

# Phenomenology of B-meson mixing and decay constants

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# 1. Introduction

- # Flavour-violating and CP-violating processes allow us to test high energy physics
  - \* Tests limited by precision.
  - \* Standard Model (SM) predictions for those observables depend on a few parameters → overconstrain those parameters.

## Test the SM

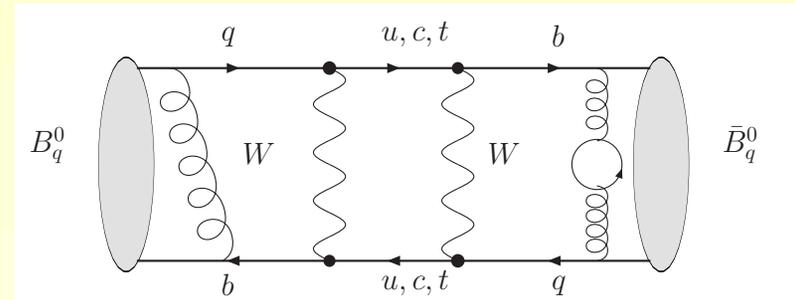
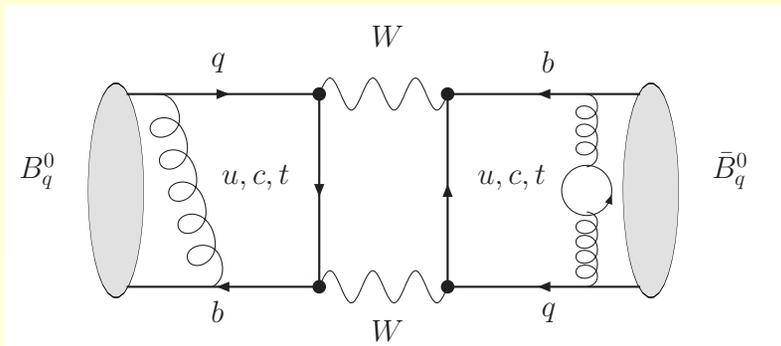
- # Already several  $2 - 3 \sigma$  tensions between flavour data and SM predictions
- # Phenomenological goals:
  - \* Determination of fundamental parameters of the SM: quark masses, Cabibbo-Kobayashi-Maskawa matrix elements.
  - \* Unveiling New Physics (NP) effects. **Even before non-SM particles directly produced at LHC.**
  - \* Constraining NP models.

# 1. Introduction

# Interplay flavour physics with direct searches for new physics and electroweak precision studies

→ Which is the correct extension of the SM?

## 2. Neutral $B$ mixing



- $B_0$  mixing parameters determined by the off diagonal elements of the mixing matrix

$$i \frac{d}{dt} \begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix} = \begin{pmatrix} M^{s/d} - \frac{i}{2} \Gamma^{s/d} \end{pmatrix} \begin{pmatrix} |B_{s/d}(t)\rangle \\ |\bar{B}_{s/d}(t)\rangle \end{pmatrix}$$

$$\Delta M_{s/d} \propto |M_{12}^{s/d}|$$

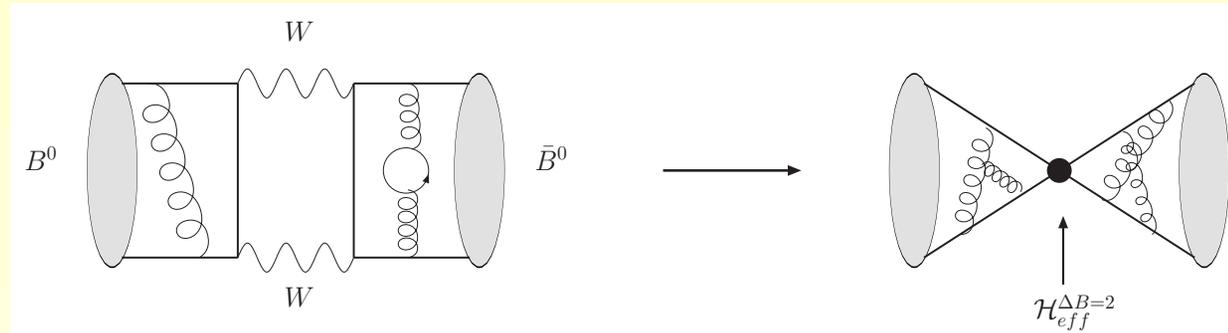
$$\Delta \Gamma_{s/d} \propto |\Gamma_{12}^{s/d}|$$

New physics can significantly affect  $M_{12}^{s/d} \propto \Delta M_{s/d}$

- \*  $\Gamma_{12}$  dominated by CKM-favoured  $b \rightarrow c\bar{c}s$  tree-level decays.

## 2.1. Mixing parameters in the Standard Model

# In the Standard Model



$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

\* Non-perturbative input

$$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1 | B_q^0 \rangle(\mu) \quad \text{with} \quad O_1 \equiv [\bar{b}^i q^i]_{V-A} [\bar{b}^j q^j]_{V-A}$$

In terms of decay constants and bag parameters

$$\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}$$

\* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio  $\implies$  **very accurate calculation**

## 2.1. Mixing parameters in the Standard Model

Experimentally: Mass differences very well measured.

$$\Delta M_d|_{exp.} = (0.507 \pm 0.005)ps^{-1} \quad \Delta M_s|_{exp.} = (17.77 \pm 0.12)ps^{-1}$$

HFAG 09

CDF

Experimentally: Decay width differences still have large errors.

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_d = 0.010 \pm 0.037 \quad \left(\frac{\Delta\Gamma}{\Gamma}\right)_s = 0.15 \pm 0.07$$

HFAG 09

## 2.2. $B_0$ mixing beyond the SM

# Comparison of experimental measurements and theoretical predictions can constraint some **BSM** parameters and help to understand **BSM** physics.

# Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

# The most general **Effective Hamiltonian** describing  $\Delta B = 2$  processes is

$$\mathcal{H}_{eff}^{\Delta B=2} = \sum_{i=1}^5 C_i Q_i + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i \quad \text{with}$$

$$Q_1^q = \left( \bar{\psi}_b^i \gamma^\nu (\mathbf{I} - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j \gamma^\nu (\mathbf{I} - \gamma_5) \psi_q^j \right) \quad \text{SM}$$

$$Q_2^q = \left( \bar{\psi}_b^i (\mathbf{I} - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j (\mathbf{I} - \gamma_5) \psi_q^j \right) \quad Q_3^q = \left( \bar{\psi}_b^i (\mathbf{I} - \gamma_5) \psi_q^j \right) \left( \bar{\psi}_b^j (\mathbf{I} - \gamma_5) \psi_q^i \right)$$

$$Q_4^q = \left( \bar{\psi}_b^i (\mathbf{I} - \gamma_5) \psi_q^i \right) \left( \bar{\psi}_b^j (\mathbf{I} + \gamma_5) \psi_q^j \right) \quad Q_5^q = \left( \bar{\psi}_b^i (\mathbf{I} - \gamma_5) \psi_q^j \right) \left( \bar{\psi}_b^j (\mathbf{I} + \gamma_5) \psi_q^i \right)$$

$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathbf{I} \pm \gamma_5) \rightarrow (\mathbf{I} \mp \gamma_5)$$

where  $\psi_b$  is a heavy b-fermion field and  $\psi_q$  a light ( $q = d, s$ ) fermion field.

- $C_i, \tilde{C}_i$  Wilson coeff. calculated for a particular **BSM** theory
- $\langle \bar{B}^0 | Q_i | B^0 \rangle$  calculated on the **lattice**

## 2.2. $B_0$ mixing beyond the SM

### # Some examples:

**F. Gabbiani et al**, Nucl.Phys.B477 (1996), **D. Bećirević et al**, Nucl.Phys.B634 (2002); general SUSY models

**M. Ciuchini and L. Silvestrini**, PRL 97 (2006) 021803; SUSY

Constraints on the mass insertions ( $|Re(\delta_{23}^d)_{RR}| < 0.4$ ,  $|(\delta_{23}^d)_{LL}| < 0.1, \dots$ )

**M. Blanke et al**, JHEP 12(2006) 003; Little Higgs model with T-parity

$\Delta M_q$  can be used to test viability of the model. To constrain and test the model in detail  $\Delta M_s / \Delta M_d$  and  $\Delta \Gamma_q$ .

**Lunghi and Soni**, JHEP0709(2007)053; Top Two Higgs Doublet Model

Constraints on  $\beta_H$  (ratio of vev's of the two Higgs) and  $m_{H^+}$

**M. Blanke et al**, JHEP0903(2009)001; Warped Extra Dimensional Models

Constraints on the KK mass scale: anarchic approach seems implausible, generally  $M_{KK} > 20 TeV$  but can be as low as  $M_{KK} \simeq 3 TeV$  (moderate fine tuning).

## 2.2. $B_0$ mixing beyond the SM

### # Some examples:

W. Altmannshofer et al, 0909.1333; SUSY flavor models

Identify useful flavour observables ( $S_{\psi\phi}$ ,  $B_s \rightarrow \mu^+ \mu^-$ , ...) to exclude some SUSY models and/or distinguish them from LHT and RS models. Updated analysis of bound on flavor violating terms in the SUSY soft sector.

