In Memoriam Robert Brout...

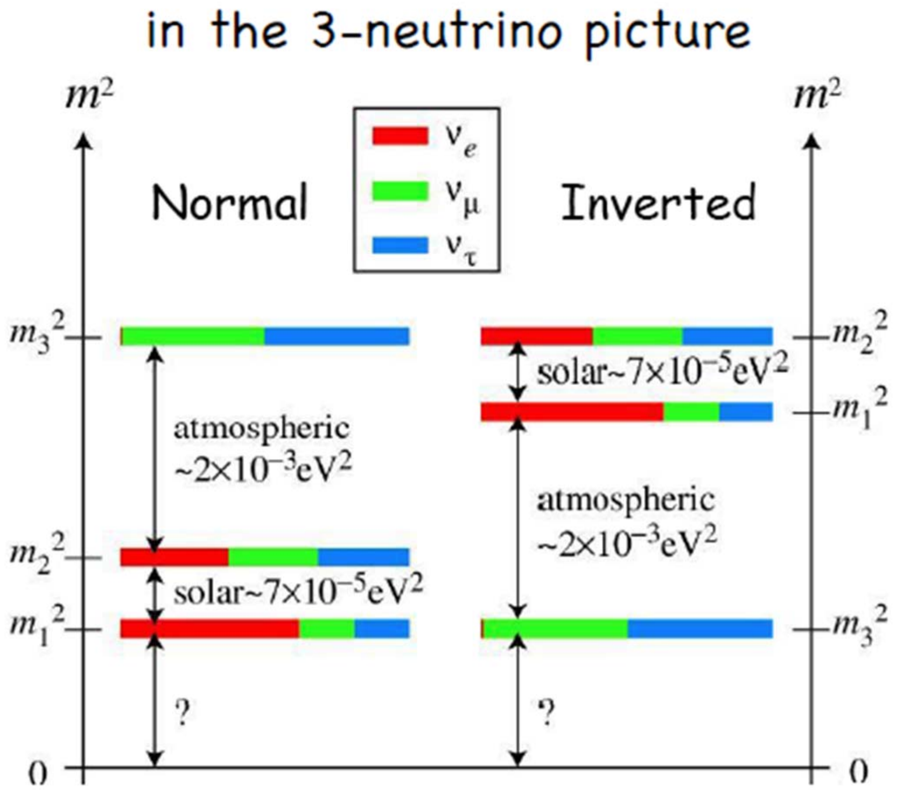
Why Neutrinos are different …

- Very low mass
- Large leptonic mixing
- Leptonic number conserved or not ?
  ….With link to matter-antimatter asymmetry

J.-M. Frère

Based on work with Maxim Libanov, Serguey Troitsky, Emin Nugaev,Ling Fu-Sin

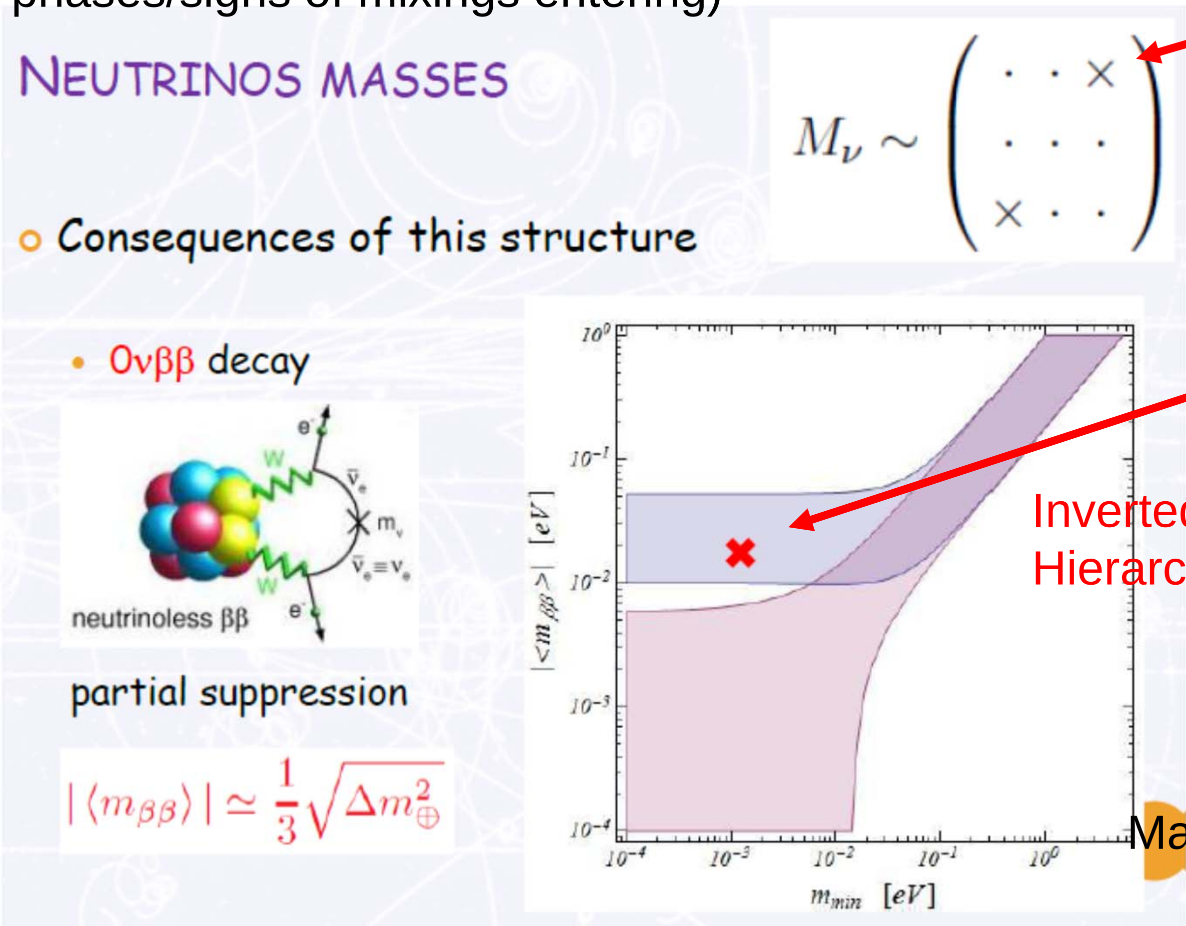
What we now from oscillations



Our Generic prediction : large mixings,
inverted hierarchy
suppressed neutrinoless double beta decay

