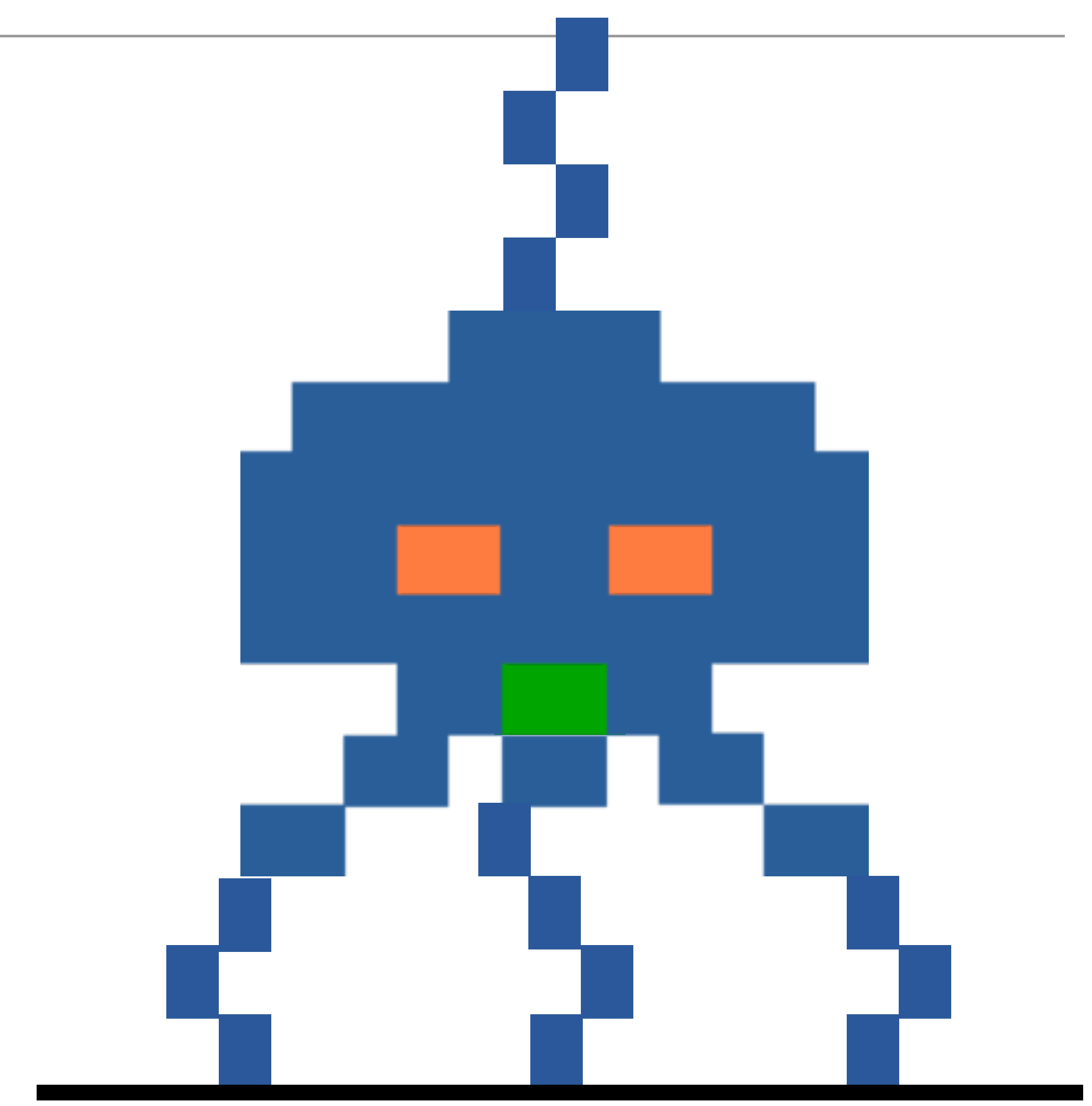
Report on

Workshop on the Hadronic Light-by-light Contribution to the Muon Anomaly $(g-2)$

held @ Institute for Nuclear Theory, University of Washington, Seattle

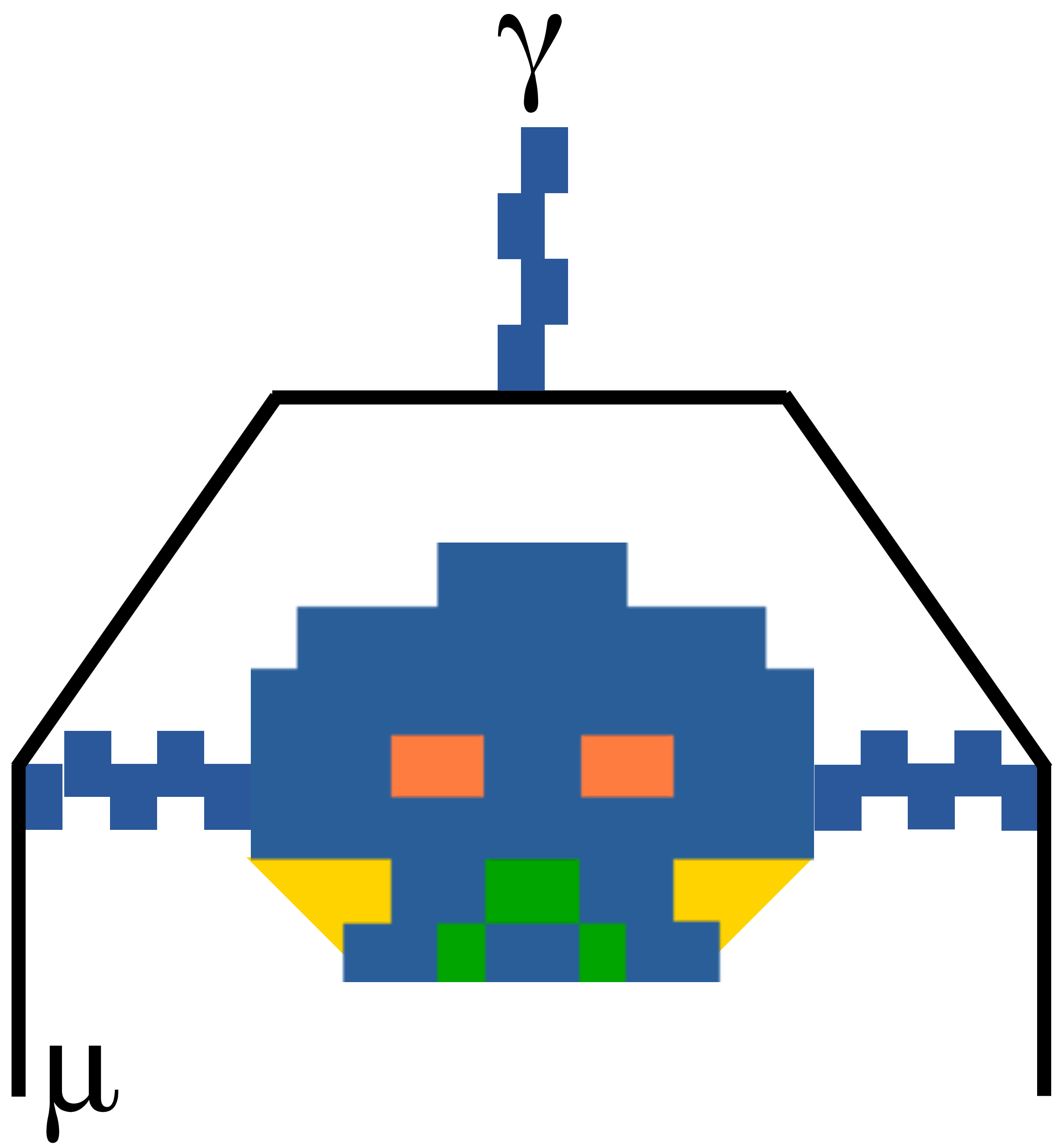


Andreas S. Kronfeld
Fermilab
10 March 2011
Fermilab Theoretical Physics Seminar

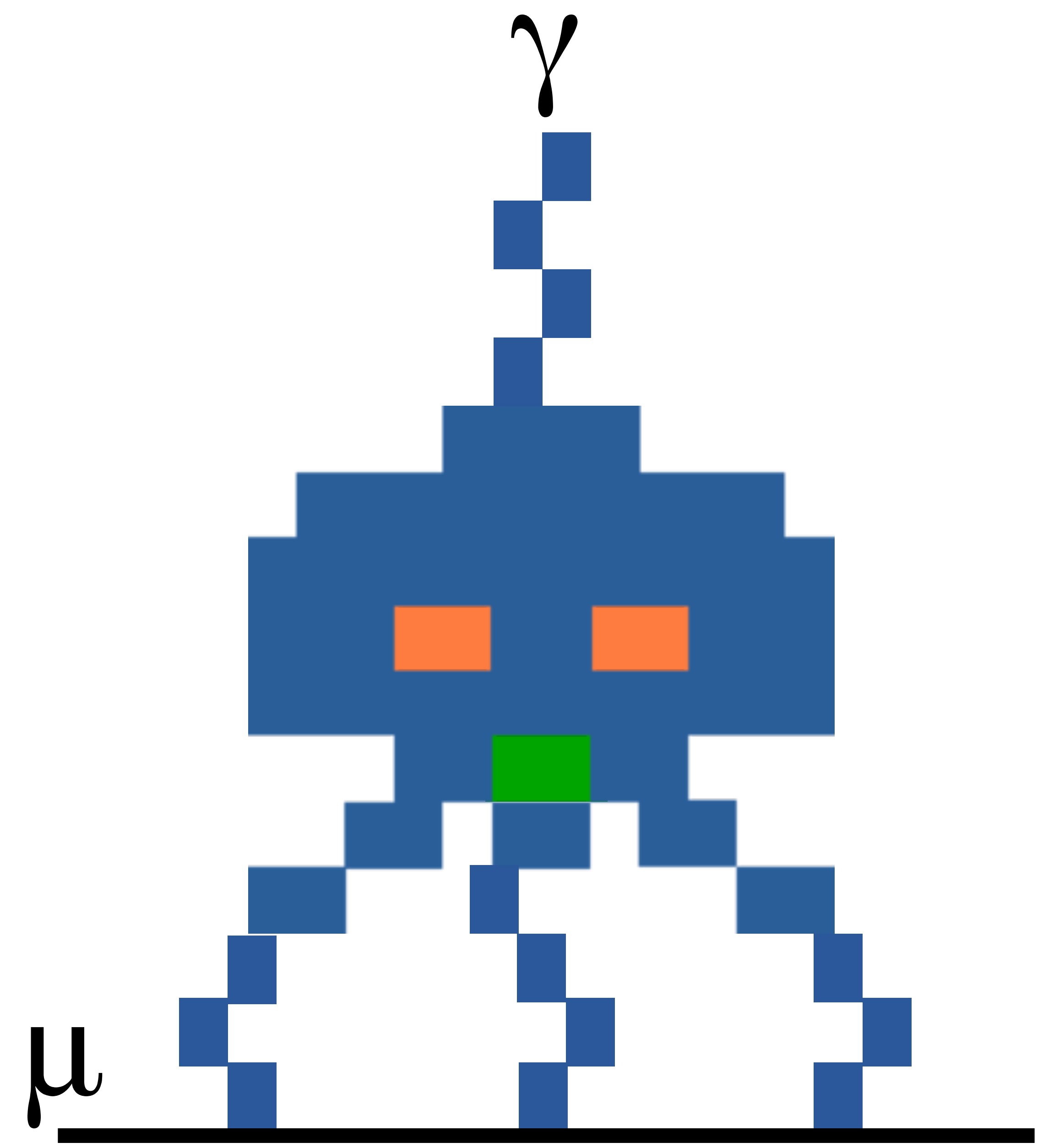
Not Your Usual Seminar

- I went to this workshop to learn more about the Standard Model theory of muon $(g-2)$.
- As a BSM curmudgeon, I haven't taken the famous "discrepancy" too seriously:
  - on the one hand, the discrepancy is evidence for susy; yet, on the other, ...
  - … the agreement provides a strong constraint on susy [Bechtle *et al.*, arXiv:0907.2589].
- Still possible for me to learn a lot about QCD in one week (but I still know less than Bill).
- Barring Tea Party effects, the BNL apparatus is coming here for a new experiment.