## 2.2. $B_0$ mixing beyond the SM

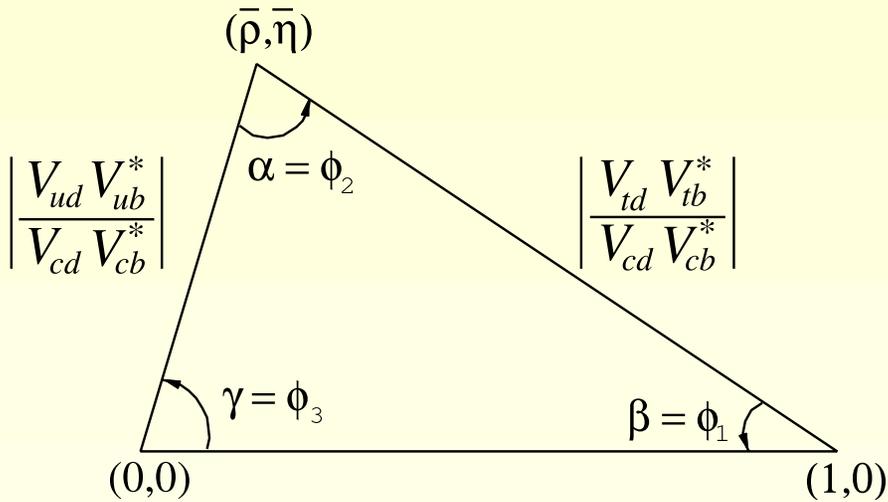
# NP effects in  $B^0 - \bar{B}^0$  mixing can be parametrized by

$$\langle B_q^0 | H_{eff}^q | \bar{B}_q^0 \rangle = A_q^{SM} + A_q^{NP} = C_{B_q^0} e^{2i\phi_{B_q^0}} A_q^{SM}$$

- \* The mixing phase also governs mixing-induced CP violation in exclusive channels like  $B_s \rightarrow J/\psi\phi$ .

## 2.3. Unitarity Triangle analyses

# For  $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \rightarrow$  **CKM unitarity triangle**.



Can use the following set of parameters

$$\lambda \equiv |V_{us}|, |V_{cb}|, R_t \text{ and } \beta$$

where

$$R_t \equiv \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|, \quad V_{td} = |V_{td}| e^{-i\beta}$$

# Within the SM and CMFV

$$R_t = \xi \frac{1}{\lambda} \sqrt{\frac{m_{B_s^0}}{m_{B_d^0}}} \sqrt{\frac{\Delta M_{B_d^0}}{\Delta M_{B_s^0}}} \quad \sin 2\beta = S_\psi K_S$$

## 2.3. Unitarity Triangle analyses

\* Mixing-induced CP asymmetry

$$\mathcal{A}_{\psi K_S} = \frac{\Gamma(\bar{B}_d^0(t) \rightarrow \psi K_S) - \Gamma(B_d^0(t) \rightarrow \psi K_S)}{\Gamma(\bar{B}_d^0(t) \rightarrow \psi K_S) + \Gamma(B_d^0(t) \rightarrow \psi K_S)} \simeq S_{\psi K_S} \sin(\Delta M t) - C_{\psi K_S} \cos(\Delta M t)$$

# In the presence of NP those relations are modified by

$$R_t = \xi \frac{1}{\lambda} \sqrt{\frac{m_{B_s^0}}{m_{B_d^0}}} \sqrt{\frac{\Delta M_{B_d^0}}{\Delta M_{B_s^0}}} \sqrt{\frac{C_{B_s^0}}{C_{B_d^0}}} \sin(2\beta + 2\phi_{B_d^0}) = S_{\psi K_S}$$

with the NP parameters defined as

$$\langle B_q^0 | H_{eff}^q | \bar{B}_q^0 \rangle = A_q^{SM} + A_q^{NP} = C_{B_q^0} e^{2i\phi_{B_q^0}} A_q^{SM}$$

### 3. Hints of New Physics in the Flavour Sector

# Most observations in the flavour sector are consistent with SM expectations but ...

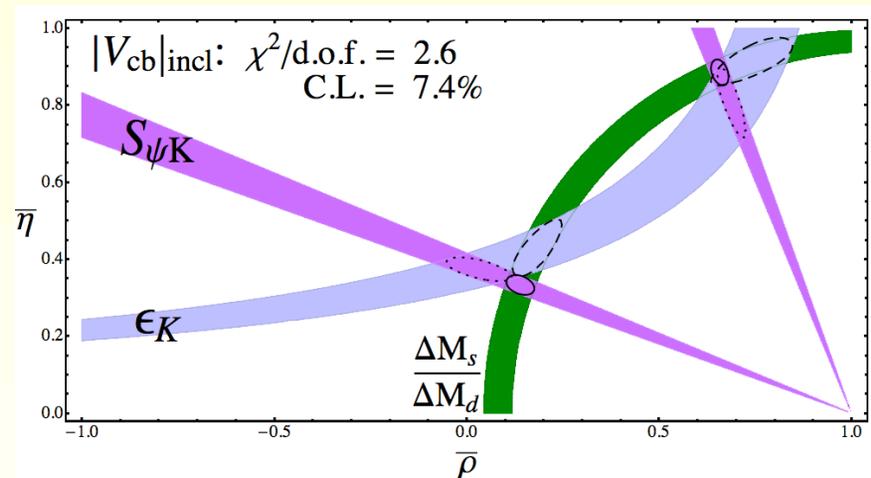
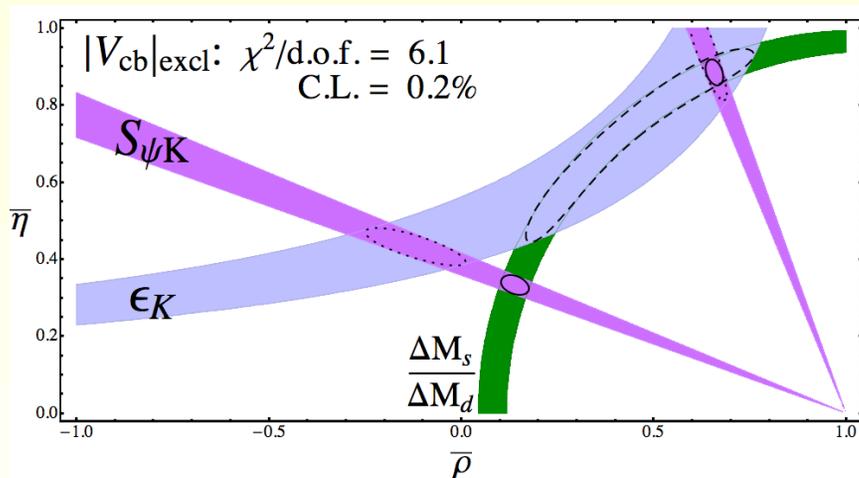
... there are currently several  $2 - 3\sigma$  tensions that may indicate New Physics.

### 3.1. Tension in the CKM unitarity triangle

**UT fit:** Global fit to the CKM unitarity triangle using experimental and theoretical constraints.

2 – 3 $\sigma$  tension in the CKM description

- \* Tension is between the three most precise constraints: the  $K^0 - \bar{K}^0$  mixing parameter  $\epsilon_K$ , the ratio of mass differences  $\Delta M_{B_s} / \Delta M_{B_d}$  describing  $B^0 - \bar{B}^0$  mixing and  $\sin(2\beta)$ .



Laiho, Van de Water and Lunghi, arXiv:0910.2928

\*\* Degree of tension depend on  $|V_{cb}|$

$$|V_{cb}^{exc.}(\text{latt. average})| = (38.6 \pm 1.2) \times 10^{-3} \quad |V_{cb}^{inc.}| = (41.6 \pm 0.6) \times 10^{-3}$$

### 3.1. Tension in the CKM unitarity triangle

2 – 3 $\sigma$  tension in the CKM description

\*\* Independent of (controversial)  $|V_{ub}|$

$$|V_{ub}^{exc.}(\text{latt. average})| = (3.42 \pm 0.37) \times 10^{-3} \quad |V_{ub}^{inc.}| = (4. - 4.5) \times 10^{-3}$$

\* If we assume no NP at tree-level at current precision

→ tension can be a sign of NP either in  $K^0$  or  $B^0$  mixing.

\*\* Current data prefer NP in  $K^0$  mixing.

\* Constraints from  $\varepsilon_K$ ,  $\Delta M_d/\Delta M_s$ , and  $|V_{ub}/V_{cb}|$  limited by lattice errors for  $|V_{cb}|_{excl.}$ ,  $\xi$ , and  $|V_{ub}|_{excl.}$

### 3.1. Tension in the CKM unitarity triangle

E. Lunghi and A. Soni, arXiv:0912.0002: UT analysis without using semileptonic decays

- \*  $|V_{ub}|$  and  $|V_{cb}|$  inclusive and exclusive disagree by  $\approx 2\sigma$   
→ eliminate the  $|V_{cb}|$  constraint from the analysis in favor of

$$f_{B_s^0} \sqrt{\hat{B}_{B_s^0}} \text{ or } \mathcal{B}r(B \rightarrow \tau\nu) \times f_{B_d}^{-2}$$

- \*  $1.8\sigma$  tension observed. Slight preference for NP in  $B_d^0$  mixing.
- \* Improvement in  $f_{B_s^0} \sqrt{\hat{B}_{B_s^0}}$  and  $f_B$  will help a lot to identify the origin of the tension.

### 3.2. Mixing in the $B_s$ system: the $S_{J/\psi\phi}$ asymmetry

Lenz and Nierste, JHEP 06, 072 (2007)

# Study the mixing-induced CP asymmetry.

$$\mathcal{A}_{J/\psi\phi} = \frac{\Gamma(\bar{B}_s^0(t) \rightarrow J/\psi\phi) - \Gamma(B_s^0(t) \rightarrow J/\psi\phi)}{\Gamma(\bar{B}_s^0(t) \rightarrow J/\psi\phi) + \Gamma(B_s^0(t) \rightarrow J/\psi\phi)} = S_{J/\psi\phi} \sin(\Delta M t) - C_{J/\psi\phi} \cos(\Delta M t)$$

#  $B_s$  mixing phase  $\beta_s$  responsible for this asymmetry in the SM

$$\langle B_s | H_{eff}^{SM} | \bar{B}_s \rangle = A_s^{SM} e^{-2i\beta_s}$$

$$(S_{J/\psi\phi})_{SM} = \sin(2|\beta_s|) = \sin \left( 2 \left| \arg \left( \frac{-V_{ts} V_{tb}^*}{V_{cs} V_{cb}} \right) \right| \right) \approx 0.04$$

# World average based on flavour-tagged analyses of  $B_s \rightarrow J/\psi\phi$  decays in CDF and DØ is  $2.2\sigma$  different from SM predictions

$$(S_{J/\psi\phi})_{exper.} \approx 0.81_{-0.32}^{+0.12}$$

\* Expect improvements of experimental measurements of  $S_{J/\psi\phi}$  asymmetry in CDF, DØ, LHCb, ATLAS and CMS.

