

A New Way to Measure Spin

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work with Michael Ramsey-Musolf,
U. Wisconsin. [arxiv:1008.5151](https://arxiv.org/abs/1008.5151)

New Energies, New Particles

- What will we find at the Terascale?
- Most of the proposed UV completions have similar signatures at hadron colliders:
 - Strongly coupled states
 - Pair creation enforced by new symmetry
 - Missing energy from dark matter candidate

New Energies, New Particles

- What will we find at the Terascale?
 - Supersymmetry?
 - Extra Dimensions?
 - Technicolor?
 - None of the above?
- Most of the proposed UV completions have similar signatures at hadron colliders:
 - Strongly coupled states
 - Pair creation enforced by new symmetry
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Breaking the Degeneracy

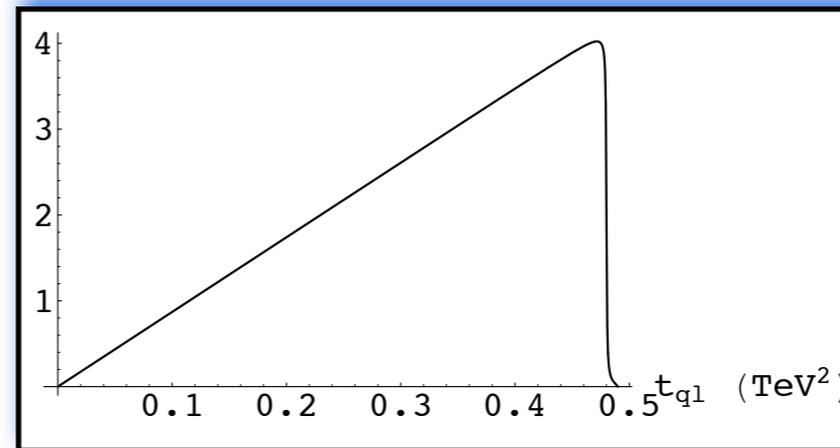
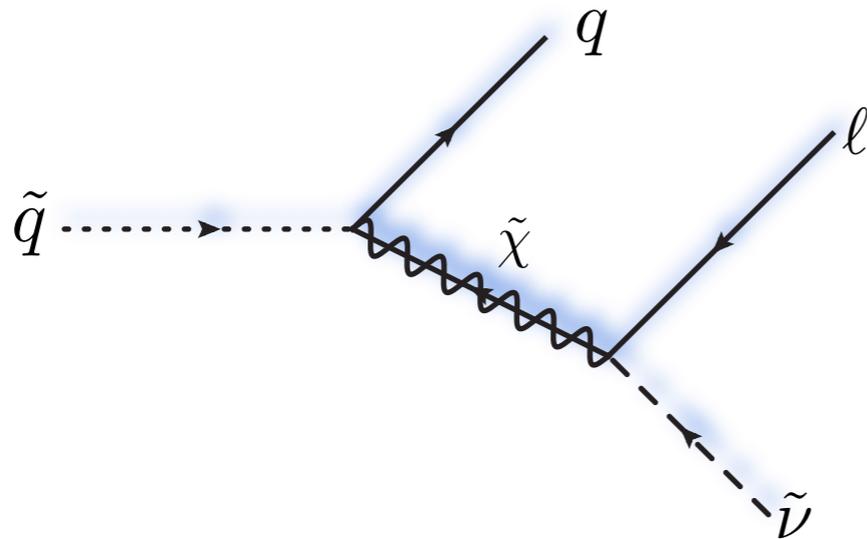
- To determine the underlying theory, we need to know the basic properties of the newly discovered particles:
 - Mass hierarchy
 - Decay channels
 - SM gauge charges
 - Spin

Spin Measurements

- Many techniques devised distinguish specific models from one another:
 - Total cross section: *i.e.* $\sigma_{\text{SUSY}} < \sigma_{\text{UED}}$
 - Spectrum of states: *i.e.* tower of Kaluza-Klein modes in UED
- If you had a linear collider, can use threshold scans,
 - Both scalars and vector bosons have $\sigma \propto \beta$

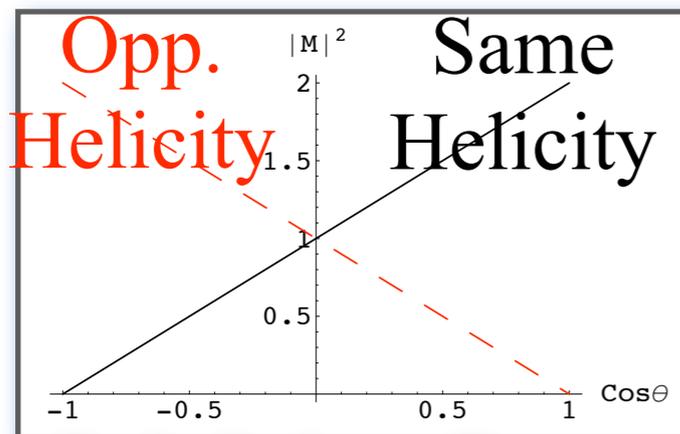
Spin Measurements

- Other techniques look at long decay chains

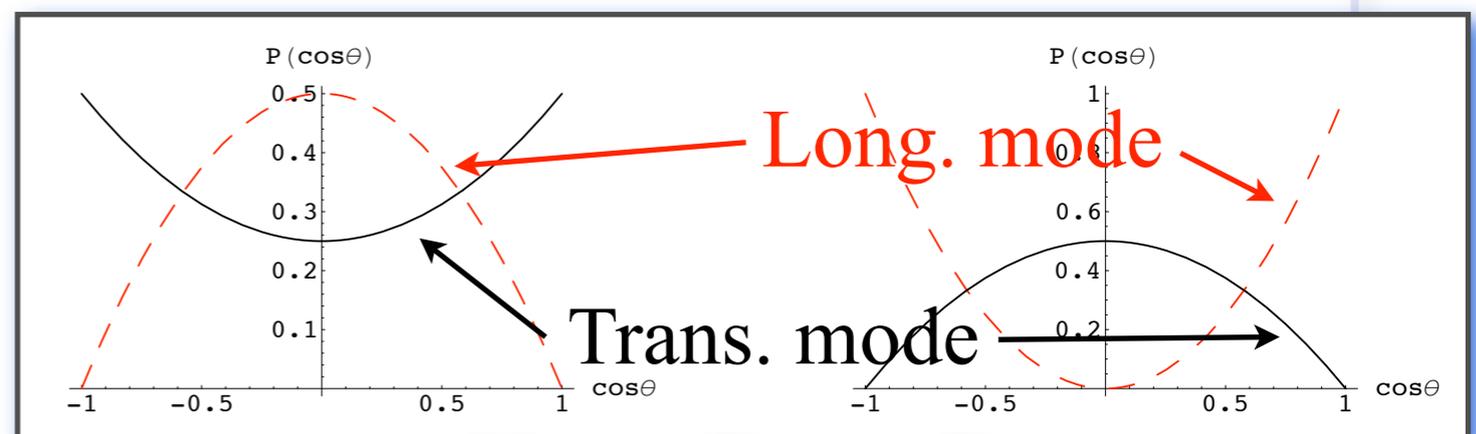


hep-ph/0605296

- But this requires chiral couplings, and depends on the spins of the final states (perhaps unknown).



