Lattice QCD, the CMSSM and dark matter

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Apply the Feynman-Hellman theorem

- Working with renormalized quantities obtained from the lattice

$$f_{T_s} = m_s \tfrac{d}{dm_s} \log m_N = (m_K^2 - \tfrac{1}{2} m_\pi^2) \tfrac{d}{dm_K^2} \log m_N$$

$$f_{T_u} = f_{T_d} = \tfrac{1}{2} m_\ell \tfrac{d}{dm_\ell} \log m_N = \tfrac{1}{2} m_\pi^2 \tfrac{d}{dm_\pi^2} \log m_N$$

- These two quantities are key in estimating dark matter cross sections with the nucleon

$$f_{T_q} = \tfrac{m_q}{m_N} \langle N | \bar{q} q | N \rangle$$

- Obviously we need a parametric description of m_N in terms of m_π and m_K

Chiral expansion

$$m_B = M^{(0)} - C^{(1)}_{Bl} \times (m_\pi^2/2) - C^{(1)}_{Bs} \times (m_K^2 - m_\pi^2/2)$$
$$+\sigma^\pi_{BB}(m_\pi, \Lambda) + \sigma^K_{BB}(m_K, \Lambda) + \sigma^\eta_{BB}(m_\eta, \Lambda)$$

- With the nonanalytic terms σ evaluated in finite range regularization

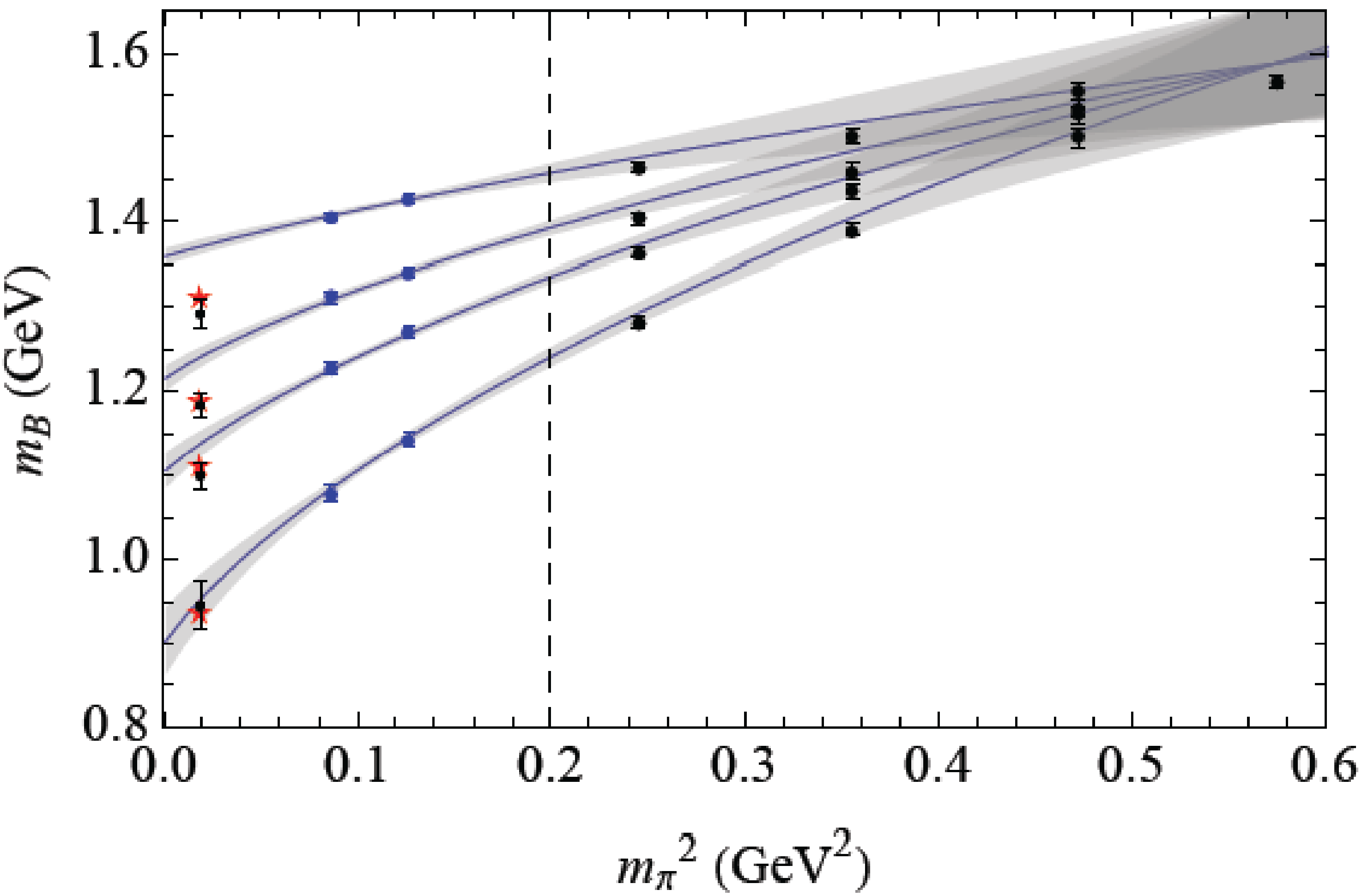
B	$C^{(1)}_{Bl}$	$C^{(1)}_{Bs}$
N	$2\alpha_M + 2\beta_M + 4\sigma_M$	$2\sigma_M$
Λ	$\alpha_M + 2\beta_M + 4\sigma_M$	$\alpha_M + 2\sigma_M$
Σ	$\frac{5}{3}\alpha_M + \frac{2}{3}\beta_M + 4\sigma_M$	$\frac{1}{3}\alpha_M + \frac{4}{3}\beta_M + 2\sigma_M$
Ξ	$\frac{1}{3}\alpha_M + \frac{4}{3}\beta_M + 4\sigma_M$	$\frac{5}{3}\alpha_M + \frac{2}{3}\beta_M + 2\sigma_M$

