Cosmic Magnetic Fields from Level Crossing Fermions and Sphalerons

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ES, *Fermion Level Crossing and Magnetic Fields in the Early Universe*, *to appear*,

A.J. Long, ES, T. Vachaspati, *Leptogenesis and Primordial Magnetic Fields* [arxiv:1309.2315].

Motivation

Level Crossing and Fermion Currents
- Level Crossing
- Electroweak Sphalerons
- Dirac Hamiltonian and Fermion Modes
- Electromagnetic Current
- Magnetic Field

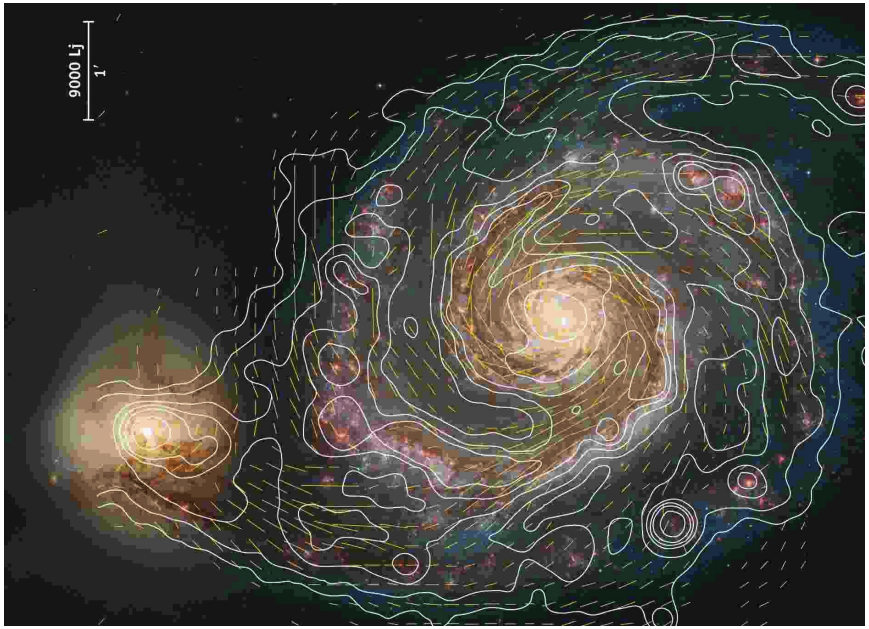
Leptogenesis and Magnetic Fields
- Leptogenesis
- Nonperturbative Processes
- Equilibrium Solution
- Magnetic Field Evolution
- Matter Genesis and Magnetic Fields

Conclusions

MOTIVATION

Galactic Magnetic Fields

- Galactic magnetic fields have been observed.



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- Seed magnetic fields are required to explain the origin of these fields.

- Seed fields can be of primordial origin.

- Cosmic magnetic fields might be traced in the extragalactic regions, e.g. using TeV blazars.

• Cosmic magnetic fields might have been generated during electroweak phase transition and inflation.

• Helicity can be related to the baryon asymmetry of the universe in the electroweak baryogenesis scenario Cornwall '97, Vachaspati '01.

• Can magnetogenesis occur within the standard model or beyond the standard model physics?

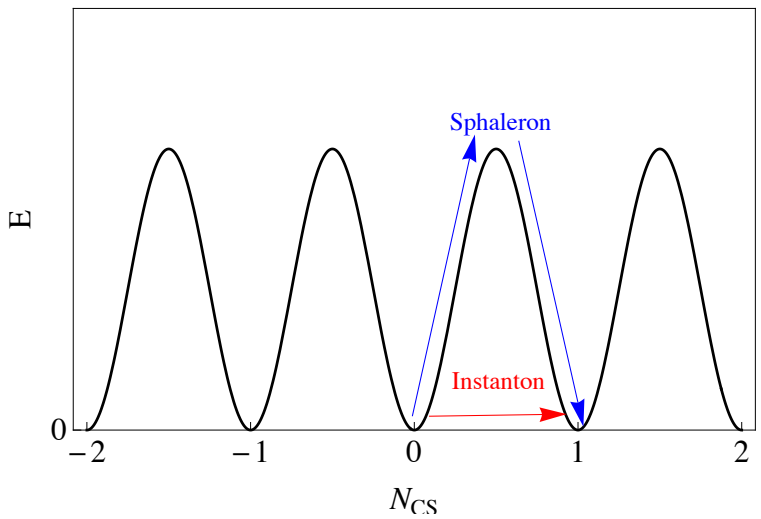
LEVEL CROSSING AND FERMION CURRENTS

Topology of Gauge Theories

- Non-Abelian gauge theories exhibit non-trivial vacuum configuration and topology.

