Measuring the Higgs boson self-coupling at high energy e^+e^- colliders

UB, PRD 80, 013012 (2009)

1. Introduction
2. ZHH production
3. $\nu\bar{\nu}HH$ production
4. Sensitivity limits
5. Conclusions

Ulrich Baur
State University of New York at Buffalo

1 – Introduction

- If it exists, the Standard Model (SM) Higgs boson will be discovered at the LHC
- The LHC promises complete coverage of Higgs decay scenarios
- Quantitatively at the LHC: measure
  - ☞ M_H to 0.1%
  - ☞ Γ_H to $\leq 10\%$
  - ☞ $\sigma\times$ Br to 10%

- what remains to be done: determine Higgs potential

$$V(\eta_H) = \frac{1}{2} m_H^2 \eta_H^2 + \lambda v \eta_H^3 + \frac{1}{4} \tilde{\lambda} \eta_H^4,$$

η_H: physical Higgs field, $v = (\sqrt{2}G_F)^{-1/2}$,
SM: $\tilde{\lambda} = \lambda = \lambda_{SM} = m_H^2/(2v^2)$
☞ λ and $\tilde{\lambda}$ are *per se* free parameters

- to measure λ ($\tilde{\lambda}$), experiments must observe HH (HHH) production
  - ☞ HHH cross sections too small to probe $\tilde{\lambda}$ at any machine considered so far
  - ☞ concentrate on λ in the following
- radiative corrections to HHH coupling:
  - ☞ SM: $-4\% - -11\%$ for 120 GeV $< M_H <$ 200 GeV (Yuan *et al.*)
  - ☞ can be up to 100% in general 2HDM
  - ☞ MSSM: up to 8% for light stop squarks (Hollik *et al.*)

- The measurement of the Higgs self-coupling, λ, is one of the benchmarks which is used to gauge the performance of the ILC
- Past investigations have focused on a very light Higgs boson ($m_H = 120$ GeV) with $\sqrt{s} = 500$ GeV
- and the background was estimated using shower Monte Carlos
- Here, I present calculations using `MadEvent`
  - ☞ for $m_H = 120$ GeV, $m_H = 140$ GeV and $m_H = 180$ GeV (disfavored in recent `GFITTER` fits)
  - ☞ $\sqrt{s} = 500$ GeV, 1 TeV and 3 TeV
  - ☞ for $e^+e^- \to ZHH \to jj4b$ and $e^+e^- \to \nu\bar{\nu}HH$
  - ☞ with the backgrounds, including the non-resonant diagrams, calculated using exact matrix elements (whereever possible)

2 – ZHH Production

- I focus on $ZHH \to jj4b$ and require that the jj system is compatible with a Z boson, and the $4b$'s form two pairs which are compatible in invariant mass with a Higgs boson:

$$|M_Z - m(jj)| < 8 \text{ GeV}$$

$$100\ (120) \text{ GeV} < m(b\bar{b}) < 126\ (150) \text{ GeV}$$

for $m_H = 120\ (140)$ GeV.

- require 4 tagged b-quarks
- include minimal detector effects by Gaussian smearing (ILC detector expectations):

$$\frac{\Delta E}{E}(\text{had}) = \frac{0.405}{\sqrt{E}}, \qquad \frac{\Delta E}{E}(\text{lep}) = \frac{0.102}{\sqrt{E}},$$

- cuts:

$$E_{j(b)} > 15 \text{ GeV}, \qquad 5^\circ < \theta(j(b), \text{beam}) < 175^\circ$$

$$\theta(j(b), j'(b')) > 10^\circ \qquad \theta(j, b) > 10^\circ$$

- assume a b-tagging efficiency of $\epsilon_b = 0.9$, and charm and light quark/gluon jet misidentification probabilities of $P_{c\to b} = 10\%$, $P_{j\to b} = 0.5\%$
- also investigate $\epsilon_b = 0.8$ and $P_{c\to b} = 2\%$, $P_{j\to b} = 0.1\%$
- take energy loss of b-quarks into account via a parametrized function
- m_{HH} distribution is sensitive to λ

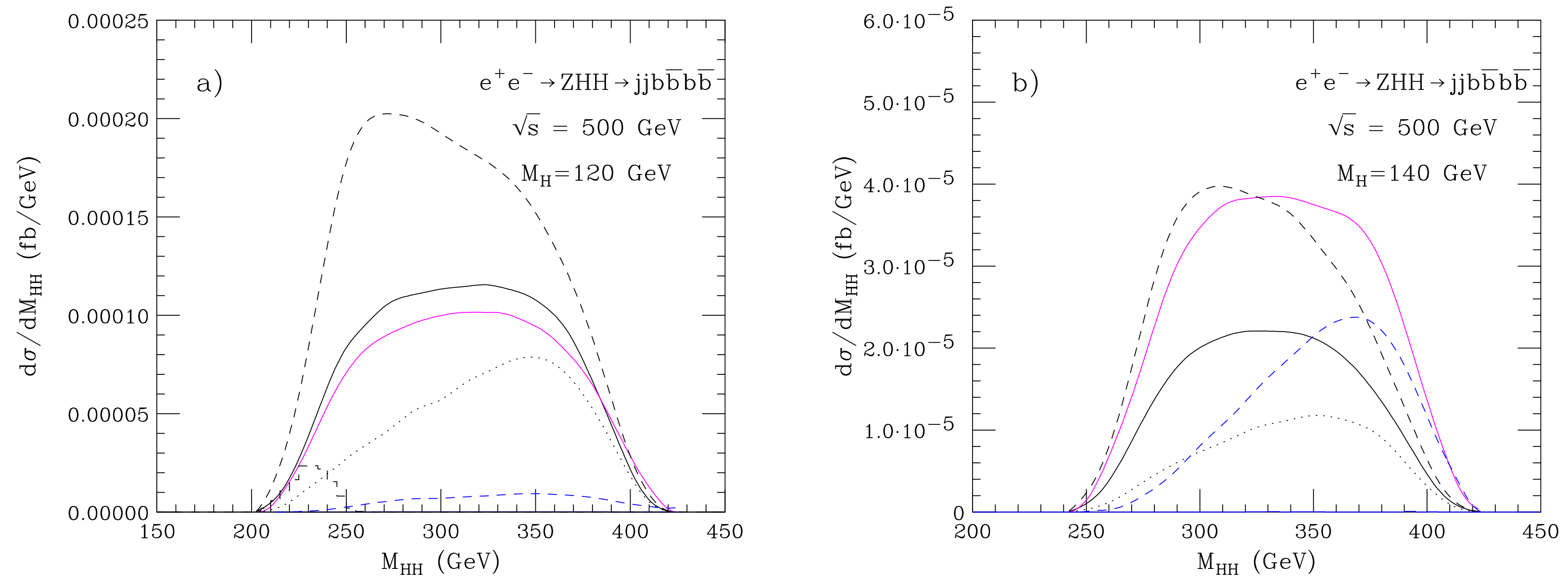
- main backgrounds:
  - ☞ non-resonant diagrams (≈ 8500 $\mathcal{O}(\alpha^6)$, $\mathcal{O}(\alpha_s^4\alpha^2)$ and $\mathcal{O}(\alpha_s^2\alpha^4)$ diagrams)
  - ☞ $jjb\bar{b}c\bar{c}$ ($b\bar{b}4j$) production with two mis-identified charm (light quark/gluon) jets (7300 [15600] diagrams)
  - ☞ assume b-jet charge can be measured with 100% efficiency (expectation for ILC: $\approx 90\%$)
- results: solid black: SM signal,

  magenta: SM resonant and non-resonant diagrams

  dash (dots): $\Delta\lambda_{HHH} = (\lambda/\lambda_{SM} - 1) = +1$ (-1)