Table from Young 0907.0239

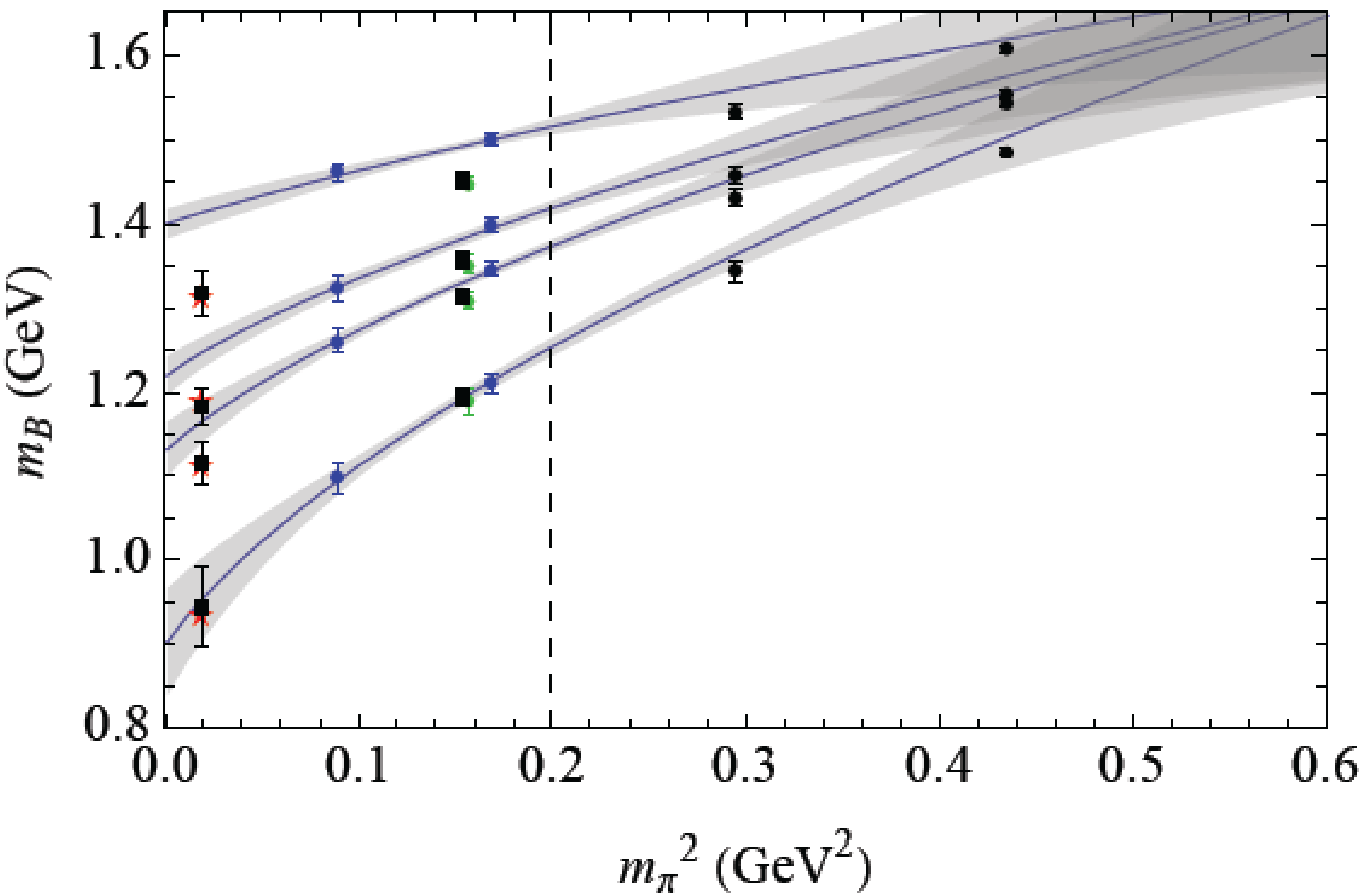
Application

- In [Young, Thomas 2010, 0901.3310] these fits were performed
- based on baryon masses obtained from
- two 2+1 flavor lattice QCD datasets:
- LHPC, 0806.4549
- PACS-CS, 0807.1661
- Infinite volume extrapolation was performed

Fit to LHPC data



Fit to PACS-CS data



Estimates

- Using these fits, Young and Thomas obtain:

$$f_{T_s} = 0.033(16)(4)(2), \quad f_{T_u} = f_{T_d} = 0.025(5)(2)(2)$$

- Toussaint & Freeman [0905.2432] obtain by their method

$$f_{T_s} = 0.063(6)(8)$$

- Example 2-flavor determination (overlap) [JLQCD, 1011.1964]:

$$f_{T_s} = 0.032(8)(22)$$

- These are to be compared with the estimates that were used in the CMSSM dark matter study [Ellis, Olive, Savage 08]:

$$f_{T_s} = 0.36(8), \quad f_{T_u} = 0.027(3), \quad f_{T_d} = 0.039(5)$$

- Since Higgs exchange plays a key role, and the lattice gives a much smaller value of f_{T_s} , it is of interest to see what this does to the cross section

