THE UNIVERSITY OF CHICAGO

The Charge Radius of the Proton

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Enrico Fermi Institute, The University of Chicago

Based on Richard J. Hill, GP

PRD **82** 113005 (2010) [arXiv:1008.4619]

and *in preparation*

Outline

- Introduction: 5σ discrepancy
- Comparison of extractions methods
- Model independent extraction
- Conclusions and outlook

Introduction: 5σ discrepancy

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^N(q^2)q_\nu\right]u(p_i)$$

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- Sachs electric and magnetic form factors ($t = q^2 = -Q^2$)

$$G_E^N(t) = F_1^N(t) + \frac{t}{4m_N^2}F_2^N(t)\,, \quad G_M^N(t) = F_1^N(t) + F_2^N(t)\,.$$

$$G_E^p(0) = 1,\ G_E^n(0) = 0,\ G_M^p(0) = \mu_p \approx 2.793,\ G_M^n(0) = \mu_n \approx -1.913$$

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- The slope of G_E^p

$$\langle r^2\rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0} \quad \text{or} \quad G_E^p(q^2) = 1 + \frac{q^2}{6}\langle r^2\rangle_E^p + \ldots,$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

Charge radius from atomic physics

- $G_E^p(t)$ and $G_M^p(t)$: input for precision QED observables for bound proton lepton systems

$$\langle p(p_f)|\sum_q e_q\,\bar{q}\gamma^\mu q|p(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2^p(q^2)q_\nu\right]u(p_i)$$

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- For a charged point particle: $F_1(0) = 1$ and $F_2(0) = 0$
  Amplitude for $p + \ell \to p + \ell$

$$i\mathcal{M} \approx \frac{ie_\ell\, e_p}{q^2} \chi_p^\dagger \chi_p \chi_\ell^\dagger \chi_\ell \quad \Rightarrow \quad U(r) = -Z\alpha/r$$

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- Including q^2 corrections

$$i\mathcal{M} \approx \frac{ie_\ell\, e_p}{q^2}q^2\left[F_1(0)\left(\frac{1}{8m_p^2}+\frac{1}{8m_\ell^2}\right) + \left.\frac{dF_1^p}{dq^2}\right|_{q^2=0} + \frac{F_2^p(0)}{4m_p^2}\right]\chi_p^\dagger\chi_p\chi_\ell^\dagger\chi_\ell$$

- Proton structure corrections

$$U(r) = 4\pi Z\alpha\,\delta^3(r)\left(\left.\frac{dF_1^p}{dq^2}\right|_{q^2=0} + \frac{F_2^p(0)}{4m_p^2}\right) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

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- The change in the energy $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell \right)$

$$\begin{aligned} \Delta E_{r_E^p} &= \int d^3r\, \psi(r)^\dagger\, U(r)\, \psi(r) = \frac{2\pi Z\alpha}{3} (r_E^p)^2 |\psi(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell\, 0} \end{aligned}$$

- Charge radius effects $\propto m_r^3$

Charge radius from Classic Lamb shift

- For electronic hydrogen: measured value Lunden and Pipkin '86

$$E_{2s} - E_{2p_{1/2}} = 1.057845(9)\,\mathrm{GHz} = 0.00437490(4)\,\mathrm{meV}$$

compared to

$$\Delta E_{r_E^p} = 0.0000008\,(r_E^p)^2\frac{\mathrm{meV}}{\mathrm{fm}^2}$$

Proton radius effects at a level of 10^{-4}
Experimental uncertainty at a level of 10^{-5}

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$$E_{2s} - E_{2p_{1/2}} \approx -205\,\mathrm{meV}$$

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- *Muonic hydrogen can potentially give the best measurement of r_E^p!*

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  - $e - p$: hydrogen
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  - Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p} = 0.84184(67)$ fm
  - CODATA value [Mohr et al. arXiv:0801.0028]
    $r_E^p = 0.8768(69)$ fm
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- **5σ discrepancy!**
- We can also extract it from electron-proton scattering data

  What does PDG say?

What does the PDG say?

