Self-Interacting Neutrinos and the MSW Resonance for Photons

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Outline

- Particle oscillations
- Established foundations & challenging frontiers
- Testing the frontiers of this paradigm
  - Neutrino oscillations: Neutrinos interacting with neutrinos
  - Photon oscillations: Faraday rotation in twisting magnetic fields
- Summary

Why interference?

- The first place to look for new physics!
- If amplitude for any process is $A=A_{old}+\varepsilon A_{new}$, the leading effect due to the new physics is in the cross-term $2\mathrm{Re}(\varepsilon A_{old}A^*_{new})$, i.e. in the interference effects
- In some sense, particle oscillations are due to the interference between the classical and quantum effects
- Thus it is very useful to explore uncharted territory

Particle oscillations

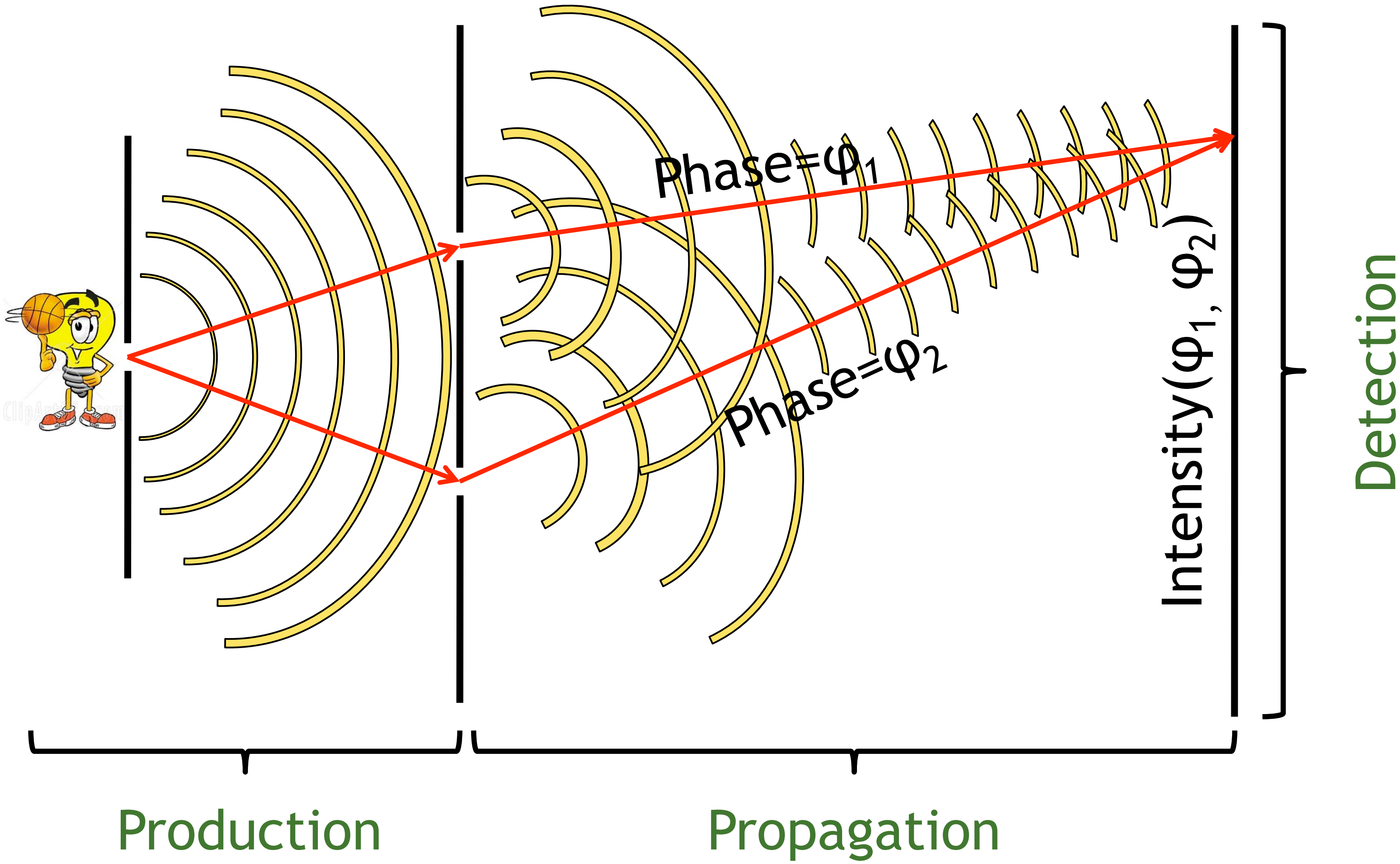
Basic set-up

- Initial state is a superposition of n≥2 eigenstates
- Each state propagates independently
- A linear superposition of states is observed