### 3.2. Mixing in the $B_s$ system: the $S_{J/\psi\phi}$ asymmetry

- \* Possible new phases in  $B_s$  decays would lead to correlated effects between  $\Delta B = 2$  processes and  $b \rightarrow s$  decays

$$(S_{J/\psi\phi}) = \sin(2|\beta_s| - 2\phi_{B_s^0})$$

→ need to improve measurements of CP-violation in  $b \rightarrow s$  penguin decays.

- # Enhancement of the asymmetry can be found in RSc and GMSSM.  
Also supersymmetric flavour models with significant right-handed curr.  
**Buras, arXiv:0910.1032**

### 3.3. Measurement of $Br(B_{s,d} \rightarrow \mu^+ \mu^-)$

One of the main targets of flavour physics is measuring the highly suppressed decay  $Br(B_s \rightarrow \mu^+ \mu^-)$ .

\* **CDF (DØ)** bounds  $Br(B_s \rightarrow \mu^+ \mu^-) \leq 3.3(5.3) \times 10^{-8}$ ,  
 $Br(B_d \rightarrow \mu^+ \mu^-) \leq 1 \times 10^{-8}$

\* The **SM** prediction for these branching ratios is given by

$$Br(B_q \rightarrow \mu^+ \mu^-)_{SM} = \tau_{B_q} \frac{G_F^2}{\pi} \eta_Y^2 \left( \frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 f_{B_q}^2 m_\mu^2 m_{B_q} |V_{tb}^* V_{tq}|^2 Y^2(x_t)$$

\*\* Uncertainty dominated by error in  $f_{B_q}$ : 9-11%

\* The most precise way of extracting these branching ratios is from (in the **SM**)

$$\frac{Br(B_q \rightarrow \mu^+ \mu^-)}{\Delta M_q} = \tau(B_q) 6\pi \frac{\eta_Y}{\eta_B} \left( \frac{\alpha}{4\pi M_W \sin^2 \theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

\*\* Uncertainty dominated by error in  $\hat{B}_q$ : 5-9%

### 3.3. Measurement of $Br(B_{s,d} \rightarrow \mu^+ \mu^-)$

\* **CDF (DØ)** bounds  $Br(B_s \rightarrow \mu^+ \mu^-) \leq 3.3(5.3) \times 10^{-8}$ ,  
 $Br(B_d \rightarrow \mu^+ \mu^-) \leq 1 \times 10^{-8}$

\* Using lattice determinations of  $\hat{B}_q$  **HPQCD**, PRD80 (2009) 014503

$$\rightarrow Br(B_s \rightarrow \mu^+ \mu^-) = (3.19 \pm 0.19) \times 10^{-9} \text{ and}$$
$$Br(B_d \rightarrow \mu^+ \mu^-) = (1.02 \pm 0.09) \times 10^{-10}$$

\*\* An error of 0.14 in  $Br(B_s \rightarrow \mu^+ \mu^-)$  is coming from  $\hat{B}$  uncertainty.

\* **Scalar operators** in the effective hamiltonian can enhance branching ratios to current experimental bounds (**example**: Higgs penguin).

\* In some models there is a strong correlation between  $Br(B_q \rightarrow \mu^+ \mu^-)$  and  $\Delta M_{B_q^0}$  (**example**: MSSM with MFV and large  $\tan\beta$ .)

\*\* Testing the correlation predicted by those kind of models needs a reduction of errors in the theoretical prediction for  $\Delta M_s^{SM}$   
 $\rightarrow$  need smaller lattice errors for the non-perturbative inputs.

### 3.3. Measurement of $Br(B_{s,d} \rightarrow \mu^+ \mu^-)$

# Tests of MFV: In the SM model and CMFV models, the following **model independent relation** hold with  $r = 1$  **Buras, PLB566 (2003) 115**

$$\frac{Br(B_s \rightarrow \mu^+ \mu^-)}{Br(B_d \rightarrow \mu^+ \mu^-)} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r$$

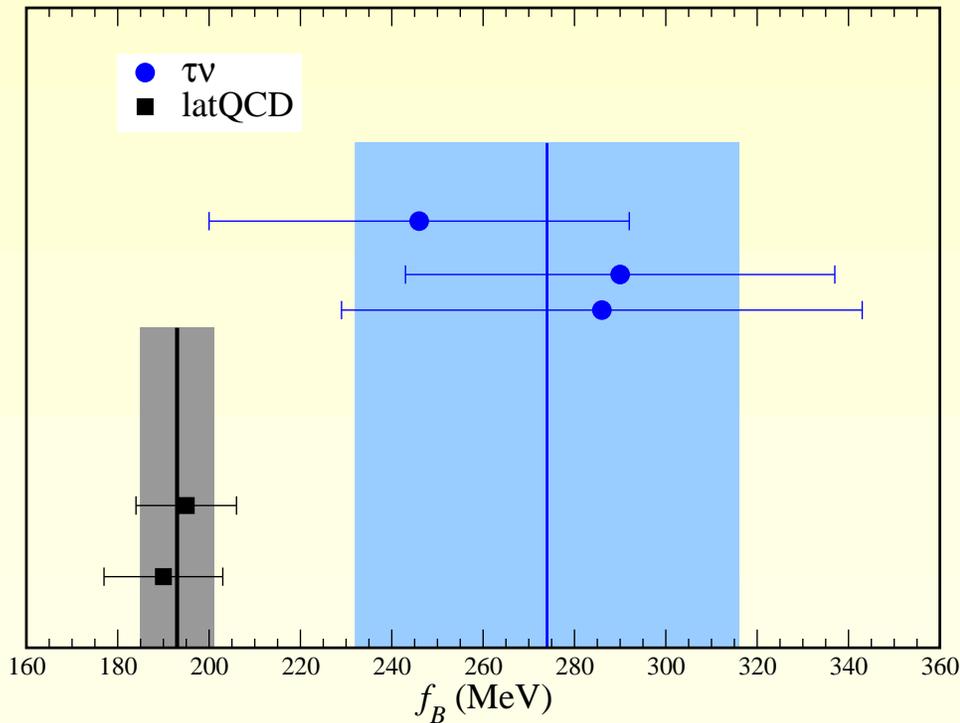
Any deviation from this relation ( $r \neq 1$ ) would indicate **NP** effects.

Supersymmetry, little Higgs models, extra space dimensions ...  
discussed in **Buras, arXiv:0910.1032**

$$\text{LHT: } 0.3 \leq r \leq 1.6, \text{ RSc: } 0.6 \leq r \leq 1.3$$

\* **LHCb** can reach the **SM** level for this branching ratio.

### 3.4. $B \rightarrow \tau \nu$ leptonic decay



1.9 $\sigma$  discrepancy between  $f_B$  values from lattice (HPQCD and FNAL/MILC) and experiment (using  $V_{ub}$  from lattice QCD)

A. Kronfeld, PHENO '09

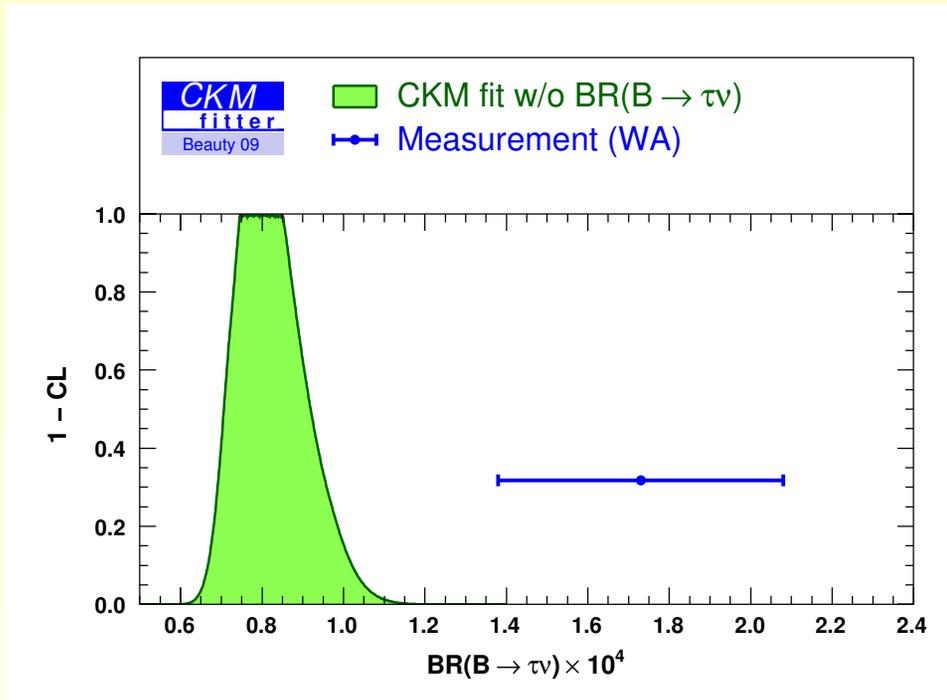
$$\mathcal{B}r(B^+ \rightarrow \tau^+ \nu)_{SM} = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 f_{B^+}^2 |V_{ub}|^2 \tau_{B^+}^+$$

\* **Differences:** Fermion discretization describing  $b$  quarks.

