WIMP-nucleon scattering with heavy particle effective theory

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(electroweak charged)

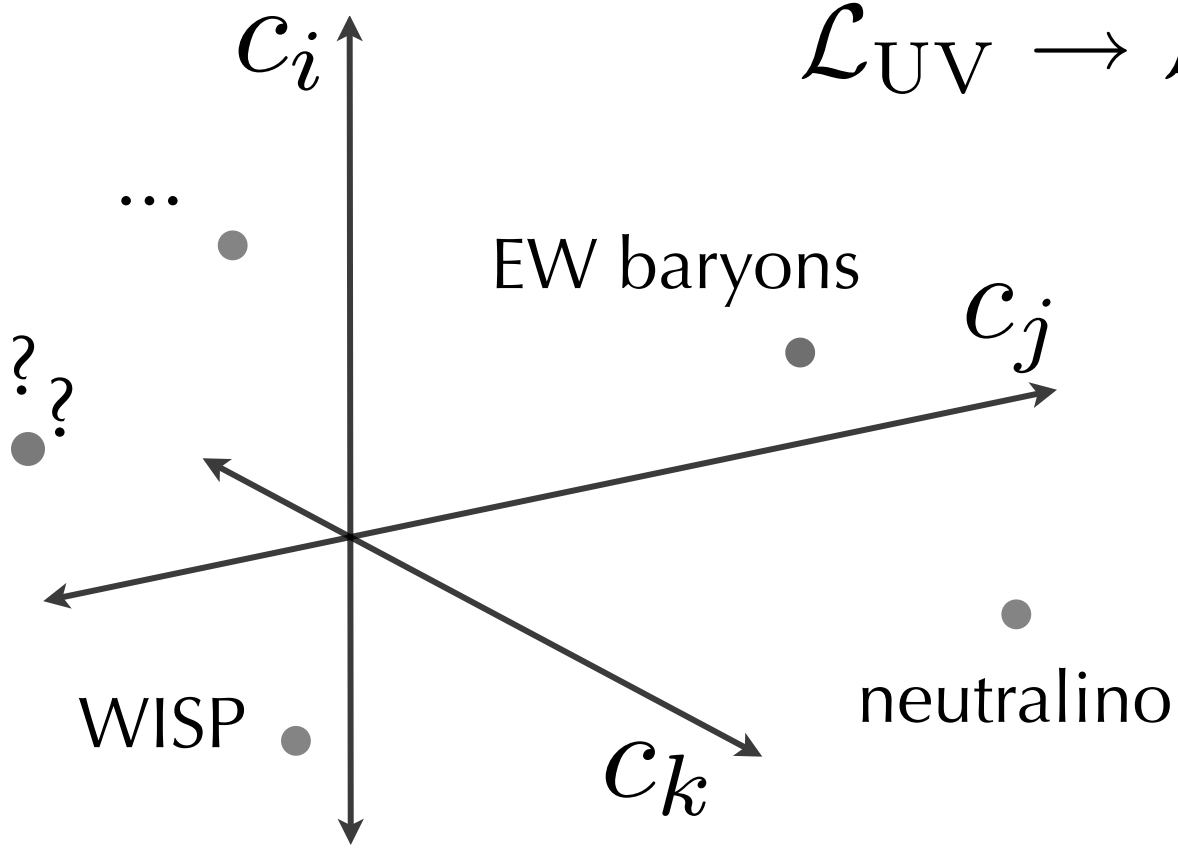
WIMP-nucleon scattering with heavy WIMP effective theory

Richard Hill, MS, 1111.0016
Johannes Heinonen, Richard Hill, MS, 1208.0601
Richard Hill, MS, 1309.4092
and work in progress

Theory space of electroweak charged dark matter

(for direct detection)

$$\mathcal{L}_{\rm UV} \to \mathcal{L}_{\rm EW} = \sum_i c_i \mathcal{O}_i$$



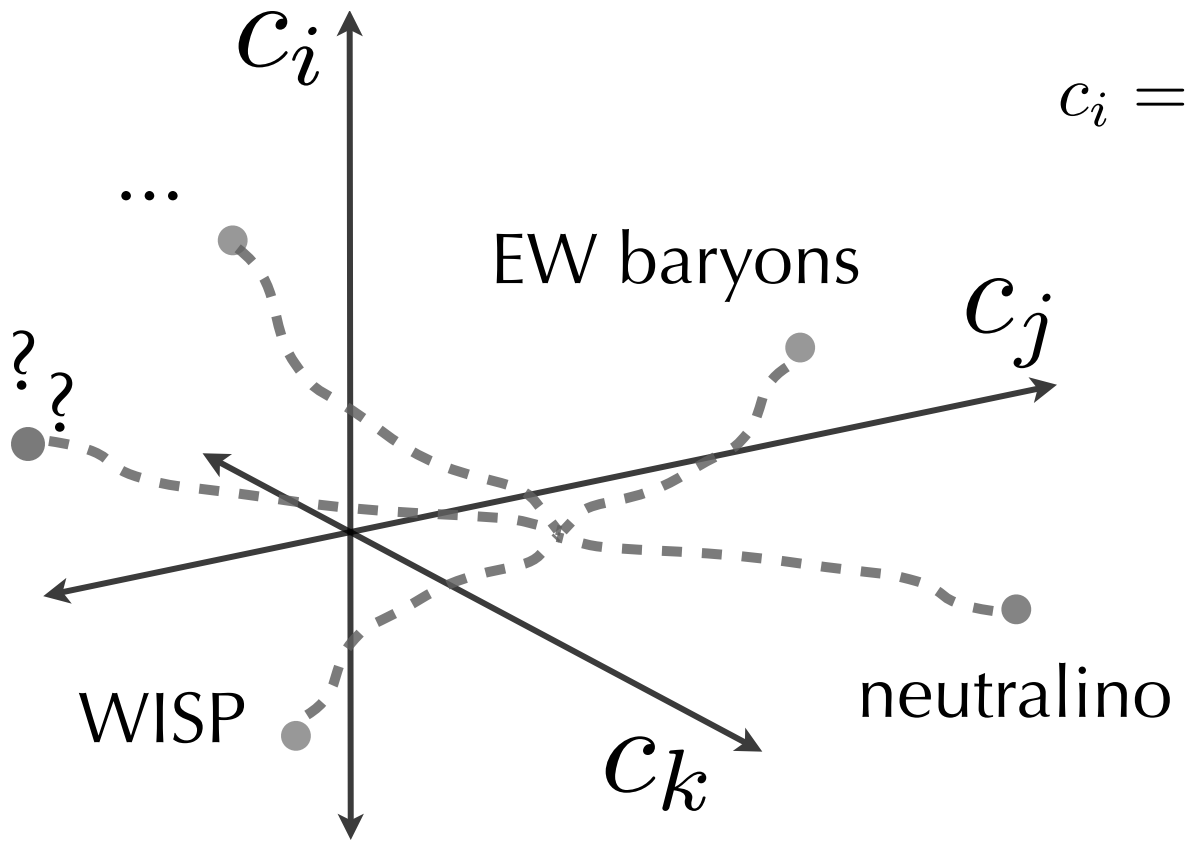
a. model, predictive (<12)
b. vanilla EFTs, inclusive (12)
c. predictive and inclusive?

universality of heavy particle interactions: hydrogen spectroscopy (NRQED), Isgur-Wise function (HQET)

...flows to a unique point in the heavy mass limit.

$$(m_W \ll M)$$

$$c_i = c_{i,0} + c_{i,1} \frac{m_W}{M} + \ldots$$



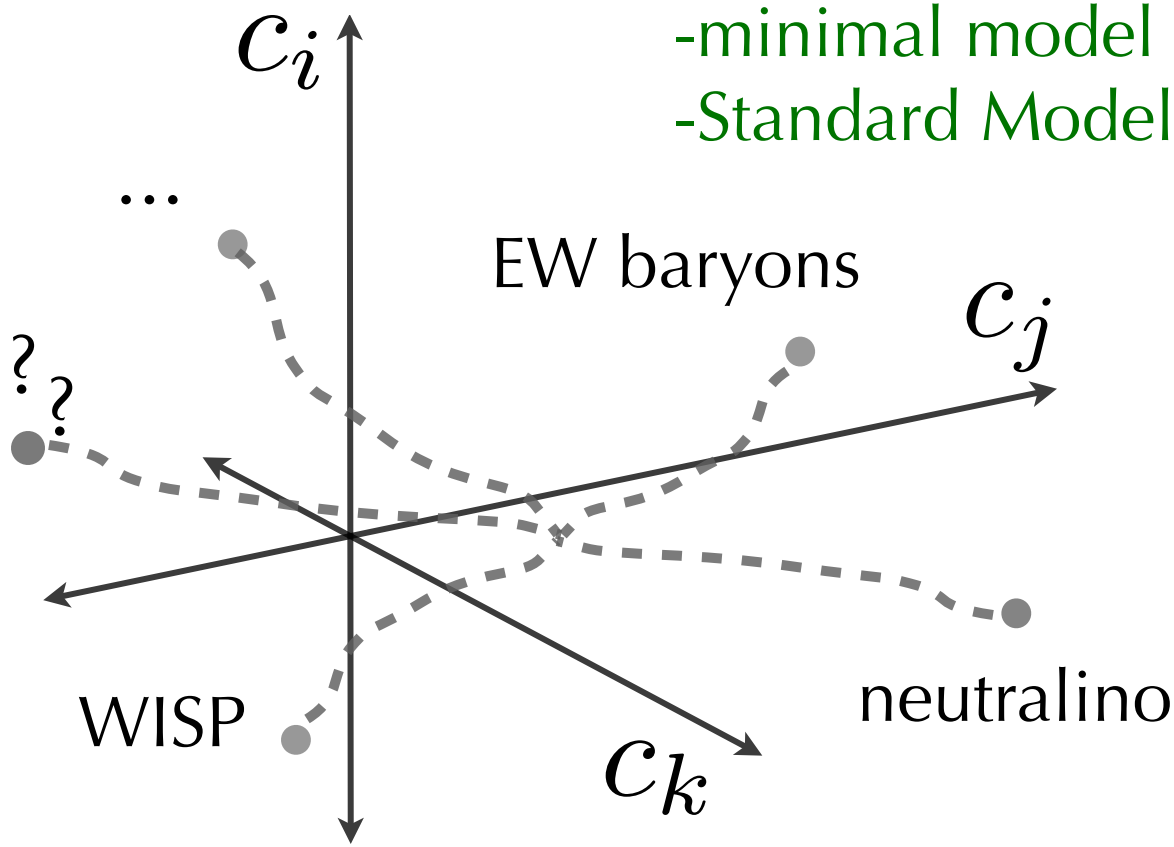
Compute the direct detection cross section for this universal point using heavy particle effective theory.