- The workshop was on hadronic light-by-light, but hadronic vacuum polarization matters too.

Feynman Diagrams as Space Invaders



Hadronic vacuum polarization



Hadronic light-by-light

Outline

- Experiments (at BNL & Fermilab) in a nutshell
- Beyond the Standard Model
- Some basics of the theory
- Models of QCD
- Data-driven estimates
- Prospects with lattice gauge theory
- Perspectives

The Muon $(g-2)$ Collaboration

from BNL E821 to FNAL E989

BNL E821

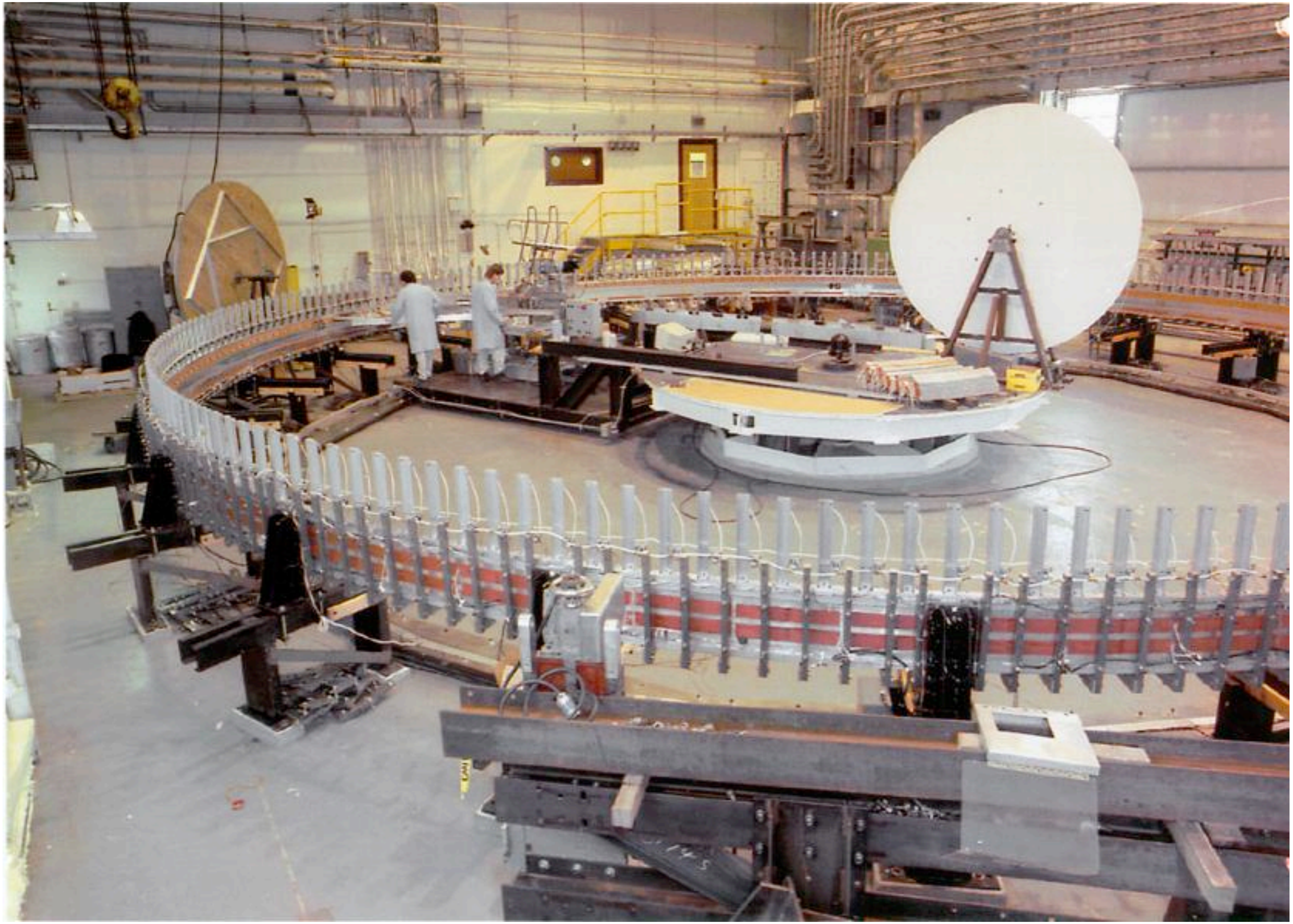
B. Lee Roberts

- Inject longitudinally polarized muons into storage ring and measure spin precession:

$$\omega_a = \omega_s - \omega_c = \mp \frac{e}{m}\left[\underbrace{\frac{g-2}{2}B}_{\text{bend}} - \left(\frac{g-2}{2} - \frac{1}{\gamma^2-1}\right)\underbrace{v \times E}_{\text{focus}}\right]$$

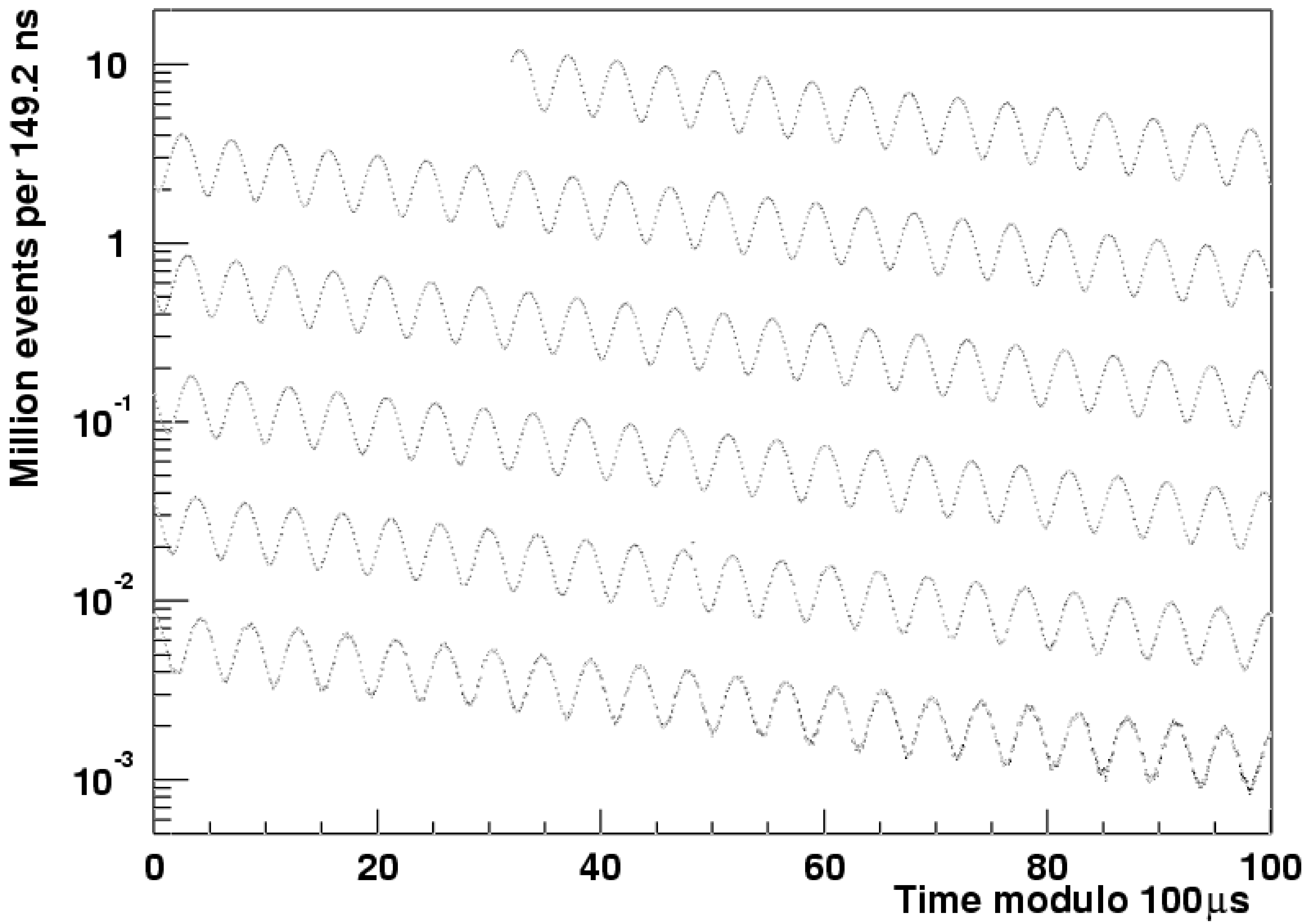
where $\omega_{s(c)}$ is spin (cyclotron) angular frequency. Forthwith, $a_\mu = (g - 2)/2$.