Cross section

- The spin-independent cross section is given by

$$\sigma = \frac{4}{\pi} m_p^2 f^2$$

$$f = m_p \{ \sum_q f_{T_q} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} f_{TG} \sum_Q \frac{\alpha_{3Q}}{m_Q} \}$$

$$f_{TG} = 1 - \sum_q f_{T_q}$$

- The parameters α_{3q}, α_{3Q} come from tree-level exchange (Higgses and squarks) in the CMSSM between the neutralino and the quarks inside the proton

The parameters from CMSSM

$$\alpha_{3i} = -\frac{1}{2(m_{1i}^2 - m_\chi^2)} \mathrm{Re}\left[(X_i)(Y_i)^*\right] - \frac{1}{2(m_{2i}^2 - m_\chi^2)} \mathrm{Re}\left[(W_i)(V_i)^*\right]$$
$$- \frac{g m_{q_i}}{4 m_W B_i} \left\{ \left(\frac{D_i^2}{m_{H_2}^2} + \frac{C_i^2}{m_{H_1}^2} \right) \mathrm{Re}\left[\delta_{2i}\left(g Z_{\chi 2} - g' Z_{\chi 1}\right)\right] + D_i C_i \left(\frac{1}{m_{H_2}^2} - \frac{1}{m_{H_1}^2} \right) \mathrm{Re}\left[\delta_{1i}\left(g Z_{\chi 2} - g' Z_{\chi 1}\right)\right] \right\}$$