- Chern-Simons number is a topological invariant

$$N_{\rm CS} = -\frac{g^2}{32\pi^2}\int_{\mathcal{M}} d^4x\ {\rm Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}). \tag{1}$$



- Yang-Mills instantons describe tunneling between successive degenerate vacua.

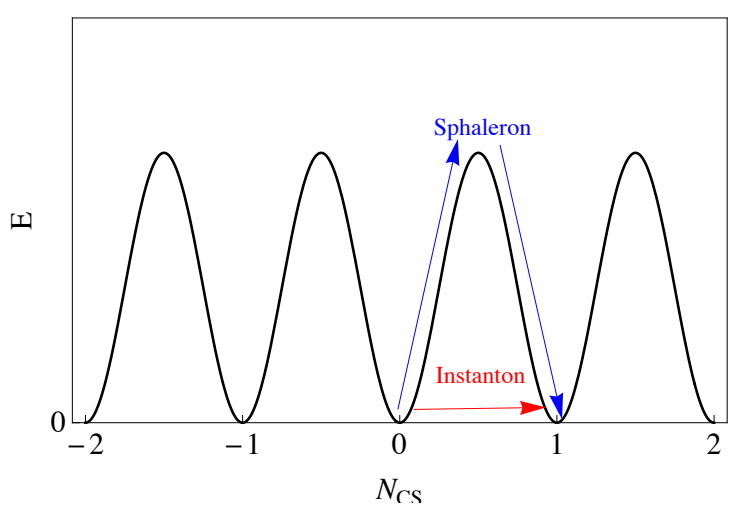
Belavin,Polyakov,Schwartz,Tyupkin '75.

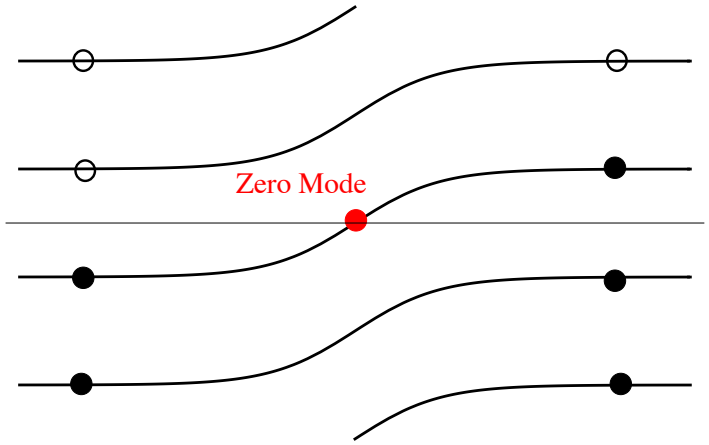
- Sphalerons describe saddle point solutions of Yang-Mills equations.

Manton '83; Klinkhamer&Manton '84

Level Crossing

• There exist fermion zero mode solutions in the topologically nontrivial gauge field backgrounds, such as instantons and sphalerons. Callan,Dashen,Gross '76; 't Hooft '76





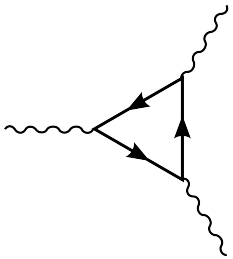
• The Atiyah-Singer index theorem relates the number of zero modes of the Euclidian Dirac operator to the Chern-Simons number Atiyah&Singer '63; Atiyah&Singer '68

$$\mathrm{Index}[\not{D}] = \Delta N_{\mathrm{CS}}. \tag{2}$$

Anomalies

- Some classical conservation laws can be anomalous (violated quantum mechanically), e.g., axial charge, lepton number, baryon number, etc.