Heavy WIMP limit is interesting.

Universality/Simplicity

-minimal model input

-Standard Model anatomy: Higgs,lattice



Viable phenomenology

-bounds pushed by LHC null results

-relic density

Anatomy of direct detection calculations

UV theory: don't know, don't care (mostly)

M — One or two heavy electroweak multiplets (HPET)
heavy particle formalism

$\approx$

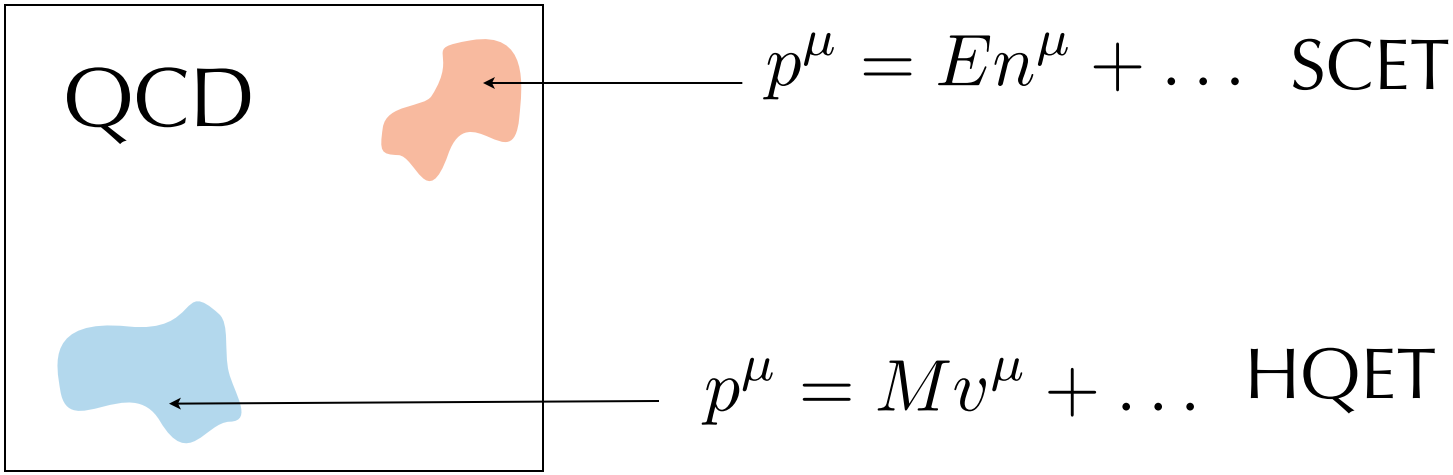
m_W — Complete weak scale matching
careful with renormalization, new integral basis

m_b, m_c — QCD anatomy of direct detection.
resum large logs, theoretical uncertainty

m_N — Matrix elements
impact of lattice improvements

$$\sigma(\mathrm{SM}) + \mathcal{O}(m_W/M, m_b/M_w, \Lambda_{QCD}^2/m_c^2)$$

What is heavy particle effective theory?



Obtain useful EFTs **from full theory**
- introduce field redefinitions
- use strategy of regions

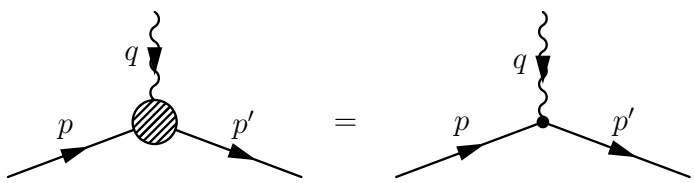
Construct EFTs **from scratch**?
- full theory may be unknown (WIMPs) or non-existent (proton)

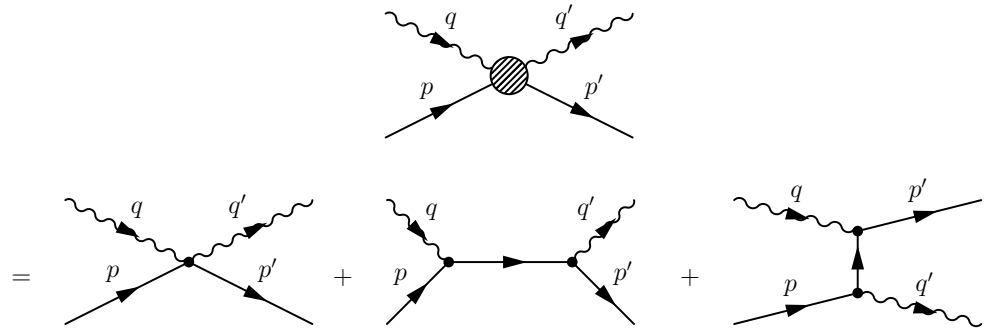
Need to know symmetries and building blocks

Obscured Lorentz symmetry $\longrightarrow$ obscured field representations

(there is a fixed time-like vector v)

- Match amplitudes or Poincare generators from full theory





$$c_F = 1 + \bar{F}_2$$

$$c_D = 1 + 2\bar{F}_2 + 8\bar{F}_1'$$

$$c_{A2} = 4\bar{F}_2 + 2\bar{F}_2^2 + 16\bar{F}_1' + 32\bar{f}_{2,0}$$

$$c_S = 1 + 2\bar{F}_2$$

$$c_S = 2c_F - 1$$

$$32c_{X4} = 3 + 4\bar{F}_2 - 64\bar{f}_{2,0}$$

$$32c_{X4} = -1 - 4c_F(1 - c_F) + 4c_D - 2c_{A2}$$

- Reparameterization Invariance (incorrect ansatz)

Need to know symmetries and building blocks

Identified Lorentz symmetry $\longrightarrow$ identified field representations

representation that keeps v fixed

- Little group for massive particles
- Boosts act as rotations (method of induced representations)

$$\vec{k} = \vec{r}h - t\vec{p} + \frac{i\vec{\Sigma} \times \vec{D}}{M + \sqrt{M^2 - \vec{D}^2}} + \mathcal{O}(g)$$

usually: $\vec{k} = \vec{r}h - t\vec{p} + i\Sigma$

- Equivalent: non-linearly realized subgroup

Heinonen, Hill, MS [1208.0601]

Need to know symmetries and building blocks

little group for massive particles ~ SO(3): spin = 0,1/2,1,...

embed in Dirac spinor-vectors with constraints

$$v_{\mu_1}\phi_v^{\mu_1\cdots\mu_n} = 0, \quad \not{v}\phi_v^{\mu_1\cdots\mu_n} = \phi_v^{\mu_1\cdots\mu_n}, \quad \gamma_{\mu_1}\phi_v^{\mu_1\cdots\mu_n} = 0$$

self-conjugacy leads to a parity:

$$\phi_v(x) \to \mathcal{C}\phi_v^*(x), \quad v^\mu \to -v^\mu$$

Now apply the QFT recipe

Write everything possible with building blocks

$$(\phi_v, v^\mu, D^\mu, \gamma^\mu, ...)$$

consistent with symmetries.

e.g. real scalar triplet with Y=0.

$$\mathcal{L}_\phi = \phi_v^* \Big\{ iv\cdot D - c_1 \frac{D_\perp^2}{2M} + c_2 \frac{D_\perp^4}{8M^3} + g_2 c_D \frac{v^\alpha [D_\perp^\beta, W_{\alpha\beta}]}{8M^2} + i g_2 c_M \frac{\{D_\perp^\alpha, [D_\perp^\beta, W_{\alpha\beta}]\}}{16M^3}$$
$$+ g_2^2 c_{A1} \frac{W^{\alpha\beta} W_{\alpha\beta}}{16M^3} + g_2^2 c_{A2} \frac{v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta}}{16M^3} + g_2^2 c_{A3} \frac{\mathrm{Tr}(W^{\alpha\beta} W_{\alpha\beta})}{16M^3}$$
$$+ g_2^2 c_{A4} \frac{v_\alpha v^\beta \mathrm{Tr}(W^{\mu\alpha} W_{\mu\beta})}{16M^3} + g_2^2 c'_{A1} \frac{\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A2} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu W_{\nu\alpha} W_{\rho\sigma}}{16M^3}$$
$$+ g_2^2 c'_{A3} \frac{\epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}(W_{\mu\nu} W_{\rho\sigma})}{16M^3} + g_2^2 c'_{A4} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu \mathrm{Tr}(W_{\nu\alpha} W_{\rho\sigma})}{16M^3} + \dots \Big\} \phi_v$$

