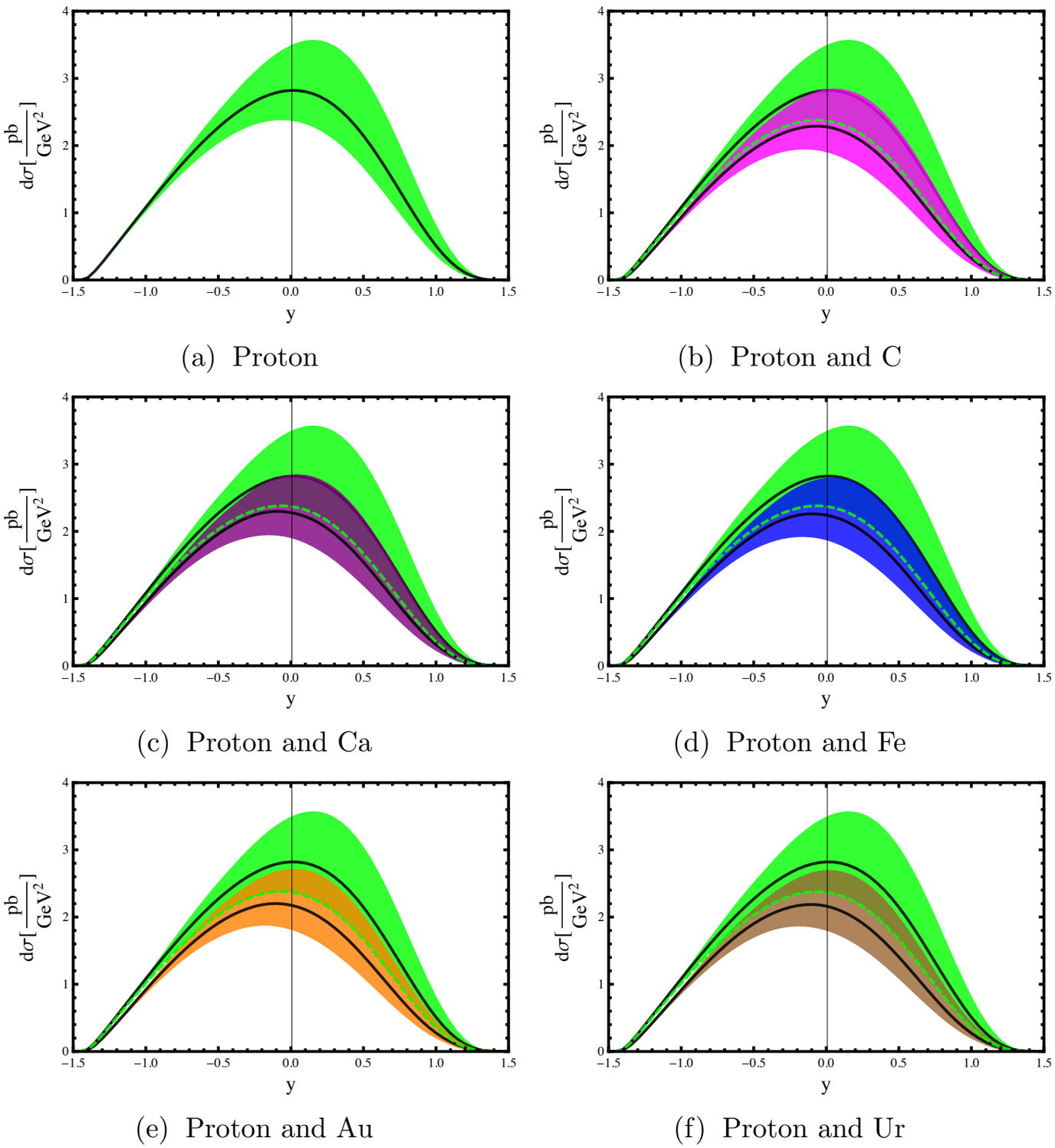
NNLL resummation

$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$y = 0$$

Jet rapidity distributions for various nuclei



(a) Proton (b) Proton and C (c) Proton and Ca (d) Proton and Fe (e) Proton and Au (f) Proton and Ur