- $\pi^0 \to \gamma\gamma$ puzzle was solved by noticing the axial anomaly Adler '69; Bell&Jackiw '69



$$\partial_\mu J_5^\mu = -\frac{e^2}{16\pi^2}\, F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{3}$$

- Both lepton and baryon numbers are anomalous in the Standard Model. 't Hooft '76

$$\partial_\mu J_L^\mu = \partial_\mu J_B^\mu = \frac{g^2}{32\pi^2} W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu}. \tag{4}$$

Electroweak Sphalerons

- For the electroweak theory, the sphaleron configuration that interpolates between, e.g., $N_{\rm CS} = 1$ and $N_{\rm CS} = 0$ has been constructed. Manton '83

- The gauge field:

$$T^a A_i^a = -\frac{i}{g} f(r)(\partial_i U) U^{-1} \ . \tag{5}$$

- The Higgs field:

$$\Phi = [1 - h(r)] \begin{pmatrix} 0 \\ e^{-i\mu} \cos\mu \end{pmatrix}_\tau + h(r) U \begin{pmatrix} 0 \\ 1 \end{pmatrix}_\tau \ , \tag{6}$$

with

$$U = \begin{pmatrix} e^{i\mu}(\cos\mu - i \sin\mu \cos\theta) & e^{i\phi} \sin\mu \sin\theta \\ -e^{-i\phi} \sin\mu \sin\theta & e^{-i\mu}(\cos\mu + i \sin\mu \cos\theta) \end{pmatrix}, \tag{7}$$

a convenient choice for the parameter $\mu \in [0, \pi]$

$$\mu(t) = \pi[1 - \tanh(2t/r_s)]/2. \tag{8}$$

Dirac Hamiltonian

- The Dirac Hamiltonian

$$H_D(\mu) = \boldsymbol{\sigma} \cdot (\mathbf{p} - gT^a \mathbf{A}^a)\,. \tag{9}$$

- At $\mu = \pi/2$ ($N_{\rm CS} = 1/2$)

$$H_D(\pi/2) = \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \left(i\partial_r - \frac{i}{r}\boldsymbol{\sigma} \cdot [\mathbf{L} + f(r)\mathbf{T}] \right) - \frac{i}{r} f(r)\mathbf{T} \cdot \hat{\mathbf{r}}. \tag{10}$$

•There exists a singlet (**L=0**) normalizable state with zero energy: The sphaleron zero mode Rigwald '88

$$\Psi^0_{\rm sph}(r) = C_0\; e^{-2\int_0^r dx f(x)/x}\; \chi, \tag{11}$$

with

$$\chi = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}_s \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_\tau - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_s \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_\tau \right]. \tag{12}$$

Free Hamiltonian

- The free Hamiltonian

$$H_D^{\text{free}} = \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \left(i\partial_r - \frac{i}{r}[\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2] \right), \tag{13}$$

- The eigenstates

$$\Psi_{\ell m}^{\text{free}}(r, \theta, \phi, t) = e^{-iEt} \left[J_\ell(Er)\, \Omega_{\ell m}^{+} - iJ_{\ell+1}(Er)\, \Omega_{\ell m}^{-} \right], \tag{14}$$

with

$$\Omega_{\ell m}^{\pm} = \frac{1}{\sqrt{2\ell+1}} \begin{pmatrix} \pm\sqrt{\ell \pm m + 1/2}\; Y_\ell^{m-1/2}(\theta, \phi) \\ \sqrt{\ell \mp m + 1/2}\; Y_\ell^{m+1/2}(\theta, \phi) \end{pmatrix}, \tag{15}$$

