5D UED: Flat and Flavorless

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Fermilab Theory Seminar

Outline

- Motivation
  - Anarchic flavor in a warped extra dimension (RS-GIM)
- Flat extra dimension with KK-parity
  - How to add fermion masses ("split UED")
  - $\rightarrow$ Light fermionic modes
  - $\rightarrow$ (Pseudo)Goldstones
- Bounds from flavor physics
  - No analog of RS-GIM
  - How to (not) circumvent these bounds $\Rightarrow$ RS
- Summary

Hierarchy Problem vs. Flavor Physics

- "Tension" between EWSB & flavor

Unitarity

$m_{Higgs} \lesssim 1$ TeV

Flavor

$\Lambda_{new} \gtrsim 10^4 - 10^5$ TeV

$\Rightarrow$ We need new physics $\lesssim 1$ TeV to ensure that Higgs is light, but it must not violate flavor.

- Does a model naturally respect flavor?

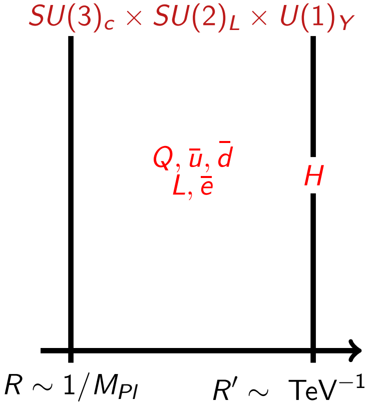
$\Rightarrow$ In general: No. In some cases: Yes.

Reminder: Flavor in RS

Randall-Sundrum

- warped metric
$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$
- Higgs on the IR brane.
- SM fields in the bulk.

- Hiss mass is suppressed by $\frac{R'}{R} \sim 10^{16}$



Flavor in RS: The fermion wave functions

- What about flavor in this model?
- Give fermions a bulk mass

$$S \supset \int d^5x \left(\frac{R}{z}\right)^4 \frac{c^i}{z} \bar{\Psi}_i \Psi_i \qquad \text{(for } Q, u, d\text{)}$$

$\Rightarrow$ KK-decomposition

$$\Psi = \begin{pmatrix} \chi \\ \bar{\psi} \end{pmatrix} = \sum_n \begin{pmatrix} g_n(y)\chi_n(x) \\ f_n(y)\bar{\psi}_n(x) \end{pmatrix} \begin{matrix} \leftarrow \text{lefthanded} \\ \leftarrow \text{righthanded} \end{matrix}$$

equations of motion

$$g_n' - \frac{2-c}{z} g_n - m_n f_n = 0$$
$$f_n' - \frac{2+c}{z} f_n + m_n g_n = 0$$

Flavor in RS: The fermion wave functions

Zero mode solutions

$$g_0 \sim \left(\frac{z}{R}\right)^{2-c} \qquad \text{and} \qquad f_0 \sim \left(\frac{z}{R}\right)^{2+c}$$

- Obtain a chiral spectrum by imposing boundary conditions

$$\begin{array}{lllll} [++]: & f_0 \equiv 0 & @ & (z = R, R') & \Rightarrow & g_0 - \text{mode} \\ [--]: & g_0 \equiv 0 & @ & (z = R, R') & \Rightarrow & f_0 - \text{mode} \end{array}$$

- Normalization, $\int dz \left(\frac{R}{z}\right)^4 g_0(z)^2 = 1$, tells us where the mode is localized

$$g_0 \begin{cases} \text{UV} & \text{for } c > 1/2 \\ \text{IR} & \text{for } c < 1/2 \end{cases}, f_0 \begin{cases} \text{UV} & \text{for } c < -1/2 \\ \text{IR} & \text{for } c > -1/2 \end{cases}$$

Flavor in RS: (Anarchic) Yukawa couplings

Normalized zero mode solutions

$$g_0 \sim z^{2-c} f(c) \qquad \text{and} \qquad f_0 \sim z^{2+c} f(-c)$$

$$\text{with } f(c) = \sqrt{\frac{1-2c}{1-(R'/R)^{2c-1}}}$$

- $f(c)$ strongly hierarchical for SM fermions.
- Yukawa couplings: after EWSB

$$\mathcal{L}_Y = -\frac{v}{\sqrt{2}} \frac{R^4}{R'^3} \left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c. \right] \Big|_{z=R'}$$

$\tilde{Y}_u, \tilde{Y}_d$ are both $\mathcal{O}(1)$, anarchic flavor matrices.

