Factorization at the Tevatron and LHC: From PDFs to Initial-State Jets

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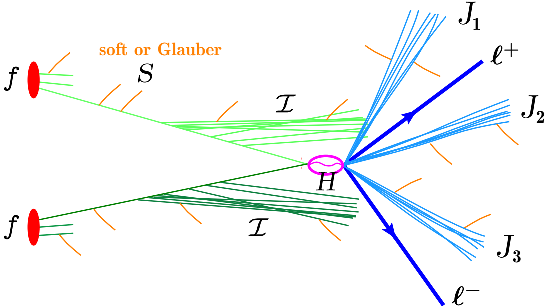
In collaboration with: Iain Stewart, Frank Tackmann
Carola Berger and Claudio Marcantonini

arXiv: 0910.0467, 1002.2213, 1004.2489, 1005.4060, more to appear

Outline

Jet Vetos and ISR

Typical Hard Interaction at Tevatron or LHC



- The Goal: disentangle new physics H from experimental signal
- Our goal: encode signal of jets, leptons and photons in theory calculations
- Factorization: “cross section is a combination of separate pieces.”

$$d\sigma = \underbrace{\text{PDFs} \otimes \text{ISR}} \otimes \text{hard interaction} \otimes \text{FSR} \otimes \text{soft radiation}$$

MC:	initial-state parton shower	matrix-element generator	final-state parton shower	hadronization underlying event

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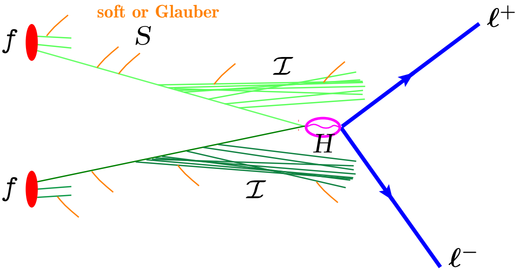

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Focus on the 0-jet case

Higgs and Jet Vetos at LHC

$gg \longrightarrow H \longrightarrow WW \longrightarrow \ell\nu\ell\bar{\nu}$

- Strong discovery potential
- Large $\sim 40 : 1$ background from $t\bar{t} \longrightarrow WWb\bar{b}$
- Cannot reconstruct Higgs invariant mass ($\nu\bar{\nu}$)

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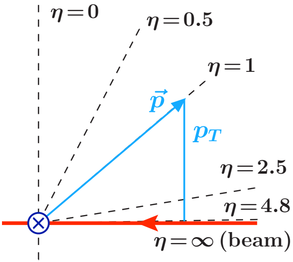
Use jet veto to obtain 0-jet sample

1. Run a jet algorithm
2. Veto all events that have jets with $p_T > p_T^{\rm cut}$ and $|\eta| < \eta^{\rm cut}$

   CMS: $p_T^{\rm cut} = 25\,\mathrm{GeV}, \eta^{\rm cut} = 2.5$

   ATLAS: $p_T^{\rm cut} = 20\,\mathrm{GeV}, \eta^{\rm cut} = 4.8$



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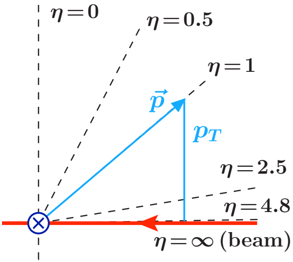
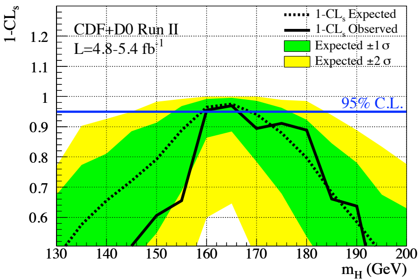


- Reduces $t\bar{t}$ background by factor ~ 400

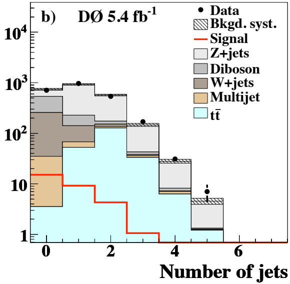
$gg \to H \to \gamma\gamma$

- Large background from jets faking a photon (e.g. $\pi^0 \to \gamma\gamma$)
- May reduce this by imposing a jet veto

$H \to WW$ at Tevatron



[CDF+DØ (arXiv:1001.4162)]



[DØ (arXiv:1001.4481)]

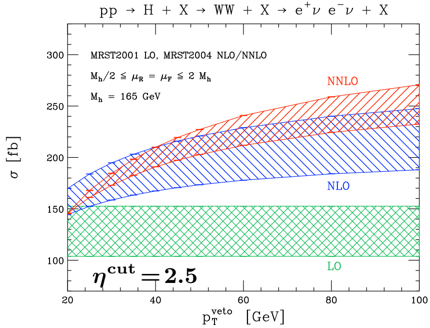
Tevatron excludes $m_H \simeq 165\,\mathrm{GeV}$ at 95% CL

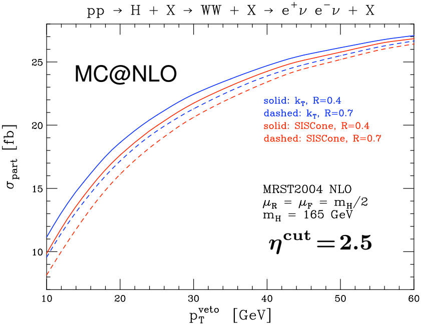
- $p_T^{\mathrm{cut}} = 15\,\mathrm{GeV}$, $\eta^{\mathrm{cut}} = 2.4 - 2.5$

  CDF: Combine separate 0-jet, 1-jet and ≥ 2-jet samples

  DØ: Number of jets used as input to neural network
- Sensitivity dominated by 0-jet sample
- Exclusion requires reliable theory predictions and uncertainties
- Theory uncertainties have been questioned [Baglio, Djouadi (arXiv:1003.4266)]

$gg \to H \to WW$ with 0 Jets





[Anastasiou, Dissertori, Stöckli, Webber]

