Physical Results from 2+1 Flavor Lattice Domain Wall QCD

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(RBC and UKQCD Collaborations)

Fermilab

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- 2+1 flavor dynamical Domain Wall Fermion simulations
- pion and kaon spectrum: quark masses, decay constants, Chiral Perturbation Theory LECs
- kaon bag parameter

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and
`arXiv:0804.0473[hep-lat]` (to appear in Phys. Rev. **D**)

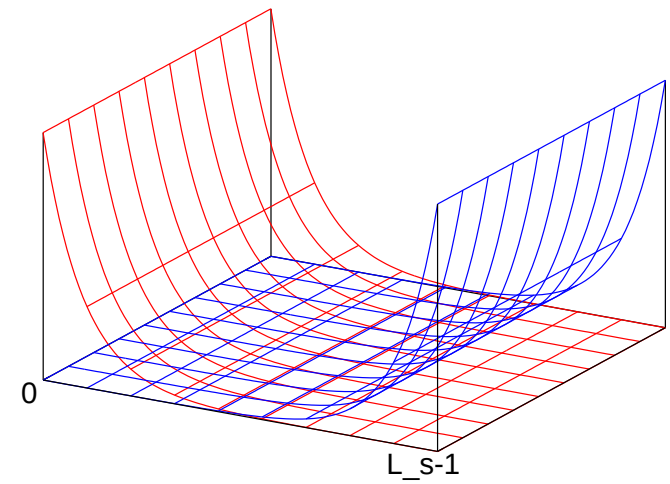
- Chiral Perturbation Theory (χPT)
  - effective theory
  - expansion in light quark masses and momenta
  - application: light meson masses and decay constants, neutral kaon mixing (heavy-light meson, baryon χPT)

- Which quarks are considered as light? Convergence radius? NLO, NNLO, . . .
  - only up, down ($m_{u,d} \approx$ 2 – 7 MeV): **SU(2)**$\times$**SU(2)**
  - up, down, strange ($m_s \approx$ 70 – 120 MeV): **SU(3)**$\times$**SU(3)**

- lattice QCD & χPT
  - χPT tool for lattice simulations
    extrapolation to physical quark masses, partially quenching
  - lattice QCD can probe χPT (now)
    quark masses can be varied in lattice simulations
  - determine light quark masses, f_π, f_K, B_K, low-energy constants

(Why) Domain Wall fermions

- different lattice fermions
  - ∗ Wilson fermions and improved versions
  - ∗ staggered fermions
  - ∗ domain wall fermions (DWF)
  - ∗ overlap-fermions
- DWF
  - ∗ fermion fields have a 5th dimension of extent L_s
  - ∗ *left* and *right* handed fermions on slice 0 and $L_s - 1$
  - ∗ propagation through 5th dimension:
    residual chiral symmetry breaking (m_{res})
    - chiral symmetry breaking under control
    - reduces (wrong chirality) operator mixing (B_K)
    - non-perturbative renormalization (quark masses, B_K)

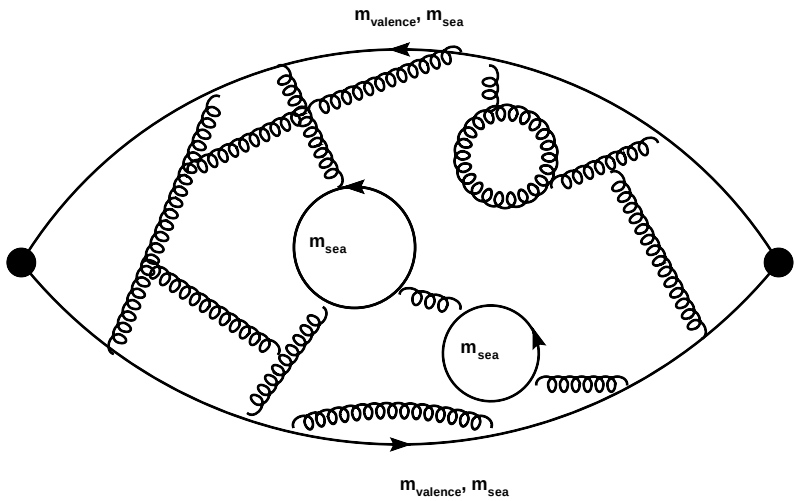


$N_f = 2 + 1$ Domain Wall Fermion ensembles

- $N_f = 2 + 1$ ensembles with Iwasaki gauge action
  - $L^3 \times T \times L_s = 24^3 \times 64 \times 16$
  - preliminary: $32^3 \times 64 \times 16$, in preparation: $48^3 \times 64 \times 16$
- Rational Hybrid Monte Carlo algorithm [Clark, Kennedy]
- $24^3 \times 64 \times 16$ - ensemble
  - $\beta = 2.13$ (Iwasaki gauge action)
  - $a^{-1} = 1.72(3)$ GeV (from Ω^-), $aL \approx 2.74$ fm
  - dynamical light quark mass: $m_l \in \{0.005, 0.01, 0.02, 0.03\}$
  - dynamical heavy quark mass: $m_h = 0.04$
  - lightest dynamical $m_\pi \approx 330$ MeV, $\widetilde{m}_l : \widetilde{m}_h = 1/5$
  - $m_{\text{res}} = 0.00315(02)$: $\widetilde{m}_X = m_X + m_{\text{res}}$
- valence quark mass (partial quenching): $m_{\text{val}} \in \{0.001, 0.005, 0.01, 0.02, 0.03, 0.04\}$

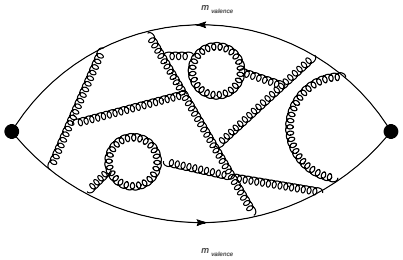
$$\Rightarrow \text{lightest valence: } m_\pi \approx 240\,\text{MeV}, \widetilde{m}_{\text{val}} \approx 1/10\,\widetilde{m}_h$$

Partial Quenching



- dynamically simulated quark masses: m_{sea}
- "measurements" done at different quark masses m_{valence}
- unitary case for $m_{\text{valence}} = m_{\text{sea}}$
- Partially Quenched χPT (Rupak/Shoresh, Sharpe/Shoresh,. . . , Sharpe/van de Water, . . .)

- different from quenched simulations: no dynamical fermions



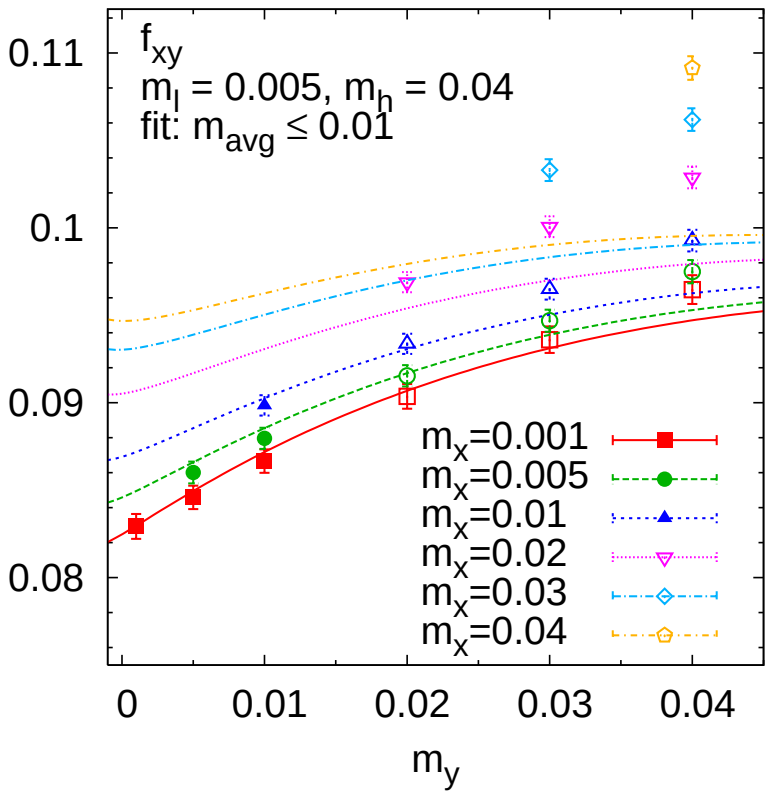
SU(3) χPT for meson masses and decay constants

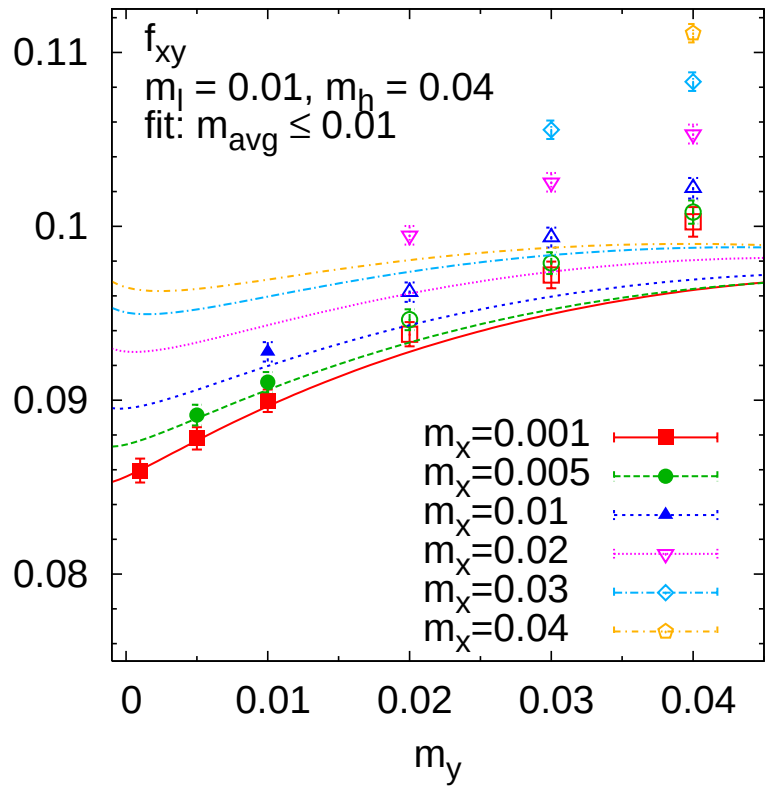
- measure m_{PS}, f_{PS} at different sea (m_l, m_h) and valence (m_x, m_y) quark masses
- SU(3) $\times$ SU(3) PQχPT (NLO):

$$
\begin{aligned}
m_{xy}^2 &= \frac{\chi_x + \chi_y}{2}\Bigg\{1 + \frac{48}{f_0^2}(2L_6^{(3)} - L_4^{(3)})\bar{\chi} + \frac{8}{f_0^2}(2L_8^{(3)} - L_5^{(3)})(\chi_x + \chi_y) \\
&\quad + \Big[\ldots \times \log(\chi_x), \log(\chi_y), \log(\chi_\eta)\Big]\Bigg\} \\
f_{xy} &= f_0\Bigg\{1 + \frac{24}{f_0^2}L_4^{(3)}\bar{\chi} + \frac{4}{f_0^2}L_5^{(3)}(\chi_x + \chi_y) + \Big[\ldots \times \log(\chi_x + \chi_l), \log(\chi_x + \chi_h), \\
&\quad \log(\chi_y + \chi_l), \log(\chi_y + \chi_h), \log(\chi_x), \log(\chi_y), \log(\chi_\eta)\Big]\Bigg\}
\end{aligned}
$$

- $\chi_X = 2B_0(m_X + m_{\mathrm{res}})$, $\bar{\chi} = 2B_0\left(\frac{2m_l + m_h}{3} + m_{\mathrm{res}}\right)$