Pol. Spinor Decay



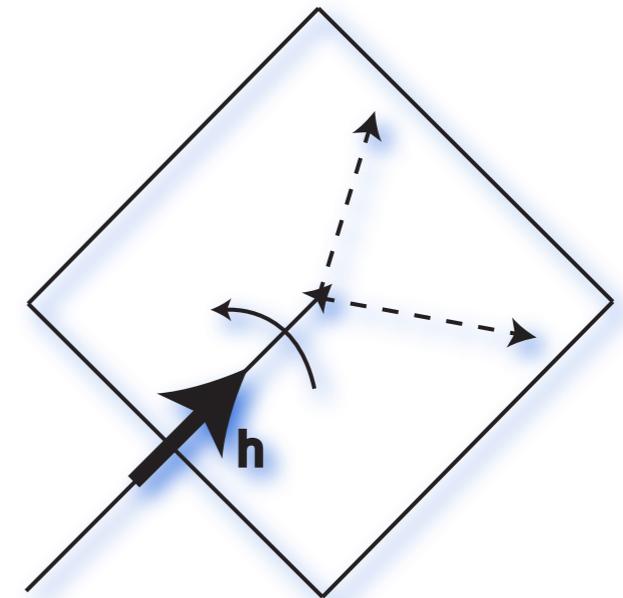
Vector Boson Decay

Model Independence

- Interfering helicity states of a decaying particle leave an imprint in the decay plane's distribution

$$\mathcal{M} \propto e^{iJ_z\phi} = e^{ih\phi}$$

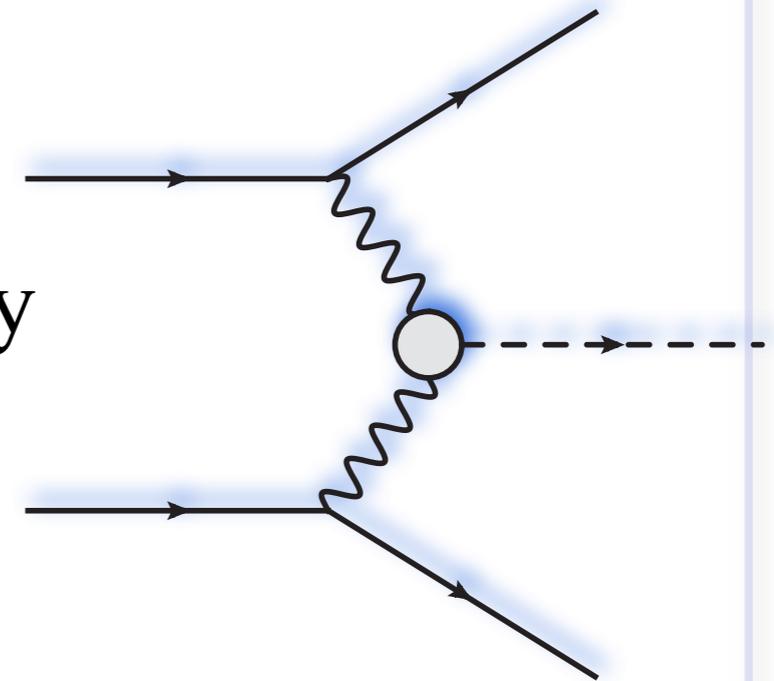
$$\begin{aligned} |\mathcal{M}|^2 &= \left| \sum_h e^{ih\phi} \mathcal{M}_h(\phi=0) \right|^2 \\ &= A_0 + \dots + A_n \cos n\phi \\ n &= 2 \times \text{spin} \end{aligned}$$



- ‘Model Independent,’ but still requires reconstruction of decay plane which is not always possible.

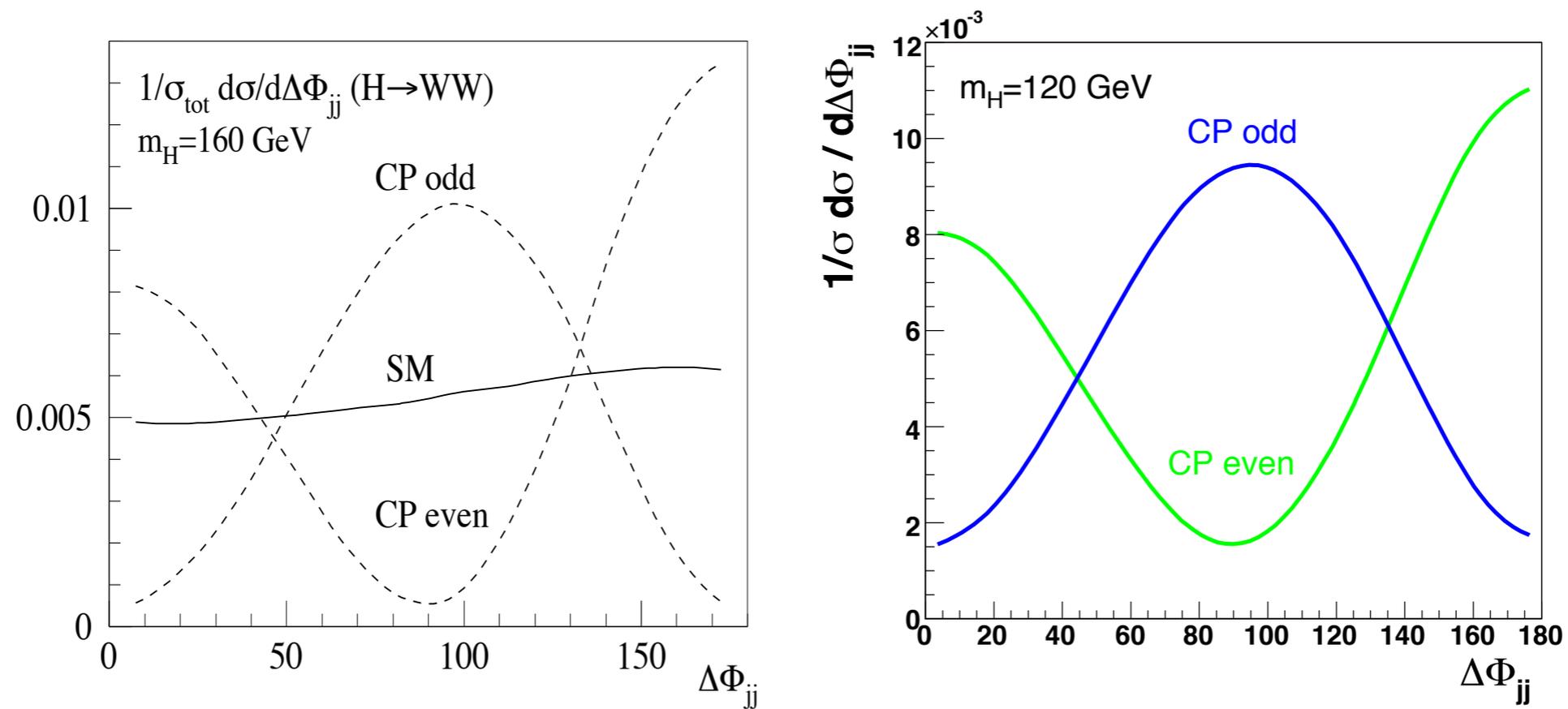
Inspiration from Higgs Measurements

- Proposal from Zeppenfeld *et. al.*
- Consider on-shell Higgs production from Vector Boson Fusion (VBF)
- Azimuthal angular dependence comes from gauge boson helicity
- Presence of various cos/sin modes depends on how these helicities can be combined.
- *i.e.* on the Lorentz structure of the matrix element for Higgs production