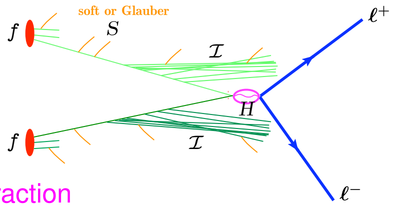
0-jet and inclusive Higgs production are very different:

- Phase space restrictions lead to large logarithms

  $$\sigma(p_T^{cut}) \sim \sigma_0[1 + \alpha_s \ln^2(p_T^{cut}/m_H) + \alpha_s^2 \ln^4(p_T^{cut}/m_H) + \dots]$$

  - Need to be summed for reliable predictions
  - Parton-shower Monte Carlo only sums leading logs
- Signal sensitive to choice of jet algorithm

Hadronic Initial-State Radiation



Jet veto restricts ISR

$\longrightarrow$ study ISR by varying veto (p_T^{cut})

ISR is important:

- Affects luminosity available for hard interaction
- Could contaminate signal (incoming gluons radiate a lot)
- Modeled by initial-state parton shower in Monte Carlo (not as well understood as final state parton shower)

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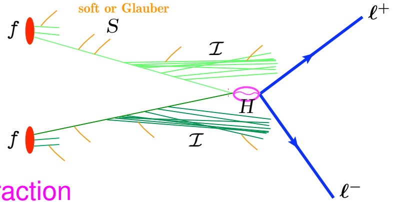

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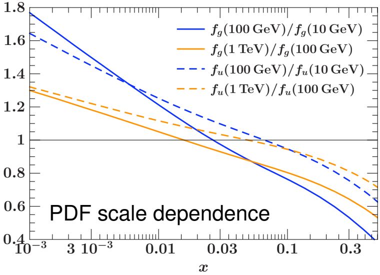
PDFs and ISR:

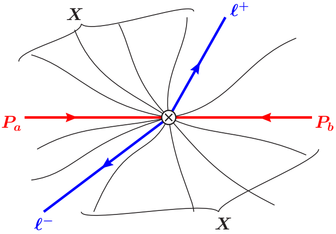
- Running of PDFs due to ISR
- Proper scale to evaluate PDFs?
- PDFs cannot describe ISR ($\mathcal{I}$)



Factorization with a Jet Veto

Drell-Yan $pp \to X\ell^+\ell^-$

Simplest case to study jet vetos and ISR



Lepton kinematics

Invariant mass $Q^2 = (p_{\ell^+} + p_{\ell^-})^2$

Rapidity Y

Mom. fraction $x_a = \frac{Q}{E_{\rm cm}} e^{Y}$

$x_b = \frac{Q}{E_{\rm cm}} e^{-Y}$

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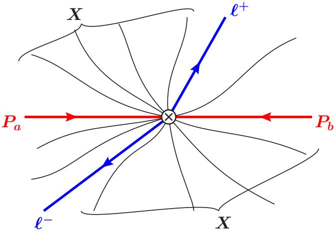

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Factorization theorem, inclusive in X [Collins, Soper, Sterman; Bodwin]

$$\frac{d\sigma}{dQ\,dY} = \sum_{i,j=\{g,q,\bar{q}\}} \int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu) \hat{\sigma}_{ij}\Big(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, Q, \mu\Big)$$

- Separates perturbative $\hat{\sigma}_{ij}$ from nonperturbative physics f_i
- PDF evolutions sum single logs $\alpha_s^n \ln^m(\Lambda_{\rm QCD}/Q)$
- $\hat{\sigma}_{ij}$ contains jets, initial-state radiation etc.
- At tree level $x_{a,b} = \xi_{a,b}$, beyond $x_{a,b} \leq \xi_{a,b}$

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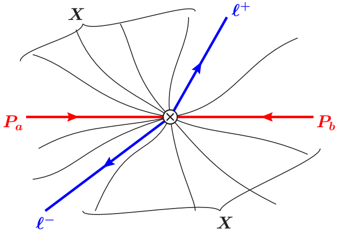

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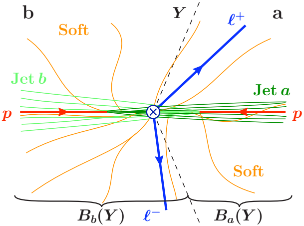
What happens when we impose a jet veto?

Definition of Beam Thrust

$$\tau_B = \frac{1}{Q}\sum_k |\vec{p}_{kT}| e^{-|\eta_k - Y|}$$

$$= \frac{1}{Q}\Big[\underbrace{e^{-Y}\sum_{\eta_k<Y}(E_k + p_k^z)}_{B_b^+(Y)} + \underbrace{e^{Y}\sum_{\eta_k>Y}(E_k - p_k^z)}_{B_a^+(Y)}\Big]$$



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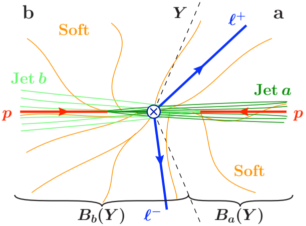

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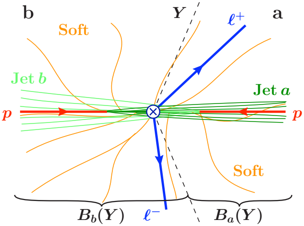

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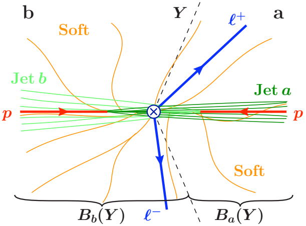

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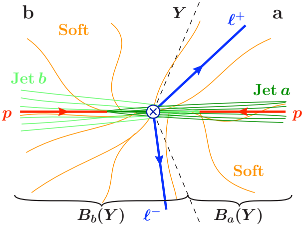

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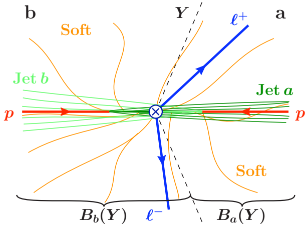