SU(3) PQχPT I: decay constants f_{xy}

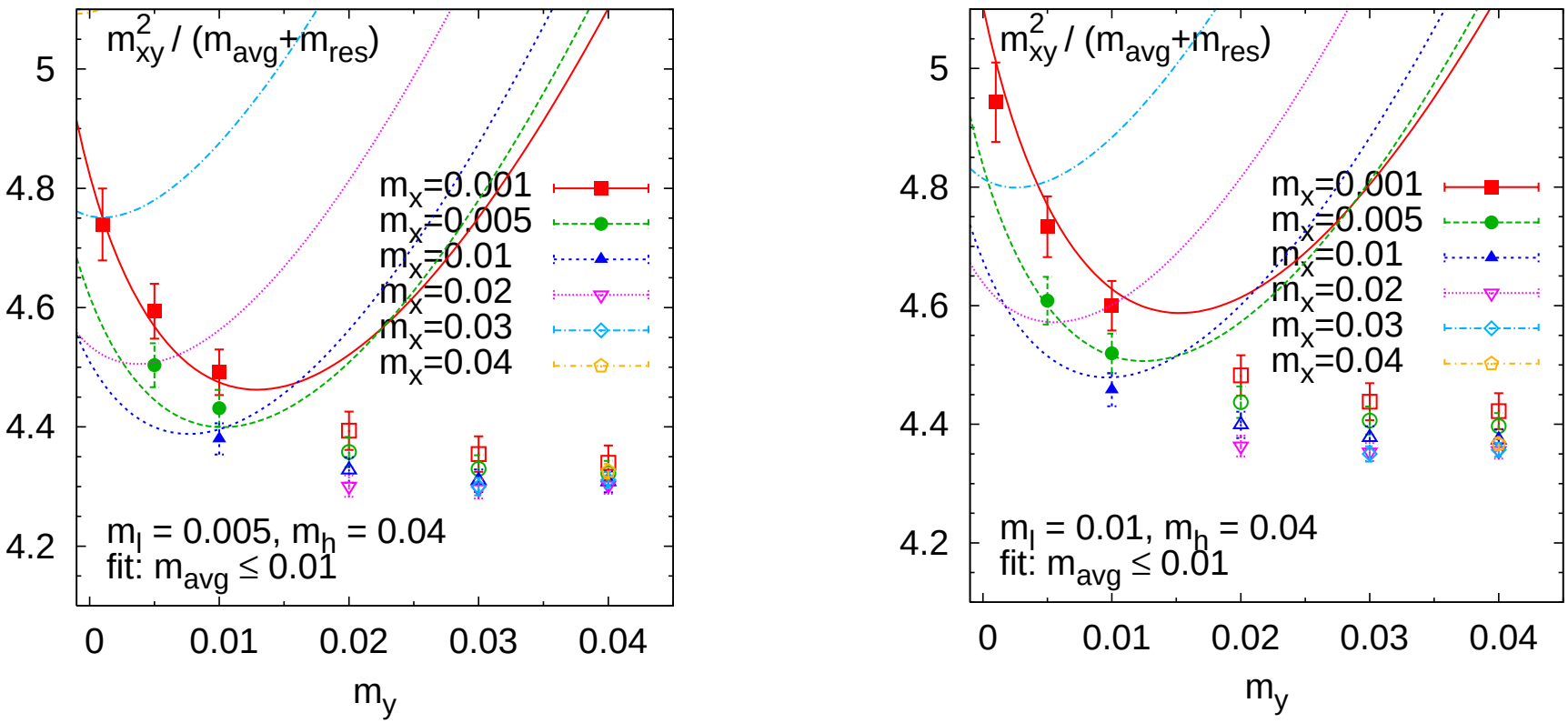




fit range $m_{\text{avg}} \leq 0.01$ corresponds to (deg.) PS-mass of 420 MeV
(only *filled symbols* inside fit range)

combined fits: 2 light sea quark masses, f_{xy} **and** m_{xy}^2

SU(3) PQχPT II: meson masses m_{xy}^2



$$\frac{m_{xy}^2}{(\widetilde{m}_x + \widetilde{m}_y)/2}$$

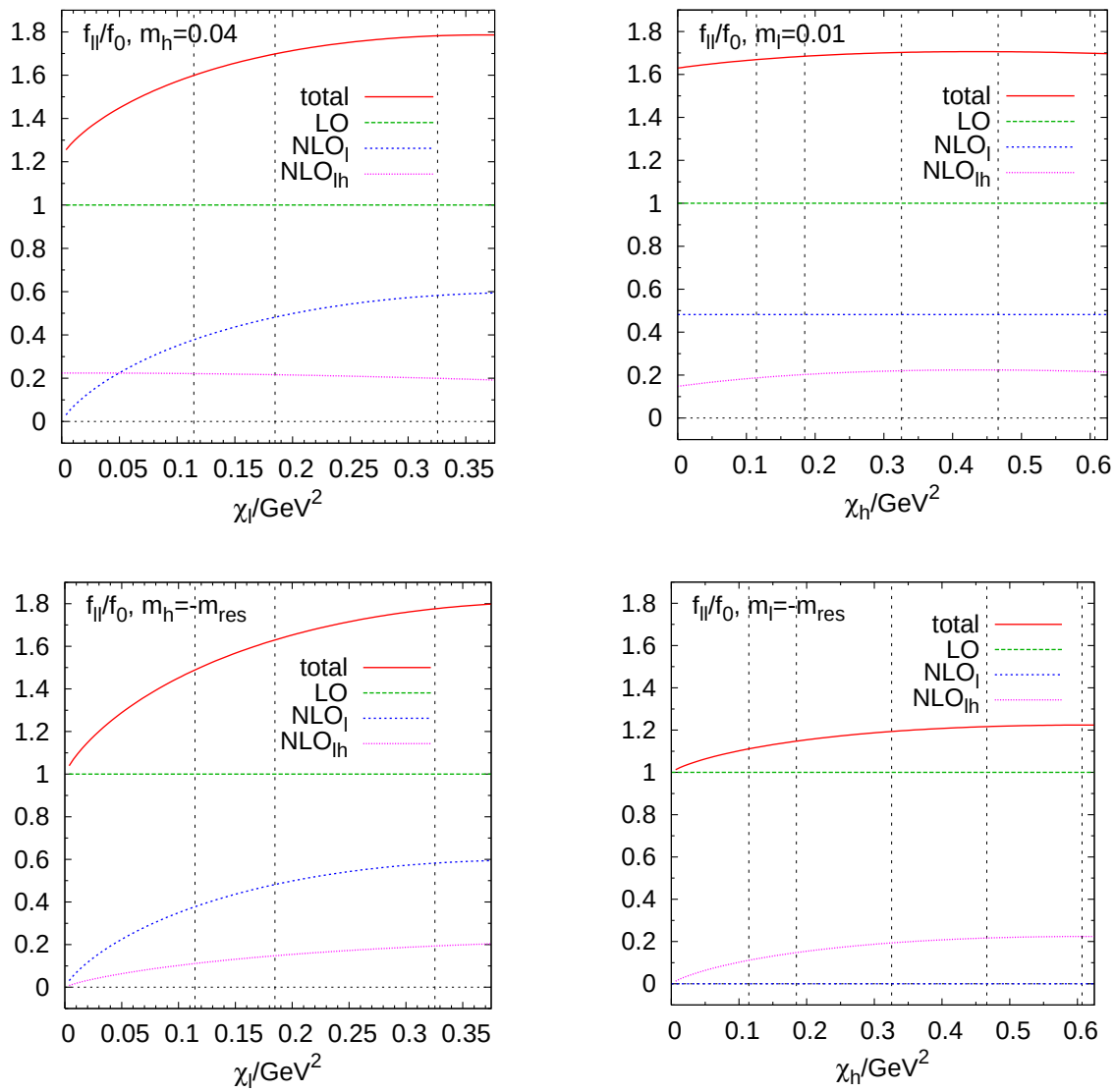
SU(3) low-energy constants

	$L_4^{(3)}$	$L_5^{(3)}$	$L_6^{(3)}$	$L_8^{(3)}$	$(2L_8^{(3)} - L_5^{(3)})$	$(2L_6^{(3)} - L_4^{(3)})$
	1.4(0.8)(–)	8.7(1.0)(–)	0.7(0.6)(–)	5.6(0.4)(–)	2.4(0.4)(–)	0.0(0.4)(–)
Bijnens (pheno)						
NLO	$\equiv 0$	14.6	$\equiv 0$	10.0	5.4	$\equiv 0$
NNLO	$\equiv 0$	9.7(1.1)	$\equiv 0$	6.0(1.8)	2.3	$\equiv 0$
MILC, 2+1 stagg. fermions, (LO, NLO, anal. NNLO + NNNLO)						
2004	-0.7(3.0)(3.0)	10.9(3.0)(3.0)	–	–	1.6(1.0)(2.0)	4.4(2.0)(4.0)
2007	1.3(3.0)$(^{+3.0}_{-1.0})$	13.9(2.0)$(^{+2.0}_{-1.0})$	2.4(2.0)$(^{+2.0}_{-1.0})$	7.8(1.0)(1.0)	2.6(1.0)(1.0)	3.4(1.0)$(^{+2.0}_{-3.0})$
PACS-CS, 2+1 impr. Wilson fermions (LO, NLO)						
	-0.4(1.0)(–)	14.3(0.7)(–)	–	–	-2.1(0.3)(–)	1.0(0.2)(–)

LEC's in 10^{-4} at $\Lambda_\chi \;=\; 770\,\text{MeV}$

fit range sufficient to extract LEC's, but did not include the strange quark mass

Convergence?



- NLO-fits not working up to the strange quark mass
  $(m_x = 0.001, m_y = 0.04 \;\Rightarrow\; m_{xy} \approx 554\,\text{MeV})$

- including NNLO-terms

  * complete formulae available BIJNENS et al.,
    work in progress, will try to apply with 32^3 data

  * just include analytic NNLO-terms
    $(\chi_x + \chi_y)^2$, $(\chi_x - \chi_y)^2$, $\overline{\chi}^2$, $\overline{\chi}(\chi_x + \chi_y)$, $\overline{\chi^2}$
    * still right behaviour in light quark mass region?? non-analytic terms???
    * limited number of data points (sea quark mass)

- chiral symmetry only for up- and down-quarks: SU(2) $\times$ SU(2)

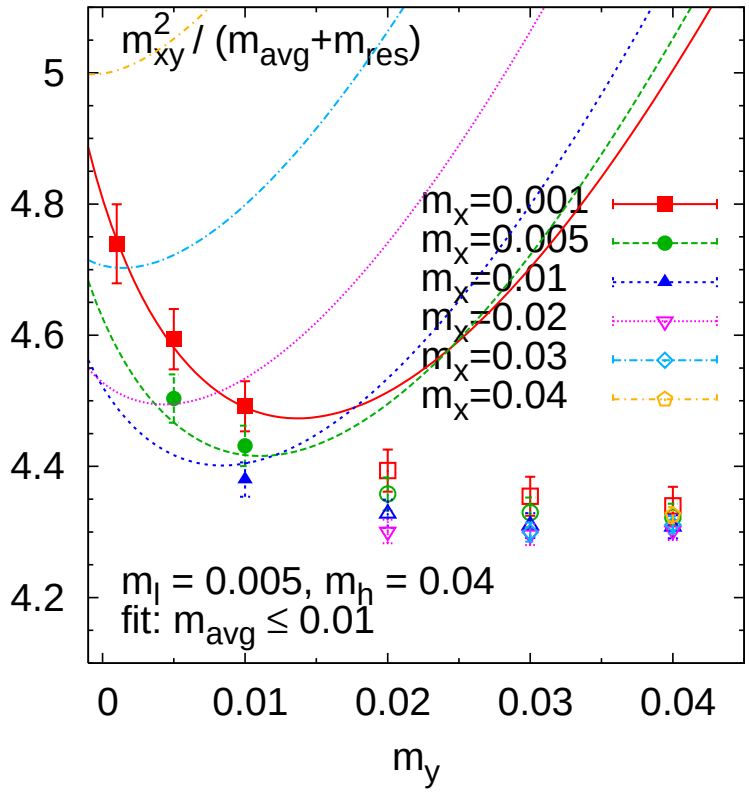
SU(2) PQχPT

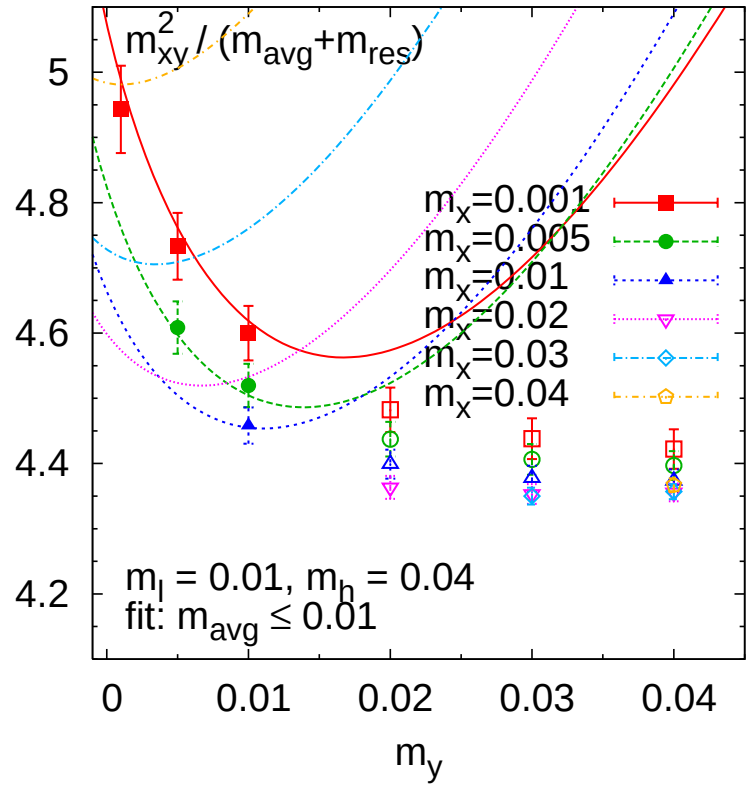
$$
\begin{aligned}
m_{xy}^2 &= \frac{\chi_x + \chi_y}{2}\Bigg\{1 + \frac{32}{f^2}(2L_6^{(2)} - L_4^{(2)})\chi_l + \frac{8}{f^2}(2L_8^{(2)} - L_5^{(2)})(\chi_x + \chi_y) \\
&\quad + \Big[\ldots \times \log(\chi_x), \log(\chi_y)\Big]\Bigg\} \\
f_{xy} &= f\Bigg\{1 + \frac{16}{f^2}L_4^{(2)}\chi_l + \frac{4}{f^2}L_5^{(2)}(\chi_x + \chi_y) \\
&\quad + \Big[\ldots \times \log(\chi_x + \chi_l), \log(\chi_y + \chi_l), \log(\chi_x), \log(\chi_y)\Big]\Bigg\} \\
\chi_X &= 2B\Big(m_X + m_{\text{res}}\Big)
\end{aligned}
$$