- Electron energy distribution correlated with muon spin s:
  - measure number of electrons above some energy threshold.
- Measure B field early and often.
- Choose "magic" muon momentum so that electric term drops out (*i.e.*, is really, really small).



3.6 billion μ^- decays

G.W. Bennett *et al.* [Muon (g–2) Collaboration], hep-ex/0602035



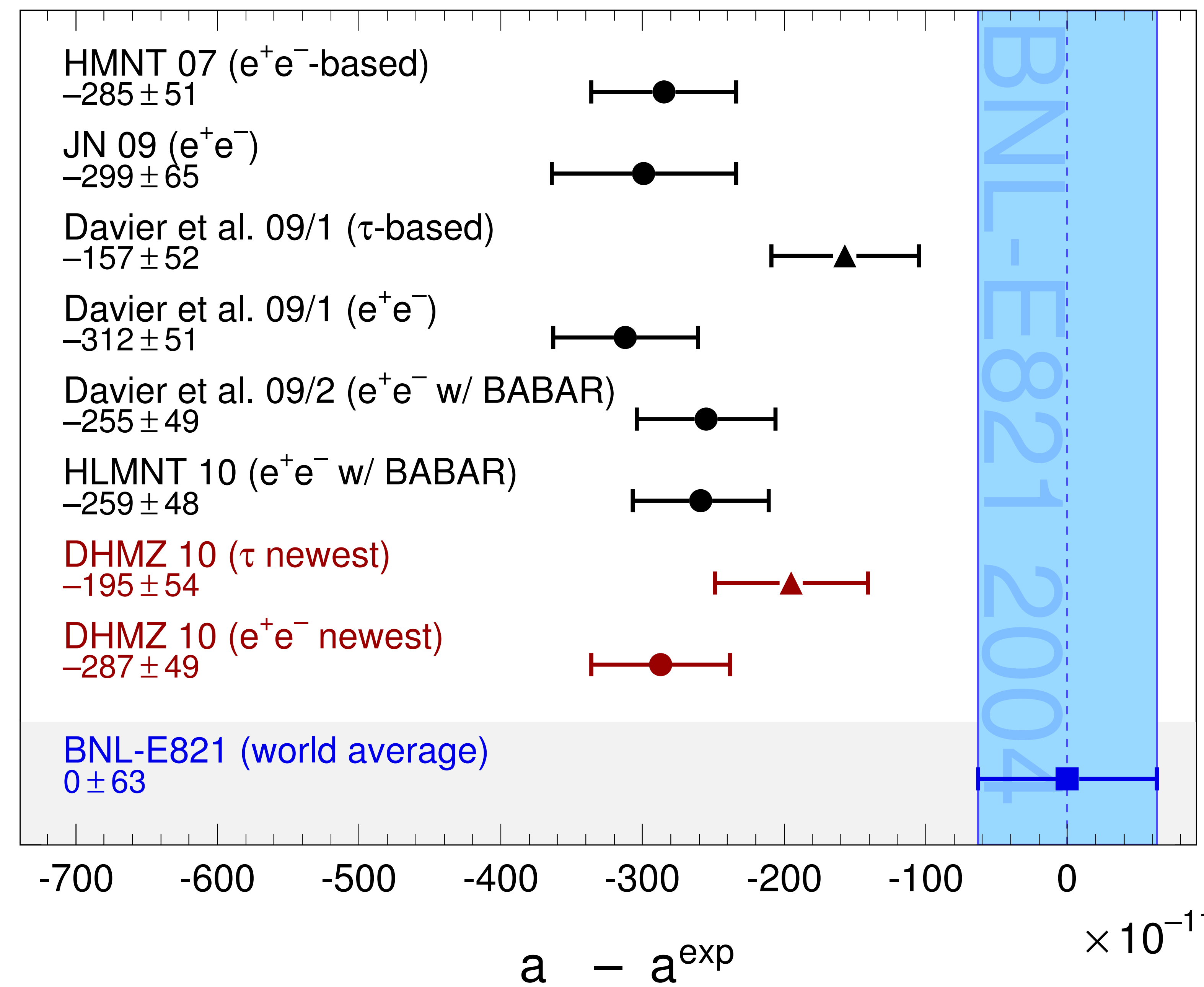
Fermilab E989

(verbatim) B. Lee Roberts

- Relocate the $(g-2)$ storage ring to Fermilab;
- Use the many proton storage rings to form the ideal proton beam;
- Use one of the antiproton rings as a 900 m decay line to produce a pure muon beam;
- Accumulate 21 times the statistics;
- Improve the systematic errors;
- Final goal: at least a factor of 4 more precise over E821;
- 2010 Christmas present.

Results and Forecasts for a_μ

how	$10^{11}a_\mu$	$10^{11}\times$error
E821 μ^+	116 592 03–	90
E821 μ^-	116 592 14–	90
E821 $\mu^\pm$	116 592 080	63
SM(τ)	116 591 894	54
SM(e^+e^-)	116 591 802	49
HVP (lo)	6 923	42
HL×L	105	26
E989 μ^+	116 59– ——	**16**