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- Require $\tau_B \ll 1$:
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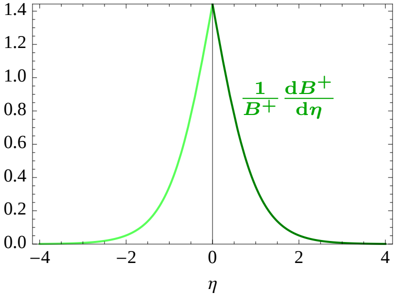
- Limited detector reach $\eta_{\rm det}^{\rm Tevatron} = 4$ no problem. Worst case scenario: unmeasured $B_a^+ \approx \sum_{\eta_k > \eta_{\rm det}} 2E_k e^{-2\eta_k} \lesssim 2\,\mathrm{TeV} e^{-8} = 0.7\,\mathrm{GeV}$

Beam Thrust as Jet Veto

Beam thrust $\tau_B = \frac{1}{Q}[e^{-Y} B_b^+(Y) + e^{Y} B_a^+(Y)]$



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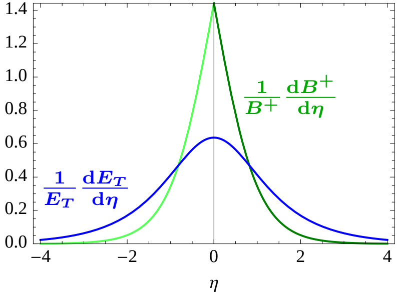
Compare to:

- Transverse energy $E_T = \sum_k |\vec{p}_{kT}|$ is not as strong a veto
- Transverse momentum $\vec{p}_T = \sum_k \vec{p}_{kT}$ is not a jet veto



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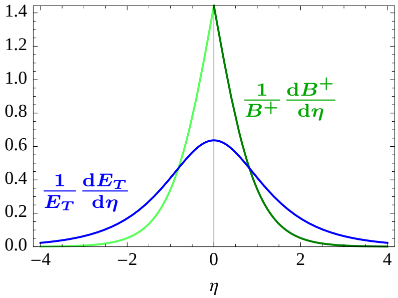
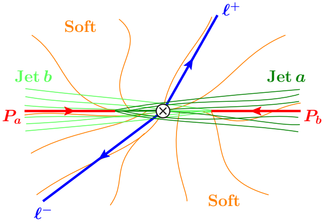


Benefits of beam thrust as jet veto

- No jet algorithm dependence
- Summation of large $\alpha_s^n \log^m \tau_B$ (beyond parton shower and leading logs)
- Theory treatment of soft effects (beyond hadronization and underlying event models)
- Better handle on theory uncertainties

Isolated Drell-Yan



Factorization theorem for $\tau_B \ll 1$

[Stewart, Tackmann, WW]

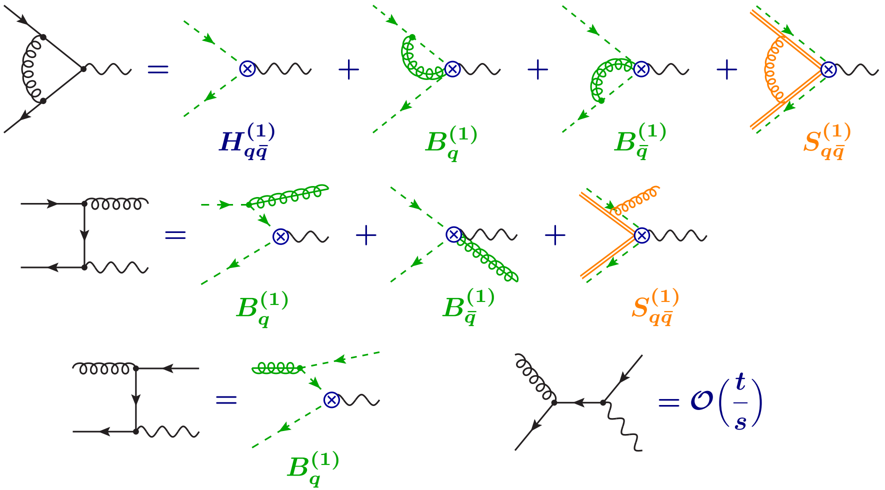
Derived using Soft-Collinear Effective Theory

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}\tau_B} = \sum_{ij=q\bar{q},\bar{q}q} H_{ij}(Q,\mu) \int \mathrm{d}t_a\, B_i(t_a, x_a, \mu) \int \mathrm{d}t_b\, B_j(t_b, x_b, \mu)$$
$$\times S_B\Big(Q\,\tau_B - \frac{t_a + t_b}{Q}, \mu\Big)\Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}}{Q}, \tau_B\Big)\Big]$$

H	hard function	virtual hard corrections	$\mu_H = Q$
B	beam function	virtual & real energetic ISR	$\mu_B = \sqrt{\tau_B}Q$
S	soft function	virtual & real soft radiation	$\mu_S = \tau_B Q$

- Each function depends on only one scale $\longrightarrow$ use RGE to sum large logs

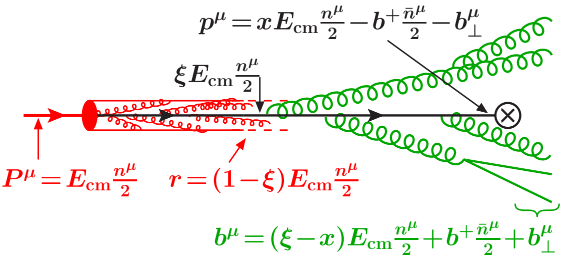
Factorization in Pictures



- Jet veto forces real radiation to be collinear to beams or soft
- Jet veto enhances graphs with t-channel singularity
  $\longrightarrow$ Corresponding logarithms will be summed

Beam Functions

Physical Picture of the Initial State



- Light cone coordinates: $n^\mu = (1, 0, 0, 1),\ \bar{n}^\mu = (1, 0, 0, -1)$
- Colliding parton has spacelike virtuality

$$-p^2 = x E_{\mathrm{cm}}\, b^+ + \vec{b}_\perp^{\,2} \geq 0$$

(real radiation implies $b^+ = b^0 - b^3 > 0$)

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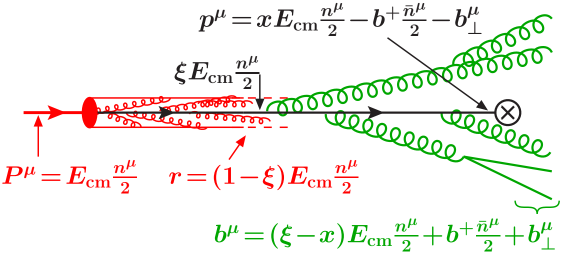

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$$-p^2 = \underbrace{x E_{\rm cm}\, b^+}_{t} + \vec{b}^2 \geq 0$$

(real radiation implies $b^+ = b^0 - b^3 > 0$)

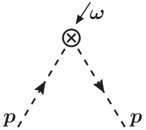
- Our measurement of τ_B depends on b^+
  - Beam function depends on x and t
  - Typically $-p^2 \geq t \gg \Lambda^2_{\rm QCD}$, so parton in initial-state jet!