Generic prediction : **large mixings,**
inverted hierarchy
suppressed neutrinoless double beta decay

Neutrino-less double beta decay controlled by weighted sum of masses
(with phases/signs of mixings entering)



Automatically get

Inverted Hierarchy

Mass scale

based on **arXiv:1006.5196** , to appear in JHEP
And work with M Libanov, S. Troitsly, E Nugaev, FS Ling

Why Neutrinos are Different

How LHC could confirm (but not exclude) the model

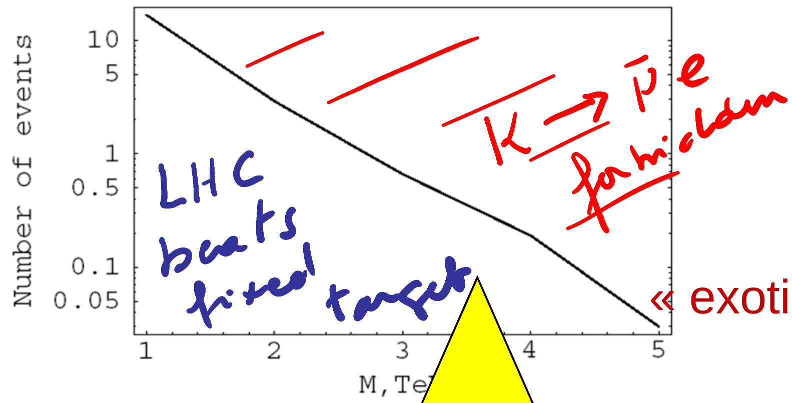
How LHC can compete with
fixed target Lepton Flavour Violation expts

From now on …

An « ordinary » Z'
(with same
couplings as Z)

e+ e- pairs

for later …



« exotic » Z'

Fig. 1. Num… …uc-
tion as a function … M, with
$\kappa = M/(100\text{TeV})$.

μ+ e- pairs
(>> μ- e+)

In a nutshell:

Previous work :

- One family in 6D and proper boundary conditions → 3 families in 6D
- At lowest order in Cabibbo mixing Charged fermion masses are
  - Diagonal
  - Strongly hierarchical

NOW

- At same order, we get 4D Majorana neutrinos with
  - Antidiagonal mass matrix
- This yields, in a generic way:
  - Large mixings in the neutrino sector
  - Inverted Hierarchy
  - Pseudo- Dirac structure (further suppression of neutrinoless double beta decay)
- Not as automatic, but typical : measurable Θ_{13}

At LHC, this can result in std model like Z'
and later more exotic signals ($Z'^{\pm} \rightarrow \mu^+ e^- \gg \mu^- e^+$)

A very few words about extra dimensions … start with ONE extra spatial dim.

What are Zero Modes ?

$$A = 0,1,2,3,4,5$$
$$\mu,\nu = 0,1,2,3$$

Dirac equation in N+1 dimensions,
For a fermion interacting with a field Φ:

$$i\partial_A\,\gamma^A\Psi = \Phi\,\Psi$$

(or $m\,\Psi$)

For ONE compact extra dim

$0 \leq y \leq 2\pi R$

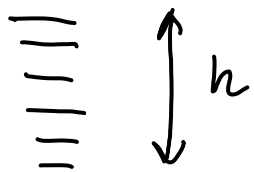
$$\Psi(x^\mu, y) = \sum \Psi_n(x^\mu) e^{i\frac{ny}{2\pi R}}$$

$$i\partial_\nu\,\gamma^\nu\Psi_n(x^\mu)e^{i\frac{ny}{2\pi R}} = (\underbrace{\frac{n}{2\pi R}i\gamma_5 + \Phi}_{\text{effective mass}})\Psi_n(x^\mu)e^{i\frac{ny}{2\pi R}}$$

→ Kaluza-Klein tower

effective mass ($\gg$ 1 TeV)

$= 0 \Rightarrow$ zero modes.



$x^4, x^5 \to \begin{cases} r \\ \varphi \end{cases}$

ℓ

Kaluza - Klein "Tower"

n

For 2 compact extra dim

$$\Psi(x^\mu, x^4, x^5) = \sum_{n,l} \psi_{n,l}(x^\mu)\; f_{n,l}(x^4, x^5)$$

$$\underbrace{i\partial_\nu\, \gamma^\nu \psi_{n,l}(x^\mu)\; f_{n,l}(x^4, x^5)}_{\text{4-d Dirac eq.}} = \psi_{n,l}(x^\mu)\underbrace{(\Phi - i\partial_4\, \gamma^4 - i\partial_5\, \gamma^5)}_{\text{large effective mass}}\; f_{n,l}(x^4, x^5)$$

$\gg 1$ TeV

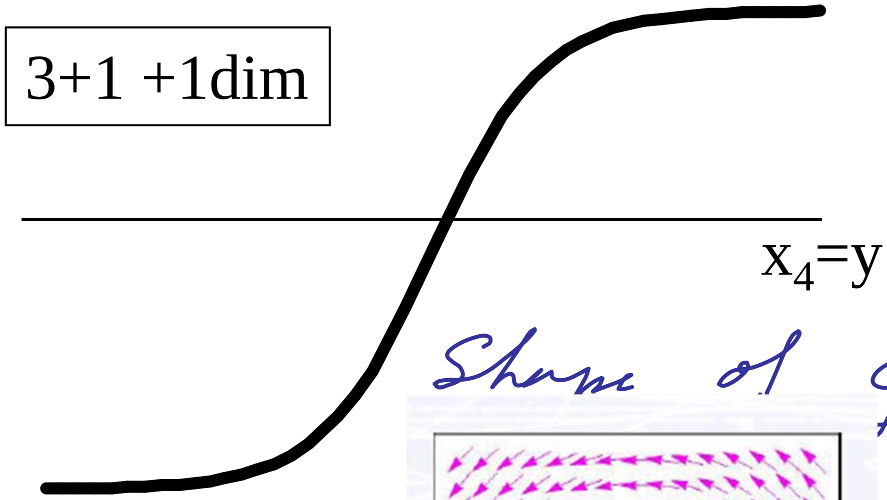
$l = 0 \to$ zero mode

Look for zero modes …

Use of dimensional reduction obtain 3+1-dim chiral spinors (L) : use of topological singularities in the extra dimensions to get zero modes, break LR symmetry.