Inspiration from Higgs Measurements

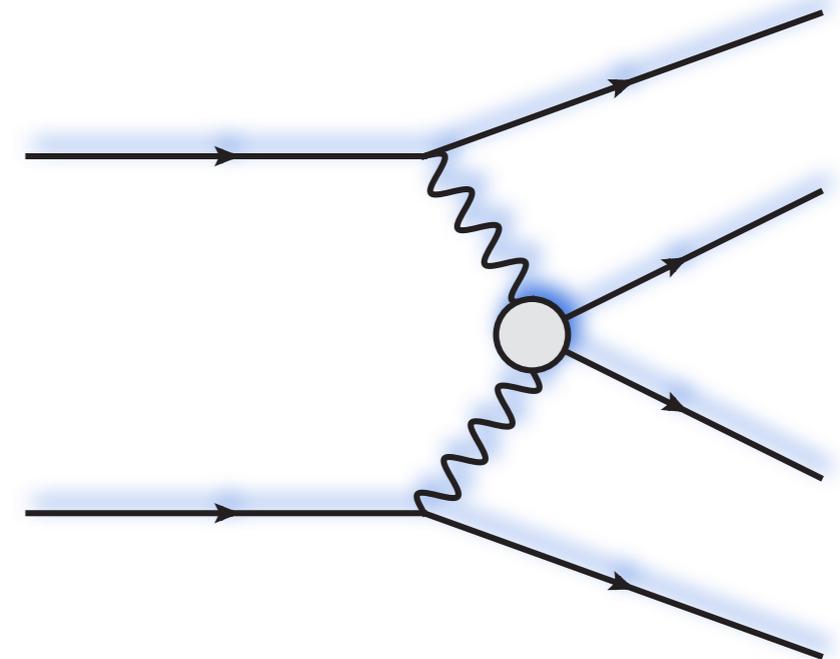
- It's been shown that $d\sigma/d\Delta\phi$ sensitive to CP-properties of Higgs coupling



- SM background has no $\cos 2\Delta\phi$ mode.

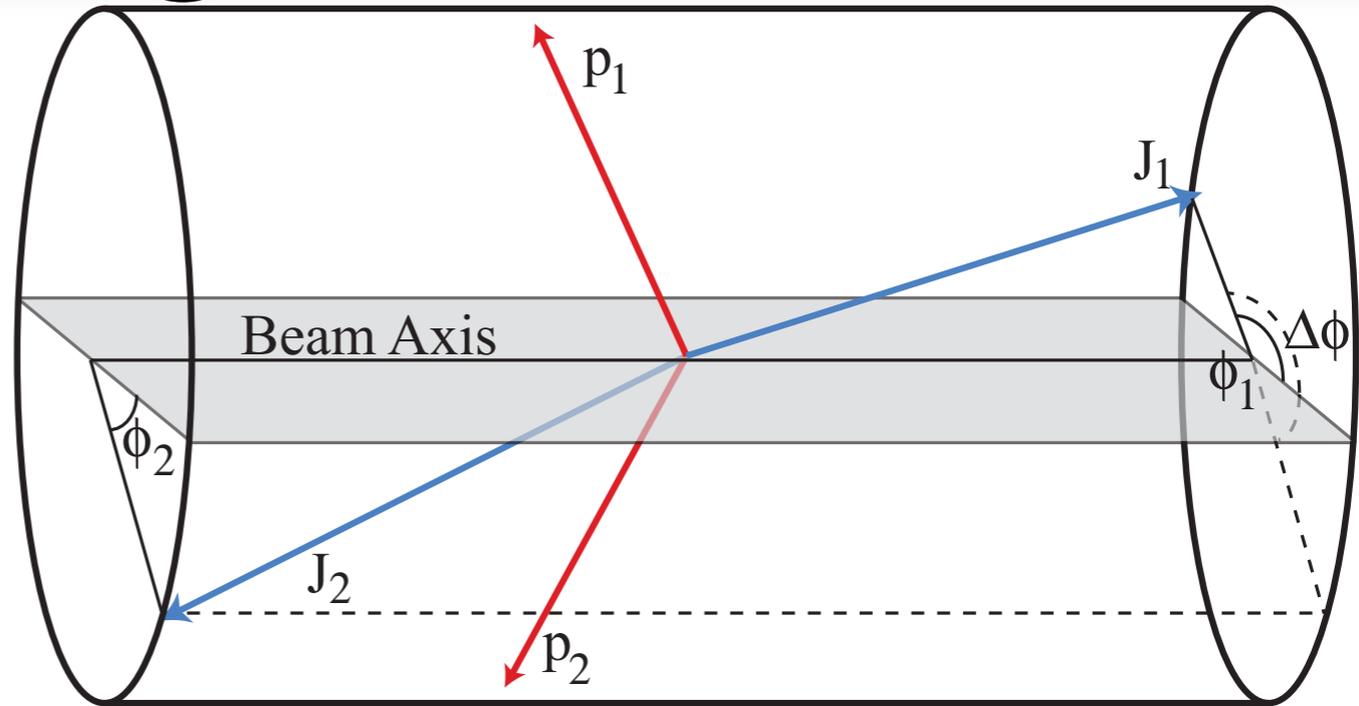
A New Way to Measure Spin

- Generalize from single-particle production to pair-production.
- Lorentz structure of pair-production matrix element fixes which combinations of vector boson polarizations can contribute to process
- This is made measurable through the *difference* in azimuthal angle of the forward jets $\Delta\phi$



The Big Picture

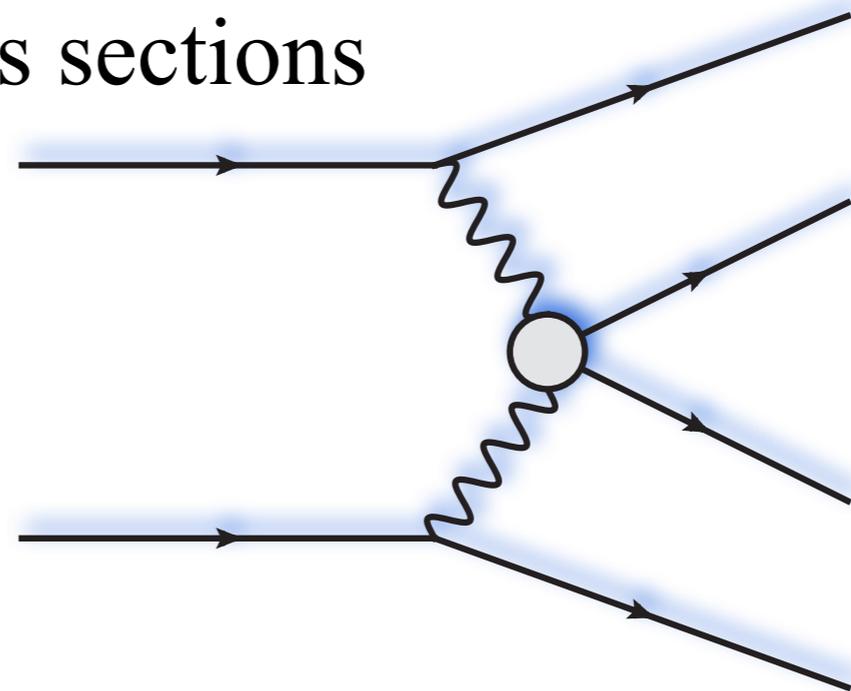
- Since the spin measurement relies on the kinematics of jets J_1/J_2 , the only requirements on new physics (p_1/p_2) is that we can trigger on it (and identify the forward jets)
- For this introductory study, we assume both these problems can be ignored
- Clearly, we are not experimentalists.