Flavor in RS: The SM masses

$\Rightarrow$ Mass matrices are given by

$$\begin{aligned} m_u &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_u f_u \\ m_d &= \frac{v}{\sqrt{2}} f_q \tilde{Y}_d f_d \end{aligned} \qquad \text{with} \qquad f_c = \text{diag}[\{f(c_i\}]$$

- Now usual SM prescription applies

$$m^{SM} = U_L m U_R^\dagger \qquad \text{and} \qquad V_{CKM} = U_{Lu}^\dagger U_{Ld}$$

$$\Rightarrow \left(m_{u,d}^{SM} \right)_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{u_i,d_i}$$

$\Rightarrow$ We get mass hierarchy, but what about new FCNC contributions?

Flavor in RS: Hierarchy of f_i's

- First let us check of what order the f_i's are.
- For left-handed fields: f_i's are determined from V_{CKM}

$$|U_{ij}| \sim \frac{f_i}{f_j} \qquad \Rightarrow \qquad |(V_{CKM})_{ij}| = |(U_{Lu}^\dagger U_{Ld})_{ij}| \sim \frac{f_{q_i}}{f_{q_j}} \qquad i \leq j.$$

$$V_{CKM} \sim \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 \\ \lambda & 1-\frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & \lambda^2 & 1-\frac{\lambda^2}{2} \end{pmatrix}$$

$$\frac{f_{q_2}}{f_{q_3}} \sim \lambda^2, \quad \frac{f_{q_1}}{f_{q_3}} \sim \lambda^3 \qquad \text{with } \lambda \sim \sin\theta_c \sim 0.2$$

Flavor in RS: Hierarchy of f_i's

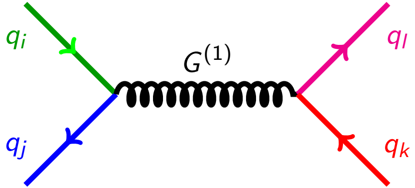
- Right-handed f_{u_i,d_i}'s fixed by fermion masses hierarchy:

$$\left(m_u^{SM}\right)_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{u_i}$$
$$\left(m_d^{SM}\right)_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{d_i}$$

$$\frac{f_{u_1}}{f_{u_3}} \sim \frac{m_u}{m_t}\frac{1}{\lambda^3}, \quad \frac{f_{u_2}}{f_{u_3}} \sim \frac{m_c}{m_t}\frac{1}{\lambda^2}$$
$$\frac{f_{d_1}}{f_{u_3}} \sim \frac{m_d}{m_t}\frac{1}{\lambda^3}, \quad \frac{f_{d_2}}{f_{u_3}} \sim \frac{m_s}{m_t}\frac{1}{\lambda^2}, \quad \frac{f_{d_3}}{f_{u_3}} \sim \frac{m_b}{m_t}$$

Flavor in RS: New Contributions to FCNC

- Strongest bound: FCNC due to KK-gluon exchange:



$$\sim \; g^{ij}_{L,u}\, \bar{u}^i_L \gamma^\mu G^{(1)}_\mu u^j_L \; + \; g^{kl}_{L,d}\, \bar{d}^k_L \gamma^\mu G^{(1)}_\mu d^l_L \; + \; (L \to R)$$

- Plug in the wave function: In original basis = diagonal

$$g_x \approx g_{s*} \Big[\underbrace{-\frac{1}{\log R'/R}}_{\text{universal}} + \underbrace{f_x^2\, \gamma(c_x)}_{\text{non-universal: } \gamma \sim 1} \Big]$$

$\Rightarrow$ Universal part does not contribute: $U^\dagger \mathbb{1}\, U = \mathbb{1}$.