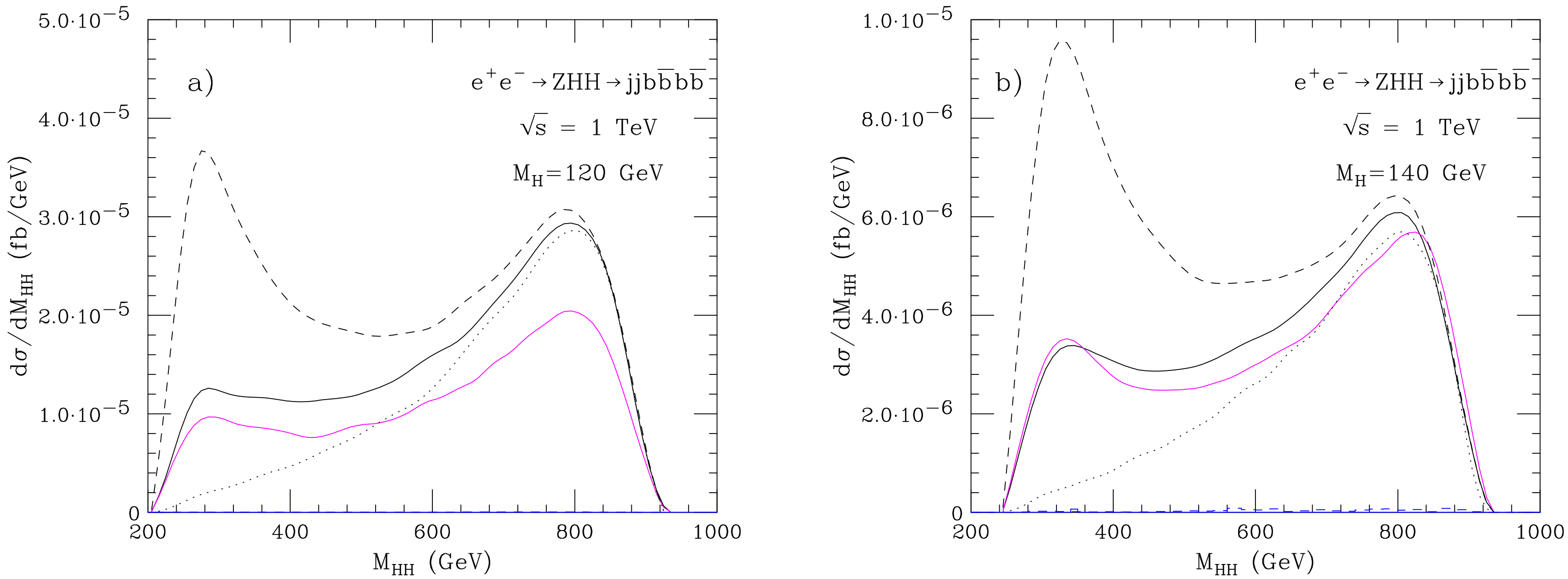
  dashed blue: $jjb\bar{b}c\bar{c}$ background

$\sqrt{s} = 500$ GeV



- The $jjb\bar{b}c\bar{c}$ and non-resonant backgrounds for $m_H = 140$ GeV are much larger than for $m_H = 120$ GeV
- The cross section for $m_H = 140$ GeV is tiny ($Br(H \to b\bar{b}) \approx 30\%$)
- black dashed histogram: combinatorial background from pairing the wrong b and $\bar{b}$

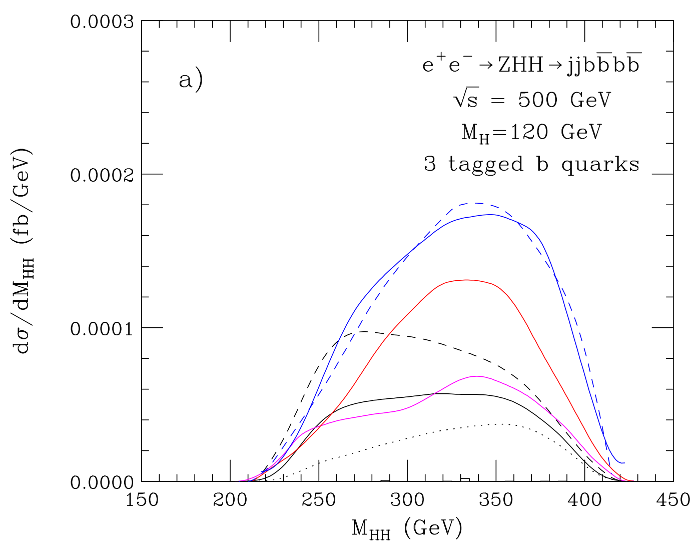
$\sqrt{s} = 1$ TeV

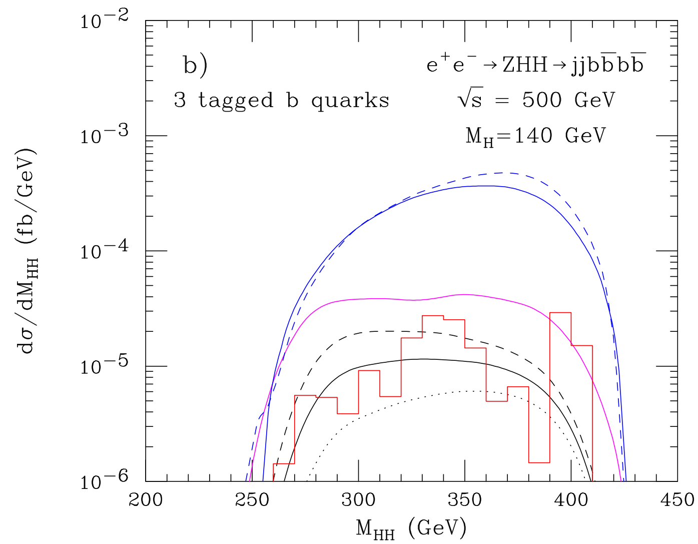


- The background for $m_H = 140$ GeV is significantly smaller for $\sqrt{s} = 1$ TeV
- The $b\bar{b}4j$ background is negligible at both $\sqrt{s} = 500$ GeV and 1 TeV