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	DOCUMENT ID		TECN	COMMENT
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$ep \to ep$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$ep \to ep$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		ep + Coul. corrections
0.847 ±0.008	MERGELL	96		ep + disp. relations

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

0.877 ±0.024	WONG	94		reanalysis of Mainz ep data
0.865 ±0.020	MCCORD	91		$ep \to ep$
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0.800 ±0.025	FREREJACQ...	66		$ep \to ep$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$ep \to ep$

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Comparison of extractions methods

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  Work in recent years on combining data sets
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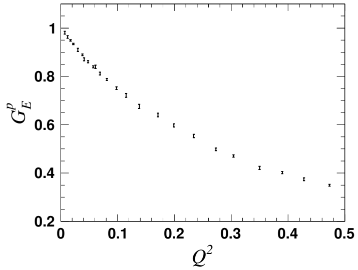

Data from [Arrington et al. arXiv:0707.1861]

- We don't know the functional form of G_E^p

How to extract r_E^p?

- How to extract r_E^p from G_E^p? Usually use either

  1) model dependent form for G_E^p, e.g. poles+continuum form

     problem: how to estimate model dependance?

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z expansion

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  $G_E^p(t)$ is analytic outside a cut $t \in [4m_\pi^2, \infty]$
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- We can map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

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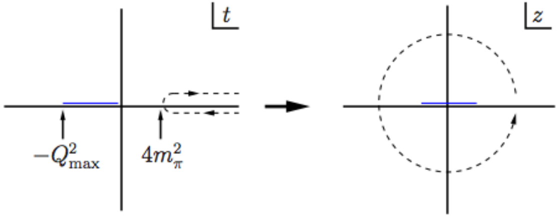

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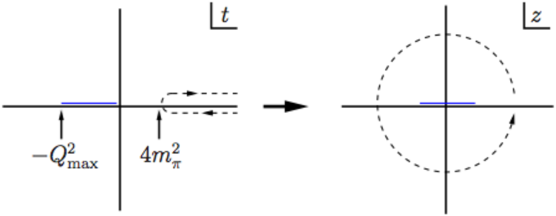
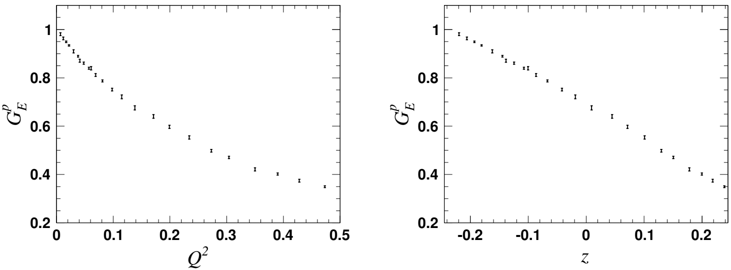


- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

z expansion

- The curvature is smaller in the z variable



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z expansion

- Standard tool in analyzing **meson** transition form factors
  - Bourrely et al. NPB **189**, 157 (1981)
  - Boyd et al. arXiv:hep-ph/9412324
  - Boyd et al. arXiv:hep-ph/9508211
  - Lellouch arXiv:hep-ph/9509358
  - Caprini et al. arXiv:hep-ph/9712417
  - Arnesen et al. arXiv:hep-ph/0504209
  - Becher et al. arXiv:hep-ph/0509090
  - Hill arXiv:hep-ph/0606023
  - Hill arXiv:hep-ph/0607108
  - Bourrely et al. arXiv:0807.2722 [hep-ph]
  - Bharucha et al. arXiv:1004.3249 [hep-ph]
  - ...
- Not applied to **nucleon** form factors

Comparison of series expansions

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4) z expansion with a constraint on a_k: $|a_k| \le 10$

Comparison of series expansions

r_E^p in $10^{-18} m$

polynomial

continued fraction

z expansion (no bound)

z expansion ($|a_k| \leq 10$)

Comparison of series expansions

	r_E^p in $10^{-18} m$
	$k_{\max} = 1$
polynomial	836^{+8}_{-9}
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Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\rm max} = 1$	2
polynomial	836^{+8}_{-9}	867^{+23}_{-24}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}
z expansion ($\lvert a_k\rvert \le 10$)	918^{+9}_{-9}	868^{+28}_{-29}