HPQCD 09, PRD80(2009)014503: NRQCD.

FNAL/MILC 09, PoS LATTICE2008 278 (2008): Fermilab formulation.

### 3.4. $B \rightarrow \tau \nu$ leptonic decay



2.4 $\sigma$  discrepancy between SM prediction for  $Br(B \rightarrow \tau \nu)$  from UT fit (relies on several lattice inputs  $f_{B_{d,s}^0}$ ,  $\hat{B}_{B_{d,s}^0}$ ,  $f_{B_{d,s}^0} \sqrt{\hat{B}_{B_{d,s}^0}}$ ) and experimental average BaBar, Belle

CKM fitter, Moriond 09, Beauty 09

$$Br(B^+ \rightarrow \tau^+ \nu)_{exp} = (1.73 \pm 0.35) \times 10^{-4}$$

$$Br(B^+ \rightarrow \tau^+ \nu)_{CKM fit} = (0.80_{-0.11}^{+0.16}) \times 10^{-4}$$

# Alternative extraction of SM prediction

$$\frac{Br(B^+ \rightarrow \tau^+ \nu)_{SM}}{\Delta M_{B_d^0}} = \frac{3\pi}{4\eta_B S_0(x_t) \hat{B}_d^0} \frac{m_\tau^2}{m_W^2} \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 \left|\frac{V_{ub}}{V_{td}}\right|^2 \tau_B^+$$

with  $\left|\frac{V_{ub}}{V_{td}}\right| = \left(\frac{1}{1-\lambda^2}\right)^2 \frac{1+R_t^2-2R_t \cos\beta}{R_t^2}$

$$Br(B^+ \rightarrow \tau^+ \nu)_{SM} = (0.80 \pm 0.12) \times 10^{-4}$$

### 3.4. $B \rightarrow \tau \nu$ leptonic decay

- # Discrepancy can be due to **charged Higgs**, but not a natural explanation. Could increase or decrease **SM  $\mathcal{B}_r$** .
- \* Most **MSSM** scenarios would lead to a suppression of the branching fraction.
- \* **Example**: Limits in the **2HDM  $\tan \beta - m_{H^+}$**  plane

$$(m_{H^+} / \tan \beta > 3.3 \text{ GeV})$$

- # Reducing experimental errors will be difficult at **LHCb**. Good prospects for a **super-B** factory

## 4. Lattice calculation of $B^0$ mixing parameters and decay constants

- # **Hints of discrepancies** between SM expectations and some flavour observables (see, for example, **A. Buras**, talk at EPS-HEP 2009 or **R. Van de Water**, plenary talk at Lattice 2009)

These analyses depend on several theoretical inputs including:

$f_{B_q^0} \sqrt{\hat{B}_{B_q^0}}$ ,  $f_{B_q^0}$ , and the SU(3) breaking mixing parameter  $\xi$ :

- # Comparison of  $\Delta M_s$  and  $\Delta\Gamma$  with experiment also provides bounds for **NP** effects
- # Bag parameters  $B_{B_s}$  and  $B_{B_d}$  can be used for theoretical predictions  
predictions of  $\mathcal{B}r(B \rightarrow \mu^+ \mu^-)$  and  $\mathcal{B}r(B^+ \rightarrow \tau^+ \nu)$

## 4.1. Some details of the lattice formulations and simulations

HPQCD, PRD80 (2009) 014503

**Unquenched:** Fully incorporate vacuum polarization effects

$$\text{MILC } N_f^{sea} = 2 + 1$$

**u,d,s** **Asqtad** action: improved staggered quarks  $\implies$  errors  $\mathcal{O}(a^2\alpha_s)$ ,  
 $\mathcal{O}(a^4)$

- \* good chiral properties
- \* accessible dynamical simulations

**b** **NRQCD:** Non-relativistic QCD improved through  $\mathcal{O}(1/M^2)$ ,  $\mathcal{O}(a^2)$   
and leading relativistic  $\mathcal{O}(1/M^3)$

- \* Simpler and faster algorithms to calculate  $b$  propagator

### Improved gluon action

- \* For further reduction of discretization errors

## 4.1. Some details of the lattice formulations and simulations: Parameters of the simulation

- # Lattice spacing: Two different values  $a \simeq 0.12 \text{ fm}, 0.09 \text{ fm}$ .  
Extracted from  $\Upsilon$  2S-1S splitting.
- # Bottom mass: Fixed to its physical value from  $\Upsilon$  mass.
- # Light masses: We work with full QCD points ( $m_{valence} = m_{sea}$ ).
  - \* Strange mass: Very close to its physical value (from Kaon masses).
  - \* up, down masses: six different values ( $m_{\pi}^{min.} \simeq 230 \text{ MeV}$ )  

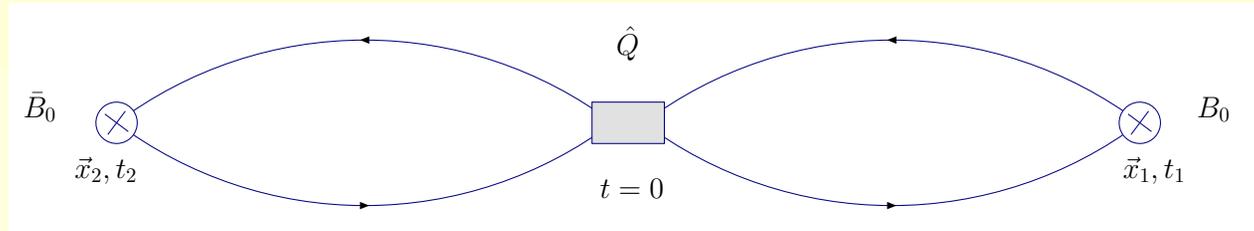
$\rightarrow$  chiral regime
- # Renormalization and matching to the continuum: One-loop.

$$\langle O_1 \rangle^{\overline{MS}} \propto (1 + \rho_{LL}\alpha_s) \langle O_1 \rangle^{latt.} + \rho_{LS}\alpha_s \langle O_2 \rangle^{latt.}$$

with  $O_1 = [\bar{b}\gamma_{\mu}(1 - \gamma_5)q] [\bar{b}\gamma_{\mu}(1 - \gamma_5)q]$  and  $O_2 = [\bar{b}(1 - \gamma_5)q] [\bar{b}(1 - \gamma_5)q]$ .

## 4.1. Some details of the lattice formulations and simulations

# Need 3-point (for any  $\hat{Q} = Q_X, Q_X^{1j}$ ) and 2-point correlators



$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}_1, t_1) [\hat{Q}](0) \Phi_{\bar{B}_q}^\dagger(\vec{x}_2, -t_2) | 0 \rangle$$

$$C^{(B)}(t) = \sum_{\vec{x}} \langle 0 | \Phi_{\bar{B}_q}(\vec{x}, t) \Phi_{\bar{B}_q}^\dagger(\vec{0}, 0) | 0 \rangle$$

- $\Phi_{\bar{B}_q}(\vec{x}, t) = \bar{b}(\vec{x}, t) \gamma_5 q(\vec{x}, t)$  is an interpolating operator for the  $B_q^0$  meson.
- \* Use smearing functions  $\phi$  to increase overlap with the ground state

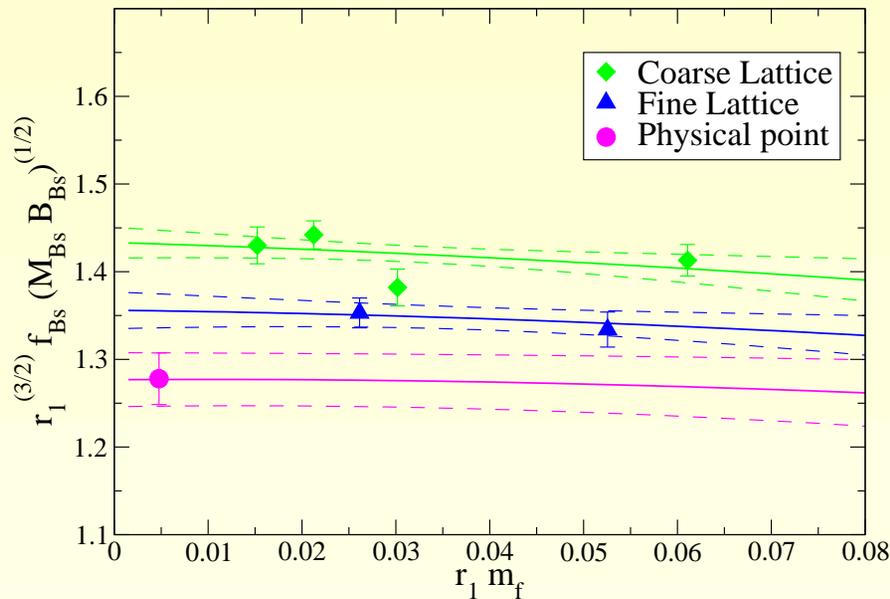
$$\Phi_{\bar{B}_q}(t) = \bar{b}(\vec{x}_2, t) \gamma_5 \phi(|\vec{x}_2 - \vec{x}_1|) q(\vec{x}_1, t)$$