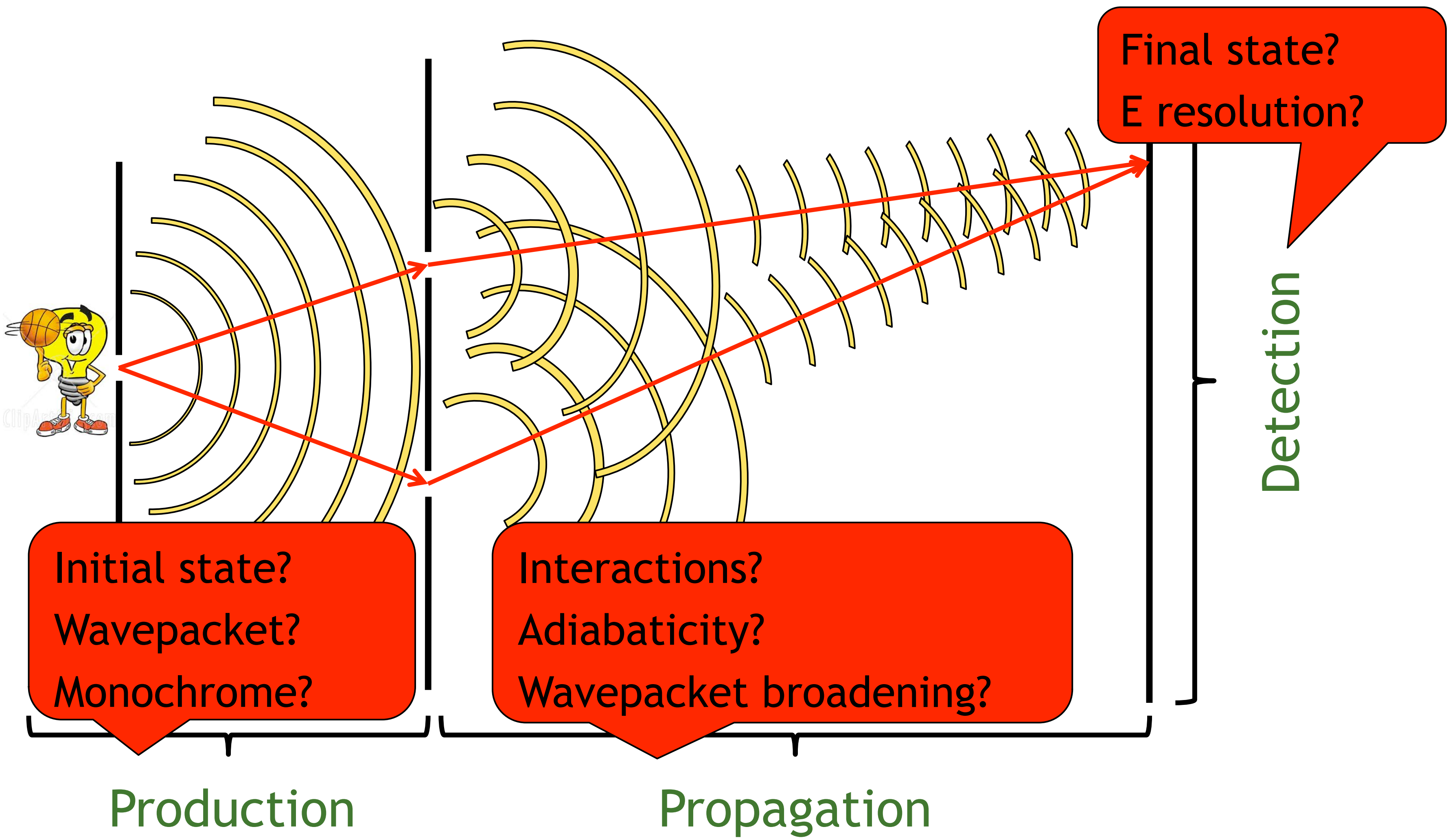
Main features

- Observed "intensity" related to mass/energy differences
- Symmetry properties probed
- Properties of the "medium" probed

Wave interference



Common issues



Developments

- Neutrinos
  - High mono-chromaticity/Mossbauer […Kopp et al, Bilenky et al]
  - Wavepacket (de)coherence […+ Kayser & Kopp]
  - New interactions (NSI) […]
  - Evolution in turbulent media [many papers…]
  - Nonlinear evolution of SN neutrinos [many papers…]
- Radio waves
  - The possibility of resonant conversion of polarization states, aka Adiabatic Faraday Effect [Broderick & Blandford…]

We are in a position to test the frontiers of this paradigm

Today, I will discuss two such possibilities

Neutrinos

Neutrino oscillations

- Neutrino oscillation usually involves only 2 terms
  - Mass matrix / 2E
  - MSW potential (due to electrons)

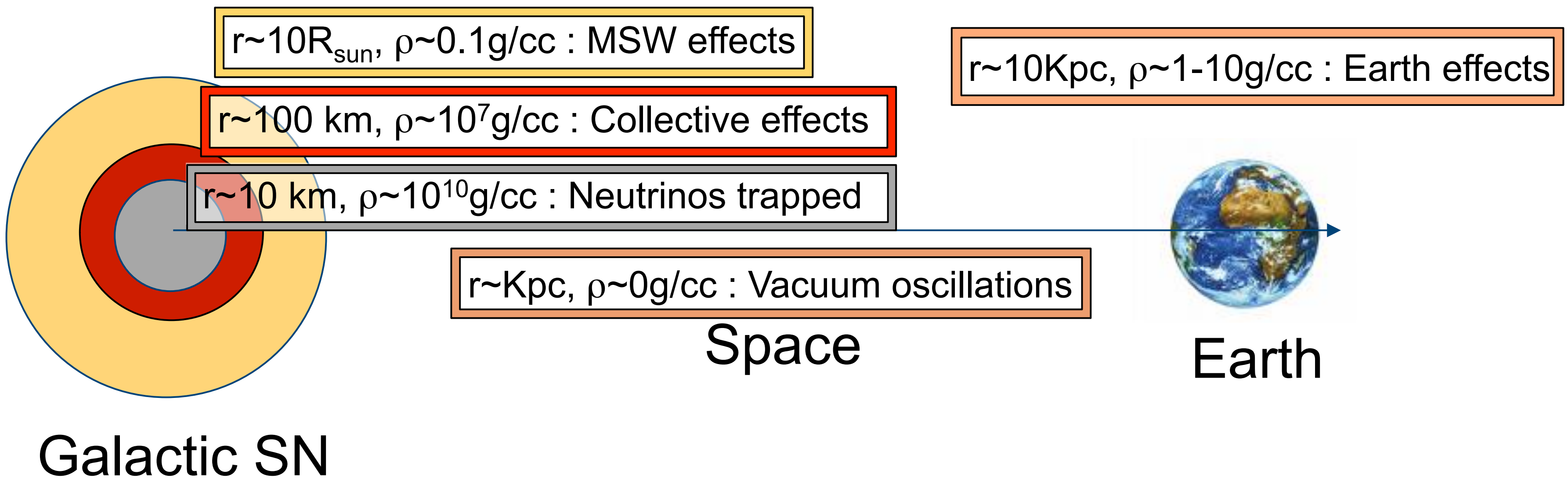
- We are familiar with the phenomenology
  - Vacuum oscillation [Pontecorvo]
  - In-medium oscillation [Wolfenstein]
  - Resonant conversion [Mikheyev,Smirnov]
  - Non-adiabatic resonant conversion [Parke]

Neutrino oscillations in SN

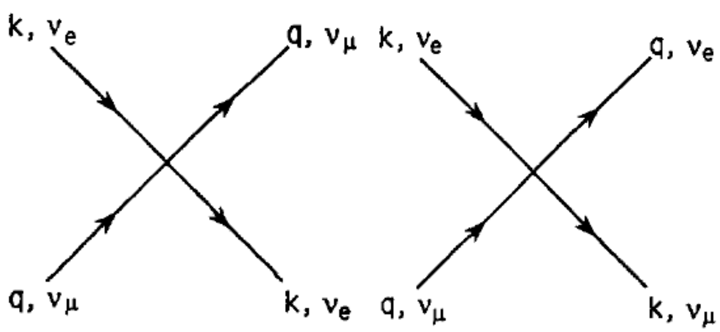
- In a SN, neutrinos are very dense and therefore neutrinos interact with each other quite frequently
- This creates a MSW-like potential from neutrinos
  - Flavor non-trivial
  - Coupled neutrino oscillations
- Neutrino flavor spectra swap in some energy ranges
- These are called “Collective Effects”