3+1 +1dim

e.g

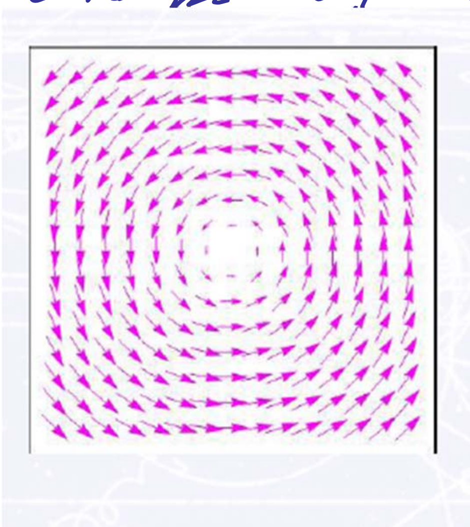


Solitonic background: index theorem localizes one chiral Fermion ; Alternatively, orbifold

Shape of Φ field :

3+1 +2 dim

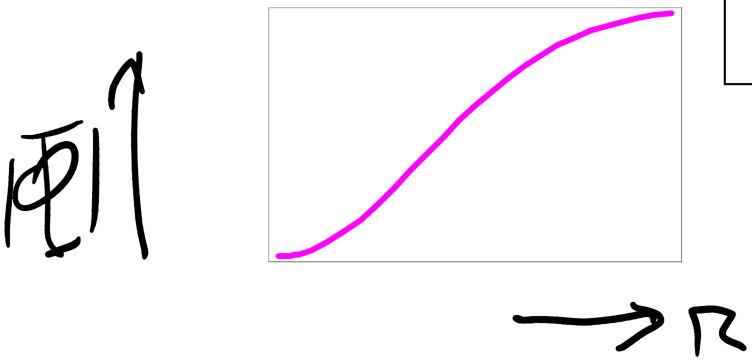
$$\Phi = e^{\,i\, n\varphi}$$



Vortex with winding number n localizes n chiral massless fermion modes in 3+1

3 families from one in 5+1 dim

$$\Phi = e^{\,i\,n\varphi}$$



we assume a background scalar field Φ providing a vortex in the 2 extra dimensions;
It vanishes at the origin– where we live!

The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable ϕ

For some reason, n=3 !!!

The transition from 6-D to the zero- modes of 4D dramatically affects

$\bar{\Psi}\,\Phi\,\frac{1-\Gamma_7}{2}\,\Psi$

Start from 4 * 2-components

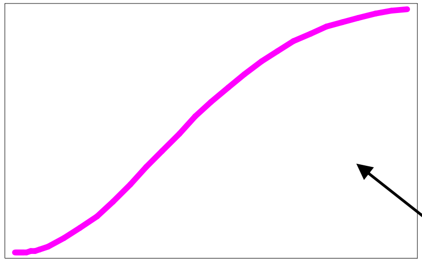
$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

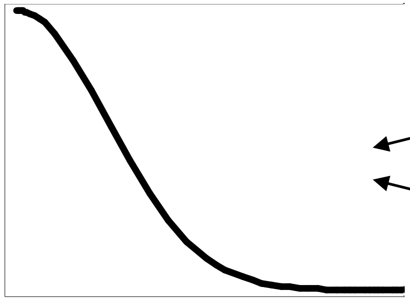
Each « zero » (massless) mode only has 2-spinor degrees of freedom:
For instance,

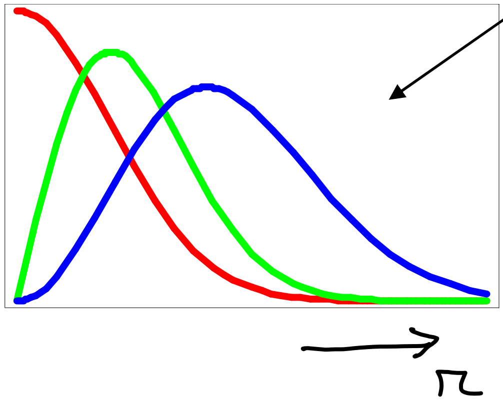
redundant
(SAME ψ_L)