$\Rightarrow$ Non-universal part: New source of FCNCs.

Flavor in RS: RS-GIM

- How big is the size of flavor violation?
- Rotate with $U \sim f_i/f_j$
  $\Rightarrow$ Off-diagonal KK-gluon couplings

$$g^{ij} \sim g_{s*} f_i f_j \qquad (\text{for } q, u, d)$$

RS - GIM $\hat{=}$ g^{ij} is automatically suppressed

- for L: by ratios of CKM-elements.
- for R: by mass hierarchy.

(For getting numbers, I will assume $f_3 \sim 1$.)

Flavor in RS: How strong is RS-GIM?

- How strong is this suppression for $\Delta F = 2$ operators?

Effective Hamiltonian

$$\mathcal{H} = C^1(\bar{q}_L^i q_L^j)(\bar{q}_L^k q_L^l) + C^4(\bar{q}_R^i q_L^k)(\bar{q}_L^l q_R^j) + C^5(\bar{q}_R^i q_L^l)(\bar{q}_L^k q_R^j)$$

- Strongest bound comes from the Kaon system:
  $|C_K^4|$ suppressed by $10^4 - 10^5$ TeV.

$\Rightarrow$ in RS:

$$C_K^4 \sim \frac{g_{s*}^2}{M^2} f_{q1} f_{q2} f_{d1} f_{d2} \sim \frac{g_{s*}^2}{M^2} \frac{m_d m_s}{m_t^2}$$

$$\Rightarrow \boxed{M \sim 20 \text{ TeV}}$$

$\Rightarrow$ Can we implement this in flat space?
Does a similar mechanism exist in UED?

Why look at a flat extra dimension?

- UED models do NOT address the hierarchy problem!
- Nevertheless, interesting for model building and LHC phenomenology

  $\rightarrow$ KK-parity provides a dark matter candidate

  $\rightarrow$ UED can fake SUSY-spectra (1st KK-level $\approx$ SUSY spectrum)

  $\rightarrow$ UED can fake gauge mediation signals (photons + missing E_T)

  $\rightarrow$ ...

$\Rightarrow$ UED is an interesting "straw man" to compare to SUSY.

How well could we distinguish these two theories at the LHC?

What do we want to do?

Goal:

A model of UED with KK-parity with

1. anarchic Yukawas
2. mass hierarchies from localization

(like RS-GIM)

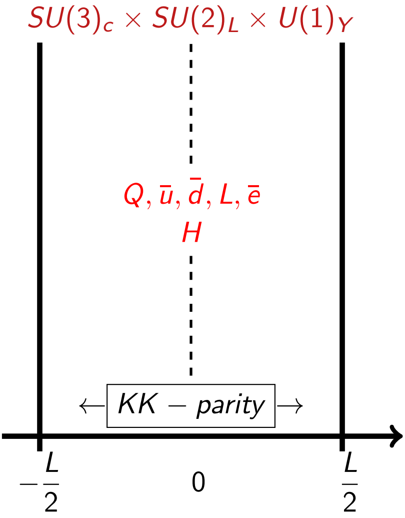
UED with KK-parity

UED

- Flat metric
  $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$
- All SM fields in the bulk.

- SM fields are flat.
- Add KK-parity: $y \to -y$
  + Improves EWPC.
  + DM candidate.
- Usually, flavor put in by hand.

$\to$ Is there a UED-GIM?



UED with KK-parity: Fermions

- Let's add a bulk mass for the fermions like we did in RS:

$$S = \int d^4x \int dy \left[\frac{i}{2} (\bar{\Psi}\, \Gamma^M \overleftrightarrow{\partial}_M \Psi) - m \bar{\Psi}\Psi \right]$$

- The wave functions obey

$$\frac{dg_n}{dy} + m\, g_n - m_n\, f_n = 0$$
$$\frac{df_n}{dy} - m\, f_n - m_n\, g_n = 0$$

- KK-parity: $y \to -y$

$\Rightarrow$ g_n and f_n have opposite KK-parity.