Beam Function Definition

Quark PDF

$$f_q(\omega/P^-) = \theta(\omega) \int \frac{\mathrm{d}y^+}{4\pi}\, e^{-\mathrm{i}\omega y^+/2} \Big\langle P^- \Big| \bar\psi\Big(y^+\frac{\bar n}{2}\Big) W\Big(y^+\frac{\bar n}{2}, 0\Big) \frac{\bar{n}\!\!\!/}{2} \psi(0) \Big| P^- \Big\rangle$$

$$= \theta(\omega) \Big\langle P^- \Big| \bar\chi_n(0)\, \delta(\omega - \overline{\mathcal{P}}_n)\, \frac{\bar{n}\!\!\!/}{2} \chi_n(0) \Big| P^- \Big\rangle$$



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- momentum space version
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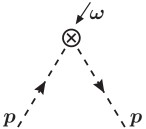
$$= \theta(\omega)\Big\langle P^-\Big|\bar\chi_n(0)\,\delta(\omega-\overline{\mathcal{P}}_n)\,\frac{\bar{n}\!\!\!/}{2}\chi_n(0)\Big|P^-\Big\rangle$$



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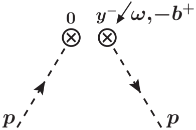

Quark beam function

$$B_q(t,\omega/P^-)$$

$$= \frac{\theta(\omega)}{\omega}\int\frac{\mathrm{d}y^-}{4\pi}\,e^{\mathrm{i}ty^-/2\omega}\Big\langle P^-\Big|\bar\chi_n\Big(y^-\frac{n}{2}\Big)\delta(\omega-\overline{\mathcal{P}}_n)\,\frac{\bar{n}\!\!\!/}{2}\chi_n(0)\Big|P^-\Big\rangle$$

- Soft Wilson line for $[y^-, 0]$ was factored out and moved to soft function
- Relate B and f by OPE for $y^- \to 0$

One-Loop Matching Calculation

$$B_i(t, x, \mu) = \sum_j \int \frac{\mathrm{d}\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mu\Big) f_j(\xi, \mu) \Big[1 + \mathcal{O}\Big(\frac{\Lambda^2_{\mathrm{QCD}}}{t}\Big)\Big]$$

Can perform matching using any external states, we use a quark or gluon:

- Tree level: $B_i(t, x, \mu) = \delta(t) f_i(x, \mu)$

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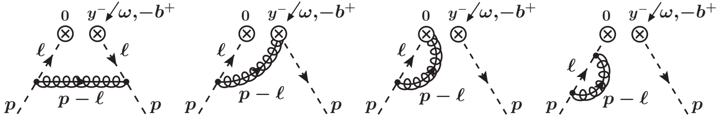
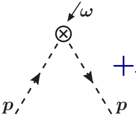
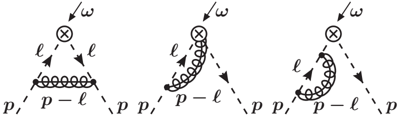
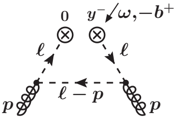
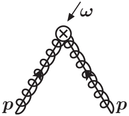
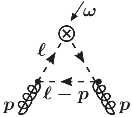


$= \mathcal{I}^{(1)}_{qq} \otimes$  $+\mathcal{I}^{(0)}_{qq} \otimes \Big($  $\Big)$

 $= \mathcal{I}^{(1)}_{qg} \otimes$  $+\mathcal{I}^{(0)}_{qq} \otimes$ 

One-Loop Matching Result

$$B_i(t,x,\mu) = \sum_j \int \frac{\mathrm{d}\xi}{\xi}\, \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mu\Big) f_j(\xi,\mu)$$

$$\begin{aligned}\mathcal{I}_{qq}(t,z,\mu) &= \delta(t)\,\delta(1-z) + \frac{\alpha_s(\mu)C_F}{2\pi}\,\theta(z)\\ &\quad\times\Big\{\frac{2}{\mu^2}\Big(\frac{\ln t/\mu^2}{t/\mu^2}\Big)_+ \delta(1-z) + \frac{1}{\mu^2}\Big(\frac{1}{t/\mu^2}\Big)_+ \Big[P_{qq}(z) - \frac{3}{2}\,\delta(1-z)\Big]\\ &\quad + \delta(t)\Big[\Big(\frac{\ln(1-z)}{1-z}\Big)_+ (1+z^2) - \frac{\pi^2}{6}\,\delta(1-z) + \theta(1-z)\Big(1-z-\frac{1+z^2}{1-z}\ln z\Big)\Big]\Big\}\end{aligned}$$

$$\mathcal{I}_{qg}(t,z,\mu) = \frac{\alpha_s(\mu)T_F}{2\pi}\,\theta(z)\Big\{\frac{1}{\mu^2}\Big(\frac{1}{t/\mu^2}\Big)_+ P_{qg}(z) + \delta(t)\Big[P_{qg}(z)\Big(\ln\frac{1-z}{z} - 1\Big) + \theta(1-z)\Big]\Big\}$$

- $P_{qq}(z)$ and $P_{qg}(z)$ are the AP splitting functions

One-Loop Matching Result

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- $P_{qq}(z)$ and $P_{qg}(z)$ are the AP splitting functions
- Large double logs

$$\int^{t_{\max}} \mathrm{d}t\,\frac{1}{\mu^2}\Big(\frac{\ln t/\mu^2}{t/\mu^2}\Big)_+ = \frac{1}{2}\ln^2\frac{t_{\max}}{\mu^2}$$