$$D_\perp^\mu = D^\mu - v^\mu v\cdot D$$

Boosts: $c_1 = c_2 = 1,\ c_M = c_D$

$$\mathcal{L}_{\phi,\mathrm{SM}} = \phi_v^* \Big\{ c_H \frac{H^\dagger H}{M} + \cdots + c_Q \frac{t_J^a \bar{Q}_L \tau^a \not{v} Q_L}{M^2} + c_X \frac{i \bar{Q}_L \tau^a \gamma^\mu Q_L \{t_J^a, D_\mu\}}{2M^3} + c_{DQ} \frac{\bar{Q}_L \not{v} i v \cdot D Q_L}{M^3}$$
$$+ c_{Du} \frac{\bar{u}_R \not{v} i v \cdot D u_R}{M^3} + c_{Dd} \frac{\bar{d}_R \not{v} i v \cdot D d_R}{M^3} + c_{Hd} \frac{\bar{Q}_L H d_R + h.c.}{M^3} + c_{Hu} \frac{\bar{Q}_L \tilde{H} u_R + h.c.}{M^3}$$
$$+ g_3^2 c_{A1}^{(G)} \frac{G^{A\,\alpha\beta} G^A_{\alpha\beta}}{16M^3} + g_3^2 c_{A2}^{(G)} \frac{v_\alpha v^\beta G^{A\,\mu\alpha} G^A_{\mu\beta}}{16M^3} + g_3^2 c_{A1}^{(G)\prime} \frac{\epsilon^{\mu\nu\rho\sigma} G^A_{\mu\nu} G^A_{\rho\sigma}}{16M^3} + g_3^2 c_{A2}^{(G)\prime} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu G^A_{\nu\alpha} G^A_{\rho\sigma}}{16M^3}$$
$$+ \ldots \Big\} \phi_v$$

Boosts: $c_Q = c_X$

Real scalar parity enforced, e.g. omitted $\frac{\phi_v^* \bar{Q}_L \not{v} Q_L \phi_v}{M^2}$

Other applications:

- NRQED up to $1/M^4$, RPI fails at this order
- Four-fermion operators for low-energy lepton-nucleon scattering Hill, Lee, Paz, MS [1212.4508]

- Four-fermion operators for DM-nucleon scattering (cf. (to do) Galilean invariant basis of Fitzpatrick et al. [1203.3542])

Focus on universal limit

M heavy, self-conjugate electroweak multiplet(s) of arbitrary spin

$$iD_\mu = i\partial_\mu + g_1 Y B_\mu + g_2 W^a t^a$$

electroweak charges

mixing through H

$$\mathcal{L} = \bar{h}\left[iv\cdot D - \delta m - f(H)\right]h + \mathcal{O}(1/M)$$

residual mass matrix

spin, mass,...

$$M + \delta m = (M, M', M', M', M')$$

$$\mathcal{L}_{H\bar{h}h} = -\bar{h}_S\left[yH^\dagger\frac{(h_{D_1} - ih_{D_2})}{\sqrt{2}} + y'H^T\frac{(h_{D_1} + ih_{D_2})}{\sqrt{2}}\right] + \text{h.c.}$$

electroweak charges

mixing through H

$$\mathcal{L} = \bar{h}\left[iv \cdot D - \delta m - f(H)\right] h + \mathcal{O}(1/M)$$

residual mass matrix

spin, mass,...

Pure states $\delta m = f(H) = 0$

- $SU(2)_W$ triplet, Y=0 i.e. wino-like

- self-conjugate combinations of $SU(2)_W$ doublets with Y=|1/2|, i.e. higgsino-like

- parameter-free, beyond specification of t^a and Y

electroweak charges

mixing through H

$$\mathcal{L} = \bar{h}\left[iv \cdot D - \delta m - f(H)\right] h + \mathcal{O}(1/M)$$

residual mass matrix

spin, mass,...

Mixed states $\delta m, f(H) = (\Delta, \kappa)$ $\Delta =(M - M')/2$

- singlet-doublet mixture i.e. bino-higgsino SUSY fixes kappa

- triplet-doublet mixture i.e. wino-higgsino

- two-parameters, interplay between loop- and mass- suppression

Weak scale matching

M

$$\mathcal{L} = \bar{h}\left[iv\cdot D - \delta m - f(H)\right]h + \mathcal{O}(1/M)$$

$\approx$

μ_t

match: integrate out t, W, Z, h, goldstones

$$\mathcal{L}_{\chi,\mathrm{SM}} = \frac{1}{m_W^3}\bar{\chi}\chi\left\{\sum_q\left[c_{1q}^{(0)}O_{1q}^{(0)} + c_{1q}^{(2)}v_\mu v_\nu O_{1q}^{(2)\mu\nu}\right] + c_2^{(0)}O_2^{(0)} + c_2^{(2)}v_\mu v_\nu O_2^{(2)\mu\nu}\right\}$$

$$O_{1q}^{(0)} = m_q\bar{q}q\,, \qquad O_2^{(0)} = (G_{\mu\nu}^A)^2\,,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q}\left(\gamma^{\{\mu}iD^{\nu\}} - \frac{1}{d}g^{\mu\nu}i\not{D}\right)q\,, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_\lambda + \frac{1}{d}g^{\mu\nu}(G_{\alpha\beta}^A)^2$$

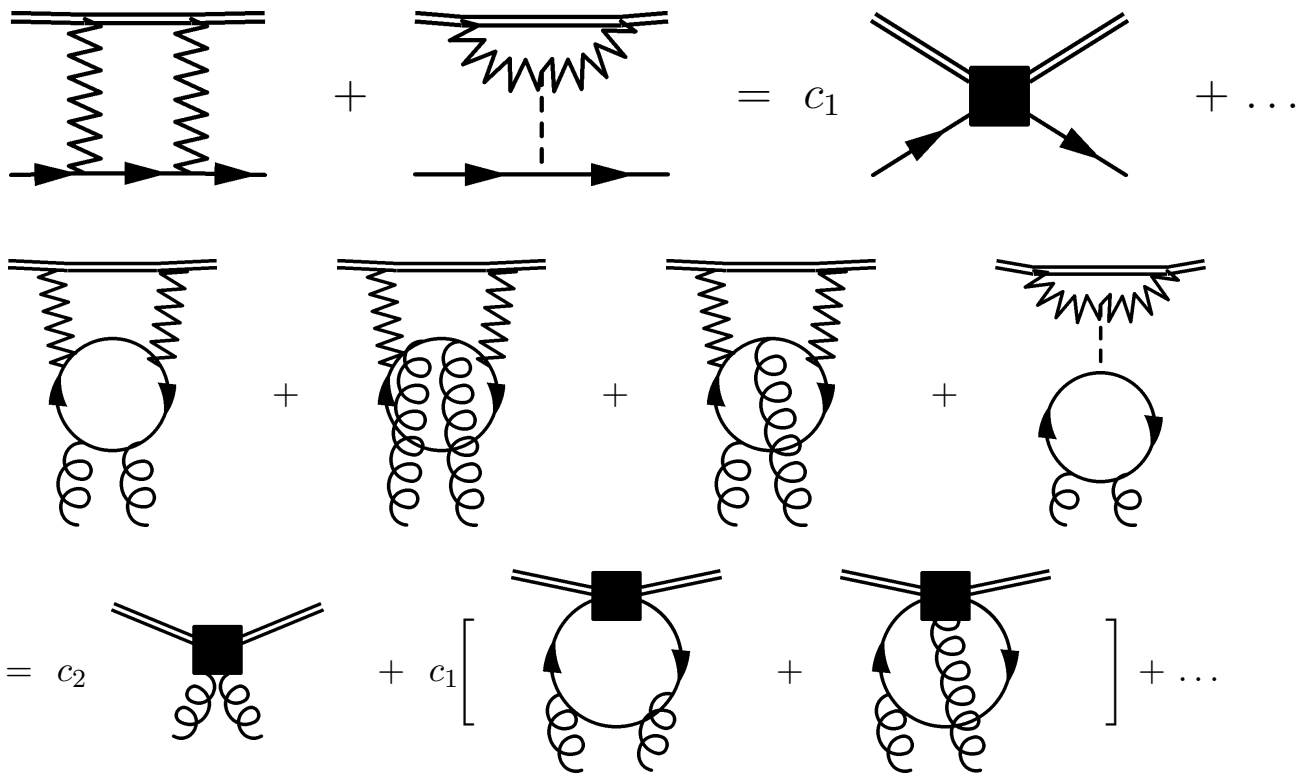
Complete leading-order basis for self-conjugate WIMP, spin-independent, low-velocity scattering (12 operators, closed under renormalization).