The SN neutrino program

- Calculate an initial neutrino spectrum
- Calculate the changed spectrum due to oscillation effects
- Calculate flux at detector
- Construct variables that distinguish different physics/astro scenarios
- Wait for a SN…



A bare-bones explanation



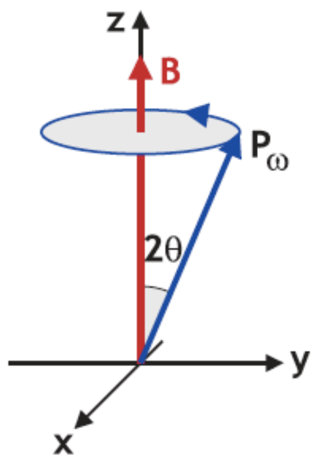
Pantaleone (PLB, 1992)

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{vmatrix} |\nu_e(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_e(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \end{vmatrix} = V_2 \begin{vmatrix} |\nu_e(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_e(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \end{vmatrix}$$

$$V_{2\nu} = \sqrt{2}G_F\xi\frac{1}{V}\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

- **Hilbert space is 2-particle (N-particle in general)**
- **The “Hamiltonian” is NOT flavor blind**

Neutrino-Neutrino interactions

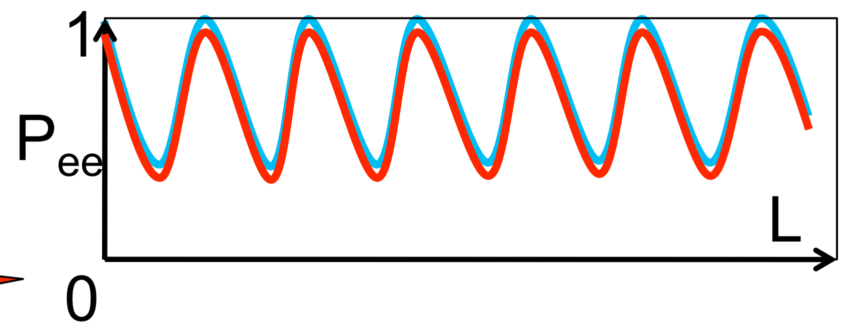


Hamiltonian is dual to a gyroscope

Neutrino pair-conversions

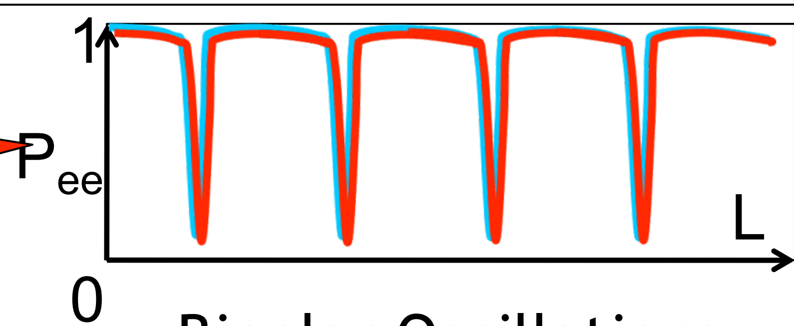
$\nu_e\bar{\nu}_e \Leftrightarrow \nu_x\bar{\nu}_x$

Neutrinos of all energies oscillate together



Synchronized Oscillations

Neutrinos of all energies flip to the lighter mass eigenstate



Bipolar Oscillations

SN collective effects

Early papers: Pantaleone, Samuel, Kostelecky in 1992-95

Seminal papers: Duan, Fuller, Carlson and Qian in 2005, 2006

Pendulum Analogy: Hannestad, Raffelt, Sigl and Wong, arXiv:astro-ph/0608695

More than 100 papers on collective effects by:
Abazajian, Balantekin, Beacom, Bell, Blennow, Carlson, Dasgupta, Dighe, Dolgov, Duan, Esteban-Pretel, Fogli, Friedland, Fuller, Gava, Goswami, Hannestad, Hansen, Kneller, Kostelecky, Lisi, Lunardini, Marrone, McLaughlin, Mirizzi, Pantaleone, Pastor, Pehlivan, Qian, Raffelt, Samuel, Serpico, Semikoz, Sigl, Smirnov, Stodolsky, Tamborra, Tomas, Volpe, Wong …

How to predict the final spectra, given the initial spectra.

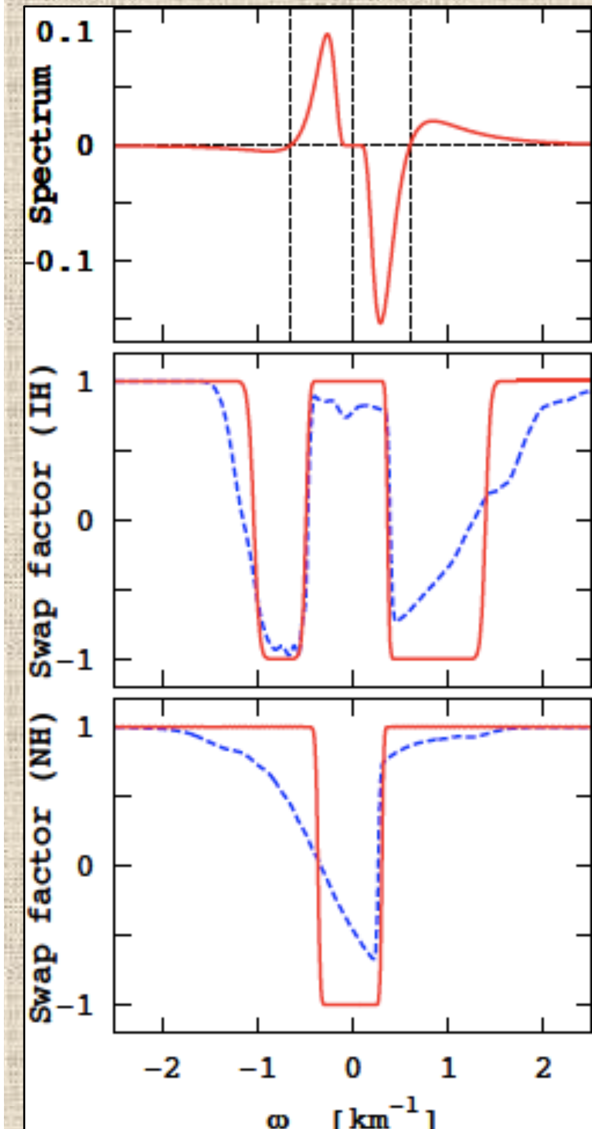
Notation

- We have the flux spectrum f(E) for each flavor
- However, let's use $\omega=\Delta m^2/2E$ as the x-axis variable
- Moreover, let's label antineutrinos with $-\omega$
- Define

$$g(\omega)=\begin{cases} f_e(E)-f_x(E) & \text{for neutrinos} \\ f_x(E)-f_e(E) & \text{for antineutrinos} \end{cases}$$