Comparison of series expansions

r_E^p in $10^{-18}m$

	$k_{\max} = 1$	2	3
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}

Comparison of series expansions

r_E^p in $10^{-18} m$

	$k_{\max} = 1$	2	3	4
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}
continued fraction	882^{+10}_{-10}	869^{+26}_{-25}	–	–
z expansion (no bound)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+64}_{-69}	1022^{+102}_{-114}
z expansion ($\lvert a_k \rvert \leq 10$)	918^{+9}_{-9}	868^{+28}_{-29}	879^{+38}_{-59}	880^{+39}_{-61}

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	$k_{\max} = 1$	2	3	4	5
polynomial	836^{+8}_{-9}	867^{+23}_{-24}	866^{+52}_{-56}	959^{+85}_{-93}	1122^{+122}_{-137}
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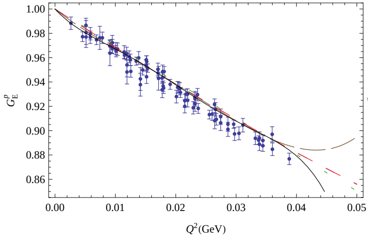
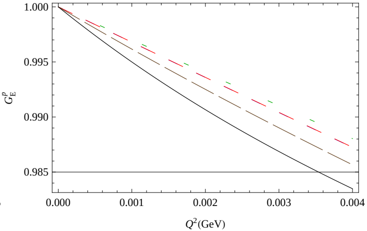
Conclusions:

- Fits with two parameters agree well
- As we increase $k_{\max}$ the errors for the first three fits grow
- For the continued fraction fit for $k_{\max} > 3$ the slope is not positive
- To get a meaningful answer we must constrain a_k. How?

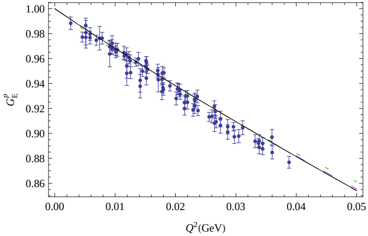
Comparison of Taylor and constrained z fits

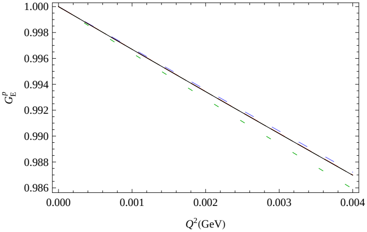
- Taylor fit





- Constrained z fit





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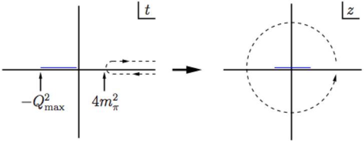
Model Independent Extraction

Analytic structure and a_k

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



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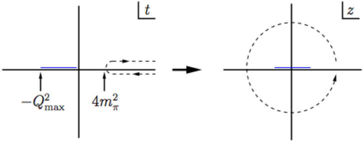

- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k
- $G(t) = \sum_{k=0}^{\infty} a_k\, z(t)^k$, z^k are orthogonal over $|z| = 1$

$$\begin{aligned} a_0 &= G(t_0) \\ a_k &= \frac{2}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}}\, \mathrm{Im}\, G(t) \sin[k\theta(t)]\,, \quad k \geq 1 \\ \sum_k a_k^2 &= \frac{1}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} |G|^2 \end{aligned}$$

- How to constrain $\mathrm{Im}\, G(t)$?

Size of a_k: vector dominance ansatz

- The isovector and isoscalar form factors are

$$G_E^{(0)} = G_E^p + G_E^n\,, \quad G_E^{(1)} = G_E^p - G_E^n$$

- Assume vector dominance anstaz [Hohler NPB **95**, 210 (1975)]

$$F_i^{(I=0)} \sim \frac{\alpha_i m_\omega^2}{m_\omega^2 - t - i\Gamma_\omega m_\omega}\,, \quad F_i^{(I=1)} \sim \frac{\beta_i m_\rho^2}{m_\rho^2 - t - i\Gamma_\rho m_\rho}\,,$$

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- Taking $t_0 = 0$ we find
  $\sum_k a_k^2/a_0^2 \approx 6$ for $G_E^{(1)}$, $\sum_k a_k^2/a_0^2 \approx 58$ for $G_E^{(0)}$

- Since ω is narrow: $\Gamma_\omega/m_\omega \approx 1\% \Rightarrow \sum_k a_k^2/a_0^2$ is large
  In fact for an infinitely narrow pole, it diverges!