(to do)

$$c_i = c_{i,0} + c_{i,1}\frac{m_W}{M} + \ldots$$

Weak-scale matching: pure case

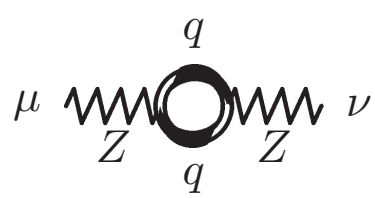
previous computations: Essig [0710.1668]; Cirelli,Fornengo,Strumia [0512090]; Hisano, Ishiwata, Nagata, Takesako [1104.0228]



spin-2 gluon matching is new

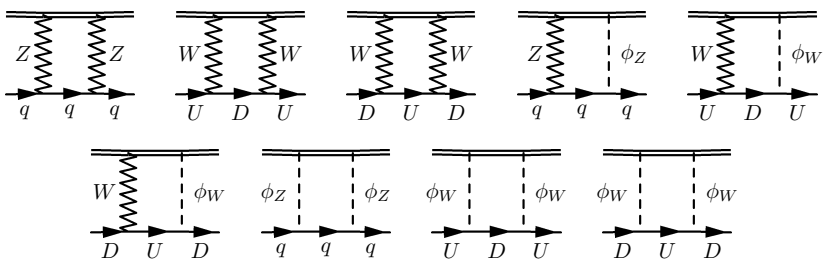
toolbox: infrared subtraction with dimreg, background field method in Fock-Schwinger gauge

$\Pi^{\nu\mu}_{(ZZ)}(L) =$



Weak-scale matching: mixed case

-intermediate Higgs and goldstone bosons, e.g., two-boson exchange



$$J(m_V, M, \delta) = \int (dL) \left[\frac{1}{-v \cdot L - \delta} + \frac{1}{v \cdot L - \delta} \right] \frac{1}{(L^2 - m_V^2)^2} \frac{1}{L^2 - M^2} \,,$$

$$J^\mu(p, m_V, M, \delta) = \int (dL) \left[\frac{1}{-v \cdot L - \delta} + \frac{1}{v \cdot L - \delta} \right] \frac{1}{(L^2 - m_V^2)^2} \frac{1}{L^2 + 2L \cdot p - M^2} L^\mu$$

- tree-level Higgs exchange

$$\sim \frac{g_2 \kappa^2}{\sqrt{\kappa^2 + (\Delta / 2 m_W)^2}} \bar{\chi} \chi h^0$$

- one-boson exchange diagrams require renormalization

toolbox: on-shell renormalization scheme, new basis of heavy particle integrals with non-zero residual masses (relevant to low energy e-p scattering) (to do) Hill, Lee, Paz, MS [1212.4508]

Weak-scale matching: on-shell renormalization

$$\mathcal{L} = \bar{h}^{\rm bare}\left[iv\cdot D - M^{\rm bare} - f^{\rm bare}(H)\right]h^{\rm bare}$$

$$= \bar{h}\left[iv\cdot D - M - f(H)\right]h + \bar{h}\left[\delta Z_h iv\cdot D - \delta M - \delta f(H)\right]h$$

Fix counterterms by enforcing conditions on the residual mass matrix (two point functions).

h, ϕ_W, ϕ_Z

W, Z

$$[\delta M]_{22} + \mathrm{Re}[\Sigma_2(\delta_0^{(+)})]_{22} - \delta_0^{(+)}[\delta Z]_{22} = 0\,,$$

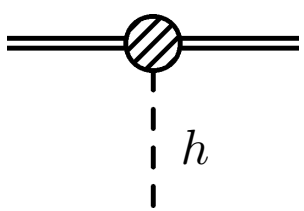
$$[\delta M]_{33} + \mathrm{Re}[\Sigma_2(0)]_{33} = 0\,,$$

$$[\delta M]_{11} + \mathrm{Re}[\Sigma_2(\delta_\pm)]_{11} - \delta_0^{(0)}[\delta Z]_{11} = 0\,,$$

$$[\delta M]_{13} + \mathrm{Re}[\Sigma_2(0)]_{13} = 0\,,$$

$$[\delta M]_{23} + \mathrm{Re}[\Sigma_2(0)]_{23} = 0\,.$$

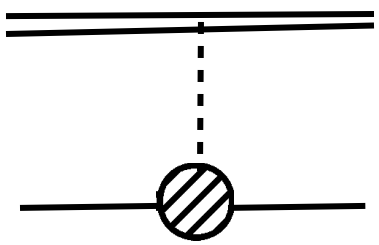
Three point function will be fixed (and finite).



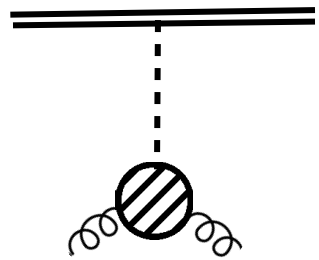
$$\sim \delta v + \delta Z + \delta\kappa + \ldots$$

Weak scale matching: subleading class of diagrams

We neglect contributions from loop corrections to SM Higgs couplings:

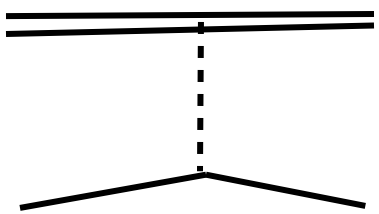


(one-loop)

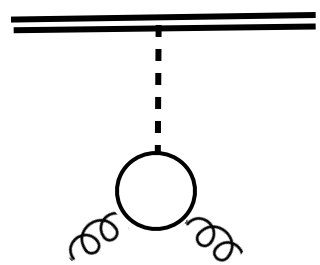


(two-loop)

These are <u>always</u> subleading compared to



(tree)



(one-loop)

Numerical impact on cross sections of O(1-10%) is generous.

Standard Model anatomy of direct detection

μ_t $c_i(\mu_t)$

μ_b

μ_c

μ_0 $c_i(\mu_0)$

spin-2 matrix elements depend on PDFs (check)

- Resums logs $\alpha_s(\mu_0)\log\frac{m_t}{\mu_0}$
- Theoretical uncertainty

spin-0 matrix elements defined in 3-flavor theory

Generic component, relevant to collider constraints on contact interactions (to do)

QCD module: technology

Running

$$\frac{d}{d\log\mu}O_i^{(S)} = -\sum_j \gamma_{ij}^{(S)} O_j$$

$$c_2^{(0)}(\mu) = c_2^{(0)}(\mu_t)\frac{\frac{\beta}{g}[\alpha_s(\mu)]}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

$$c_1^{(0)}(\mu) = c_1^{(0)}(\mu_t) - 2[\gamma_m(\mu) - \gamma_m(\mu_t)]\frac{c_2^{(0)}(\mu_t)}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

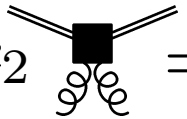
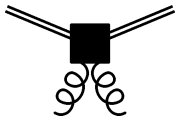
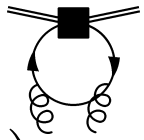
$$\hat{\gamma}^{(0)} = \frac{\alpha_s}{4\pi}\left(\begin{array}{ccc|c} 0 & & & 0 \\ & \ddots & & \vdots \\ & & 0 & 0 \\ \hline 32 & \cdots & 32 & -2\beta_0 \end{array}\right) + \ldots$$

$$\hat{\gamma}^{(2)} = \frac{\alpha_s}{4\pi}\left(\begin{array}{ccc|c} \frac{64}{9} & & & -\frac{4}{3} \\ & \ddots & & \vdots \\ & & \frac{64}{9} & -\frac{4}{3} \\ \hline -\frac{64}{9} & \cdots & -\frac{64}{9} & \frac{4n_f}{3} \end{array}\right) + \ldots$$

Tarrach [82]; Grinstein, Randall [89]; Vogt, Moch, Vermaseren [04]; Ovrut, Schnitzer [82]; Inami, Kubota, Okada [83], Chetyrkin, Kniehl, Steinhauser [98]

QCD module: technology

Matching

c_2  $= \tilde{c}_2$  $+ \tilde{c}_1$ 

$(n_f = 4)$ $(n_f = 5)$

(to do)

$$c_2^{(0)}(\mu_b) = \tilde{c}_2^{(0)}(\mu_b)\left(1 + \frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\right)$$
$$- \frac{\tilde{a}}{3}\tilde{c}_{1b}^{(0)}(\mu_b)\left[1 + \tilde{a}\left(11 + \frac{4}{3}\log\frac{m_b}{\mu_b}\right)\right] + \mathcal{O}(\tilde{a}^3),$$
$$c_{1q}^{(0)}(\mu_b) = \tilde{c}_{1q}^{(0)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$
$$c_2^{(2)}(\mu_b) = \tilde{c}_2^{(2)}(\mu_b) - \frac{4\tilde{a}}{3}\log\frac{m_b}{\mu_b}\tilde{c}_{1b}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$
$$c_{1q}^{(2)}(\mu_b) = \tilde{c}_{1q}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a})$$

familiar from h to gg

Tarrach [82]; Grinstein, Randall [89]; Vogt, Moch, Vermaseren [04]; Ovrut, Schnitzer [82]; Inami, Kubota, Okada [83], Chetyrkin, Kniehl, Steinhauser [98]

QCD module: higher-order $\alpha_s(\mu)$ corrections

μ_t weak scale matching: LO (largest uncertainty)

RG: NLO, NLO

μ_b threshold matching: NLO

RG: NLO, NLO

μ_c threshold matching: NLO

RG: NNNLO, NNNLO

pay close attention to charm scale

(under control; fix this scale)

