WARPED PENGUINS

Yuhsin Tsai

Fermilab / Cornell

Fermilab Theory Seminar, 18 Aug 2011

BG: Wall paper from www.layoutspa

This talk includes the two papers

Warped Penguins

arXiv:1103.0240

Csaba Csaki, Yuval Grossman, Flip Tanedo, YT

The Birds and the Bs in RS

Work in progress

Monika Blanke, Bibhushan Shakya, Flip Tanedo, YT

Why are warped penguins interesting?

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Flavor & Finiteness

For the finiteness

One generic feature of extra dimension models:

They are non-renormalizable!

$$\mathcal{M} = a_0 + a_1 \log(\Lambda/M) + a_2(\Lambda/M) + a_3(\Lambda/M)^2 + \ldots$$

For the finiteness

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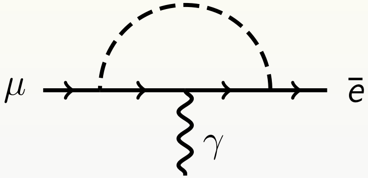
$$\mathcal{M} = a_0 + a_1 \log(\Lambda/M) + a_2(\Lambda/M) + a_3(\Lambda/M)^2 + \ldots$$

The cutoff dependance is important.

Penguins live in extra dimension

Penguin diagram is a loop induced process.



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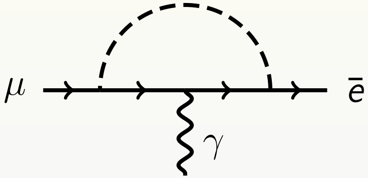

In SM

Renormalizable, the leading diagram must be finite.

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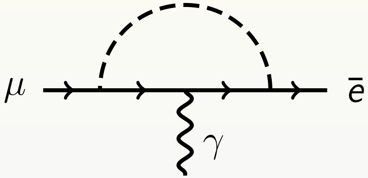
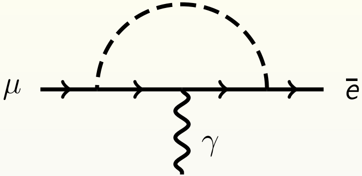


In extra dimension

Non-renormalizable, the leading diagram must be...?

The finiteness issue

Would XD penguins explode?



Previous Analysis

RS Penguinswith brane-higgs loop is UV sensitive!

- Cannot get a physical result.

The finiteness issue

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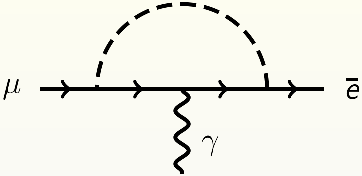
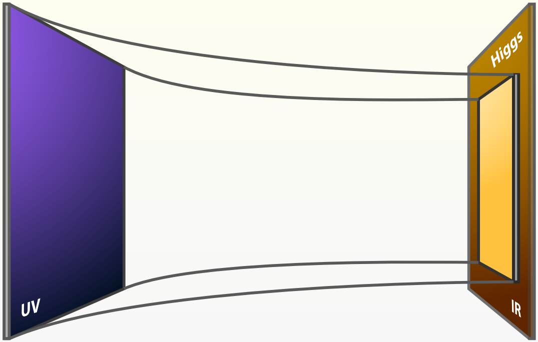


Previous Analysis

The one-loop RS penguins are FINITE.

- Can get a physical result.
- Interesting Yukawa bounds.

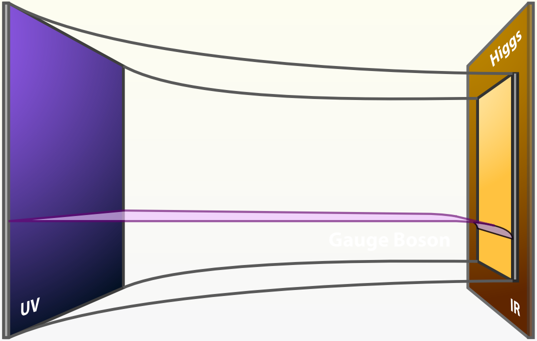
Randall-Sundrum in one slide



$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

Randall, Sundrum (99);

Randall-Sundrum in one slide



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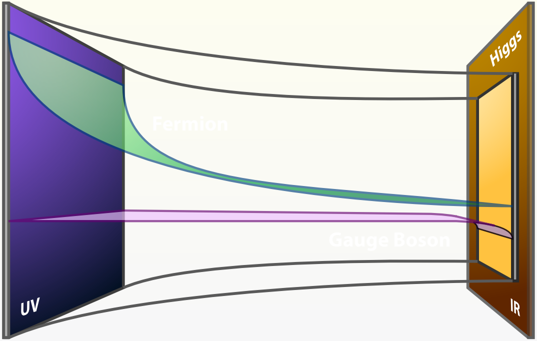