f, B, $L_i^{(2)}$ depend on (background) m_h

SU(2) PQχPT I: meson masses

arXiv:0804.0473[hep-lat], to appear in Phys. Rev. **D**

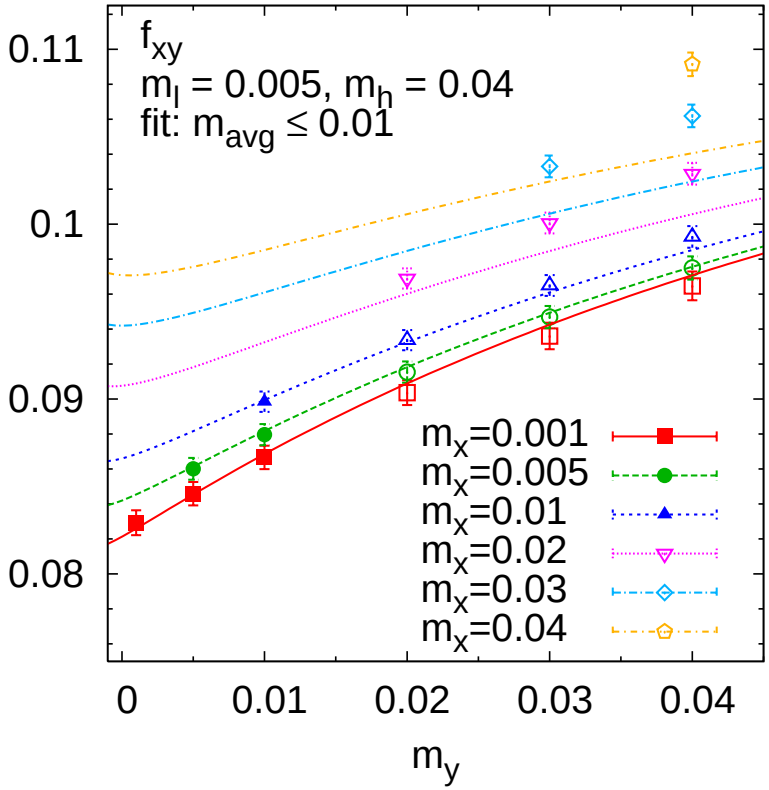


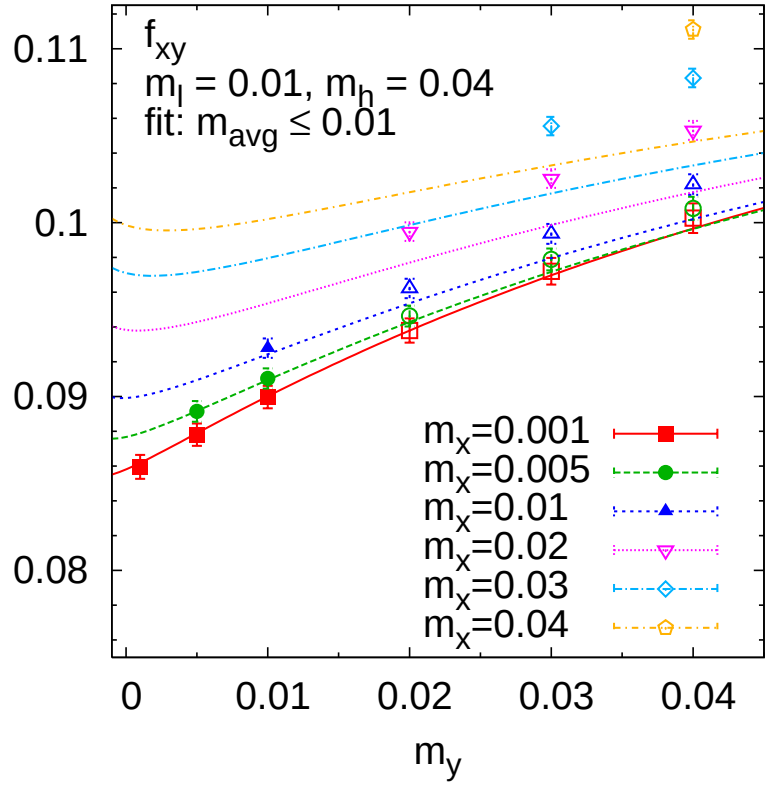


from $m_\pi = 135\,\text{MeV} \Rightarrow m_{\text{ud}}^{\overline{\text{MS}}}(2\,\text{GeV}) = 3.72(16)\text{MeV}$ (NPR)

fit range $m_{\text{avg}} \leq 0.01$, combined fits (only *filled symbols* inside fit range)

SU(2) PQχPT II: decay constants

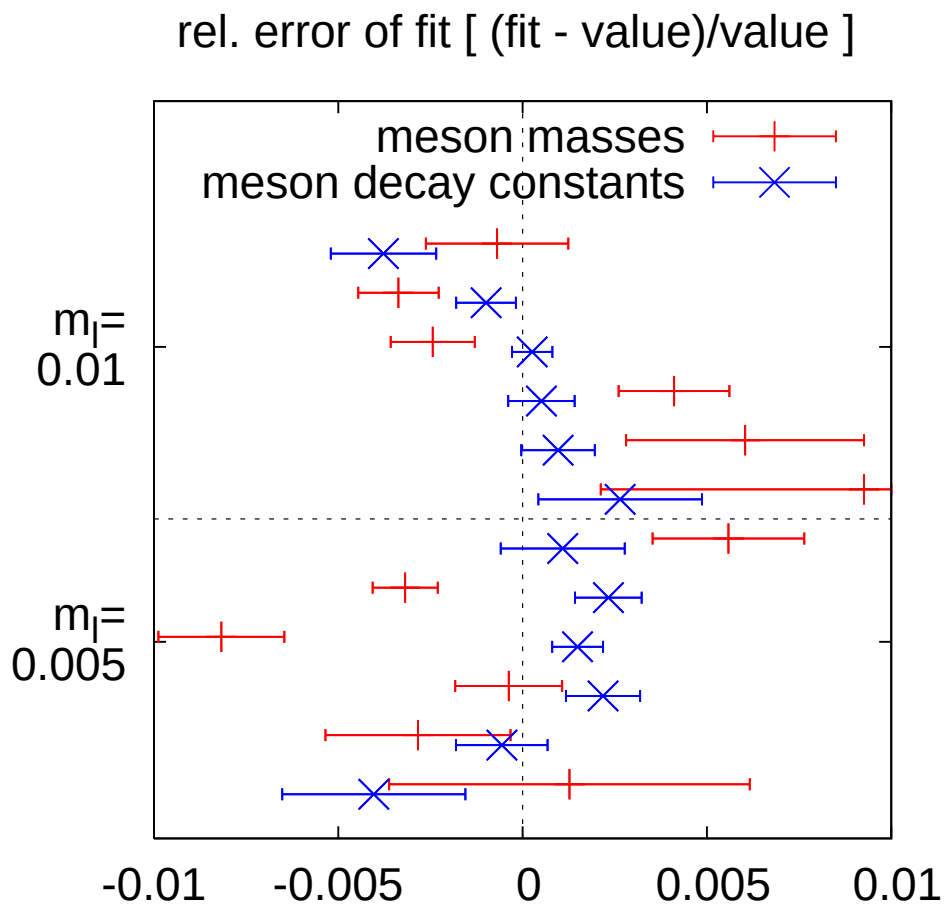




at m_{ud}: $f_\pi \;=\; 124.1(3.6)$ MeV (130.7)

fit range $m_{\text{avg}} \leq 0.01$, combined fits (only *filled symbols* inside fit range)

- uncorrelated $\chi^2/\text{d.o.f.} = 0.3$
- 2x45(blocked) configurations
- 2x12 data-points
- not fitting to an *exact* theory (NLO, NNLO, . . .)
- maximal deviation $\leq 1\%$
- acceptable for fit range $m_{\text{PS}} \leq 420\,\text{MeV}$



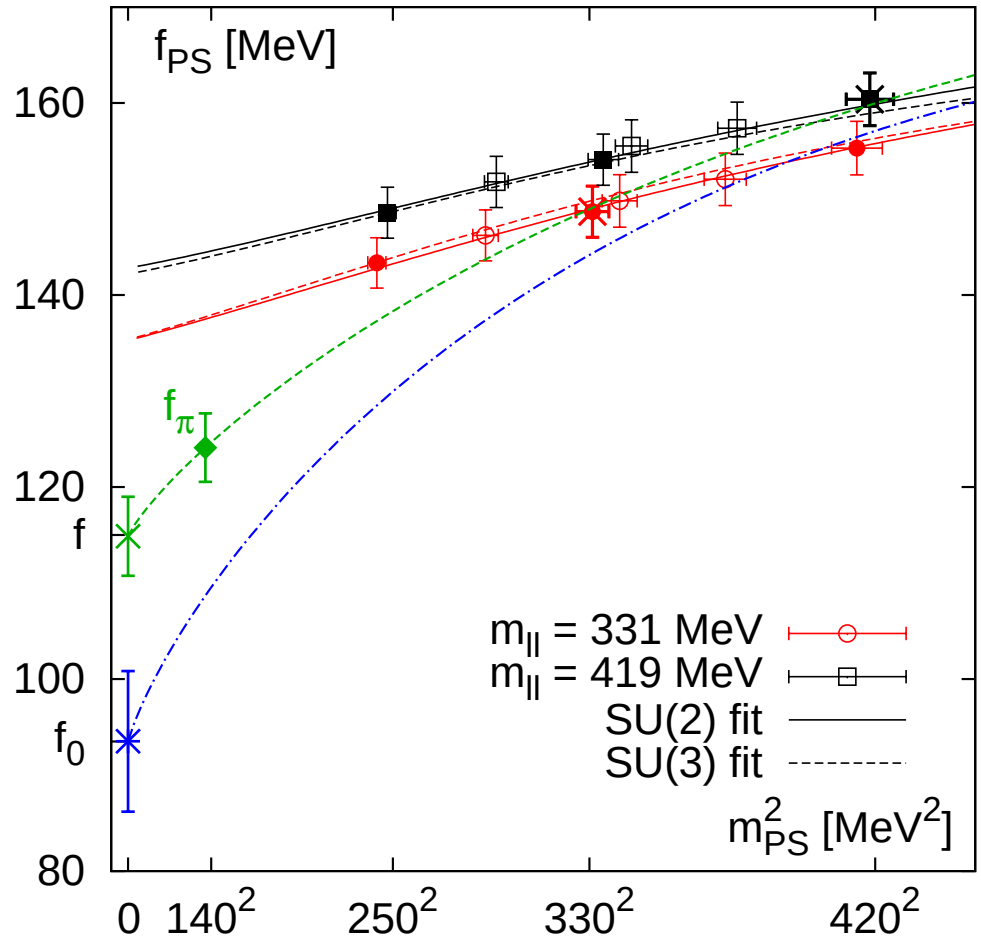
SU(2) low-energy constants

- convert B_0, f_0, LECs from SU(3)-case to SU(2)
  (1-loop matching in Gasser, Leutwyler, 1985, 2-loop: Gasser et al. 2006)
- compare SU(2)-LECs at scale $m_\pi = 139$ MeV: $\bar{l}_{3,4}$

	B	f	$\bar{l}_3$	$\bar{l}_4$
SU(2)×SU(2)	2.414(61)	0.0665(21)	3.13(33)	4.43(14)
SU(3)×SU(3)	2.457(78)	0.0661(18)	2.87(28)	4.10(05)
MILC ($N_f = 2+1$)			0.6(1.2)	3.9(5)
MILC, pure NLO			2.85(07)	–
PACS-CS (SU(3)-fit)			3.47(11)	4.21(10)
PACS-CS (SU(2)-fit)			3.14(23)	4.04(19)
ETMC ($N_f = 2$)			3.44(08)(35)	4.61(04)(11)
CERN ($N_f = 2$)			3.0(5)	–
phenom.			2.9(2.4)	4.4(0.2)

SU(3) vs SU(2)