## 4.1. Some details of the lattice formulations and simulations

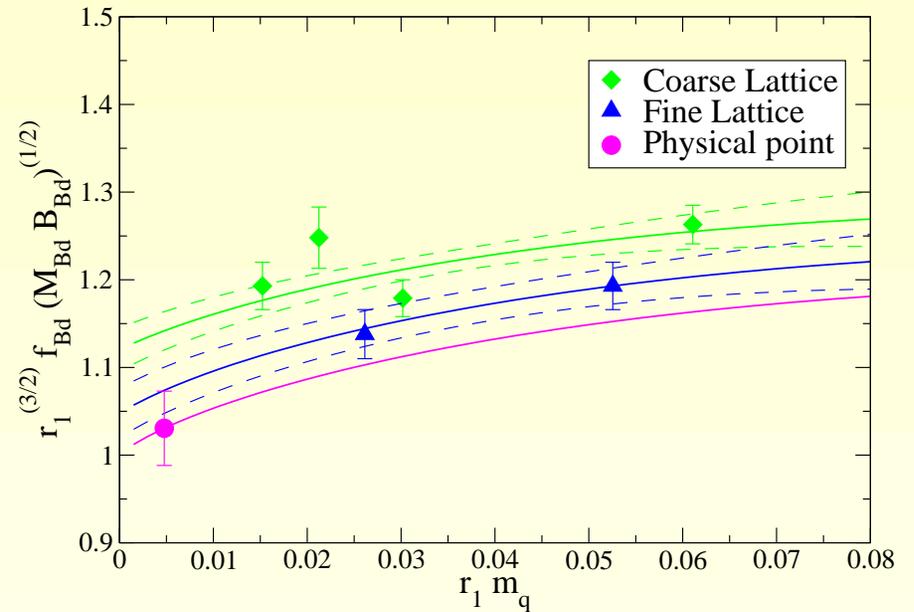
We carried out **simultaneous** fits of the 3-point and 2-point correlators using **Bayesian** statistics to the forms  $\rightarrow$  extract  $\langle O_X \rangle$  and  $f_{B_s(d)}$ .

$$C^{(4f)}(t_1, t_2) = \sum_{j,k=0}^{N_{exp}-1} A_{jk} \zeta_i \zeta_j (-1)^{j \cdot t_1} (-1)^{k \cdot t_2} e^{-E_B^{(j)}(t_1-1)} e^{-E_B^{(k)}(t_2-1)}$$
$$C^B(t) = \sum_{j=0}^{N_{exp}-1} \zeta_j (-1)^{j \cdot t} e^{-E_B^{(j)}(t-1)}$$

## 4.2. Results: $f_{B_q} \sqrt{M_{B_q} B_{B_q}}$



$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 266(6)(17) \text{MeV}$$

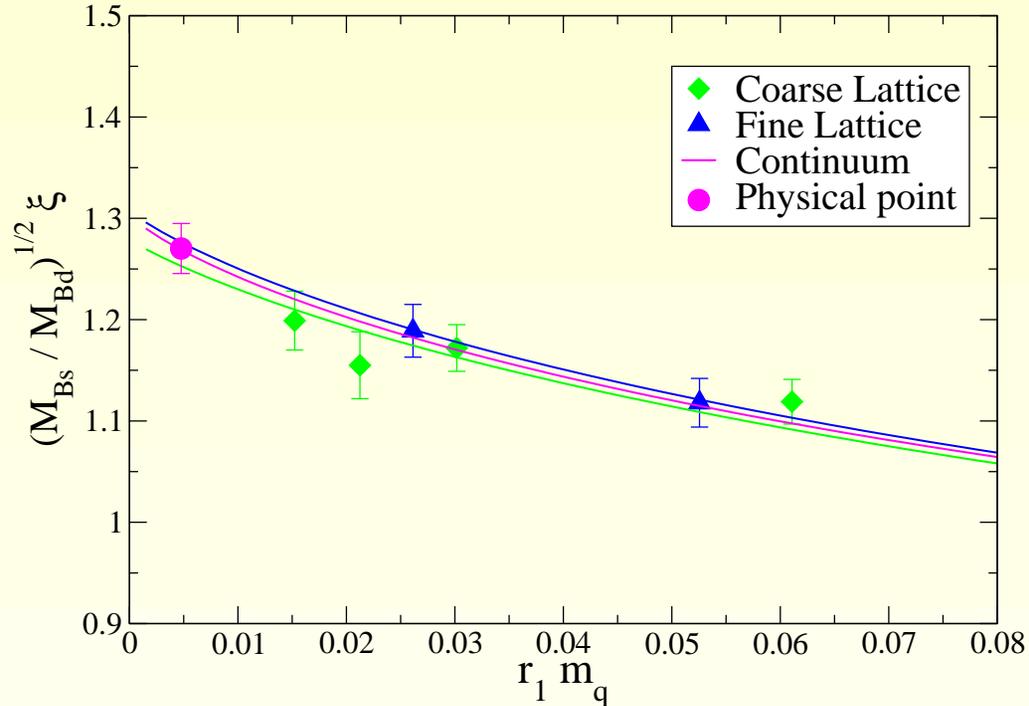


$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 216(9)(12) \text{MeV}$$

**Chiral+continuum extrapolations:** NLO Staggered CHPT.

- \* accounts for NLO quark mass dependence.
- \* accounts for light quark discretization effects through  $\mathcal{O}(\alpha_s^2 a^2 \Lambda_{QCD}^2)$   
 $\rightarrow$  remove the dominant light discretization errors

## 4.2. Results: $\xi \sqrt{\frac{M_{B_s}}{M_{B_d}}}$

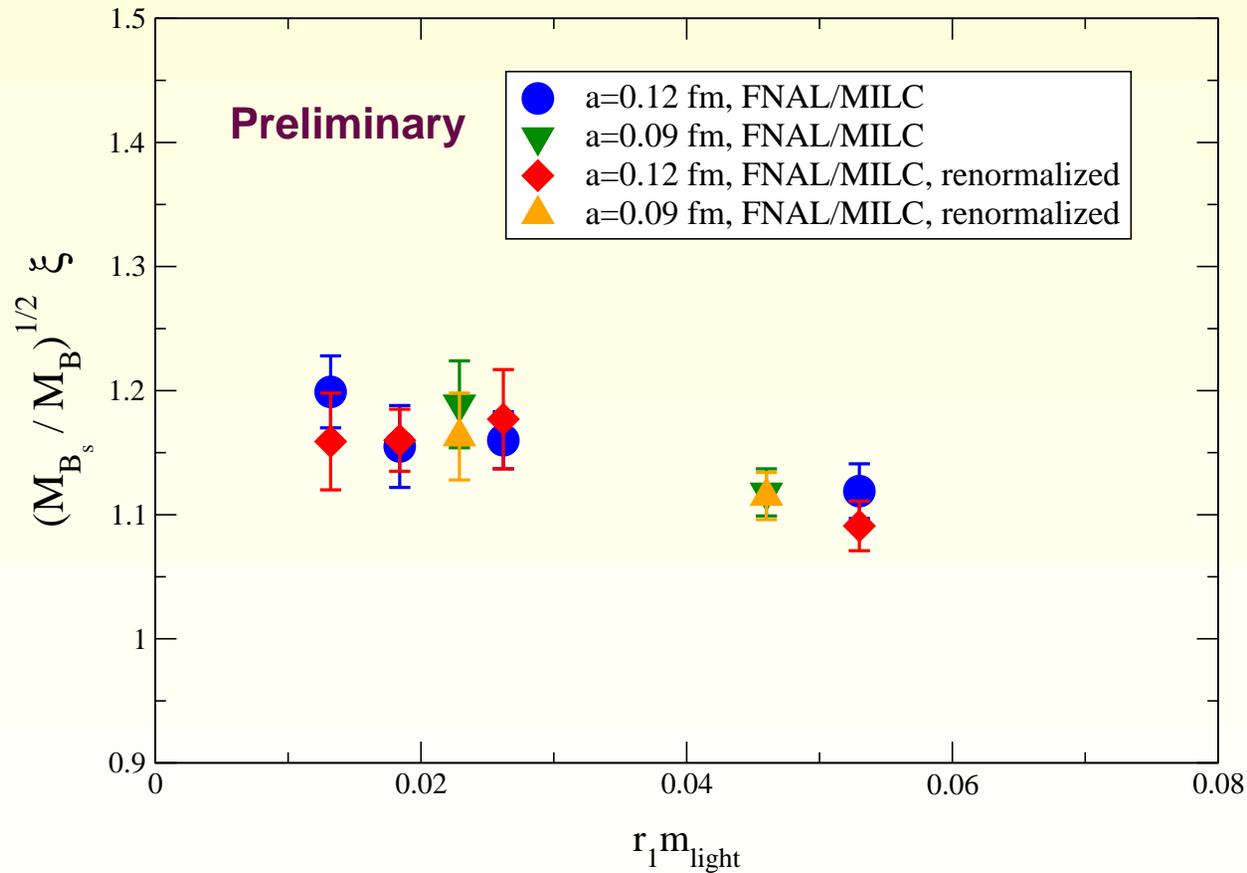


$$\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} = 1.258(25)(21) \quad \Rightarrow \quad \left| \frac{V_{td}}{V_{ts}} \right| = 0.214(1)(5)$$

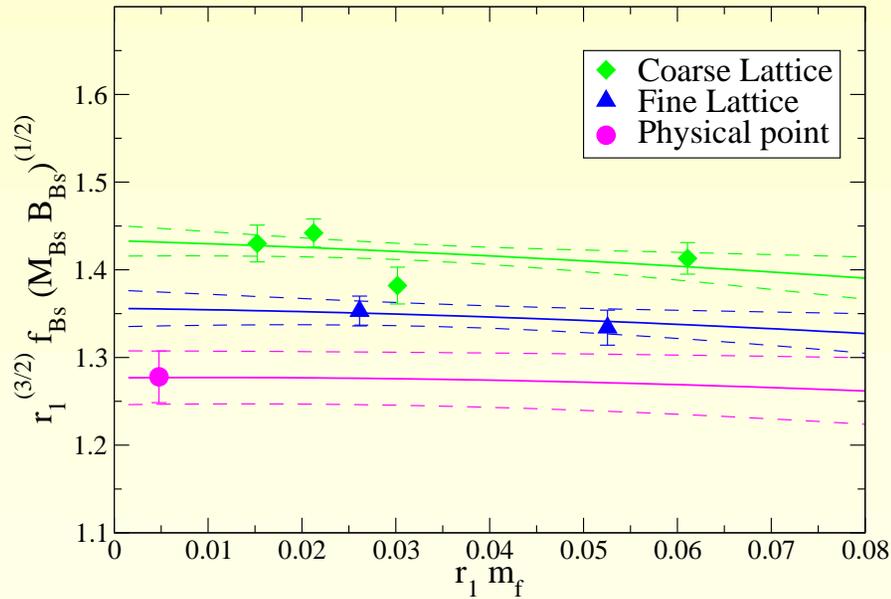
\* Previous value used in UT fits and another analyses (HPQCD/JLQCD):  
 $\xi = 1.20 \pm 0.06$