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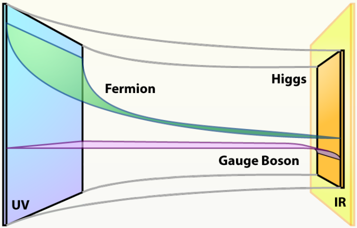
Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99); Grossman, Neubert (00); Gherghetta, Pomarol (00); **Bulk Higgs:** Agashe, Contino, Pomarol (04); Davoudiasl, Lille, Rizzo (05)

The pros and cons

Flavor changing & the mass hierarchy



4D Yukawa Coupling: $Y_* \bar{L}_i H E_j \times f_{Li}(R') f_{Ej}(R')$

4D Gauge Coupling: $g_{ii} \bar{L}_i \not{Z} L_i \times \int_R^{R'} dz \left(\frac{R}{z}\right)^4 f_{Li}(z) f_Z(z) f_{Li}(z)$

Anarchic Flavor in RS

For an interesting model, we want...

- $Y^*_{ij} = Y_* \text{Ⓐ}_{ij}$ is an ancharic matrices with $\mathcal{O}(1)$ numbers.
  $\Rightarrow$ wave function decides the hierarchy.

- M_{kk} is not too heavy. $\Rightarrow$ relevant to LHC.

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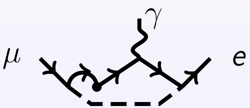
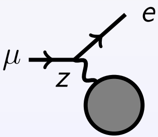
Allowed by flavor constraints?

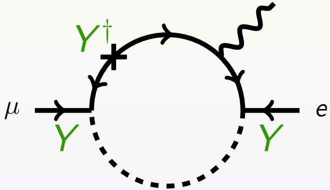




Lepton Flavor Violation : Loop

Controlled by two dominant parameters

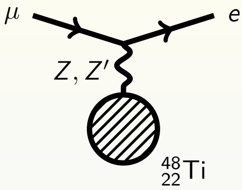
Flavor is dominantly controlled by: Y_* and M_{KK}

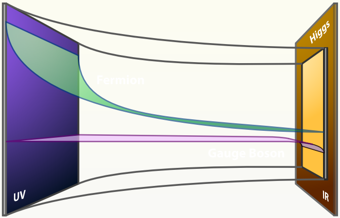


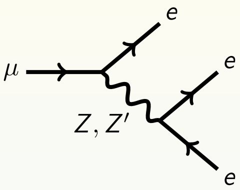
$$\mathcal{M}_{\text{loop}} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 f_L {Y_*}^3 f_{-E}$$
$$\sim \left(\frac{1}{M_{\text{KK}}}\right)^2 {Y_*}^2 m$$

Lepton Flavor Violation : Tree

Two dominant parmeters







$$\mathcal{M}_{\text{tree}} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 \left(\frac{1}{Y_*}\right)$$

If we increase Y_*, must maintain SM mass spectrum

$\Rightarrow$ push fermion profiles to UV

$\Rightarrow$ Less overlap with the FCNC part of the Z

Complementary tree- and loop-level bounds

Possible tension between tree- and loop-level processes

In the lepton sector:

- Tree-level bound: $\left(\frac{3\ \mathrm{TeV}}{M_{\mathsf{KK}}}\right)^2 \left(\frac{2}{Y_*}\right) < 0.5,$

- Penguin bound: $\left|\ aY_*{}^2 + b\ \right| \left(\frac{3\ \mathrm{TeV}}{M_{\mathsf{KK}}}\right)^2 \leq 0.015$

Can test anarchic flavor ansatz.

Penguins into Details

Answering the questions of Finiteness & Flavor

Penguins in a warped XD

- 5D calculation
- Diagram parade

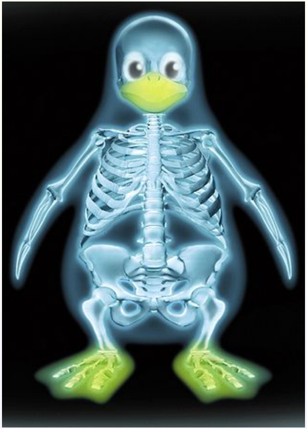
The flavor bounds

- Lepton sector
- Quark sector

The finiteness

- The finiteness
- Matching 5D to 4D

Conclusion



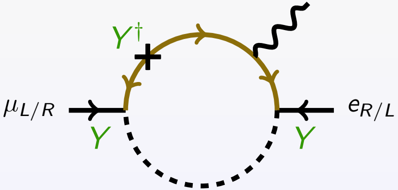
Penguins in a Warped Extra Dimension

Operator analysis: $\mu \to e\gamma$

Match to 4D EFT:

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} \; Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