- SM values and compilation from Andreas Höcker, arXiv:1012.0055

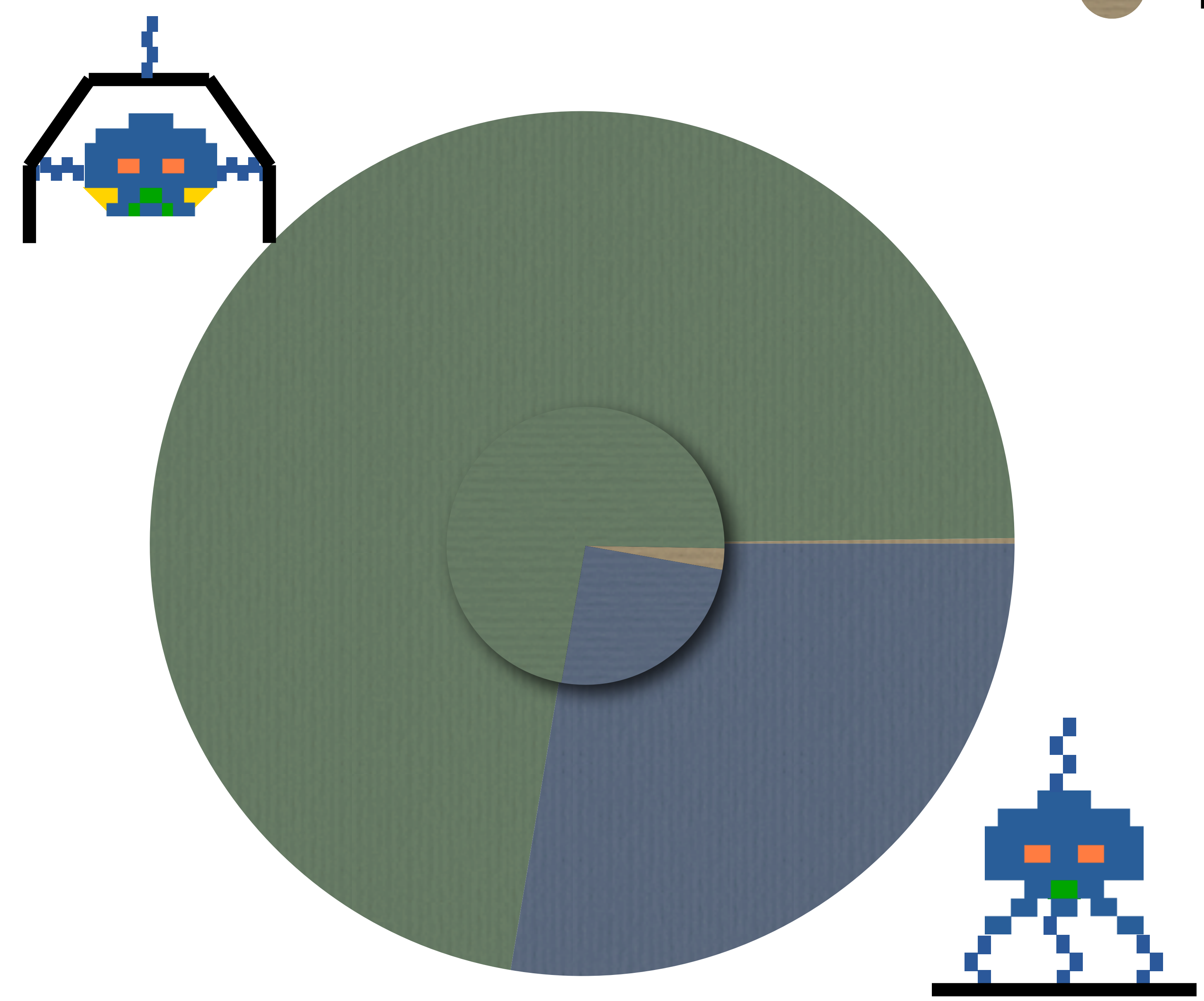
Error Budgets for Muon $(g-2)$

error $\propto$ perimeter; area $\propto$ weight in sum in quadrature

stats
syst

BNL E821 → FNAL E989

HL×L
HVP
EW

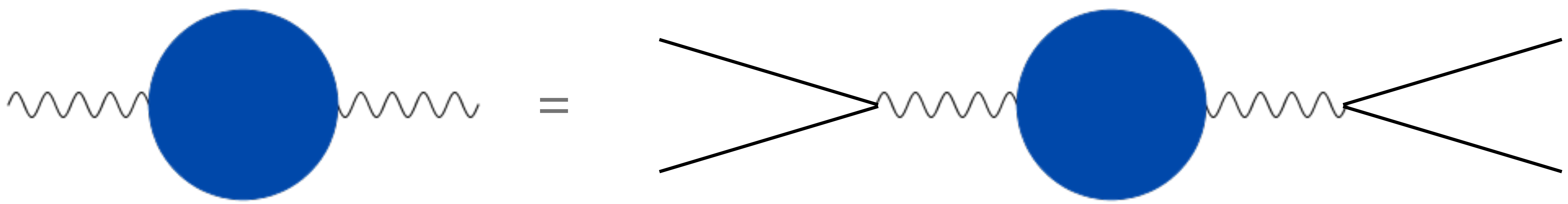


Standard Model Calculation

HVP from $e^+e^- \to$ hadrons *vs*. hadronic τ decay

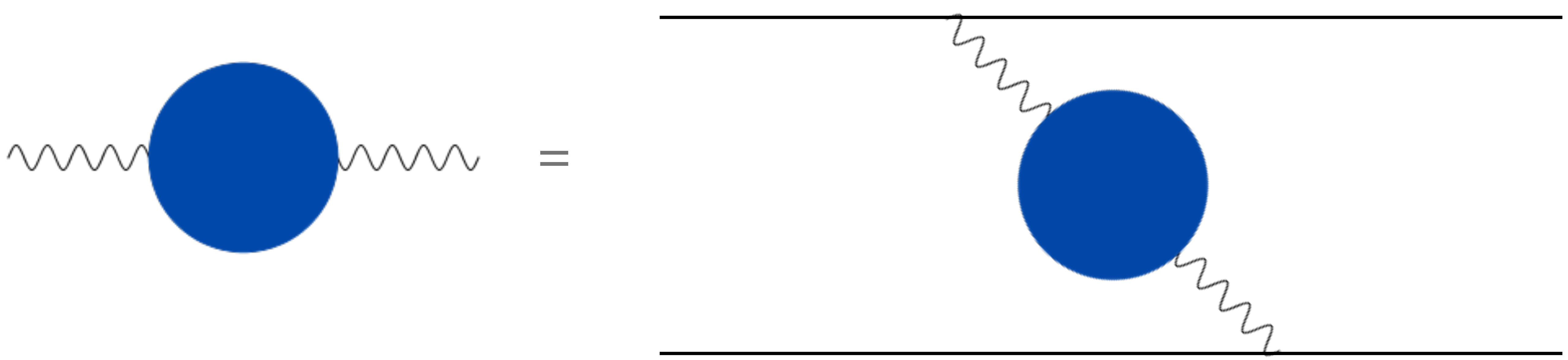
F. Jegerlehner

- The cross section for $e^+e^- \to$ hadrons contains the needed vacuum polarization:



– radiative corrections

- The partial width for $\tau \to$ hadrons contains W VP (related to γ VP by isopin):



$\oplus$ isospin corrections

- Jegerlehner & Szafron [arXiv:1101.2872] find that energy-dependence of mixing in the 2×2 ϱ-γ propagator can resolve the discrepancy. See also Benayoun *et al.*, arXiv:0907.5603.

Sociology

- E989 proponents receive many questions about HL×L (*e.g.*, P5, Intensity Frontier Review):
  - HL×L relies on models and indirect experimental information;
  - "recuperating" from sign mistakes (FORM's form for $\varepsilon^{\mu\nu\varrho\sigma}$; mismatch notes/code);
  - hence, the INT workshop.
- Even with a resolution between HVP(e^+e^-) and HVP(τ), E989 will warrant a dramatic improvement in the uncertainty on HVP:
  - my pie imagined $10^{11}a_\mu^{\mathrm{HVP}} = 6900 \pm 12\ (42{\div}3.5)$ & $10^{11}a_\mu^{\mathrm{HL\times L}} = 100 \pm 7\ (26{\div}3.7)$;
  - hence, some future workshop.

Explaining the Anomalous Anomaly BSM

Explanations beyond the Standard Model

Bill Marciano

- Discrepancy in $10^{11} a_\mu$ is 278±80 [Höcker, arXiv:1012.0055].
- Generic susy is $\mathrm{sign}(\mu)\ 260\ (\tan\beta/8)\ (200\ \mathrm{GeV}/M_{\mathrm{susy}})^2$; “fits like a glove”.
- Multi-Higgs models; extra dimensions, ….
- Dark photon with $m_A \approx 10\text{–}150$ MeV and $\alpha' = 10^{-8}$:
    - would be seen the first weekend of planned searches at JLab or Mainz.
- Insanely light Higgs, $m_H < 10$ MeV [Kinoshita & Marciano (1990)]:
    - Why doesn’t everyone know why every decade of m_H is ruled out?

Theory: Amplitudes and their Constraints

Hadronic Vacuum Polarization

- Integral over space-like momenta [Blum, hep-lat/0212018 (PRL)]:

$$a_\mu^{\mathrm{HVP}} = \frac{\alpha}{2\pi}\int_0^\infty dt\,\frac{64t^2}{(t+\sqrt{t^2+4t})^4\sqrt{t^2+4t}}\,2\pi\alpha\left[\Pi(m_\mu^2 t)-\Pi(0)\right]$$

where $t = q^2/m_\mu^2$ (Euclidean—or Weinberg's—conventions).

- Integral over time-like momenta $s = -q^2 > 0$:

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\int_{4m_\pi^2}^\infty ds\,K(s)R(s) \qquad R(s) = \frac{\sigma(e^+e^-\to \text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}$$

- Split (both) integrals into data (experimental or numerical) portion & pQCD portion.

- Vacuum polarization function $\Pi(q^2)$ is defined by (J_{em} for quarks only)

$$\Pi^{\mu\nu}(q^2) = (q^\mu q^\nu - \delta^{\mu\nu} q^2)\Pi(q^2) = \int d^4x\, e^{iq\cdot x} \langle J^\mu_{\mathrm{em}}(x) J^\mu_{\mathrm{em}}(0)\rangle$$

which is very smooth: space-like q^2!!!

- At time-like q^2, dispersion relations can relate this function to its imaginary part, and then the optical theorem to the total cross section:

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_0^\infty ds \frac{\Im\Pi(-s)}{s(s+q^2+i0^+)} = \frac{q^2}{\pi}\int_0^\infty ds \frac{\alpha(s)R(s)}{3s(s+q^2+i0^+)}$$

take jagged resonance regions from experiment; rest from pQCD.

Hadronic Light-by-light Amplitude

- The contribution to (g–2) is [*e.g.*, arXiv:0901.0306]

$$a_\mu^{\mathrm{HL\times L}} = \frac{e^2}{24 m_\mu} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \mathcal{K}_{\mu\lambda\nu\rho\sigma}(p, k_1, k_2, k_3) \, \frac{\partial}{\partial q_\mu} \Pi^{\lambda\nu\rho\sigma}(q, k_1, k_2, k_3) \Big|_{k_2 = k_1 - k_3 - q,\; q = 0}$$

where QED readily yields

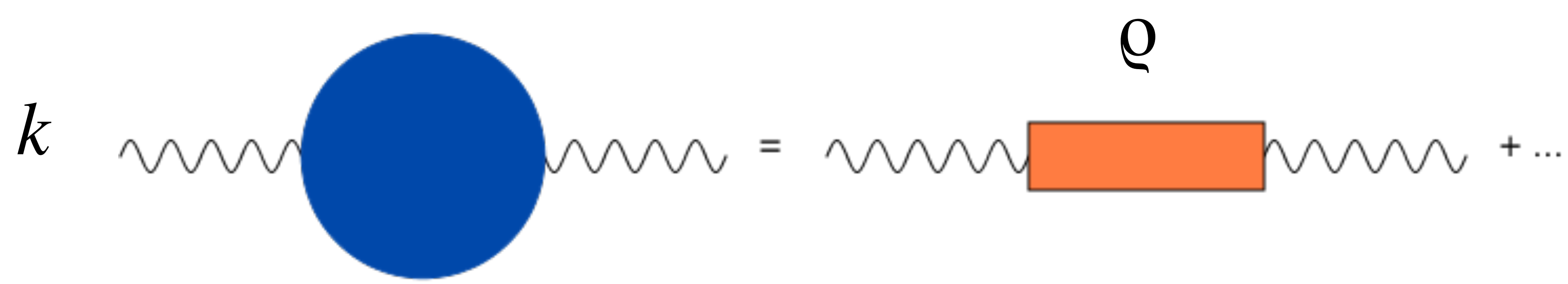
$$\mathcal{K}_{\mu\lambda\nu\rho\sigma}(p, k_1, k_2, k_3) = \frac{\mathrm{tr}\{[i\not{p} - m_\mu]\sigma_{\mu\lambda}[i\not{p} - m_\mu]\gamma_\nu[i(\not{p} + \not{k}_1) - m_\mu]\gamma_\rho[i(\not{p} + \not{k}_3) - m_\mu]\gamma_\sigma\}}{k_1^2 k_2^2 k_3^2 [(p + k_1)^2 + m_\mu^2][(p + k_3)^2 + m_\mu^2]}$$