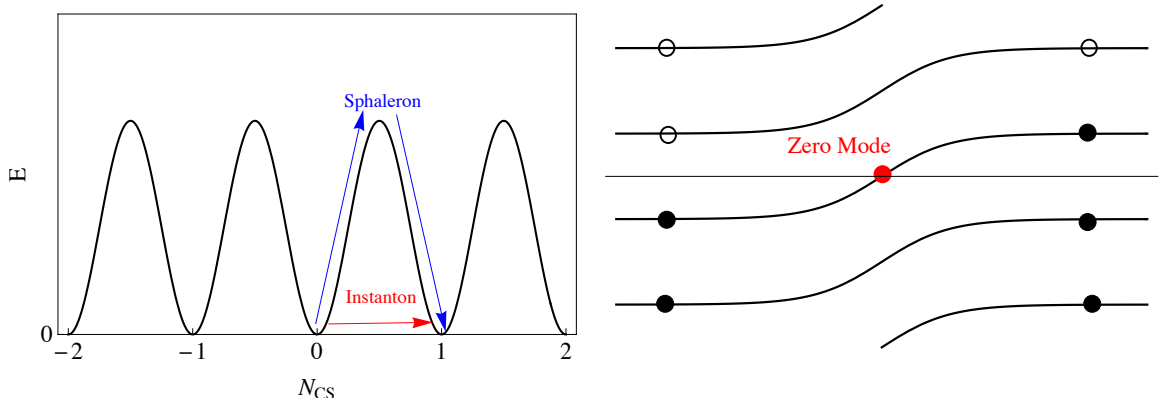
The Spectrum

- In the limit $f(r) \to 1$

$$H_D \to \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \left(i\partial_r - \frac{i}{r} \boldsymbol{\sigma} \cdot [\mathbf{L} + \mathbf{T}] \right) - \frac{i}{r} \mathbf{T} \cdot \hat{\mathbf{r}} \tag{16}$$

$$= H_D^{\text{free}} = \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \left(i\partial_r - \frac{i}{r} [\mathbf{J}^2 - (\mathbf{L} + \mathbf{T})^2 - \mathbf{S}^2] \right) . \tag{17}$$

- The spectrum is the same as the free case except $L \to L' \equiv L + T$: $\Psi \to \Psi^{\text{free}}_{\ell' m'}$.
- Only the singlet state $\ell = 0$ contributes to the level crossing.



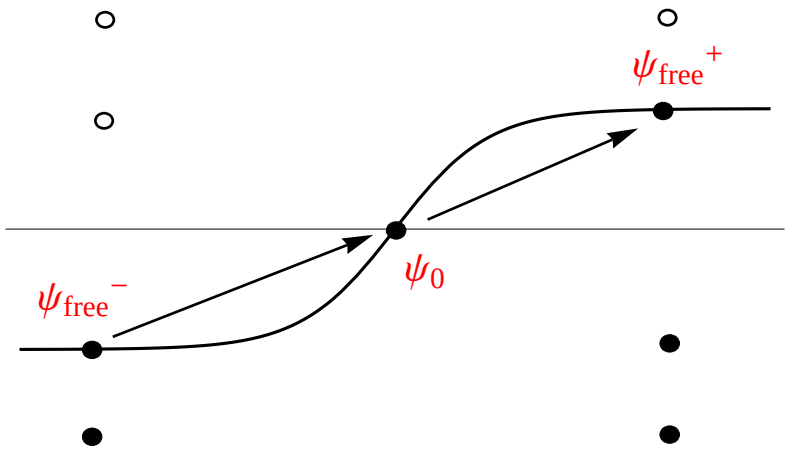
- The full time dependent spectrum cannot be found analytically.