- Y_{ij} is a spurion of $\mathrm{U}(3)^3$ lepton flavor
- Indices on a_{ij} and b_{ij} encode bulk mass dependence



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Flavor structure

- $a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j}$ gives a generic contribution

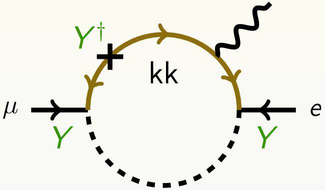
  Depends 'only' on Y_* and M_{KK}

- New: $b_{ij} Y_{ij}$ is aligned up to structure of b_{ij}

  $f_i Y_{ij} f_j \sim m_{ij}$, so this term is almost diagonal in the mass basis

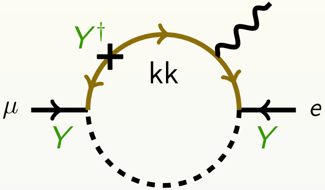
  This depends on the *particular* flavor structure of the anarchic Y

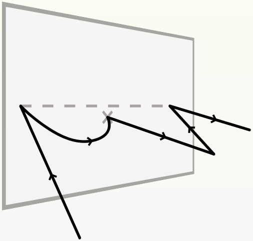
4D KK vs. 5D



$$\sum_{kk=1}^{N_1} \sum_{kk=1}^{N_2} \int d^4k$$

4D KK vs. 5D





$$\sum_{kk=1}^{N_1} \sum_{kk=1}^{N_2} \int d^4k$$

$$\int_R^{R'} dz \int d^4k$$

The 5D propagator

A mixed propagator with momentum & position space

One example: brane-to-brane $\mathrm{SU}(2)_L$ fermion propagator:

$$\Delta(p,\, z = R',\, z' = R',\, c) \;=\; i\not{p}\, \left[\frac{\pi\, R'^5}{2R^4} \frac{\widetilde{S}_c^+\, \widetilde{S}_c^-}{S_c^+} \right].$$

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The propagator carries Bessel functions:

$$\begin{aligned}
\widetilde{S}_c^{\pm} &= \quad J_{c\pm\frac{1}{2}}(pR')\, Y_{c\mp\frac{1}{2}}(pR') - J_{c\mp\frac{1}{2}}(pR')\, Y_{c\pm\frac{1}{2}}(pR') \\
S_c^{+} &= \quad J_{c+\frac{1}{2}}(pR)\, Y_{c+\frac{1}{2}}(pR') - J_{c+\frac{1}{2}}(pR')\, Y_{c+\frac{1}{2}}(pR)
\end{aligned}$$

The 5D propagator

The meaning of the bessel functions

- The Bessel function part contains

$$e^{p(x-x')}$$

as the usual propagator in position space.

- It also contains zero- and KK-mode wave functions:

$$\Delta(p,\, R',\, R',\, c) \sim i\,\frac{\not{p}}{p^2}\, f_c^{(0)} f^{(0)}{}_c + \sum_{n=1}^{\infty} \frac{i\,\not{p}}{p^2 + (n/R')^2}\, f^{(n)} f^{(n)}.$$

The ∞ is important.

How to calculate the 5D loop?

Feynman's parametrization with Bessel functions??

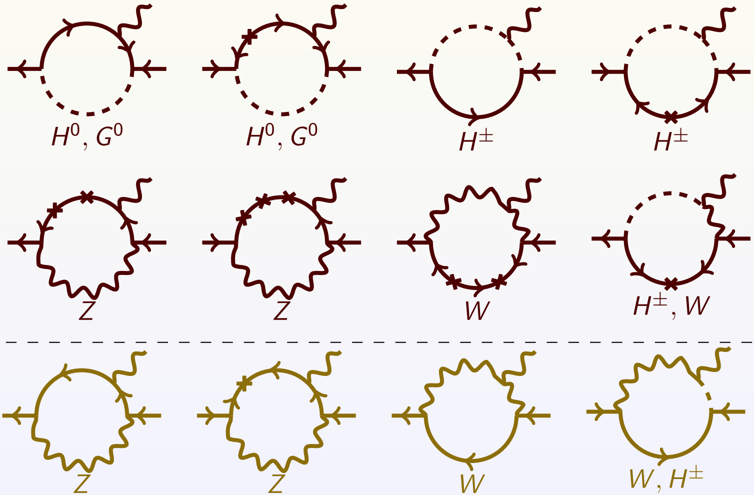
You must be kidding...however, we can

- Taylor expand the propagator into powers of the external momentum $(p^\mu,\ q^\mu)$. $\frac{1}{(k^2+2k\cdot q)} = \frac{1}{k^2}\left(1 - \frac{2k\cdot q}{k^2} + \ldots\right)$.
- Isolate the $p^\mu + p_e^\mu$ terms. Get the coefficient a.
  $\epsilon_\mu \mathcal{M}^\mu \sim a\,\epsilon_\mu \bar{u}\,(\,p^\mu + p_e^\mu - (m_\mu + m_e)\gamma^\mu\,)\,u$
- Obtain the a numerically.
  $a \times R'^2 \frac{e}{16\pi^2}\frac{v}{\sqrt{2}}\left(f_L YY^\dagger Y f_E\right)\bar{u}\not{\epsilon}u$

Diagrams for $\mu \to e\,\gamma$: a and b coefficients

Yes, we actually calculated all of these...