Size of a_k: Vector dominance ansatz

- Recall

$$
\begin{aligned}
a_0 &= G(t_0) \\
a_k &= \frac{2}{\pi}\int_{t_{\rm cut}}^{\infty} \frac{dt}{t-t_0}\sqrt{\frac{t_{\rm cut}-t_0}{t-t_{\rm cut}}}\,{\rm Im}\,G(t)\sin[k\theta(t)]\,, \quad k\geq 1 \\
\sum_k a_k^2 &= \frac{1}{\pi}\int_{t_{\rm cut}}^{\infty} \frac{dt}{t-t_0}\sqrt{\frac{t_{\rm cut}-t_0}{t-t_{\rm cut}}}|G|^2
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- For $G(t)=1/(t-m_V^2)$, $\sum_k a_k^2/a_0^2$ diverges!

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- But ${\rm Im}\,G(t+i0) = -i\pi\delta(t-m_V^2)$
  $\Rightarrow |a_k/a_0| \leq 2\sqrt{(t_{\rm cut}-t_0)/(m_v^2-t_{\rm cut})}$
  Taking $t_0=0$: $|a_k|<1.3$ for $G_E^{(0)}$,$|a_k|<0.78$ for $G_E^{(1)}$

- Conclusion: $|a_k|\leq 10$ is a very conservative estimate for this ansatz

Size of a_k: $\pi\pi$ continuum

- $\pi\,\pi$ is the lightest state that can contribute to $\mathrm{Im}\, G_E^{(1)}$

$$\mathrm{Im}\, G_E^{(1)}(t) = \frac{2}{m_N \sqrt{t}} \left(t/4 - m_\pi^2\right)^{\frac{3}{2}} F_\pi(t)^* f_+^1(t)$$

$F_\pi(t)$ pion form factor, $f_+^1(t)$ is a partial amplitude for $\pi\pi \to N\bar{N}$ [Federbush et al. Phys. Rev. **112**, 642 (1958), Frazer et al. Phys. Rev. **117**, 1609 (1960), Belushkin et al. arXiv:hep-ph/0608337]

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- Since they share the same phase up to $t < 16m_\pi^2$, we can use $|F_\pi|$ (We will assume phase equality through ρ peak)
- Using $|F_\pi(t)|$ data from
  - NA7 experiment [Amendolia et al. PLB **138**, 454 (1984)]
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- For $t_0 = 0$: $a_0 \approx 2.1$ $a_1 \approx -1.4$, $a_2 \approx -1.6$, $a_3 \approx -0.9$, $a_4 \approx 0.2$ Using $|\sin(k\theta)| \leq 1$ in the integral gives $|a_k| \lesssim 2.0$ for $k \geq 1$.

Size of a_k: $t > 4m_N^2$ region

- For the region $t > 4m_N^2$ we can use $e^+e^- \to N\bar{N}$ data, e.g.
  - $p - \bar{p}$: BES collaboration [Ablikim et al. arXiv:hep-ex/0506059]
  - $n - \bar{n}$: FENICE experiment [Antonelli et al. NPB **517**, 3 (1998)]

- We find a very small contribution from this region
  - $|\delta a_k| \lesssim 0.006 + 0.002$ for the proton
  - $|\delta a_k| \lesssim 0.013 + 0.025$ for the neutron

Size of a_k: Summary

- In all of the above $|a_k| \leq 10$ appears very conservative

Size of a_k: Summary

- In all of the above $|a_k| \leq 10$ appears very conservative
- In practice we will find $|a_k| \sim 2$
- Final results are presented for both $|a_k| \leq 5$ and $|a_k| \leq 10$

Results: Low Q^2 data

- Using low $Q^2 < 0.04\,\mathrm{GeV}^2$ data [Rosenfelder arXiv:nucl-th/9912031]