- SU(2) chiral limit better controlled
- SU(2) plus kaon. . .
- NLO SU(3) not sufficient for m_s . . .
- . . . but only one value of m_s so far



SU(2)×SU(2) for the kaon-sector

- only use chiral symmetry properties of the up- and down-quarks (SU(2) $\times$ SU(2))
- to NLO, we will ignore terms $(m_\pi/m_K)^4$
  - PDG: 95 ± 25 MeV $\gg 2.5 \sim 5.5$, MeV ?
  - our simulation, dynamical: $0.04 + m_{\text{res}} \gg \{0.005, 0.01\} + m_{\text{res}}$??
- we only have one dynamical strange quark mass (at the moment)
  using partially quenched as well for the moment
- Sharpe, Zhang (1996), Booth (1995), Roessl (1999)

$$\mathcal{L}_{\pi\pi} = \frac{f^2}{8}\mathrm{Tr}\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger + \frac{f^2 B}{4}\mathrm{Tr}\Big(M^\dagger\Sigma + M\Sigma^\dagger\Big)$$

$$\mathcal{L}_{\pi K} = D_\mu K^\dagger\, D^\mu K - M_K^2 K^\dagger K$$

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \qquad \Sigma = \xi^2 = \exp\frac{\mathrm{i}}{f}\begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}$$

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger,$$

$$K \to UK, \qquad D_\mu K \to U D_\mu K$$

$$D_\mu K = \partial_\mu K + V_\mu K, \qquad V_\mu = \frac{1}{2}\Big(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\Big)$$

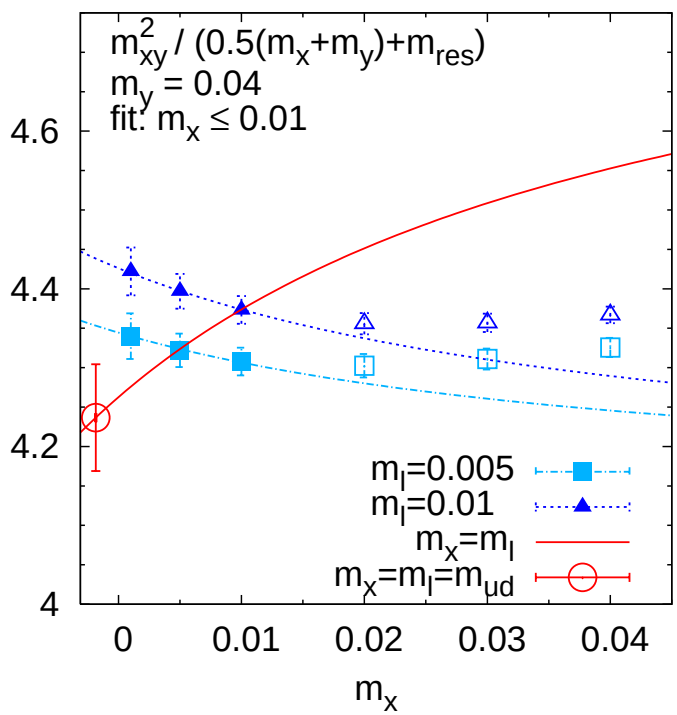
$$m_{xh}^2 = B^{(K)}(m_h)\widetilde{m}_h\left\{1 + \frac{\lambda_1(m_h)}{f^2}\chi_l + \frac{\lambda_2(m_h)}{f^2}\chi_x\right\}$$

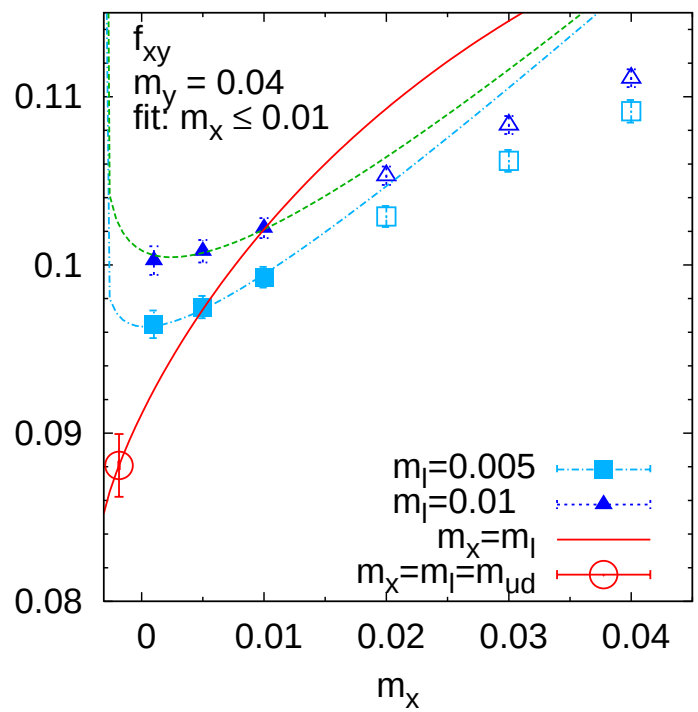
$$f_{xh} = f^{(K)}(m_h)\left\{1 + \frac{\lambda_3(m_h)}{f^2}\chi_l + \frac{\lambda_4(m_h)}{f^2}\chi_x - \frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda_\chi^2} + \frac{\chi_l - 2\chi_x}{4}\log\frac{\chi_x}{\Lambda_\chi^2}\right]\right\}$$

$$m_{xh}^2 = B^{(K)}(m_h)\widetilde{m}_h\left\{1 + \frac{\lambda_1(m_h)}{f^2}\chi_l + \frac{\lambda_2(m_h)}{f^2}\chi_x\right\}$$

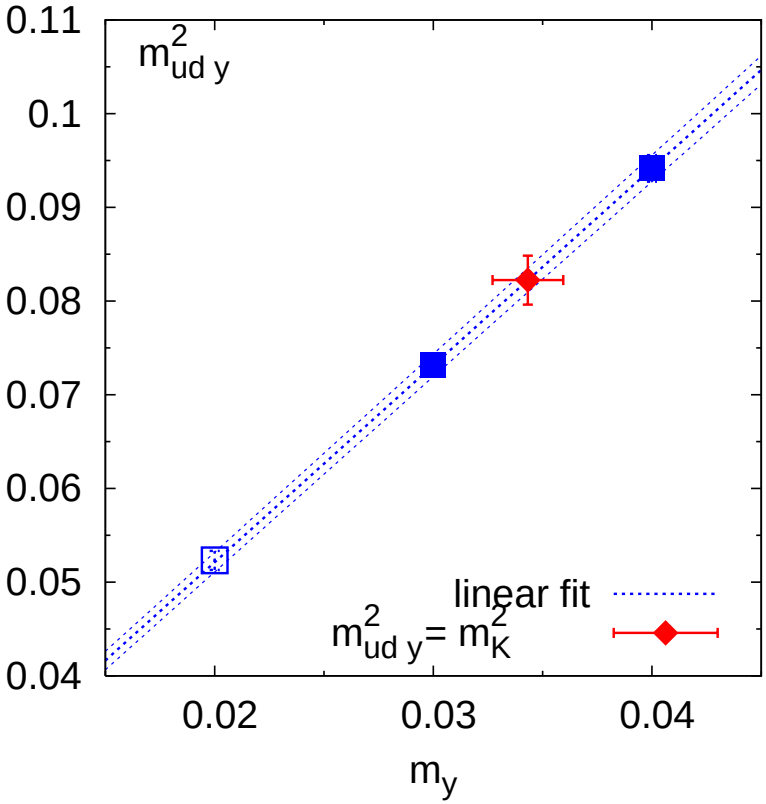
$$f_{xh} = f^{(K)}(m_h)\left\{1+\frac{\lambda_3(m_h)}{f^2}\chi_l+\frac{\lambda_4(m_h)}{f^2}\chi_x-\frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda^2}+\frac{\chi_l-2\chi_x}{4}\log\frac{\chi_x}{\Lambda^2}\right]\right\}$$

- f, B from SU(2) $\times$ SU(2) fit, fixed, $m_{l,x} \leq 0.01$
- extrapolate to m_{ud}



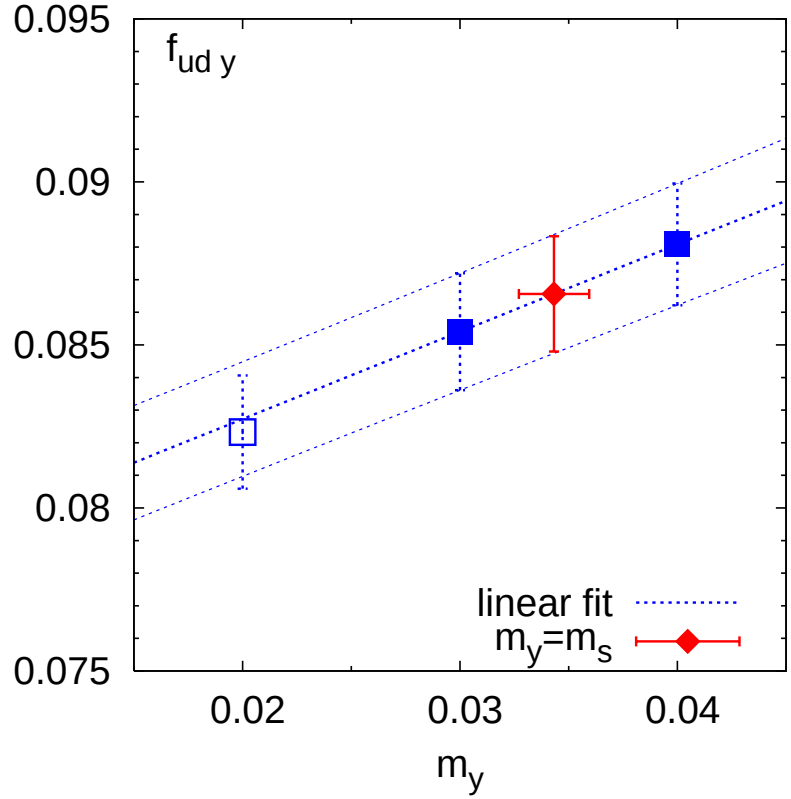


m_s from physical m_K^2



- $m_s^{\overline{MS}}(2\,\text{GeV}) = 107.3(4.4)$ MeV (NPR)
- $m_{ud} : m_s = 1 : 28.8(4)$

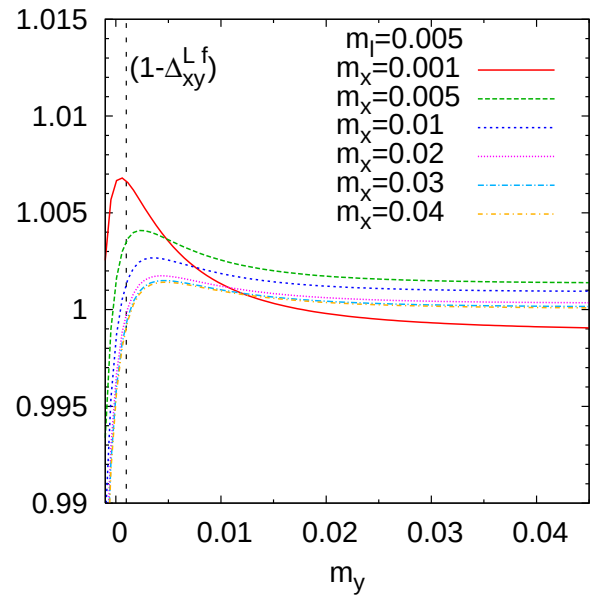
interpolate f_K to m_s

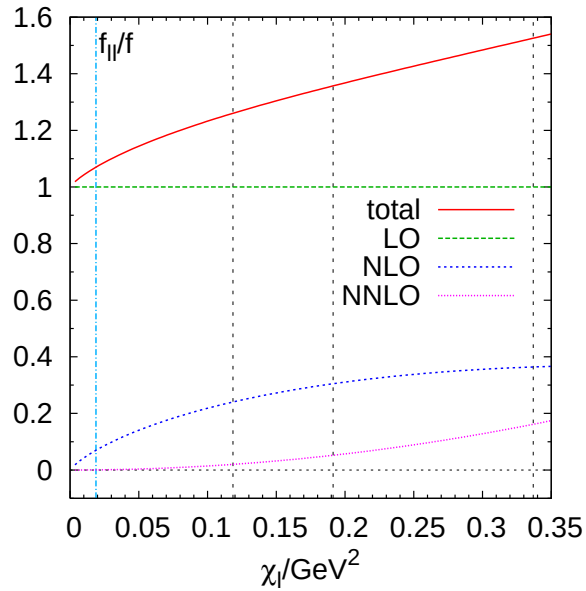


- $f_K = 149.6(3.6)$
  (159.8 MeV)
- $f_K/f_\pi = 1.205(18)$
  (1.223)