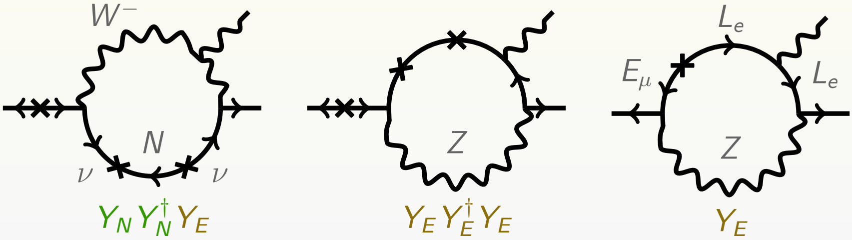
$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F^{(0)}_{\mu\nu}$$



The Flavor Bounds

Leading order $\mu \to e\,\gamma$

The following diagrams with external mass insertions dominate[1]



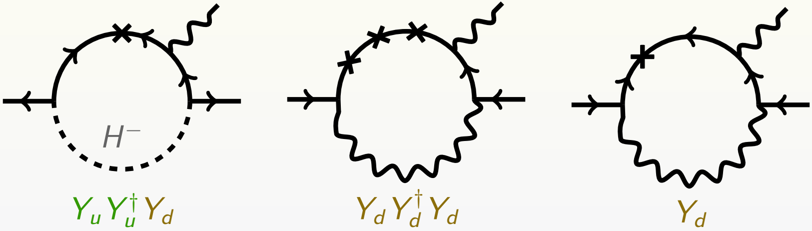
Three coefficients (a_W, a_Z, b) with arbitrary relative signs

Defined $aY_*^3 = \sum_{k,\ell} a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j}$ and $bY_* = \sum_{k,\ell} (U_L)_{ik} b_{k\ell} Y_{k\ell} (U_R^\dagger)_{\ell j}$

[1] We thank Martin Beneke for pointing this out.