Scalars vs. Spinors

- Choose simple models: 500 GeV ‘R-hadron’ $SU(3)_C$ triplets with spin-0 or spin-1/2
- Looking at 2 R-hadrons + 2 jet events
- Avoids trigger and background problems
- Large production cross sections

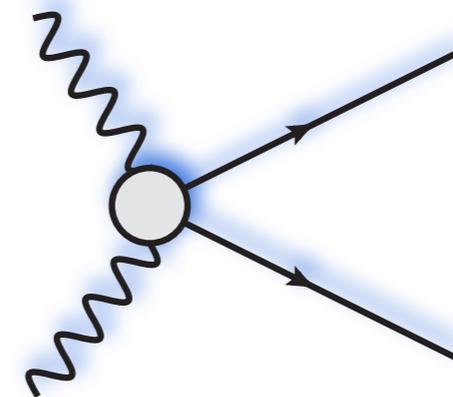
We’ll consider the VBF production matrix elements



Scalars vs. Spinors

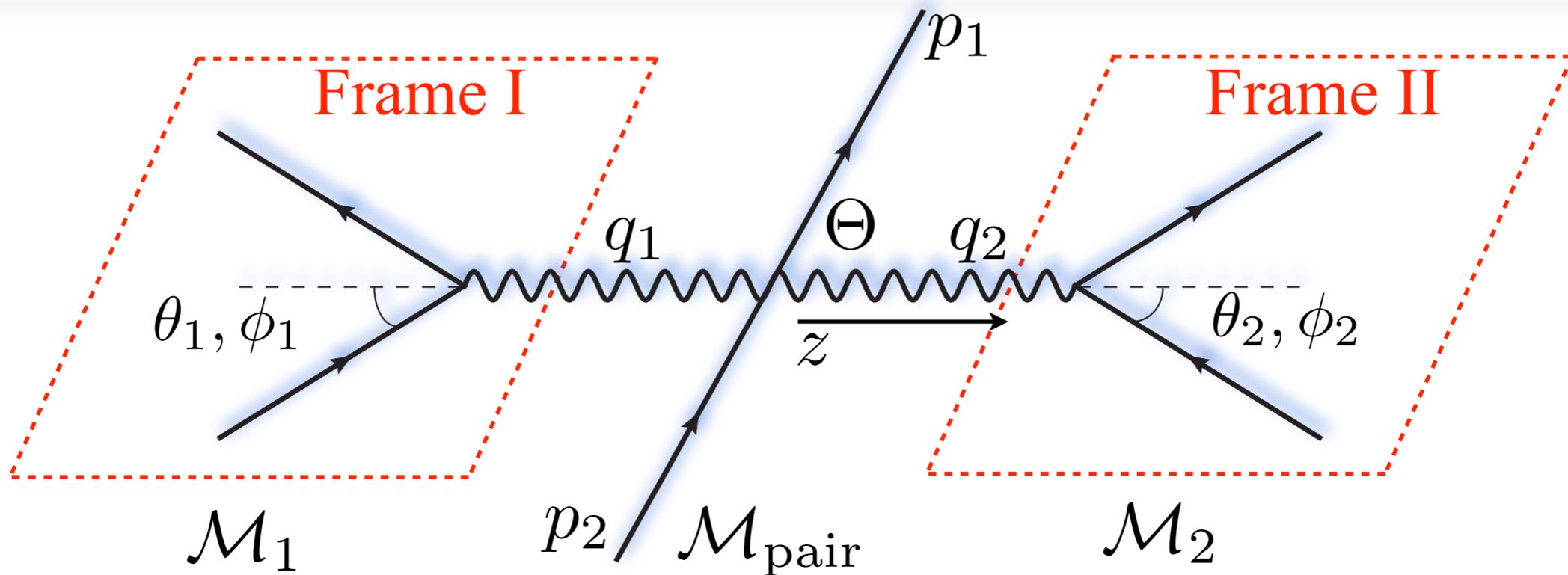
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production matrix elements



(This turns out to be an over-simplification)

VBF Kinematics



- With these choices, ϕ dependence made clear:

$$\epsilon_{1/2} \propto e^{+i\phi_{1/2}}, e^{-i\phi_{1/2}}, e^{0 \times i\phi_{1/2}}$$

- I've drawn quark initial states only, but anti-quark and gluon contribute as well.

Azimuthal Angle

- Can expand out dependence on ϕ_1, ϕ_2 :

$$|\mathcal{M}|^2 = \left| \sum_{h_1, h_2 = \pm, 0} \mathcal{M}_1 \mathcal{M}_2 \mathcal{M}_{\text{pair}} e^{i(h_1 \phi_1 + h_2 \phi_2)} \right|^2$$

$$\begin{aligned} |\mathcal{M}|^2 = & f_1 + f_2 \cos \phi_1 + f_3 \cos \phi_2 + f_4 \cos 2\phi_1 + f_5 \cos 2\phi_2 \\ & + f_6^+ \cos(\phi_1 + \phi_2) + f_6^- \cos(\phi_1 - \phi_2) + f_7^+ \cos(2\phi_1 + \phi_2) + f_7^- \cos(2\phi_1 - \phi_2) \\ & + f_8^+ \cos(\phi_1 + 2\phi_2) + f_8^- \cos(\phi_1 - 2\phi_2) + f_9^+ \cos 2(\phi_1 + \phi_2) + f_9^- \cos 2(\phi_1 - \phi_2) \\ & + \bar{f}_2 \sin \phi_1 + \bar{f}_3 \sin \phi_2 + \bar{f}_4 \sin 2\phi_1 + \bar{f}_5 \sin 2\phi_2 \\ & + \bar{f}_6^+ \sin(\phi_1 + \phi_2) + \bar{f}_6^- \sin(\phi_1 - \phi_2) + \bar{f}_7^+ \sin(2\phi_1 + \phi_2) + \bar{f}_7^- \sin(2\phi_1 - \phi_2) \\ & + \bar{f}_8^+ \sin(\phi_1 + 2\phi_2) + \bar{f}_8^- \sin(\phi_1 - 2\phi_2) + \bar{f}_9^+ \sin 2(\phi_1 + \phi_2) + \bar{f}_9^- \sin 2(\phi_1 - \phi_2) \end{aligned}$$

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$$|\mathcal{M}|^2 = f_1 + f_6^- \cos \Delta\phi + f_9^- \cos 2\Delta\phi$$

After integrating out

f_9^-

- We will show that f_6^- gets a contribution from cuts.
- Look at the coefficient of $\cos 2\Delta\phi$ instead.

- From
$$|\mathcal{M}|^2 = \left| \sum_{h_1, h_2 = \pm, 0} \mathcal{M}_1 \mathcal{M}_2 \mathcal{M}_{\text{pair}} e^{i(h_1 \phi_1 + h_2 \phi_2)} \right|^2$$

we're interested in

$$f_9^- = (\mathcal{PS}) \sum_{\substack{h_1, h'_1, h_2, h'_2 \\ |h_i - h'_i| = 2}} \left(\mathcal{M}_1(h_1) \mathcal{M}_2(h_2) \mathcal{M}_{\text{pair}}(h_1, h_2) \right) \times \\ \left(\mathcal{M}_1(h'_1) \mathcal{M}_2(h'_2) \mathcal{M}_{\text{pair}}(h'_1, h'_2) \right)^*$$

VBF Cuts

- Our measurement technique requires that we isolate VBF events from other production
- Use the cuts from Zeppenfeld *et. al.*
 - R-hadron cuts to ensure they remain in the barrel

$$\eta_{j_1} \cdot \eta_{j_2} < 0, \quad |\eta_j| \leq 5, \quad |\eta_{j_1} - \eta_{j_2}| \geq 4.2$$

$$p_{T,j_1} \geq 30 \text{ GeV}, \quad p_{T,j} \geq 20 \text{ GeV}, \quad M_{jj} \geq 500 \text{ GeV}$$

$$|\eta_{R\text{-hadron}}| < 2.1, \quad p_{T,R\text{-hadron}} > 50 \text{ GeV}$$

Scalars

- Factoring out production matrix elements:

$$f_9^- = (\mathcal{PS})\mathcal{M}_1(+1)\mathcal{M}_1(-1)^*\mathcal{M}_2(-1)\mathcal{M}_2(+1)^* \\ \times \left[\mathcal{M}_{\text{pair}}(+1, -1)\mathcal{M}_{\text{pair}}(-1, +1)^* + (1 \leftrightarrow -1) \right]$$