$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix}$$

Field Content

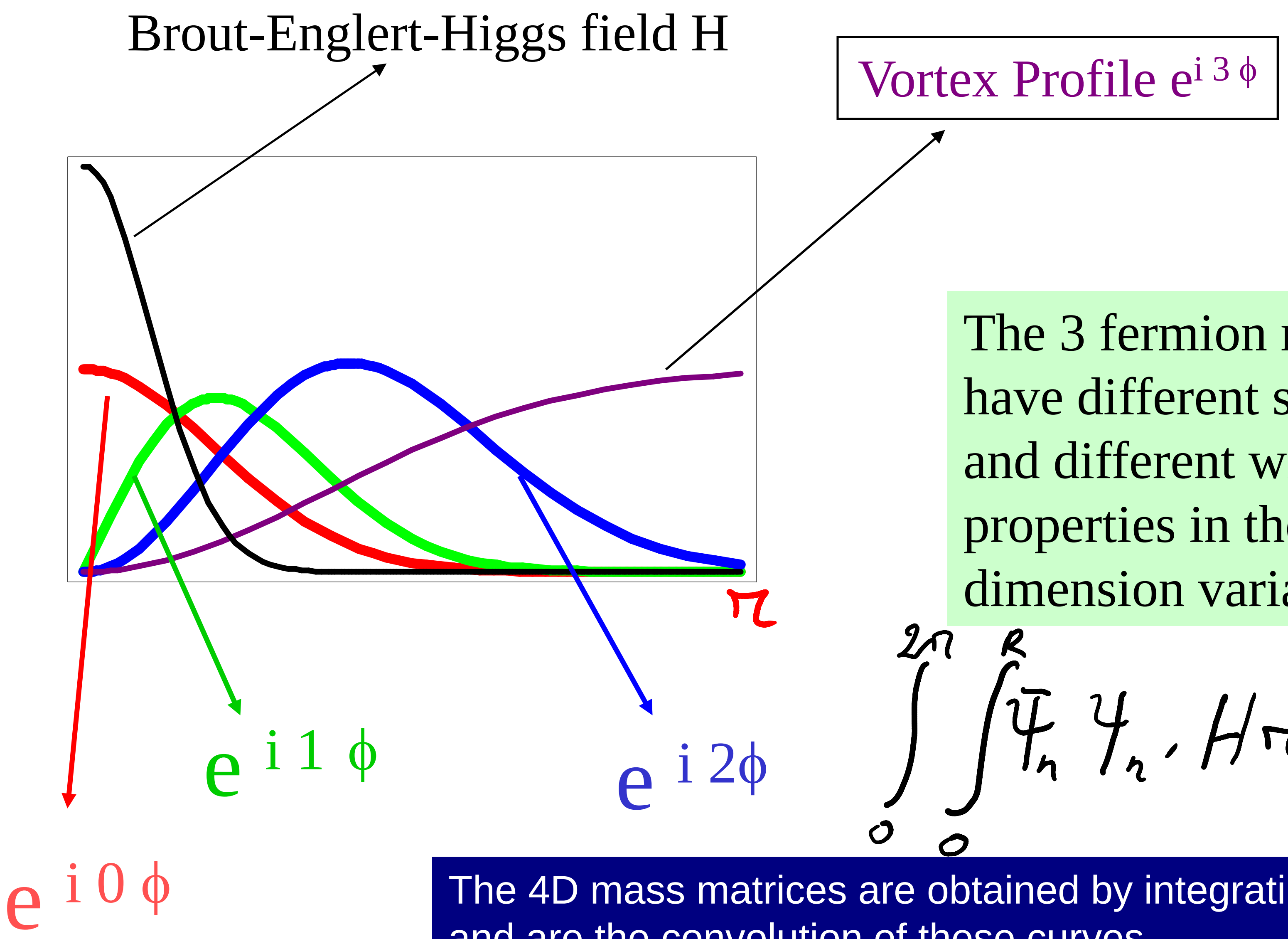


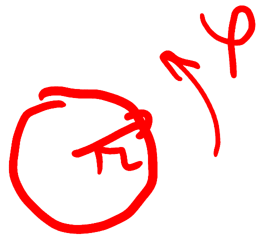




fields		profiles	charges		representations	
			$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$F(r)e^{i\theta}$ $F(0)=0,\ F(\infty)=v_\Phi$	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	X	$X(r)$ $X(0)=v_X,\ X(\infty)=0$	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
scalar	H	$H(r)$ $H(0)=v_H,\ H(\infty)=0$	-1	$+1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	Q	3 L zero modes	axial $+3/2$	$+1/6$	$\mathbf{2}$	$\mathbf{3}$
fermion	U	3 R zero modes	axial $-3/2$	$+2/3$	$\mathbf{1}$	$\mathbf{3}$
fermion	D	3 R zero modes	axial $-3/2$	$-1/3$	$\mathbf{1}$	$\mathbf{3}$
fermion	L	3 L zero modes	axial $+3/2$	$-1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	E	3 R zero modes	axial $-3/2$	-1	$\mathbf{1}$	$\mathbf{1}$

Table 1: Scalars and fermions with their gauge quantum numbers. For convenience, we describe here also the profiles of the classical scalar fields and fermionic wave functions in extra dimensions.





The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable ϕ

$$\int_0^{2\pi}\int_0^{R} \bar{\Psi}_n \Psi_n \cdot H\, r\, dr\, d\varphi$$

The 4D mass matrices are obtained by integrating r and ϕ, and are the convolution of these curves

For Quarks and Dirac fermions,
we get a mass matrix like :

$$\begin{pmatrix} small & \varepsilon & \\ & medium & \varepsilon \\ & & large \end{pmatrix}$$

Generation number n =	Winding
1	2
2	1
3	0

Additional couplings involving the vortex field, with winding $e^{i\phi}$ can give the small Cabibbo mixings ε

The scheme is very constrained, as the profiles are dictated by the equations, instead of being imposed by hand, like in multilocalisation

Neutrinos ARE different

In the same context (0th order in Cabibbo mixing), we will get indeed (see later):

$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & m \\ \cdot & \mu & \cdot \\ m & \cdot & \cdot \end{pmatrix}$$

Where m >> μ
After 45° 1-3 rotation and 23 permutation, this leads to an inverted hierarchy, (minute solar mass difference found between the heavier neutrinos)

$$M_\nu \sim \begin{pmatrix} m & \cdot & \cdot \\ \cdot & -m & \cdot \\ \cdot & \cdot & \mu \end{pmatrix}$$

The – sign may be absorbed in the mixing matrix, *but contributes destructively to the effective mass for neutrinoless double beta decay* (Pseudo-Dirac structure when full Cabibbo-like mixing is introduced)

WHY the difference? --- return in more detail to the 6D spinors,

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

Zero modes

$$\bar{\Psi}\, \phi\, \frac{1-\Gamma_7}{2}\, \Psi \quad \Rightarrow \quad \begin{pmatrix} 0 \\ \psi_L \\ \psi_L \\ 0 \end{pmatrix} \quad \text{solutions } (L)$$

$$\bar{\Psi}\, \phi\, \frac{1+\Gamma_7}{2}\, \Psi \quad \Rightarrow \quad \begin{pmatrix} \psi_R \\ 0 \\ 0 \\ \psi_R \end{pmatrix} \quad (R)$$

.