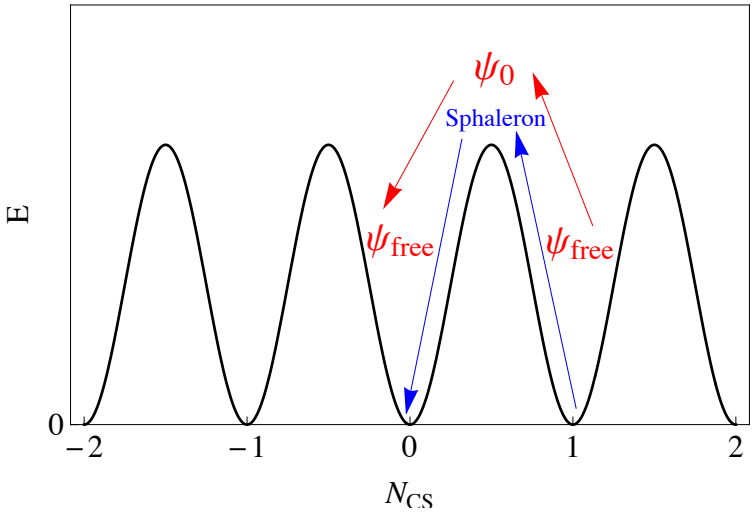
Level Crossing Fermion Configuration

- Similar to Manton's sphaleron configuration, we propose a level crossing fermion configuration:

$$\Psi_{\rm L}(\mathbf{x},t) = C(t)\; e^{-i\int_0^t dt' E(t')}\; e^{-[8\mu(t)\overline{\mu}(t)/\pi^2]\int_0^r dx f(x)/x}\;\chi, \tag{18}$$

with $E(t) \equiv E_0[1-2\mu(t)/\pi]$





$$\Psi_{\rm L}(x, t\to-\infty) \equiv \Psi^-_{\rm free} \propto e^{iE_0 t}\;\chi,$$
$$\Psi_{\rm L}(x, t=0) = \Psi^0_{\rm sph}(r)$$
$$\Psi_{\rm L}(x, t\to\infty) \equiv \Psi^+_{\rm free} \propto e^{-iE_0 t}\;\chi.$$

Electromagnetic Current

- In the unitary gauge

$$\Phi_{\text{unitary}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{19}$$

the electromagnetic charge operator is simply $q = g\ T^3 + g'\ \frac{Y_L}{2}$.

- But, Higgs field is not in this gauge:

$$\Phi = \begin{pmatrix} h(r)e^{i\phi}\sin\mu\sin\theta \\ e^{-i\mu}\left[\cos\mu + ih(r)\sin\mu\cos\theta\right] \end{pmatrix}. \tag{20}$$

- The electromagnetic charge operator in an arbitrary gauge

$$q = g\ n^a T^a + g'\ \frac{Y_L}{2}, \tag{21}$$

with

$$n^a = -\frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi}. \tag{22}$$

Electromagnetic Current

- The unit vectors n^a:

$$n^a = n^a(\mu, h(r), \theta, \phi). \tag{23}$$

- The electromagnetic current density is

$$j^\nu_{\rm em} = \overline{\Psi}_L(\sigma^\nu \otimes q)\Psi_L = \frac{1}{2} j^\nu_{Y_L} + n^1\, j^\nu_{W^1} + n^2\, j^\nu_{W^2} + n^3\, j^\nu_{W^3}. \tag{24}$$

Hypercharge and weak currents

$$j^\nu_{Y_L} = g'\ Y_L\ \overline{\Psi}_L(\sigma^\nu \otimes I)\Psi_L, \tag{25}$$

$$j^\nu_{W^a} = \frac{1}{2} g\ \overline{\Psi}_L(\sigma^\nu \otimes \tau^a)\Psi_L. \tag{26}$$

Electromagnetic Current

- We found that

$$j^i_{Y_L} = \frac{1}{2} g' \; Y_L \; j(r,t) \; \mathrm{tr}(\sigma^i) = 0, \tag{27}$$

$$\mathbf{j}_{W^1} = \frac{g}{2} j(r,t)(-\sin\theta\cos\phi \; \hat{\mathbf{r}} - \cos\theta\cos\phi \; \hat{\boldsymbol{\theta}} + \sin\phi \; \hat{\boldsymbol{\phi}}), \tag{28}$$

$$\mathbf{j}_{W^2} = \frac{g}{2} j(r,t)(-\sin\theta\sin\phi \; \hat{\mathbf{r}} - \cos\theta\sin\phi \; \hat{\boldsymbol{\theta}} - \cos\phi \; \hat{\boldsymbol{\phi}}), \tag{29}$$

$$\mathbf{j}_{W^3} = \frac{g}{2} j(r,t)(-\cos\theta \; \hat{\mathbf{r}} + \sin\theta \; \hat{\boldsymbol{\theta}}), \tag{30}$$