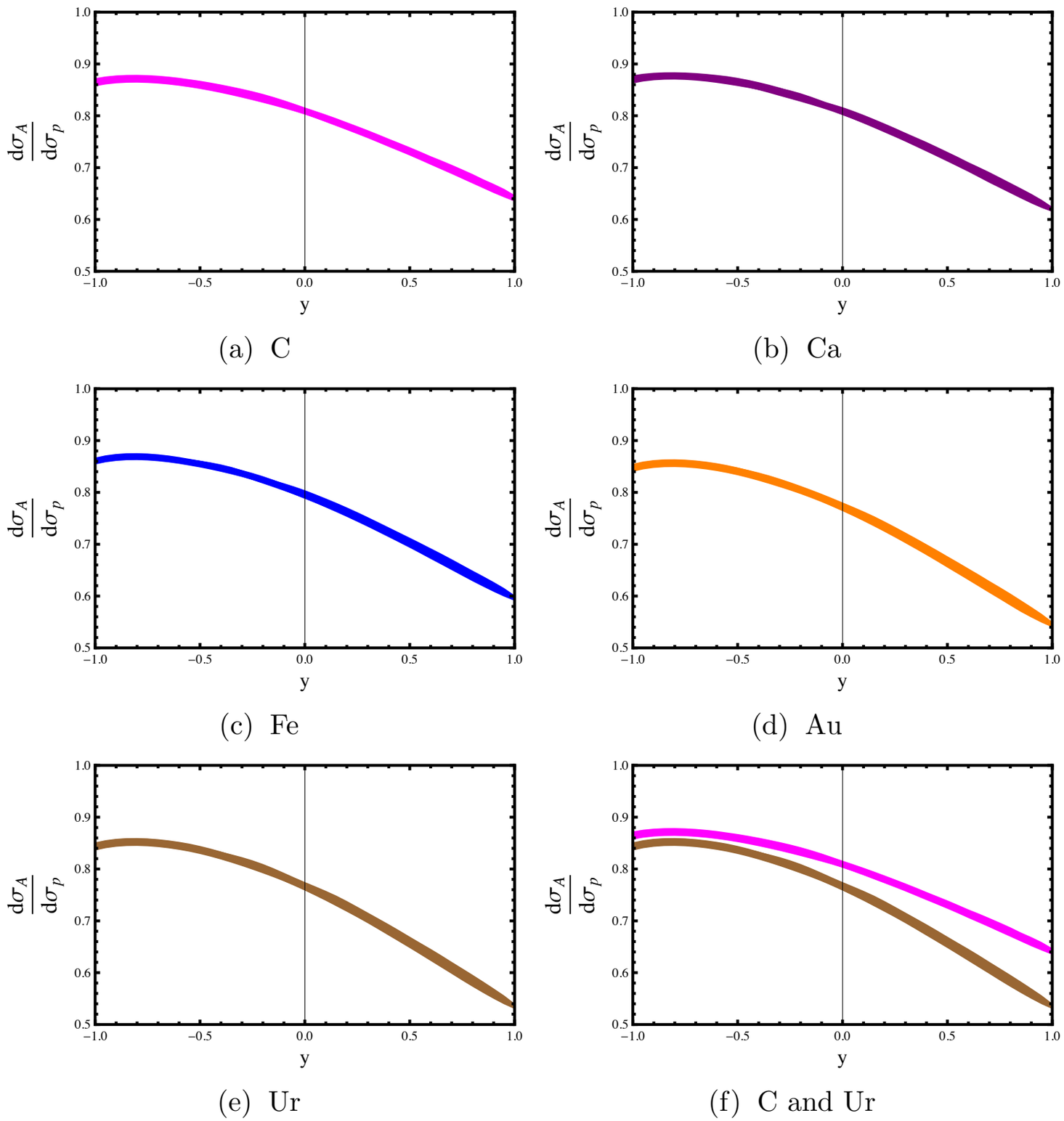
NNLL resummation

$$Q_e = 90 \text{ GeV},$$

$$P_{J_T} = 20 \text{ GeV}$$

$$\tau_1 = 1.5 \text{ GeV}$$

Jet rapidity: Nucleus to proton ratios



(a) C (b) Ca (c) Fe (d) Au (e) Ur (f) C and Ur

$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

NNLL resummation

$$Q_e = 90 \text{ GeV},$$

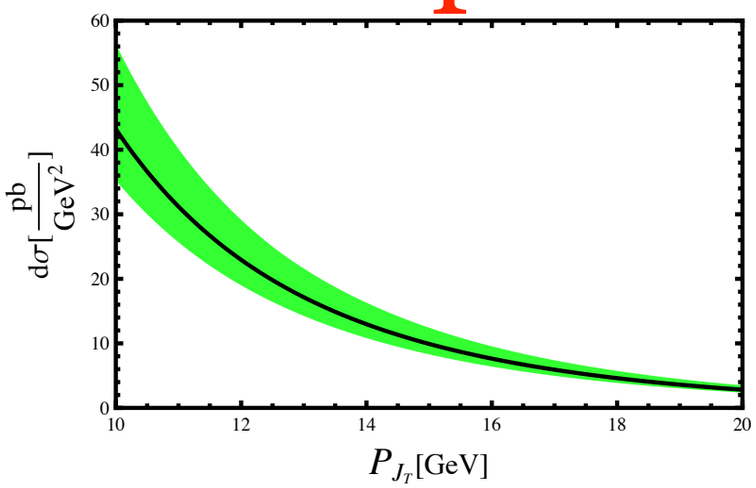
$$P_{J_T} = 20 \text{ GeV}$$

$$\tau_1 = 1.5 \text{ GeV}$$

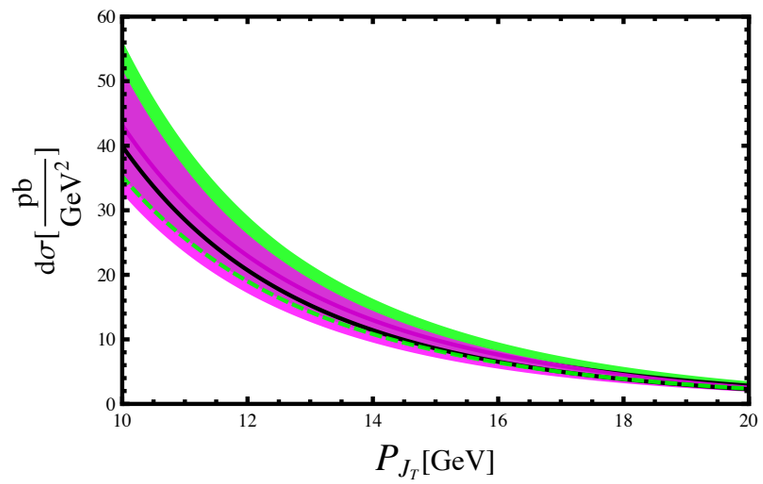
$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