- The $e^+e^- \to ZHH \to jj4b$ rate is very small
- The signal rate can be increased by requiring $\geq$ 3 tagged b-quarks instead of 4 b-tags
  gain: factor ≈ 1.4

$$\sqrt{s} = 500 \text{ GeV}$$

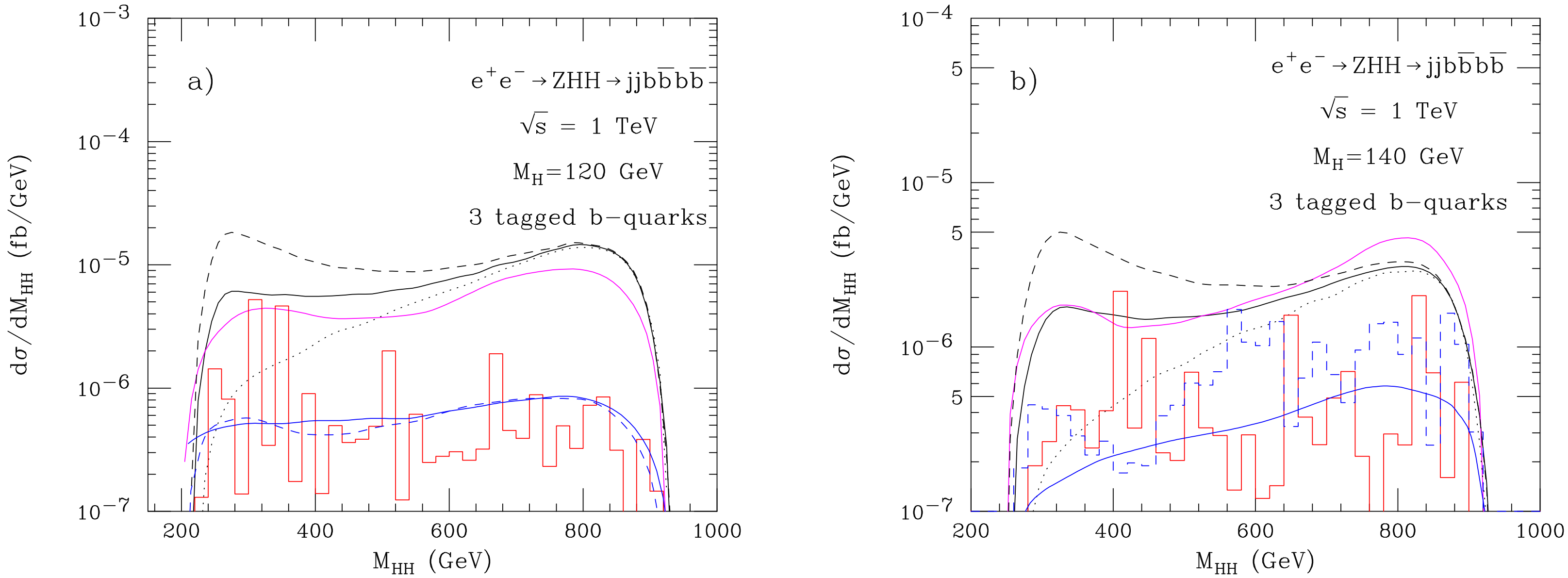




- require a jj pair consistent with Z, a $b\bar{b}$ pair, and a bj pair consistent with a Higgs

- lines:
  solid (dashed) blue: $b\bar{b}cjjj$ ($b\bar{b}c\bar{c}jj$)
  red: $b\bar{b}4j$
  all others have the same meaning as before
- Unfortunately, at $\sqrt{s} = 500$ GeV, the gain is more than compensated by the increase in the $b\bar{b}4j$ and $jjb\bar{b}c\bar{c}$ background.
- In addition, $b\bar{b}cjjj$ production contributes to the background
- Note: the background cross section is $\propto \alpha_s^4$ and thus carries a substantial renormalization scale uncertainty
  $\rightarrow$ either need NLO QCD corrections calculated for backgrounds (good luck with that!) or have to measure backgrounds

$\sqrt{s} = 1$ TeV



- background much more favorable at $\sqrt{s} = 1$ TeV; however, it is still non-negligible

3 – $\nu\bar{\nu}HH$ Production

- Consider $\nu\bar{\nu}HH \to \nu\bar{\nu}4b$ for $m_H = 120$ GeV and $m_H = 140$ GeV first
- Require that the four b's form two pairs which are compatible with a Higgs boson

$$100\ (120)\ \mathrm{GeV} < m(b\bar{b}) < 126\ (150)\ \mathrm{GeV}$$

for $m_H = 120\ (140)$ GeV.

- require ≥ 3 tagged b-quarks
- include $ZHH \to \nu_l\bar{\nu}_l 4b$ with $l = \mu,\ \tau$

- use same basic cuts and resolutions as before
- In addition, we require

$$\not{p}_T > 15 \text{ GeV}$$

- main backgrounds:
  - ☞ non-resonant diagrams (≈ 2300 $\mathcal{O}(\alpha^6)$, and $\mathcal{O}(\alpha_s^2\alpha^4)$ diagrams)
  - ☞ $\nu\bar{\nu}b\bar{b}c\bar{c}$ ($\nu\bar{\nu}b\bar{b}jj$) production with two mis-identified charm (light quark/gluon) jets (900 [2100] diagrams)
- other backgrounds: $4b$ and $b\bar{b}jj$ production with the missing transverse momentum originating from jet mismeasurements and the energy loss of b-quarks
  require $\not{p}_T > 15$ GeV
- Furthermore: $e^+e^- \to e^+e^-b\bar{b}b\bar{b}$ where both electrons are missed (not calculated yet)