where

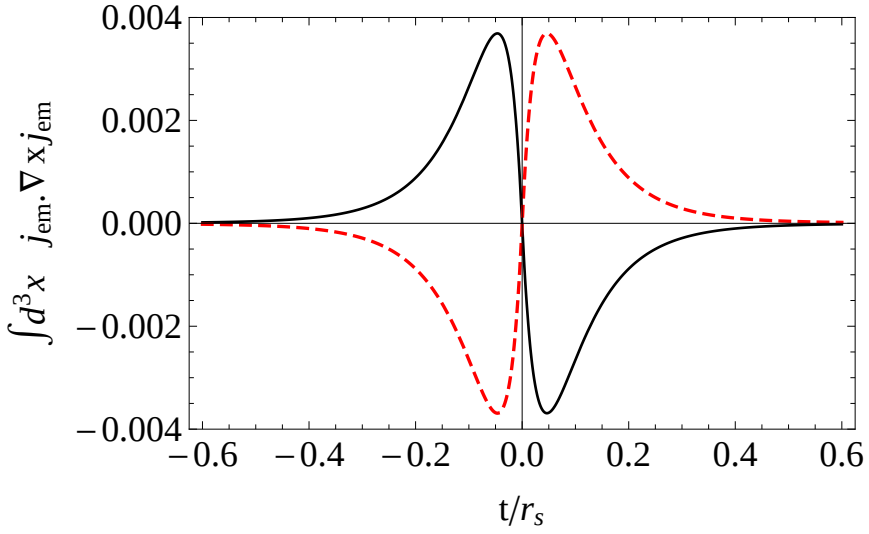
$$j(r,t) \equiv |C(t)|^2 \; e^{-[16\mu(t)\overline{\mu}(t)/\pi^2] \int_0^r dx f(x)/x}. \tag{31}$$

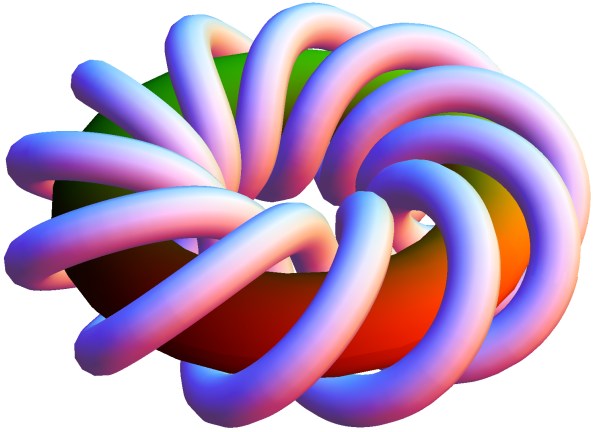
- Thus, the electromagnetic current density

$$j^i_{\mathrm{em}} = n^1 \; j^i_{W^1} + n^2 \; j^i_{W^2} + n^3 \; j^i_{W^3}. \tag{32}$$

Magnetic Field

- We also found that the electromagnetic current produced during level crossing is linked.





- This is a hint for helical magnetic fields (still in progress).

- The magnetic field amplitude

$$B_{\rm initial} \sim |j_{\rm em}| r \sim g|C_0|^2 r_s/2 \sim 0.17 g/(2r_s^2) \sim 10^{12}\,G. \tag{33}$$