Systematic Errors

- finite volume effects
  - $(aL)^3 \approx (2.74\text{fm})^3$
  - FV-χPT: small effect
- continuum extrapolation
  - flat estimate $(a\Lambda_{\text{QCD}})^2 \approx 4\%$
  - data at 2nd a preliminary, improve/justify error
- $\boldsymbol{m_s \neq m_h}$
  - 15% too large m_h
  - estimated effect by SU(3) $\longrightarrow$ SU(2) conversion
- χPT extrapolation
  - fit range $m_l \leq 0.02$
  - added analytic NNLO terms





$$
\begin{aligned}
f &= 114.8(4.1)_{\text{stat}}(8.1)_{\text{syst}}\ \text{MeV}\,, \\
B^{\overline{\text{MS}}}(2\,\text{GeV}) &= 2.52(0.11)_{\text{stat}}(0.23)_{\text{ren}}(0.12)_{\text{syst}}\ \text{GeV}\,, \\
\Sigma^{\overline{\text{MS}}}(2\,\text{GeV}) &= \Big(255(8)_{\text{stat}}(8)_{\text{ren}}(13)_{\text{syst}}\ \text{MeV}\Big)^3\,, \\
\bar{l}_3 &= 3.13(0.33)_{\text{stat}}(0.24)_{\text{syst}}\,, \\
\bar{l}_4 &= 4.43(0.14)_{\text{stat}}(0.77)_{\text{syst}}\,, \\
f_\pi &= 124.1(3.6)_{\text{stat}}(6.9)_{\text{syst}}\ \text{MeV}\,, \\
f_K &= 149.6(3.6)_{\text{stat}}(6.3)_{\text{syst}}\ \text{MeV}\,, \\
f_K/f_\pi &= 1.205(0.018)_{\text{stat}}(0.062)_{\text{syst}}\,, \\
m_{ud}^{\overline{\text{MS}}}(2\,\text{GeV}) &= 3.72(.16)_{\text{stat}}(.33)_{\text{ren}}(.17)_{\text{syst}}\ \text{MeV}\,, \quad \text{(NPR)} \\
m_s^{\overline{\text{MS}}}(2\,\text{GeV}) &= 107.3(4.4)_{\text{stat}}(9.7)_{\text{ren}}(4.9)_{\text{syst}}\ \text{MeV}\,, \\
m_{ud} : m_s &= 1 : 28.8(0.4)_{\text{stat}}(1.6)_{\text{syst}}
\end{aligned}
$$

PRELIMINARY results from $32^3 \times 64 \times 16$ lattices PRELIMINARY

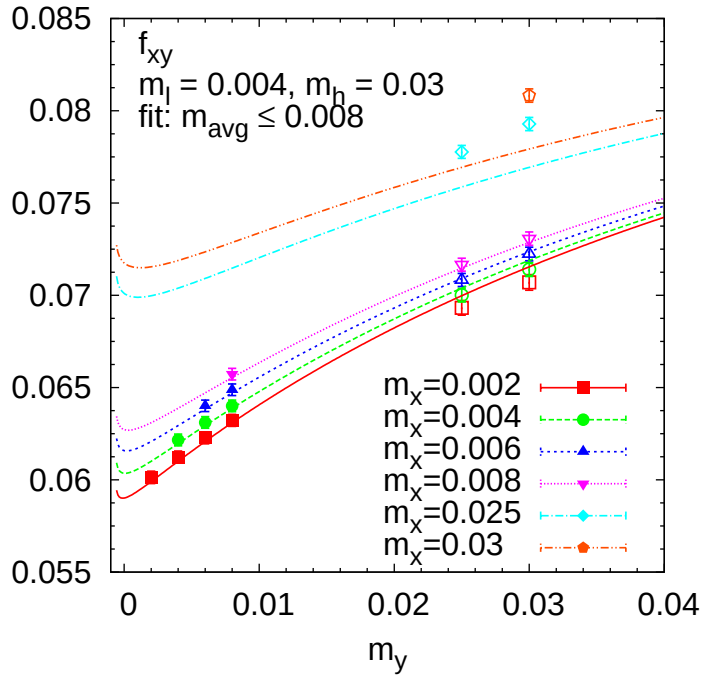
- Iwasaki-gauge action $\beta = 2.25$
- 3 dynamical light quark masses $m_l = 0.004, 0.006, 0.008$, $m_s = 0.03$
- (dynamical) pion masses: $\approx 310,\ 365\ ,\ 420$ MeV
- lightest PQ pion mass ≈ 235 MeV ($m_x \in \{0.002, 0.004, 0.006, 0.008, 0.025, 0.03\}$)
- $m_{\text{res}} = 6.65(13) \cdot 10^{-4}$
- $a^{-1} = 2.42(4)$ GeV (from r_0/a, input: $r_0 = 0.47$ fm)
- $a \approx 0.08$ fm, $aL \approx 2.6$ fm
- FV-effects? $m_{ll}L \approx 4.1$–5.5, (smallest $m_{xx}L \approx 3.1$)

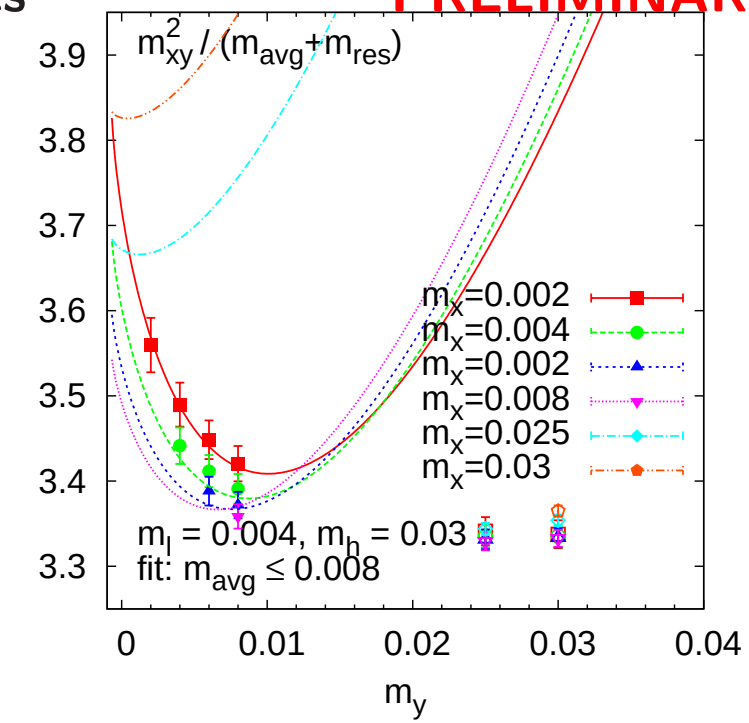
. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .

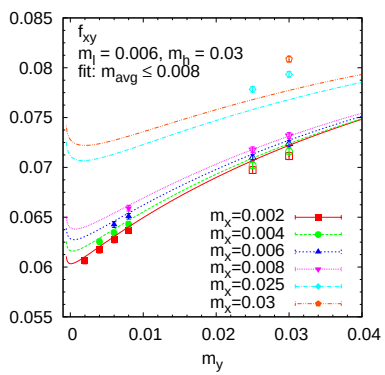
PRELIMINARY

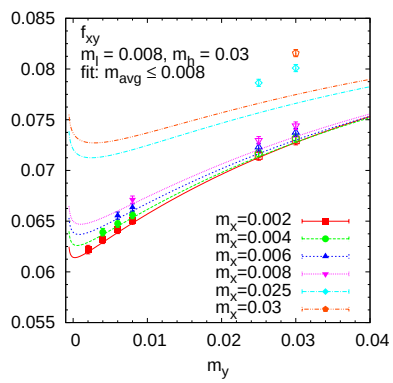
SU(2)-PQχPT fits

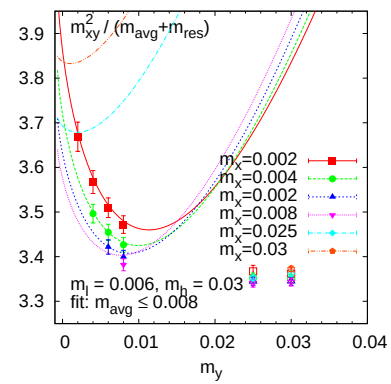
PRELIMINARY

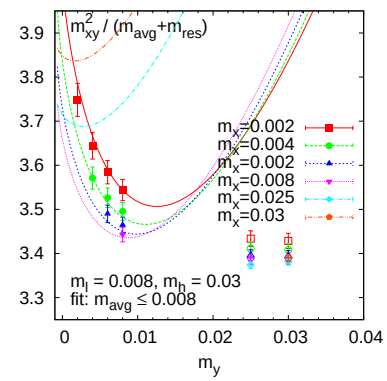










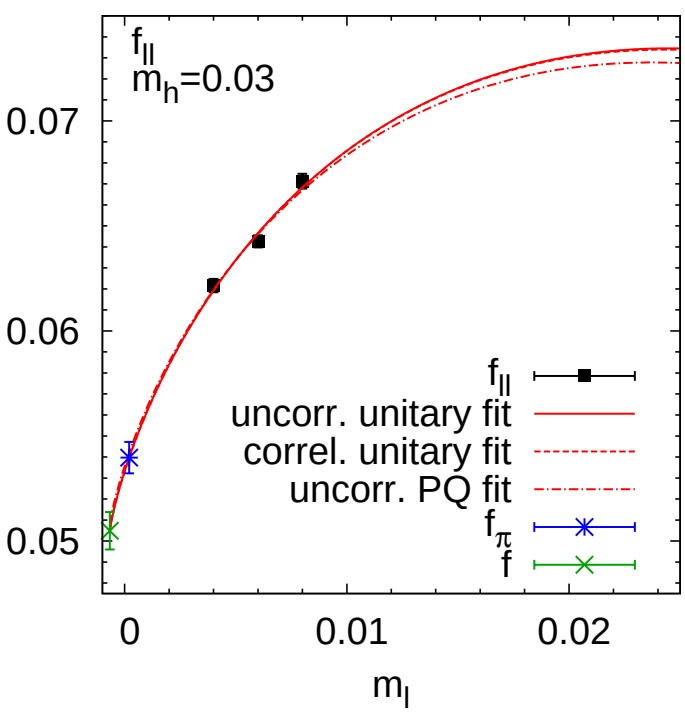


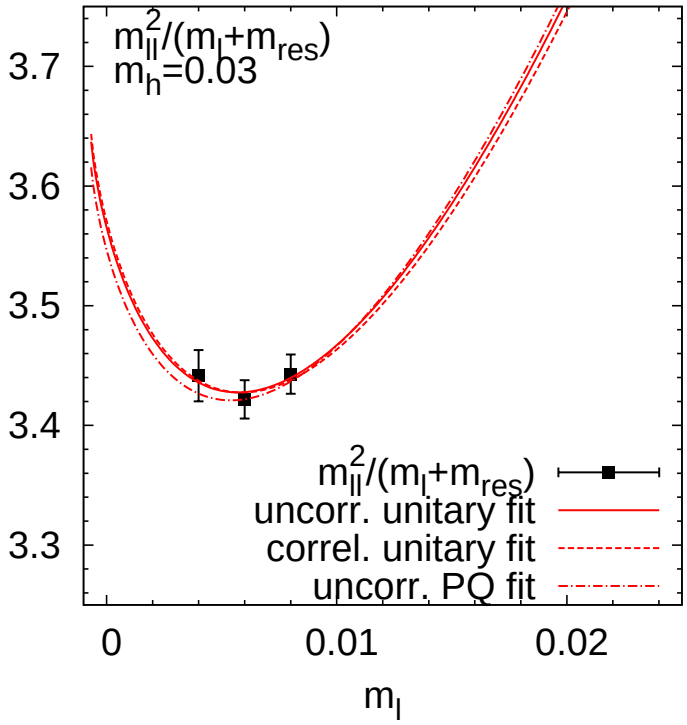
. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .

PRELIMINARY

fits to unitary points

PRELIMINARY





6 data points/4 fit-parameters (2 LO-, 2 NLO-LECs)

only 2x2 blocks in block-diag. correlation matrix

unitary-uncorrelated, unitary-correlated fits agree with PQ-uncorr. fits

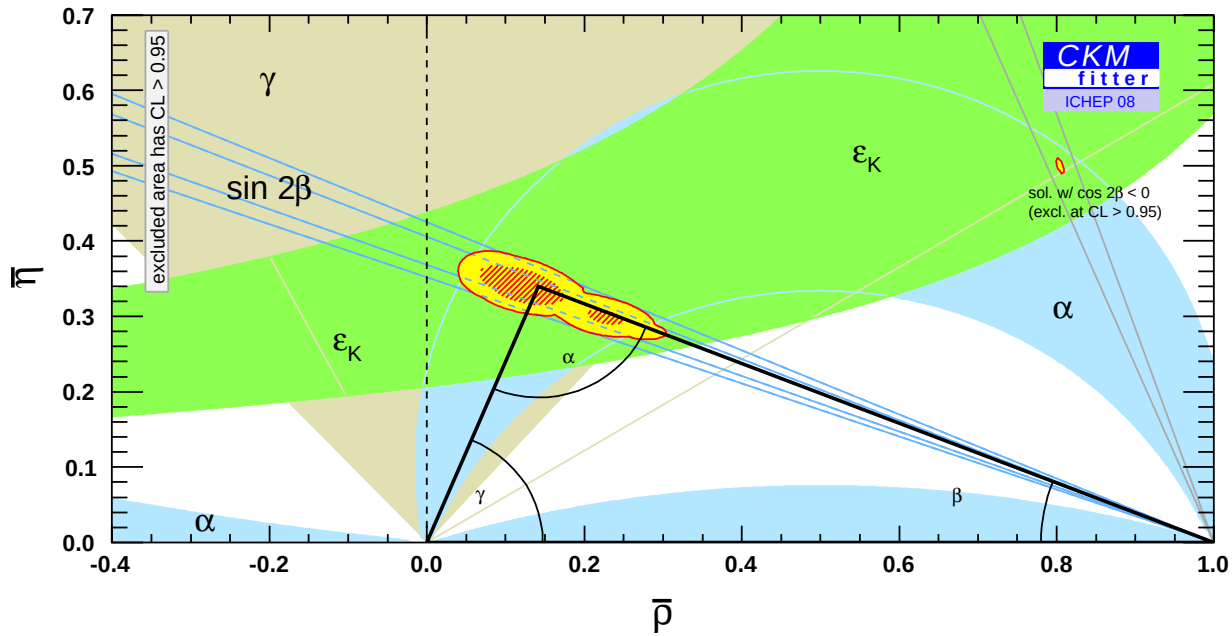
. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .

Part II: Neutral Kaon Mixing ϵ_K and B_K

$$B_K(\mu) \;=\; \langle \bar{K}^0 | Q^{\Delta S=2} | K^0 \rangle \;/\; (\tfrac{8}{3} f_K^2 m_K^2)$$

$$|\epsilon_K| \;=\; C_\epsilon \, \hat{B}_K \lambda^2 \bar{\eta}^2 |V_{cb}|^2 \Big[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} S_0(x_c) \Big]$$

PDG '08: $|V_{cb}| \;=\; 0.0412(11)$ 2.7% $\rightarrow \quad \delta |V_{cb}|^4 \;\simeq\; \delta B_K^{\text{lat}}$



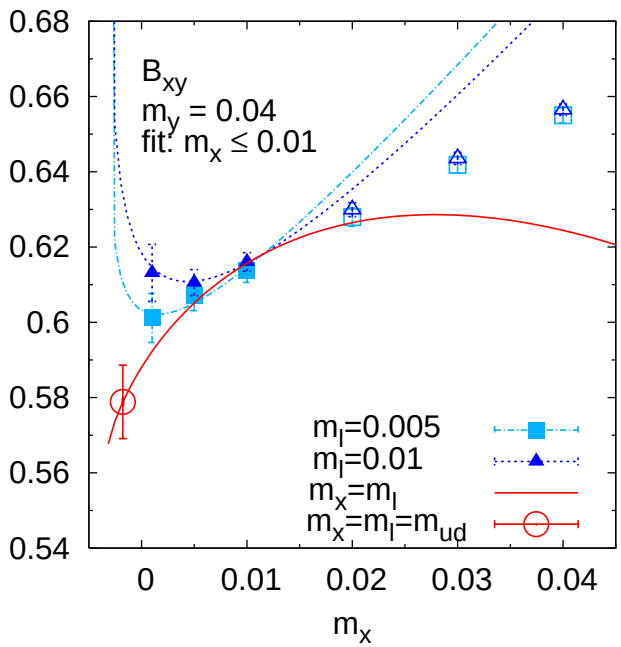
(CKM fitter 2008, prelim.)

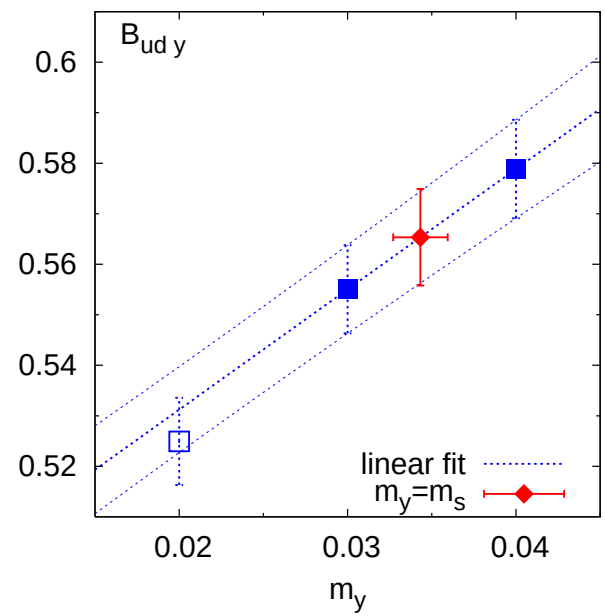
SU(2)×SU(2) for the kaon sector: B_K

Phys. Rev. Lett. **100** (2008) 032001

$$B_K = B_0^{m_h}\left\{1 + C_1^{m_h}\chi_l + C_2^{m_h}\chi_x - \frac{\chi_l}{2(4\pi f)^2}\log\frac{\chi_x}{\Lambda^2}\right\}$$

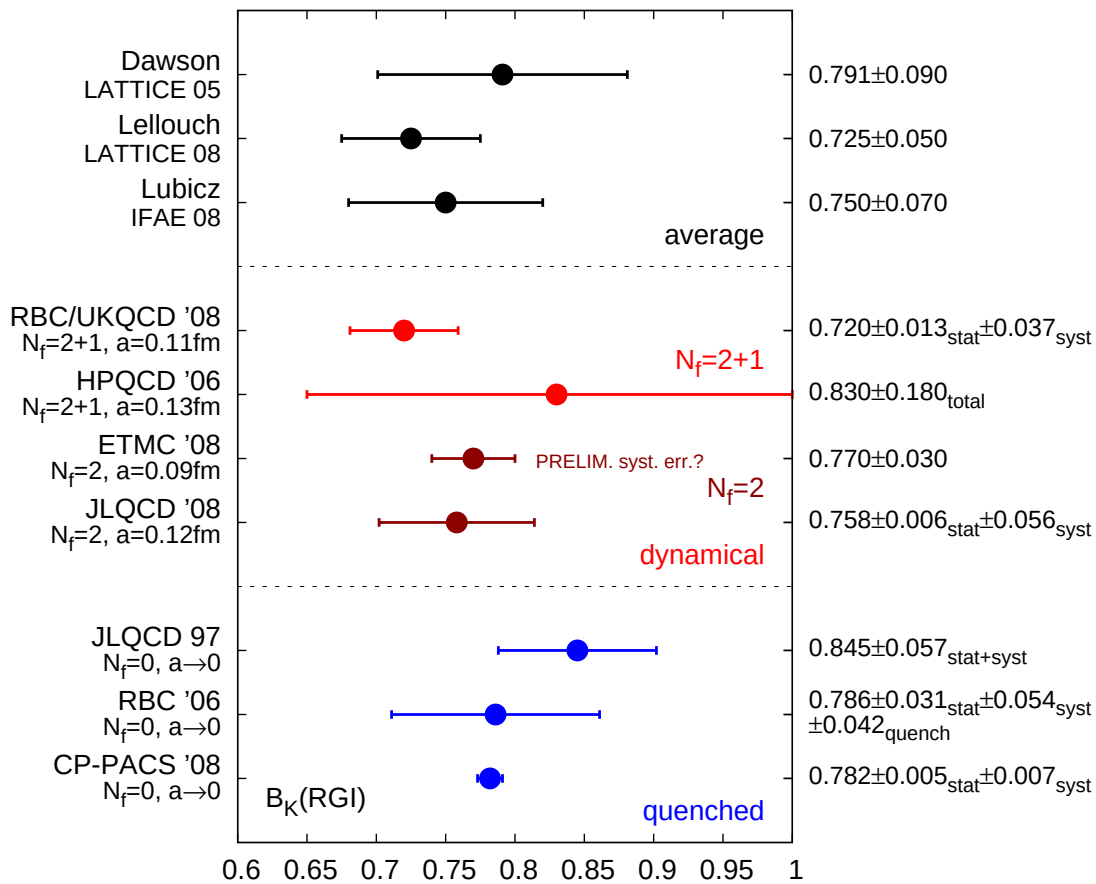
- f, B from SU(2) × SU(2) fit, fixed, $m_l \leq 0.01$
- extrapolate to m_{ud}
- interpolation to m_s: $B_K^{\text{lat}} = 0.565(10)$





- $B_K^{\mathrm{RI}}(2\,\mathrm{GeV}) = 0.514(10)_{\mathrm{stat}}(07)_{\mathrm{ren}}(24)_{\mathrm{syst}}$
- $B_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.524(10)_{\mathrm{stat}}(13)_{\mathrm{ren}}(25)_{\mathrm{syst}}$
- $\hat{B}_K = 0.720(13)_{\mathrm{stat}}(14)_{\mathrm{ren}}(34)_{\mathrm{syst}}$
- systematic error:
  - finite volume: 1%
  - continuum extr: 4%
  - fixed sea strange: 1%
  - χPT: 2%
- comb. uncert.: 5.1%
- $\Delta B_K \simeq \Delta |V_{cb}|^4$
- $\epsilon_K = \bar{\epsilon}_K + i\xi$

Soni, Lunghi (08)
and
Buras, Guadagnoli (08)



- SU(2)×SU(2) for kaon-sector more reliable
- estimates for LECs, light quark masses
- B_K with 5.1% error (combined)
- Outlook:
  - extending statistics on 24^3 ensembles (330, 420 MeV)
  - enough data to compare with NNLO SU(3)×SU(3)
  - 2nd dynamical strange quark mass needed
  - continuum extrapolation with $a \approx 0.08$ fm ongoing

computations were done on:
QCDOC Edinburgh, RBRC, USDOE
and BG/P at Argonne (ESP, INCITE)

BACKUP

Systematic Errors

- finite volume $(aL)^3 \approx (2.74\,\text{fm})^3$
- (missing) continuum extrapolation $a \approx 0.11\,\text{fm}$
- χPT - extrapolation
- dynamical heavy quark $\boldsymbol{m_h} \neq \boldsymbol{m_s}$ physical strange quark mass

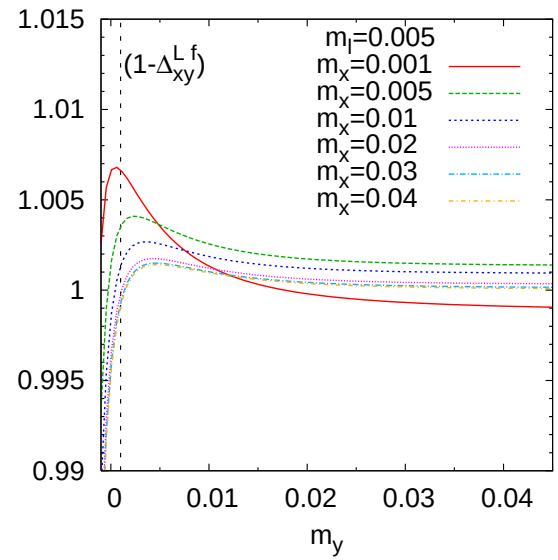
Systematic Errors I: Finite Volume

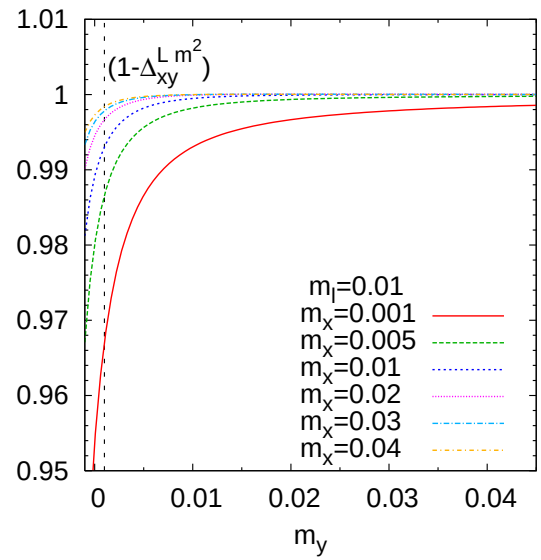
- using FV-corrected PQχPT formulae

Gasser, Leutwyler

- assign difference in measured quantity as syst. error (typical size $\approx$ 1 – 2%, larger effect (6%) on $\bar{l}_3$)
- compare at unitary points with "resummed Lüscher-formula" of Colangelo, Dürr, Haefeli

	R_m [%]		$-R_f$ [%]	
m_π [MeV]	PQχPT	CDH	PQχPT	CDH
331	0.09(.01)	0.13(.04)	0.36(.03)	0.32(.00)
419	0.03(.00)	0.04(.01)	0.10(.01)	0.09(.00)