$\Rightarrow$ The mass term violates KK-parity, unless $m \to -m$

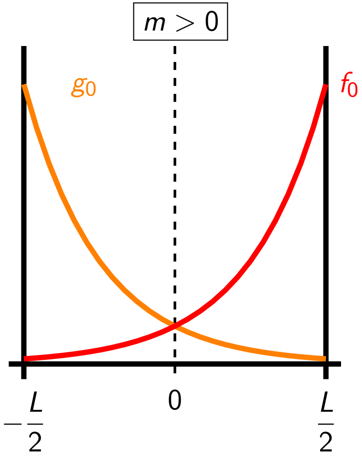
UED with KK-parity: Fermion zero mode

- Zero mode solutions

$$g_0 \sim e^{-my}$$
$$f_0 \sim e^{+my}$$

(BC $\rightarrow$ Chiral spectrum)

$\Rightarrow$ not KK-Parity invariant.



UED with KK-parity

- To maintain KK-parity while allowing bulk masses

$$m = m(y) = \begin{cases} \mu & , y < 0 \\ -\mu & , y > 0 \end{cases}$$

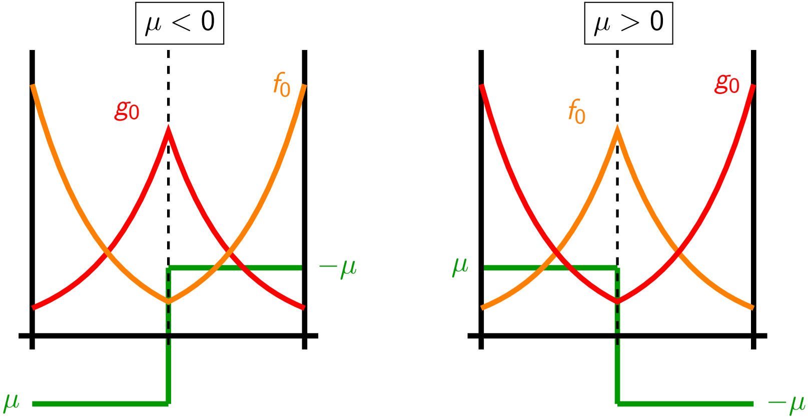
- For $y \neq 0$, we still have

$$g_n' + m(y)\, g_n - m_n f_n = 0$$
$$f_n' - m(y)\, f_n - m_n g_n = 0$$

$\Rightarrow$ Invariant under KK-parity: $m(y) \to -m(y)$

UED with KK-parity: Fermion zero mode

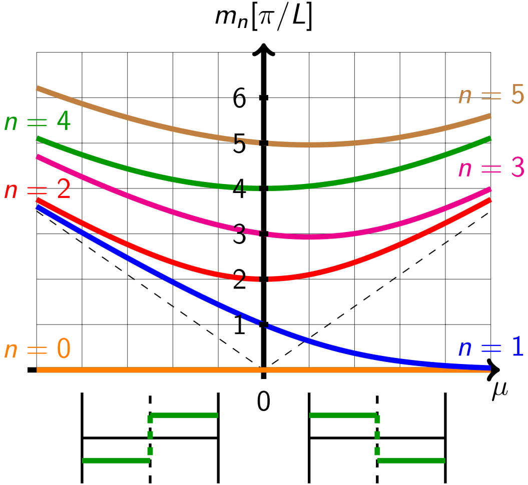
Different localization, depending on sign of μ.



$\Rightarrow$ Could give mass hierarchy due to small overlap.

Fermion spectrum: For a LH zero mode

- Let's examine the complete fermion spectrum:
  Lefthanded zero mode g_0.