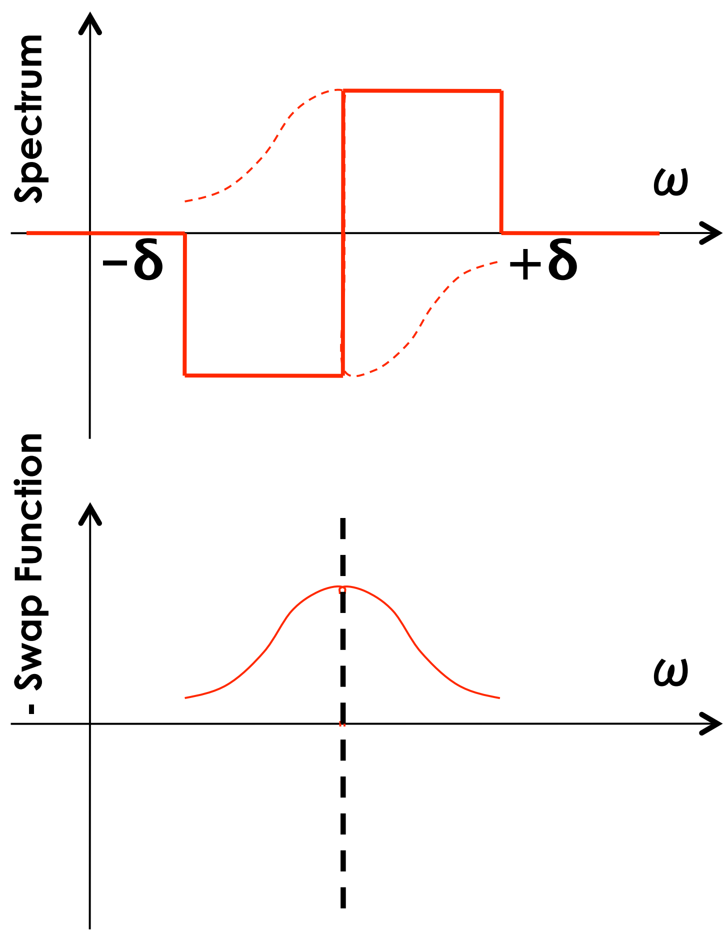
- Now we have put the all the relevant spectral information in a single function $g(\omega)$
- How does this function look? Let's see…

In the g(ω) variable…



- $g(\omega)=0$ where fluxes equal
- "Swaps" around every "± crossing"
- Each swap flanked by two "splits"
- Splits not always washed out completely by multi-angle effects
- Let's answer some questions now…
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?

Fixed initial neutrino density μ



- "Box" spectrum at finite μ
- Spectrum oscillates to the dotted lines and back
- Swap function looks like a Lorentzian centered at the crossing at any instant!
  - Collective motion
  - May be we can solve this analytically?
  - Let's try…

"Deriving" the Lorentzian

- The system has EOM

$$\dot{\mathbf{P}}_\omega = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\int d\omega_1\,\mathbf{P}_{\omega_1}) \times \mathbf{P}_\omega$$

- Ansatz:

$$\mathbf{P}_\omega(t) = \begin{pmatrix} -\sin\varphi\, L(\omega) \\ -\frac{\omega}{\Gamma}\, 2\sqrt{1-\cos\varphi}\, L(\omega) \\ 1-(1-\cos\varphi)\, L(\omega) \end{pmatrix} g(\omega)$$