and QCD not-so-readily provides

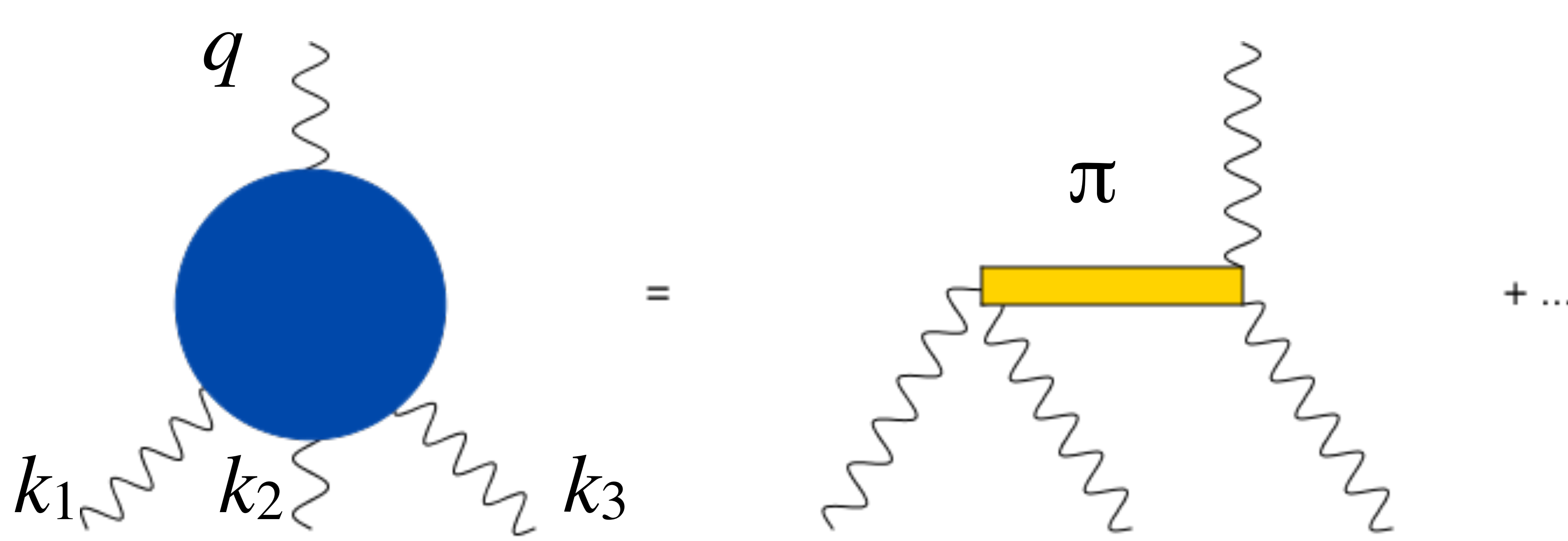
$$\Pi^{\lambda\nu\rho\sigma}(q, k_1, k_2, k_3) = \int d^4 x_1 \, d^4 x_2 \, d^4 x_3 \, e^{-i(k_1 x_1 - k_2 x_2 - k_3 x_3)} \left\langle J^\lambda_{\mathrm{em}}(0) J^\nu_{\mathrm{em}}(x_1) J^\rho_{\mathrm{em}}(x_2) J^\sigma_{\mathrm{em}}(x_3) \right\rangle$$

Dominant contributions

- Hadronic vacuum polarization is dominated by the rho meson (VMD):

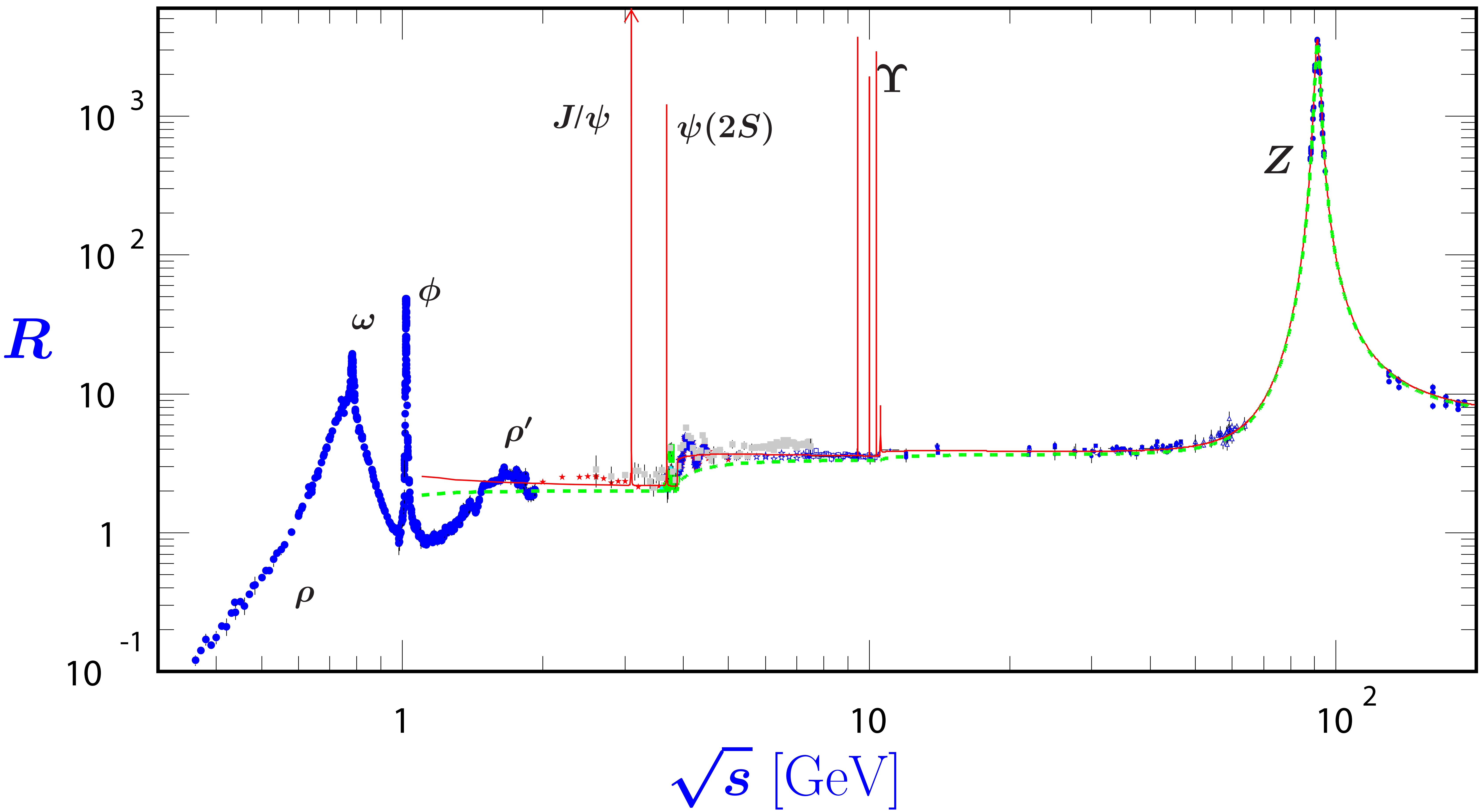


- Hadronic light-by-light amplitude is dominated by π (and η, η') exchange (normalized by the anomaly; well described by Wess-Zumino Lagrangian)



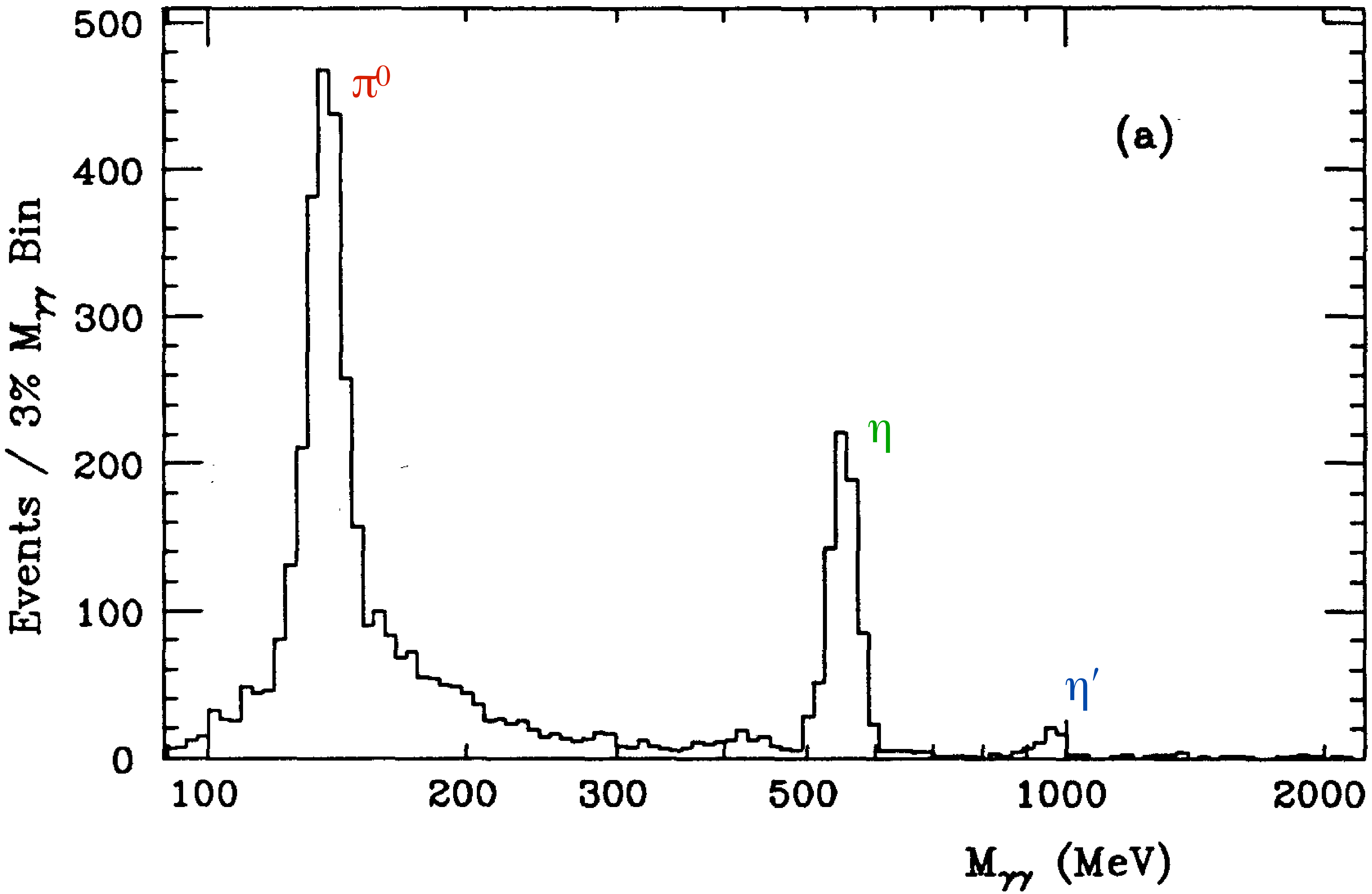
- Of course, the uncertainty is dominated by the other contributions … .