$$x_* \in [0.2, 0.7]$$

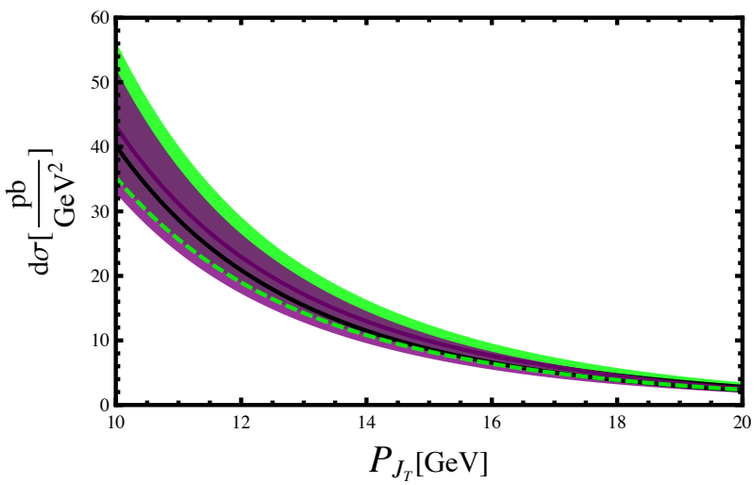
Jet pT distributions for various nuclei



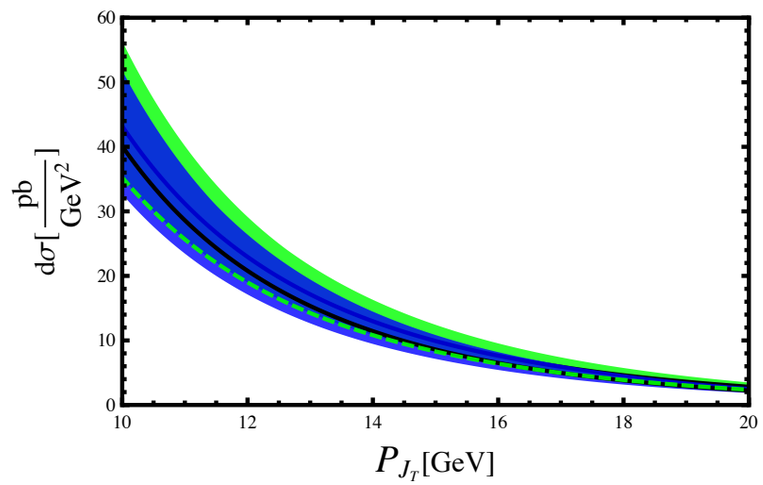
(a) Proton



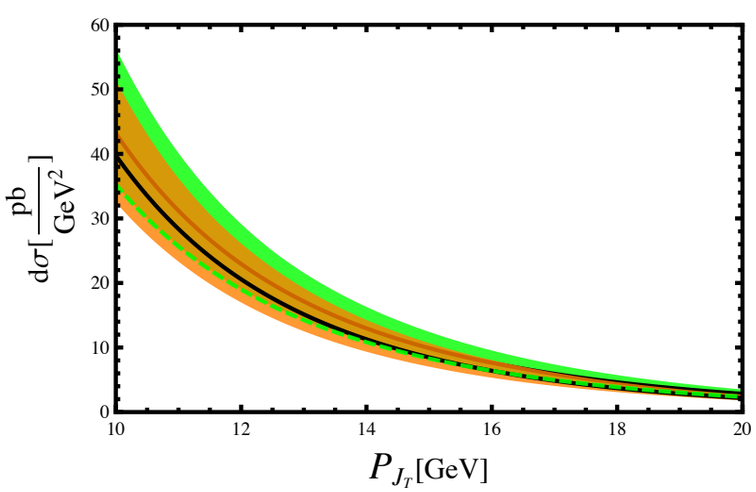
(b) Proton and C



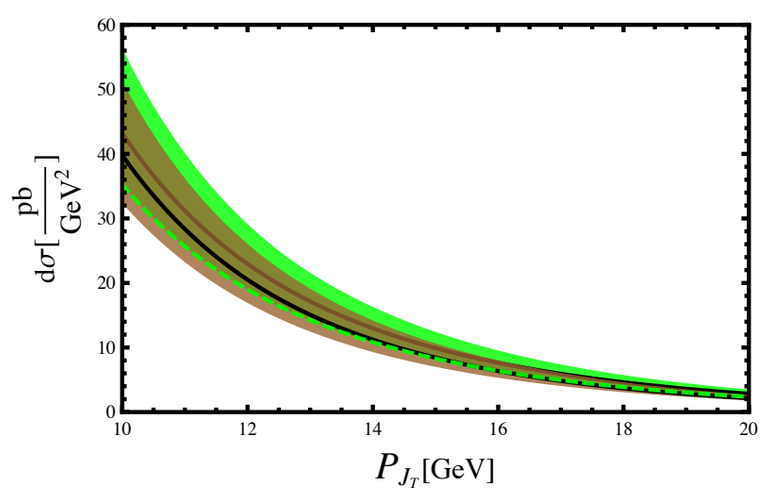
(c) Proton and Ca



(d) Proton and Fe



(e) Proton and Au



(f) Proton and Au

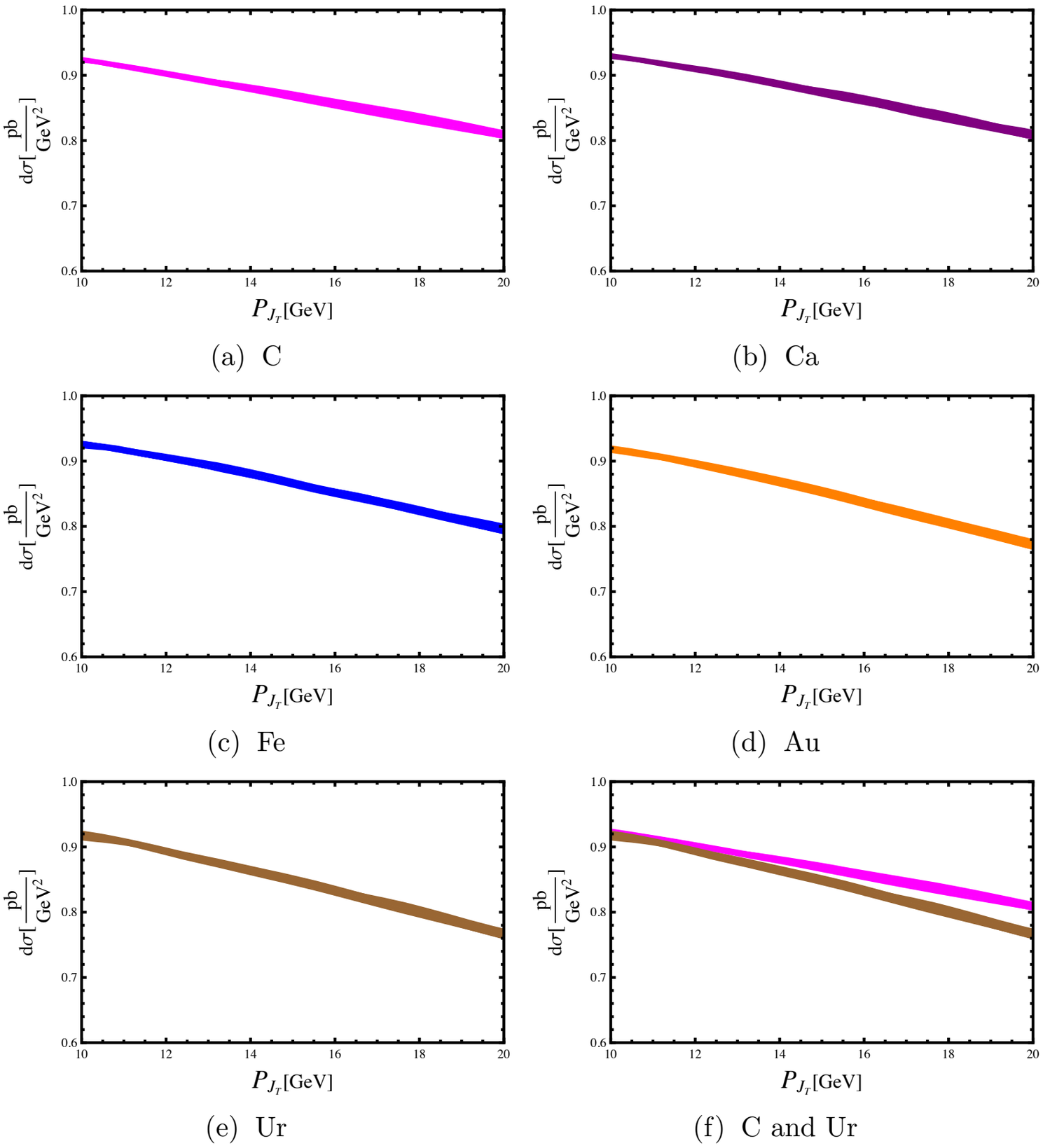
NNLL resummation