- Should evaluate $B = \mathcal{I} \otimes f$ at $\mu \simeq \sqrt{t}$

Quark Beam Function: Renormalization

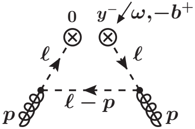
- Use the RGE to evolve to a different scale, which sums these logs

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} B_i(t, x, \mu) = \int \mathrm{d}t' \, \gamma_B^i(t - t', \mu) \, B_i(t', x, \mu)$$

The anomalous dimension is

$$\gamma_B^i(t, \mu) = -2\Gamma_{\mathrm{cusp}}^i(\alpha_s) \frac{1}{\mu^2} \left(\frac{1}{t/\mu^2} \right)_+ + \gamma_B^i(\alpha_s) \, \delta(t)$$

- No mixing between B_q and B_g,
  Mixing graph has no UV divergences
- Structure at all orders follows from Wilson-line renormalization and consistency of the RGE



- Very different from PDF evolution which sums single logs and changes ξ

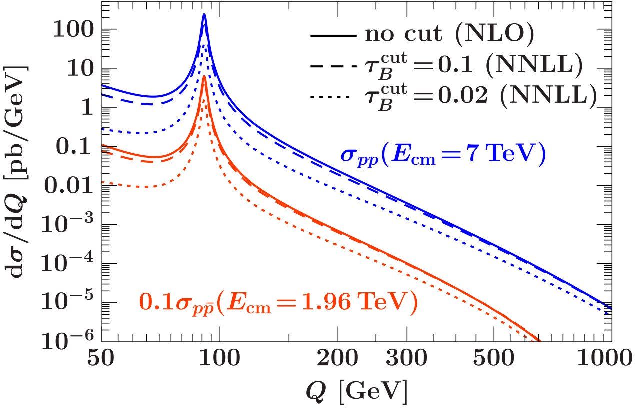
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} f_i(\xi, \mu) = \sum_k \int \frac{\mathrm{d}\xi'}{\xi'} P_{jk}\left(\frac{\xi}{\xi'}, \mu \right) f_k(\xi', \mu)$$

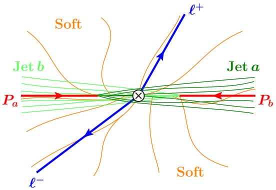
Drell-Yan and Higgs with Jet Veto

Drell-Yan With Jet Veto

Compare cross section [using MSTW2008 NLO PDFs]

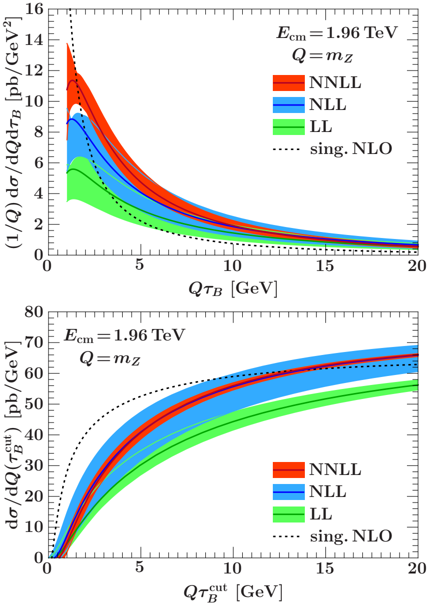
- without a cut at NLO
- with jet veto $\tau_B^{\text{cut}} = \{0.1, 0.02\}$ at NNLL





$pp \to X\ell^+\ell^-$

Beam Thrust Cross Section for Drell-Yan



- Most of the radiation at small τ_B
- Summation of $\ln \tau_B$ important
- NLO is singular in IR ($\tau_B \to 0$) Regulated by resummation
- Soft function perturbative in tail, nonperturbative below peak
- Resummed perturbation series converges

	match	running non-cusp	running cusp
NLO	1-loop	-	-
LL	0-loop	-	1-loop
NLL	0-loop	1-loop	2-loop
NNLL	1-loop	2-loop	3-loop

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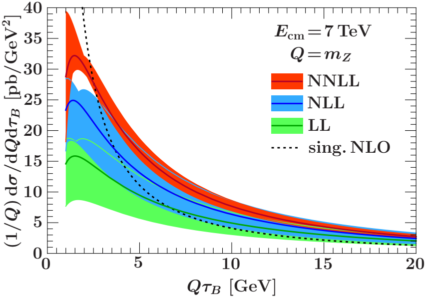

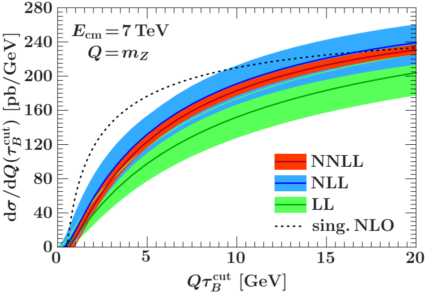

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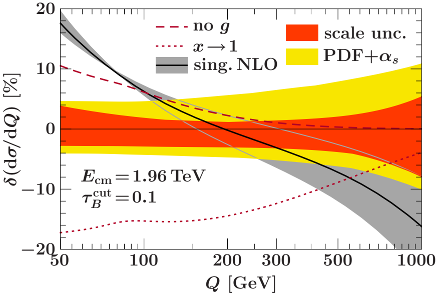
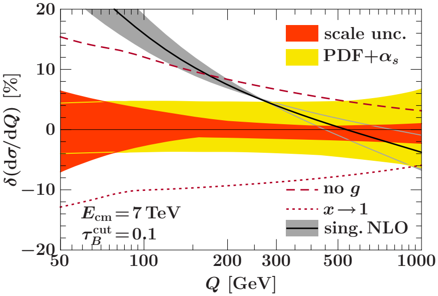
Drell-Yan Theory Uncertainties





Uncertainties relative to NNLL

- Scale uncertainties envelope of separate μ_H, μ_B, μ_S variation
- NLO uncertainty does not capture difference with NNLL