PDG: $e^+e^- \rightarrow$ hadrons



Crystal Ball: π^0, η, and η' in $\gamma\gamma \to \gamma\gamma$(1988)

SLAC-PUB-4580, Fig. 2 (see also Fig. 8)



Estimates of HL×L from Models of QCD

Apology

- Most of the following slides follow the dreadful format “so-and-so gave a nice talk in which he* showed this nice plot”.

- Just without the nice plots.

- * At this workshop, all speakers were “he”.

Glasgow Consensus

Prades, de Rafael, Vainshtein [arXiv:0901.0306]

- Combining several ingredients (covered below), PRV find $10^{11} a_\mu^{\text{HL×L}} = 105 \pm 26$:
  - $10^{11} a_\mu^{\text{HL×L}}(\pi, \eta, \eta') = 114 \pm 13$ [MV ≈ (ENJL+OPE) ± max.ENJL];
  - $10^{11} a_\mu^{\text{HL×L}}(a_1, \text{etc.}) = 15 \pm 10$ [MV ± 10×MV];
  - $10^{11} a_\mu^{\text{HL×L}}(\text{scalars}) = -7 \pm 7$ [ENJL ± inflated ENJL];
  - $10^{11} a_\mu^{\text{HL×L}}(\text{dressed } \pi \text{ loop}) = -19 \pm 19$ [ENJL ± inflated ENJL];
  - add error estimates in quadrature.

Extended Nambu–Jona-Lasinio & Chiral Quark Models

Hans Bijnens (work with Pallante & Prades)

- The chiral quark model has a pion field (χPT) constituent-like quark field:
  - quark captures short-distance QCD, but freezes out at long distances;
  - pion captures long-distance constraints of chiral symmetry;
  - need great care to avoid double counting of long & short (>1 invariant!).
- NJL adds to this four-quark interactions whose bubble sums generate non-NG mesons.
- Thus, combo incorporates obviously needed ingredients: pion & other meson exchange + quark loop.
- Hayakawa, Kinoshita, Sanda: meson models, VMD, hidden local symmetry.

Chiral approach and resonance dominance

Andreas Nyffeler

- The BPP and HKS papers simplify the pion exchange amplitude

$$\mathcal{A} \propto F_{\pi\gamma^*\gamma^*}\big((q_1+q_2)^2, q_1^2, q_2^2\big)\,\frac{1}{(q_1+q_2)^2 - m_\pi^2}\,F_{\pi\gamma^*\gamma}\big((q_3+q_4)^2, q_3^2, 0\big)$$

with $F_{\pi\gamma^*\gamma^*}\left((q_1+q_2)^2, q_1^2, q_2^2\right) \approx F_{\pi\gamma^*\gamma^*}\left(m_\pi^2, q_1^2, q_2^2\right)$.

- Off-shell effects should enter. How large are they?

- Can be estimated only using resonance models, and in a model calculation of HL×L, this is not an essentially new ingredient: estimates $10^{11}a_\mu^{\text{HL}\times\text{L}}$(off shell) ≈ 35–40.

- NB: *magnetic susceptibility* $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{F_{\mu\nu}}$ constrains meson exchanges [Belyaev & Kogan, 1984]; can be calculated in lattice gauge theory.

Using Constraints from Operator Product Expansion

Arkady Vainshtein; Kiril Melnikov

- In the limit $k_1^2 \approx k_2^2 \gg k_3^2 \gg \Lambda^2_{\mathrm{QCD}}$, the OPE relates $\mathrm{FT}\langle VVVV \rangle$ to $\mathrm{FT}\langle AVV \rangle$ [hep-ph/0312226]:
  - fixes normalization of pseudoscalar and axial-vector exchanges in these kinematics;
  - in particular, $\lim\limits_{q^2 \gg \Lambda^2} F_{\pi\gamma^*\gamma^*}(q^2, q^2) = \dfrac{8\pi^2 f_\pi^2}{N_c q^2}$ matches low-energy normalization from anomaly;
  - facilitates introduction of a model *function* to interpolate between limits (in contrast to model Lagrangians of other approaches);
  - MV choose an Ansatz; you could choose yours.
- Despite any limitations of MV's Ansatz, it should be clear that model Lagrangians in other approaches should satisfy their OPE constraint.

Holographic QCD

Oscar Catà; Deog Ki Hong

- Exploit (conjectured) duality between d-dimensional strongly-coupled gauge theories and $(d+1)$-dimensional weakly-coupled gravity:
  - incorporates large N_c & (conformal) short-distance behavior w/ Lagrangian;
  - few parameters (3 new for Catà; no new for Hong);
  - becomes a model when a dilation factor $e^{-\Phi(x)}$ is chosen.
- Focus on $F_{\pi\gamma\gamma^*}$ form factor: obtain numerical results for pseudoscalar exchange in very good agreement with other approaches.
- Hong also obtains non-strange $10^{11}a_\mu^{\mathrm{HVP}} = 4705$ *vs*. 5141 ± 38 from *BaBar* data.

Two-loop Chiral Perturbation Theory

Michael Ramsay-Musolf

- Notes that χPT provides useful, model-independent constraint of pion contribution:
  - pion pole term yields $\ln^2$; single $\ln$ from $\pi \to e^+e^-$; last LEC from lattice
  - BR($\pi \to e^+e^-$) from KTeV 2007 should reduce uncertainty in single $\ln$.
- Resonances built up from higher-order contributions:
  - MRM + students computing full 2-loop χPT HL×L.
- Pion loops will need further LECs from pion charge radius and pion polarizability.
- This seems like a hard way to gain real improvement, but I think these calculations could guide chiral extrapolation of QED+QCD method.

Schwinger-Dyson Equations (DSE)

Richard Williams

- Start with (exact) Dyson-Schwinger eq'ns for dressed propagators, vertex, 4-pt function.
- Introduce "model" functions (*e.g.*, Maris-Tandy) that satisfy—
  - Ward identities;
  - good agreement with phenomenology in other applications;
  - good agreement with lattice calculations (in Landau gauge).
- Keep large N_c part in DSE resummation (i.e., neglect non-planar and 2- & 3-gluon vtx).
- Results: $10^{11}a_\mu^{\text{HVP}} = 6700$ & $10^{11}a_\mu^{\text{HL}\times\text{L}} = 217 \pm 91$ [arXiv:1012.3886] or 147 ± 91 [this talk?]; compare: $10^{11}a_\mu^{\text{HVP}} = 6923 \pm 42$ [data] & $10^{11}a_\mu^{\text{HL}\times\text{L}} = 105 \pm 26$ [consensus, arXiv:0901.0306].

Guiding HL×L with Experimental Measurements

What Do Data Say about HL×L?

Fred Jegerlehner

- HL×L contains a $\gamma \to \gamma^*\gamma^*\gamma^*$ amplitude, which can be related—by analyticity and optical theorem—to cross sections for $\gamma^{(*)}\gamma^{(*)} \to$ hadrons.
- Crystal Ball (1988) $\gamma\gamma \to$ hadrons spectrum shows clear peaks for π, η, & η' but nothing else.
- Primakoff effect ($\gamma N \to \pi^0 \to \gamma\gamma$) yields pion part of $\gamma\gamma\gamma\gamma^*$.
- Central π^0 production in e^+e^- (CELLO, CLEO, BaBar, …) yield pion part of $\gamma^{(*)}\gamma^*\gamma\gamma$.
- Axial-vector mesons require off-shell photon(s) (Lee-Yang theorem): data are "sparse".
- Scalar mesons seen in $\gamma\gamma \to \pi\pi$; tensor mesons needed too….
- Need to connect data with 0, 2, or 4 photons off shell to amplitude with 3 off shell: models inevitably enter: they should be compatible with measurements mentioned here.

Meson Transition Form Factors at BaBar

Achim Denig

- Test onset of perturbative QCD behavior for form factors [Brodsky, Lepage]:

$$F_{\pi\gamma^{(*)}\gamma^{(*)}}(q_1^2, q_2^2) = \int_0^1 dx\, T(x; q_1^2, q_2^2)\phi(x)$$

where $T(x;q^2)$ is hard scattering amplitude $\gamma^{(*)}\gamma^{(*)} \to q\bar{q}$, $\phi(x)$ is the distribution amplitude.

- (My opinion): more likely to shed light on $\phi(x)$ than on HL×L:
  - interesting, but beyond the scope of this talk.
- Medium and low q^2 measurements will (see above) provide constraints for models.

Future Measurements at KLOE/DAΦNE

Dario Moricciani [KLOE Collaboration]; Henryk Czyż

- KLOE-2 detector will study ϕ region, including 2-photon physics.
- Latter are distinquished from the huge ϕ signal by tagging $e^{\pm}$ at small angles.
- Should clear up some discrepancies from older experiments, improve slope of $\pi\gamma\gamma$ form factor, and shed light on scalar [Moricciani].
- Important tool is the EKHARA event generator: take model form factors to generate events and then compare output to data [Czyż].