$$Q_e = 90 \text{ GeV}$$

$$y = 0$$

$$\tau_1 = 1.5 \text{ GeV}$$

Jet pT: Nucleus to proton ratios



(a) C (b) Ca (c) Fe (d) Au (e) Ur (f) C and Ur

$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

NNLL resummation

$$Q_e = 90 \text{ GeV}$$

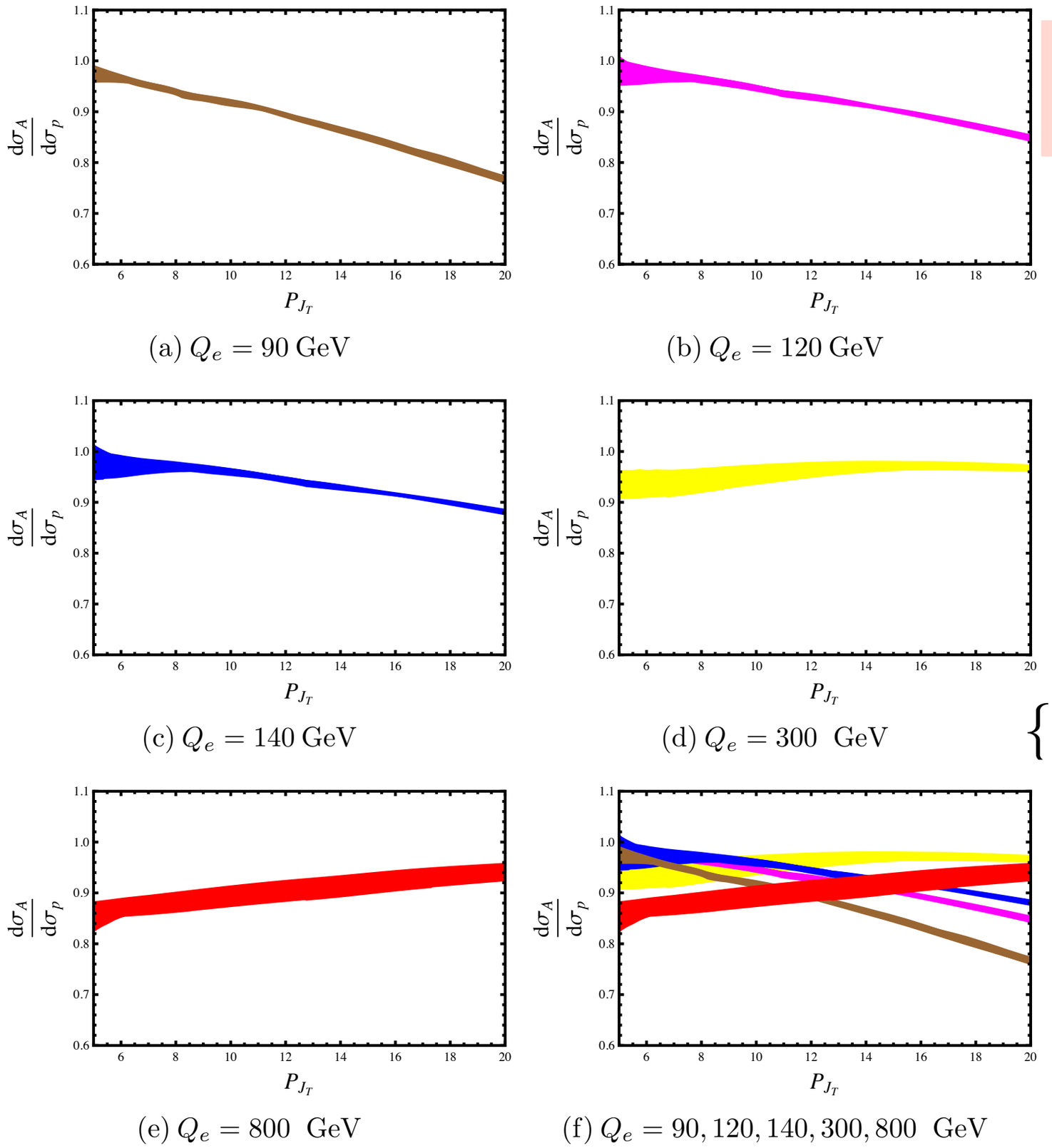
$$y = 0$$

$$\tau_1 = 1.5 \text{ GeV}$$

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

$$x_* \sim [0.1, 0.3]$$