- In an abelian theory, easy to write down the matrix elements for transverse polarizations:

$$\mathcal{M}_{\text{scalar}} \propto (\epsilon_1 \cdot \epsilon_2) - 4 \left[\frac{(p_1 \cdot \epsilon_1)(p_1 - q_1) \cdot \epsilon_2}{q_1^2 - 2p_1 \cdot q_1} + \frac{(p_1 \cdot \epsilon_2)(p_1 - q_2) \cdot \epsilon_1}{q_2^2 - 2p_1 \cdot q_2} \right]$$

- Invariant under $\epsilon_{1/2}^+ \leftrightarrow \epsilon_{2/1}^-$
- (Also true in non-abelian calculation)

Scalars

- Since $\mathcal{M}(+1, -1) = \mathcal{M}(-1, +1)$,

$$\underline{f_9^- \propto \mathcal{M}(+1, -1)\mathcal{M}(-1, +1)^* > 0}$$

- Prediction: $\cos 2\Delta\phi$ mode for scalars must be positive
- Recall this is a consequence of how polarization vectors enter into $\mathcal{M}_{\text{pair}}$ calculation
 - *i.e.* Lorentz structure

Spinors

- Need to repeat the calculation for spinors
 - (Spoiler: f_9^- will be negative)
- Straightforward for on-shell abelian example:

$$\mathcal{M}(\pm 1, \mp 1) = i\bar{u} \left[\frac{\not{\epsilon}_1^\pm (\not{p}_1 - \not{q}_1 + M) \not{\epsilon}_2^\mp}{q_1^2 - 2p_1 \cdot q_1} + \frac{\not{\epsilon}_2^\mp (\not{p}_1 - \not{q}_2 + M) \not{\epsilon}_1^\pm}{q_2^2 - 2p_1 \cdot q_2} \right] v$$

- After lots of tediousness, you can prove that

$$f_9^- \propto -\frac{64m^2}{s} \left(1 + \frac{4m^2}{s\sqrt{1-4m^2/s}} \tanh^{-1} \sqrt{1-4m^2/s} \right) < 0$$

(Remember: Abelian)

Non-abelian Spinors

- Can divide $\mathcal{M}(+1, -1)\mathcal{M}(-1, +1)^*$ into symmetric ($d_{abc}d_{abd}T^cT^d$) and antisymmetric ($f^{abc}f^{abd}T^cT^d$) parts
- Symmetric part reproduces the abelian example:

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- Simulations of full process find $f_9^- < 0$
- Naive calculation of antisymmetric part finds positive and *large* contribution

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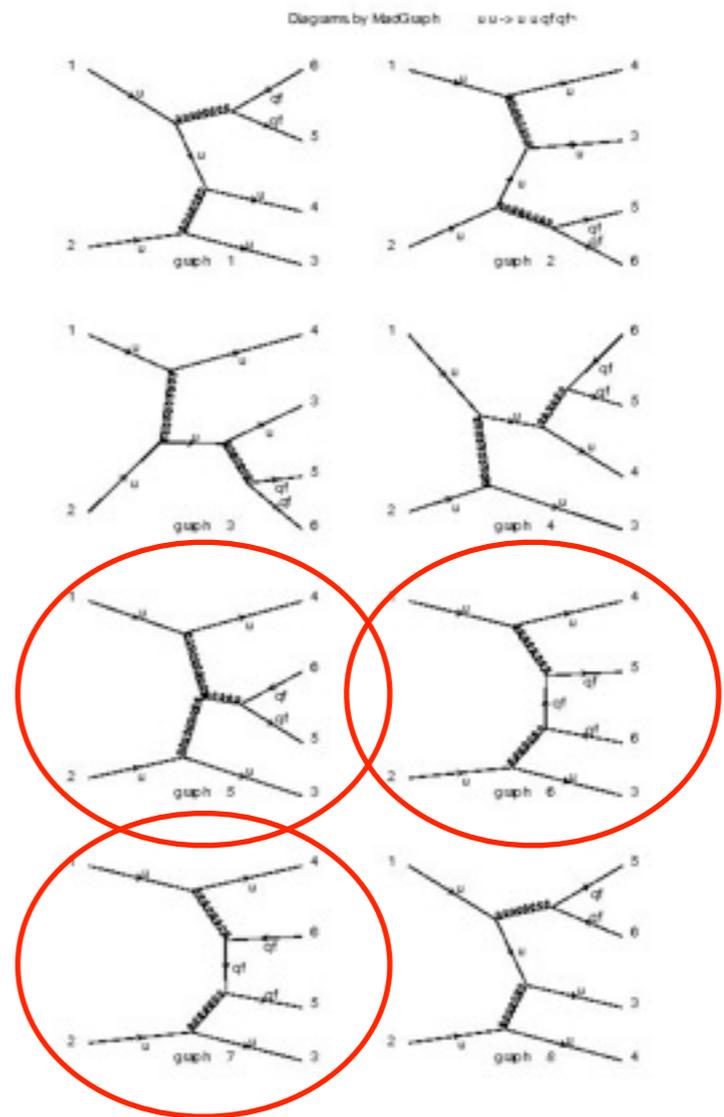
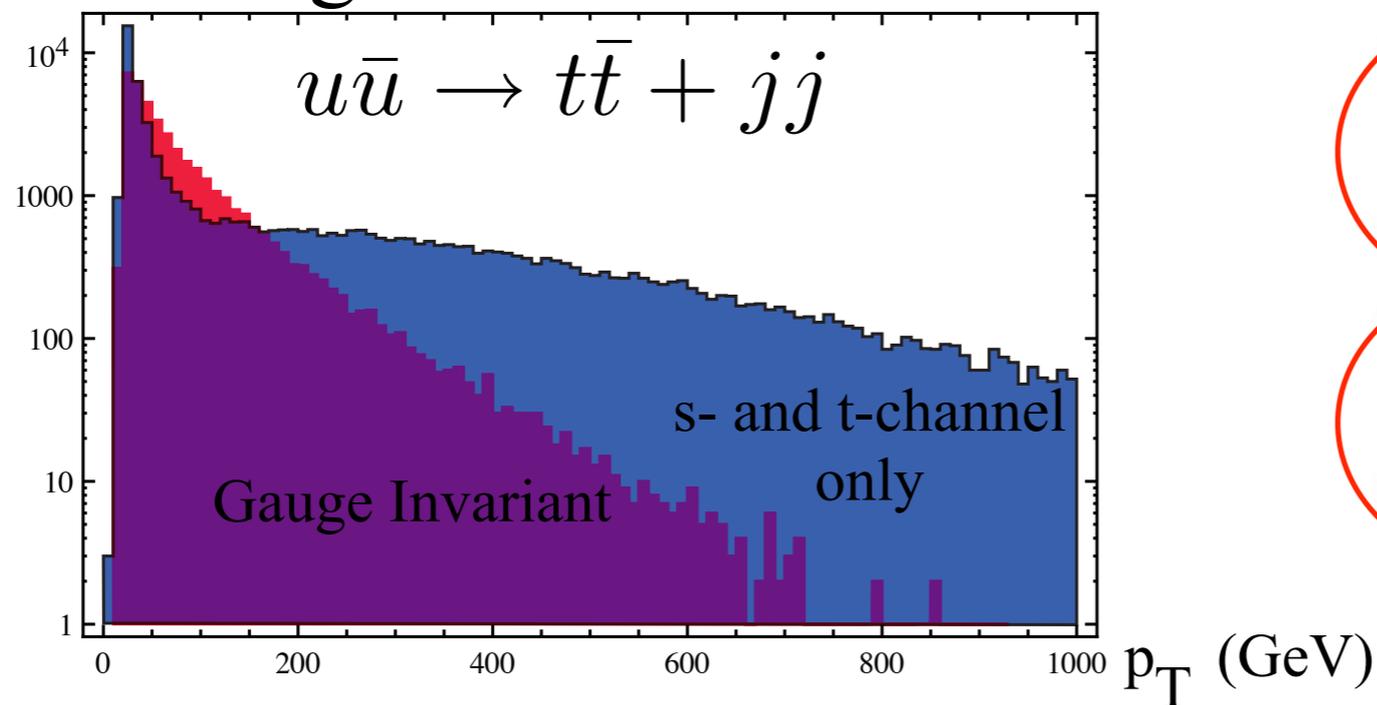
WHAT?