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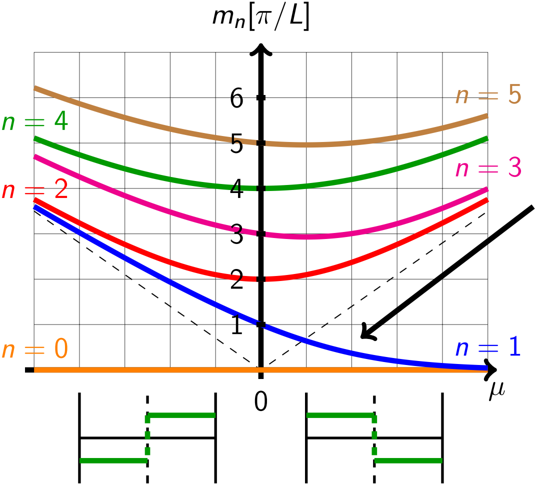


$$m_1 \approx 2\,\mu\, e^{-\mu L/2}$$

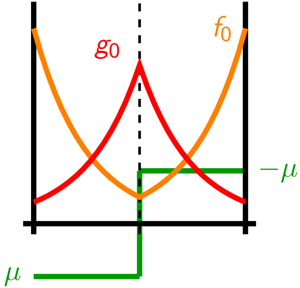
$\Rightarrow$ too light
(we need $\mu L \approx 10$)

Fermion spectrum: Origin of the extra light mode

- Origin: LH and RH modes have opposite behavior

For LH zero mode

$$\begin{aligned} g_n' + m g_n - m_n f_n &= 0 \\ f_n' - m f_n - m_n g_n &= 0 \end{aligned} \qquad \text{with BC: } f_n(\pm L/2) = 0$$



BC: $f_0 \neq 0$ $\Rightarrow$ gets heavy

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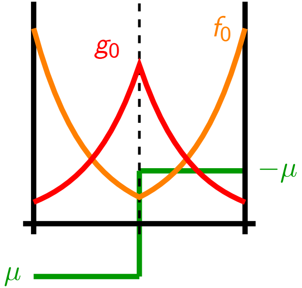
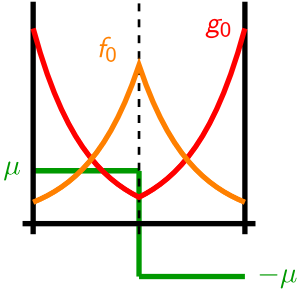


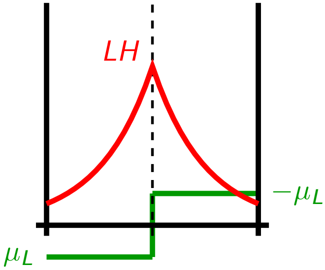
BC: $f_0 \neq 0 \quad \Rightarrow$ gets heavy



BC: $f_0 \approx 0 \Rightarrow f_0$ remains light

Fermion spectrum: Consequences

- What does this mean for our model?
  - Need to choose $\mu_L < 0$ to localize LH in the middel



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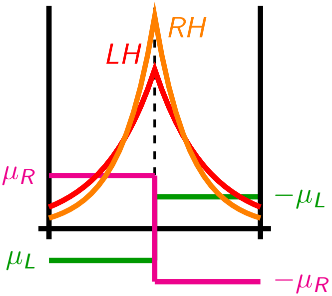
  - For same reason we need RH in middle ($\mu_R > 0$)



Fermion spectrum: Consequences

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  - Need to choose $\mu_L < 0$ to localize LH in the middel
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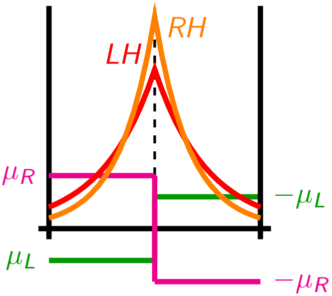
$\Rightarrow$ Need all SM localized at $y = 0$ to avoid light modes.



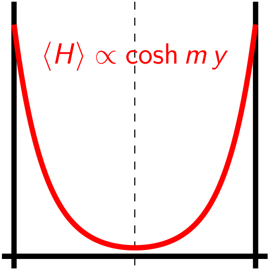
$\Rightarrow$ No small overlap: How will we get the hierarchy now?