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

First error assuming $|a_k| \leq 5$, first+second $|a_k| \leq 10$

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- For now explore ways of reducing the error by including:
  - High Q^2 data
  - proton and neutron data
  - proton, neutron and $\pi\,\pi$ data

Results: Low+High Q^2 data

- To reduce the error include both low an high Q^2 data
  Use tables from [Arrington et al. arXiv:0707.1861]
  We fit with $k_{\rm max} = 10$, $t_0 = 0$, $|a_k| \leq 10$

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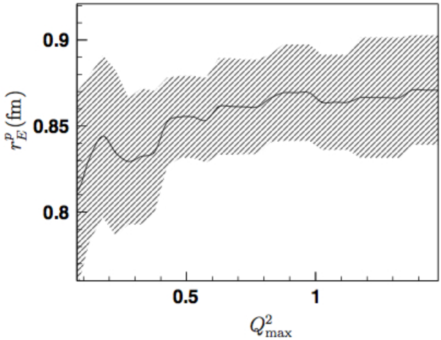

- Beyond $Q^2 \gtrsim \mathrm{few} \times 0.1\,\mathrm{GeV}^2$ the impact of additional data is minimal

$$\text{For } Q^2_{\max} = 0.5\,\mathrm{GeV}^2 : \quad r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

Results: Proton and Neutron data

- Including neutron data $\Rightarrow$ fit $G_E^{(0)}$ and $G_E^{(1)}$ separately
  For isoscalar $t_{\rm cut} = 9m_\pi^2 \Rightarrow$ smaller value of $|z|_{\rm max}$
- Using
  - G_E^p up to $Q^2_{\rm max} = 0.5\,{\rm GeV}^2$
  - 20 data points for G_E^n
  - Neutron charge radius from [PDG 2010]

$$\langle r^2 \rangle_E^n = -0.1161(22)\,{\rm fm}^2\,.$$

- We get

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,{\rm fm}$$

Results: Proton, Neutron and $\pi\,\pi$ data

- $\pi\,\pi$ data allows us to set $t_{\rm cut} = 16 m_\pi^2$ for $G_E^{(1)}$

$$G_E^{(1)}(t) = G_{\rm cut}(t) + \sum_k a_k z^k(t, t_{\rm cut} = 16 m_\pi^2, t_0)$$

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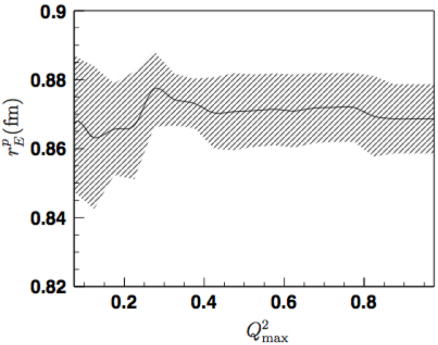

- We get: $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

  Last error 30% normalization for $f_1^+(t)$

Results: Summary

- Proton low: $Q^2 < 0.04\,\mathrm{GeV}^2$

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

- Proton high: $Q^2 < 0.5\,\mathrm{GeV}^2$

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Comparison to the Literature: PDG table

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

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0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

- We will consider several highly cited extractions

- [Rosenfelder arXiv:nucl-th/9912031] : $r_E^p = 0.880 \pm 0.015\,\mathrm{fm}$
  [Simon et al. NPA **333**, 381 (1980)] $r_E^p = 0.862 \pm 0.012\,\mathrm{fm}$

- [Sick arXiv:nucl-ex/0310008]] : $r_E^p = 0.895 \pm 0.010 \pm 0.013\,\mathrm{fm}$
  [Blunden et al. arXiv:nucl-th/0508037] $r_E^p = 0.897 \pm 0.018\,\mathrm{fm}$