$$L(\omega) = \frac{\Gamma^2}{\Gamma^2+\omega^2}$$

- This is a merely a parametrization, and putting it back in EOMs we get

$$\dot{\varphi} = \Gamma\sqrt{2(1-\cos\varphi)}$$

$$\Gamma = \frac{\delta}{\sqrt{-1+e^{2\delta/\mu}}}$$

- EOM of a pendulum
- Width is exponential in μ

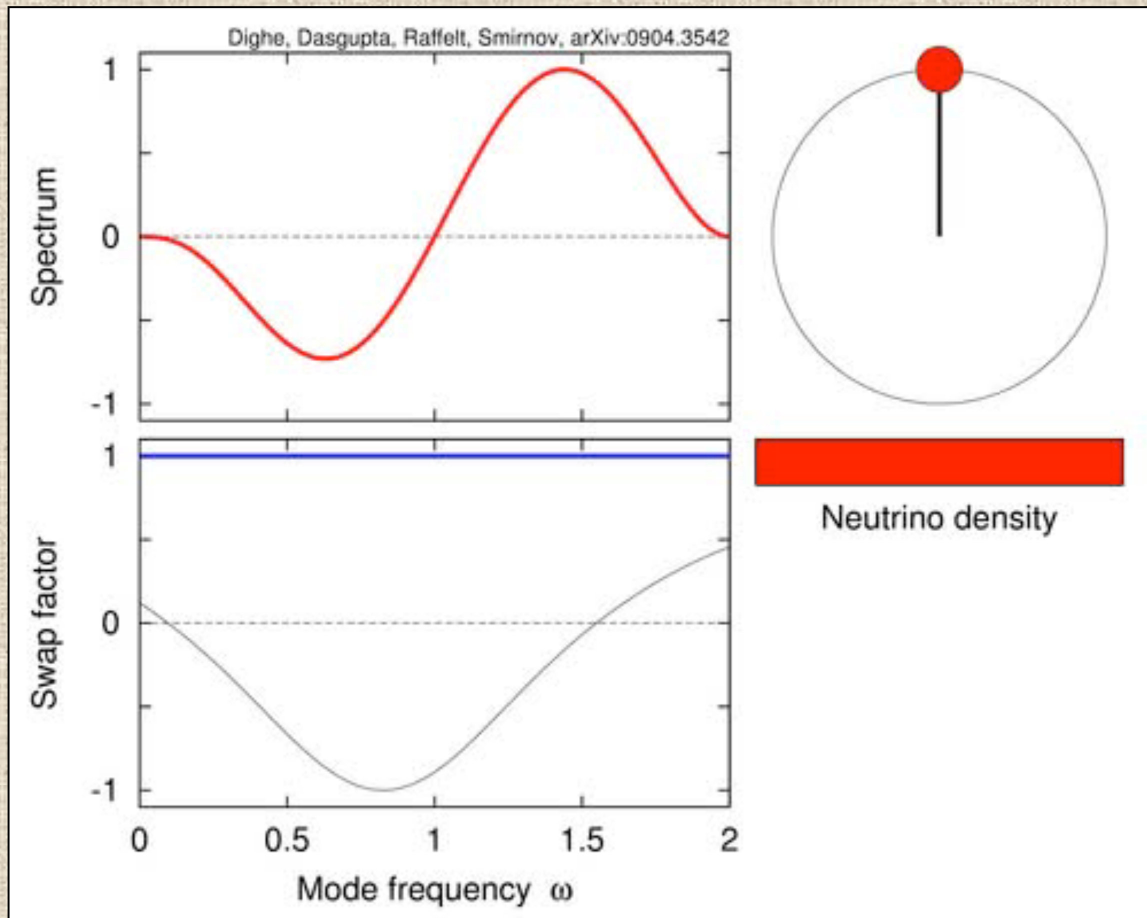
Changing neutrino density μ

- We know that as we decrease μ (mimicking decreasing neutrino density away from the core) the pendulum damps and relaxes to lowest energy configuration
- This system involves an adiabatic invariant that *roughly* relates the width of split ω_s to width of Lorentzian Γ

$$\omega_s = \frac{\pi}{4}\Gamma \frac{2}{1+\sqrt{1+\frac{\pi^2}{4}\frac{\mu}{2\delta}}} = \frac{\delta}{\sqrt{-1+e^{2\delta/\mu}}} \frac{\pi/2}{1+\sqrt{1+\frac{\pi^2}{4}\frac{\mu}{2\delta}}}$$

Dasgupta, Dighe, Raffelt, Smirnov, 0904.3542(PRL) + longer version in preparation

Collective spectral splits



Dasgupta, Dighe, Raffelt and Smirnov, arXiv:0904.3542 (PRL)
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Some comments

- We showed that there is a pendulum like oscillation about a crossing
- As μ decreases, this pendulum eventually tips over if it is inverted, i.e. +ive crossings for IH, -ive crossings for NH
- Thus there are swaps around a crossing (B.P conserved)
- Width of the swap is related to Γ and depends on initial μ: wider swaps for larger initial μ. Exponentially thin swaps for small. Also depends on δ, i.e. box width

Analogy to Spin Magnetic Resonance

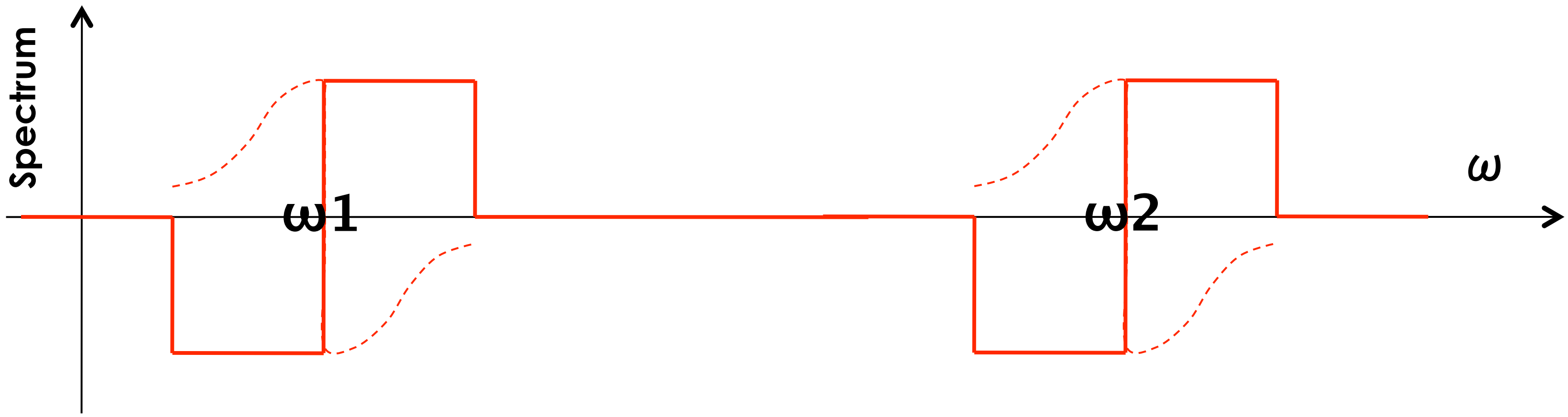
- We break the collective magnetic field into a parallel and perpendicular component, drop the former

$$\dot{\mathbf{P}}_{\omega} = (\omega\mathbf{B} + \mu\int d\omega_1\, \mathbf{P}^{\parallel}_{\omega_1} + \mu\int d\omega_1\, \mathbf{P}^{\perp}_{\omega_1}) \times \mathbf{P}_{\omega}$$

- For ω=0, P is on-resonance (the mode has the same frequency as the transverse magnetic field)!
- Others are slightly off-resonance by ω, and their amplitude falls off as a Lorentzian, as in SMR

But that's still only two splits …

- What happens if two copies of the box are put far apart in ω-space?

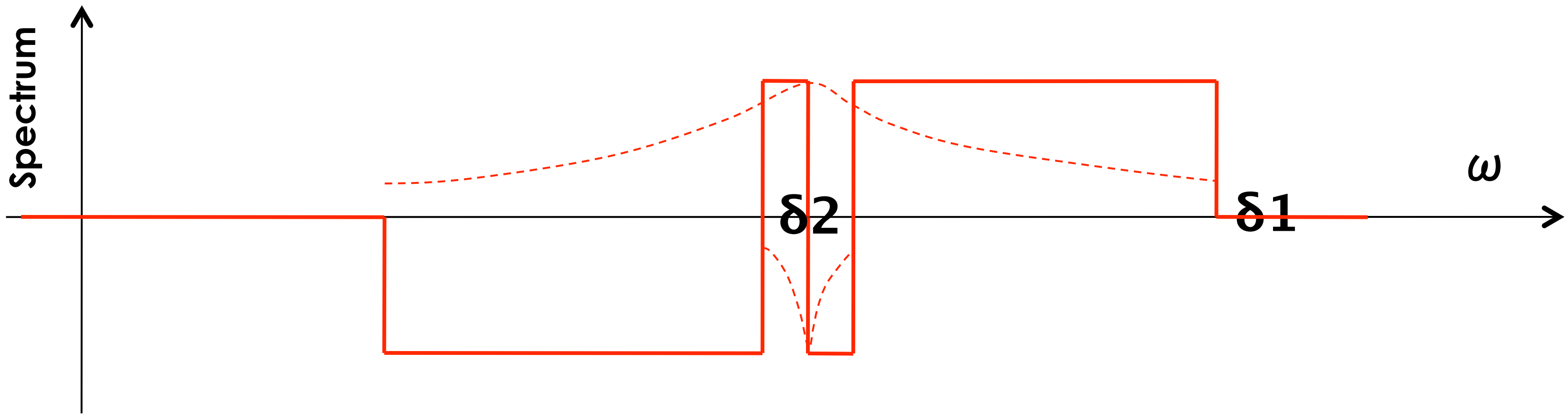


- Each box acts like an independent pendulum; the transverse field due to the other is averaged to zero

$$\dot{\mathbf{P}}_\omega = (\omega\mathbf{B} + \mu\int \mathrm{d}\omega_1\,\mathbf{P}_{\omega_1} + \mu\int \mathrm{d}\omega_2\,\mathbf{P}_{\omega_2}) \times \mathbf{P}_\omega$$