Leading order $b \to s\,\gamma$

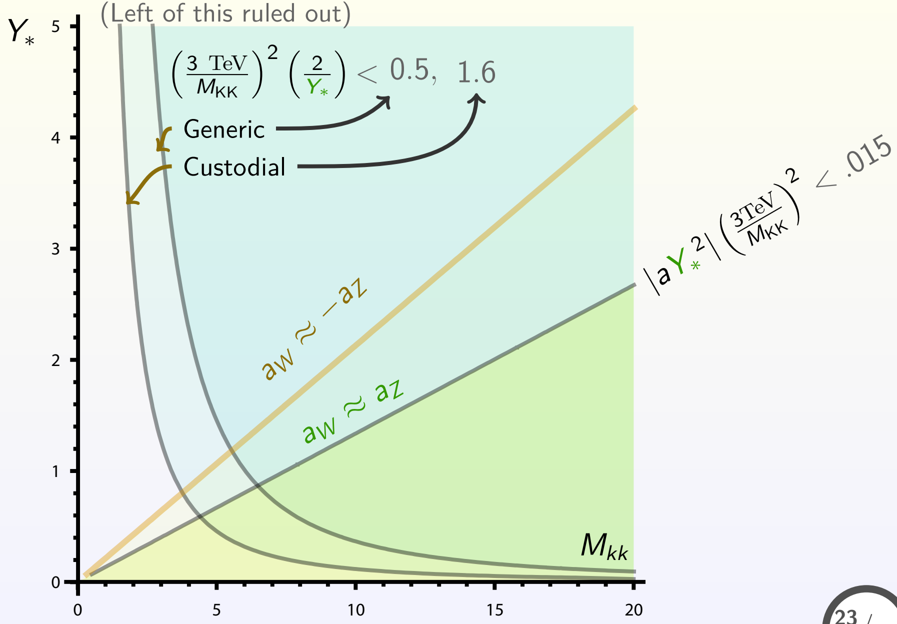
Different from the lepton case



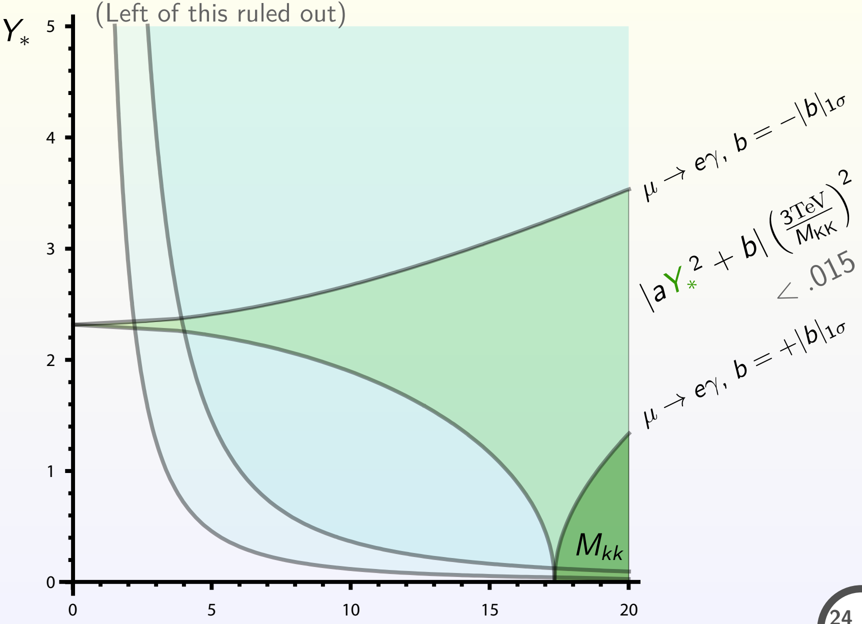
Three coefficients (a_H, $a_{Z,g}$, b) with arbitrary relative signs

Defined $aY_*^3 = \sum_{k,\ell} a_{k\ell}\,Y_{ik}\,Y_{k\ell}^\dagger\,Y_{\ell j}$ and $bY_* = \sum_{k,\ell}(U_L)_{ik}\,b_{k\ell}\,Y_{k\ell}(U_R^\dagger)_{\ell j}$

Bounds for $\mu \to e\,\gamma$: $b = 0$

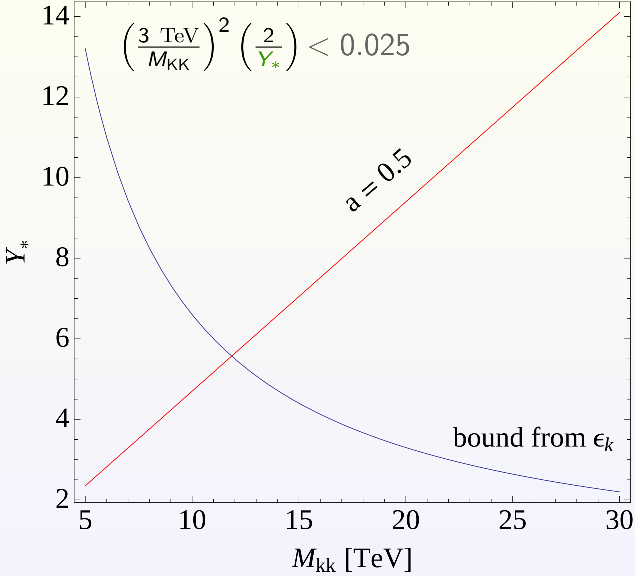


Bounds for $\mu \to e\,\gamma$: $b \neq 0$



Bounds for $b \to s\gamma$: $b = 0$

(PRELIMINARY)



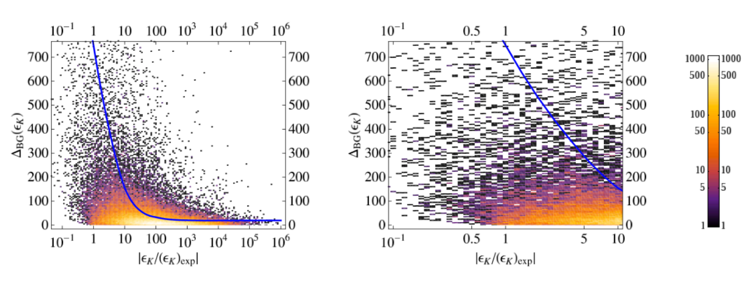
$\left|a Y_*^{\ 2}\right| \left(\frac{3\text{TeV}}{M_{KK}}\right)^2 < 0.1$

The strongest bound comes from the ϵ_K in the $K\bar{K}$ mixing.

Some tuning is necessary

M. Blankea, A. Burasa, B. Dulinga, S. Goria, and A. Weiler (08)

The tuning of ϵ_K in the RS model with Custodial Protection.