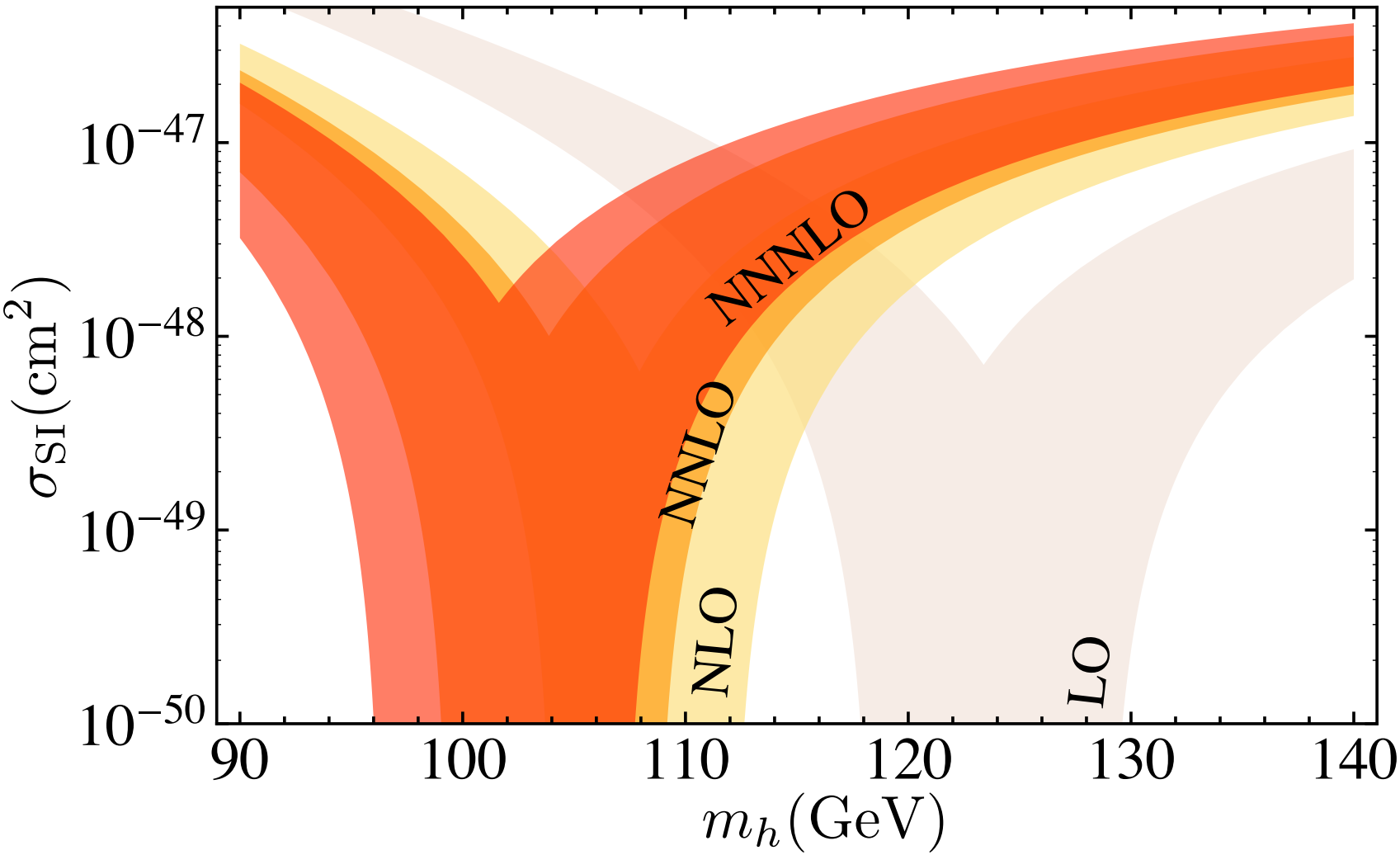
μ_0 NNNLO in the spin-0 gluon matrix element

$$m_N = (1-\gamma_m)\sum_q \langle N|m_q\bar{q}q|N\rangle + \frac{\beta}{2g}\langle N|(G^a_{\mu\nu})^2|N\rangle$$

m_N } Matrix elements

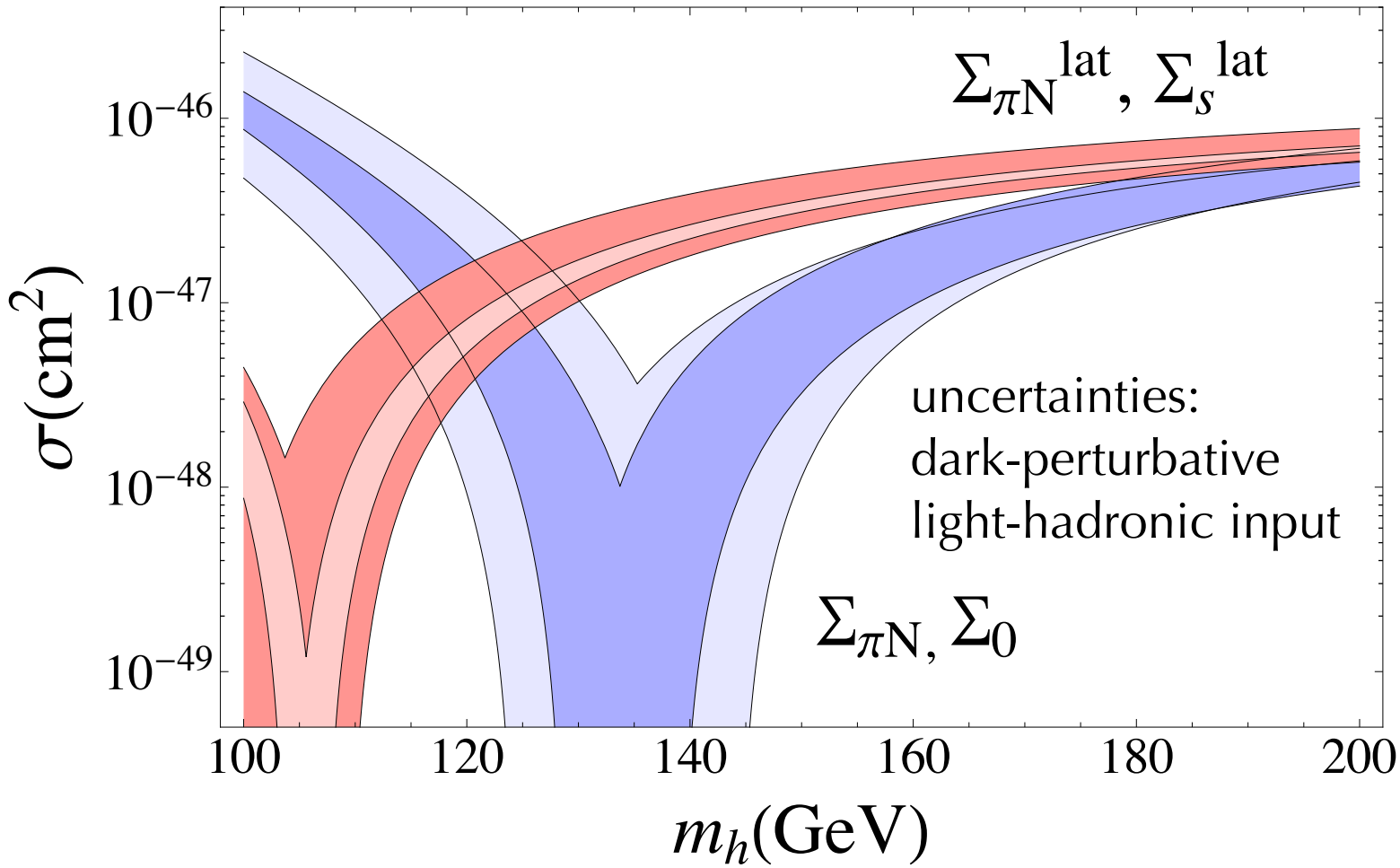
$$\sigma(\mathrm{SM}) + \mathcal{O}(m_W/M, m_b/M_w, \Lambda^2_{QCD}/m_c^2)$$

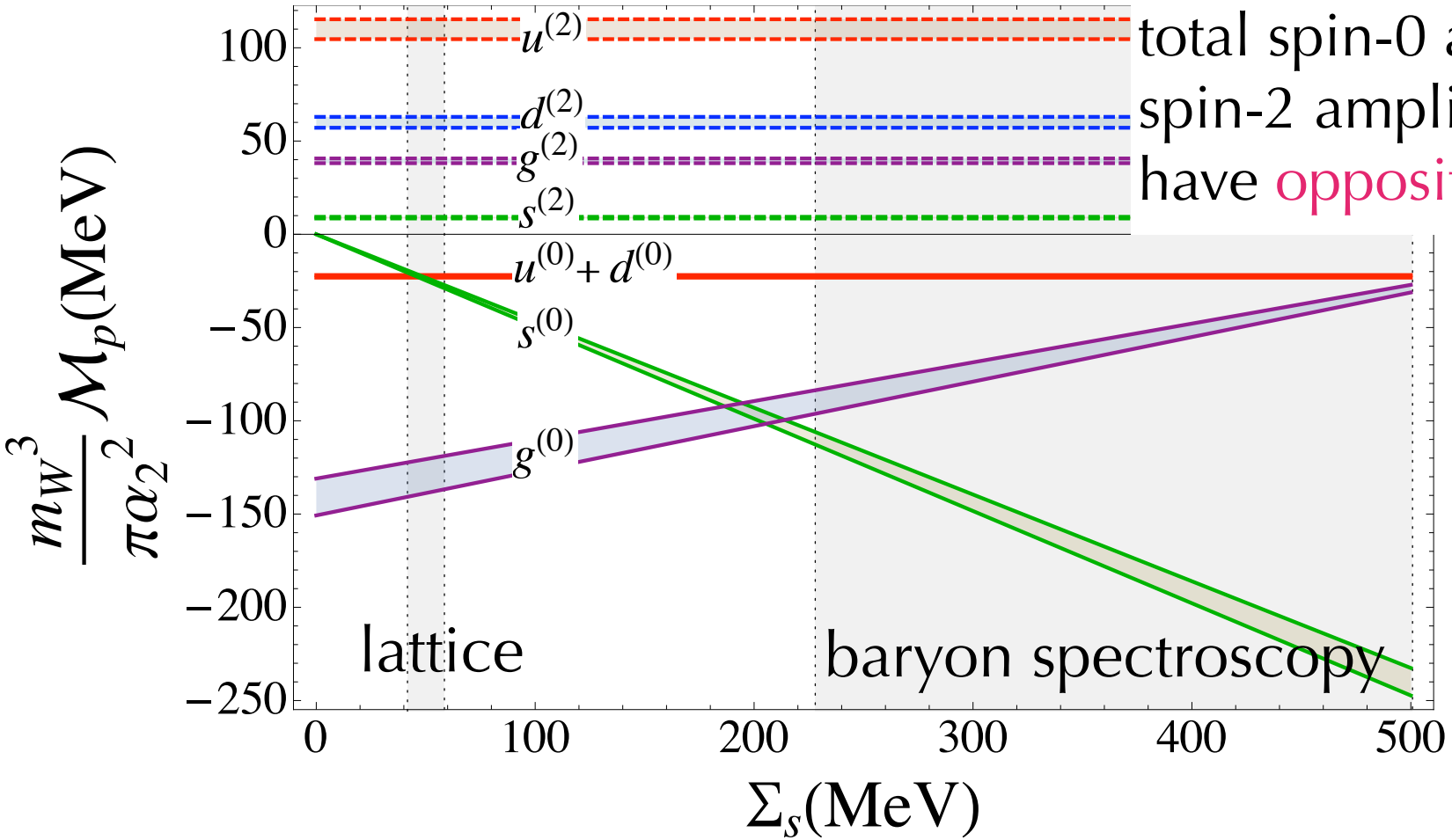
Impact of perturbative QCD
(pure triplet)