From [Ellis, Olive, Savage 08]

Line 1: squark exchange
Lines 2&3: Higgses exchange (note proportional to quark mass)
X,Y etc. are functions of couplings and mixing matrix elements of CMSSM

CMSSM

- Specified by 5 high-scale parameters

$$A_0, \quad m_0, \quad M_{1/2}, \quad \tan\beta, \quad \mathrm{sign}(\mu)$$

- Two-loop RGEs used to evolve all couplings down to electroweak scale
- Diagonalization of mass matrices generates masses and mixings that we need
- Set of 13 benchmark models considered, from [Battaglia et al. 03], all consistent with WMAP

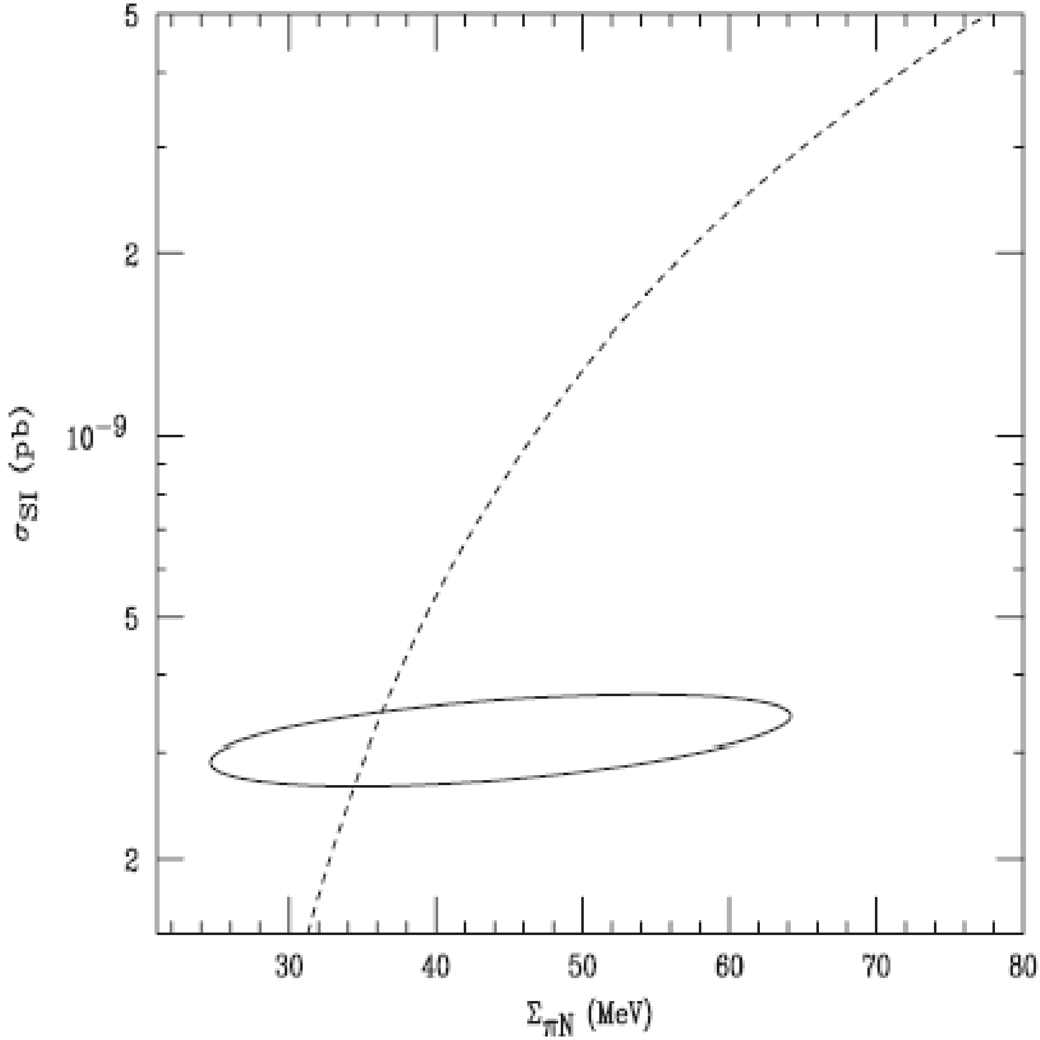


FIG. 3. A comparison of our results (solid ellipse) for model *C*, versus the traditional approach (dashed line) which relates the strange-quark sigma commutator to the light-quark one through Eq. (1) with $\sigma_0 = 36$ MeV.

An amusing fact

TABLE I. Example breakdown of quark flavor contributions to σ_{SI}. This is for model L at the maximum value of its cross section, within our 95% CL region.

model	q	α_{3q}/m_q	f^p_{Tq} or f^p_{TG}	f^p_q/f_p
L (maxσ_{SI})	u	-1.019×10^{-09}	0.0280	0.0105
$\sigma_{SI} =$	d	-1.302×10^{-08}	⋯	0.1342
2.8×10^{-9} pb	c	-1.031×10^{-09}	0.8751	0.0261
	s	-1.522×10^{-08}	0.0689	0.3633
$\sigma_\ell = 52.5$ MeV,	t	-1.936×10^{-09}	0.8751	0.0462
$\sigma_s = 64.6$ MeV	b	-1.670×10^{-08}	⋯	0.3984

How it happens

$$\begin{aligned}\theta^\mu_\mu &= \sum_{uds} m_q \bar{q} q + \sum_{cbt} m_Q \bar{Q} Q - \frac{7\alpha_s}{8\pi} G \cdot G \\ &= \sum_{uds} m_q \bar{q} q + \frac{9}{2} \sum_{cbt} m_Q \bar{Q} Q\end{aligned}$$