## 4.2. Results: $\xi \sqrt{\frac{M_{B_s}}{M_{B_d}}}$

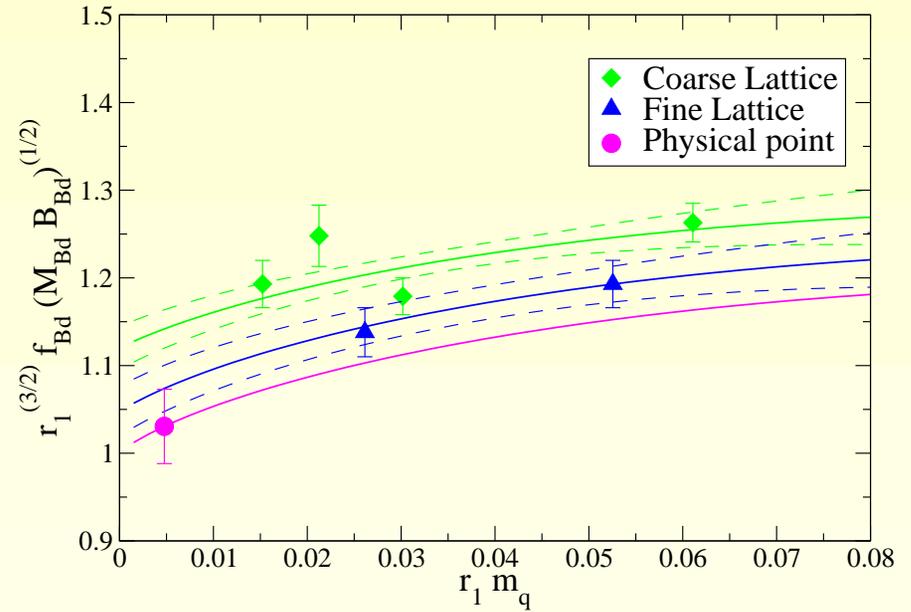
# Comparison of final **HPQCD**, PRD80 (2009) 014503 and preliminary **FNAL/MILC**, PoS LATTICE 2009, 245 (2009)



## 4.2. Results: $f_{B_q} \sqrt{M_{B_q}}$



$$f_{B_s} = 231(15) \text{ MeV}$$



$$f_{B_d} = 190(13) \text{ MeV}$$

$$\frac{f_{B_s}}{f_{B_d}} = 1.226(26)$$

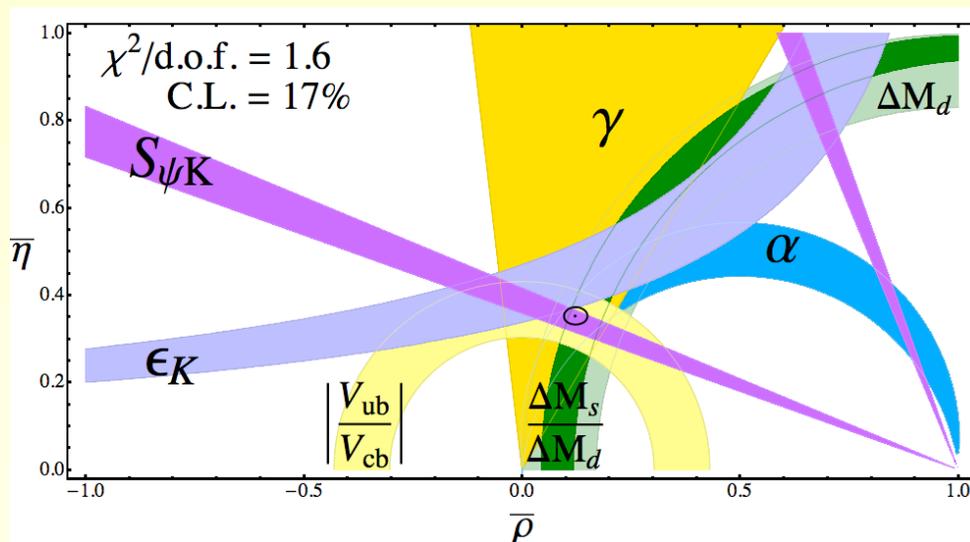
\* To be compared with preliminary **FNAL/MILC**, PoS LATTICE 2008, 278 (2008)

$$f_{B_s} = 243(11) \text{ MeV}$$

$$f_{B_d} = 195(11) \text{ MeV}$$

## 5. Impact of up-to-date lattice averages on UT.

Laiho, Lunghi & Van de Water, arXiv:0910.2928



\*  $2 - 3\sigma$  tension

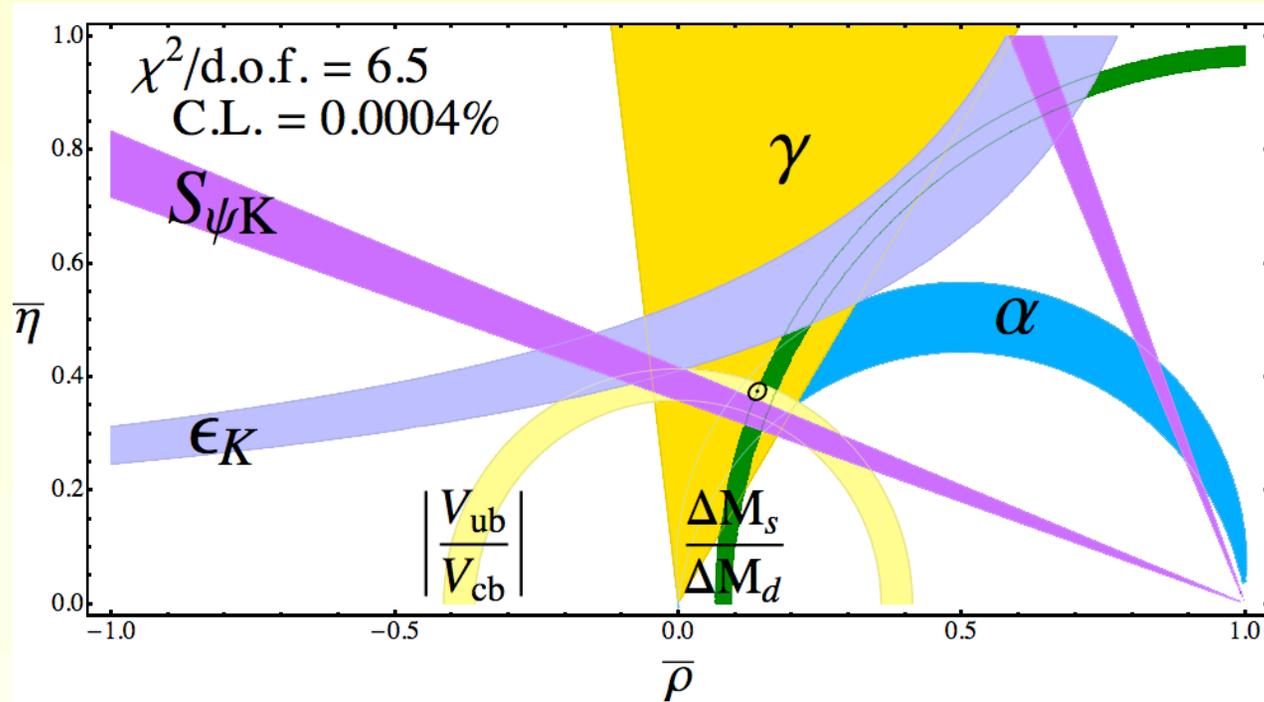
\* If we assume no NP at tree-level at current precision  
→ tension can be a sign of NP either in  $K^0$  or  $B^0$  mixing.

\*\* Current data prefer NP in  $K^0$  mixing.

\* Constraints from  $\epsilon_K$ ,  $\Delta M_d/\Delta M_s$ , and  $|V_{ub}/V_{cb}|$  limited by lattice errors for  $|V_{cb}|_{excl.}$ ,  $\xi$ , and  $|V_{ub}|_{excl.}$ .