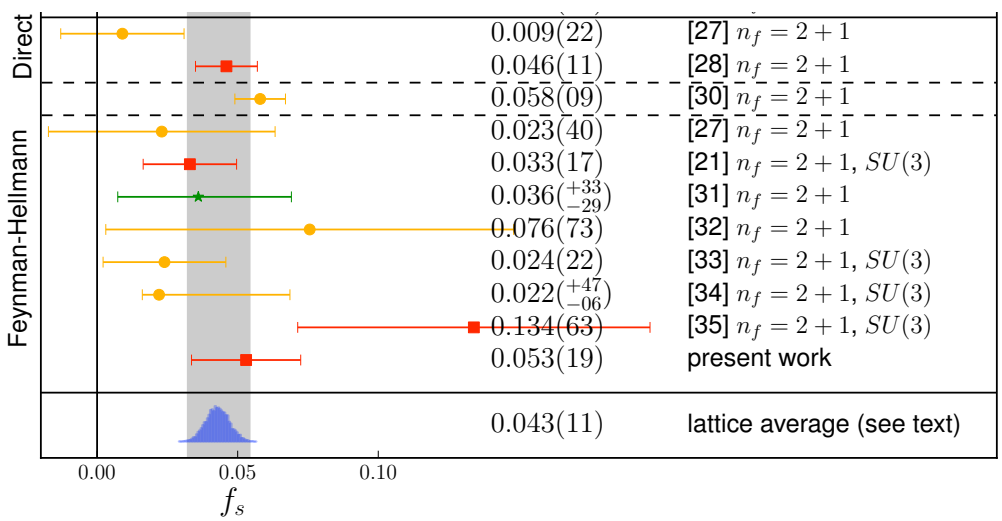
perturbative corrections in the RG running from μ_c to μ_0 and in the spin-0 gluon matrix element

Hadronic uncertainties



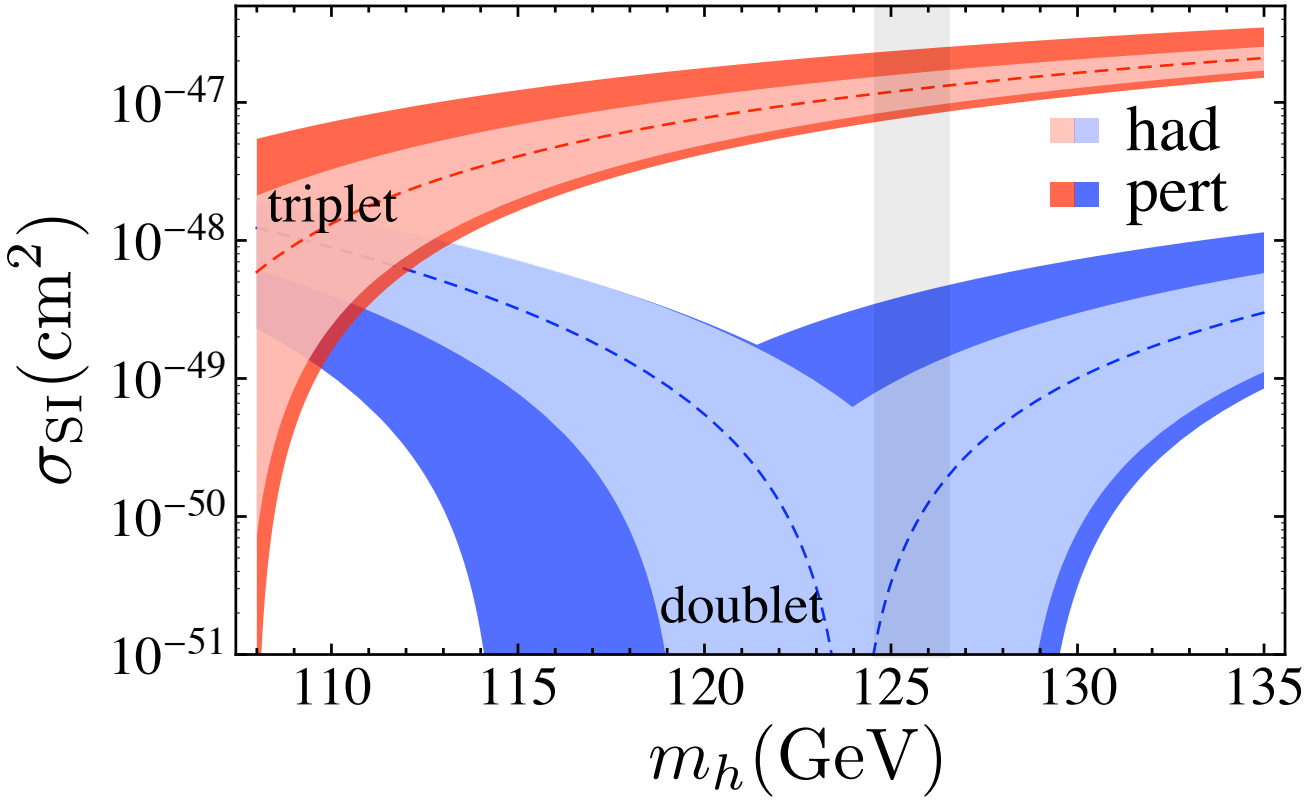


total spin-0 and spin-2 amplitudes have opposite signs



Junnarkar, Walker-Loud [1301.1114]

Pure states: parameter-free cross sections



(!) Cancellation between spin-0 and spin-2 amplitudes.

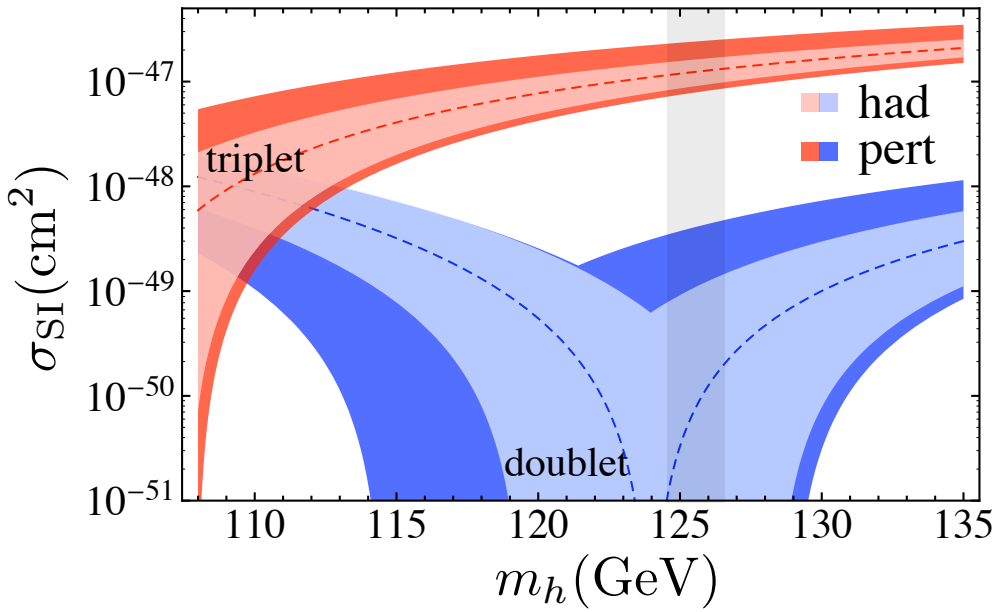
Leading order in $\alpha_s(\mu_t), m_W/M, m_b/m_W, \Lambda^2_{\rm QCD}/m_c^2$ (to do)