- Assuming flux freezing

$$B_0 \sim B_{\rm initial}/(1+z_{\rm ew})^2 \sim 10^{-20}\,G. \tag{34}$$

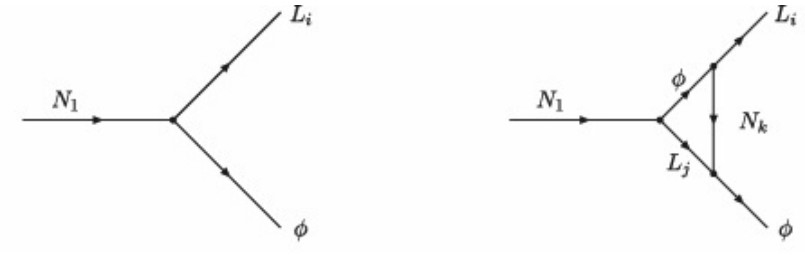
LEPTOGENESIS AND MAGNETIC FIELDS

Leptogenesis

- In the standard leptogenesis scenario Fukugita&Yanagida '86

$$\mathcal{L}_{\rm int} = -(\lambda^\nu)^{ij}\epsilon^{ab}(L^\dagger)_{a,i}(\Phi^*)_b(N_R)_j + \text{h.c.} \tag{35}$$

- The decay channels: $N_R \to L_i\Phi$, $L_i^c\Phi^c$.



- CP-violating phases in the Yukawa matrix λ^ν can bias the decay $N_R \to L_i\Phi$ or $L_i^c\Phi^c$

$$\epsilon_{\rm cp} = \frac{(\Gamma_L - \Gamma_{L^c})}{(\Gamma_L + \Gamma_{L^c})} \lesssim O(|\lambda^\nu|^2) \sim 10^{-6}. \tag{36}$$

Leptogenesis

- Out of equilibrium decays of N_R produce net lepton number.
- The particle number asymmetry can be related to the chemical potential

$$n_X - n_{X^c} \approx g_s(gT^3/6)(\mu_X/T). \tag{37}$$

- Leptogenesis provides the initial condition

$$\mu_{L^i} = \mu_0, \qquad 2\mu_\Phi = 3\mu_0, \qquad \mu_{Q^i} = \mu_{u_R^i} = \mu_{d_R^i} = \mu_{e_R^i} = 0, \tag{38}$$

with

$$\mu_0/T \approx \epsilon_{\rm cp} n_{N_R}/T^3 \lesssim 10^{-6}. \tag{39}$$

- As a fiducial value, let us take $M_{N_R} = 10^{10}$ GeV.

Sphalerons

- **Weak sphalerons** equilibrate particle species with weak isospin at a rate $\Gamma_{\rm w} \sim 25\alpha_{\rm w}^5 T$, Moore, Hu, Muller '98; DOnofrio, Rummukainen, Tranberg '12

$$\sum_i \left(3\mu_{Q^i} + \mu_{L^i}\right) = 0. \tag{40}$$

- **Strong sphalerons** equilibrate particle species with color at a rate $\Gamma_{\rm s} \sim 100\alpha_{\rm s}^5 T$, McLarren, Mottola, Shaposhnikov '91; Arnold, Son, Yaffe '97; Moore '97

$$\sum_i \left(2\mu_{Q^i} - \mu_{u_R^i} - \mu_{d_R^i}\right) = 0. \tag{41}$$

- For comparison, the expansion rate of the universe

$$H = 1.66\sqrt{g_*}\,\frac{T^2}{M_P}. \tag{42}$$

- Both processes are fast: $\Gamma_{\rm w} \gg H$, $\Gamma_{\rm s} \gg H$.

"Hypercharge Sphaleron"

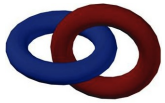
- The Abelian anomalies cannot occur in vacuum.

$$\partial_\mu j^\mu_{e_R} = -\frac{g'^2}{16\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} = \frac{g'^2}{4\pi^2}\mathbf{E_Y}\cdot\mathbf{B_Y}, \tag{43}$$

- There is no "hypercharge sphaleron" solution in vacuum since $\pi_3[U(1)]$ is trivial.