What happens when they are close

- What happens if two "boxes" are put close together in the ω-space



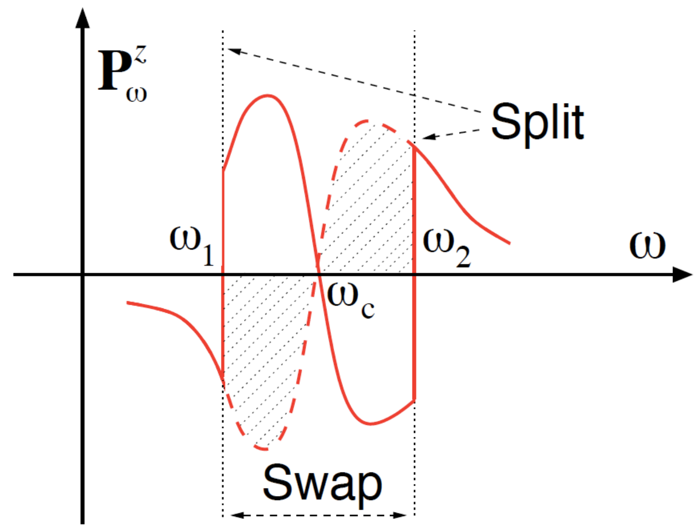
- The inner block acts like a superimposed oscillator on the bigger one. The inner swap-width is exponentially small

What's special about the box?

- **Short answer: Nothing!**
- **Long answer: Although any function around the crossing works fine, doing the integrals is harder/impossible. Also, the uniqueness and stability of the solution is not guaranteed**

General case

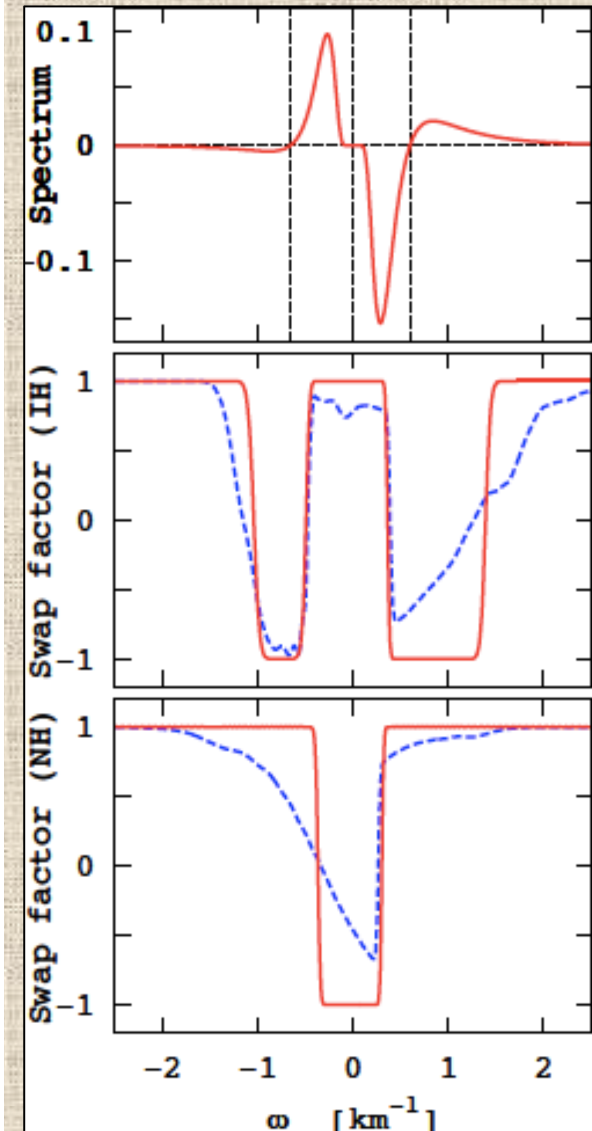
- The basic picture …



  - One swap for every ± crossing for IH/NH
  - Width of each swap depends exponentially on μ and also on the δ for the block around that crossing
  - Each swap approx. preserves lepton number B.P locally

- When blocks are close more complicated things can happen, and it would be interesting to study…