Comparison to the Literature

- Using low $Q^2 < 0.04\,\mathrm{GeV}^2$ data we find

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,\mathrm{fm}$$

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- Rosenfelder used a taylor series

$$G_E^p(q^2) = 1 + a_1 \frac{q^2}{t_\mathrm{cut}} + a_2 \left(\frac{q^2}{t_\mathrm{cut}}\right)^2 + \dots$$

but a_2 was not fitted, instead it was taken from higher Q^2 data [Borkowski et al. NPA **222**, 269 (1974)]: $a_2^\mathrm{Taylor}/t_\mathrm{cut}^2 = 0.014(4)\,\mathrm{fm}^4$ (similar procedure was used in [Simon et al. NPA **333**, 381 (1980)])

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- Using this value we find

$$r_E^p = 0.878 \pm 0.008^{+0.047}_{-0.039} \quad \textbf{Taylor}$$

errors are from data and $a_2^{\rm Taylor}/t_{\rm cut}^2$ only

- Compatible with

$$r_E^p = 0.877^{+0.031}_{-0.049} \pm 0.011\,{\rm fm} \quad \textbf{z expansion}$$

Comparison to the Literature

- Using the continued fraction expansion
  Sick and Blunden and Sick have found

[Sick arXiv:nucl-ex/0310008]] : $r_E^p = 0.895 \pm 0.010 \pm 0.013\,\mathrm{fm}$
[Blunden et al. arXiv:nucl-th/0508037] $r_E^p = 0.897 \pm 0.018\,\mathrm{fm}$

- Their error estimate relies on model datasets

- We find the expansion becomes unstable
  when including more then 2 parameters

Comparison to the Literature: Summary

- Previous studies have underestimated the error on r_E^p
- "Race to the bottom":
  It's very important to have *reliable* error estimates
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  There seems to be a push to get a smaller error by all means
- Case in point: New result from A1 experiment
  [Bernauer et al. arXiv:1007.5076 [nucl-ex]]

For the spline group we obtain the values

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.875(5)_{\rm stat.}(4)_{\rm syst.}(2)_{\rm model}\,{\rm fm},$$
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.775(12)_{\rm stat.}(9)_{\rm syst.}(4)_{\rm model}\,{\rm fm}$$

and for the polynomial group

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.883(5)_{\rm stat.}(5)_{\rm syst.}(3)_{\rm model}\,{\rm fm},$$
$$\langle r_M^2 \rangle^{\frac{1}{2}} = 0.778(^{+14}_{-15})_{\rm stat.}(10)_{\rm syst.}(6)_{\rm model}\,{\rm fm}.$$

Despite detailed studies the cause of the difference between the two model groups could not be found. Therefore, we give as the final result the average of the two values with an additional uncertainty of half of the difference

$$\langle r_E^2 \rangle^{\frac{1}{2}} = 0.879(5)_{\rm stat.}(4)_{\rm syst.}(2)_{\rm model}(4)_{\rm group}\,{\rm fm},$$
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Future Directions

- The z expansion can be applied to other data sets and also to fits of cross sections

- Can be applied to other nucleon form factors $G_M^{p,n}$, The axial-vector form factor F_A
[Bhattacharya, Hill, GP *in preparation*]

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- The z expansion can be applied to other data sets and also to fits of cross sections
- Can be applied to other nucleon form factors $G_M^{p,n}$, The axial-vector form factor F_A [Bhattacharya, Hill, GP *in preparation*]
- But wait, what about the 5σ discrepancy ?

The recent discrepancy

- The recent discrepancy:
  - Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p} = 0.84184(67)$ fm
  - CODATA value [Mohr et al. arXiv:0801.0028]
    $r_E^p = 0.8768(69)$ fm
    extracted mainly from (electronic) hydrogen
- Our results
  - Proton data only

$$r_E^p = 0.870 \pm 0.023 \pm 0.012 \, \mathrm{fm}$$

  - Proton, neutron and $\pi\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \, \mathrm{fm}$$

are more consistent with the CODATA value

Lamb shift in muonic hydrogen

- PSI group measured for the *first time*
  the Lamb shift in muonic hydrogen
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  but unfortunately they need to relay on theorists to extract r_E^p...
- They measured [Pohl et al. Nature **466**, 213 (2010)]

$$\Delta E = 206.2949 \pm 0.0032 \text{ meV}$$

- Comparing to the theoretical expression
  [Pachucki PRA **60**, 3593 (1999), Borie PRA **71**(3), 032508 (2005)]

$$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3 \text{ meV}$$

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$$r_E^p = 0.84184(67) \text{ fm}$$