Jet pT: Uranium to proton ratios



(a) $Q_e = 90$ GeV (b) $Q_e = 120$ GeV (c) $Q_e = 140$ GeV (d) $Q_e = 300$ GeV (e) $Q_e = 800$ GeV (f) $Q_e = 90, 120, 140, 300, 800$ GeV

$$R_A(\tau_1, P_{J_T}, y) = \frac{d\sigma_A(\tau_1, P_{J_T}, y)}{d\sigma_p(\tau_1, P_{J_T}, y)}$$

NNLL resummation

$$y = 0$$

$$\tau_1 = 1.5 \text{ GeV}$$

$$Q_e = \{90, 120, 140, 300, 800\} \text{ GeV}$$

$$x_* = \frac{e^y P_{J_T}}{Q_e - e^{-y} P_{J_T}}$$

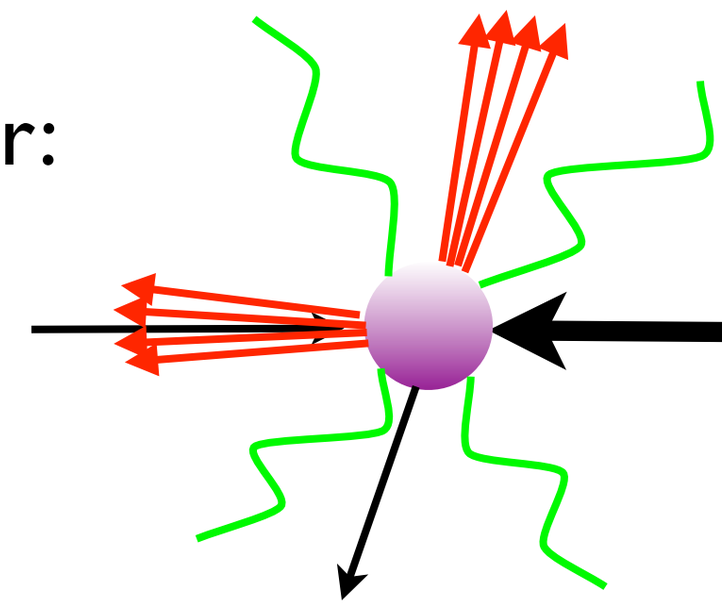
Non-perturbative Region

- Hierarchy of scales:

$$\frac{d^3\sigma}{dy dP_{JT} d\tau_1} \sim H \otimes B_A \otimes J \otimes \mathcal{S}, \qquad P_{J_T} \gg \sqrt{\tau_1 P_{J_T}} \gg \tau_1$$

- Soft function becomes non-perturbative for:

$$\tau_1 \sim \Lambda_{\rm QCD}$$



- Soft function model

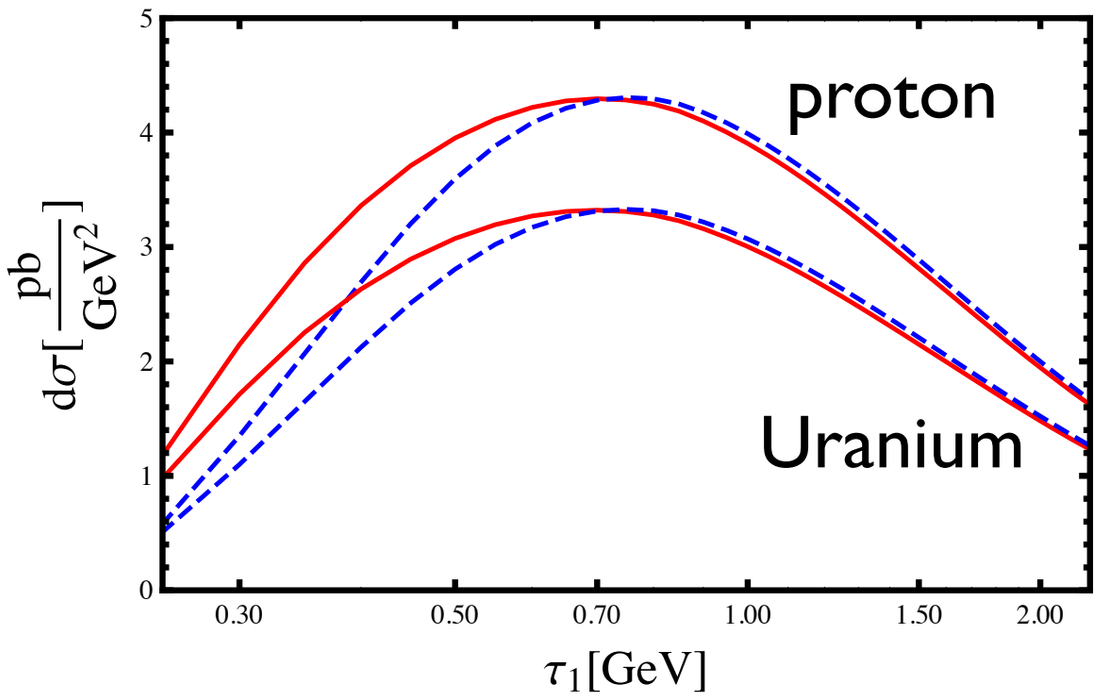
(Ligeti, Stewart, Tackmann)