$\longrightarrow$ Need resummation for central value and reliable uncertainties

Higgs Production With Jet Veto

- Use jet veto to remove $t\bar{t}$ background for $H \to WW \to \ell\nu\ell\bar{\nu}$
- Don't know m_H and cannot measure Y, so we use

$$\tau_B = \sum_k \frac{|\vec{p}_{k\,T}|}{m_H} e^{-|\eta_k - Y|} \quad \longrightarrow \quad \mathcal{T}_B^{\text{cm}} = \sum_k |\vec{p}_{k\,T}|\, e^{-|\eta_k|}$$

- $\mathcal{T}_B^{\text{cm,cut}} \sim 10\,\mathrm{GeV}$ similar background rejection as jet algorithm veto
- Narrow width approximation for Higgs

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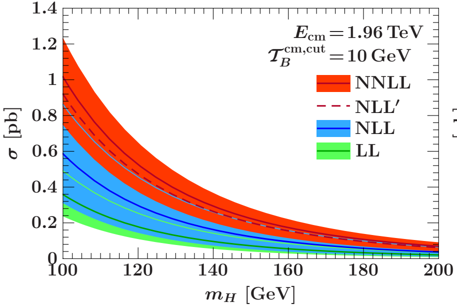
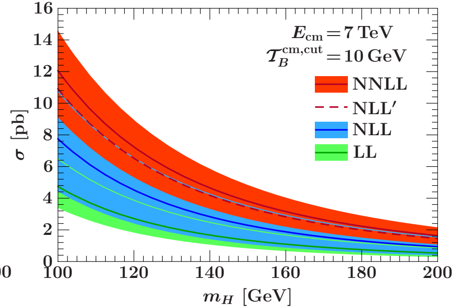
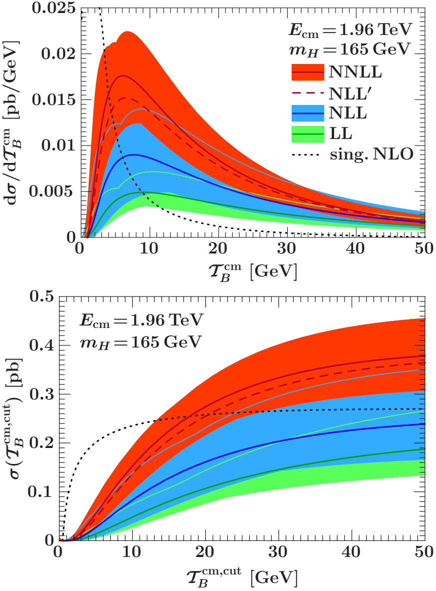
$gg \to H$ with a jet veto





Beam Thrust Cross Section for Higgs Production



Similar features as Drell-Yan, but...

- Gluons instead of quarks: $C_F \to C_A$
- Resummation of $\ln \mathcal{T}_B^{\rm cm}/m_H$ more important
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- Radiation peaked at larger $\mathcal{T}_B^{\rm cm}$ and larger tail

  $\rightarrow$ more affected by jet veto

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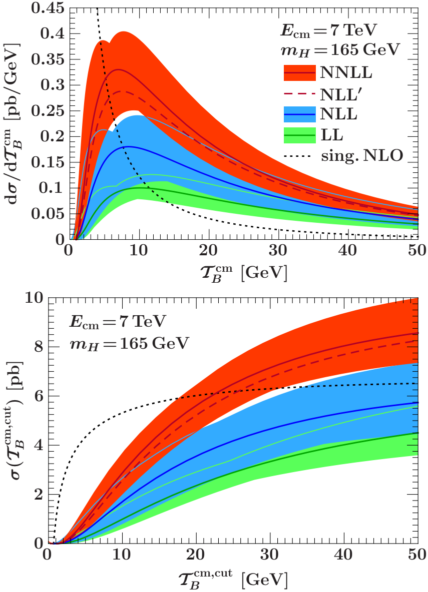

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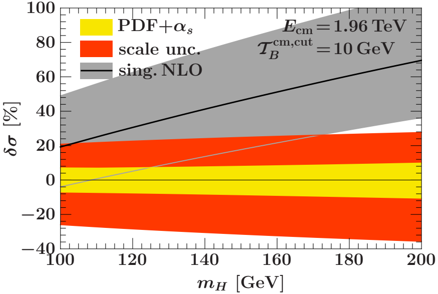

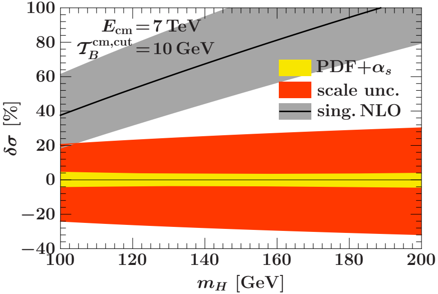

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N-Jettiness

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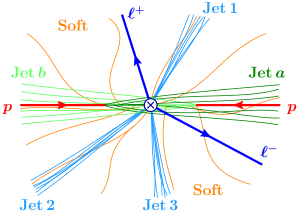

N-jet signal

- Find signal jets using a jet algorithm
  - $< N$ jets: not signal $\longrightarrow$ throw away
  - N jets: could be signal $\longrightarrow$ keep
  - $> N$ jets: likely background $\longrightarrow$ throw away
- kinematic variable better than jet algorithm for veto in calculations

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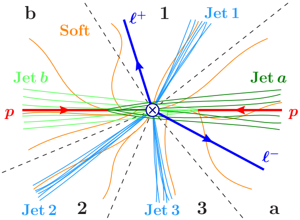

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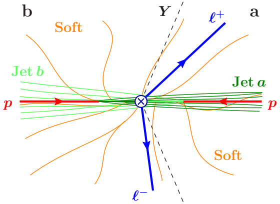
N-jettiness
$$\tau_N = \frac{2}{Q^2} \sum_k \min\Big\{ q_a \!\cdot\! p_k, q_b \!\cdot\! p_k, q_1 \!\cdot\! p_k, \ldots, q_N \!\cdot\! p_k \Big\}$$