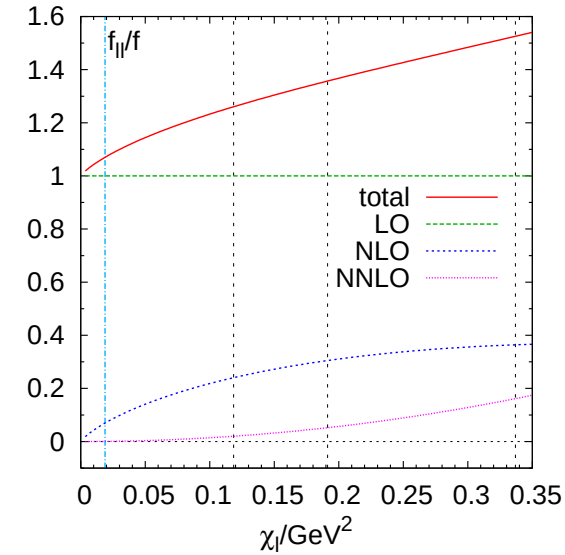


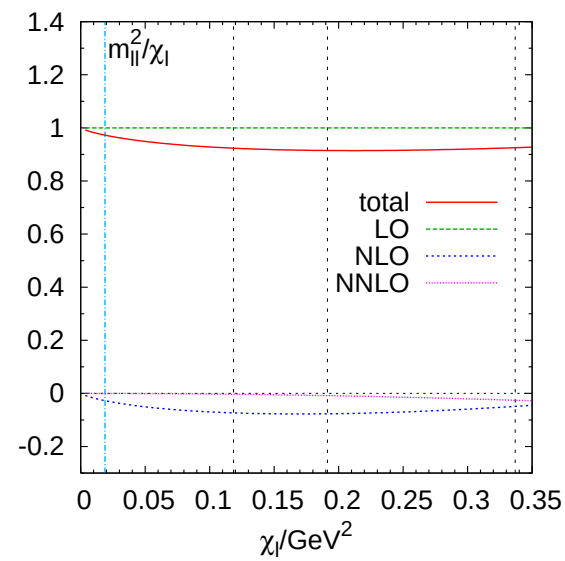
Systematic Errors II: χPT extrapolation

- influence of NNLO?
- 4 possible NNLO terms, incl. first 3

$$(\chi_x + \chi_y)^2,\ (\chi_x - \chi_y)^2,\ (\chi_x + \chi_y)\chi_l,\ \chi_l^2$$

- extended fit range to $m_{\text{avg}} \leq 0.02$
- good convergence behaviour
- assigned (doubled) difference as syst. error
  (typical size $\approx$ 2 – 4 % (but large (17%) effect on $\bar{l}_4$)





Systematic Errors III:

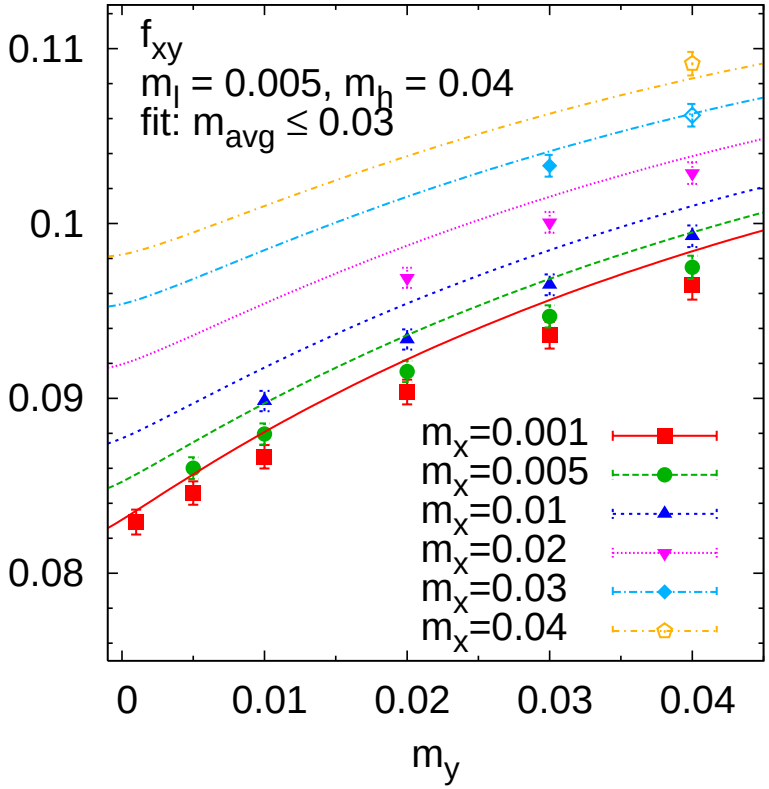
Continuum Extrapolation

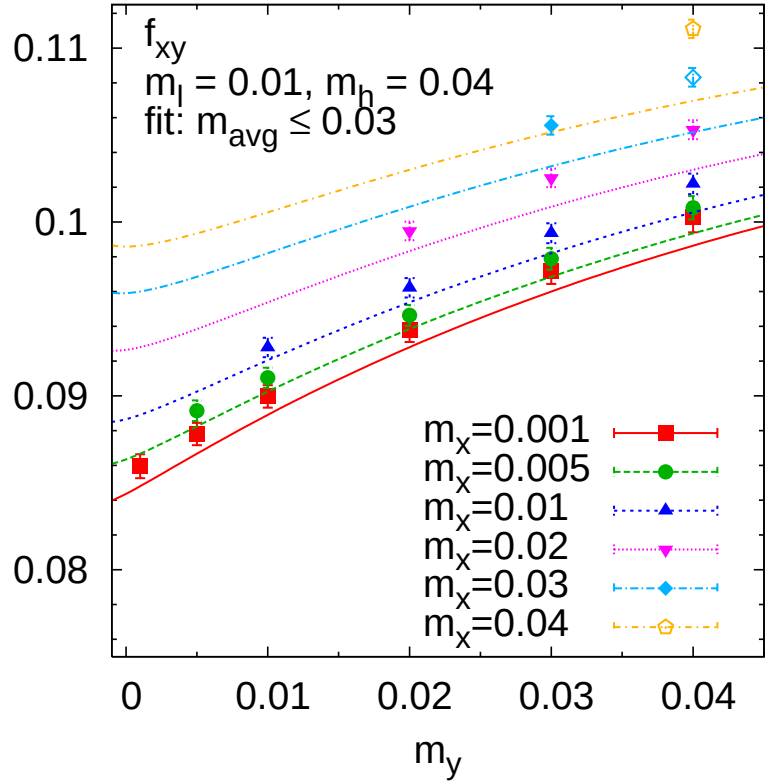
- currently only one lattice spacing available
- assign flat $(a\Lambda_{\text{QCD}})^2 \approx$ 4% error
- better (smaller?) estimate from 32^3-data, $a \approx 0.08$ fm

$$m_s \neq m_h$$

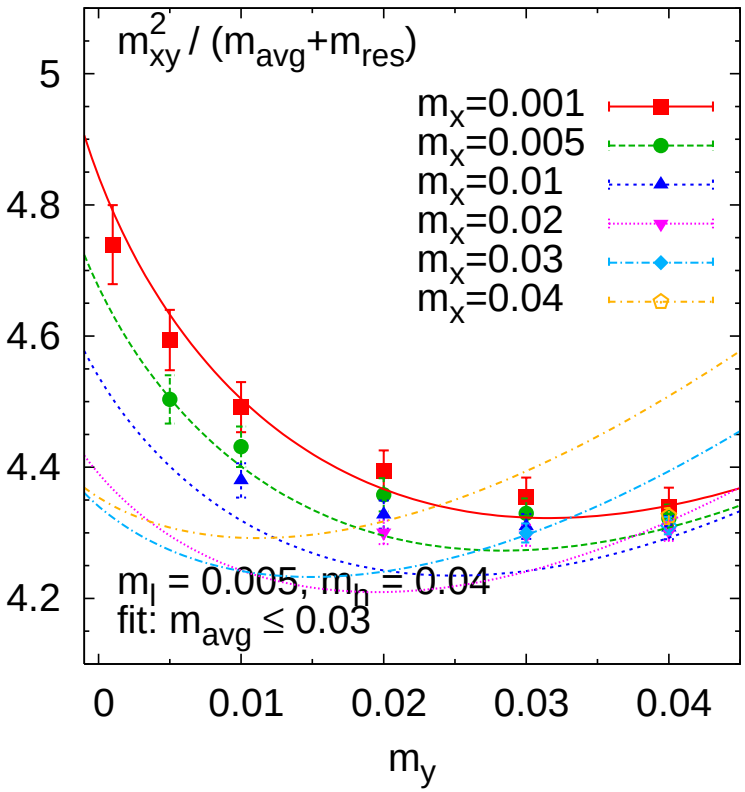
- 15% too large m_h
- estimated effect by conversion SU(3) $\rightarrow$ SU(2)
  (typical size $< 1\%$, except 2% for m_s)

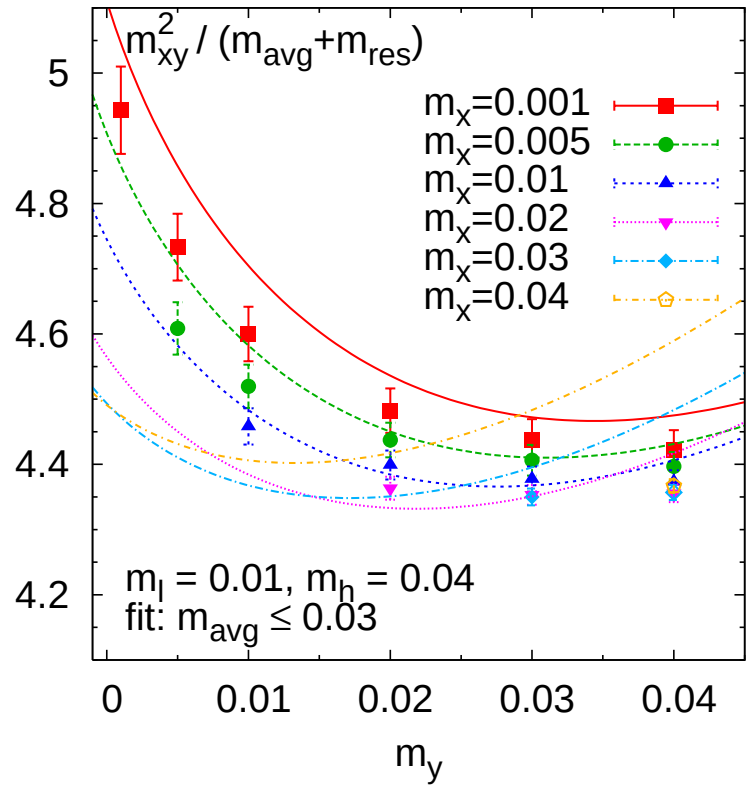
SU(3) PQχPT, fit range $m_{\text{avg}} \leq 0.03$





(combined) NLO-fits not working up to m_s


$m_{xy}^2 / (m_{avg}+m_{res})$
m_x=0.001
m_x=0.005
m_x=0.01
m_x=0.02
m_x=0.03
m_x=0.04
m_l = 0.005, m_h = 0.04
fit: $m_{avg} \leq 0.03$
5
4.8
4.6
4.4
4.2
0
0.01
0.02
0.03
0.04
m_y