$$\mathcal{S}(k_a, k_J, \mu_S) = \int dk'_a \int dk'_J \, \mathcal{S}_{\rm part.}(k_a - k'_a, k_J - k'_J, \mu_S) S_{\rm mod.}(k'_a, k'_J)$$

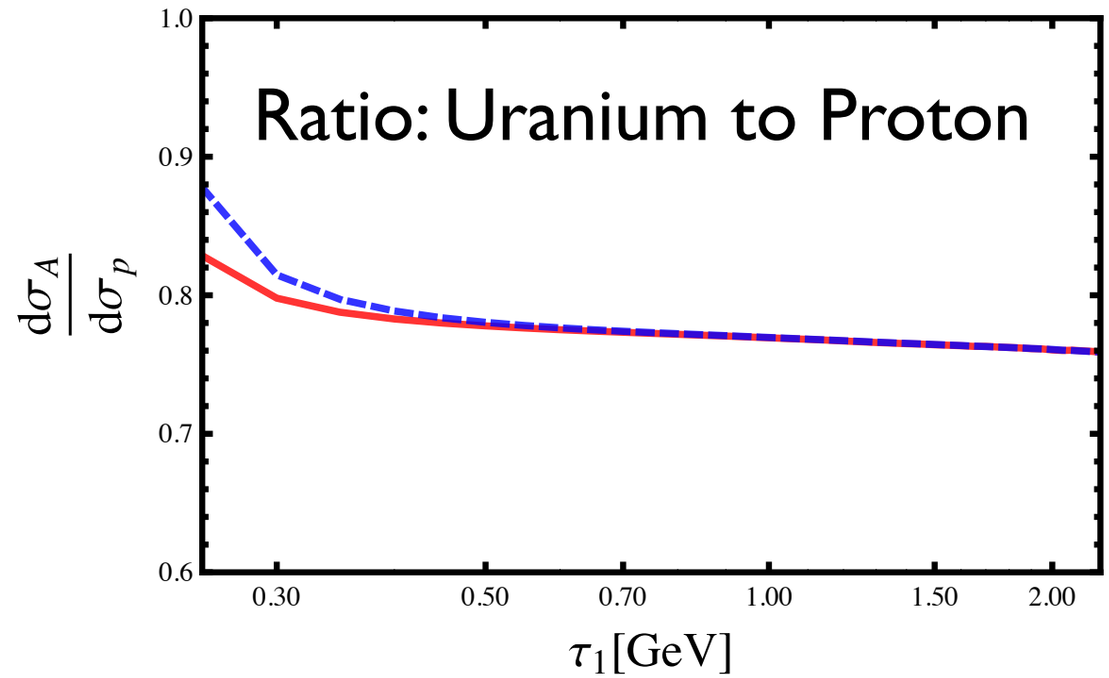
perturbative soft function

model

Soft Function Models



(a)