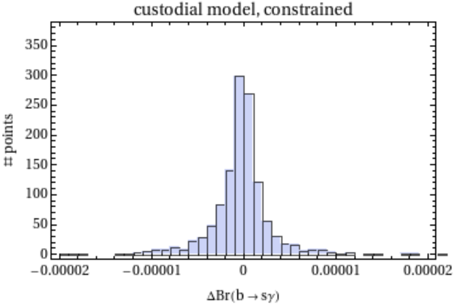
with $M_{kk} = 2.5\,\mathrm{TeV}$, $0.3 < |y| < 3$.

Question

How many points can pass the tree-level & $b \to s\gamma$ bound?

A PRELIMINARY result

The size of $B \to X_s\gamma$ in the RS model with custodial protection.



Using 1228 points passing the $\Delta F = 2$ bound with $0.3 < |y| < 3$.
Allowed $\Delta\mathrm{Br}(\mathrm{b} \to \mathrm{s}\gamma) < (0.4 \pm 0.7) \cdot 10^{-4}$.

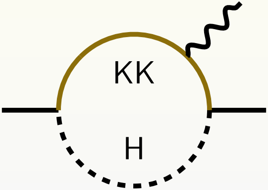
NOT surprised

Most of the points pass the bound due to small Yukawas.

Divergent?

The finiteness in the KK picture

Naïve NDA in mass basis	KK	SM
Loop integral $(\sum) d^4 k$	$+5$	$+4$
Gauge invariance $(p + p')$	-1	-1
Fermion propagators	-1	-2
Higgs propagator	-2	-2
Total degree of divergence	0	-2

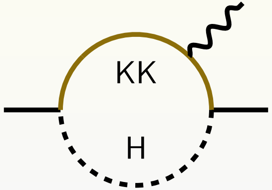


Hard to see the finiteness in KK.

Infinitely large Yukawa matrix with the KK tower.

The finiteness in the KK picture

Naïve NDA in mass basis	KK	SM
Loop integral $(\sum) d^4k$	$+5$	$+4$
Gauge invariance $(p+p')$	-1	-1
Even number of k	0	-1
Fermion propagators	-2	-2
Higgs propagator	-2	-2
	0	-2
Yukawa $\sim 1 + M_i^{-1}$	-1	
Total degree of divergence	-1	-2



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The finiteness in 5D

Some tricks...

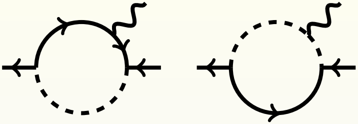
- Large momentum pulls the vertices together in 5D.

  Loop with mass insertion $\Rightarrow$ shrink to the brane.
  Loop in the bulk $\Rightarrow$ 5D theory without compactification.

- fermion: k^0, scalar: k^{-1}, vector: k^{-1}.

- Bulk vertices (dz): k^{-1}.

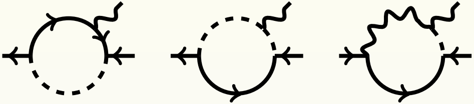
Finiteness: bulk 5D fields



	Neutral	Charged
Loop integral (d^4k)	$+4$	$+4$
Gauge invariance ($p + p'$)	-1	-1
Bulk boson propagator	-1	-2
Bulk vertices (dz)	-3	-3
Overall z-momentum	$+1$	$+1$
Derivative coupling	0	$+1$
Mass insertion/EOM	-1	-1
Total degree of divergence	-1	-1

Note: this all carries over to the KK picture

Finiteness: brane-localized Higgs



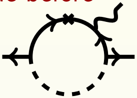
	Neutral	Charged	W–$H^{\pm}$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p+p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Derivative coupling	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

Divergent?

No, they're finite!

Matching the 5D to 4D KK

One mistake people made before

When calculating  in the KK picture, if we do

$$\mathcal{M} = \sum_{n=1}^{N} \int_0^{\infty} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

The leading order result is

$$\mathcal{M} \propto \frac{1}{(M_{KK})^2} \times \frac{v^2}{M_{kk}^2}$$

which is different from the 5D result $\mathcal{M} \propto R'^2$!

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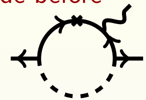
When calculating  in the KK picture, if we do

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which is different from the 5D result $\mathcal{M} \propto R'^2$!

5D $\neq$ KK !?

The KK result should be the same as 5D

The problem comes from the

Wrong UV limit!

Momentum cutoff $\lesssim$ KK cutoff.

e.g., the 5D propagator in a flat extra dimension can be expand into:

$$(\not{p} + i\gamma^5\partial_z)\,\frac{\cos p(2\pi - z)}{\sin p\pi R} = (\not{p} + i\gamma^5\partial_z)\left[\frac{1}{2p^2} + \sum_{n=0}^{\infty}\frac{\cos(znL)}{p^2 - (n/L)^2}\right]$$

The KK result should be the same as 5D

The problem comes from the

Wrong UV limit!

Momentum cutoff $\lesssim$ KK cutoff.