- results: solid black: SM signal,

  magenta: SM resonant and non-resonant diagrams

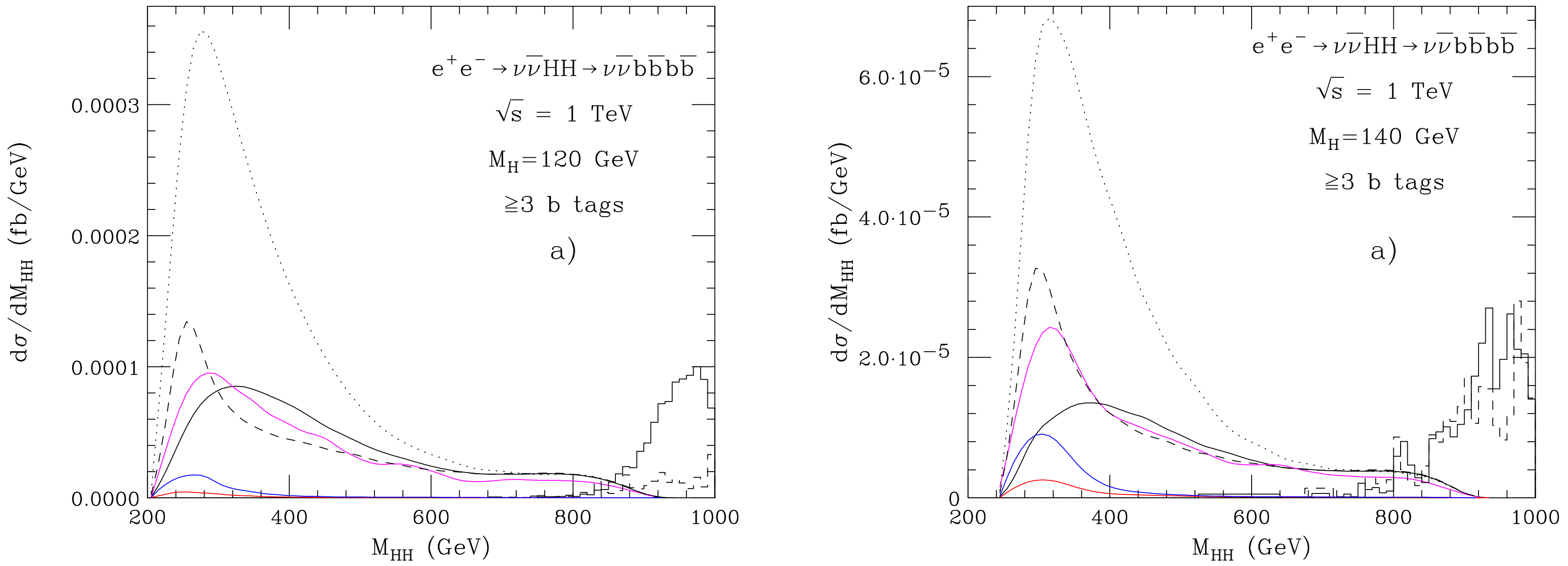
  dash (dots): $\Delta\lambda_{HHH} = (\lambda/\lambda_{SM} - 1) = +1$ (-1)

  blue: $\nu\bar{\nu}b\bar{b}c\bar{c}$ background

  red: $\nu\bar{\nu}b\bar{b}jj$ background

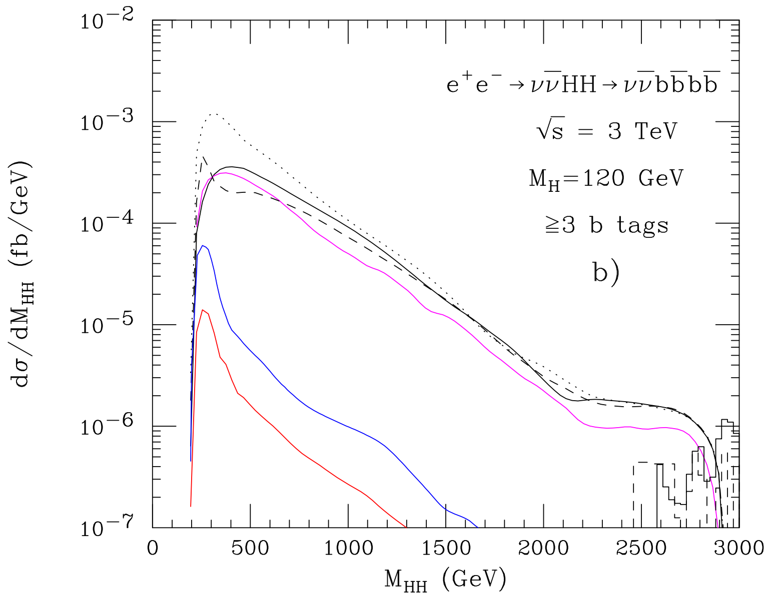
  solid (dashed) histogram: $4b$ ($b\bar{b}jj$) background

$\sqrt{s} = 1$ TeV

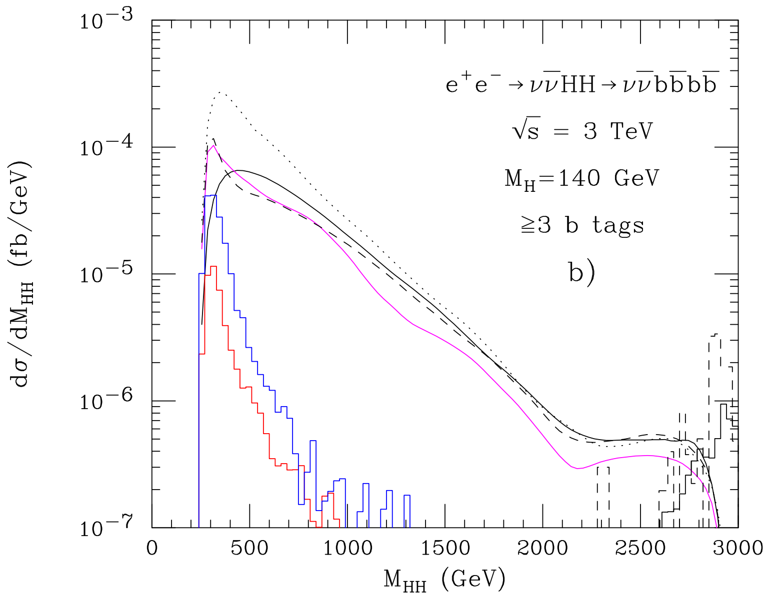


- The $\nu\bar{\nu}b\bar{b}c\bar{c}$ and $\nu\bar{\nu}b\bar{b}jj$ backgrounds are small
- The $4b$ and $b\bar{b}jj$ backgrounds pose no threat to the measurement of λ
- non-resonant contributions can easily be mistaken for a positive anomalous Higgs self-coupling

$\sqrt{s} = 3$ TeV



$e^+e^- \to \nu\bar{\nu}HH \to \nu\bar{\nu}b\bar{b}b\bar{b}$
$\sqrt{s}$ = 3 TeV
M_H=120 GeV
≥3 b tags
b)
$d\sigma/dM_{HH}$ (fb/GeV)
M_{HH} (GeV)



$e^+e^- \to \nu\bar{\nu}HH \to \nu\bar{\nu}b\bar{b}b\bar{b}$
$\sqrt{s}$ = 3 TeV
M_H=140 GeV
≥3 b tags
b)
$d\sigma/dM_{HH}$ (fb/GeV)
M_{HH} (GeV)

- For $m_H = 140$ GeV: $B(H \to WW^* \to 4f) \approx 50\%$, $B(WW^* \to 4j) \approx 46\%$

  → one can significantly increase the signal cross section by taking into account the $\nu\bar{\nu}b\bar{b}4j$ final state