WHY the difference? --- return in more detail to the 6D spinors,

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

In each case, the massless mode only has 2-spinor degrees of freedom:
For instance,

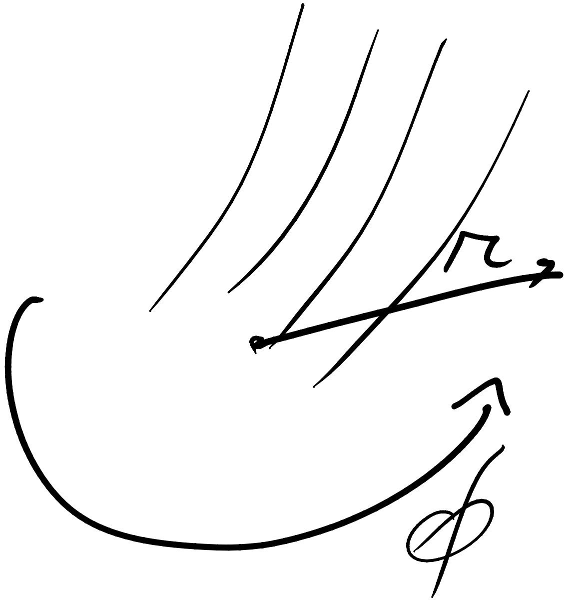
$f\, e^{i\phi}$ Same ψ

$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\, e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix}$$

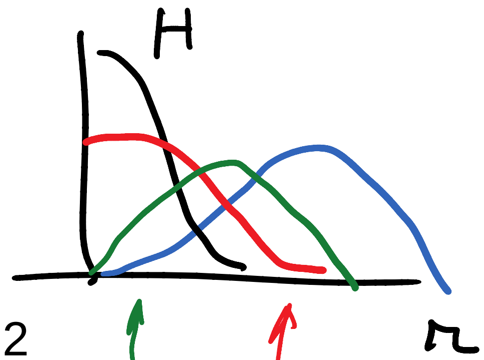
WHY the difference?

It comes from rotation invariance in the 2 extra dimensions…

Vortex



$$\int_0^{2\pi} d\phi \, \bar{\Psi}_i(\phi)\, \Psi_j(\phi)\, H$$



WHY the difference? --- return in more detail to the 6D spinors,

For the charged spinors, we have both L and R spinors bound to the vortex.

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

L $\bar{R}$

$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix} \qquad R \sim \sum_n \begin{pmatrix} f_{n-1}[r)\ e^{i(1-n)\phi}\chi_{Rn}(x^\mu) \\ 0 \\ 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\chi_{Rn}(x^\mu) \end{pmatrix}$$

Dirac mass

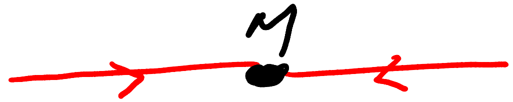
$$\bar{R} L = \sum_{n,n'} \bar{R}_{n'} L_n$$

$$\Rightarrow \int dr \int_0^{2\pi} d\phi\, f_{3-n}\, f^{*}_{3-n'}\, e^{i(n-n')\phi} \to \delta(n-n')$$

Effective Lagrangian : integrate over r and φ, diagonal

For neutrinos (using only Majorana-type 4D mass term)
we will get (through see-saw discussed later)

$$\overline{L^c}\, L \Rightarrow \sum_{n\,n'} \overline{(L_{n'})^c}\, L_n$$



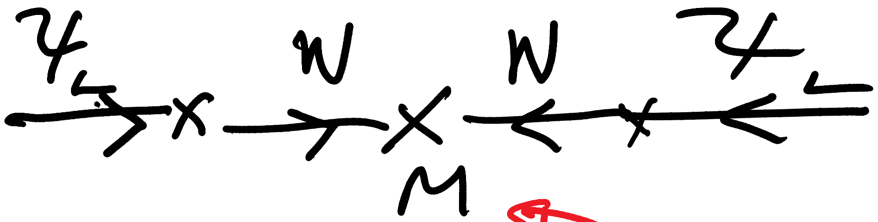
$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix}$$

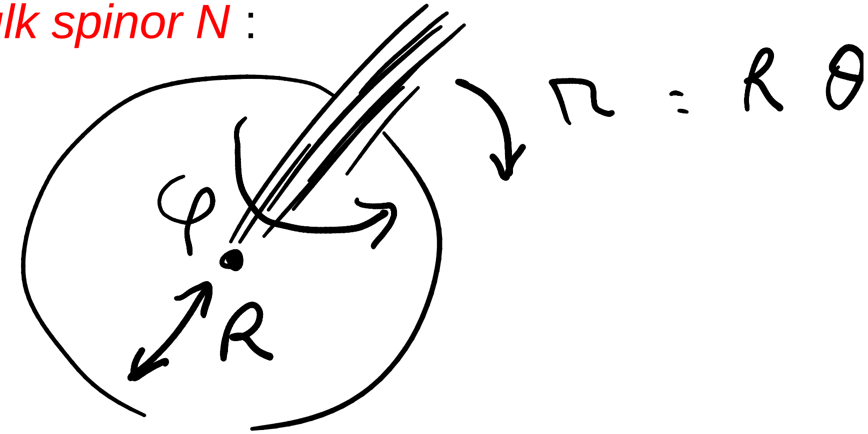
$$\Rightarrow \int_0^{2\pi} d\phi \ e^{i(4-n-n')\phi}$$

$$\Rightarrow \delta(n+n'-4)$$

$$\Rightarrow \begin{array}{cc} \begin{pmatrix} & & m \\ & \mu & \\ m & & \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

The Majorana term can be traced to the « Majorana mass term » in the Lagrangian (not to be confused with a non-existant 6D Majorana spinor) through a see-saw
It leads to a contribution proportional to the effective propagator after summing over a *large number of proper modes of the bulk spinor N* :





$M \, \overline{N^c} N$

$\Longrightarrow$ 4D

Each mode n
of the bulk spinor N

$$\frac{M}{\not{p}^2 - \left(\frac{2\pi n}{R}\right)^2 - M^2}$$

small m

$\rightarrow M \gg 1/R$

OR

$\rightarrow M \ll 1/R$ (GeV ok)

fields		profiles	charges		representations	
			$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$F(\theta)e^{i\phi}$ $F(0)=0,\ F(\pi)=C_{\pi F}$	$+1$	0	$\mathbf{1}$	$\mathbf{1}$
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scalar	H	$H(\theta)$ $H(0)=v_H,\ H(\pi)=0$	-1	$+1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	L_+, L_-	3 L zero modes	$(3,0)$	$-1/2$	$\mathbf{2}$	$\mathbf{1}$
fermion	E_+, E_-	3 R zero modes	$(0,3)$	-1	$\mathbf{1}$	$\mathbf{1}$
fermion	N	massive modes $\chi_{\lambda,m}$, $\xi_{\lambda,m}$	0	0	$\mathbf{1}$	$\mathbf{1}$
scalar	S_+	(composite field)	-1	0	$\mathbf{1}$	$\mathbf{1}$
scalar	S_-	(composite field)	2	0	$\mathbf{1}$	$\mathbf{1}$

Table 1: Field content of the model (scalars and leptons only).