Localizing the Higgs

- To obtain hierarchy:
  Need to exponentially localize the Higgs at the boundaries
  (or put it directly on the boundary)

Localizing the Higgs

- Add bulk potential
  $\mathcal{V} = m^2 |H|^2$
- Add boundary potentials
  $V \propto \lambda(|H|^2 - v^2)^2$



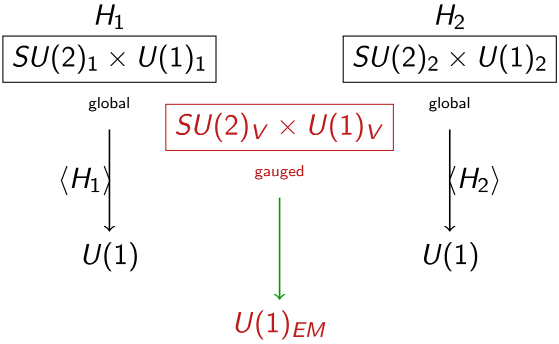
$\Rightarrow$ Gives hierarchy:

$$\mathcal{L}_Y \approx -\frac{v}{\sqrt{2}} \left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c. \right] \Big|_{y=\pm\frac{L}{2}}$$

- BUT, also gives very light, KK-odd mode.

Localizing the Higgs: Two site model

- Where are these (Pseudo)Goldstones from?
- For simplicity, consider a 2 site model: $H \to H_1$ & H_2
- Symmetry structure of 2 site model:



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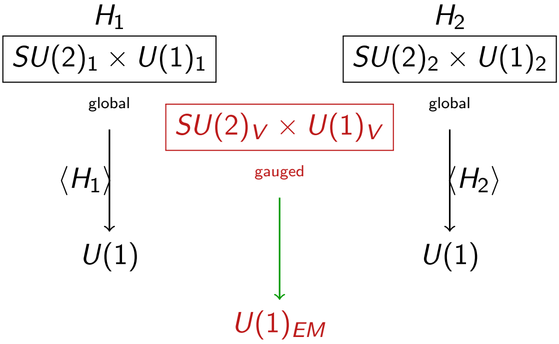


- Two independent H_i + global symmetries $\doteq$ 6 Goldstones
  - 3 KK-even: $\pi_{\text{even}} \sim \pi_1 + \pi_2 \to$ get eaten
  - 3 KK-odd: $\pi_{\text{odd}} \sim \pi_1 - \pi_2 \to$ remain in spectrum!

Mass of Pseudo-Goldstones

Global $[SU(1)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$ is explicitly broken by

- Having localized Higgs, not 2-site model (small correction)

$$\langle H \rangle \propto \cosh(m\,y) \quad \Rightarrow \quad m_0 \propto m\, e^{-mL/2}$$

- Gauging $SU(2)_V \times U(1)_V$

$\rightarrow U(1)_A$ remains unbroken by this: π^0_{odd} does not get a mass.

- Introducing Yukawa couplings

$$\mathcal{L}_Y \sim \bar{\Psi}_q H_1 \Psi_u + \bar{\Psi}_q H_2 \Psi_u + (down)$$

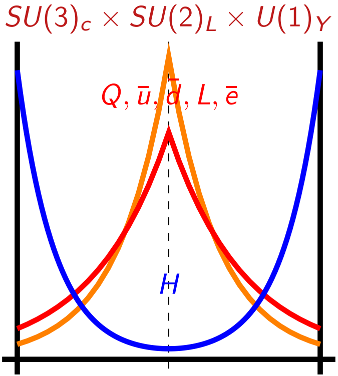
Coleman-Weinberg potential

$\Rightarrow$ All the KK-odd Goldstones get mass from fermion (and gauge) loops.