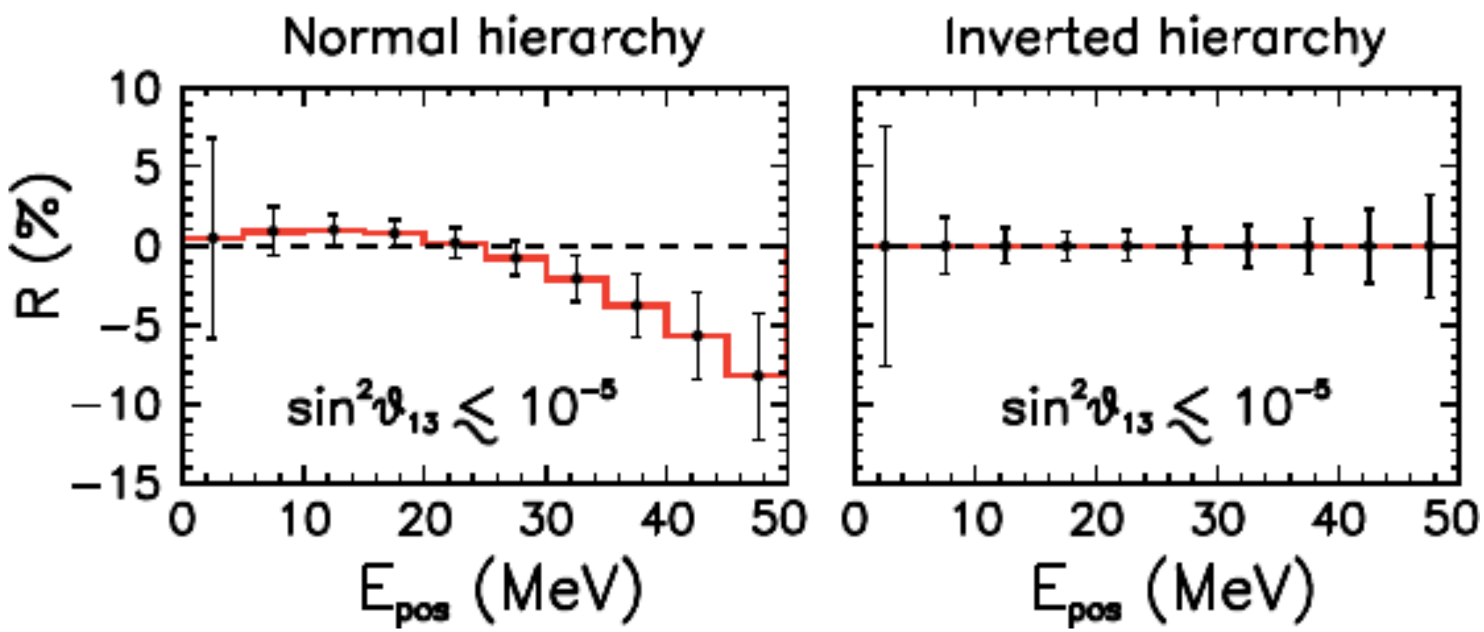
Recap



- Swaps around every "± crossing"
- Each swap flanked by two splits
- Splits not always washed out completely by multi-angle effects
- We have answered the questions…
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?

NH/IH determination

- **Look at the early signal (< 1 sec) in antineutrinos using ratio of events at two WC detectors**



Dasgupta, Dighe, Mirizzi, arXiv:0802.1481(PRL)

Outlook for SN neutrinos

- An area of brisk activity
- More work needed with greater sophistication
- Hopefully we will learn enough by the time a SN explodes!

Photons

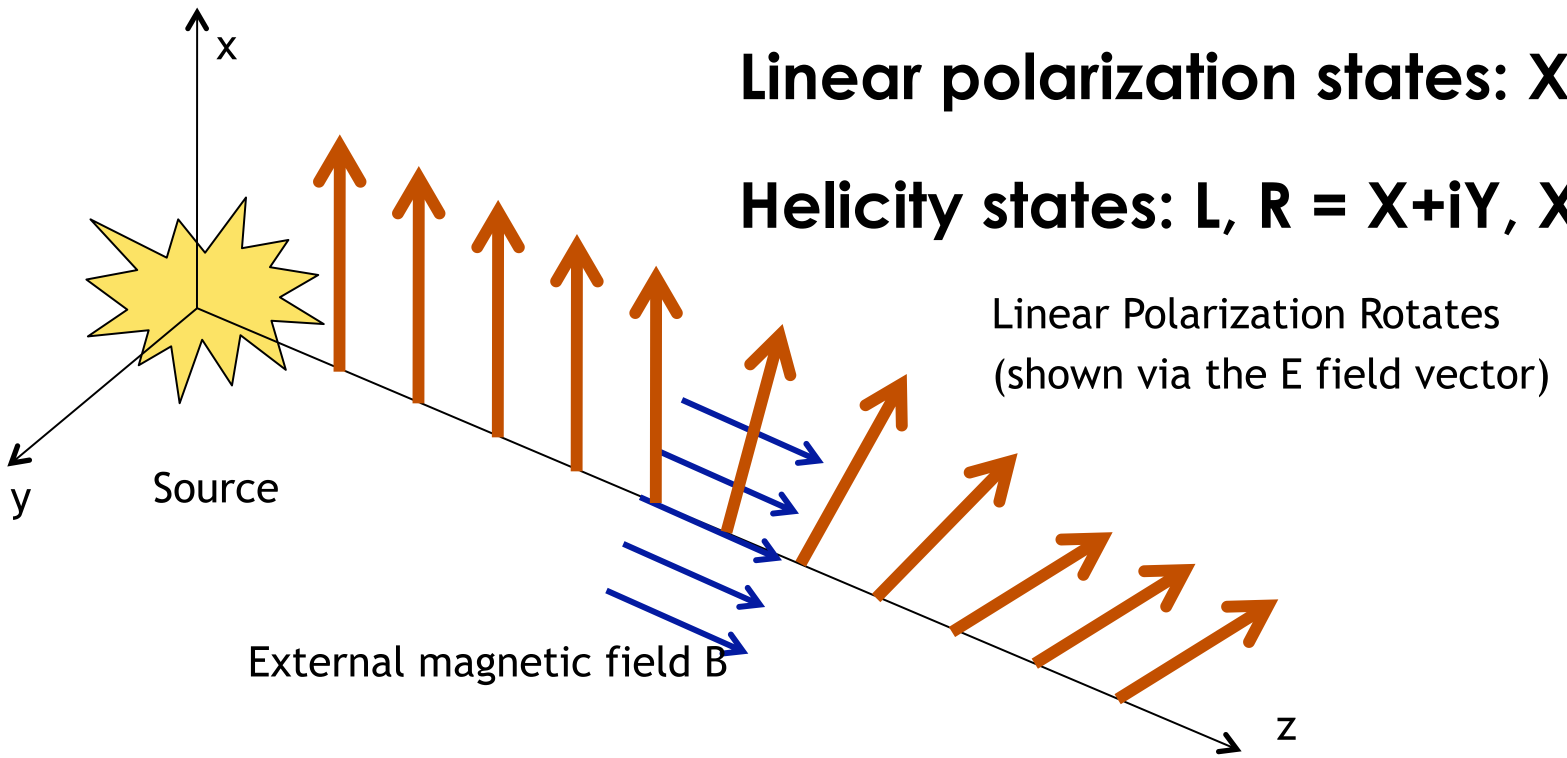
Do photons oscillate?