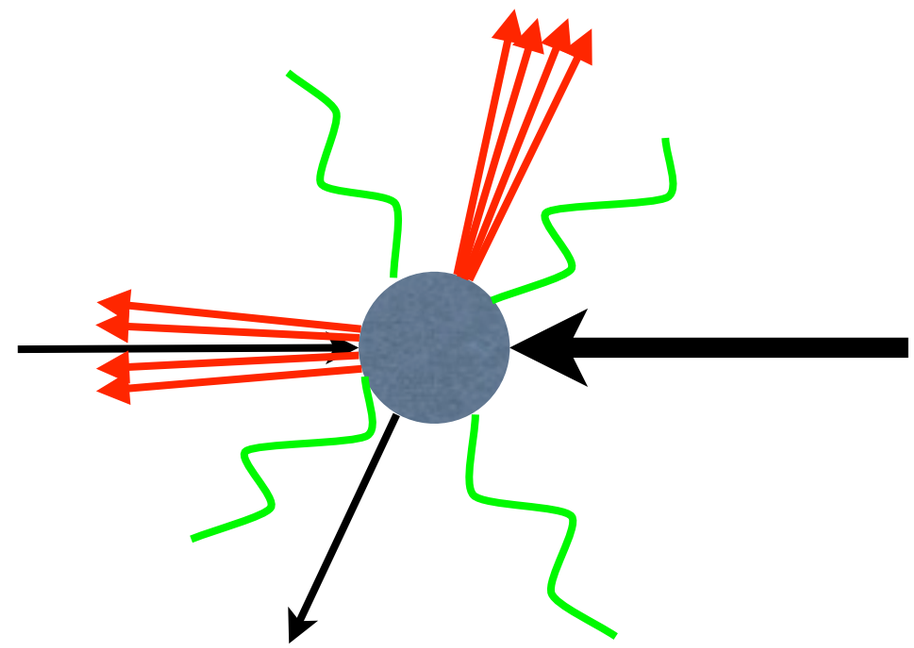
(b)

- Model dependence in the non-perturbative region.
- Models converge to perturbative result.
- Model dependence largely cancels in the ratio.

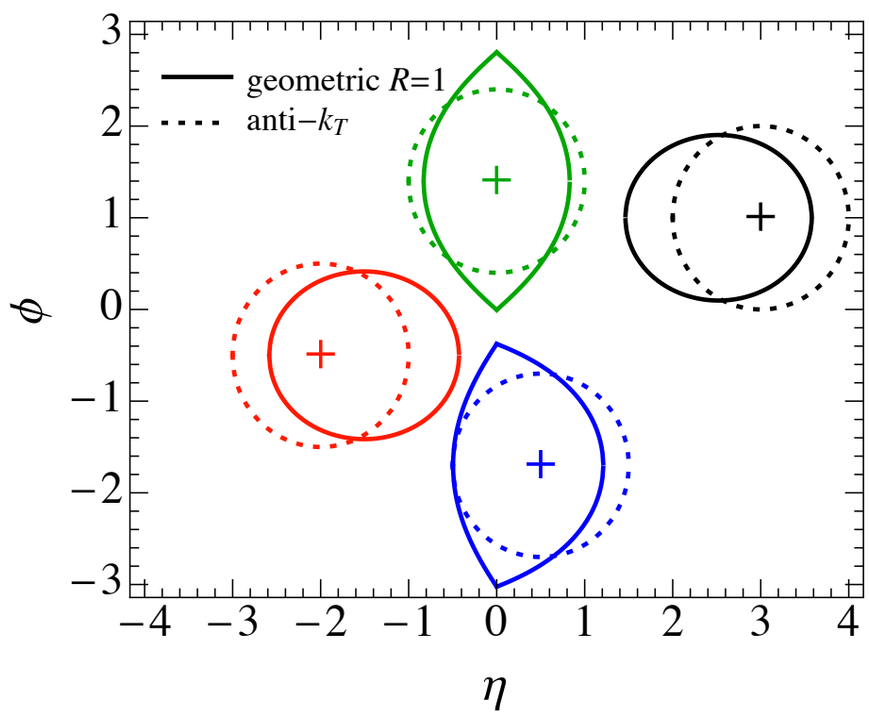
Jet Shape analysis

- Jet momentum:

$$P_J = \sum_k p_k \, \theta\left(\frac{2q_A \cdot p_k}{Q_a} - \frac{2q_J \cdot p_k}{Q_J}\right)$$



Can act as jet shape parameter



(b) Anti-k$_T$ and geometric R for $R = 1$.

(Jouttenus, Stewart, Tackmann, Waalewijin)

$$Q_J = 2\rho(R, \eta_J)\, E_J$$

- Choice can affect geometric shape of jet.
- One can exploit this property to also employ jet shape analyses to study nuclear modification.

Conclusions

- Studied factorization of gauge boson production in association with multiple jets near threshold.

- Used N-jettiness event shape formalism that avoids jet algorithm dependence.

- Numerical results obtained for photon + 2jets at NLL level of accuracy.

- Significant K-factors obtained in the large meff region.

Conclusions

- Event shape based analysis of exclusive jet production can be a useful tool to probe nuclear dynamics.

- Allows for jet shape analysis while also giving information on wide-angle soft radiation.

- Probe nuclear dynamics through distributions in the space:

$$\{A, Q_e, P_{J_T}, y, \tau_1\}$$

- Leading twist results given with NNLL resummation for various nuclear targets.

- Many directions can be pursued along these lines...