- Also take into account $H \to ZZ^* \to 4j$ ($B(H \to ZZ^*) \approx 10\%$ for $m_H = 140$ GeV)

- require

$$|m_H - m(4j)| < 20 \text{ GeV}$$

  and one un-tagged jet pair with

$$|m_W - m(jj)| < 8 \text{ GeV}$$

- main backgrounds: non-resonant $\nu\bar{\nu}b\bar{b}4j$, $\nu\bar{\nu}4c$ and $\nu\bar{\nu}b\bar{b}c\bar{c}jj$ production

- problem: these are $2 \to 8$ processes with $> 10^5$ Feynman diagrams
  - ☞ too many diagrams for `MadEvent` (takes more than 200h CPU time (3ghz Xeon) to *generate* diagrams
  - ☞ `WHIZARD` could not compile code (compilation terminated after > 48h)
  - ☞ `HELAC-PHEGAS` bombed with a glibc error
  - ☞ `SHERPA` can't handle it in its current version (V1.1), but, according to Frank Krauss will be able to so in V1.2
  - ☞ `CARLOMAT` should be able to handle it, but is not publically available (author never replied to my email request)

- A substantial portion of the contribution of the non-resonant $\nu\bar{\nu}b\bar{b}4j$ diagrams should come from the off-shell $W^* \to jj$ pair.
  ➞ most of the non-resonant effects can be captured by calculating

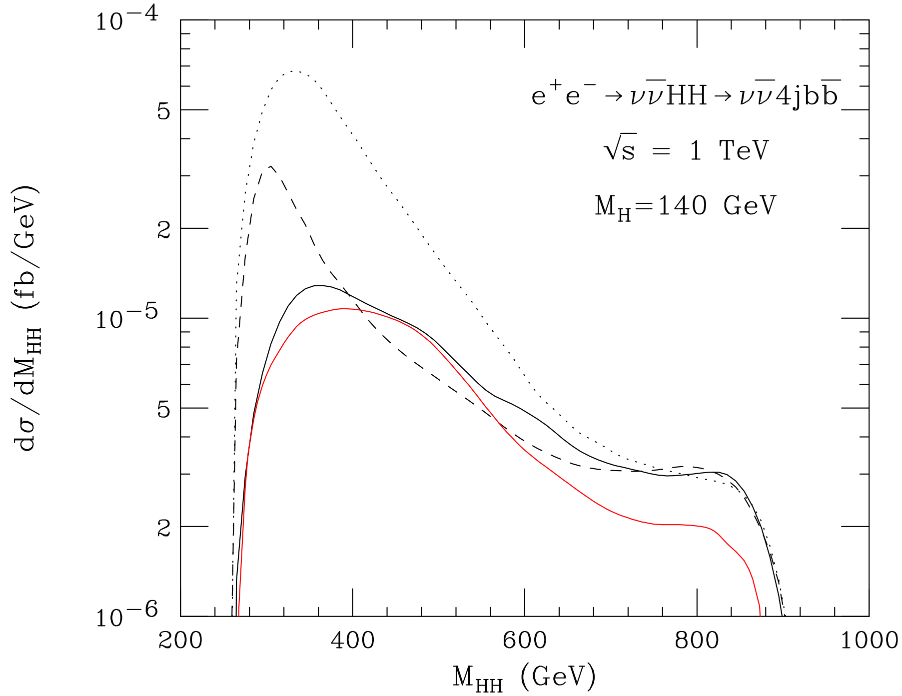
$$e^+e^- \to \nu\bar{\nu}Wjjb\bar{b} \qquad \text{with} \qquad W \to jj$$

or

$$e^+e^- \to \nu\bar{\nu}H4j \qquad \text{with} \qquad H \to b\bar{b}$$

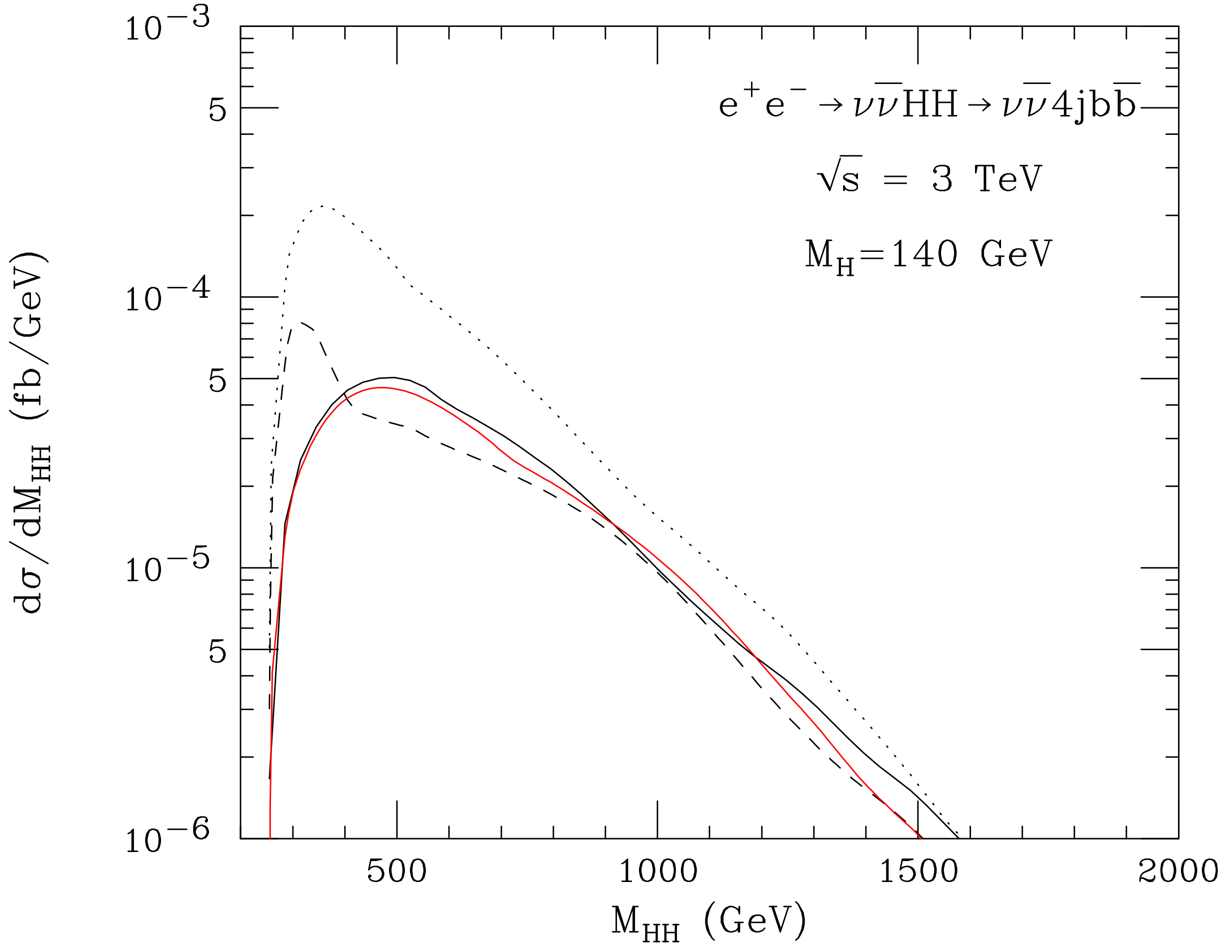
- Calculate $e^+e^- \to \nu\bar{\nu}Wjjb\bar{b}$, $W \to jj$ here (about 7000 Feynman diagrams)
- Expect $\nu\bar{\nu}b\bar{b}c\bar{c}jj$ and $\nu\bar{\nu}4c$ backgrounds to be small (as in $\nu\bar{\nu}b\bar{b}c\bar{c}$ case)
- The $b\bar{b}4j$ background (with $\not{p}_T$ from jet mis-measurement and energy loss of the b-quarks) is very small