- In some sense, of course! Interference effects are path-length dependent variation in intensity!
- But do photons show "flavor oscillation"? Essentially the question is: Are there two kinds of photons with different refractive indices?
- Birefringence: Different polarizations, different n
- Vacuum birefringence is small (QED effect)
- In medium can be large (calcite crystal, or magnetized plasma,…)

Faraday effect

Linear polarization states: X,Y

Helicity states: L, R = X+iY, X-iY



- **When B field is longitudinal, L and R are propagation states and L and R have different refractive indices, naturally X and Y rotate as they propagate**

Resonant Faraday effect

- **For a longitudinal external B field the mixing angle between helicity and propagation states is small**

$$\mathrm{H} = \frac{\omega_{\mathrm{p}}^2}{2\omega} \begin{pmatrix} b_{\|} & -\frac{1}{2}\,\mathrm{e}^{-\mathrm{i}2\varphi}\,b_{\perp}^2 \\ -\frac{1}{2}\,\mathrm{e}^{\mathrm{i}2\varphi}\,b_{\perp}^2 & -b_{\|} \end{pmatrix}$$

- **If B is primarily transverse, that is not the case! In this case one hits an MSW resonance (Hamiltonian is degenerate)**
- **Here, one can get resonant conversion between L and R if there is (are) B field reversal(s) on the trajectory**

Jump across levels

- **Probability of L to remain L (or switch propagation states)**

$$P_{\mathrm{j}} = \mathrm{e}^{-\pi\gamma/2}$$

$$\gamma = \frac{\omega_{\mathrm{p}}^2}{2\omega}\,\frac{b_\perp^4}{|2b'_{||}|} = \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{c}}^3}{4\omega^4}\,\ell_B$$

- **The two extreme limits are 0 (Ordinary Faraday effect) and 1 (Adiabatic Faraday effect)**
- **Lower frequencies are more adiabatic**

Faraday rotation measure

- **Unitary evolution** $\mathbf{A}_{\mathrm{D}} = \mathsf{U}_{\mathrm{tot}}\mathbf{A}_0$

$$\mathsf{U}_{\mathrm{tot}} = \mathcal{S}\ \exp\left(-\mathrm{i}\int_0^{\mathrm{D}} \mathsf{H}\,\mathrm{d}z\right)$$

- **Net rotation of polarization**

$$\phi = \frac{1}{2\omega}\int_0^{\mathrm{D}} \mathrm{d}z\,\omega_{\mathrm{p}}^2\,b_{\|} = \frac{e^3}{2\omega^2 m_e^2}\int_0^{\mathrm{D}} n_e\mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell}$$

Adiabatic Faraday rotation measure

- **Unitary evolution**

$$\mathbf{A}_{\mathrm{D}} = \mathsf{U}_{\mathrm{tot}} \mathbf{A}_0$$

$$\mathsf{U}_{\mathrm{tot}} = \mathsf{U}_2 \mathsf{U}_{\mathrm{flip}} \mathsf{U}_1$$

$$\mathsf{U}_{1,2} = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\,\phi_{1,2}} & 0 \\ 0 & \mathrm{e}^{+\mathrm{i}\,\phi_{1,2}} \end{pmatrix}$$

$$\mathsf{U}_{\mathrm{flip}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

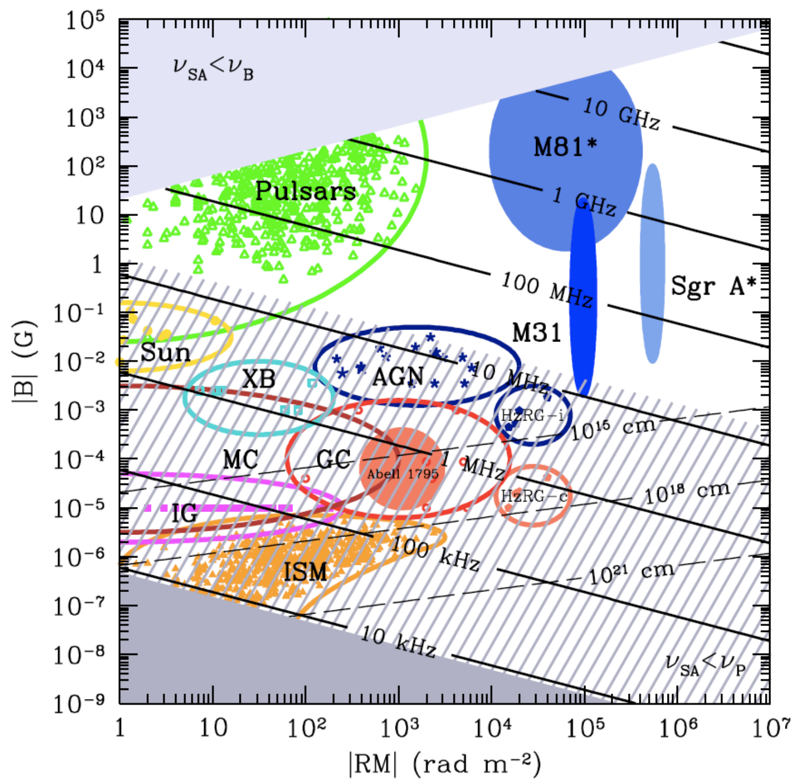
- **Net rotation of polarization**

$$\mathrm{RM}^{\mathrm{ad}} = \pm \frac{e^3}{8\pi^2 m_e^2} \int_0^{\mathrm{D}} n_e |\mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}|$$

the overall sign is determined by sign(B_z) at the end

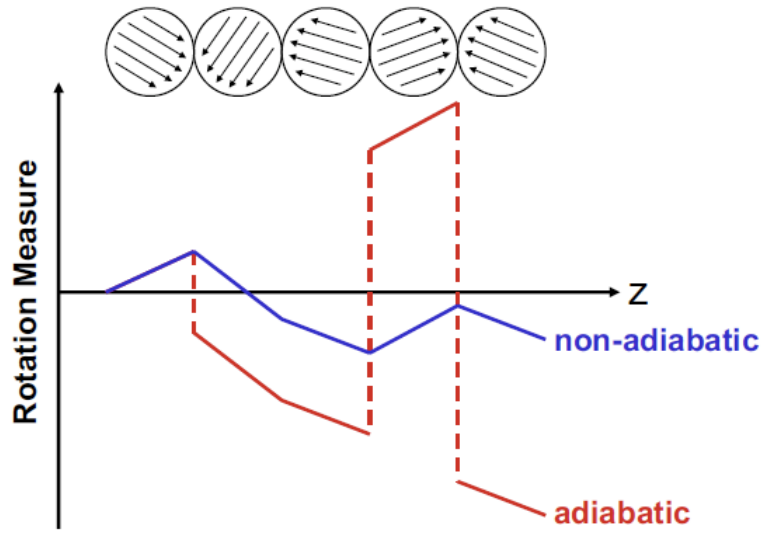
Possible detection and applications

- **Proof of principle**
- **A beautiful extension of a classic effect**
- **Probing geometry of the magnetic fields**



Broderick and Blandford, arXiv:0911.3909 (ApJ)

Multiple reversals



Dasgupta and Raffelt, arXiv:1006.4158 (PRD)

- **Enhanced effect: Sqrt(N) (ordinary) vs. N (adiabatic)**
- **Elliptic polarization at the crossover frequency**

Outlook for MSW effect in radio waves

- Perhaps we will see this new effect in the coming years using large radio telescopes
- Good polarization capability at low frequencies is the main requirement
- May turn out to be an important tool in astronomy

Summary

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- Testing the frontiers of this paradigm
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