Recap: Setup

UED with KK-parity

- Flat metric
  $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$
- All SM fields in the bulk.
- KK-parity: $y \to -y$



- Fermion bulk mass:
  $m(y) = -m(-y)$
- Higgs boundary potentials

$\to$ Fermions and Higgs localized at different points

Flavor in UED with KK-parity

- Now, we can proceed analogous to the RS case:

Normalized zero mode solutions

$$g_0, f_0 \sim f(c) \exp\left[-c\left(\frac{|y|}{L} - \frac{1}{2}\right)\right] \qquad \text{with } c = \mu L$$

with $f(c) = \sqrt{\frac{c}{e^c - 1}} = \frac{1}{\log R'/R} f_{RS}(c_{RS})$

- Yukawa couplings are given by

$$\mathcal{L}_Y \approx -\frac{v}{\sqrt{2}}\left[\bar{\Psi}_q \tilde{Y}_u \Psi_u + \bar{\Psi}_q \tilde{Y}_d \Psi_d + h.c.\right]\Big|_{y=\pm L/2}$$

- $\tilde{Y}_u, \tilde{Y}_d$ are both $\mathcal{O}(1)$, anarchic flavor matrices.

Flavor in UED with KK-parity

$\Rightarrow$ Mass matrices are given by

$$m_u = \frac{v}{\sqrt{2}} f_q \tilde{Y}_u f_u \qquad \text{with} \qquad f_c = \text{diag}[\{f(c_i)\}]$$
$$m_d = \frac{v}{\sqrt{2}} f_q \tilde{Y}_d f_d$$

- Now usual SM prescription applies

$$m^{SM} = U_L m U_R^\dagger \qquad \text{and} \qquad V_{CKM} = U_{Lu}^\dagger U_{Ld}$$

$$\Rightarrow \left(m_{u,d}^{SM}\right)_{ii} \sim \frac{v}{\sqrt{2}} Y_* f_{q_i} f_{u_i,d_i}$$

$\Rightarrow$ We got flavor hierarchy, but what about new FCNC contributions?

Flavor in UED with KK-parity: Check the hierarchy of f_i's

- For left-handed fields: f_{q_i}'s are determined from the diagonalization matrices

$$|U_{ij}| \sim \frac{f_i}{f_j} \qquad \Rightarrow \qquad |(V_{CKM})_{ij}| \sim \frac{f_{q_i}}{f_{q_j}} \qquad i \leq j.$$

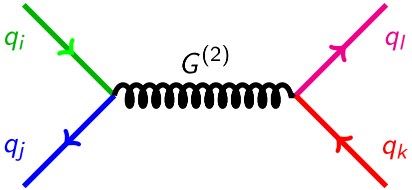
$$\frac{f_{q_2}}{f_{q_3}} \sim \lambda^2, \quad \frac{f_{q_1}}{f_{q_3}} \sim \lambda^3 \qquad \text{with } \lambda \sim \sin\theta_c \sim 0.2$$

- Fermion masses hierarchy fixes right-handed f_{-u_i,d_i}'s

$$\frac{f_{u_1^c}}{f_{u_3^c}} \sim \frac{m_u}{m_t}\frac{1}{\lambda^3}, \quad \frac{f_{u_2^c}}{f_{u_3^c}} \sim \frac{m_c}{m_t}\frac{1}{\lambda^2}$$

$$\frac{f_{d_1^c}}{f_{u_3^c}} \sim \frac{m_d}{m_t}\frac{1}{\lambda^3}, \quad \frac{f_{d_2^c}}{f_{u_3^c}} \sim \frac{m_s}{m_t}\frac{1}{\lambda^2}, \quad \frac{f_{d_3^c}}{f_{u_3^c}} \sim \frac{m_b}{m_t}$$

Flavor in UED with KK-parity: FCNC



- Plug in the wave function: In original basis = diagonal

$$g_x \approx g^{4D}\sqrt{2}\Big[\underbrace{1}_{\text{universal}} - \underbrace{f_x^2\,\gamma(c_x)}_{\text{non-universal}}\Big]$$

- BUT, ere $\gamma \propto \frac{e^c}{c^3}$, but to obtain mass hierarchy we need $c \sim 1 \ldots 15$.

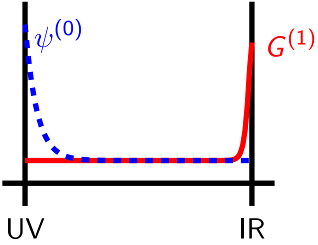
Unlike RS: $\gamma \neq \mathcal{O}(1)$ $\Rightarrow$ NO protection from FCNCs.