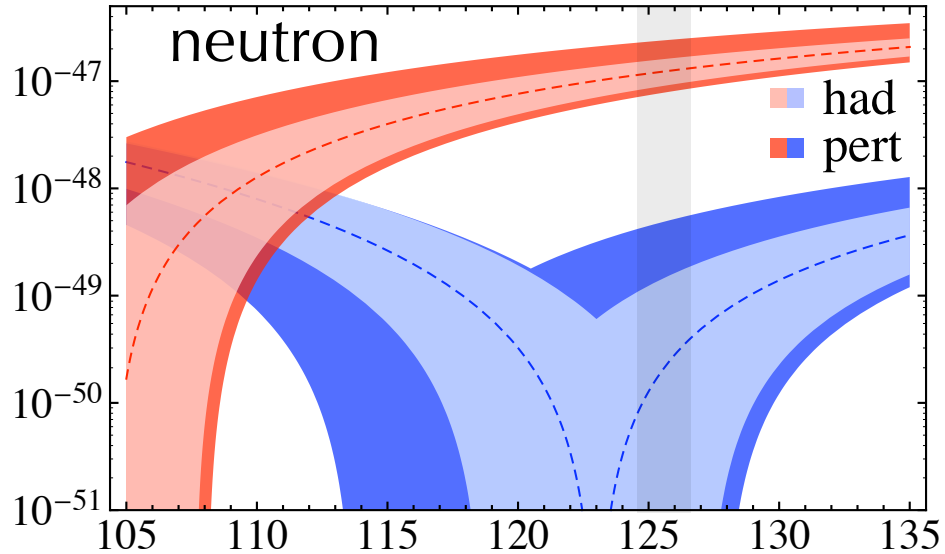
triplet: $\sigma_{\rm SI} = 1.3^{+1.2+0.4}_{-0.5-0.3} \times 10^{-47}\,\rm cm^2$

doublet: $\sigma_{\rm SI} \lesssim 10^{-48}\,\rm cm^2 \quad (95\%\,\rm C.L.)$

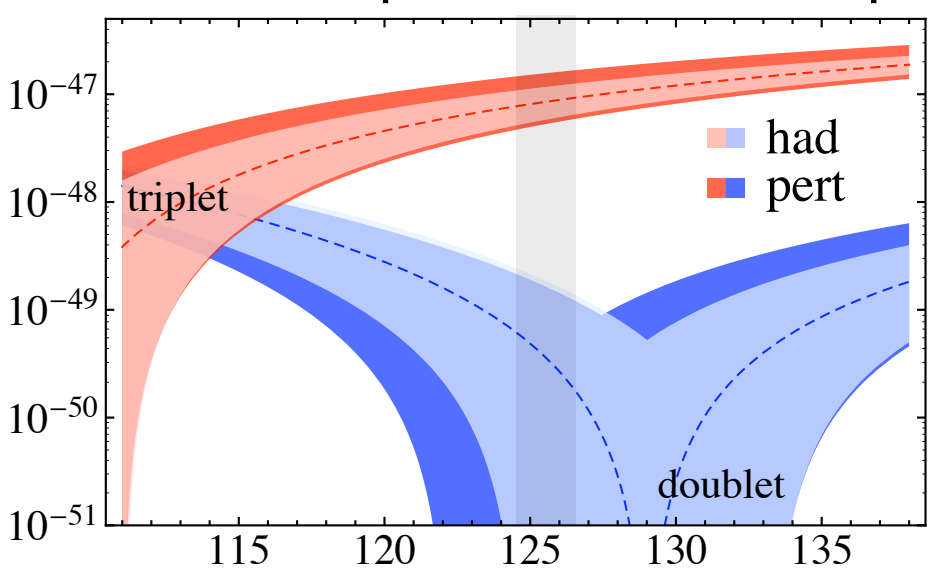
Previous estimates orders of magnitude off and lack uncertainties.

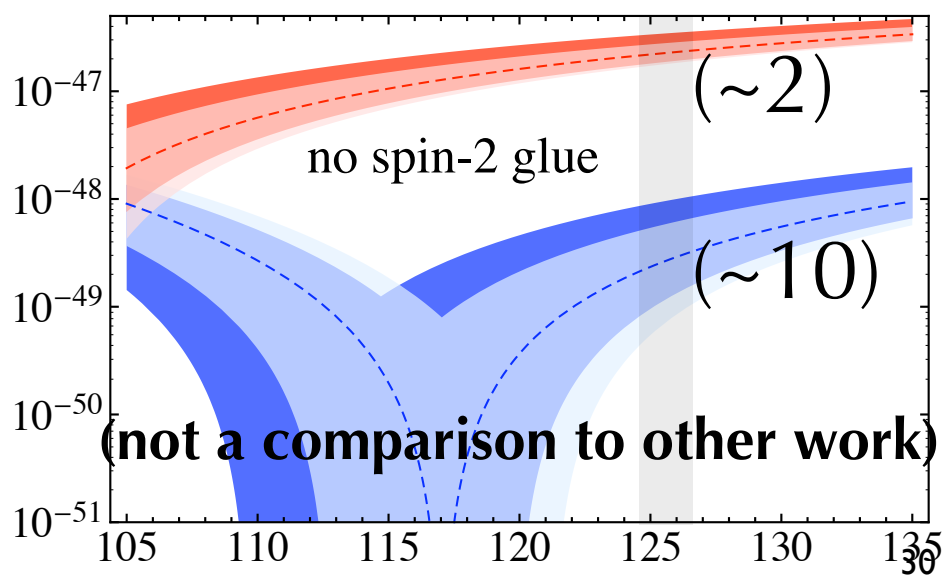
Pure states: checks



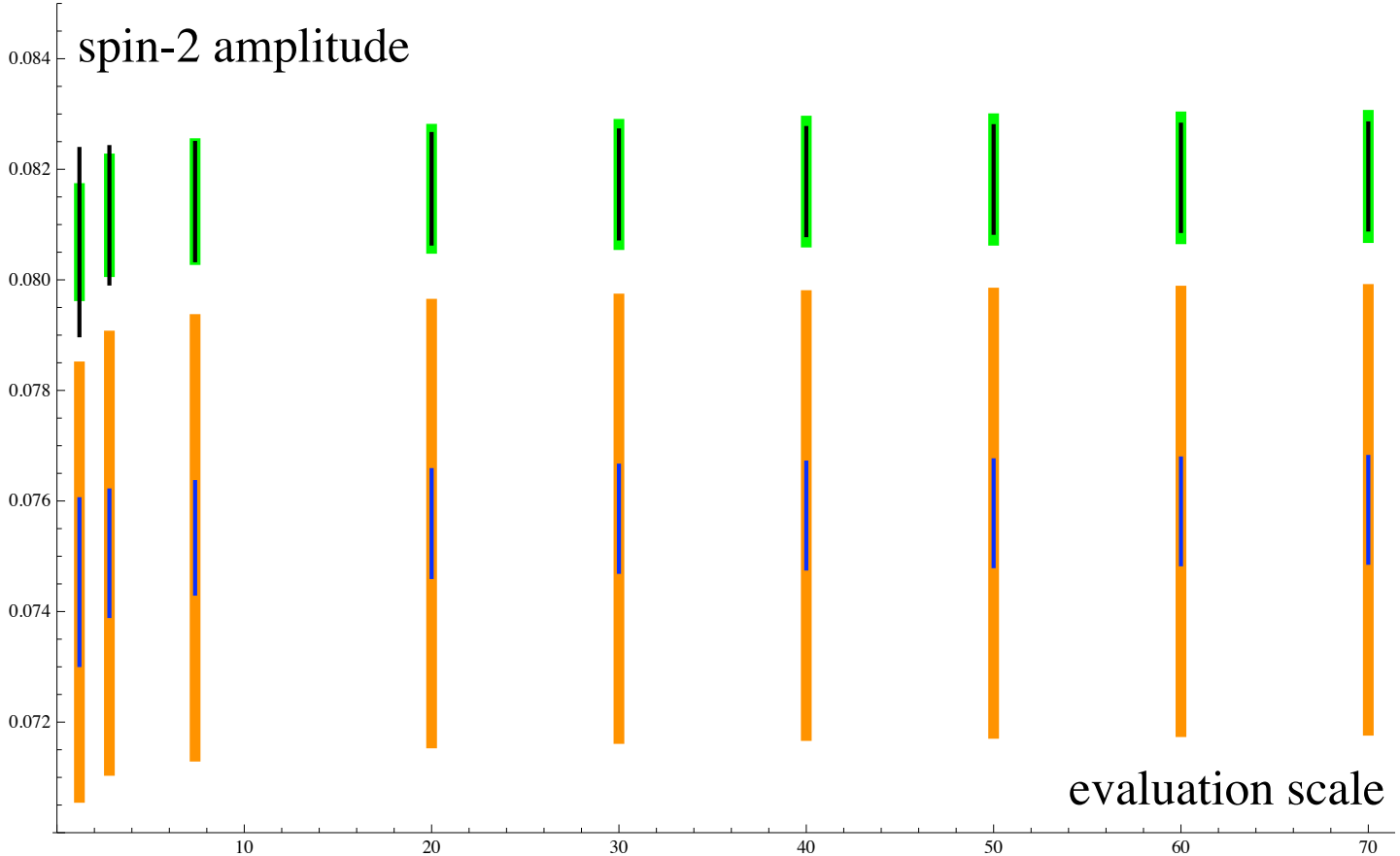


$\alpha(\mu_c)$
corrections (partial, scheme dependent)