$\sqrt{s} = 1$ TeV



- Non-resonant diagrams substantially reduce the cross section at large values of m_{HH}
- The non-resonant diagrams not included in $\nu\bar{\nu}Wjjb\bar{b}$ jet production may well affect the cross section to a similar degree

$\sqrt{s} = 3$ TeV



$e^+e^- \to \nu\bar{\nu}HH \to \nu\bar{\nu}4jb\bar{b}$
$\sqrt{s} = 3$ TeV
M_H=140 GeV
$d\sigma/dM_{HH}$ (fb/GeV)
M_{HH} (GeV)
10^{-3}
5
10^{-4}
5
10^{-5}
5
10^{-6}
500
1000
1500
2000

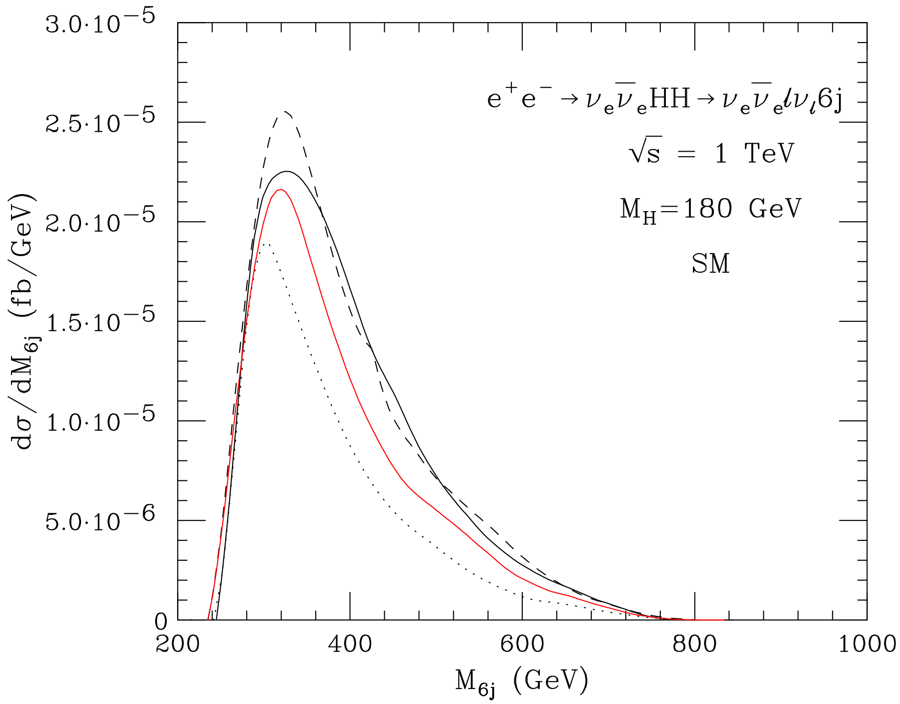
$m_H = 180$ GeV

- For $m_H = 180$ GeV, $B(H \to WW) \approx 93\%$
- Final states with the largest branching ratios are $HH \to 4W \to \ell^\pm \nu_\ell 6j$ ($\ell = e,\, \mu$) ($Br \approx 24\%$) and $HH \to 4W \to 8j$ ($Br \approx 19\%$) $\to$ concentrate on those final states
- $e^+e^- \to \ell^\pm \not{p}_T 6j$:
  require standard cuts and, in addition, 3 or more jet pairs consistend with a W, with two of the pairs

  $$160 \text{ GeV} < m([jj][jj]) < 200 \text{ GeV}$$
- main backgrounds: non-resonant diagrams and $W6j$ production (about 21,000 Feyman diagrams)
- use $6j$ invariant mass distribution to search for anomalous Higgs self-couplings

- A full calculation of the $2 \to 10$ process $e^+e^- \to \ell^\pm \not{p}_T 6j$ is currently not feasible
- to get an idea of how important the non-resonant diagrams are, I calculate $e^+e^- \to \nu\bar{\nu}\ell\nu_\ell jjH$ with $H \to WW \to 4j$ (1,300 Feynman diagrams) and $e^+e^- \to \nu\bar{\nu}4jH$ with $H \to WW \to \ell\nu_\ell jj$ (20,000 Feynman diagrams)
- results for $e^+e^- \to \nu_e\bar{\nu}_e HH \to \nu_e\bar{\nu}_e\ell^\pm\nu_\ell 6j$:
  black line: SM signal
  dashed black line: $\nu_e\bar{\nu}_e\ell^\pm\nu_\ell jjH$, $H \to WW \to 4j$
  dotted curve: $\nu_e\bar{\nu}_e 4jH$, $H \to WW \to \ell^\pm\nu_\ell jj$
  red line: average of $\nu_e\bar{\nu}_e 4jH$, $H \to WW \to \ell^\pm\nu_\ell jj$ and $\nu_e\bar{\nu}_e\ell^\pm\nu_\ell jjH$, $H \to WW \to 4j$