We can then introduce some extra terms, carrying winding, to generate the Cabibbo-like mixings,

For charged leptons

$$\frac{\mathcal{L}_E}{\sqrt{-\det g_{AB}}} = \sum_{S^l_+} Y_l^+(S^l_+) S^l_+ H \bar{L} \frac{1+\Gamma_7}{2} E + \sum_{S^l_-} Y_l^-(S^l_-) S^l_- H \bar{L} \frac{1-\Gamma_7}{2} E + \text{h.c.}$$

For neutrinos

$$\frac{\mathcal{L}_D}{\sqrt{-\det g_{AB}}} = \sum_{S_+} Y_\nu^+(S_+) \tilde{H} S_+ \bar{L} \frac{1+\Gamma_7}{2} N + \sum_{S_-} Y_\nu^-(S_-) \tilde{H} S_- \bar{L} \frac{1-\Gamma_7}{2} N + \text{h.c.}$$

$$S_+ = \Phi^*,\ X^*, X^{*2}\Phi,\ \ldots$$
$$S_- = X^2,\ X\Phi,\ \Phi^2,\ \ldots$$

Example : for charge leptons, on a sphere:

$$3C\begin{pmatrix} \delta^4\beta/3 & E\delta^3/3\beta & \cancel{F\delta^2} \\ E'\delta^3/3 & \delta^2/2 & E\delta^2/2 \\ \cancel{G\delta^2} & E'\delta^2/2 & 1 \end{pmatrix}$$

β = geom coeff. → here 1,1 (≈ 1)

δ = vortex size → $\sim .1$ (.14 here)

E Yukawas off. diag. → 7

E' → 2

F, G } → 0

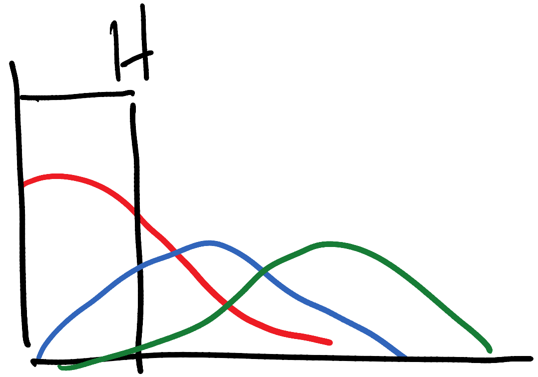
C = main Yukawa = 1.6

Neutrino matrix ;

$$\begin{pmatrix} 0 & 25 & -45.1 \\ 25 & -3 & 0 \\ -45.1 & 0 & 0 \end{pmatrix} \ \mathrm{meV}$$

rem: $|M_{22}/M_{13}|$ given by dynamics.

Results of this simple approximation:



$m_e \sim 6\ \mathrm{MeV}$

$m_\mu \sim 85\ \mathrm{MeV}$

$m_\tau \sim 1.7\ \mathrm{GeV}$

{-51.9358,51.2292,-2.29345}

$$U_{PMNS} = \begin{pmatrix} 0.78 & 0.62 & 0.1 \\ -0.47 & 0.68 & -0.57 \\ -0.42 & 0.40 & 0.82 \end{pmatrix}$$

$$m_\nu = \begin{pmatrix} |-51.9| \\ |51.2| \\ |-2.3| \end{pmatrix} \mathrm{meV}$$

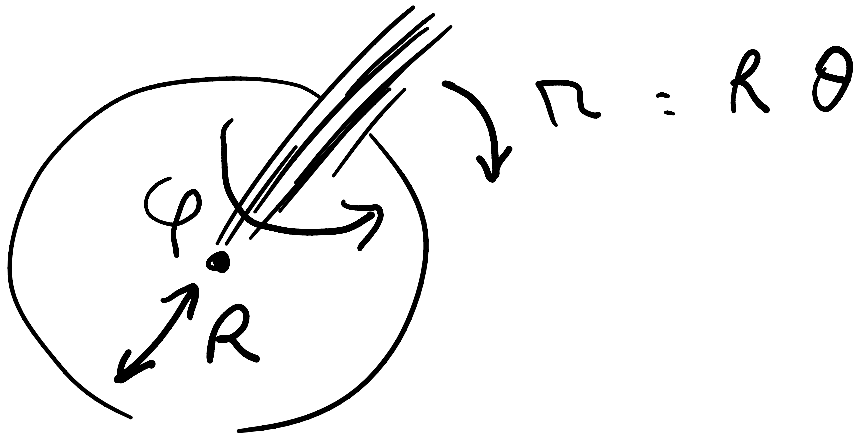
$$\langle m_{\beta\beta\,\mathrm{eff}} \rangle = 19.3\ \mathrm{meV}$$

Some other developments :

- compactification of the 2 extra dim on a sphere instead of a plane (avoid localisation of gauge bosons) – spinors modified, but conclusions kept (already mentioned) with extra scale 1/R

- phenomenological implications of the excited modes..