One way to do the integral,

$$\lim_{N\to\infty} \sum_{n=1}^{N} \int_0^{N\, M_{KK}} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

This gives the same result as in 5D.

The KK result should be the same as 5D

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$$\lim_{N\to\infty} \sum_{n=1}^{N} \int_0^{N\, M_{KK}} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

This gives the same result as in 5D. $\quad$ 5D $=$ KK

Conclusion

Warped penguins are important.

- Cutoff structure
- Flavor bounds

Full 5D calculations are useful.

- Easier to see the cutoff structure.
- Avoid mistakes from the KK sum.

One loop penguins are finite.

- Can be easily seen in the 5D way.
- The two loop result decides the cutoff scale.

Interesting flavor bounds on Y_*, M_{kk}.

- Tension from the loop- and tree-level bounds.
- Will have more results for the quark sector.

Backup Slides

The allowed cutoff scales

When will the 1-loop diagrams really be the leading ones?

$$\mathcal{M} = \mathcal{M}_{\mathrm{NDA}} \left(a + \frac{y^2}{16\pi^2}\, b + \ldots \right), \quad \text{need } a \gtrsim \frac{1}{16\pi^2}\, b.$$

For $\mu \to e\,\gamma$, a is suppressed by $(vR')^2$. In the WORST CASE:

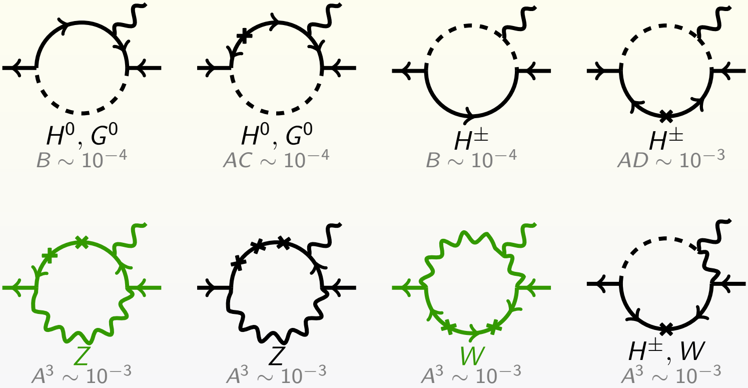
- Bulk-higgs: $b = \log(\Lambda R') \Rightarrow \log(\Lambda R') \leq 7$...ok
- Brane-higgs: $b = \Lambda R' \Rightarrow \Lambda R' \leq 7$...dangerous

For $b \to s\,\gamma$, a is ~ 0.5. In the WORST CASE:

- Bulk-higgs: $b = \log(\Lambda R') \Rightarrow \log(\Lambda R') \lesssim 80$...ok
- Brane-higgs: $b = \Lambda R' \Rightarrow \Lambda R' \lesssim 80$...ok

The 2-loop calculation is necessary for $\mu \to e\,\gamma$ in the brane-higgs case.

$\mu \to e\,\gamma$: a coefficient



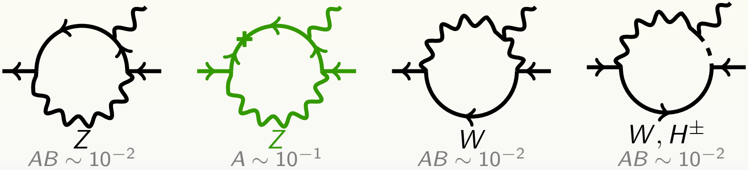
A. Mass insertion $\sim 10^{-1}$ per insertion (cross)

B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)

C. Higgs/Goldstone cancellation $\sim 10^{-3}$ (H^0, G^0 diagram only)

D. Proportional to charged scalar mass $\sim 10^{-2}$

$\mu \to e\,\gamma$: b coefficient

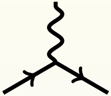


A. Mass insertion $\sim 10^{-1}$ per insertion (cross)

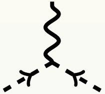
B. No sum over internal flavors $\sim 10^{-1}$

5D Feynman rules

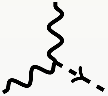
See our paper for lots of appendices on performing 5D calculations.