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- But the theoretical expression should be

  [Friar Annals Phys. **122**, 151 (1979),

  Eides et al. Theory of Light Hydrogenic Bound states, Springer]

  $$\Delta E = 209.9779(49) - 5.2262(r_E^p)^2 + 0.00913 \langle r^3 \rangle_{(2)} \text{ meV}$$

  where $\langle r^3 \rangle_{(2)}$ is the third Zemach moment

- [Zemach Phys. Rev. **104**, 1771 (1956)]

  $$\langle r^3 \rangle_{(2)} \equiv \int d^3r\, d^3s\, \rho(r)\rho(s)|r - s|^3$$

  ρ electric charge distribution

Third Zemach Moment

- In the Breit frame G_E^p is the Fourier transform of ρ

$$
\begin{aligned}
(r_E^p)^2 &= \int d^3r\, \rho(r)|r|^2 \\
\langle r^3\rangle_{(2)} &\equiv \int d^3r\, d^3s\, \rho(r)\rho(s)|r-s|^3
\end{aligned}
$$

- If we **assume** one parameter model for G_E^p
  the two parameters are related, otherwise they are not
- The correct formula for the Lamb shift has two unknowns!
  $\Rightarrow$ use the CODATA value of r_E^p and solve for $\langle r^3\rangle_{(2)}$

Third Zemach Moment

- The result [De Rújula arXiv:1008.3861]

$$[\langle r^3 \rangle_{(2)}]^{1/3} = 3.32 \pm 0.21 \text{ fm} \quad \textbf{muonic hydrogen}$$

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  Can investigate using the z expansion!

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  [Hill, GP *in preparation*]
- Formula for ΔE relies on loop diagrams with propagating proton

  Is that reliable?

  [Hill, GP *in preparation*]

New Physics?

- It is possible that the discrepancy is due to New Physics...
- "Dark photon" coupling to the photon via kinetic mixing

  Jaecakel, Roy [arXiv:1008.3536v2]

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- Other explanations?

Conclusion and Outlook

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- Recent discrepancy in the extraction the proton charge radius between muonic and regular hydrogen

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- We presented model independent extraction of the charge radius from $e - p$ scattering data using the z expansion

  - $r_E^p = 0.870 \pm 0.023 \pm 0.012$ fm using just proton scattering data
  - $r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$ adding neutron data
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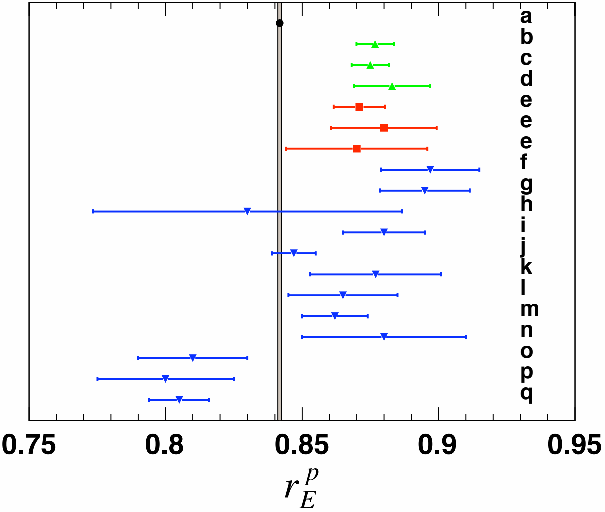
  - $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$ adding $\pi\pi$ data
- Previous extractions seem to have underestimated the error
- The results are compatible with CODATA value of $r_E^p = 0.8768(69)$ fm
- Discrepancy might be due to higher correlations of proton charge distribution



muonic hydrogen (circle and vertical band)

electronic hydrogen (green triangles)

electron scattering employing the z expansions (red squares)

previous electron scattering extractions (blue downward triangles)

Future Directions

- Applying z expansion to the magnetic and axial-vector form-factors
- Model independent extraction of the third Zemach moment

  from $e - p$ scattering data
- A model independent analysis using NRQED
- Resolution of the discrepancy?