Non-abelian Spinors

- Naive calculation ignores cuts (integrates over all of phase space) and only used VBF t - and s -channel diagrams (not gauge invariant)
- Analytic calculation can't easily take into account the cuts
 - *i.e.* 4-body phase space is hard
- Full calculation involves hundreds of diagrams
- Rather than reinvent the wheel, use Calchep!

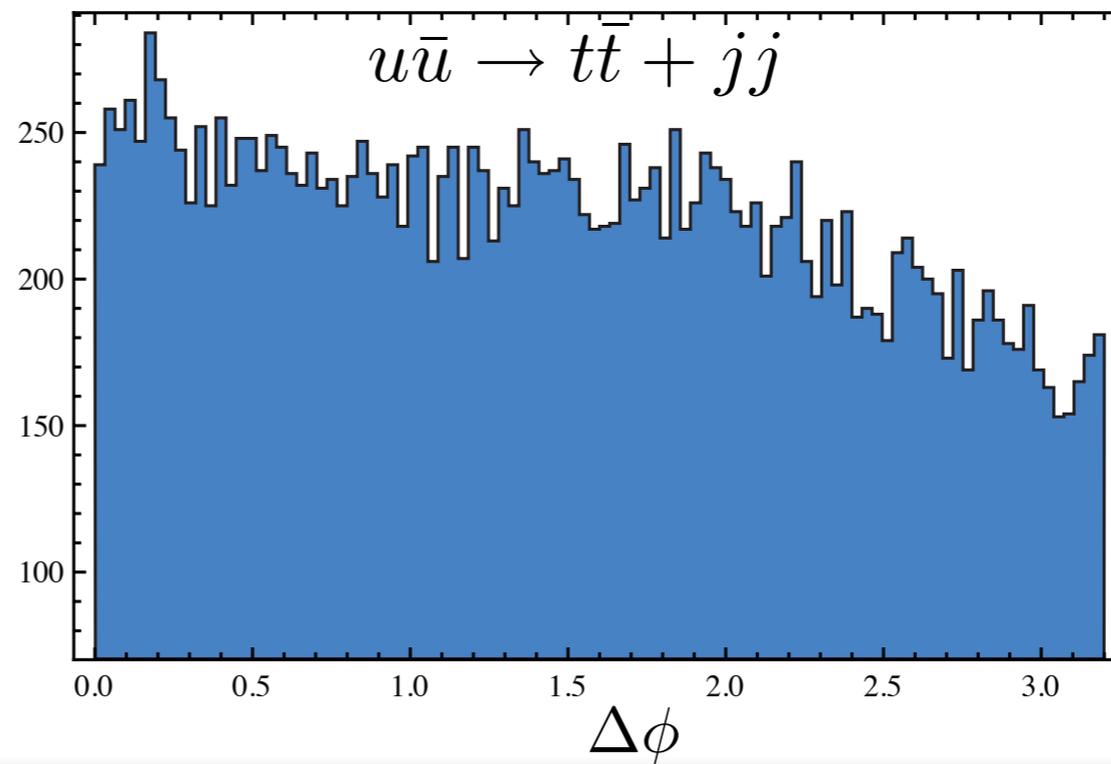
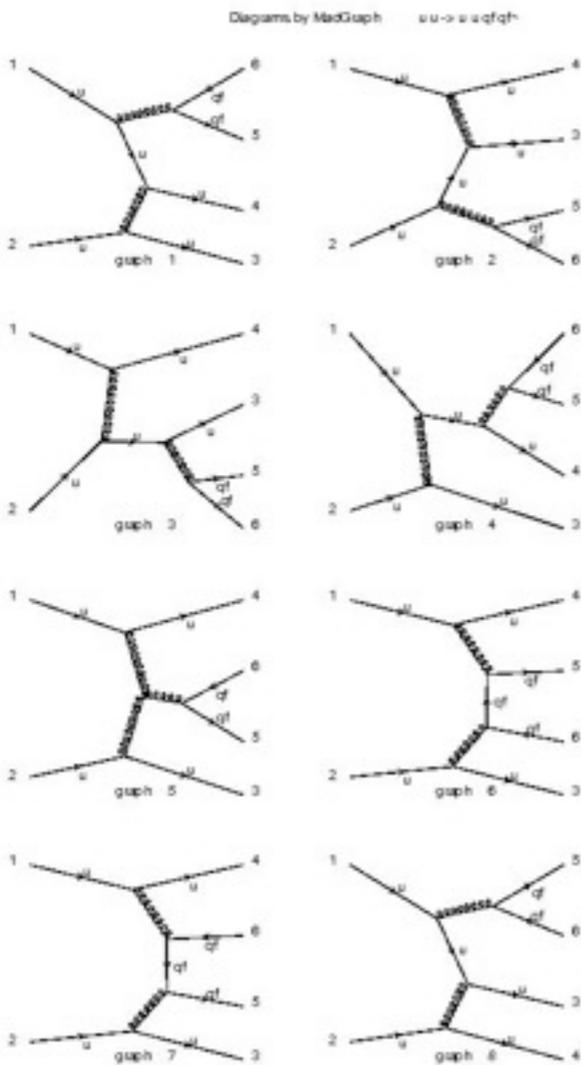
Gauge Invariance

- We were calculating only s - and t -channel diagrams
- This is not a gauge invariant quantity.
- Using CalcHep, can compare distributions with and without all diagrams.