•The Abelian Chern-Simons number per unit volume is simply the helicity of the magnetic field

$$h_Y \equiv \lim_{V\to\infty} \frac{1}{V}\int_V d^3x\, \mathbf{Y}\cdot\mathbf{B_Y}.$$



- In plasma, one expects hypermagnetic knots that can mediate Abelian anomaly dubbed "hypercharge sphaleron" with the rate $\Gamma_Y \sim \alpha_Y^4 T \gg H$.

$$\sum_i \left(18\mu_{L^i} + 6\mu_{Q^i} - 48\mu_{u^i_R} - 12\mu_{d^i_R} - 36\mu_{e^i_R}\right) = 0. \tag{44}$$

Chemical Equilibrium

- Assuming all Yukawa interactions are slow, $\Gamma_{\rm Yuk} \ll H$

$$\mu_Q = -\frac{7}{44}\mu_0 \,, \;\; \mu_L = \frac{21}{44}\mu_0 \,, \;\; \mu_{u_R} = -\frac{\mu_0}{11} \,, \;\; \mu_{d_R} = -\frac{5}{22}\mu_0 \,, \;\; \mu_{e_R} = \frac{9}{22}\mu_0 \,, \;\; \mu_\Phi = \frac{3\mu_0}{2} \,.$$

- Baryon chemical potential

$$\mu_{\rm bar} = -\frac{7}{11}\mu_0 \,. \tag{45}$$

- Required lepton asymmetry

$$\frac{\mu_0}{T} = -\frac{11}{7}\frac{\mu_{\rm bar}}{T} = -\frac{66}{7}\frac{2\pi^2}{45}g_*\frac{n_{\rm bar}}{s} \approx -5\times 10^{-8} \,. \tag{46}$$

Magnetic Helicity

- Integrating the Abelian anomaly equation

$$h_Y = -\frac{2\pi}{\alpha_Y} \lim_{V\to\infty} \frac{1}{V} \int_V d^3x\, j^0_{e_R} = -\frac{2\pi}{\alpha_Y} \left(n_{e_R} - n_{e^c_R} \right). \tag{47}$$

- Using $\mu_{e_R} = \frac{9}{22}\mu_0$ and assuming that the observed baryon asymmetry is due to leptogenesis

$$\frac{h_Y}{T^3} = \frac{2\pi^2 g_*}{35\alpha_Y} \frac{n_{\rm bar}}{s} \sim 10^{-6}. \tag{48}$$

- $h_Y > 0$ indicates that the produced hypercharge magnetic fields carry **right-handed helicity**. Long, E.S., Vachaspati '13

Magnetic Field Evolution

- Magnetic fields are dissipated if the primordial plasma is not turbulent.
- Helicity is conserved also in a turbulent magnetic field evolution.
- The energy density of the magnetic fields

$$\rho_B(t_0) \sim \frac{\alpha_Y^4}{\epsilon_{\rm cp}\ \alpha_{\rm w}^2}\ \frac{h_Y}{\xi_0}. \tag{49}$$

- The initial coherence length for the magnetic fields: $\xi_i \sim 1/(\alpha_Y T_i)$.
- Assuming turbulence, present coherence length can be estimated Kahniashvili, Tevzadze, Brandenburg, Neronov '12; Saveliev, Jedamzik,Sigl '13

$$\xi_0 \sim 5 \times 10^{19}\ \mathrm{cm} \sim 15\ \mathrm{pc}. \tag{50}$$

- Using $\rho_B = B^2/8\pi$

$$B_0 \sim 5 \times 10^{-19} \left(\frac{10^{-6}}{\epsilon_{\rm cp}}\right)^{1/2} \mathrm{Gauss}. \tag{51}$$

Other Matter Genesis Mechanisms

- Electroweak baryogenesis produces left handed magnetic fields. Vachaspati '01
- Affleck-Dine baryogenesis: $\mu^0_{\rm bar} > 0$ Affleck&Dine '85 $\rightarrow$ left handed magnetic fields?
- Cosmic magnetic fields could be used to distinguish between, e.g. xogenesis and darkogenesis models, where the observed dark matter abundance is related to the baryon asymmetry of the universe. Buckley&Randall '10; Shelton&Zurek '10; Haba&Matsumoto '10

CONCLUSIONS

Conclusions

• Electromagnetic currents are produced during fermion level crossing in the electroweak sphaleron background.

• These currents can lead to cosmic magnetic fields.

• If baryogenesis is due to leptogenesis, then magnetic fields with right handed helicity are expected.

• Depending on the initial conditions, one can expect magnetic fields with right or left helicity or no magnetic fields.

• Observation of cosmic magnetic fields and their helicity can be a unique handle on the early universe physics.