using NSV relation. Then

$$\begin{aligned}\langle N|\theta^\mu_\mu|N\rangle &= m_N = \sum_{uds} \langle N|m_q \bar{q} q|N\rangle + \frac{9}{2} \sum_{cbt} \langle N|m_Q \bar{Q} Q|N\rangle \\ &= m_N \sum_{uds} f_{T_q} + \frac{9}{2} \sum_{cbt} \langle N|m_Q \bar{Q} Q|N\rangle\end{aligned}$$

Assuming each $\langle N|m_Q \bar{Q} Q|N\rangle$ identical, we get

$$\langle N|m_Q \bar{Q} Q|N\rangle = \frac{2}{27} m_N (1 - \sum_{uds} f_{T_q})$$

Conclusions

- Application of the Feynman-Hellman theorem to lattice QCD determinations of nucleon mass
- Together with chiral fits
- Allows for accurate determinations of f_{T_s}
- This has provided much more reliable estimates of cross sections with dark matter in the CMSSM
- Clear follow-up is to

1. Improve the fits with new lattice data
2. Explore other dark matter models (NMSSM)
3. Other lattice methods (see advertisement next slide)

MAIN RESULT: Smaller cross sections, with certainty

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- "Disconnected contributions to nucleon form factors with chiral fermions"
- Ronald Babich, Richard Brower, Mike Clark, Saul Cohen, George Fleming, JG, James Osborn, Claudio Rebbi
- Follow-up to work of (" – JG) with clover fermions [1012.0562]
- The hope is that chiral fermions reduce the mixing with nonsinglet contribution:

$$f_{Ts} = \frac{\tilde{m}_s^0 + 2\Delta(\tilde{m}_s^0 - \tilde{m}_l^0)/3}{M_N^0}\left[\langle N|\bar{s}s|N\rangle_0 - \Delta\langle N|(\bar{u}u + \bar{d}d - 2\bar{s}s)|N\rangle_0/3\right]$$

$$\Delta = Z_8/Z_0 - 1.$$

From Babich et al. 2010