Computing HVP and HL×L with Lattice Gauge Theory

Lattice QCD for $g\text{–}2$

- With lattice QCD, one can compute FT$\langle V_\mu(x) V_\nu(0)\rangle$ or FT$\langle V_\mu(x) V_\nu(y) V_\rho(z) V_\sigma\rangle$ (from first principles) and convolute the result with QED Feynman diagrams.

- In addition to usual worries (continuum limit, physical pion cloud), need $q \sim m_\mu$, so might expect to need box-size a few times $\pi/m_\mu \sim 6$ fm.

- Structure in Green functions expected at two QCD scales: $m_\pi \approx 1.3 m_\mu$ and $m_\rho \approx 7 m_\mu$; also need to match onto pQCD regime.

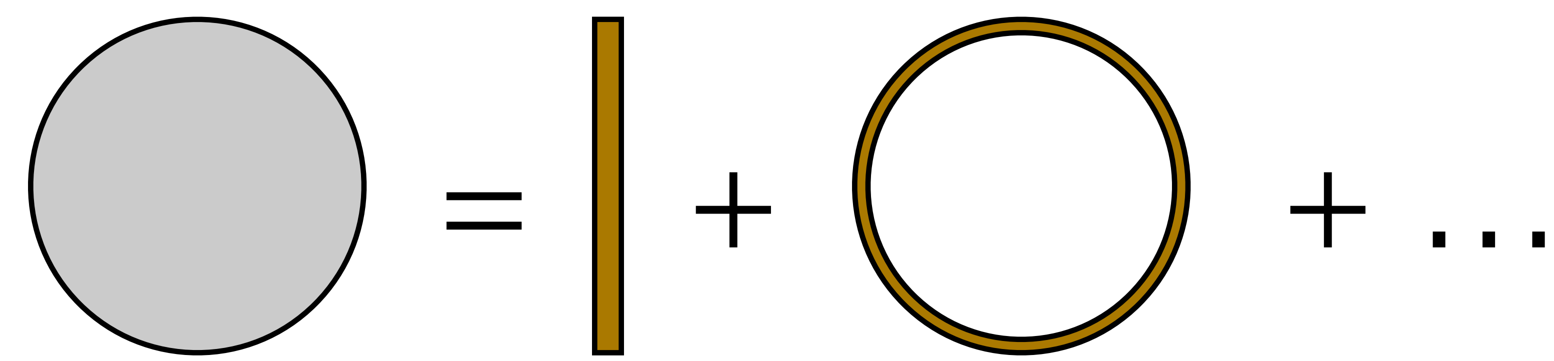
- HVP 2-pt function has 2 (1) form factors; HL×L has 138 (43 by gauge symmetry; 32 in $g\text{–}2$).

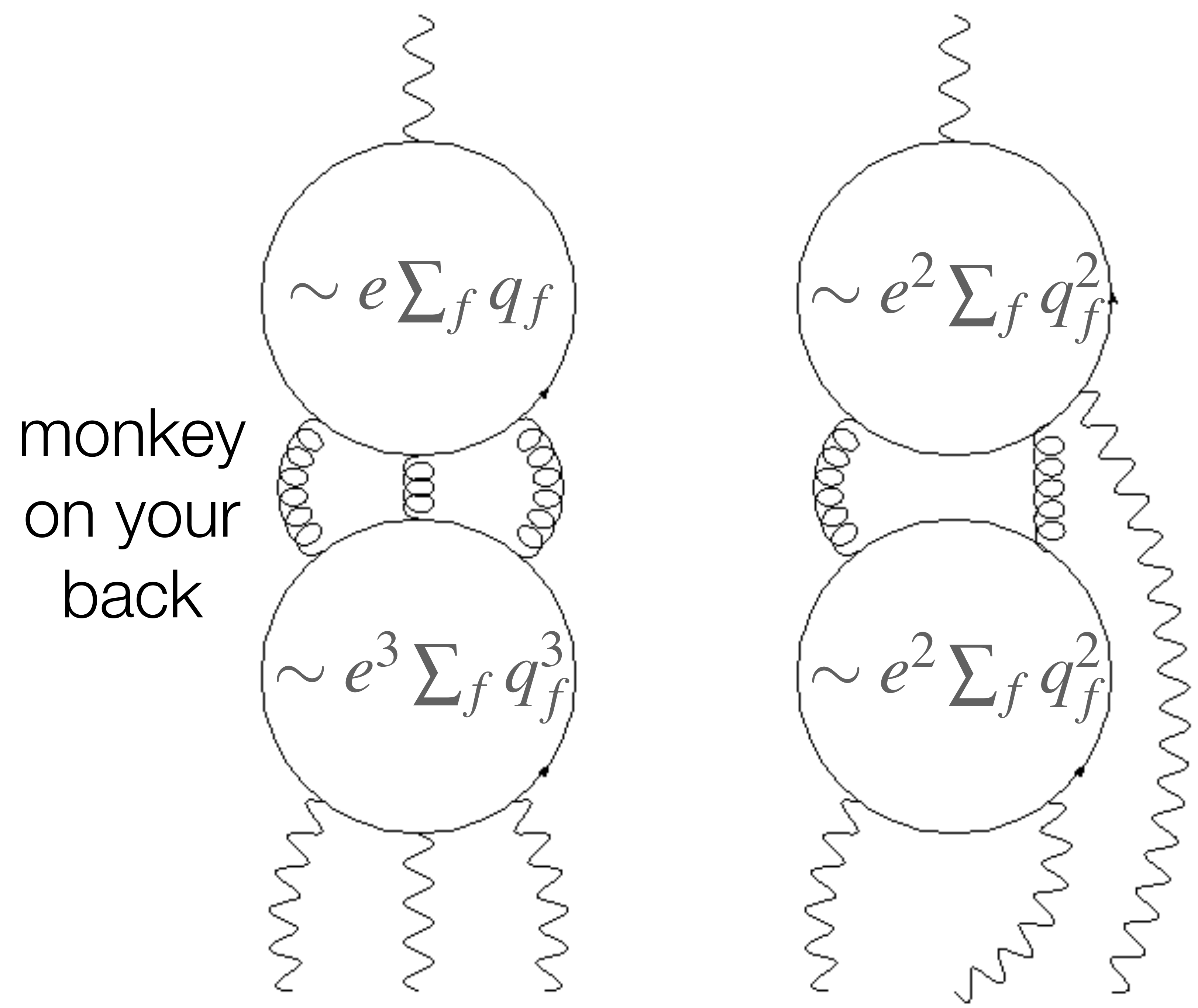
- In the end, need only two numbers, HVP (≈ 7000) to 0.2%, HL×L (≈ 100) to 5%, to match measurement of approved experiment Fermilab E989.

- Probably need cleverness, not just brute force.

Sea Quarks are Necessary for g–2

- Not just for processes sketched in the top figure (for both vacuum polarization and HL×L).
- All fermion lines/loops connected to initial or final state must be treated separately:
    - “disconnected diagrams”—
    - present because photon is flavor singlet;
    - really, really demanding.
- As far as I know, no one has attempted a fully disconnected calculations for HL×L or HVP.





QCD+QED: Direct Calculation of HL×L

Tom Blum

- Computing $\mathrm{FT}\langle VVVV\rangle$ seems difficult and unnecessarily so.
- Need one number: the (hadronic part of the) muon's magnetic form factor at $q^2 = 0$.
- Compute $F_2(0)$ in lattice QCD+QED (QED quenched for now):
  - need subtraction to eliminate some QED renormalization parts;
  - successful in pure QED for muon, not for electron—signal ~ $(m_{\text{leg}}/m_{\text{loop}})2$, noise same;
  - in QCD+QED, muon suffers from the same problem—constituent $m_{\text{loop}} \sim m_{\mu}$.
- Smells like a promising way forward; see also Blum's talk at ⟨Lattice||Experiment⟩.