- constraint on B-E-H boson (Libanov and Nugaev: LIGHT)



IMPORTANT : « family number » (n) is approximatively conserved ! - $e^{in\phi}$ plays somewhat like a U(1) horizontal symmetry

2 extra dim : → ll gauge bosons, possess 2 types of Kaluza- Klein excitations in particular, Z and Gluons

Almost the same couplings as Z

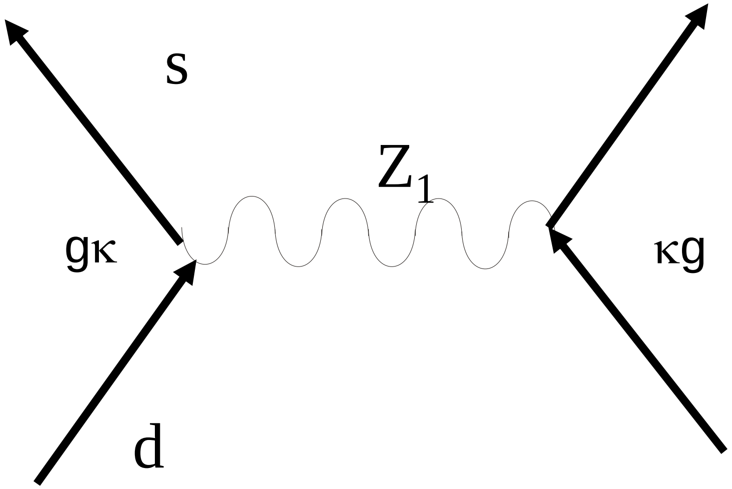
- radial Z'_0 (approx. flavour conserving) → From now on …

- angular : $Z'_{\pm 1}$ behaves like $e^{i\phi}$ → for later …

Flavour violating

$Z'_{\pm 1}$ thus carries « family number »

« family number » (n) is approximatively conserved ! - somewhat like U(1) horizontal symmetry $e^{i\phi}$



u e

c μ

$Z_{\S\, 1}$ thus carries « family number »

Flavour conserving

Flavour violating,

LIMITS

Family number conserving

→ LIMITS

Typical limit

$K_L \rightarrow \mu^- e^+$ or $\mu^+ e^-$ B.R. $< 10^{-12}$

Expect thus typical mass scale $M_{Z1} / \kappa > (10^{12})^{1/4} M_Z = \kappa\, 100$ TeV

In fact, the small overlap of wave functions implies

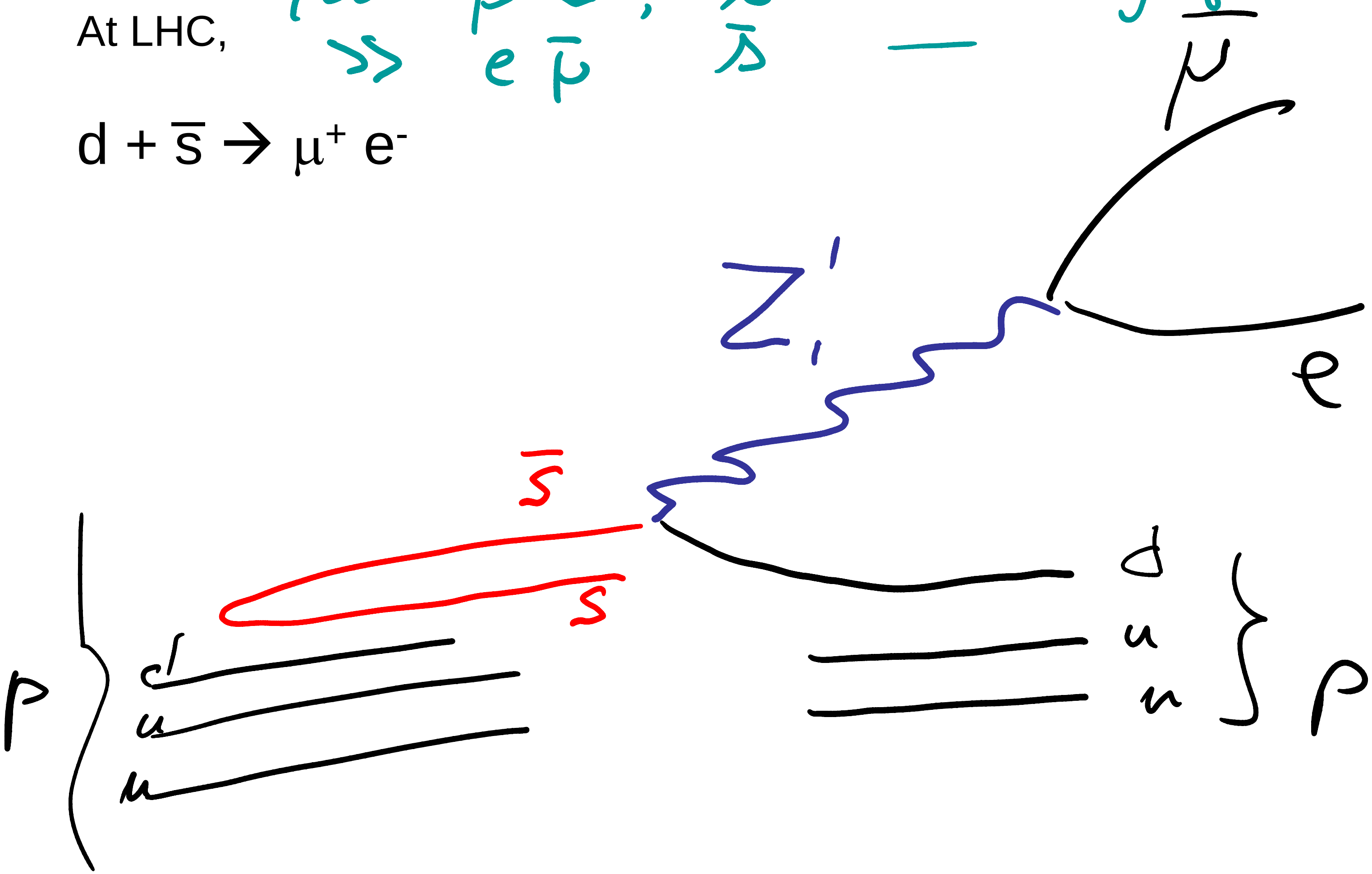
some suppression of the coupling; $\kappa \ll 1$

→ bound becomes $M(Z_1) > \kappa$ 100 TeV

→ Take κ from .01 to 0.5 → Plot for $M(Z_1)$>1TeV--

At LHC, Nes $\bar{\mu} e$, s in underlying event
$\gg$ $e \bar{\mu}$ $\bar{s}$ —

$d + \bar{s} \rightarrow \mu^+ e^-$



$$pp \longrightarrow \underbrace{\bar{\mu}\, e}_{\text{Nes.}} + s + X$$
$\hookrightarrow \Lambda, \bar{K}, \ldots$