## 5. Impact of up-to-date lattice averages on UT

When lattice QCD uncertainties become smaller

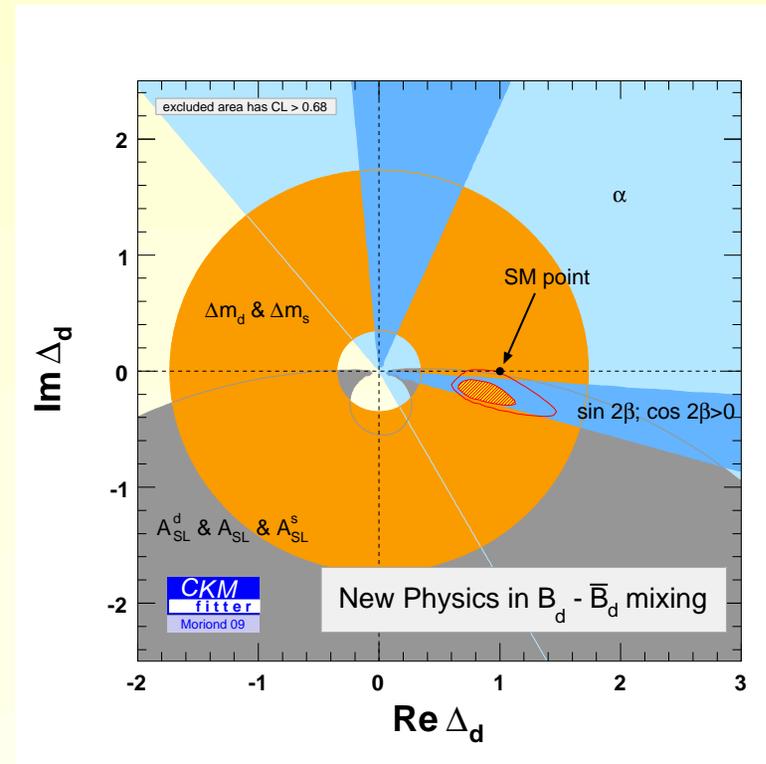
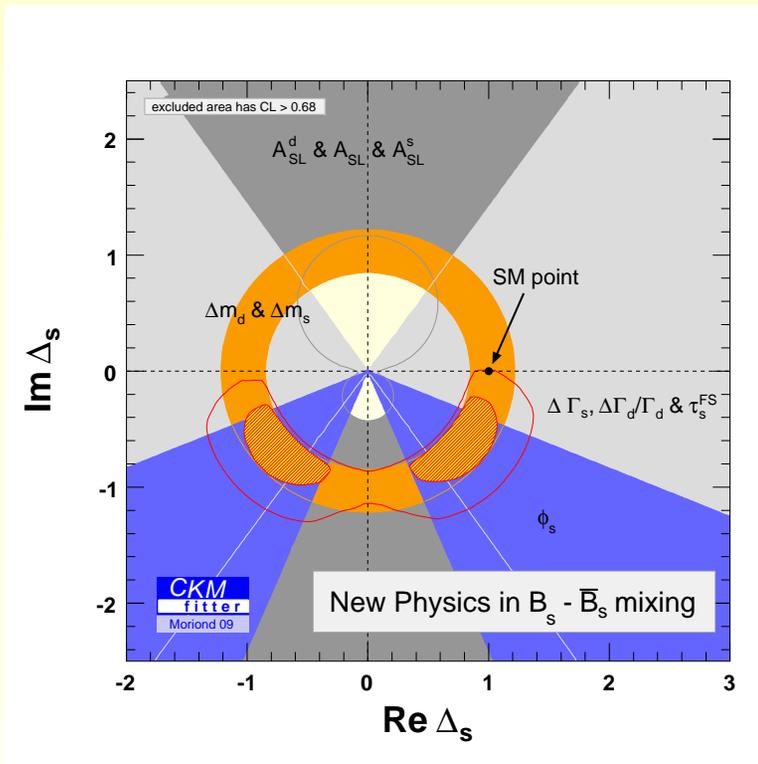


- \* Lattice QCD errors are reduced to 1% keeping central values.
- \* Use only exclusive  $|V_{cb}|$ .

Could see NP with a high significance!

## 5.1. Hints of New Physics in neutral $B$ mixing

CKMfitter:  $\langle B_q^0 | M_{12}^{SM+NP} | \bar{B}_q^0 \rangle = \Delta_q^{NP} \langle B_q^0 | M_{12}^{SM} | \bar{B}_q^0 \rangle$  V. Tisserand, 0905.1572



$1.9\sigma$ : Tension driven by the exp. measurement ( $2\beta_s, \Delta\Gamma_s$ ).

$2.1\sigma$ : Tension between  $\sin(2\beta)$  and  $|V_{ub}|/\tau\nu$

\* Tree-level mediated decays through a Four Flavor Change ( $b \rightarrow q_i \bar{q}_j q_k$ ) are SM

\* NP effects in oscillation parameters, weak phases, semi-leptonic asymmetries and  $B$  lifetime differences parametrized through  $\Delta$

## 6. Future plans for lattice analyses of $B^0$ mixing and decay constants

# Reduction of errors for  $f_{B_q}$ ,  $f_{B_q} \sqrt{B_{B_q}}$ ,  $\xi$ : smaller lattice spacing ( $a = 0.06, 0.045$ ), more statistics, improved renormalization methods, improved actions, better fitting and smearing methods ...

# Calculation of matrix elements needed for  $\Delta\Gamma_q$  **Lenz and Nierste**,  
JHEP0706 (2007) 072

$$\left(\frac{\Delta\Gamma}{\Gamma}\right) = \left(\frac{1}{245\text{MeV}}\right)^2 \left[ 0.170 \left(f_{B_q}^2 B_{B_q}\right) + 0.059R^2 \left(f_{B_q}^2 \tilde{B}_S R^2\right) - 0.044 f_{B_q}^2 \right]$$

\* Useful to impose constraints on BSM building, **M. Blanke et al**, LHT

\* Allows a theoretical prediction for

$$(A_{SL}^s)_{SM} \equiv \frac{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) - \Gamma(B_s^0 \rightarrow l^- X)}{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) + \Gamma(B_s^0 \rightarrow l^- X)} = \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)$$

$(A_{SL}^s)_{SM} \sim 10^{-5}$  **Lenz and Nierste**, JHEP 06 (2007) 072

## 6. Future plans for lattice analyses of $B^0$ mixing and decay constants

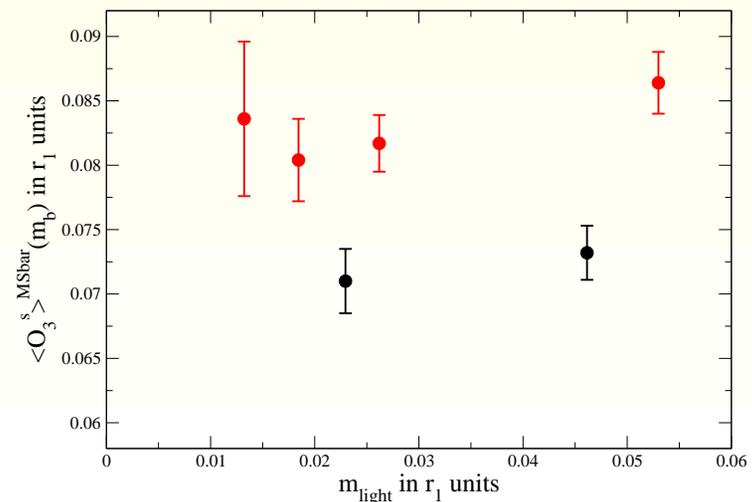
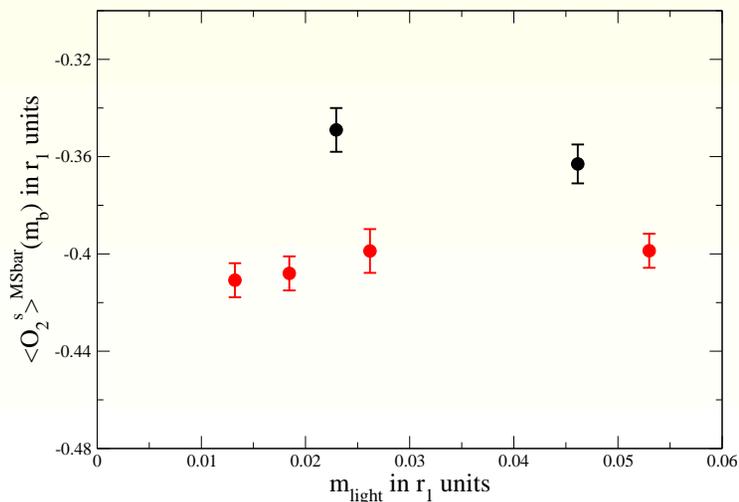
to compare with the value of the asymmetry in the presence of NP

Z. Ligeti, M. Papucci and G. Perez, PRL 97 (2006) 101801

$$A_{SL}^s = -\frac{\Delta\Gamma_s}{\Delta M_s} \frac{S_{\psi\phi}}{C_{B_s^0}} \simeq -(2.6 \pm 1.0) \times 10^{-3} \frac{S_{\psi\phi}}{C_{B_s^0}}$$

\*\* Even  $S_{\psi\phi} \simeq 0.1$  would lead to an order of magnitude enhancement relative to SM.

\* Some preliminary results HPQCD



## 6. Future plans for lattice analyses of $B^0$ mixing and decay constants

- # Calculation of matrix elements corresponding to operators that only appear in **BSM** theories.
  - \* Only quenched calculation available **Becirevic et al**, JHEP **04** (2002) 025
  - \* Straightforward extension of previous calculation
    - **FNAL/MILC**: work in progress
- # Analysis of short-distance contributions to  $D^0 - \bar{D}^0$  mixing
  - \* Also provides strong constraints on **BSM** physics **E. Golowich, J. Hewett, S. Pakvasa and A. Petrov**, PRD **76** (2007)
  - \* **FNAL/MILC** already working on extending their calculation to  $D^0 - \bar{D}^0$  mixing

## 7. More conclusions

- # High precision measurements/calculations of low energy observables allow to indirectly probe very short-distances.
- \* Test **SM** and **BSM** theories
- \* Learning about the flavour structure of the new physics.