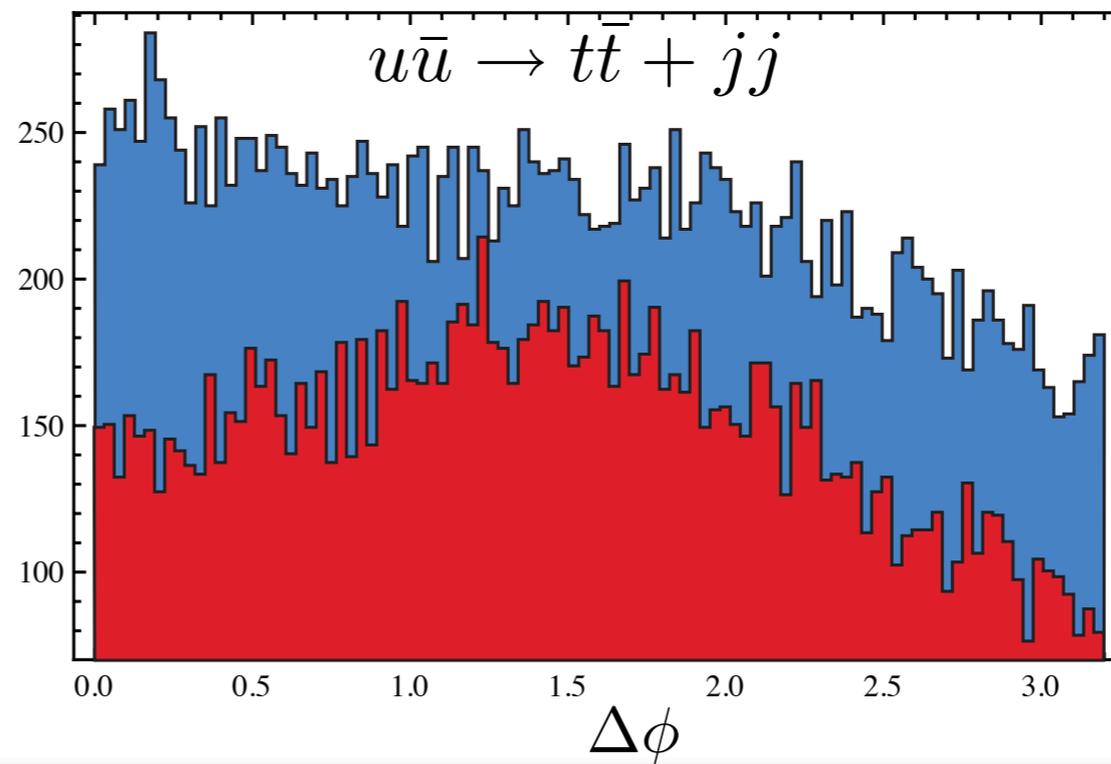
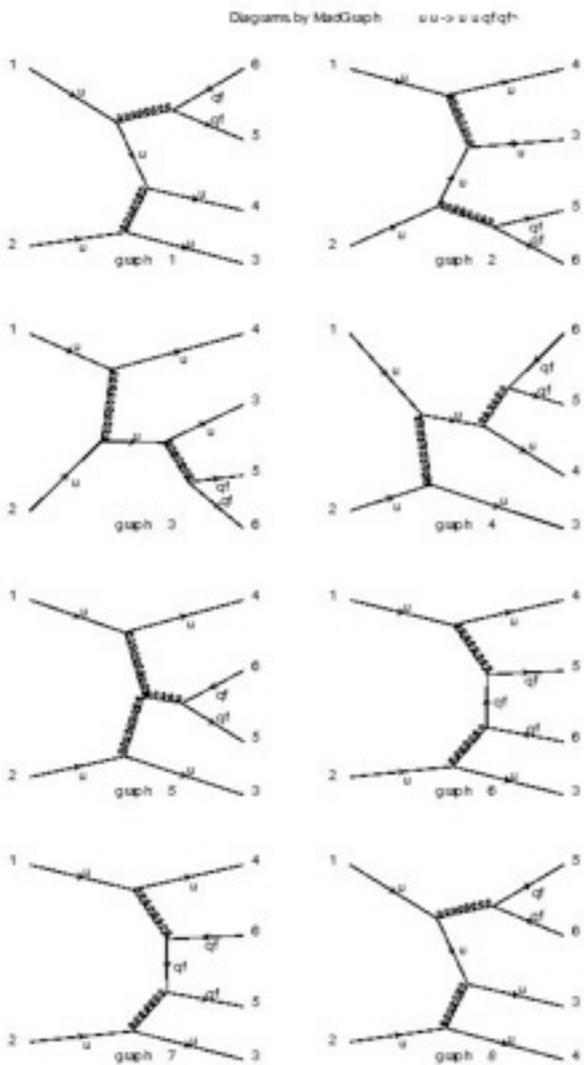
Gauge Invariance

- Lesson learned: gauge invariance is important!
 - Need to include all diagrams, not just the ones that ‘look like VBF’
 - Cancellations between all diagrams remove spurious high p_T tail, as a result:



Gauge Invariance

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Results

- Use MadGraph/MadEvent for background-free simulation:

$$pp \rightarrow 2(R - \text{hadrons}) + jj$$

- Apply VBF-isolating cuts:

$$\eta_{j_1} \cdot \eta_{j_2} < 0, \quad |\eta_j| \leq 5, \quad |\eta_{j_1} - \eta_{j_2}| \geq 4.2$$

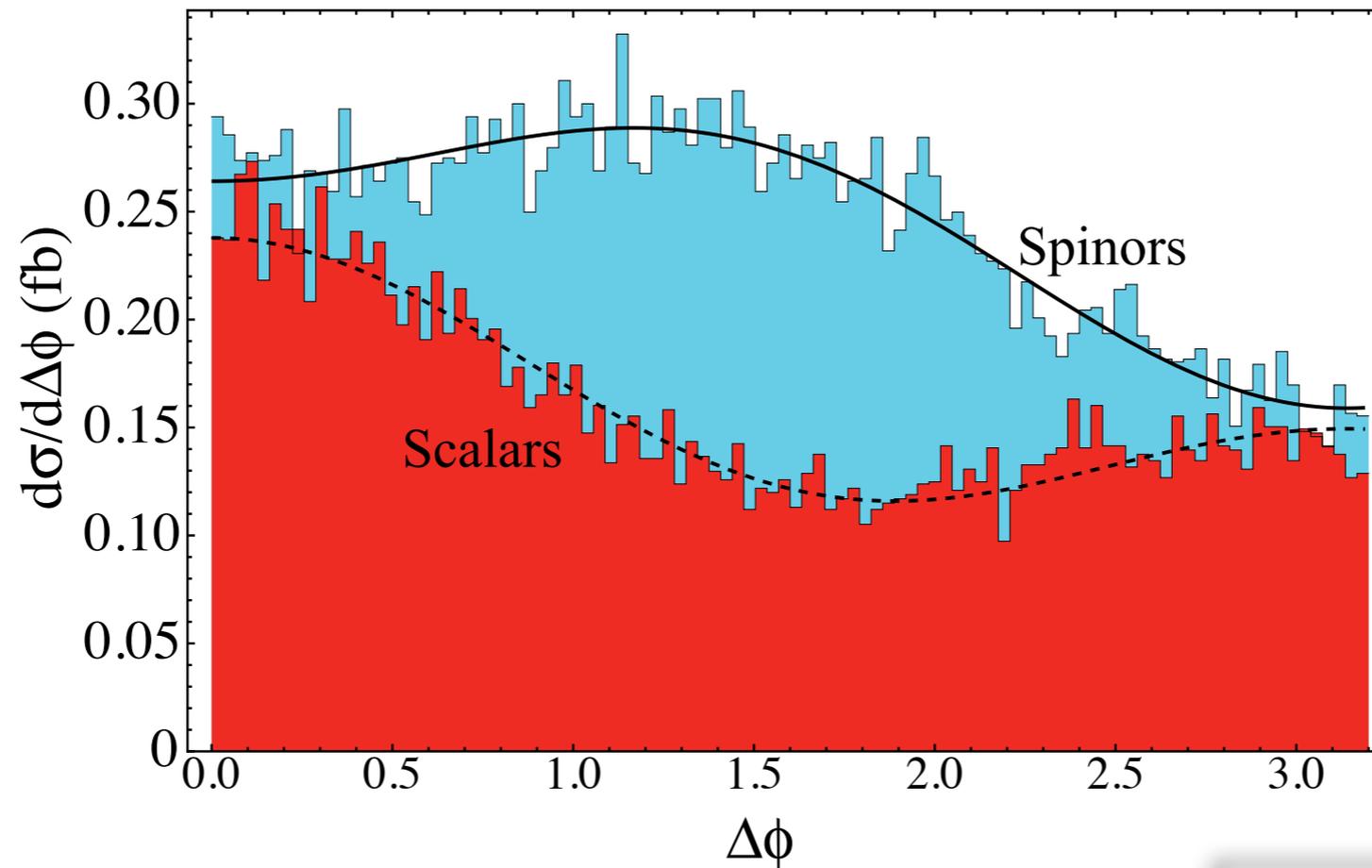
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- Total cross sections ($\sqrt{s} = 10 \text{ TeV}, m = 500 \text{ GeV}$):

$$\sigma_{\text{spinor}} = 33 \text{ fb}, \quad \sigma_{\text{scalar}} = 21 \text{ fb}$$