- $q_i = q_i^0(1, \hat{n}_i)$ are massless reference momenta for the beams and jets
- Q scale of hard interaction

$$q_i \!\cdot\! p_k = q_i^0 E_k (1 - \cos\theta_{ik})$$

- "$\min$" associates particles with closest beam or jet
- Large contributions to τ_N from particles with $E_k \sim Q$ *and all* $\theta_{ik} \sim 1$ $\longrightarrow \tau_N \ll 1$ vetos additional unwanted jets

Examples of N-Jettiness



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$N = 0$ for Drell-Yan: $\quad \tau_0 = \tau_B$ for $q^\mu_{a,b} = Q e^{\pm Y}/2\,(1, 0, 0, \pm 1)$

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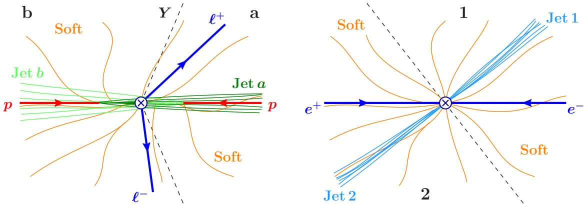

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$N = 2$ for $e^+e^- \to jets$

- choose $q^\mu_{1,2} = Q/2\,(1,\pm\hat{t})$ where $\hat{t}$ is thrust axis
- $\tau_2^{ee} = 1 - T$ where T is thrust, in two-jet limit $\tau_2^{ee} \ll 1$

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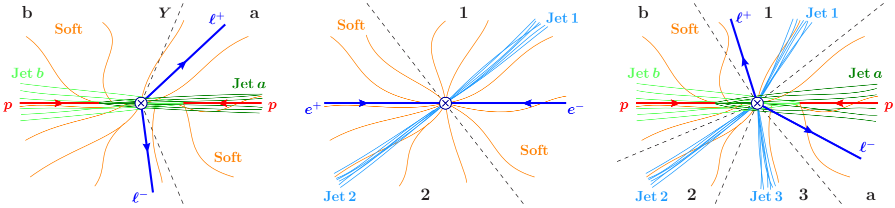

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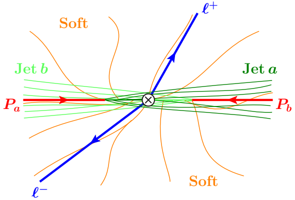
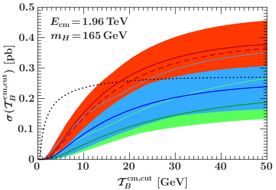
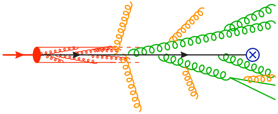
Nice properties of τ_N

- Jet algorithm dependence is power suppressed $\tau_N^{\text{alg.1}} = \tau_N^{\text{alg.2}} + \mathcal{O}(\tau_N^2)$
- Collinear radiation always associated with its own beam or jet
  $\longrightarrow$ Inclusive beam and jet functions

Conclusions







Measurements need to veto jets

- Reduce background in $H \to WW \to \ell\nu\ell\bar{\nu}$
- Study initial-state radiation (compare exp., MC)
- Jet veto leads to large logs, must be summed

Factorization with a Jet Veto

- Use beam thrust τ_B as jet veto (no jet alg.)
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- Describe extracting parton out of proton and formation of initial-state jet
- Match onto PDFs $B = \mathcal{I} \otimes f$

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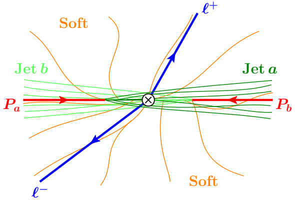

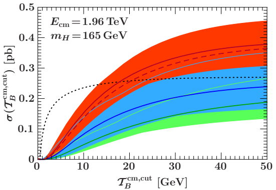

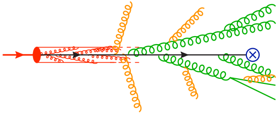

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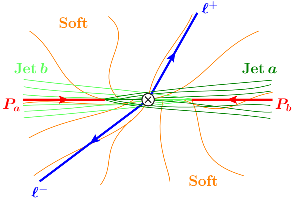

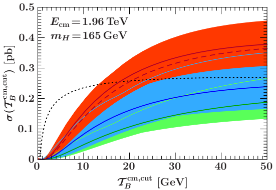

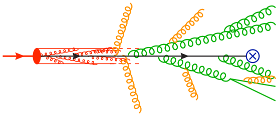

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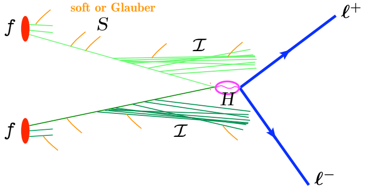
Towards Factorization at the LHC

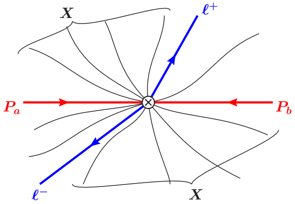
Factorization separates scales:

- Hard new physics from Standard Model
- Pertubative from nonperturbative physics
- Allows us to sum large logarithms



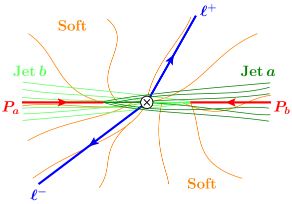
Archetypal case: **Drell-Yan** $pp \to X\ell^+\ell^-$

Inclusive



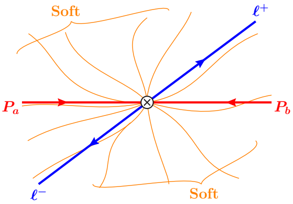
Inclusive in X
Can't study ISR

"Isolated"