The origin of RS-GIM

- Why does this work in RS, but not in UED?
- In RS: Flavor violation comes from the coupling to KK-gluons

$$g_x = g^{5D}\int_R^{R'} dz\sqrt{-g}\,\left[\psi^{(0)}(z)\right]^2 G^{(1)}(z) \approx g_{s*}\left[-\frac{1}{\log R'/R} + f_x^2\,\gamma(c_x)\right]$$

(In appropriate coordinates)



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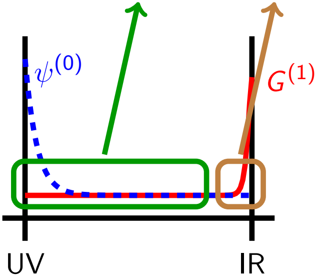
(In appropriate coordinates)

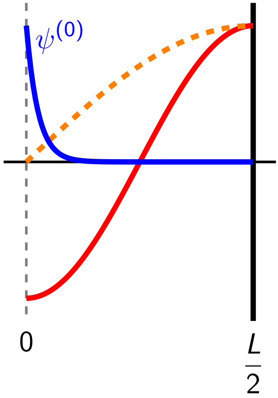
universal!



RS-GIM originates in well-separated wave functions

Why "UED-GIM" does not exist

- In UED the KK-gluon wave functions are not localized (at least not strongly enough):



$G^{(2)}$: normal UED

($\tilde{G}^{(2)}$: UED with "brane" terms at $y = 0$)

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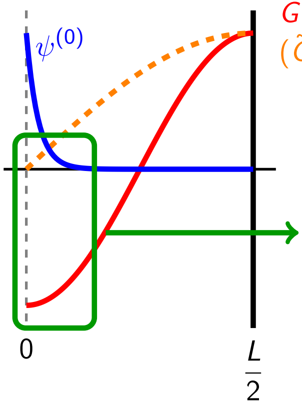

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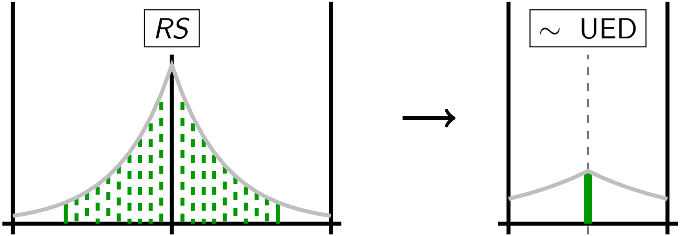
overlap too large/non-universal!

To work in UED need to localize fermions even more!

RS vs. UED with KK-parity

- We can justify brane localized terms at $y = 0$:

$\rightarrow$ Think of UED as integrated out RS



$\Rightarrow$ Effect of integrating out: Add "boundary" kinetic term

RS vs. UED with KK-parity

- Add "boundary" kinetic term

$$S_{\text{boundary}} = \int d^5x \; \delta(y) \left\{ \frac{i}{2} \bar{\Psi} \, \Gamma^\mu \overleftrightarrow{\partial}_\mu \Psi \right\} \kappa L + \text{ gauge kinetic term}$$

- Gauge kinetic terms turn out not to matter much.
- For fermions: only changes the function $f(c, \kappa) = \sqrt{\frac{c}{(1 + c\kappa)e^c - 1}}$.

$\rightarrow$ Can suppress $f \sim 1/\kappa$, while keeping $\gamma \sim \frac{e^c}{c} \sim 1$.

However, this is basically the low energy version of RS and not UED!

Conclusion

- UED with KK-parity can localize fermions (but only in the middle).
- Localizing the Higgs on the boundaries gives Pseudo-Goldstones: Get masses at loop level ($\rightarrow$ no problem ?)

$\Rightarrow$ Obtain flavor hierarchy, but no protection from FCNC.

- Can work around this, but at cost of obtaining low-energy RS.

Anarchic flavor in UED is difficult.

The End

Thank you.