$\sqrt{s} = 1$ TeV

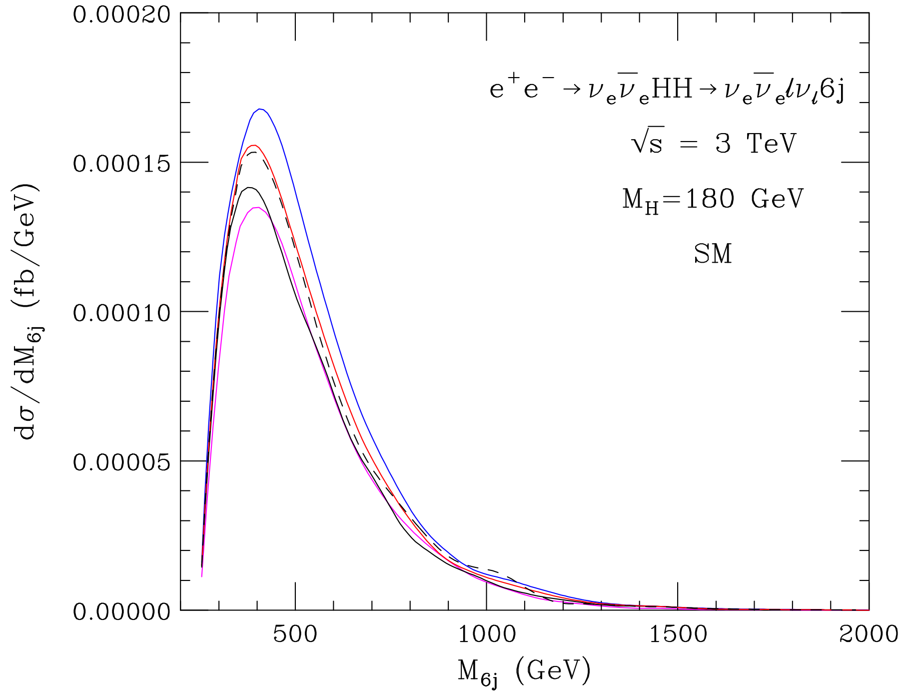


- The nonresonant diagrams in $e^+e^- \to \nu\bar{\nu}\ell\nu_\ell jjH$ with $H \to WW \to 4j$ significantly enhance the cross section near threshold.

  This is expected since the $\ell\nu_\ell$ invariant mass cannot be constrained (3 ν's in the final state)

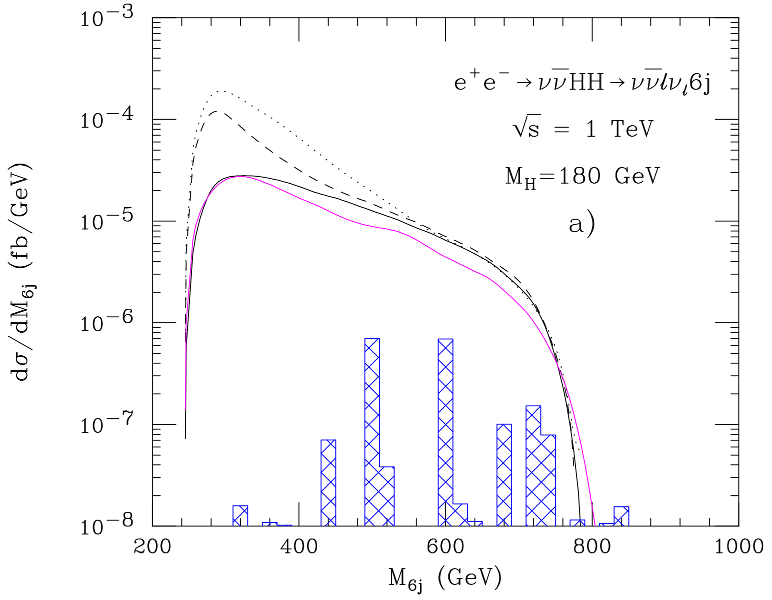
- The non-resonant diagrams in $\nu_e\bar{\nu}_e 4jH$, $H \to WW \to \ell^\pm\nu_\ell jj$ production reduce the cross section by a factor $1.5-2$
- The averaging procedure used here ignores a very large number of Feynman diagrams which still may have a significant impact on the cross section
- Justification: compare SM $e^+e^- \to \nu_e\bar{\nu}_e\ell^\pm\nu_\ell jjH$, $H \to WW \to 4j$ cross section (black dashed line) with averaging (red line) the $e^+e^- \to \nu_e\bar{\nu}_e\ell^\pm\nu_\ell WH$, $W \to jj$, $H \to WW \to 4j$ (blue line) and the $e^+e^- \to \nu_e\bar{\nu}_e WjjH$, $W \to \ell\nu_\ell$, $H \to WW \to 4j$ cross section (black solid line)

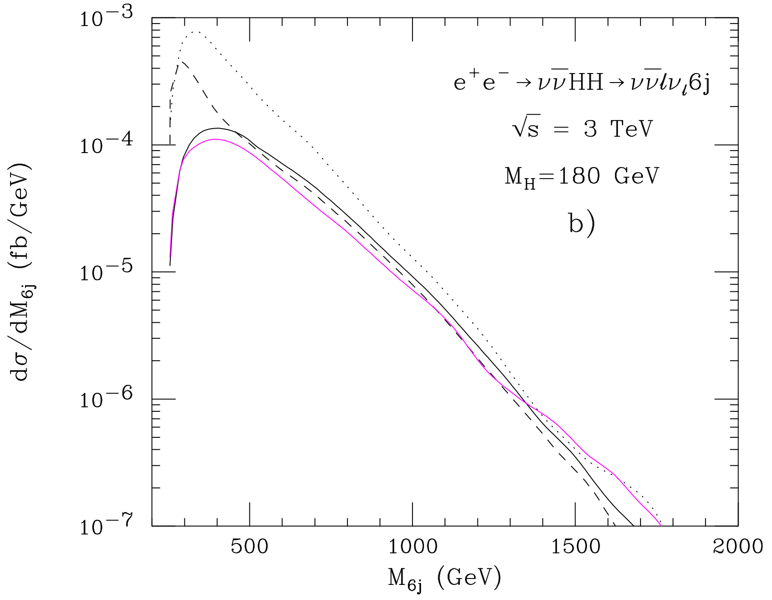
$\sqrt{s} = 3$ TeV



- red and black dashed lines agree with a few precent
- The magenta line shows the $\nu_e\bar{\nu}_e HH$, $HH \to 4W \to \ell\nu_\ell 6j$ SM signal cross section