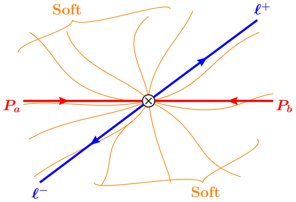
No central jets

Threshold



Only soft radiation
Tiny cross section

Threshold Drell-Yan



Threshold limit

$$\tau = \frac{Q^2}{E_{\rm cm}^2} \longrightarrow 1$$

$$Y \longrightarrow 0$$

$$1 \geq \xi_{a,b} \geq x_{a,b} \longrightarrow 1$$

- Large logs: $\sigma \sim \sigma_0[1 + \alpha_s \ln^2(1-\tau) + \alpha_s^2 \ln^4(1-\tau) + \dots]$
- Factorization theorem:

$$\frac{d\sigma}{dQ^2} = \sum_{ij=q\bar{q},\bar{q}q} H_{ij}^{\rm thr}(Q^2,\mu) \int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu) S^{\rm thr}\Big[Q\Big(1-\frac{\tau}{\xi_a\xi_b}\Big),\mu\Big]$$

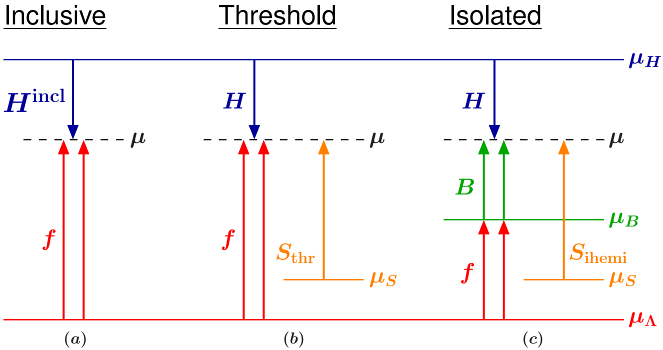
- Resum logs using RGE:

$$H^{\rm thr} \sim 1 + \alpha_s \ln^2 Q/\mu^2 + \dots \qquad S^{\rm thr} \sim 1 + \alpha_s \ln^2[(1-\tau)Q^2/\mu^2] + \dots$$

- There are $\mathcal{O}(1-\tau)$ corrections, so only valid for $\tau \to 1$.

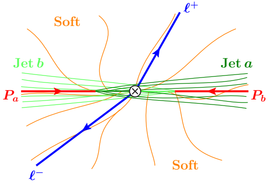
  [Becher, Bonciani, Campbell, Catani, Chiu, de Florian, Grazzini, Huston, Kelley, Kidonakis, Laenen, Mangano, Manohar, Nason, Neubert, Oderda, Sterman, Stirling, Trentadue, Trott, Xu, ...]

Resummation and the RGE



- Inclusive: can use the same PDFs for different μ_H
- Threshold: resum $\ln^2(1-\tau) = \ln^2(1 - Q^2/E^2_{\mathrm{cm}})$
- Isolated: resum $\ln^2 \tau_B$
- Expect beam functions **in general** from RGE consistency, when constraining hadronic final state and away from threshold

How to Veto Jets



Option 1: use a jet algorithm

- Run a jet algorithm to find all jets in the event
- Reject events containing jets with e.g. $p_T > 15\,\mathrm{GeV}$ and $|\eta| < 2.5$
- Phase-space restrictions lead to large logarithms. Schematically:

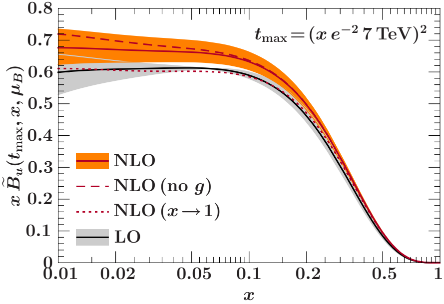
$$\sigma = \sigma_0(1 + \alpha_s \ln^2 p_T^{\mathrm{cut}} + \alpha_s^2 \ln^4 p_T^{\mathrm{cut}} + \dots)$$

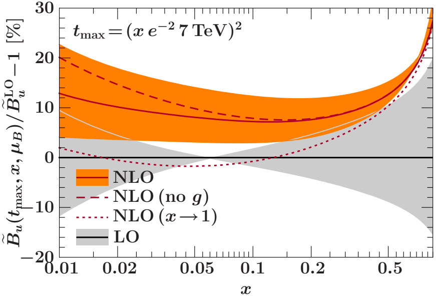
- Phase-space restrictions are very complicated: Usually rely on parton shower Monte Carlo to resum leading logs.

Option 2: use a kinematic variable

- Phase-space restrictions easier in calculations
- Systematic resummation of phase-space logs (beyond leading log)
- Theory treatment of soft effects (beyond hadronization and underlying event models)

Up-Quark Beam Function at NLO

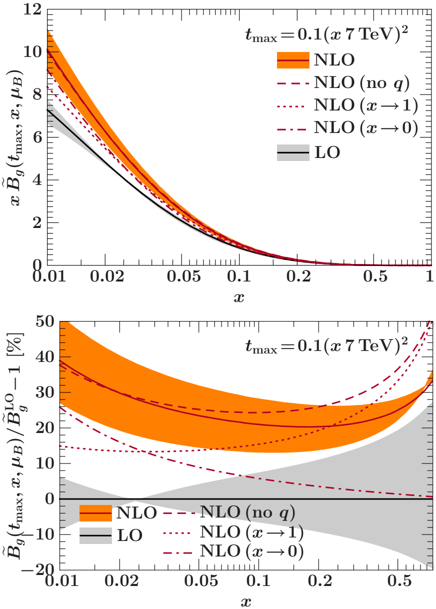




Up Quark

- We plot
  $\widetilde{B}(t_{\max}, x, \mu) = \int^{t_{\max}} B(t, x, \mu)$
  $\sqrt{t_{\max}} = e^{-y_{\mathrm{cut}}} Q = e^{-2} x 7\,\mathrm{TeV}$
- Vary $\sqrt{t_{\max}}/2 \leq \mu \leq 2\sqrt{t_{\max}}$
- Absolute and relative plots
- Perturbative corrections $\mathcal{O}(10\%)$
- Gluon contribution $\mathcal{I}_{qg}$ important for $x \lesssim 0.1$ (Gluon PDF is steeper)

Gluon Beam Function at NLO



Gluon

- Much larger NLO corrections since $C_A/C_F = 9/4$
- Quark contribution important for $x \gtrsim 0.1$, but here B_g is small