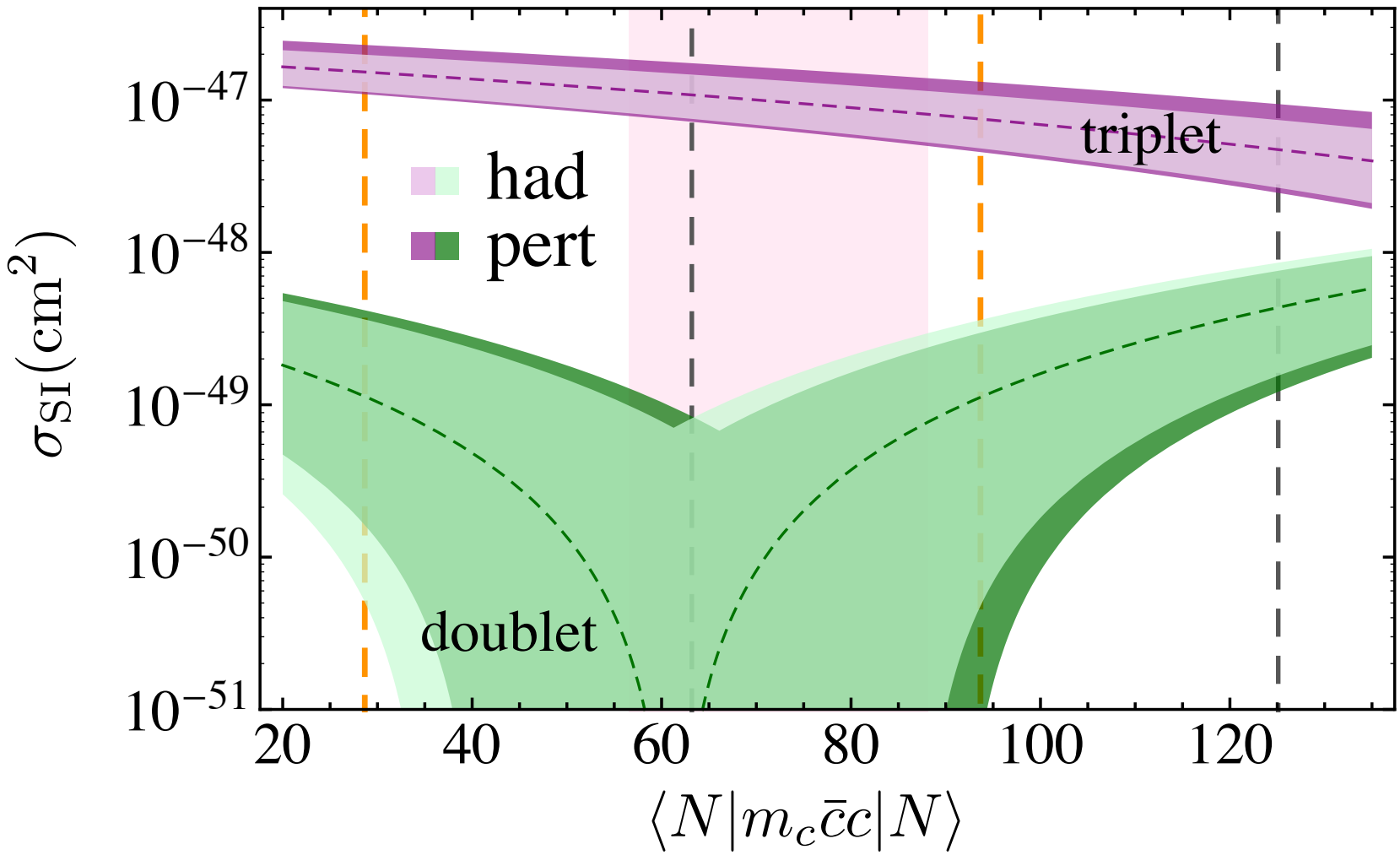
Pure states: spin-2 gluon contributions



spin-2 amplitude shift neglecting: bottom (~1%), charm (~10%), strange (~10%), down (~30%), up (~50%), glue (~10-20%)

Impact of charm scalar matrix element determination

evaluate in n_f=4 theory



perturbative QCD estimate: Junnarkar, Walker-Loud [1301.1114]

lattice: Freeman *et al.* [1204.3866], Gong *et al.* [1304.1194]

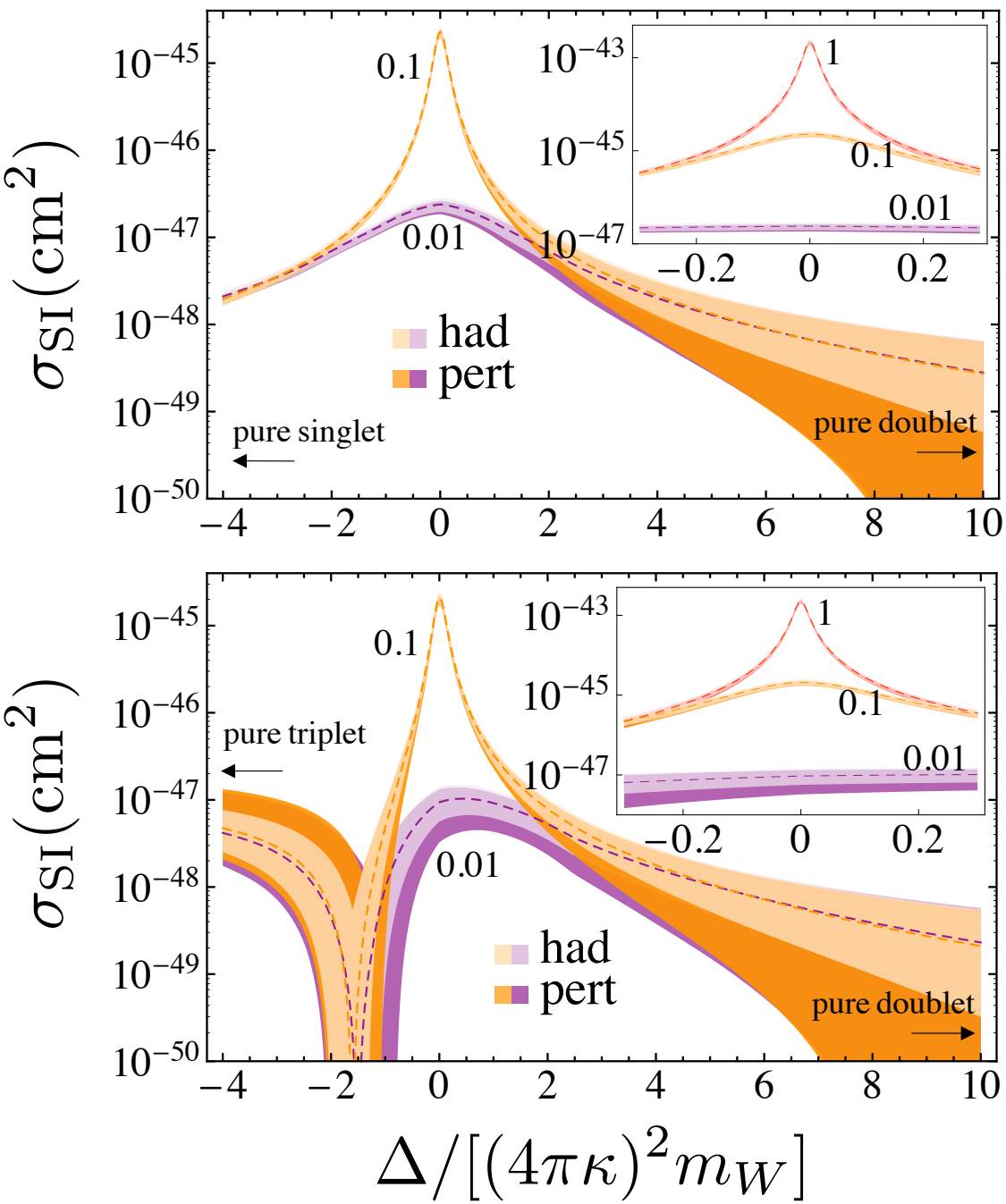
(to do)

upper bound vs prediction

triplet: $\sigma_{\rm SI} = 1.3^{+1.2+0.4}_{-0.5-0.3} \times 10^{-47}\,{\rm cm}^2$

doublet: $\sigma_{\rm SI} \lesssim 10^{-48}\,{\rm cm}^2 \quad (95\%\,{\rm C.L.})$

Mixed-state cross sections



Interplay of loop- and mass-suppressed contributions

Pure states generic

Cancellations generic

interference important

robust? (to do)

SUSY fixes kappa: (to do)

b-h: $\kappa \leq \tan\theta_W/2$

w-h: $\kappa \leq 1/2$

correlate with (to do)

other searches

Conclusions

Develop heavy particle effective theory and calculational techniques to evaluate the universal cross section.

A simple target to guide future direct searches.

Standard Model anatomy: Higgs mass, lattice, QCD effects, (cancellations).

charm

generic

THANKS!

What is RPI?

Identities lead to an equivalent formulation of boosts

$$\phi_v(x) \to W(B, iD)\phi_v(x') = \boxed{B} B^{-1} W(B, iD)\phi_v(x')$$

$$v \to v = \boxed{B} B^{-1} v$$

$$\boxed{\phi_v \to B^{-1} W(B, iD)\phi_v \qquad v \to B^{-1} v = v + q/M}$$

The RPI transformation

- equivalent for free fields, but not in interacting case
- takes the field outside of assumed representation space

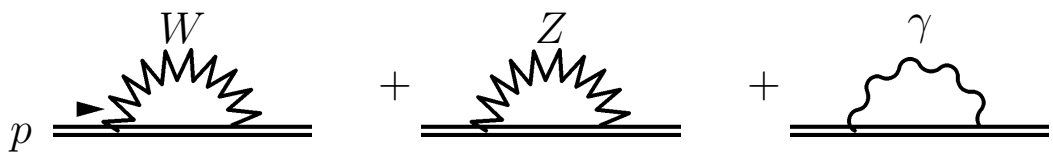
$$\not{v}\psi_v \neq \psi_v$$

- yields an incorrect coefficient relation for c_{X4}

$$32 c_{X4} = 4c_F^2 - \boxed{6} c_F Z + 4 Z c_D - 2c_{A2} - Z^2$$

e.g. Triplet mass splitting due to EWSB

$-i\Sigma(p) =$



$+ \ldots .$

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2 \frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170\,\mathrm{MeV}) Q^2$$

universal in the class