## Error budget for $B^0$ mixing parameters

source of error	$f_{B_s} \sqrt{\hat{B}_{B_s}}$	$f_{B_d} \sqrt{\hat{B}_{B_d}}$	$\xi$
stat. + chiral extrap.	2.3	4.1	2.0
residual $a^2$ extrap. uncertainty	3.0	2.0	0.3
$r_1^{3/2}$ uncertainty	2.3	2.3	—
$g_{B^* B \pi}$ uncertainty	1.0	1.0	1.0
$m_s$ and $m_b$ tuning	1.5	1.0	1.0
operator matching	4.0	4.0	0.7
relativistic corr.	2.5	2.5	0.4
Total	6.7	7.1	2.6

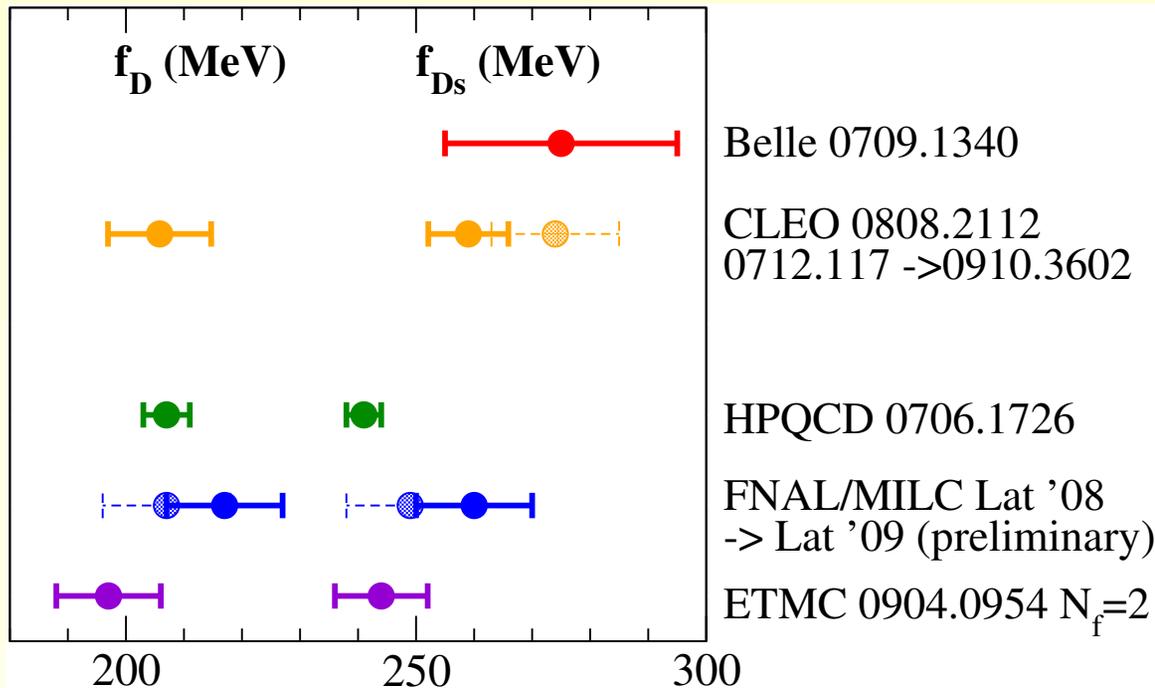
## 5.2.2 Value of the UT angle $\sin(2\beta)$

The value of the UT angle  $\sin(2\beta)$  obtained from  $b \rightarrow q\bar{q}s$  penguin decays is lower than from tree-level  $b \rightarrow c\bar{c}s$  (expected to be less sensitive to NP).

- \* For example,  $\sin(2\beta)_{B \rightarrow \phi K^0}$  is  $1.3\sigma$  from tree-level average (including, for example,  $\sin(2\beta)_{\Psi K_S}$ ).
- \* This tension can not be resolved at LHCb (only some clues from  $B_S \rightarrow \phi\phi$ ). Need Super Belle at KEK and Super-B machine at Frascati.
- \* Need better measurements of  $b \rightarrow q\bar{q}s$  penguin decays.

## 5.2.4 The $f_{D_s}$ puzzle

R. van de Water (Lattice09)



2008  $3.6\sigma$  discrepancy in  $f_{D_s}$  between HPQCD and experiment. Agreement in

$$\frac{f_K, f_\pi, f_D, m_D, m_{D_s}, 2m_{D_s} - m_{\eta_c}}{2m_D - m_{\eta_c}}.$$

2009  $2.3\sigma$  discrepancy

lattice(average)-exper.(average)

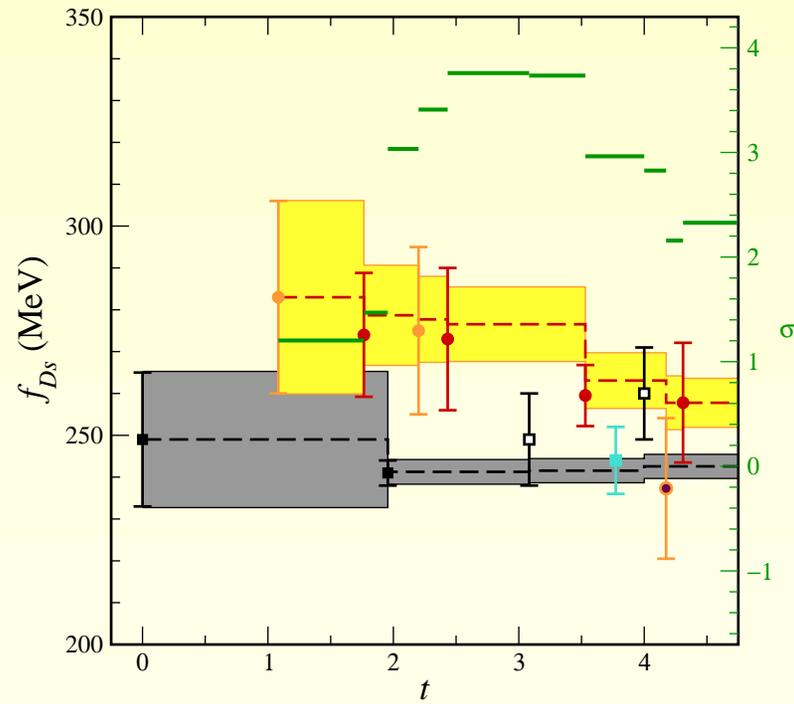
New CLEO, BaBar and FNAL/MILC results.

- \* Leptonic decays occurs at tree level  $\rightarrow$  disagreement difficult to accommodate in BSM.
- \* Models with charged Higgs or leptoquarks can work Kronfeld and Dobrescu  $\rightarrow$  signal in  $D \rightarrow K(\pi)l\nu$ .

## 5.2.4 The $f_{D_s}$ puzzle

- \* Lattice calculations.
  - \*\* **HPQCD** has redetermined the scale that converts lattice quantities to physical units  $r_1$ . New value will make their value lower by  $1 - 1.5\sigma \rightarrow$  disagreement under  $2\sigma$ . Update soon.
  - \*\* Include effects of sea **charm** since errors are around 1%
  - \*\* Need lattice results with different fermion formulations.
- \* Some experimental issues.
  - \*\* Experiment uses  $|V_{cs}| = |V_{ud}|$ .
  - \*\* Better understanding of **radiative corrections**.
- \* **BES-III** should measure  $f_D$  and  $f_{D_s}$  with  $\sim 1\%$  precision.

## 5.2.4 The $f_{D_s}$ puzzle



Andreas Kronfeld

## 5.2.6 Clarification of $\mu$ anomalous magnetic moment, $(g - 2)_\mu$ anomaly

The measured  $(g - 2)_e$  is in excellent agreement with **SM** but measured  $(g - 2)_\mu$  is significantly larger ( $3.1\sigma$ ) than predicted.

- \* Hadronic (non-perturbative) contributions to  $(g - 2)_\mu$  make the comparison of data and theory a bit problematic.
- \* New experiments are being designed to reduce the experimental error by a factor of 5.
- \* Need theoretical improvements too. Goal in light-by-light contribution: a 20-40% reduction of the error.
- \* **Example:** Confirmation of exper. measurements  $\rightarrow$  favour the **MSSM** over **LHT**.

## 2.2. $B_0$ mixing beyond the SM

### # Some examples:

Isidori, Nir and Perez, Ann. Rev. Nucl. Part. Sci 2010:

Bounds on representative dimension-six  $\Delta F = 2$  operators.

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum \frac{c_i^{(d)}}{\Lambda^{(d-4)}} O_i^{(d)}$$

Operator	Bounds on $c_{ij}$ ( $\Lambda = 1$ TeV)		Observables
	Re	Im	
$(\bar{c}_L \gamma^\mu u_L)^2$	$5.6 \times 10^{-7}$	$1.0 \times 10^{-7}$	$\Delta m_D;  q/p , \phi_D$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$5.7 \times 10^{-8}$	$1.1 \times 10^{-8}$	$\Delta m_D;  q/p , \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	$3.3 \times 10^{-6}$	$1.0 \times 10^{-6}$	$\Delta m_{B_d}; S_{B_d \rightarrow \psi K}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$8.8 \times 10^{-7}$	$2.6 \times 10^{-7}$	$\Delta m_{B_d}; S_{B_d \rightarrow \psi K}$
$(\bar{b}_L \gamma^\mu s_L)^2$	$6.0 \times 10^{-5}$	$6.0 \times 10^{-5}$	$\Delta m_{B_s}$
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	$1.6 \times 10^{-5}$	$1.6 \times 10^{-5}$	$\Delta m_{B_s}$

# Effects of NP in some flavour observables involving neutral meson mixing parameters

(Altmannshofer et al., arXiv:0909.1333 and Buras, EPS-HEP 2009)

	AC	RVV2	AKM	$\delta_{LL}$	FBMSSM	LHT	RS
$D^0 - \bar{D}^0$	★★★	★	★	★	★	★★★	?
$S_{\psi\phi}$	★★★	★★★	★★★	★	★	★★★	★★★
$S_{\phi K_S}$	★★★	★★	★	★★★	★★★	★	?
$B_s \rightarrow \mu^+ \mu^-$	★★★	★★★	★★★	★★★	★★★	★	★

AC = SUSY flavour model with right-handed currents

RVV2 = SUSY flavour model with right-handed currents

AKM = SUSY flavour model with right-handed currents

$\delta_{LL}$  = SUSY flavour model with only left-handed currents

FBMSSM = flavour blind MSSM

LHT = Little Higgs models with T-parity.

RS = Randall Sundrum models