Adopting the averaging procedure introduced above





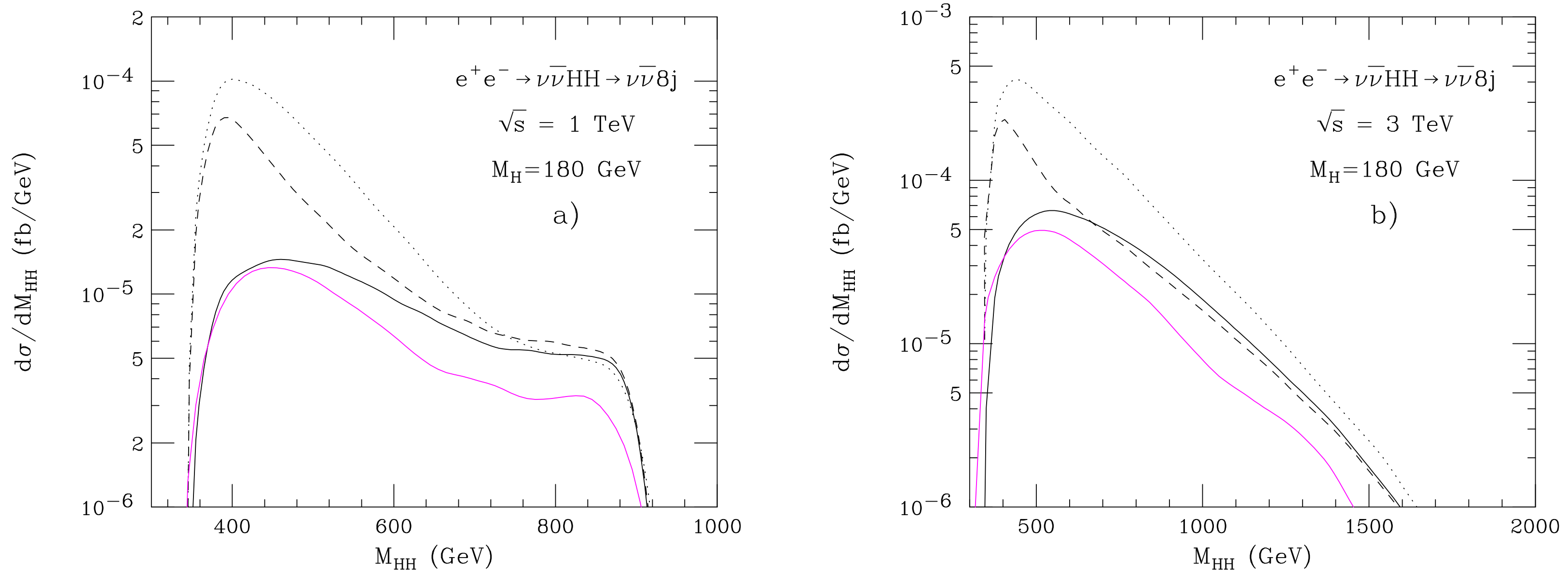
The $W6j$ background is very small

$HH \to 8j$

- Require 4 jet pairs consistent with W's
- The jet pairs are required to form two groups of $4j$ systems which satisfy

$$160 \text{ GeV} < m([jj][jj]) < 200 \text{ GeV}$$

- Main background: non-resonant diagrams
- Estimate by calculating $e^+e^- \to \nu\bar{\nu}4jH$ with $H \to WW \to 4j$ and the previously used averaging procedure
- black solid line: SM signal
  dashed and dotted lines: $\Delta\lambda_{HHH} = (\lambda/\lambda_{SM} - 1) = \pm 1$
  magenta: including non-resonant diagrams



- non-resonant diagrams have a big effect
- there is no guarantee that the approximation used here is a good approximation

4 – Sensitivity limits

- Perform a log-likelihood test
- Assume a 10% systematical uncertainty on cross section (probably optimistic)
- assume $\int \mathcal{L}dt = 1$ ab^{-1} (corresponds to 5 years of running at ILC design luminosity)
- no (marignal) gain from including final states with only 3 tagged b-quarks for $\sqrt{s} = 500$ GeV (1 TeV)
  → considering a working point with a somewhat reduced b-tagging efficiency, but a much reduced light/charm misidentification probabilty may help
- for $m_H = 140$ GeV and $\nu\bar{\nu}HH$ production, the $HH \to b\bar{b}WW^*$ final states are included in the analysis

- for $m_H = 180$ GeV, both the $\nu\bar{\nu}8j$ and $\nu\bar{\nu}\ell\nu_\ell 6j$ final states are taken into account
- 68% CL limits:
  - ☞ $ZHH \to jj4b$:
    $\sqrt{s} = 500$ GeV, $m_H = 120$ GeV: $-0.41 < \Delta\lambda_{HHH} < 0.44$
    $\sqrt{s} = 500$ GeV, $m_H = 140$ GeV: $-6.8 < \Delta\lambda_{HHH} < 2.1$
    $\sqrt{s} = 1$ TeV, $m_H = 120$ GeV: $-0.45 < \Delta\lambda_{HHH} < 0.53$
    $\sqrt{s} = 1$ TeV, $m_H = 140$ GeV: $-1.0 < \Delta\lambda_{HHH} < 1.1$
  - ☞ $\nu\bar{\nu}HH$:
    $\sqrt{s} = 1$ TeV, $m_H = 120$ GeV: $-0.21 < \Delta\lambda_{HHH} < 0.30$
    $\sqrt{s} = 1$ TeV, $m_H = 140$ GeV: $-0.38 < \Delta\lambda_{HHH} < 0.94$
    $\sqrt{s} = 1$ TeV, $m_H = 180$ GeV: $-0.29 < \Delta\lambda_{HHH} < 0.55$
    $\sqrt{s} = 3$ TeV, $m_H = 120$ GeV: $-0.12 < \Delta\lambda_{HHH} < 0.14$
    $\sqrt{s} = 3$ TeV, $m_H = 140$ GeV: $-0.19 < \Delta\lambda_{HHH} < 0.15$
    $\sqrt{s} = 3$ TeV, $m_H = 180$ GeV: $-0.20 < \Delta\lambda_{HHH} < 0.16$

Random Remarks

- The values reported for $m_H = 120$ GeV, $\sqrt{s} = 500$ GeV agree quite well this those of a fast simulation by SiD (Tim Barklow)
- A full simulation, however, gives limits which are about a factor 2 worse (SiD, private communication)

☞ limits from $\nu\bar{\nu}HH$ production are significantly more stringent at $\sqrt{s} =$ 1 TeV than those from ZHH production

- At a 500 GeV e^+e^- machine, one can measure the Higgs boson self-coupling only if the Higgs mass is close to the current lower experimental limit

☞ At CLIC ($\sqrt{s} = 3$ TeV), limits can be improved by up to a factor of 1.5 if 3 ab^{-1} (5 years of running at design luminosity) can be achieved

☞ Whether the cuts used are adequate for CLIC remains to be seen

Measuring the Higgs selfcoupling at a Muon Collider

- All results presented here apply *mutatis mutandis* for a muon collider with the same center of mass energy and integrated luminosity
- The cross section for the direct channel $\mu^+\mu^- \to HH$ is several orders of magnitude smaller than that for ZHH and $\nu\bar{\nu}HH$ production

5 – Conclusions

- Non-resonant diagrams can significantly affect the total and differential cross sections for $e^+e^- \to jj4b$
- At a 500 GeV e^+e^- machine, one can measure the Higgs boson self-coupling only if the Higgs mass is close to the current lower experimental limit
- Non-resonant diagrams in $\nu\bar{\nu}4b$ production can mimic the effects of non-standard Higgs self-couplings
- At a 1 TeV machine, with 1 ab^{-1}, $\nu\bar{\nu}HH$ production gives more precise limits than $e^+e^- \to ZHH$.
- For CLIC ($\sqrt{s} = 3$ TeV, 3 ab^{-1}) can measure the Higgs self-coupling with a precision of $10 - 20\%$.