Two Approaches to Form Factor for $\pi\gamma^{(*)}\gamma^*$

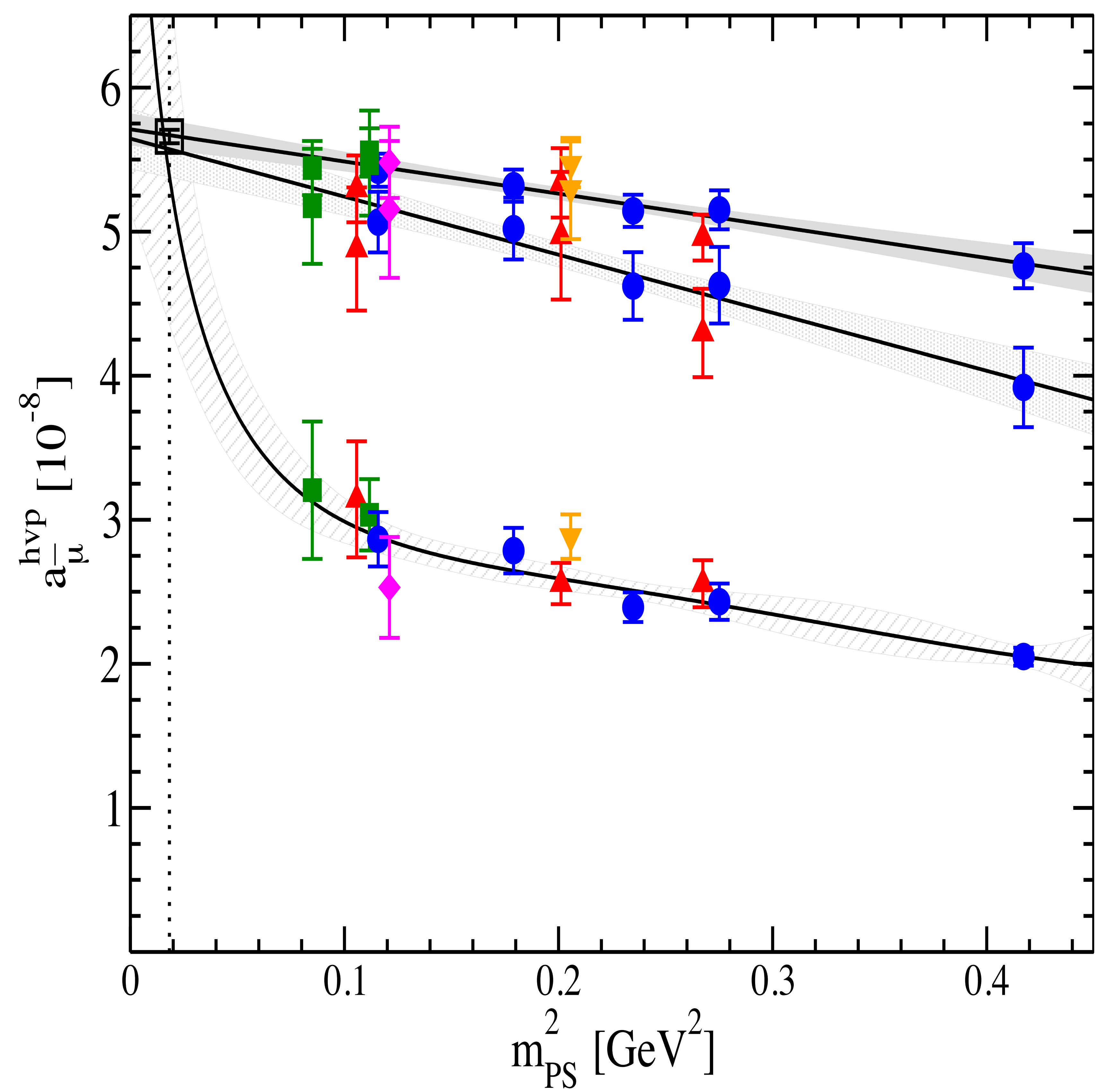
Shoji Hashimoto

- Space-like [arXiv:0912.0253]:
    - standard lattice QCD form factor techniques;
    - ABJ anomaly reproduced (most involved calculation ever) ⇒ precise pion width;
    - limited range of momentum transfer: twisted bc? constrain with unitarity & analyticity?
- Time-like [S. Cohen *et al.*, arXiv:0810.5550]:
    - exploit masses of vector mesons to get to time-like $q^2 = \boldsymbol{p}^2 - m_{\rm V}{}^2 < 0$;
    - pilot study by JLab group; new preliminary work by JLQCD.

HVP with 2 Twisted-mass Sea Quarks

Karl Jansen

- Lattice calculations of a_μ^{HVP} pioneered by Blum, Blum & Aubin.
- New, and precise, calculation of up-down contribution to HVP (data $10^8 a_\mu^{\text{HVP}} = 5.66 \pm 0.05$):
    - first attempt lacked control of chiral extrapolation: head scratching: resolution:
    - solving this problem: $10^8 a_\mu^{\text{HVP}} = 5.66 \pm 0.11$;
    - agrees with expt and error is only twice;
- Now attack with 2+1+1 flavors of sea quarks!!!



Direct Calculation of $\mathrm{FT}\langle JJJJ\rangle$

QCDSF Collaboration (Paul Rakow, Gerrit Schierholz)

- Note that, short of calculating $\mathrm{FT}\langle JJJJ\rangle$ at "all" momenta, a well-chosen subset can put constraints on models—similar & complementary to input from experiment—
  - goal of workshop participants to define "well-chosen subset";
  - QCDSF may already know.
- Rakow [QCDSF] had to cancel at short-ish notice; info from linked talk and e-mail:
  - computing *connected* diagram and see pion dominance in signal;
  - have ideas to obtain non-small disconnected diagram;
  - expect 5% calculation of HL×L [Schierholz], with HVP a by-product.

Conclusions and Outlook

Compilation of Models: Consensus?

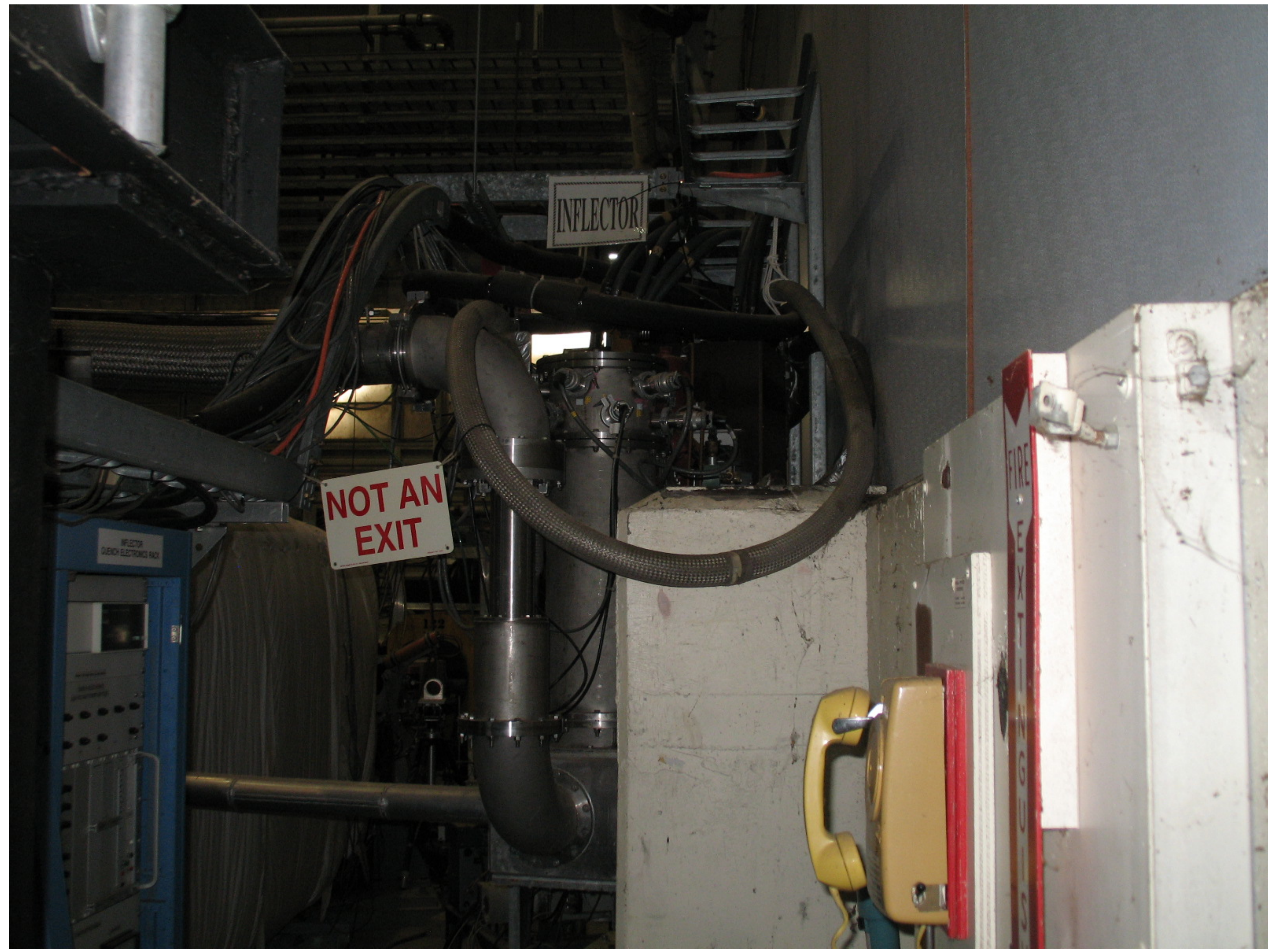
Andreas Nyffeler

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN	FGW
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16	84 ± 13
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5	–
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2	–
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13	–
π, K loops $+$subl. N_C	–	–	–	0 ± 10	–	–	–	–
other	–	–	–	–	–	–	–	0 ± 20
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3	107 ± 48
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39	191 ± 81

BPP = Bijnens, Pallante, Prades ’95, ’96, ’02; HKS = Hayakawa, Kinoshita, Sanda ’95, ’96; HK = Hayakawa, Kinoshita ’98, ’02; KN = Knecht, Nyffeler ’02; MV = Melnikov, Vainshtein ’04; BP = Bijnens, Prades ’07; MdRR = Miller, de Rafael, Roberts ’07; PdRV = Prades, de Rafael, Vainshtein ’09; N = Nyffeler ’09, JN = Jegerlehner, Nyffeler ’09; FGW = Fischer, Goecke, Williams ’10, ’11 (used values from arXiv:1009.5297v2 [hep-ph], 4 Feb 2011)

Where is the way out?

- Models are faced with several obstacles (my opinion):
  - solidification possible.
- Leaves lattice gauge theory:
  - QCD;
  - QCD+QED.



Needs for *g*–2

ASK

- Let’s assume that the monkey-on-your-back topology can be safely neglected (likely).
- Let’s assume that the HVP to needed precision comes along with HL×L (not obvious).
- Let’s focus on QCD+QED: easier to forecast one number than many form factors.
- BCHIYY find 100% error using 10^{-2} Tflop s^{-1} yr, and planning “reasonable” calculation with 10 Tflop s^{-1} yr. Target 10% (5%) needs—naïvely—a factor of 100 (400) more computing:
    - 1–5 Tflop s^{-1} yr needed.
- *Caveats*: with 100% error it is hard to foresee obstacles both surmountable and unsurmountable. Estimate is, thus, more likely to be over-pessimistic or over-optimistic than accurate.

Resources for g–2

ASK

- "Luminosity" formula: resource = f_{g-2} × budget × Moore's Law; f_{g-2} = fraction for g–2:
  - USQCD Moore's Law: $2^{t/1.6}$ Tflop s^{-1} (\$M)$^{-1}$; (now t = years since 2005/09)
  - USQCD budget experience: 2.9×$2^{t/10.5}$ \$M yr^{-1}; (omits Tea Party effects)
  - TB *et al.* are increasing f_{g-2} from 10^{-4} to 10^{-2}.
- Predict resource of 5 Tflop s^{-1} yr in 2016.
- Coincides with forecast of computing need.
- Several groups engaged: perhaps even human resource will be available.