Results



$$\frac{d\sigma_{\text{scalar}}}{d\Delta\phi} = 0.16 + 0.044 \cos \Delta\phi + 0.035 \cos 2\Delta\phi \quad (\text{fb})$$

$$\frac{d\sigma_{\text{spinor}}}{d\Delta\phi} = 0.24 + 0.053 \cos \Delta\phi - 0.033 \cos 2\Delta\phi \quad (\text{fb})$$

$$(f_9^- / f_1)_{\text{scalar}} = 0.22$$

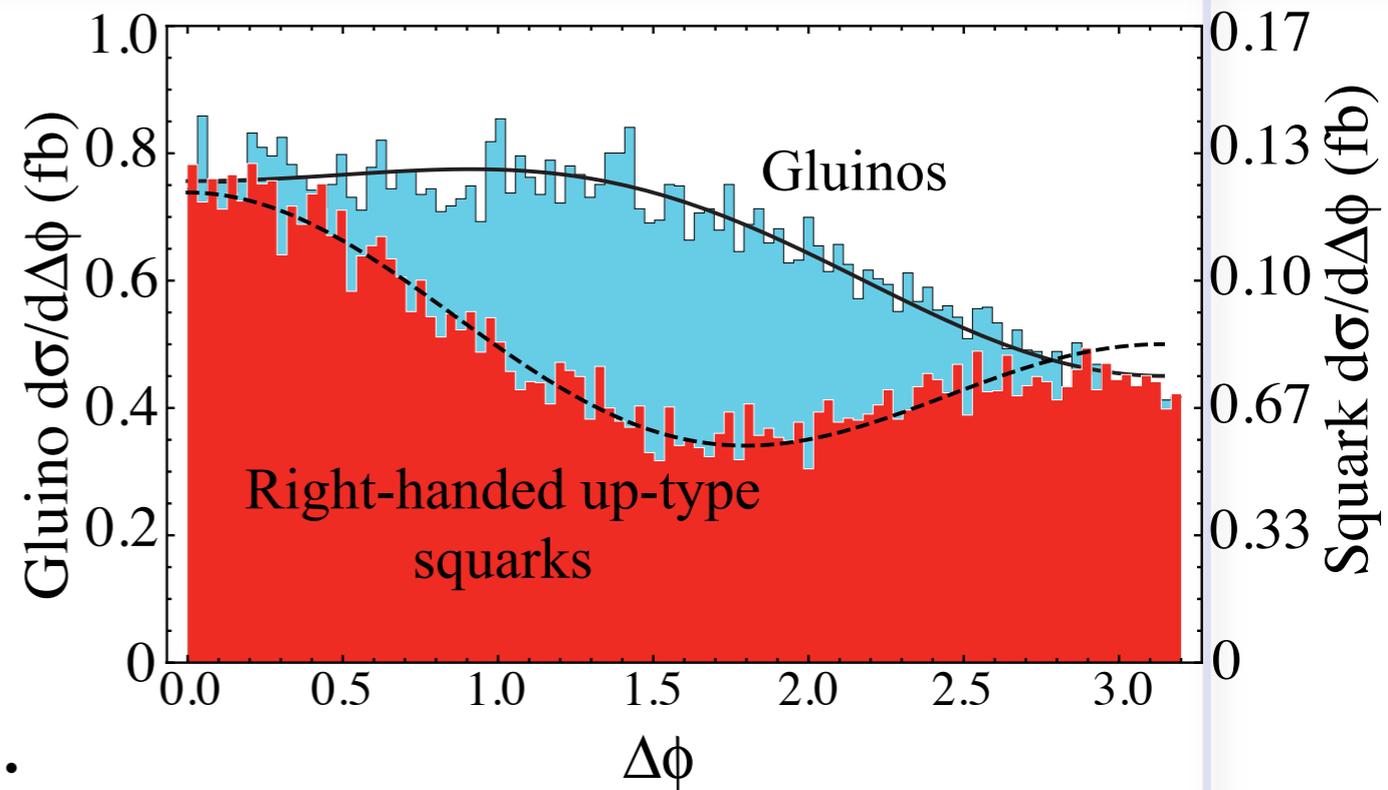
$$(f_9^- / f_1)_{\text{spinor}} = -0.14$$

SUSY Applications

- We picked a “background free” model
 - No central jets that can be confused with the forward jets that constitute our observables
- Obvious next step: SUSY gluino pairs/squark pairs
- What we have done:
 - Background-free, trigger/tagging free
MadGraph/Calchep study (*i.e.* is there a signal?)

Gluginos and Squarks

- Demonstrates that a signal is present, and that the majorana nature of the gluinos isn't a problem.
- What needs to be done:
 - Background (naive expectation: flat in $\cos 2\Delta\phi$)
 - Trigger analysis, cut optimization
 - Jet ID, including decays of gluinos/squarks



Future Work

- Top spin measurements
 - Negative f_9^- confirmed in simulations
 - Might be a background to New Physics searches
 - b-tagging necessary?
 - Tevatron search possible?
- Weak VBF events
 - Rate will be low, but signal should be present
- Vector Boson final states
 - Expectation is a positive f_9^-
 - If so, how to distinguish from scalars?

Future Work

- Other modes:

$$\begin{aligned} |\mathcal{M}|^2 = & f_1 + f_2 \cos \phi_1 + f_3 \cos \phi_2 + f_4 \cos 2\phi_1 + f_5 \cos 2\phi_2 \\ & + f_6^+ \cos(\phi_1 + \phi_2) + f_6^- \cos(\phi_1 - \phi_2) + f_7^+ \cos(2\phi_1 + \phi_2) + f_7^- \cos(2\phi_1 - \phi_2) \\ & + f_8^+ \cos(\phi_1 + 2\phi_2) + f_8^- \cos(\phi_1 - 2\phi_2) + f_9^+ \cos 2(\phi_1 + \phi_2) + f_9^- \cos 2(\phi_1 - \phi_2) \\ & + \bar{f}_2 \sin \phi_1 + \bar{f}_3 \sin \phi_2 + \bar{f}_4 \sin 2\phi_1 + \bar{f}_5 \sin 2\phi_2 \\ & + \bar{f}_6^+ \sin(\phi_1 + \phi_2) + \bar{f}_6^- \sin(\phi_1 - \phi_2) + \bar{f}_7^+ \sin(2\phi_1 + \phi_2) + \bar{f}_7^- \sin(2\phi_1 - \phi_2) \\ & + \bar{f}_8^+ \sin(\phi_1 + 2\phi_2) + \bar{f}_8^- \sin(\phi_1 - 2\phi_2) + \bar{f}_9^+ \sin 2(\phi_1 + \phi_2) + \bar{f}_9^- \sin 2(\phi_1 - \phi_2) \end{aligned}$$

- If we can identify some plane in the event, we can use ϕ_1, ϕ_2 modes, not just $\Delta\phi$
 - Not as ‘model independent’
 - Probably very hard

Conclusions

- Azimuthal angular correlations of associated forward jets in pair-production events carry spin information.
- Demonstrated in a “best case scenario.”
- Technique has significant advantages:
 - Works regardless of decay modes or structure of couplings
 - Does not require reconstruction of heavy particle kinematics
- A lot of work remains to realize this measurement at Tevatron or the LHC

Non-Relativistic Version

- Consider the symmetries of the final state radiation in QM (rather than QFT)
- The combination $\psi_{\text{spatial}}\psi_{\text{color}}$ needs to be symmetric
- Limiting case of forward jets going down the beam, $\ell = 0$ and so ψ_{spatial} symmetric
- Thus, the color indices must be symmetric
- Only $d_{abc}d_{abd}T^cT^d$ (negative f_9^-), no $f^{abc}f^{abd}T^cT^d$ (positive f_9^-)