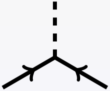
$$= ig_5 \left(\frac{R}{z}\right)^4 \gamma^\mu$$



$$= ie_5 (p_+ - p_-)_\mu$$



$$= \frac{i}{2} e_5 g_5 \, v \, \eta^{\mu\nu}$$



$$= i \left(\frac{R}{R'}\right)^3 Y_5$$

$$= \Delta_k(z, z')$$

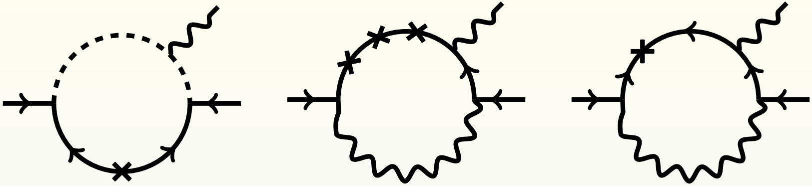
$$= -i\eta^{\mu\nu} G_k(z, z')$$

$$= \epsilon^\mu(q) f_A^{(0)}$$

$$= \frac{f_c}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} u(p)$$

$$g_5^2 = g_{SM}^2 R \ln R'/R \qquad e_5 f_A^{(0)} = e_{SM} \qquad Y_5 = RY$$

Analytic expressions



$$\mathcal{M}(1\mathrm{MI}H^{\pm}) = \frac{i}{16\pi^2}\,(R')^2\, f_{c_L}\, Y_E\, Y_N^{\dagger}\, Y_N\, f_{-c_E}\,\frac{ev}{\sqrt{2}}\cdot 2I_{1\mathrm{MI}H^{\pm}}$$

$$\mathcal{M}(3\mathrm{MI}Z) = \frac{i}{16\pi^2}\,(R')^2\, f_{c_L}\, Y_E\, Y_E^{\dagger}\, Y_E\, f_{-c_E}\,\frac{ev}{\sqrt{2}}\left(g^2\ln\frac{R'}{R}\right)\left(\frac{R'v}{\sqrt{2}}\right)^2\cdot I_{3\mathrm{MI}Z}$$

$$\mathcal{M}(1\mathrm{MI}Z) = \frac{i}{16\pi^2}\,(R')^2\, f_{c_L}\, Y_E\, f_{-c_E}\,\frac{ev}{\sqrt{2}}\left(g^2\ln\frac{R'}{R}\right)\cdot I_{1\mathrm{MI}Z}.$$

Written in terms of dimensionless integrals. See paper for explicit formulae.

Finiteness in the KK picture

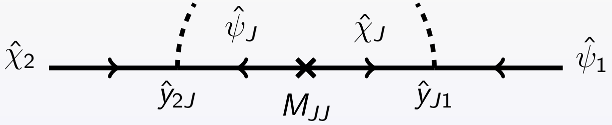
Power counting for the brane-localized Higgs

Charged Higgs: same M_W^2 cancellation argument as 5D
Neutral Higgs: much more subtle!

A basis of chiral KK fermions:

$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right) \qquad \psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right)$$

Worry about the following type of diagram:



The (KK) mass term in the propagator can be $\sim \Lambda$. Have to show that the mixing with large KK numbers is small.

Finiteness in the KK picture

Power counting for the brane-localized Higgs

A basis of chiral KK fermions:

$$\psi = \left(\psi^{(0)}_{R_i}, \psi^{(1)}_{R_i}, \psi^{(1)}_{L_i}\right) \qquad \chi = \left(\chi^{(0)}_{L_i}, \chi^{(1)}_{R_i}, \chi^{(1)}_{L_i}\right)$$

Mass and Yukawa matrices (gauge basis, $\psi M \chi + \mathrm{h.c.}$):

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\mathrm{KK},1} & m^{23} \\ 0 & 0 & M_{\mathrm{KK},2} \end{pmatrix} \qquad y \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The zeroes are fixed by gauge invariance.

$$\hat{y}_{1J}\hat{y}_{J2} = 0$$

Indices run from $1, \ldots, 9$ labeling flavor and KK number

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right)$$
$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right)$$

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\mathrm{KK},1} & m^{23} \\ 0 & 0 & M_{\mathrm{KK},2} \end{pmatrix}$$

Rotating to the mass basis, $\epsilon \sim v/M_{\mathrm{KK}}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix}$$

Now we have $y_{1J} y_{J2} \sim \epsilon$, good!

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi^{(0)}_{R_i}, \psi^{(1)}_{R_i}, \psi^{(1)}_{L_i}\right) \qquad M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\mathrm{KK},1} & m^{23} \\ 0 & 0 & M_{\mathrm{KK},2} \end{pmatrix}$$

$$\chi = \left(\chi^{(0)}_{L_i}, \chi^{(1)}_{R_i}, \chi^{(1)}_{L_i}\right)$$

Rotating to the mass basis, $\epsilon \sim v/M_{\mathrm{KK}}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix} \to \begin{pmatrix} 1 & 1+\epsilon & -1+\epsilon \\ 1+\epsilon & & \\ 1-\epsilon & & \end{pmatrix}$$

Must include 'large' rotation of m^{21} and m^{13} blocks representing mixing of chiral zero modes into light Dirac SM fermions. This mixes wrong-chirality states and does not affect the mixing with same-chirality KK modes.

Indeed, $\mathcal{O}(1)$ factors cancel: $y_{1J} y_{J2} \sim \epsilon$, good!