We saturate the bound on κ

$\kappa = 100\ \mathrm{TeV}/M_{Z1}$

(100 fb^{-1}, 14TeV)

numbers for $\mu^- e^+$ are ONE ORDER below at LHC,due to quark content of protons

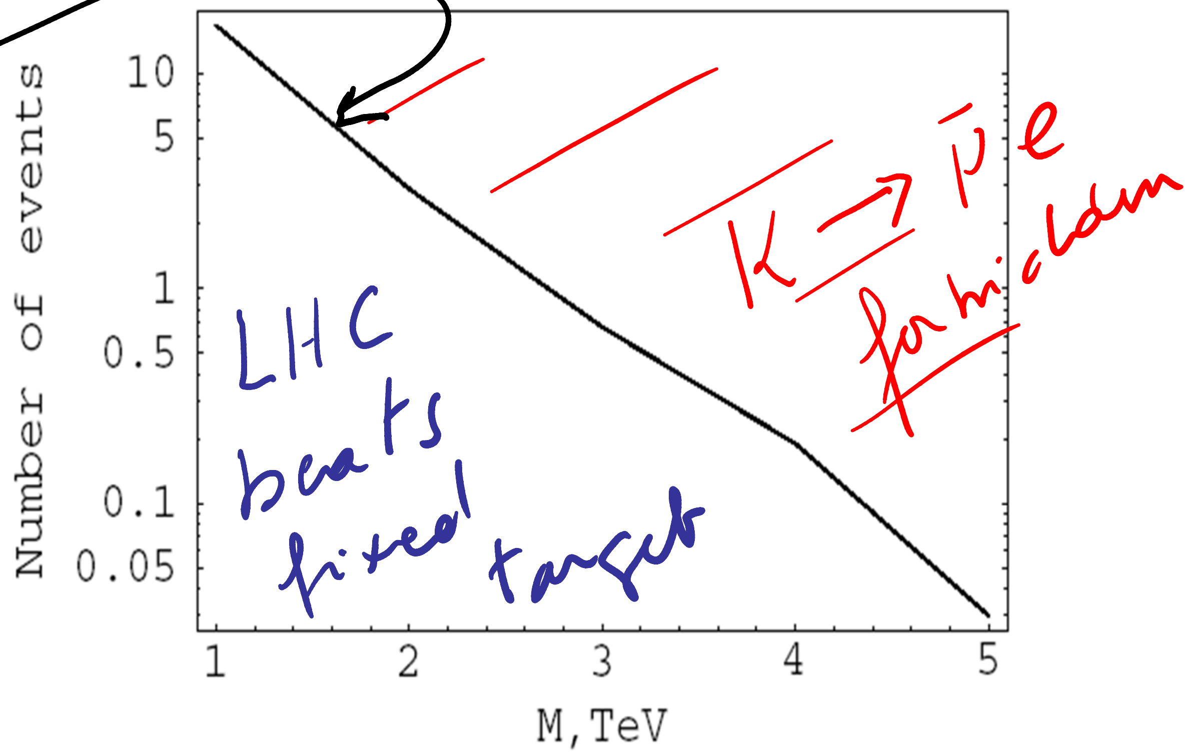


Fig. 1. Number of events for $(\mu^+ e^-)$ pairs production as a function of the vector bosons mass M, with $\kappa = M/(100\mathrm{TeV})$. (also s left in underlying event)

See.**JETP Lett.79:598-601,2004, Pisma Zh.Eksp.Teor.Fiz.79:734-737,2004**. JMF, M Libanov, S Troitsky, E Nugaev **hep-ph/0404139**

LHC thus has the potential (in a specific model, of course) to beat even the very sensitive fixed-target $K \rightarrow \mu\, e$ limit!

$\bar{t} + c$ or $\bar{b} + s$ are similarly produced by the **gluon excitations**,

Expect a **few 1000's events ---** but must consider background!

From now on …

$$M(Z'_0)= M(Z'\pm) > \kappa\ 100\ \text{TeV}$$
$$\kappa\ 0.01\ldots0.3$$

for later …

Find the Z'±,
κ suppressed

Find the Z'_0, W'_0,

…also expect gluon recurrences

An « ordinary » Z'
(with same
couplings as Z)

No κ suppression

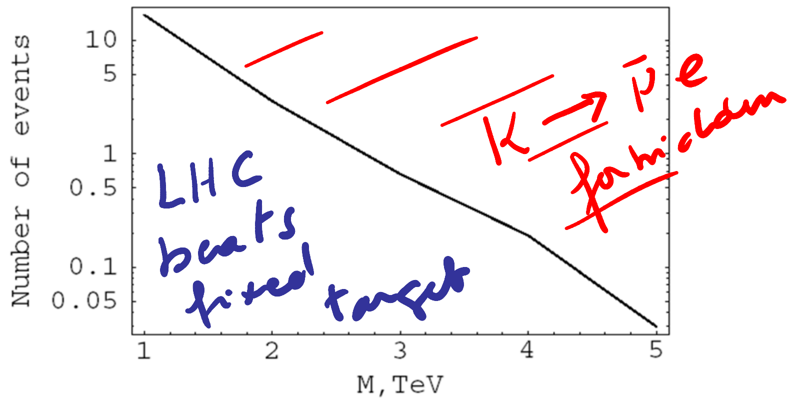


Fig. 1. Number of events for (μ^+e^-) pairs production as a function of the vector bosons mass M, with $\kappa = M/(100\text{TeV})$.

(100 fb^{-1}, 14TeV)

In a nutshell:

- One family in 6D and proper boundary conditions → 3 families in 6D
- At lowest order in Cabibbo mixing , Charged fermion masses are
  - diagonal
  - strongly hierarchical

At LHC, this can result in exotic signals ($Z' \rightarrow \mu^+ e^- \gg \mu^- e^+$)

- At same order, we get 4D Majorana neutrinos with
  - Antidiagonal mass matrix
- This yields, in a generic way:
  - Large mixings in the neutrino sector
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  - Pseudo- Dirac structure (further suppression of neutrinoless double beta decay)
- Not as automatic, but typical : measurable Θ_{13}