$m_{xy}^2 / (m_{avg}+m_{res})$
m_x=0.001
m_x=0.005
m_x=0.01
m_x=0.02
m_x=0.03
m_x=0.04
m_l = 0.01, m_h = 0.04
fit: $m_{avg} \leq 0.03$
5
4.8
4.6
4.4
4.2
0
0.01
0.02
0.03
0.04
m_y

physical quark masses

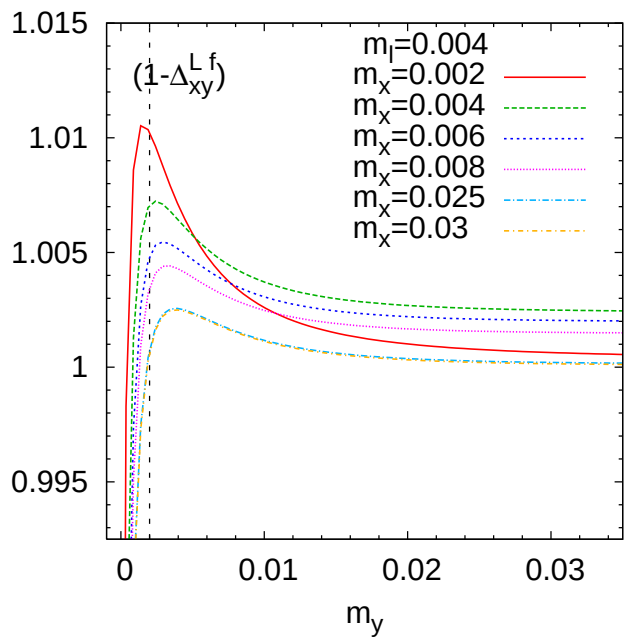
	N_f	fermion	m_{ud}/MeV	m_s/MeV m_s/m_{ud}
using non-perturbative renormalization				
	2+1	**DWF**	$\mathbf{3.72(.16)_{\text{stat}}(.33)_{\text{ren}}(.17)_{\text{syst}}}$	$\mathbf{107.3(4.4)_{\text{stat}}(9.7)_{\text{ren}}(4.9)_{\text{syst}}}$ $\mathbf{28.8(0.4)_{\text{stat}}(1.6)_{\text{syst}}}$
RBC	2	DWF	$4.25(.23)_{\text{stat}}(.26)_{\text{ren}}$	$119.5(5.6)_{\text{stat}}(7.4)_{\text{ren}}$ $28.10(.38)_{\text{stat}}$
ETMC	2	TM-Wilson	$3.85(.12)_{\text{stat}}(.40)_{\text{syst}}$	$105(3)_{\text{stat}}(9)_{\text{syst}}$ $27.3(0.3)_{\text{stat}}(1.2)_{\text{syst}}$
QCDSF	2	impr. Wilson	$4.08(.23)_{\text{stat}}(.19)_{\text{syst}}(.23)_{\text{scale}}$	$111(6)_{\text{stat}}(4)_{\text{syst}}(6)_{\text{scale}}$ $27.2(3.2)$
using perturbative renormalization				
MILC	$2+1$	stagg.	$3.2(0)_{\text{stat}}(.1)_{\text{ren}}(.2)_{\text{EM}}(0)_{\text{cont}}$	$88(0)_{\text{stat}}(3)_{\text{ren}}(4)_{\text{EM}}(0)_{\text{cont}}$ $27.2(.1)_{\text{stat}}(.3)_{\text{syst}}$
PACS-CS	$2+1$	impr. Wilson	$2.3(1.1)$	$69.1(2.5)$ $30(?)$
JLQCD	$2+1$	impr. Wilson	$3.54(^{+.64}_{-.35})_{\text{total}}$	$91.1(^{+14.6}_{-6.2})_{\text{total}}$ $25.7(?)$

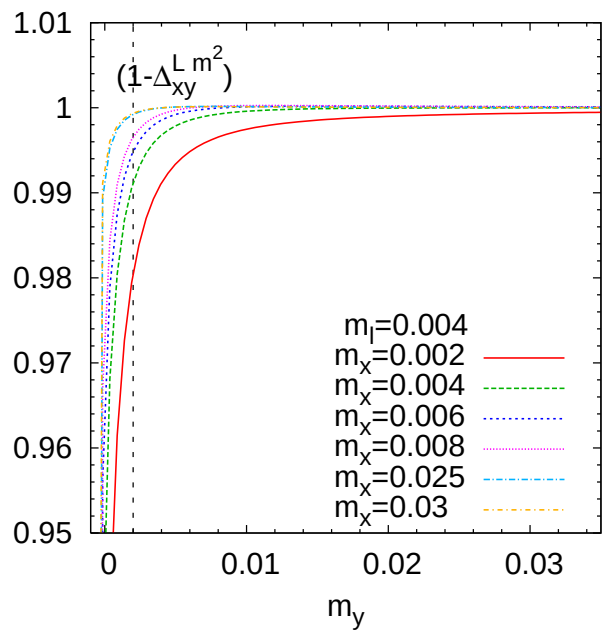
($\overline{\text{MS}}$-scheme at $\mu = 2\,\text{GeV}$)

PRELIMINARY

Finite volume correction $(2.6\,\mathrm{fm})^3$

PRELIMINARY





- correcting with SU(2) PQChPT for finite volume: [Gasser, Leutwyler]

  for $m_{ll} \approx 310\,\mathrm{MeV}$: $R_m = 0.16(.01)\%, \quad -R_f = 0.65(.04)\%$

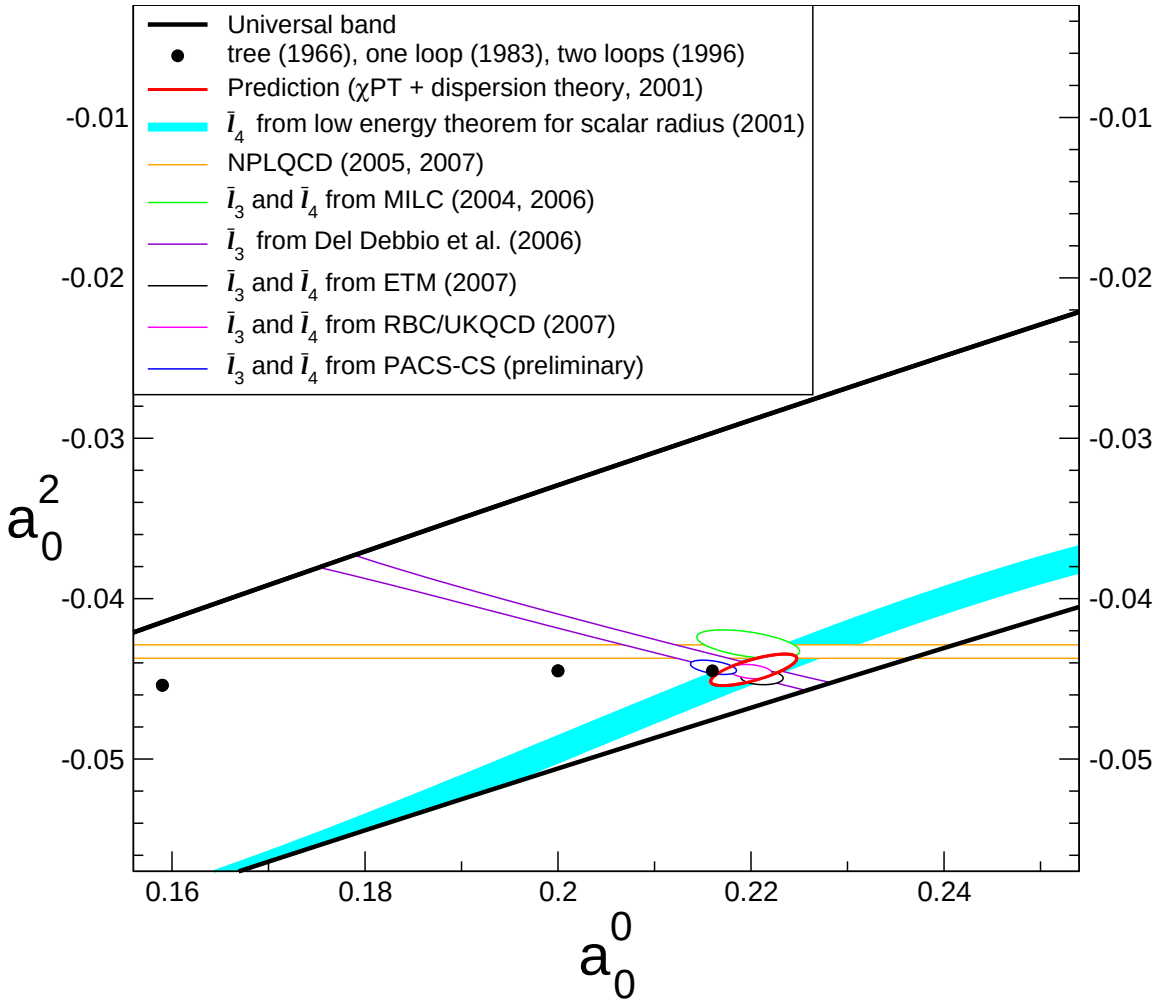
- compare with Colangelo, Dürr, Haefeli (resummed Lüscher formula):

  $R_m = 0.26(.07)\%, \quad -R_f = 0.64(.01)\%$

. . . PRELIMINARY . . . NOT YET FULL STATISTICS . . . PRELIMINARY . . .

$\pi\pi$ S-wave scattering length

plot from H. Leutwyler, `arXiv:0804.3182 [hep-ph]`



short intermission: CKM-matrix elements

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2387(4)\,\frac{f_\pi}{f_K}\sqrt{\frac{\Gamma(K\to\mu\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))}}$$

Blucher, Marciano, PDG '06

with $\frac{\Gamma(K\to\mu\nu(\gamma))}{\Gamma(\pi\to\mu\nu(\gamma))} = 1.3383(46)$ and our result for f_k/f_π we obtain:

$$\begin{aligned}
\frac{|V_{us}|}{|V_{ud}|} &= 0.2292(034)_{\text{stat}}(138)_{\text{syst}}(005)_{\text{other}} \\
|V_{us}| &= 0.2232(033)_{\text{stat}}(115)_{\text{syst}}(005)_{\text{other}} \\
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 0.9980(15)_{\text{stat}}(51)_{\text{syst}}(06)_{\text{other}} = 0.9980(54)_{\text{total}}
\end{aligned}$$

(using $V_{ud} = 0.97377(27)$, β-decay, $|V_{ub}| = 0$)

non-perturbative renormalization (Rome-Southampton)

- renormalization done at $16^3 \times 32 \times 16$ lattices (same gauge action, lattice spacing)
- match bare lattice operators to RI/MOM non-perturbatively
- perturbative matching to $\overline{\text{MS}}$ at 2 GeV
- Domain Wall Fermions:
  - $*$ control of chiral symmetry breaking
  - $*$ $\mathcal{O}(a)$-improved
  - $\longrightarrow$ operator mixing reduced
  - $\longrightarrow$ (partially) conserved axial and vector currents
- publication in preparation by RBC- and UKQCD-Collaborations
- here we are interested in $Z_m = 1/Z_S$

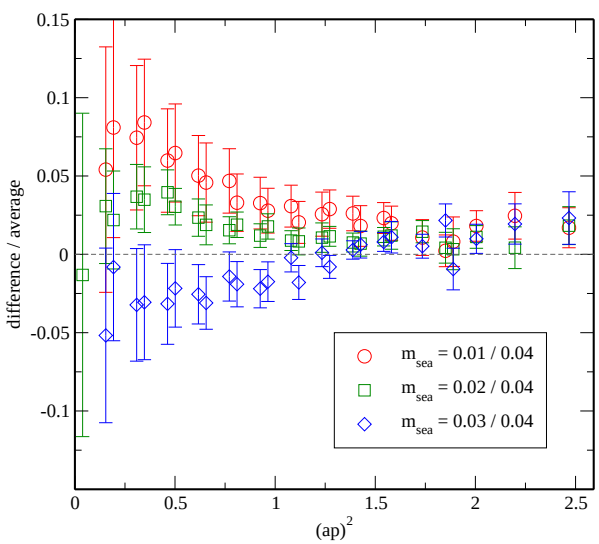
- renormalized amp. vertex functions $\Lambda_i^{\text{ren}} = Z_i/Z_q\,\Lambda_i = 1,\ i \in \{S, P, V, A, T\}$

$$Z_m^{\text{RI}} = \underbrace{\frac{Z_q}{Z_S}(p)}_{\Lambda_S}\ \underbrace{\frac{Z_A}{Z_q}(p)}_{\Lambda_A}\ \underbrace{\frac{1}{Z_A}}_{\text{hadronic ME}}$$

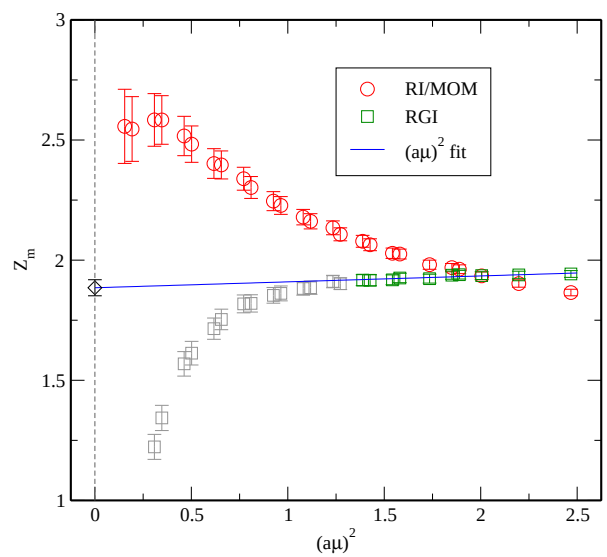
- four loop RG-running $Z_m^{\text{RGI}} = \frac{c(\alpha_s(\mu_0)/\pi)}{c(\alpha_s(\mu)/\pi)} Z_m^{\text{RI}}(\mu)$ [Chetyrkin et al., 2000]
- three loop matching RI/MOM to $\overline{\text{MS}}$

$$\boldsymbol{Z_m^{\overline{\text{MS}}}(2\,\text{GeV}) = 1.575(28)(15)(83)}$$

(error: statistical, $\Lambda_A \leftrightarrow \frac{1}{2}(\Lambda_A + \Lambda_V)$, linear vs. quadratic chiral extrap.)



comparing $\frac{1}{12}\frac{\partial \text{Tr} S^{-1}}{\partial m_v}$ and Λ_S



dividing out the running factor